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The Strategy Selection Problem on Artificial Intelligence With an Integrated VIKOR and AHP Method Under Probabilistic Dual Hesitant Fuzzy Information

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ABSTRACT Artificial intelligence (AI) is the most popular technology for searching the natural essence of human beings' intelligence. AI is influencing the paradigm of the enterprise management and product optimization. The development of AI in enterprise management provide an opportunity for all enterprises to construct automated business processes, improve the customer experience, and expand the product differentiation. Nowadays, the changeable world increases the level of difficulty to make an appropriate AI strategy for a company because of the information uncertainty and complexity. Probabilistic dual hesitant fuzzy set (PDHFS), which is a very effective tool to handle uncertain information, contains the hesitant fuzzy information and the corresponding probabilistic information. Standing in front of the more and more complex evaluation/selection problems, decision makers (DMs) could express their preference information more flexibly using the probabilistic hesitant fuzzy information. In this paper, we focus on the strategy selection problem in AI and solving it by a proposed integrated AHP and VIKOR method under probabilistic dual hesitant fuzzy information. First, we construct the probabilistic dual hesitant fuzzy comparison matrix (PDHFCM) and propose a specific transformation function for using AHP method under probabilistic dual hesitant fuzzy information. For completing the AHP, we redefine a consistency measure and propose an appropriate information-improved approach to obtain the consistent comparison matrix and the corresponding weight values simultaneously. In addition, we study the properties of PDHFS deeply and propose a new comparison method and a novel distance measure for PDHFS to distinguish the different probabilistic hesitant fuzzy information effectively. Then, we propose an integrated VIKOR and AHP method and use the method to solve the AI strategy selection problem. Finally, the availability and effectiveness of the proposed method are illustrated by a case on AI strategy selection.

INDEX TERMS AHP, artificial intelligence, probabilistic dual hesitant fuzzy comparison matrix, probabilistic dual hesitant fuzzy set, VIKOR method.

I. INTRODUCTION

Artificial Intelligence (AI) is the state-of-the-art science for studying and developing some theories, methods, techniques and applied systems to simulate, stretch and extend the human intelligence. AI is one of the most popular concepts in today's world [1], [2] albeit some problems are inevitable [3]. Almost every enterprise wants develop a long-term AI strategy for the future survival and development. The key is to determine a correct decision in the varied and complicated society. However, AI is not a completely new concept and have been proposed several decades ago. With the development of computer techniques and other information processing technology, AI has been an inevitable tendency in almost

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every aspect of real life. The future of a realm will be caught if a company implements appropriate AI strategy. Thus, it has been a more and more important decision problem to put the AI strategy into effect in a company. Actually, the AI strategy selection problem is a multi-criteria group decision-making (MCGDM). Many decision makers (DMs) are required to give their preferences and/or evaluations, which always depends on the subjective judgments of the DMs due to the imprecise data and uncertain information in hands. Facing the rapidly changing world and endlessly generated data, the uncertainty and probability of loss information are increasing. Due to the limited data processing ability of human-beings, some effective techniques have been used to help people to express, collect, aggregate and compare in the process of decision-making. Thus, some extended forms of fuzzy set [4] have been studied and applied in real decision-making problems. In order to consider different viewpoints (membership degrees and non-membership degrees) in the process of decision-making, the intuitionistic fuzzy set [5] (IFS) was proposed to assist DMs in expressing their evaluations. Considering the hesitant situation when DMs could not make a determination between several values in decision-making, the hesitant fuzzy set [6] (HFS), which provides an information form containing all uncertain information, was proposed and has been widely used in multi-criteria decision-making problems. Combing the advantages of the IFS and the HFS, dual hesitant fuzzy set [7], [8] (DHFS) was proposed to express DMs' evaluations from different viewpoints (membership degrees and non-membership degrees) and with several different crisp values belong to in each viewpoint (the sum of the maximum values in membership degrees and non-membership degrees is less than or equal to one). In fact, the DHFS is suitable for group decision-making to collect all evaluation values of DMs by membership and non-membership degrees. However, in group decision-making, if some DMs give same evaluation values in membership or non-membership, the value will be just recorded once in DHFSs. Obviously, the situation results in a loss of information and maybe influences the final decision. In order to solve the problem and help DMs to express their evaluations more flexibly and accurately, Zhu and Xu [9] added the probability information in HFS and proposed the concept of probability-hesitant fuzzy set (PHFS). Considering the information format of DHFS and the idea of PHFS, combining the advantages of DHFS and PHFS, Hao *et al.* [10] proposed the probabilistic dual hesitant fuzzy set (PDHFS). The PDHFS could contain both the aleatory uncertainty and epistemic uncertainty. According to the information format, a PDHFS is a combination of a DHFS and the corresponding probabilistic information. However, the real effect is that the PDHFS could express the evaluation information completely, especially in the process of group decision-making. Therefore, in this paper, we focus on solving the AI strategy selection problem under probabilistic dual hesitant fuzzy information.

