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Game Theory Design for Deceptive Jamming Suppression in Polarization MIMO Radar

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ABSTRACT Controlling the polarization states of transmit waveforms can improve the performance of radar systems, especially for main lobe jamming suppression applications. In this paper, we consider the design of optimal transmit polarizations for deceptive jamming suppression in the main lobe using a game theory framework. We propose a co-located polarization multiple-input multiple-output (MIMO) radar system that combines the advantages of MIMO radar and those offered by optimally choosing the transmit polarization to improve the jamming suppression performance. In the polarization MIMO radar, polarization diversity is employed in the transmit array, and 2-D vector sensors are adopted in the receive array to separately measure the horizontal and vertical components of the received signals. Furthermore, based on the concepts and advantages of game theory, we formulate a polarization design problem for this radar system as a two-player zero-sum (TPZS) game between the radar and jammers. Additionally, we propose two design methods for different cases: a unilateral game for dumb jammers, and a strategic game for smart jammers. The optimal strategy and Nash equilibrium solution for two cases are presented. The simulation results demonstrate that jamming can be effectively suppressed with the proposed radar configuration and that improved jamming suppression performance can be achieved when the transmit polarization scheme is designed using the game theory approach.

INDEX TERMS Game theory, MIMO radar, deceptive jamming, jamming suppression, polarization design, unilateral game, strategic game.

I. INTRODUCTION

With the rapid development of digital radio frequency memory (DRFM) technology [1], a DRFM repeated jammer can intercept radar transmit waveforms and generate replicas (false targets) in random range bins and Doppler cells to confuse the radar system. Because the jammer is usually located in the main lobe of the radar system, these false targets cannot be easily discriminated and suppressed in a single domain, such as the time, frequency, or space domain. Therefore, main lobe deceptive jamming has become a serious threat to radar systems [2], [3].

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To enhance radar performance, jamming suppression technology has been widely studied, and many jamming suppression methods have been proposed by radar researchers. The traditional adaptive beamforming method is can achieve effective performance in side lobe jamming. However, for main lobe jamming, adaptive beamforming methods result in the distortion of the main lobe and an increase in the side lobe level [4]. In recent years, with the rapid development of multiple-input-multiple-output (MIMO) radar theory, polarization information processing technology has been applied to MIMO radar systems (distributed and co-located) to improve the system performance [5]–[16].

Distributed polarization MIMO radar systems combine the advantages offered by polarimetric and spatial diversity

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and outperform conventional radar systems [5]–[9]. In [5], a polarization MIMO radar system with distributed antennas for target detection was proposed. The target detection performance was improved by using the optimal design of transmit polarization. The problems associated with detecting different targets and backgrounds using the distributed polarization MIMO radar system were studied in [6]–[9]. The direction-of-departure (DOD), direction-of-arrival (DOA) and polarization parameter estimation methods for the co-located polarization MIMO radar system were explored in [10]–[13]. However, few studies have focused on the potential capacity of polarization MIMO radar systems to counter deceptive jamming. A statistical method for target discrimination in an active-decoy scenario was explored in [14]. The improved target discrimination performance was achieved by optimizing transmit and receive polarization. In [15], transmit polarization optimization with co-located polarization MIMO radar for main lobe interference suppression was studied. With the optimal polarization, improved discrimination performance and interference suppression were obtained. Furthermore, a polarization optimization method based on oblique projection for main lobe interference suppression in a co-located polarization MIMO radar system was proposed in [16], and improved jamming suppression performance was achieved with the optimal transmitter and receiver polarizations.

To further enhance the deceptive jamming suppression performance, we proposed a special polarization and frequency-diverse MIMO (PFD-MIMO) radar system in [17] that combines the advantages of the traditional MIMO radar system and polarization-range domain coupling; the system exhibited better performance in deceptive jamming suppression than did a frequency-diverse array MIMO (FDA-MIMO) radar system [18] and a polarization MIMO radar. Further, we developed a polarization and frequency increment design scheme for this radar system to improve the deceptive jamming suppression performance.

However, there are some problems among these polarization design approaches. First, these approaches are mostly aimed at dumb jammers because they all assume that the polarization properties of the jamming signal (false targets) are fixed and unchanged in several coherent processing intervals (CPIs). Second, these approaches rely on obtaining accurate estimates of the jamming polarization properties from the measured data. The improvement in radar performance is very sensitive to the accuracy of these estimates. Hence, accurate estimations require a significant amount of training data, which can be expensive. Finally, as shown in [14]–[16], the effect of the transmit polarization diversity on the amplitude of the false targets is not considered.

In addition, advances in digital signal processing and computing technology have resulted in the emergence of increasingly adaptive jamming systems. For these smart jammers, the polarization characteristics of the jamming signal can be changed as needed. In this situation, the approaches mentioned above become invalid. Consequently, the task of suppression in such systems remains a notable challenge.

To date, game theory methods have been applied in a wide variety of fields, such as communications, economics, and political science [19]–[21]. However, game theory has seldom been studied in the context of radar signal processing, especially in jamming suppression applications. A game theory design approach for distributed polarization MIMO radar target detection was proposed in [22], and the performance advantage of this approach was demonstrated using numerical simulations. The interaction between a smart target and a smart MIMO radar was investigated from a game theory perspective in [23]. The interaction was modeled as a two-player zero-sum (TPZS) game. A game theory framework for the joint design of amplitudes and frequency-hopping codes for frequency-hopping waveforms was studied and two joint design algorithms (noncooperative scheme and cooperative scheme) were proposed in [24]. In [25], noncooperative game theory was applied to analyze the dynamic interaction between two adversarial adaptive control systems – a radar and a jammer.

In this paper, we propose a co-located polarization MIMO radar system that combines the advantages of a MIMO radar system with the advantages offered by optimizing the transmitting waveform polarization for main lobe deceptive jamming suppression. In the transmit array of polarization MIMO radar, a polarization diversity scheme is adopted. Additionally, 2-D vector sensors are employed in the receive array to measure the horizontal and vertical components of the received signal separately. Furthermore, based on the concepts and advantages of game theory, we formulate the polarization design problem for this proposed radar system as a TPZS game between the radar and the jammers and propose two design methods for different cases: one is a unilateral game with asymmetrical information for dumb jammers, and the other is a strategic game with symmetrical information for smart jammers. We present the solution methods of the optimal strategy and the Nash equilibrium for two cases in this paper. Based on this game theory design for the transmit polarization, an improved jamming suppression performance can be achieved in the polarization MIMO radar system.

The remainder of the paper is organized as follows. In Section II, we give a brief introduction to the polarization MIMO radar system and present the signal models for the target and jamming. Then, the jamming suppression method based on the adaptive beamforming technique and output signal-to-interference-plus-noise ratio (SINR) analysis are presented in Section III. In Section IV, we formulate and solve the polarization design problem for MIMO radar jamming suppression using a game theory framework. Next, we use the numerical simulation results to demonstrate the performance improvement of the radar system by employing the proposed design mechanism in Section V. Finally, the conclusions are drawn in Section VI.

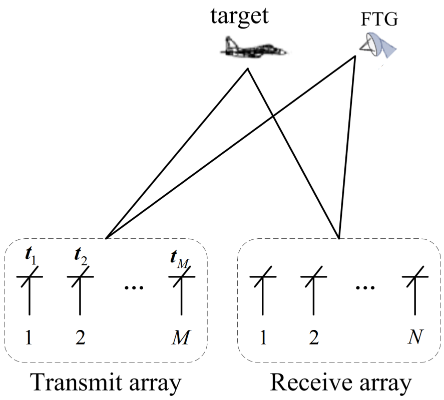

FIGURE 1. Illustration of the polarization MIMO radar system.