First, we study the comparison method of probabilistic dual hesitant fuzzy element (PDHFE). PDHFE is the preliminary element of PDHFS. The comparison method proposed by Hao *et al.* [10] could not discriminate some probabilistic dual hesitant fuzzy information reasonably under some special situations. The reason is that the comparison method just considers the mean values and standard deviation values of membership degrees and non-membership degrees in two steps respectively. In order to propose a new comparison method overcoming the drawback, a synthetical score function of PDHFE is proposed for distinguishing different PDHFEs accurately. We not only consider the mean values and the standard deviation values in membership degrees and non-membership degrees simultaneously, but also utilize the real preference degree [11] based on orness measure [12] to show the data tendency of people. In addition, considering the fact that people are usually more sensitive to negative evaluations (non-membership degrees), a parameter is added in the synthetical score function to show the attitude of DMs to non-membership degrees. The idea comes from the Prospect theory [13], which shows that people are risk aversion and loss sensitivity. The parameter can be referred to as risk factor or sensitive index. Thus, the synthetical score function could reflect the real value of the PDHFE. Based on the function, we propose a new comparison method to compare different PDHFEs. Then, according to the characteristics of PDHFEs, we propose an equiprobability distance measure. For two different PDHFEs, in the process of getting their equiprobability distance, we use a simple method to let them have same probability distributions. Moreover, in order to consider more information in distance measure, on the strength of the synthetical score function and the equiprobability distance, we propose a general distance measure of PDHFEs. The general distance measure could get the distance value more accurately. To accomplish one of the objectives of this article, the general distance measure and the synthetical score function will be used in the extended VIKOR method.

The VIKOR method is a very useful tool to handle multi-criteria decision-making (MCDM) problems [14]. One of the advantages of the VIKOR method is that the compromise solution could satisfy people's requirement better. The VIKOR method considers three different distances for getting the final compromise solution, the closeness distance, the maximum regret distance and the composite distance. Thus, the VIKOR method could provide a more appropriate decision in MCDM problems. The final decision may be one or several alternatives rather than just one optimal alternative, and that will provide the ultimate DM with more flexible options. Many works have been done to illustrate the advantages of the VIKOR method with some comparisons to other methodologies, such as TOPSIS [15], ELECTRE [16] and some simple additive weighting methods [17]. The VIKOR method has been applied to many different information forms, such as IFS [18], HFS [19] and DHFS [20]. Furthermore, many real problems have been solved by VIKOR method effectively, such as water resources planning [21], water supply to climate change and variability [22] and green supply chain management [23]. Thus, the VIKOR method has been verified to be an available and effective technique for solving MCDM problems.

In this paper, the main contribution is to propose an integrated VIKOR and AHP method under probabilistic dual hesitant fuzzy information for solving MCGDM problems. The AHP is used to determine the relative importance of DMs and criteria. In pair-wise comparison, people could give more accurate and actual evaluations. For completing the approach, we propose the probabilistic dual hesitant fuzzy comparison matrix for expressing all the preferences collected from the DMs. Besides, we define the consistency index of the comparison matrix and its calculation method. An effective method is also proposed for improving the consistency degree if the comparison matrix does not meet the requirement. In the process of calculation method and the improved method for the consistency index, we consider the membership degrees and the non-membership degrees respectively and the corresponding relative importance could be obtained.

The main construction of this paper is listed as follows: Section II contains some necessary concepts such as the DHFS, the PHFS and the PDHFS. In addition, we introduce the traditional approaches of VIKOR method and AHP. In Section III, we propose a synthetical score function and a new comparison method. Besides, based on the synthetical score function and a proposed equiprobability distance measure, a new distance measure is proposed. The main approach, i.e. the integrated VIKOR and AHP method, is proposed in Section IV. In Section V, a case analysis on AI strategy selection is described in detail for showing the effectiveness of the proposed method. Moreover, we use some requisite comparisons to show the advantages and disadvantages of the proposed method. We conclude this paper in Section VI.

II. SOME PRELIMINARY CONCEPTS

Several necessary concepts will be reviewed in this section, such as DHFS, PHFS and PDHFS. Besides, the classical AHP and VIKOR method will also be introduced briefly.