II. POLARIZATION MIMO RADAR SIGNAL MODEL

A. JAMMING-FREE MODELING

Without loss of generality, we consider a co-located MIMO radar system with M transmit antennas and N receive antennas in uniform linear arrays (ULAs), as shown in Fig. 1. Each of the transmitters is capable of transmitting a waveform of any arbitrary polarization, and each receiver employs a 2-D vector sensor to separately measure both the horizontal and vertical components of the received signal. The polarization vector of the mth transmit antenna is defined as $\boldsymbol{t}_m = \left[t_h^m, t_v^m\right]^T$, where $\|\boldsymbol{t}_m\| = 1$; each entry is a complex number; the subscripts h and v denote the horizontal and vertical components of the polarization vector, respectively; and the superscript T represents the transpose operation. The radiated signal of the mth transmit antenna is defined as

$$\boldsymbol{s}_m(t) = g_m(t)\exp(j2\pi ft)\boldsymbol{t}_m, \quad 0 \le t \le T \tag{1}$$

where $\boldsymbol{s}_m(t)$ is a 2-D column vector consisting of the horizontal and vertical components and $g_m(t)$ is the complex envelope of the mth transmit antenna. We assume that all these complex envelopes are orthonormal to each other at all mutual delays. f is the carrier frequency,T is the radar pulse duration, and t is the quick time index for the radar pulse.

Considering a far-field stationary point target located at the azimuth angle θ_t and range r_t. The time delay of the signal that is transmitted by the mth antenna reflected by the target and received by the nth antenna is written as

$$\tau_{nm} = \frac{2r_t}{c} - \frac{d_T(m-1)\sin\theta_t + d_R(n-1)\sin\theta_t}{c} \tag{2}$$

where c is the speed of light and d_T and d_R are the interspaces of the transmit and receive antennas, respectively.

After target reflection, the signal transmitted by the mth antenna and received by the nth antenna is expressed using a formulation similar to that presented in [15], [16].

$$\boldsymbol{x}_{tnm}(t) = \alpha_t \boldsymbol{S}_t \boldsymbol{t}^m g_m(t-\tau_{nm})\exp(j2\pi f(t-\tau_{nm})) \tag{3}$$

where α_t is a complex constant related to the transmitted power, the attenuation during propagation, and other factors, and $\boldsymbol{S}_t$ is the polarization scattering matrix (PSM) of the target, which can be expressed as follows.

$$\boldsymbol{S}_t = \begin{bmatrix} s_{hh} & s_{hv} \\ s_{vh} & s_{vv} \end{bmatrix} \tag{4}$$

After matched filtering with $g_m(t-\tau_{nm})\exp(j2\pi ft)$, the output signal of the target can be expressed as

$$\begin{aligned} &\boldsymbol{x}_{tnm} \\ &= \alpha_t \exp(-j2\pi f\tau_{nm})\boldsymbol{S}_t\boldsymbol{t}_m \\ &= \beta_t \exp(j2\pi\frac{d_T(m-1)\sin\theta_t}{\lambda})\exp(j2\pi\frac{d_R(n-1)\sin\theta_t}{\lambda})\boldsymbol{S}_t\boldsymbol{t}_m \end{aligned} \tag{5}$$

where $\beta_t = \alpha_t \exp(-j4\pi f r_t/c)$ and $\lambda = c/f$ is the wavelength. $N \times M$ of these vectors are obtained, and they are arranged into a single vector

$$\begin{aligned} \boldsymbol{x}_t &= [\boldsymbol{x}_{t11}^T, \boldsymbol{x}_{t12}^T, \cdots, \boldsymbol{x}_{t1M}^T, \cdots, \boldsymbol{x}_{tN1}^T, \boldsymbol{x}_{tN2}^T, \cdots, \boldsymbol{x}_{tNM}^T]^T \\ &= \beta_t \boldsymbol{a}_R(\theta_t) \otimes [\boldsymbol{H}_t\,(\boldsymbol{a}_T(\theta_t)\otimes \mathbf{1})] \\ &= \beta_t \boldsymbol{b}_t \end{aligned} \tag{6}$$

where $\otimes$ denotes the Kronecker product, $\boldsymbol{H}_t = diag(\boldsymbol{h}_t)$ is a diagonal matrix, $\boldsymbol{h}_t = [\boldsymbol{S}_t\boldsymbol{t}_1; \boldsymbol{S}_t\boldsymbol{t}_2; \cdots; \boldsymbol{S}_t\boldsymbol{t}_M]$ is the target polarization information vector, and $\mathbf{1} = [1, 1]^T$. $\boldsymbol{a}_R(\theta_t) \in C^{N\times 1}$ and $\boldsymbol{a}_T(\theta_t) \in C^{M\times 1}$ are the receive and transmit steering vectors, respectively. These vectors have the following forms.

$$\begin{aligned} \boldsymbol{a}_R(\theta) = [1, &\exp(j2\pi d_R \sin\theta/\lambda), \cdots, \\ &\exp(j2\pi d_R(N-1)\sin\theta/\lambda)]^T \end{aligned} \tag{7}$$

$$\begin{aligned} \boldsymbol{a}_T(\theta) = [1, &\exp(j2\pi d_T \sin\theta/\lambda), \cdots, \\ &\exp(j2\pi d_T(M-1)\sin\theta/\lambda)]^T \end{aligned} \tag{8}$$

Therefore, the received jamming-free snapshot of the polarization MIMO radar system can be expressed as

$$\boldsymbol{y} = \boldsymbol{x}_t + \boldsymbol{n} \tag{9}$$

where $\boldsymbol{n}$ is a white noise with zero mean and covariance matrix $\sigma_n^2\boldsymbol{I}_{2MN}$. Additionally, σ_n^2 and $\boldsymbol{I}_{2MN}$ represent the noise power and a $2MN$ dimensional identity matrix, respectively.

B. DECEPTIVE JAMMING MODELING

In practice, radar systems often face deceptive electronic countermeasures (ECMs). A false target generator (FTG) can capture, store and repeat the transmitted radar signal to deceive the radar system. We consider an FTG located at the azimuth angle θ_j and range r_j. Let $\boldsymbol{h}_{jt}$ and $\boldsymbol{h}_{jr}$ denote the transmit and receive polarization vectors of the FTG antenna, respectively. The definitions of $\boldsymbol{h}_{jt}$ and $\boldsymbol{h}_{jr}$ are the same as those for $\boldsymbol{t}_m$. The signal transmitted by the polarization MIMO radar system and captured by the FTG is

$$J(t) = \sum_{m=1}^{M} \boldsymbol{h}_{jr}^T \boldsymbol{t}_m g_m(t - \frac{\tau_j}{2} - \tau_{jm})\exp(j2\pi f(t - \frac{\tau_j}{2} - \tau_{jm})) \tag{10}$$

where $\tau_j = 2r_j/c$ and $\tau_{jm} = -d_T(m-1)\sin(\theta_j)/c$. The FTG can generate a false target in any range bin with an appropriate delay time. Suppose that the delay time is τ_0. Then, the signal radiated by the FTG and received by the nth antenna of the radar is

$$\begin{aligned} &\boldsymbol{x}_{jn}(t-\tau_{jn}-\tau_0) \\ &= \alpha_j(t)\boldsymbol{h}_{jt}\sum_{m=1}^{M}\boldsymbol{h}_{jr}^T\boldsymbol{t}_m \\ &\quad \times g_m(t-\tau_j-\tau_{jm}-\tau_{jn})\exp(j2\pi f(t-\tau_j-\tau_{jm}-\tau_{jn})) \end{aligned} \tag{11}$$

where $\tau_{jn} = -d_R(n-1)\sin(\theta_j)/c$ and $\alpha_j(t)$ is the modulation signal. $\alpha_j(t)$ includes the amplitude and velocity modulation by the FTG. Similarly, after matched filtering with $g_m(t-\tau_j-\tau_{jm}-\tau_{jn})\exp(j2\pi ft)$, the output signal of the false target can be expressed as

$$\begin{aligned} \boldsymbol{x}_{jnm} = \beta_j \exp(j2\pi\frac{d_T(m-1)\sin\theta_j}{\lambda_0}) \\ \times \exp(j2\pi\frac{d_R(n-1)\sin\theta_j}{\lambda_0})\boldsymbol{S}_j\boldsymbol{t}_m \end{aligned} \tag{12}$$

where $\beta_j = \alpha_j\exp(-j4\pi f r_j/c)$ is a complex constant related to the modulation and matched filtering gain. $\boldsymbol{S}_j = \boldsymbol{h}_{jt}\boldsymbol{h}_{jr}^T$ is called the PSM of the false target (the FTG).