A. DUAL HESITANT FUZZY SET

Considering the complexity of the world, to express the uncertain information from different viewpoints accurately, Zhu et al. introduced the concept of DHFS [7], [8], which contains the characteristics of the HFS [6] and the IFS [5]. The DHFS is constructed by two parts, one is the set of several membership values and another is the set of several non-membership values. The mathematical expression of DHFS could be denoted as:

$$D = \left\{ \left\langle x, \tilde{h}(x), \tilde{g}(x) \right\rangle | x \in X \right\}$$

where D is a DHFS, X is a finite reference set and x is one of the element in X, $\tilde{h}(x)$ and $\tilde{g}(x)$ are the membership function and the non-membership function respectively. The restricted condition of a DHFS is: 1) every value in the membership part or the non-membership part belongs to the interval .[0, 1].; 2) the sum of the maximum value in membership part and the maximum value in the non-membership part is less than or equal to one.

B. PROBABILISTIC HESITANT FUZZY SET

When the decision values are repetitive, the HFS could just use one of them to represent all values. The obvious information missing could influence the final decision in some cases. In order to avoid the missing information, one of the solutions is to import the probabilistic information in hesitant fuzzy information. Therefore, the PHFS was proposed and some properties have been studied [9], [24]. After that, Zhang *et al.* [25] studied a more general probabilistic information form and proposed a weak-PHFE. In this paper, we consider the general PHFS and its mathematical expression could be denoted as:

$$HP = \{\langle x, hp(x)\rangle | x \in X\}$$

where $hp(x)$ is constructed by several paired values as $\gamma\, |p$, X is a finite reference set and x is one of the element in X. γ is one of the membership values respect to x belongs to HP and p is the corresponding probability information. $\gamma \in [0, 1]$, $p \in [0, 1]$ and $\sum_{\gamma|p\in hp(x)} p \leq 1$.

The PHFS just considers the membership. Therefore, for completing the information form, the non-membership part needs to be considered in decision-making process.

C. PROBABILISTIC DUAL HESITANT FUZZY SET

For considering the membership degrees, the non-membership degrees and the probabilistic information simultaneously, Hao et al. proposed the concept of PDHFS [10]. The mathematical expression of PDHFS could be denoted as:

$$PD = \left\{ \left\langle x, \widetilde{hp}(x), \widetilde{gp}(x) \right\rangle | x \in X \right\}$$

where $\widetilde{hp}(x)$ and $\widetilde{gp}(x)$ are the membership and the non-membership functions respectively. $\gamma\, |p^h$ is one of the elements in $\widetilde{hp}(x)$ and $\eta\, |p^g$ is one of the elements in $\widetilde{gp}(x)$. For any $x \in X$, $\gamma, \eta \in [0, 1]$ and $\gamma^+ + \eta^+ \leq 1$, where γ^+ is the maximum value in the membership function and the η^+ is the maximum value in the non-membership function. $\sum p^h \leq 1$ and $\sum p^g \leq 1$. The pair $pd = \langle \widetilde{hp}, \widetilde{gp} \rangle$ is called the probabilistic dual hesitant element (PDHFE).

Some necessary basic operations of PDHFEs were proposed by Hao et al. [1]. Let $pd_1 = \langle \widetilde{hp}_1, \widetilde{gp}_1 \rangle$ and $pd_2 = \langle \widetilde{hp}_2, \widetilde{gp}_2 \rangle$ be any two PDHFEs, then these operations could be:

$$\begin{aligned} pd_1 \oplus pd_2 &= \bigcup_{\substack{\gamma_1 |p_1^h \in \widetilde{hp}_1,\ \eta_1 |p_1^g \in \widetilde{gp}_1, \\ \gamma_2 |p_2^h \in \widetilde{hp}_2,\ \eta_2 |p_2^g \in \widetilde{gp}_2}} \\ &\quad \times \left\langle (\gamma_1 + \gamma_2 - \gamma_1\gamma_2) \left| p_1^h p_2^h, \ \eta_1\eta_2 \right| p_1^g p_2^g \right\rangle \end{aligned}$$

$$\begin{aligned} pd_1 \otimes pd_2 &= \bigcup_{\substack{\gamma_1 \left| p_1^h \in \widetilde{h}p_1, \eta_1 \right| p_1^g \in \widetilde{g}p_1, \\ \gamma_2 \left| p_2^h \in \widetilde{h}p_2, \eta_2 \right| p_2^g \in \widetilde{g}p_2}} \times \left\langle \gamma_1\gamma_2 \left| p_1^h p_2^h, \; (\eta_1 + \eta_2 - \eta_1\eta_2) \right| p_1^g p_2^g \right\rangle \\ \lambda pd_1 &= \bigcup_{\gamma_1 \left| p_1^h \in \widetilde{h}p_1, \eta_1 \right| p_1^g \in \widetilde{g}p_1} \left\langle \left(1 - (1-\gamma_1)^\lambda\right) \left| p_1^h, \; \eta_1^\lambda \right| p_1^g \right\rangle, \\ \lambda &> 0 \end{aligned}$$

Based on the basic operations, Hao *et al.* proposed the probabilistic dual hesitant fuzzy weighted averaging (PDHFWA) operator. Let $(pd_1, pd_2, \cdots, pd_n)$ be any n PDHFEs and $(\omega_1, \omega_2, \cdots, \omega_n)$ be the corresponding weight set. $\omega_j \geq 0$ and $\sum_{j=1}^{n} \omega_j = 1$, $j = 1, 2, \cdots, n$. Then, the PDHFWA could be shown as:

$$PDHFWA\,(pd_1, pd_2, \cdots, pd_n) = \mathop{\oplus}_{j=1}^{n} \omega_j pd_j, \quad j = 1, 2, \cdots, n$$

In order to compare different PDHFEs, Bundy *et al.* [1] proposed a score function and a deviation degree of PDHFE and gave a comparison method of PDHFEs. The score function is the value that equals to the mean value of membership degrees minus the mean value of non-membership degrees. The score function could be denoted as:

$$s\,(pd) = \sum_{\gamma \left| p^h \in \widetilde{h}p \right.} \gamma \cdot p^h - \sum_{\eta \left| p^g \in \widetilde{g}p \right.} \eta \cdot p^g \tag{1}$$

where pd is a PDHFE. In fact, the score function just uses the mean values of the membership degrees and the non-membership degrees of a PDHFE. The deviation degree uses the standard deviation degree to indicate the level of information stability in a PDHFE. The deviation degree could be:

$$\sigma\,(pd) = \left(\sum_{\gamma \left| p^h \in \widetilde{h}p \right.} (\gamma - s\,(pd))^2 \cdot p^h + \sum_{\eta \left| p^g \in \widetilde{g}p \right.} (\eta - s\,(pd))^2 \cdot p^g \right)^{\frac{1}{2}} \tag{2}$$

where pd is a PDHFE. Based on the score function and the deviation degree, the comparison method could be expressed as (let pd_1 and pd_2 be any two PDHFEs):

- If $s\,(pd_1) > s\,(pd_2)$, then pd_1 is superior to pd_2, denoted as $pd_1 \succ pd_2$.
- If $s\,(pd_1) = s\,(pd_2)$, then
  1) If $\sigma\,(pd_1) > \sigma\,(pd_2)$, then pd_1 is inferior to pd_2, denoted as $pd_1 \prec pd_2$;
  2) If $\sigma\,(pd_1) = \sigma\,(pd_2)$, then pd_1 is indifferent to pd_2, denoted as $pd_1 \sim pd_2$.

In fact, the idea of the comparison method is rational because it considers firstly the mean value of PDHFE and then the standard deviation of PDHFE. However, the comparison method is divided to two parts and the deviation degree would be ignored if the score values of two PDHFEs are different. In addition, the difference of the score values may not reflect the real difference of the two PDHFEs. In Eq. (2), the score function value is used as the mean value to calculate the deviation degrees of membership degrees and non-membership degrees. It may be not rational because the score function value is the difference value of the mean value of membership degrees and the mean value of non-membership degrees. The mean values in Eq. (2) should be the mean value of membership degrees and the mean value of non-membership degrees respectively.

Next, we use an example to analyze some drawbacks of the comparison method.

Example 1: Let pd_1, pd_2 and pd_3 be three PDHFEs and

$$\begin{aligned} pd_1 &= \langle (0.5\,|0.5,\; 0.6\,|0.5)\,, (0.2\,|0.5,\; 0.3\,|0.5) \rangle, \\ pd_2 &= \langle (0.4\,|0.25,\; 0.5\,|0.25\;, 0.6\,|0.25,\; 0.7\,|0.25)\,, \\ &\qquad (0.2\,|0.5,\; 0.3\,|0.5) \rangle\,, \\ pd_3 &= \langle (0.5\,|0.4,\; 0.6\,|0.6)\,, \\ &\qquad (0.1\,|0.25,\; 0.2\,|0.25\;, 0.3\,|0.25,\; 0.4\,|0.25) \rangle\,. \end{aligned}$$