The output signals corresponding to the false target range bin are combined into the following vector:

$$\begin{aligned} \boldsymbol{x}_j &= [\boldsymbol{x}_{j11}^T, \boldsymbol{x}_{j12}^T, \cdots, \boldsymbol{x}_{j1M}^T, \cdots, \boldsymbol{x}_{jN1}^T, \boldsymbol{x}_{jN2}^T, \cdots, \boldsymbol{x}_{jNM}^T]^T \\ &= \beta_j\boldsymbol{a}_R(\theta_j)\otimes\left[\boldsymbol{H}_j\left(\boldsymbol{a}_T(\theta_j)\otimes\boldsymbol{1}\right)\right] \\ &= \beta_j\boldsymbol{b}_j \end{aligned} \tag{13}$$

where $\boldsymbol{H}_j = diag(\boldsymbol{h}_j)$ is a diagonal matrix and $\boldsymbol{h}_j = [\boldsymbol{S}_j\boldsymbol{t}_1; \boldsymbol{S}_j\boldsymbol{t}_2; \cdots; \boldsymbol{S}_j\boldsymbol{t}_M]$ is called the false target polarization information vector in this paper.

We consider the test scenario with one true target and K false targets. In this scenario, the false targets are placed in the same range bin as the true target by the FTGs. Mathematically, the polarization MIMO radar signal model can be modified as follows.

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{x}_t + \boldsymbol{x}_j + \boldsymbol{n} \\ &= \beta_t\boldsymbol{b}_t + \sum_{k=1}^{K}\beta_{jk}\boldsymbol{b}_{jk} + \boldsymbol{n} \end{aligned} \tag{14}$$

III. JAMMING SUPPRESSION PERFORMANCE ANALYSIS

In this section, we use the adaptive beamforming technique to suppress jamming [26]. The output SINR after beamforming can be expressed as

$$\mathrm{SINR} = \frac{\boldsymbol{w}^H\boldsymbol{R}_t\boldsymbol{w}}{\boldsymbol{w}^H\boldsymbol{R}_{jn}\boldsymbol{w}} \tag{15}$$

where $\boldsymbol{w} \in C^{2MN\times 1}$ is the weight vector, the superscript H represents the Hermitian transpose operation, and $\boldsymbol{R}_t$ and $\boldsymbol{R}_{jn}$ are the covariance matrixes of the target signal and the jamming plus noise signal, respectively. Obviously, equation (15) is a generalized Rayleigh quotient problem. According to the Rayleigh-Ritz theorem, we can obtain the output SINR after optimal beamforming as follows.

$$\mathrm{SINR}_{\mathrm{out}} = \lambda_{\max}\left\{\boldsymbol{R}_{jn}^{-1}\boldsymbol{R}_t\right\} \tag{16}$$

where $\lambda_{\max}$ is the largest eigenvalue of $\boldsymbol{R}_{jn}^{-1}\boldsymbol{R}_t$.

For the scenario with one true target and K false targets, $\boldsymbol{R}_t = \sigma_t^2\boldsymbol{b}_t\boldsymbol{b}_t^H$, where $\sigma_t^2 = E[|\beta_t|^2]$ represents the true target signal power. Then, equation (16) can be modified as follows.

$$\begin{aligned} \lambda_{\max}\left\{\boldsymbol{R}_{jn}^{-1}\boldsymbol{R}_t\right\} &= \lambda_{\max}\left\{\boldsymbol{R}_{jn}^{-1}\cdot\sigma_t^2\boldsymbol{b}_t\boldsymbol{b}_t^{\boldsymbol{H}}\right\} \\ &= \sigma_t^2\boldsymbol{b}_t^{\boldsymbol{H}}\boldsymbol{R}_{jn}^{-1}\boldsymbol{b}_t \end{aligned} \tag{17}$$

The inverse of the matrix $\boldsymbol{R}_{jn}$ can be expressed as

$$\boldsymbol{R}_{jn}^{-1} = \frac{1}{\sigma_n^2}\left[\boldsymbol{I}_{2MN} - \sum_{k=1}^{K}\frac{\mu_k-\sigma_n^2}{\mu_k}\boldsymbol{u}_k\boldsymbol{u}_k^H\right] \tag{18}$$

where μ_k is the eigenvalue corresponding to the kth jamming and $\boldsymbol{u}_k$ is the eigenvector of the covariance matrix $\boldsymbol{R}_{jn}$. In general, the jamming power is usually much greater than the noise power; therefore,

$$\frac{\mu_k-\sigma_n^2}{\mu_k} \approx 1. \tag{19}$$

Then, equation (16) can be simplified as follows.

$$\begin{aligned} \mathrm{SINR}_{\mathrm{out}} &\approx \frac{\sigma_t^2}{\sigma_n^2}\left[\|\boldsymbol{b}_t\|^2 - \sum_{k=1}^{K}\left|\boldsymbol{b}_t^H\boldsymbol{u}_k\right|^2\right] \\ &= \frac{\sigma_t^2}{\sigma_n^2}\left[N\|\boldsymbol{h}_t\|^2 - \sum_{k=1}^{K}\left|\boldsymbol{b}_t^H\boldsymbol{u}_k\right|^2\right] \end{aligned} \tag{20}$$

IV. POLARIZATION DESIGN USING GAME THEORY

To improve the jamming suppression performance, we should design the transmit polarization $\boldsymbol{t}_1, \boldsymbol{t}_2, \cdots, \boldsymbol{t}_M$ to maximize the output SINR. Thus, we can formulate the following optimization problem.

$$\max_{\boldsymbol{t}_1,\boldsymbol{t}_2,\cdots,\boldsymbol{t}_M} \mathrm{SINR}_{\mathrm{out}} \tag{21}$$

According to (20), the optimization problem can be rewritten as follows.

$$\max_{\boldsymbol{t}_1,\boldsymbol{t}_2,\cdots,\boldsymbol{t}_M} N\|\boldsymbol{h}_t\|^2 - \sum_{k=1}^{K}\left|\boldsymbol{b}_t^{\boldsymbol{H}}\boldsymbol{u}_k\right|^2 \tag{22}$$

When the matrix $\boldsymbol{R}_{jn}$ is a row full rank matrix, the jamming subspace spanned by the K eigenvectors is the same as the subspace spanned by the jamming steering vectors, i.e., span $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_K\} = $ span $\left\{\boldsymbol{b}_{j1}, \boldsymbol{b}_{j2}, \cdots, \boldsymbol{b}_{jK}\right\}$, and the cost function can be represented as follows.

$$N\|\boldsymbol{h}_t\|^2 - \sum_{k=1}^{K}\left|\boldsymbol{b}_t^{\boldsymbol{H}}\boldsymbol{b}_{jk}\right|^2 \tag{23}$$

In this section, we apply game theory concepts to the radar jamming suppression problem for transmit polarization design. Game theory is explained in detail in [29], [30], and a brief overview of the two-person finite strategy game is given in [20]. Therefore, we do introduce the game theory background in this paper and instead focus on the polarization design problem.

In our radar design problem, we have only two players: player 1 is the FTGs who choose the polarization characteristics of the false targets to effectively deceive the radar system, and player 2 is the MIMO radar system, which selects the transmit polarizations to improve the jamming suppression performance. This is a TPZS game [31] in which one player's gain is the other's loss. We propose two design methods: one is a unilateral game with asymmetrical information for dumb jammers, and the other is a strategic game with symmetrical information for smart jammers. Better jamming suppression performance can be achieved in the MIMO radar system with both methods for dumb and smart jammers.

We emphasize that in this paper, we do not consider the parameter estimation algorithms introduced in [27], [28]. Moreover, in the following analysis and simulations, the angles of the target and jammers are all set to known values, as are the PSM of the true target and the polarization vectors of all the jammers. In practice, all the parameters noted above should be estimated in advance.

A. UNILATERAL GAME

This subsection considers the extreme case in which player 2 can intercept player 1's strategy and the latter does not notice that it is happening, i.e., the smart radar counters dumb jammers. In this case, the available information for the two players is asymmetric. Therefore, player 2 can always choose the best response, and the TPZS game is reduced to a unilateral game.