- Considering pd_1 and pd_2. According to Eq. (1) and the comparison method, we have their score values $s\,(pd_1) = 0.3$ and $s\,(pd_2) = 0.3$. Due to $s\,(pd_1) = s\,(pd_2)$, then we need to calculate the deviation degrees of pd_1 and pd_2 according to Eq. (2), $\sigma\,(pd_1) = 0.2646$ and $\sigma\,(pd_2) = 0.2828$. Due to $\sigma\,(pd_1) < \sigma\,(pd_1)$, therefore we get $pd_1 \succ pd_2$.
- Considering pd_1 and pd_3. According to Eq. (1), we have their score values $s\,(pd_1) = 0.3$ and $s\,(pd_3) = 0.31$. If we just consider the score function values, then we get $pd_1 \prec pd_3$. But, the deviation degrees of pd_1 and pd_3 can be calculated by Eq. (2), $\sigma\,(pd_1) = 0.2646$ and $\sigma\,(pd_3) = 0.2846$, $\sigma\,(pd_1) < \sigma\,(pd_3)$. If we just consider the deviation degrees, we get $pd_1 \succ pd_3$. Because $|s\,(pd_1) - s\,(pd_3)| = 0.01$, $|\sigma\,(pd_1) - \sigma\,(pd_3)| = 0.02$. Thus, it will be not convincing to sort pd_1 and pd_3 just according the score values when the difference between the deviation degrees is bigger.

According to Example 1, the main drawbacks of the above comparison method could be shown in two aspects.

- First, the multi-steps of comparison method. The score function values and deviation degrees could not be considered simultaneously but in two different steps. In Example 1, the comparison between pd_1 and pd_2 needs two steps consist of the calculation of the score function values and the deviation degrees.
- Second, the score function value could not reflect the real value of PDHFE and the deviation degree will be ignored. In Example 1, $s\,(pd_1) < s\,(pd_3)$ and $\sigma\,(pd_1) < \sigma\,(pd_2)$, therefore we should have not determine the sort of pd_1 and pd_3 according to the comparison method mentioned above. The deviation degrees of them will be ignored if we just consider the score function values.

In this paper, one of our main works is to propose a more effective comparison method that could cover all the

TABLE 1. The 1-9 ratio scale when compare two objects.

Scale	1/9	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1	2	3	4	5	6	7	8	9
Priority degree	The smaller the scale value, the smaller the priority degree.								Equal	The bigger the scale value, the bigger the priority degree.							

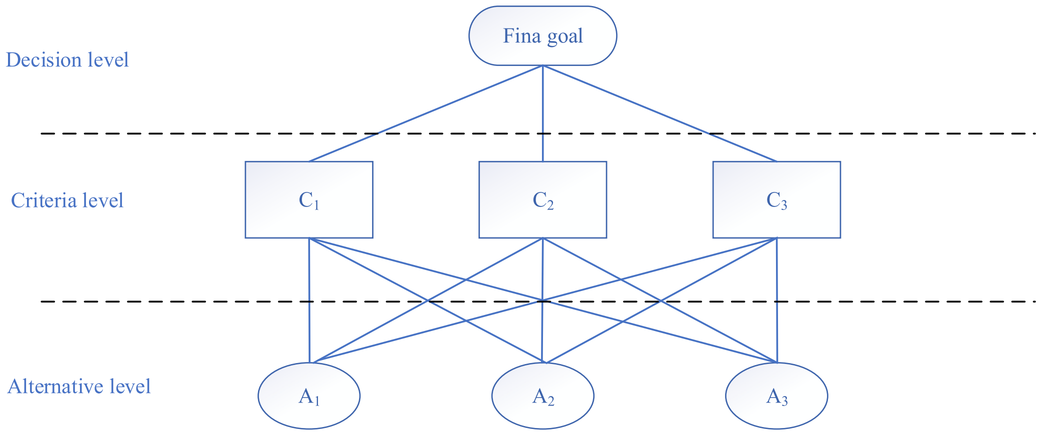

FIGURE 1. The hierarchical structure.

TABLE 2. Random consistency index [6].

m	3	4	5	6	7	8	9	10
RI	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

drawbacks expressed above. In the next section, we will give the detailed development of the new comparison method.

D. SOME MAJOR CONCEPTS ON AHP AND VIKOR

In this paper, the main contribution is to propose an integrated VIKOR and AHP method for solving MCGDM problems. Therefore, the traditional concepts of AHP and VIKOR method are necessary to be introduced in this subsection.

1) THE OUTLINE OF AHP

The basic idea of dealing with a complex problem is to decompose and simplify the problem. Based on pair-wise comparisons, the AHP could get the priorities of multiple objects. Thus, the AHP is one of the most popular methods in MCDM and used to calculate the weights of criteria in general.

The AHP is an approach which stratifies the specific problem and determines the weights of every layer by comparing every two elements in each layer [2]. The AHP could make full use of the people's limited ability that people could give accurate information when comparing two objectives. One of the obvious advantages of AHP is that it builds the hierarchy of the relative problem clearly and divide the original problem into sub-problems, which could be solved easier. From the bottom to the top, the weight of every object could be calculated by the relative comparison matrix. In this subsection, first, we recall the outline of the AHP procedure simply based on Saaty's 1-9 ratio scale [3] (see Table 1).

Step 1: Identify the corresponding problem, such as the final aim, conflicting criteria and alternatives. Then, a hierarchical structure could be constructed as Fig. 1.

Step 2: Based on upper level objects, construct some necessary judgment matrices of bottom level objects. For instance, based on criterion C_1, the judgment matrix can be got according to the pair-wise comparisons among alternatives A_1, A_2 and A_3.

Step 3: Calculate the consistency index $CI = (\lambda_{\max} - m) / (m - 1)$ of each judgment matrix. $\lambda_{\max}$ is the maximum eigenvalue of the judgment matrix and m is the number of the corresponding elements. Then, according to the random consistency index RI in Table 2, we can get the consistency ratio $CR = CI/RI$ of each judgment matrix. Besides, another consistency index called geometric consistency index (GCI) was proposed by Zadeh [4] based on a row geometric mean method [5]. Similarly, the thresholds of GCI for different m can be seen in Table 3.

Step 4: If $CR < 0.1$, then go to next step for getting corresponding weight information. If $CR \geq 0.1$, then the judgment matrix should be adjusted until its consistency ratio is less than 0.1.

TABLE 3. The thresholds of GCI for different *m*.

m	3	4	>4
$\mathrm{GCI}^{(m)}$	0.3147	0.3526	0.37

Step 5: Calculate the weight information of objects in each level. Synthesize the weight information of each alternative to the final goal, then rank all alternatives.

Considering the uncertainty of DMs, Zhu et al. proposed the hesitant analytic hierarchy process, which is based on a hesitant comparison matrix (HCM) [7]. In HCM, DMs could use several scales as in Table 1 to show their preferences.

In this paper, under the probabilistic dual hesitant fuzzy information, we define the comparison matrix (judgement matrix) and propose the adjustment process for decreasing the consistency ratio. The relevant weight information could be obtained in the adjustment process or the process of getting the consistency index. The weight information will be used in the extended VIKOR method for getting accurate decision.

2) THE MAIN IDEA OF VIKOR METHOD

The VIKOR (Serbian name: VIseKriterijumska Optimizacija I Kom-promisno Resenje) method is an effective tool in MCDM. It is also called a compromise ranking method. Its main idea is based on a compromise ranking result and the $\mathrm{L_p}$-metric in compromise programming:

$$L_{p,i} = \left\{ \sum_{j=1}^{n} \left[\omega_j \left(f_j^* - f_{ij} \right) / \left(f_j^* - f_j^- \right) \right]^p \right\}^{1/p} \quad 1 \le p \le \infty, \quad i = 1, 2, \ldots, m$$

where $L_{p,i}$ is regarded as the main fusion function, ω_j is the importance of the jth criterion, f_{ij} is the evaluation information of the ith alternative respect to the jth criterion, f_j^* and f_j^- are the best and the worst evaluation information under the jth criterion. p is the index of the weighted deviation between alternatives and the ideal solution. The set of alternatives is $A = \{a_1, a_2, \ldots, a_m\}$.

The maximum group utility of the majority, which is constructed as $p = 1$, and the minimum individual regret of the opponent, which is constructed as $p = \infty$ are used for getting the final compromise solution. The compromise solution is one or several feasible alternatives which are the closest to the ideal solution [8]. In the traditional VIKOR method, $L_{1,i}$ and $L_{\infty,i}$ are denoted as S_i and R_i respectively. Q_i is constructed by S_i and R_i. The final compromise solution can be determined using series special rules according to the three rank information S_i, R_i and Q_i. The specific procedure of the traditional VIKOR method is shown as follows:

Step 1: f_j^* and f_j^- should be first determined in the beginning of the method. The benefit and the cost criterion should be treated differently.