If the MIMO radar system knows the FTG strategy, the game is simplified to a classical radar waveform optimization problem [15]–[17] in which the radar strategy is selected to maximize the output SINR. Mathematically, this problem is formulated as

$$\max_{\boldsymbol{t}} \ \mathrm{SINR}_{\mathrm{out}} \quad \text{s.t. } \|\boldsymbol{h}_t\| = c_t \tag{24}$$

where $\boldsymbol{t} = [\boldsymbol{t}_1^T, \boldsymbol{t}_2^T, \cdots, \boldsymbol{t}_M^T]^T$ and c_t is a constant. According to (23), we can obtain the following optimization problem.

$$\min_{\boldsymbol{t}} \sum_{k=1}^{K} \left|\boldsymbol{b}_t^H \boldsymbol{b}_{jk}\right|^2 \quad \text{s.t. } \|\boldsymbol{h}_t\| = c_t \tag{25}$$

According to matrix theory, we can obtain the following transformation

$$\begin{aligned}\boldsymbol{b} &= \boldsymbol{a}_R(\theta) \otimes [\boldsymbol{H}(\boldsymbol{a}_T(\theta) \otimes \mathbf{1})] \\ &= \boldsymbol{a}_R(\theta) \otimes \mathrm{diag}\,(\boldsymbol{a}_T(\theta) \otimes \mathbf{1})\,\boldsymbol{h} \\ &= \boldsymbol{A}(\theta)\boldsymbol{B}(\boldsymbol{S})\boldsymbol{t}\end{aligned} \tag{26}$$

where $\boldsymbol{A}(\theta) \triangleq \boldsymbol{a}_R(\theta) \otimes \mathrm{diag}\,(\boldsymbol{a}_T(\theta) \otimes \mathbf{1})$ and $\boldsymbol{B}(\boldsymbol{S}) \triangleq \boldsymbol{I}_M \otimes \boldsymbol{S}$. Thus,

$$\|\boldsymbol{b}_t\| = \|\boldsymbol{A}(\theta_t)\boldsymbol{B}(\boldsymbol{S}_t)\boldsymbol{t}\| \tag{27}$$

$$\|\boldsymbol{b}_{jk}\| = \|\boldsymbol{A}(\theta_{jk})\boldsymbol{B}(\boldsymbol{S}_{jk})\boldsymbol{t}\| \tag{28}$$

$$\left|\boldsymbol{b}_t^H \boldsymbol{b}_{jk}\right| = \left|\boldsymbol{t}^H \boldsymbol{B}^H(\boldsymbol{S}_t)\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_{jk})\boldsymbol{B}(\boldsymbol{S}_{jk})\boldsymbol{t}\right|. \tag{29}$$

With the above discussion, the optimal problem can be re-expressed as follows.

$$\min_{\boldsymbol{t}} \sum_{k=1}^{K} \left|\boldsymbol{t}^{\mathrm{H}} \boldsymbol{B}^{\mathrm{H}}(\boldsymbol{S}_t)\boldsymbol{A}^{\mathrm{H}}(\theta_t)\boldsymbol{A}(\theta_{jk})\boldsymbol{B}(\boldsymbol{S}_{jk})\boldsymbol{t}\right|^2 \quad \text{s.t. } \|\boldsymbol{h}_t\| = c_t \tag{30}$$

In this case, we define

$$\boldsymbol{P}_k = \boldsymbol{B}^H(\boldsymbol{S}_t)\boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_{jk})\boldsymbol{B}(\boldsymbol{S}_{jk}). \tag{31}$$

Notably, the optimal strategy $\hat{\boldsymbol{t}}$ satisfies the following equation.

$$\left|\hat{\boldsymbol{t}}^{\mathrm{H}} \boldsymbol{P}_k \hat{\boldsymbol{t}}\right| = 0, \quad k = 1, 2, \cdots, K \tag{32}$$

Defining $\boldsymbol{Q} = \sum_{k=1}^{K} \boldsymbol{P}_k \boldsymbol{P}_k^H$, the eigen decomposition of $\boldsymbol{Q}$ can be expressed as $\boldsymbol{Q} = \boldsymbol{U}_Q \boldsymbol{\Sigma}_Q \boldsymbol{U}_Q^{\mathrm{H}}$, where $\boldsymbol{\Sigma}_Q$ and $\boldsymbol{U}_Q$ are the eigenvalue matrix and eigenvector matrix, respectively. Note that the diagonal elements of eigenvalue matrix $\boldsymbol{\Sigma}_Q$ include M small eigenvalues and M large eigenvalues, as shown in the simulation. Moreover, each eigenvector only contains two nonzero elements. Under this condition, we can obtain the optimal strategy $\hat{\boldsymbol{t}}$ composed of the nonzero entries of the eigenvectors corresponding to the M small eigenvalues.

In conclusion, the major steps in the transmit polarization design process can be summarized as follows.

Step 1: Select the initial transmit polarization for the polarization MIMO radar system, decompose the received signals with the matched filters, and estimate the angles and PSMs of all the targets.

Step 2: Construct $\boldsymbol{P}_k$ according to (31).

Step 3: Construct $\boldsymbol{Q} = \sum_{k=1}^{K} \boldsymbol{P}_k \boldsymbol{P}_k^{\mathrm{H}}$ and perform the eigen decompose of $\boldsymbol{Q}$ as $\boldsymbol{Q} = \boldsymbol{U}_Q \boldsymbol{\Sigma}_Q \boldsymbol{U}_Q^{\mathrm{H}}$.

Step 4: The optimal strategy $\hat{\boldsymbol{t}}$ is obtained from the nonzero entries of the eigenvectors corresponding to the M small eigenvalues in $\boldsymbol{U}_Q$.

B. STRATEGIC GAME

In the above analysis, the true strategies of the FTGs are assumed to remain unchanged and be accurately known by the MIMO radar system in advance, and the FTGs are not aware of this scenario. However, with advances in digital signal processing and computing technology, adaptive jamming systems usually have multiple jamming strategies and can flexibly choose these strategies according to the working strategy of the radar system. In such circumstances, the radar does not know the true strategy of the FTGs in advance, and vice versa, so the polarization design method proposed in the previous subsection will fail. Therefore, in this subsection, we study the polarization design problem for a smart radar system and smart jammers.

In practice, although each player does not know the true strategy of the other, they can obtain knowledge of all possible strategies of the other through long-term observations in advance. In the proposed radar design problem, the polarization parameters of the jamming signal (false targets) are dependent on the transmit and receive polarization vectors of the FTGs. In general, when the FTG receives the radar signal, the polarization vector of the FTG antenna is fixed, but when the FTG transmits the signals to generate the false targets, the polarization vector of the antenna can be changed as needed to improve the jamming effect. Therefore, we assume that the receive polarization vectors of all FTGs remain unchanged and that each FTG has R optional transmit polarization vectors.

$$\Psi = \{\boldsymbol{h}_{jt}^1, \boldsymbol{h}_{jt}^2, \cdots, \boldsymbol{h}_{jt}^R\} \tag{33}$$

Further, we assume that each transmitter of the MIMO radar system has L optional transmit polarization vectors

$$\Omega = \left\{\boldsymbol{t}_a^1, \boldsymbol{t}_a^2, \cdots, \boldsymbol{t}_a^L\right\} \tag{34}$$

where the definitions of $\boldsymbol{h}_{jt}^{(\cdot)}$ and $\boldsymbol{t}_a^{(\cdot)}$ are the same as those for $\boldsymbol{t}_m$. Each FTG of player 1 can independently select one of the R possible transmit polarization vectors. Similarly, each transmitter of player 2 can independently choose one of the L possible transmit polarization vectors. Therefore, for the K FTGs and the MIMO radar system with M transmit antennas, there are N_J and N_R possible pure strategies without order consideration, respectively, where

$$N_J = \frac{(R+K-1)!}{(R-1)!K!} \tag{35}$$

$$N_R = \frac{(L+M-1)!}{(L-1)!M!}. \tag{36}$$

We have now formally defined the players and their pure strategies. The next step is to define the payoff functions of both players for the $N_J \times N_R$ possible profiles of strategies.