Step 2: S_i and R_i could be calculated according to the equations:

$$S_i = \sum_{j=1}^{n} \omega_j \left(f_j^* - f_{ij} \right) \Big/ \left(f_j^* - f_j^- \right)$$

$$R_i = \max_j \left[\omega_j \left(f_j^* - f_{ij} \right) \Big/ \left(f_j^* - f_j^- \right) \right]$$

where ω_j is the weight value of the jth criterion and will be determined according to the judgment matrices of AHP method in this paper.

Step 3: Q_i could be calculated according to S_i and R_i:

$$Q_i = \nu \left(S_i - S^- \right) \Big/ \left(S^* - S^- \right) + (1 - \nu) \left(R_i - R^- \right) \Big/ \left(R^* - R^- \right)$$

where $S^- = \min_i S_i$, $S^* = \max_i S_i$, $R^- = \min_i R_i$, $R^* = \max_i R_i$. ν determines the weights of the strategy of maximum group utility and the individual regret.

Step 4: Three different ranking lists of alternatives can be got according to the values S, R and Q in decreasing order.

Step 5: A compromise solution alternative $a^{(1)}$, which has the minimum value in Q, can be determined if it satisfies two conditions:

Condition 1: $Q\left(a^{(2)}\right) - Q\left(a^{(1)}\right) \ge DQ$, where $a^{(2)}$ has the second minimum value in Q and $Q(\cdot)$ indicates the real value of alternative in Q. $DQ = 1/(m-1)$.

Condition 2: $a^{(1)}$ is also the best alternative according to S or R. This will be stable as long as the value ν can be determined.

However, if one of the two conditions could not be satisfied, the compromise solutions have to be proposed. The followings are two kinds of compromise solutions.

- The compromise solutions are $a^{(1)}$ and $a^{(2)}$ if only the condition 2 is not satisfied.
- The compromise solutions are $a^{(1)}$, $a^{(2)}$,..., $a^{(M)}$ if the condition 1 is not satisfied. $a^{(M)}$ satisfies $Q\left(a^{(M)}\right) - Q\left(a^{(1)}\right) < DQ$. The alternatives $a^{(1)}$, $a^{(2)}$,..., $a^{(M)}$ are denoted as indistinguishable.

The VIKOR method could help DMs to express their preferences and provide the compromise solutions that will meet DMs' requirement. Therefore, the VIKOR method is an effective tool in MCDM. Making full use of the characteristics of PDHFS and combining the advantages of the VIKOR method and AHP, in this paper, we propose an integrated VIKOR and AHP method under probabilistic dual hesitant fuzzy information.

III. THE NEW COMPARISON METHOD, DISTANCE MEASURE AND THE PROBABILISTIC DUAL HESITANT FUZZY COMPARISON MATRICES

The comparison of fuzzy information is a preliminary for its applications in real life. Therefore, we first introduce a synthetical score function and then propose a new comparison method to distinguish different probabilistic dual hesitant fuzzy information. The new comparison method could conquer the drawbacks of the comparison method in subsection II.C. Then, based on the synthetical score function and a proposed equiprobability distance measure, we propose a general distance measure of probabilistic dual hesitant fuzzy information.

Facing many alternatives, people could not give an effective order accurately because of the limited ability. However, people are good at comparing two objects and giving an accurate information about which one is better. Thus, under the probabilistic dual hesitant fuzzy information, it is necessary to construct the probabilistic dual hesitant fuzzy comparison matrices (PDHFCMs).

A. THE NEW COMPARISON METHOD OF PROBABILISTIC DUAL HESITANT FUZZY INFORMATION

According to the analysis in subsection II.C, the comparison method proposed by Hao *et al.* would not differentiate PDHFEs in some situations. Hence, we propose a new comparison method based on three main aspects.

- The new comparison method could consider the mean value and the stability of information simultaneously.

For calculation convenient, it is necessary to consider the mean value and the standard deviation degree simultaneously. Besides, in the new comparison method, we consider different calculation method that satisfies people's cognition better.

- The new comparison method involves the data tendency of people.

When a DM gives a data set, such as $(0.7, 0.8, 0.9)$, that indicates the DM has a tendency to one. On contrary, $(0.1, 0.2, 0.3)$ indicates the DM has a tendency to zero. The simple mean value of a data set could not reflect the real preference information that DM wants to express. Thus, in the new comparison method, we use a real preference value to replace the mean value based on orness measure [9].

- The new comparison method contains the sensibility of people to negative information.