We assume that the payoff functions of both players sum to zero. This is a reasonable assumption because the players having opposing goals. Player 1 tries to select the transmit polarization vectors that make the false targets difficult to suppress, and player 2 designs transmit polarization vectors to improve the jamming suppression performance. Such games, i.e., zero-sum games, have been widely discussed in the game theory literature [20], [29]. In our radar design problem,

$$u_1(\boldsymbol{t}_J^i, \boldsymbol{t}_R^j) = -u_2\left(\boldsymbol{t}_J^i, \boldsymbol{t}_R^j\right), \ \forall i = 1, \cdots, N_J; j=1, \cdots, N_R \tag{37}$$

where $\boldsymbol{t}_J^i \triangleq [\boldsymbol{h}_{jt1}^{(i)}; \boldsymbol{h}_{jt2}^{(i)}; \cdots; \boldsymbol{h}_{jtK}^{(i)}]^T, \boldsymbol{t}_R^j \triangleq [\boldsymbol{t}_{a1}^{(j)}; \boldsymbol{t}_{a2}^{(j)}; \cdots; \boldsymbol{t}_{aM}^{(j)}]^T$, and $\boldsymbol{h}_{jtk}^{(i)}$ and $\boldsymbol{t}_{am}^{(j)}$ are independently selected by player 1 and player 2 from the corresponding lists of all optional transmit polarization vectors Ψ and Ω, respectively.

According to (26), the inner product of $\boldsymbol{b}_t$ and $\boldsymbol{b}_{jk}$ can be represented as follows.

$$\boldsymbol{b}_t^H \boldsymbol{b}_{jk} = \boldsymbol{h}_t^H \boldsymbol{A}^H(\theta_t)\boldsymbol{A}(\theta_{jk})\boldsymbol{h}_{jk} \tag{38}$$

Since jamming is settled in the main lobe, we have $\boldsymbol{A}^{\mathrm{H}}(\theta_t)\boldsymbol{A}(\theta_{jk}) \simeq \boldsymbol{I}_{2M}$, which indicates that the spatial correlation between the true target and jamming is approximately equal to 1. Thus, for a conventional phase array radar and the MIMO radar, main lobe jamming cannot be effectively suppressed. However, for the polarization MIMO radar, equation (38) can be approximated as $\boldsymbol{b}_t^{\mathrm{H}}\boldsymbol{b}_{jk} \simeq \boldsymbol{h}_t^{\mathrm{H}}\boldsymbol{h}_{jk}$. In this case, the target steering vector and the jamming steering vector are mainly related to the polarization correlation. Jamming can be suppressed in the polarization domain using the polarization difference between the target and jamming signal. When the target polarization information vector $\boldsymbol{h}_t$ and the false target polarization information vector $\boldsymbol{h}_{jk}$ are strongly correlated, the suppression performance worsens, and vice versa. Therefore, we define the utility function for player 1 as

$$u_1\left(\boldsymbol{t}_J^i, \boldsymbol{t}_R^j\right) = \sum_{k=1}^{K}\left|\boldsymbol{b}_t^{\mathrm{H}}\boldsymbol{b}_{jk}\right|^2 \simeq \sum_{k=1}^{K}\left|\left(\boldsymbol{h}_t^j\right)^{\mathrm{H}}\boldsymbol{h}_{jk}^{ij}\right|^2 \tag{39}$$

where $\boldsymbol{h}_t^j = \boldsymbol{B}(\boldsymbol{S}_t)\boldsymbol{t}_R^j$, $\boldsymbol{h}_{jk}^{ij} = \boldsymbol{B}(\boldsymbol{S}_{jk}^i)\boldsymbol{t}_R^j$ and $\boldsymbol{S}_{jk}^i = \boldsymbol{h}_{jtk}^{(i)}\boldsymbol{h}_{jrk}^T$.

If the players choose the mixed-strategy profile (P_1, P_2), the corresponding utility function becomes

$$u_1\left(P_1, P_2\right) = \sum_{\substack{i=1,\cdots,N_J \\ j=1,\cdots,N_R}} P_1(\boldsymbol{t}_J^i)P_2(\boldsymbol{t}_R^j)\sum_{k=1}^{K}\left|\left(\boldsymbol{h}_t^j\right)^{\mathrm{H}}\boldsymbol{h}_{jk}^{ij}\right|^2. \tag{40}$$

From (37)-(40), the utility function is only related to the FTGs' transmit polarization $\boldsymbol{t}_J^i$ and the MIMO radar transmit polarization $\boldsymbol{t}_R^j$ when the receive polarization vectors of all FTGs $\boldsymbol{h}_{jrk}(k = 1, 2, \cdots, K)$ remain unchanged. These expressions for the utility functions reflect the polarimetric correlation between the target and jamming signal. Note that $\boldsymbol{h}_t^j$ and $\boldsymbol{h}_{jk}^{ij}$ are functions of the strategy $\boldsymbol{t}_R^j$ chosen by player 2. Minimizing the polarimetric correlation between the target and jamming signal improves the jamming suppression performance. This result is the goal of player 2. However, player 1 has the opposite goal, i.e., maximizing the polarimetric correlation between the target and jamming signal to enhance the jamming efficiency. Therefore, the utility function of player 2 is defined as the additive inverse of the utility function of player 1, thereby leading to a zero-sum game.

We have expressed the problem of polarization design for polarization MIMO radar as a TPZS game. Different approaches can be used to solve this problem. As noted in [29], we can follow the procedure of iterated strict dominance to find the dominant strategy. If this procedure does not provide a solution, we can search for the possible Nash equilibria for this game and pick a solution from the corresponding solution set [31]. For finite games, the existence of a Nash equilibrium has been shown in the literature [32].

Furthermore, computing the Nash equilibrium of any TPZS game can be formulated as a linear programming problem; hence, algorithms to solve this problem in polynomial time exist [33]. Thus, we can always find at least one equilibrium solution to this design problem.

This proposed strategic game design method does not require accurate estimations of jamming properties from measured data. Furthermore, this method is a one-time offline computational process implemented before radar scanning. The complexity of game theory design is typically low when compared with the dimensions of the training data required by conventional approaches. Hence, our approach is easy to implement in practice without high costs, unlike the conventional approaches discussed above.

V. NUMERICAL SIMULATIONS

In this section, we will present numerical examples to demonstrate the performance of the proposed polarization MIMO radar system and game theory design mechanism with respect to deceptive jamming suppression in the main lobe.

Consider a polarization MIMO radar system in which two ULAs with $M = N = 3$ antennas and half-wavelength spacing between adjacent antennas are used for transmitting and receiving. The test scenario includes one true target and two FTGs. The azimuth angle and range bin of the true target are 0° and 100, respectively, and the signal-to-noise ratio (SNR) is 5 dB. The azimuth angles of the two FTGs are 3° and −1°. The jamming-to-noise ratios (JNRs) of the two FTGs are 25 and 30 dB. The false targets generated by FTG 1 are in range bins 60, 100, and 140, and those of FTG 2 are in range bins 85, 100, and 115. The results reported in this section correspond to the average of 200 Monte Carlo simulations.

A. EXAMPLES FOR THE UNILATERAL GAME

This subsection concentrates on the unilateral game. In the simulations, the PSM of the true target is as follows.

$$\boldsymbol{S}_t = \begin{bmatrix} 0.3 + 1.5j & -1.9j \\ -1.4j & -0.2 \end{bmatrix}$$

Additionally, the initial transmit polarization of the polarization MIMO radar system is randomly selected. The transmit polarization vectors are as follows.

$$\begin{bmatrix} \cos 60^\circ & \cos 78^\circ & \cos 52^\circ \\ \sin 60^\circ e^{j45^\circ} & \sin 78^\circ e^{j60^\circ} & \sin 52^\circ \end{bmatrix}$$

The transmit and receive polarization vectors of two FTGs are as follows.

$$\boldsymbol{h}_{jr1} = \begin{bmatrix} \cos 13^\circ \\ \sin 13^\circ e^{j60^\circ} \end{bmatrix} \quad \boldsymbol{h}_{jt1} = \begin{bmatrix} \cos 55^\circ \\ \sin 55^\circ e^{j45^\circ} \end{bmatrix}$$
$$\boldsymbol{h}_{jr2} = \begin{bmatrix} \cos 78^\circ \\ \sin 78^\circ e^{j60^\circ} \end{bmatrix} \quad \boldsymbol{h}_{jt2} = \begin{bmatrix} \cos 52^\circ \\ \sin 52^\circ \end{bmatrix}$$

Fig. 2 shows the eigenvalues of the matrix $\boldsymbol{\Sigma}_Q$, which is composed of 3 small eigenvalues and 3 large eigenvalues. This structure is consistent with that discussed in subsection 4.1. The eigenvectors corresponding to the 3 small eigenvalues constitute the optimal transmit polarization, and output SINR improvement is demonstrated in the following experiments with the optimal transmit polarization.

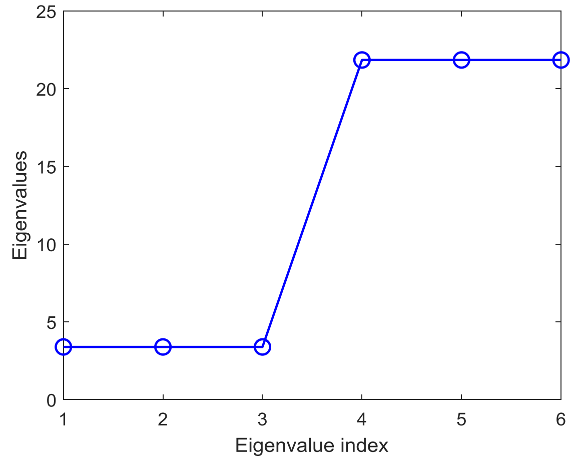

FIGURE 2. Eigenvalues of the matrix Σ_Q.

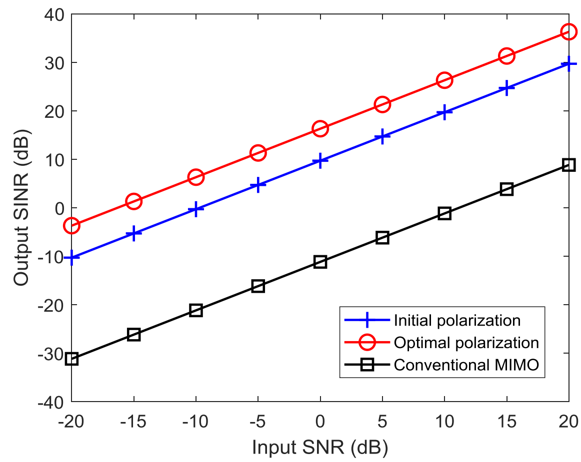

FIGURE 3. The output SINR versus the input SNR for the polarization MIMO radar and conventional MIMO radar systems.

Next, we examine how the optimal design of the transmit polarization improves deceptive jamming suppression. Fig. 3 shows the output SINR versus the input SNR in the polarization MIMO radar system. For ease of reference, the output SINR versus the input SNR for the conventional MIMO radar system (denoted as conventional MIMO, where the transmit and receive polarizations are horizontally polarized) is included. A significantly higher SINR is achieved in polarization MIMO radar compared to that for the conventional MIMO radar system. Thus, the polarization MIMO radar provides better interference robustness than the conventional MIMO radar when tackling main lobe deceptive jamming problems. This gain is expected because the proposed polarization MIMO radar combines the advantages of both the polarization diversity and MIMO radar. Additionally, the transmit polarization optimization is useful for the polarization MIMO radar.

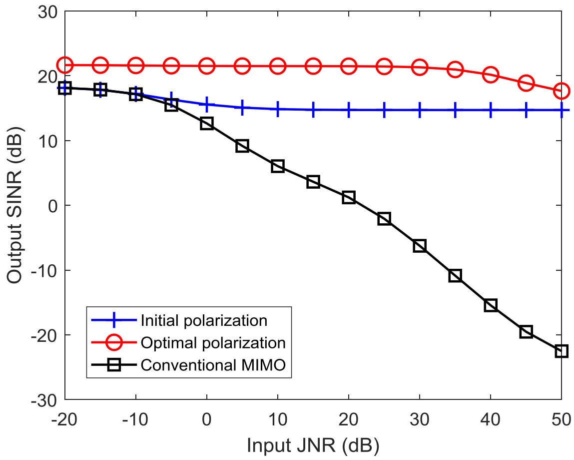

FIGURE 4. The output SINR versus the input JNR for the polarization MIMO radar and conventional MIMO radar systems.

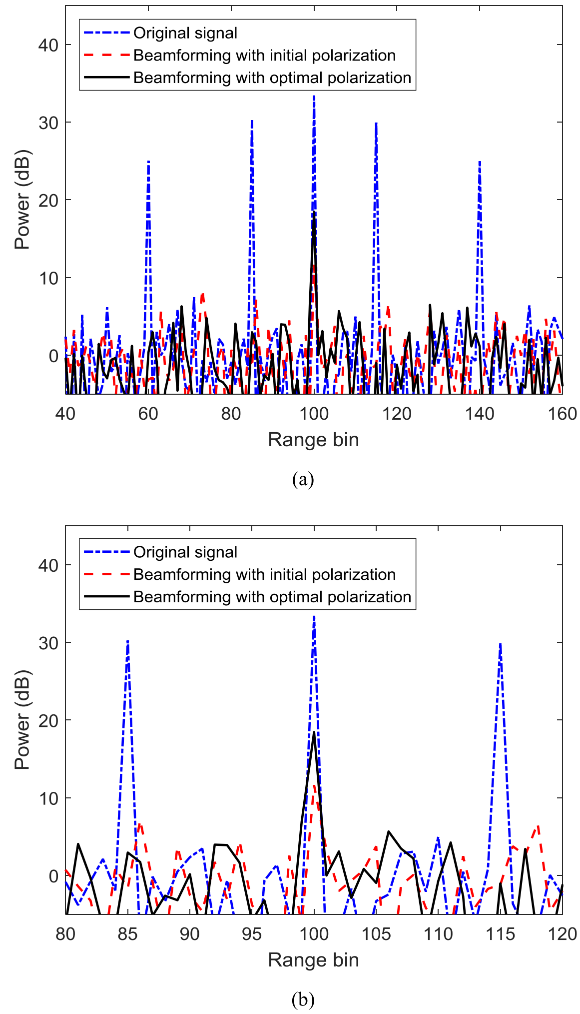

FIGURE 5. Power of signals before and after beamforming with the initial and optimal transmit polarizations: (a) Original view and (b) Enlarged view around the true target.

Fig. 4 shows the output SINR versus the JNRs of the two FTGs (SNR is fixed to 5 dB) for the polarization MIMO radar and the conventional MIMO radar systems. From this figure, we can observe that the output SINR of the polarization MIMO radar is significantly higher than the output SINR of the conventional MIMO radar at a JNR greater than 0 dB. However, as the JNR decreases, the performance difference between the polarization MIMO radar with the original polarization and the conventional MIMO radar tends to decrease, and the two curves eventually coincide. This observation agrees with the fact that the polarization MIMO radar and the conventional MIMO radar have the same performance in this simulation scenario when the JNR is less than -20 dB, i.e., the jamming power can be neglected compared to the noise power. Additionally, this figure also shows that the optimal polarization design of the transmit waveforms provides a significant performance improvement for the polarization MIMO radar for all JNR values compared with the performance of systems that transmit the original polarized waveforms.

Fig. 5a shows the output power as a function of the range bin for the polarization MIMO radar system before and after beamforming with the original and optimal transmit polarizations. Fig. 5b shows an enlarged view around the true target. The figures clearly show that all false targets are effectively suppressed, and the true target can be detected by the polarization MIMO radar system after beamforming. Furthermore, a higher target power is achieved by the polarization MIMO radar system with the optimal transmit polarization compared to that for the original polarization.

B. EXAMPLES FOR THE STRATEGIC GAME

This subsection demonstrates the performance of the proposed strategic game design method for the polarization MIMO radar system and compares the system with purely horizontally or vertically polarized radar systems. First, we present an example (denoted as case 1) that gives a pure strategy Nash equilibrium solution to the design game. Later, we will also discuss a scenario (denoted as case 2) in which the only possible Nash equilibrium solution is a mixed strategy.

It should be noted that the proposed strategic game is a finite game in which players have a finite strategy set to choose from. Furthermore, in the following simulation, for simplicity and clarity of exposition, we suppose that there are only three optional transmit polarization vectors for each FTG and for each transmitter of the MIMO radar system.

In the simulation, we define the PSM of the true target as follows.

$$\boldsymbol{S}_t = \begin{bmatrix} 1 & 0.5j \\ -0.2j & 0.9 \end{bmatrix}$$

Additionally, the initial transmit polarization vectors of the polarization MIMO radar are as follows.

$$\begin{bmatrix} \cos 10^\circ & \cos 38^\circ & \cos 52^\circ \\ \sin 10^\circ e^{j45^\circ} & \sin 38^\circ e^{j60^\circ} & \sin 52^\circ \end{bmatrix}$$

The receive polarization vectors of the two FTGs are given as follows.

$$\boldsymbol{h}_{jr1} = \begin{bmatrix} \cos 13^\circ \\ \sin 13^\circ e^{j60^\circ} \end{bmatrix} \quad \boldsymbol{h}_{jr2} = \begin{bmatrix} \cos 78^\circ \\ \sin 78^\circ e^{j60^\circ} \end{bmatrix}$$

For each FTG, we assume the following three optional transmit polarization vectors.

$$\boldsymbol{h}_{jt}^1 = [1, 0]^{\mathrm{T}} \quad \boldsymbol{h}_{jt}^2 = [0, 1]^{\mathrm{T}} \ \boldsymbol{h}_{jt}^3 = \left[\sqrt{0.5}, \sqrt{0.5}\right]^{\mathrm{T}}$$

In other words, the transmit antennas of each FTG can work in the horizontal polarization state, the vertical polarization state, or a 45° linear polarization state. Moreover, we assume that each of the transmitters of the MIMO radar system has the following three optional transmit polarization vectors.

$$\boldsymbol{t}_a^1 = [1, 0]^{\mathrm{T}} \quad \boldsymbol{t}_a^2 = [0, 1]^{\mathrm{T}} \ \boldsymbol{t}_a^3 = \left[\sqrt{0.5}, \sqrt{0.5}\right]^{\mathrm{T}}$$

Therefore, there are 6 and 10 possible pure strategies without order consideration for player 1 and player 2, respectively.

$$\begin{aligned}
\boldsymbol{t}_J^1 &= [0, 1, 0, 1]^{\mathrm{T}} \\
\boldsymbol{t}_J^2 &= [1, 0, 1, 0]^{\mathrm{T}} \\
\boldsymbol{t}_J^3 &= [\sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}]^{\mathrm{T}} \\
\boldsymbol{t}_J^4 &= [1, 0, 0, 1]^{\mathrm{T}} \\
\boldsymbol{t}_J^5 &= [1, 0, \sqrt{0.5}, \sqrt{0.5}]^{\mathrm{T}} \\
\boldsymbol{t}_J^6 &= [\sqrt{0.5}, \sqrt{0.5}, 0, 1]^{\mathrm{T}} \\
\boldsymbol{t}_R^1 &= [1, 0, 1, 0, 1, 0]^{\mathrm{T}} \\
\boldsymbol{t}_R^2 &= [0, 1, 0, 1, 0, 1]^{\mathrm{T}} \\
\boldsymbol{t}_R^3 &= [\sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}]^{\mathrm{T}} \\
\boldsymbol{t}_R^4 &= [1, 0, 1, 0, 0, 1]^{\mathrm{T}} \\
\boldsymbol{t}_R^5 &= [0, 1, 0, 1, 1, 0]^{\mathrm{T}} \\
\boldsymbol{t}_R^6 &= [\sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}, 1, 0]^{\mathrm{T}} \\
\boldsymbol{t}_R^7 &= [\sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}, \sqrt{0.5}, 0, 1]^{\mathrm{T}} \\
\boldsymbol{t}_R^8 &= [1, 0, 1, 0, \sqrt{0.5}, \sqrt{0.5}]^{\mathrm{T}} \\
\boldsymbol{t}_R^9 &= [0, 1, 0, 1, \sqrt{0.5}, \sqrt{0.5}]^{\mathrm{T}} \\
\boldsymbol{t}_R^{10} &= [1, 0, \sqrt{0.5}, \sqrt{0.5}, 0, 1]^{\mathrm{T}}
\end{aligned}$$

In reality, the radar system does not know which of these different possible transmit polarization strategies corresponds to the actual jamming signal. Hence, the radar must consider all the possible jamming strategies before designing the transmit polarizations. Given the above sets of possible strategies for each player, the next step in defining the game is computing the utility functions for different profiles. There are $6 \times 10 = 60$ profiles in this problem, and we compute the utility functions for both the players using the expressions given in the previous section.

The utility function of this game is presented in Table 1, where we specify the utilities corresponding to player 1. The utilities for player 2 are easily obtained using the zero-sum property of this game. It should be noted that in order to make the table more concise, the values in the table are given to only two decimal places.

TABLE 1. Utility function of the game (CASE 1).

Strategies	$\boldsymbol{t}_J^1$	$\boldsymbol{t}_J^2$	$\boldsymbol{t}_J^3$	$\boldsymbol{t}_J^4$	$\boldsymbol{t}_J^5$	$\boldsymbol{t}_J^6$
$\boldsymbol{t}_R^1$	0.35	8.93	4.64	8.56	8.74	4.45
$\boldsymbol{t}_R^2$	7.34	2.26	4.80	7.08	4.67	7.21
$\boldsymbol{t}_R^3$	4.63	6.81	10.08	5.73	8.43	7.37
$\boldsymbol{t}_R^4$	1.23	4.95	3.43	5.09	5.18	3.34
$\boldsymbol{t}_R^5$	3.56	2.73	3.48	4.60	3.83	4.26
$\boldsymbol{t}_R^6$	2.32	6.79	7.46	5.88	7.41	5.92
$\boldsymbol{t}_R^7$	4.98	4.25	7.53	5.38	6.62	6.28
$\boldsymbol{t}_R^8$	0.90	7.50	5.64	6.82	7.51	4.95
$\boldsymbol{t}_R^9$	5.88	2.73	5.77	5.83	5.37	6.23
$\boldsymbol{t}_R^{10}$	2.66	4.24	5.07	4.84	5.34	4.57

For this high-dimensional problem, it is difficult to iteratively find the dominant strategies and obtain the solution. In such a situation, we can directly compute the Nash equilibria for the game. Using the Gambit software tools [34] for game theory, we observe that only one equilibrium profile $(\boldsymbol{t}_J^4, \boldsymbol{t}_R^5)$ exists for this game. Note that this is a pure strategy profile.

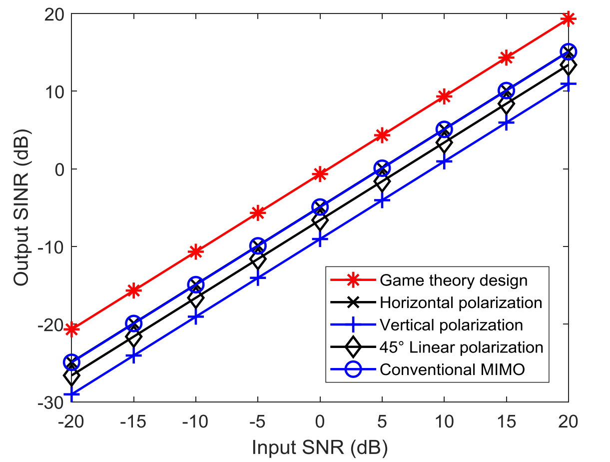

FIGURE 6. The output SINR versus the input SNR (Case 1).

After obtaining the transmit waveform polarizations using game theory design, the improvements in the radar jamming suppression performance associated with this design mechanism are shown in Figs. 6 and 7. We observe that in the polarization MIMO radar the game theory design for the transmit waveform polarizations yields a significant improvement in the output SINR when compared with the SINRs of systems that transmit only horizontal, vertical, or 45° linearly polarized waveforms. Furthermore, the proposed polarization MIMO radar has better SINR performance after transmit polarization design compared to conventional MIMO radar.

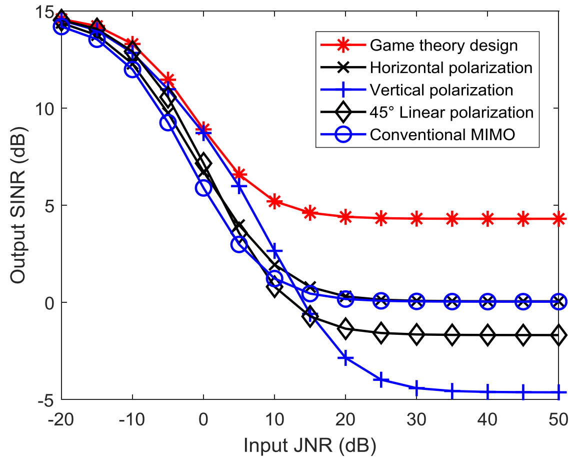

FIGURE 7. The output SINR versus the input JNR (Case 1).

It should be noted that in this example, the performance of the polarization MIMO radar system, which transmits purely horizontally polarized waveforms, is equivalent to that of the conventional MIMO radar system. This result is specific to this choice of the simulation parameters and is not always the case.

In the above problem, the Nash equilibrium solution to the game involved pure strategies for both players. However, this approach may not be true for all other choices of simulation parameters for the radar system and FTGs. Thus, we will study a problem that does not have pure strategy equilibrium solutions. For example, assume that the receive polarization vector of the second FTG in the previous problem is changed to

$$\boldsymbol{h}_{jr2} = \begin{bmatrix} \cos 50^\circ \\ \sin 50^\circ e^{-j30^\circ} \end{bmatrix}.$$

This change affects the utilities corresponding to this game. The utility function of this modified game is described in Table 2. Due to the modified utilities of the players, this zero-sum game does not have a pure strategy Nash equilibrium. Thus, the solution to this game is a unique mixed-strategy Nash equilibrium that is given as follows.

$$P_1 = \left\{0, 0, 0, \frac{27}{154}, \frac{127}{154}, 0\right\}$$
$$P_2 = \left\{0, \frac{3}{77}, 0, 0, \frac{74}{77}, 0, 0, 0, 0, 0\right\}$$

Player 1 assigns non-zero probabilities to pure strategies $\boldsymbol{t}_J^4$ and $\boldsymbol{t}_J^5$, whereas player 2 assign non-zero probabilities to pure strategies $\boldsymbol{t}_R^2$ and $\boldsymbol{t}_R^5$. This assignment shows that the other pure strategies of player 2, namely, $\boldsymbol{t}_R^1, \boldsymbol{t}_R^3, \boldsymbol{t}_R^4$, and $\boldsymbol{t}_R^6$ through $\boldsymbol{t}_R^{10}$, are dominated by the strategies $\boldsymbol{t}_R^2$ and $\boldsymbol{t}_R^5$. Hence, these strategies can be eliminated from the design problem.

In this problem, it is not straightforward to plot the performance curves because the value of the output SINR will vary for the different nondominant pure strategy pairs.

TABLE 2. Utility function of the game (CASE 2).

Strategies	t_J^1	t_J^2	t_J^3	t_J^4	t_J^5	t_J^6
t_R^1	0.49	12.26	6.37	8.69	10.47	4.59
t_R^2	4.64	1.43	3.04	4.39	2.91	4.51
t_R^3	5.87	8.64	12.78	6.97	11.14	8.61
t_R^4	0.69	5.49	3.95	4.55	5.70	2.79
t_R^5	2.07	1.88	2.84	3.12	3.18	2.77
t_R^6	2.79	8.94	9.84	6.35	9.80	6.39
t_R^7	5.00	4.52	8.16	5.40	7.26	6.31
t_R^8	1.00	10.15	7.71	6.92	9.58	5.05
t_R^9	4.59	2.12	4.91	4.54	4.51	4.94
t_R^{10}	2.20	4.56	5.65	4.38	5.92	4.11

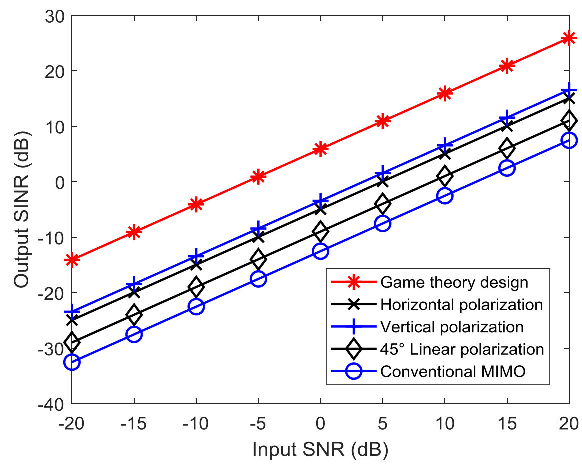

FIGURE 8. The output SINR versus the input SNR (Case 2).

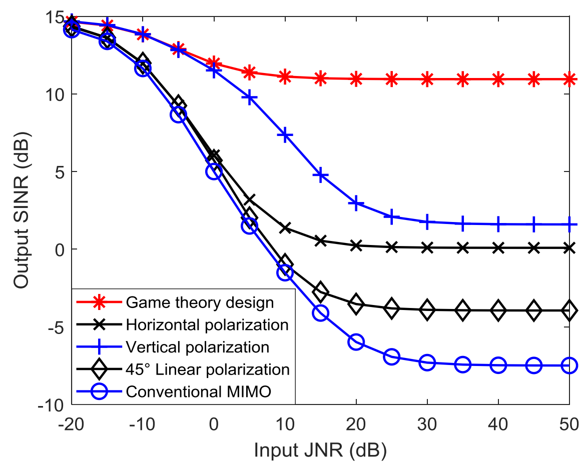

FIGURE 9. The output SINR versus the input JNR (Case 2).

We have four such pairs here; hence, we compute the following constituent SINRs: $\mathrm{SINR_{out}}|(\boldsymbol{t}_J^4, \boldsymbol{t}_R^2)$, $\mathrm{SINR_{out}}|(\boldsymbol{t}_J^4, \boldsymbol{t}_R^5)$, $\mathrm{SINR_{out}}|(\boldsymbol{t}_J^5, \boldsymbol{t}_R^2)$, and $\mathrm{SINR_{out}}|(\boldsymbol{t}_J^5, \boldsymbol{t}_R^5)$. Based on the mixing probabilities of the Nash equilibrium, we define

$$\mathrm{SINR}_{\mathrm{out}} = \sum_{i=4,5}\sum_{j=2,5} P_1(\boldsymbol{t}_J^i)P_2(\boldsymbol{t}_R^j)\mathrm{SINR}_{\mathrm{out}}\Big|(\boldsymbol{t}_J^i, \boldsymbol{t}_R^j). \quad (41)$$

Using this definition, we plot the output SINR versus the input SNR in Fig. 8, from which we observe that the system with the mixed-strategy polarimetric design outperforms the radar systems with only horizontal, vertical, or 45° linear polarizations. Further, Fig. 9 shows the output SINR versus the input JNR of the two FTGs. We note that the mixed-strategy equilibrium solution has a higher SINR for all values of the input JNR than do the other solutions when the input JNR is greater than 0 dB.

VI. CONCLUSION

In this paper, we explore the co-located polarization MIMO radar configuration and study the polarization design problem for deceptive jamming suppression in the main lobe from a game theory perspective. Instead of using identical transmit polarizations, the polarization vector of each element in the transmit array is determined to improve jamming suppression performance. Inspired by the concepts and advantages of game theory, we formulate the polarization design problem for a co-located polarization MIMO radar system as a TPZS game between the radar and the jammers and propose two design methods for different cases. With the designed transmit polarizations, improved jamming suppression performance can be achieved. Real radar data will be used to validate the results presented above in the future. Furthermore, an optimal polarization design algorithm for jamming suppression without prior knowledge of targets and interference will be studied.

In practice, the target polarization scattering characteristics will vary with time, which is not considered in this work. When the possible polarization state of the target is a finite set, we can consider the combination of real target and FTGs as one player and the radar system as the other player in the game, and then the polarization design problem can also be solved following the method presented in this paper. However, when the number of strategies possible for one (each) player is an infinite set, the game will become complex, and some other solutions must be used to solve this problem, such as the ϵ-equilibrium, Stackelberg equilibrium, and saddle-point equilibrium [31], which will be studied in future work.

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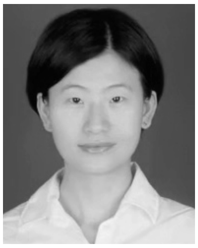

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