Based on the Prospect theory [10], in the new comparison method, we assume that DMs have a high sensibility degree to non-membership degrees (negative information).

We use the real preference degree, which proposed by Ren *et al.* [11], to get the real preference value. Let $\Upsilon = (\gamma_1, \gamma_2, \cdots, \gamma_n)$ be a set of several values belong to $[0, 1]$, the real preference value of Υ is:

$$RPV(\Upsilon) = 2mean(\Upsilon) * rpd\left(\widehat{\Upsilon}\right)$$

where $mean(\Upsilon)$ is the mean value of all values in the set Υ, $rpd(\cdot)$ is the real preference degree in [11], $\widehat{\Upsilon} = \left\{\gamma_i/_{sum(\Upsilon)}, (i = 1, 2, \cdots, n), sum(\Upsilon) = \sum_{i=1}^{n} \gamma_i\right\}$.

First, we give the definition of the synthetical score function of PDHFE, which will be used in the new comparison method.

Definition 2: Let $pd = \langle \widetilde{hp}, \widetilde{gp}\rangle$ be a PDHFE, and the synthetical score function of pd is:

$$ss(pd) = \left(RPV(\gamma) - \delta\left(\gamma \left|p^h\right.\right)\right) - \theta\left(RPV(\eta) - \delta\left(\eta \left|p^g\right.\right)\right) \tag{3}$$

where $RPV(\gamma)$ and $RPV(\eta)$ are the real preference values of the membership degree and the non-membership degree respectively. $\bar{\gamma} = \sum_{\gamma|p^h \in \widetilde{hp}} \gamma \cdot p^h$ and $\bar{\eta} = \sum_{\eta|p^g \in \widetilde{gp}} \eta \cdot p^g$. $\delta\left(\gamma \left|p^h\right.\right)$ and $\delta\left(\eta \left|p^g\right.\right)$ are the standard deviation value of the membership degrees and the non-membership degrees respectively. $\delta(\gamma) = \sqrt{\sum_{\gamma|p^h \in \widetilde{hp}} (\gamma - \bar{\gamma})^2 \cdot p^h}$ and $\delta(\eta) = \sqrt{\sum_{\eta|p^g \in \widetilde{gp}} (\eta - \bar{\eta})^2 \cdot p^g}$. The parameter $\theta (\theta > 1)$ reflects the sensibility degree of DMs to the non-membership degree (negative information). According to the characteristics of PDHFE, considering some special situations such as $pd = \langle (1|1), (0|1)\rangle$ or $pd = \langle (0|1), (1|1)\rangle$, we can easily find the value range of the synthetical score function $[-\theta, 1]$.

We let $\theta = 1.5$ in this paper for calculating conveniently.

Next, we use an example to show the availability of the synthetical score function. In addition, we could find some advantages of the synthetical score function directly from the example.

Example 3: Let pd_1, pd_2 and pd_3 be three PDHFEs and $pd_1 = \langle (0.5|0.5,\ 0.6|0.5), (0.2|0.5,\ 0.3|0.5)\rangle$,

$$\begin{aligned} pd_2 &= \langle (0.4|0.25,\ 0.5|0.25,\ 0.6|0.25,\ 0.7|0.25), \\ &\quad (0.2|0.5,\ 0.3|0.5)\rangle, \\ pd_3 &= \langle (0.5|0.4,\ 0.6|0.6), \\ &\quad (0.1|0.25,\ 0.2|0.25,\ 0.3|0.25,\ 0.4|0.25)\rangle. \end{aligned}$$

Then, according to Definition 2, we can get their synthetical score function values as $ss(pd_1) = 0.325$, $ss(pd_2) = 0.2965$, $ss(pd_3) = 0.4687$ if $\theta = 1.5$. Then, the order of the three PDHFEs is $pd_3 \succ pd_1 \succ pd_2$.

From Example 3, we can find that the result is succinct and clear. The synthetical score function can get the real value of a PDHFE and satisfy people's cognition. Next, we use a sensitive analysis to show the influence of the parameter θ. We let $\theta \in [0, 10]$ though the limitation of θ is equal or greater than one in Definition 2. Let two PDHFEs be $pd_4 = \langle (0.3|0.5,\ 0.4|0.5), (0.4|0.5,\ 0.5|0.5)\rangle$ and $pd_5 = \langle (0.2|0.5,\ 0.3|0.5), (0.2|0.5,\ 0.3|0.5)\rangle$. We can get a figure (Fig. 2) according to their synthetical score function values with the changeable θ. From Fig. 2, we can find that θ could influence the deviation of two PDHFEs. In addition,

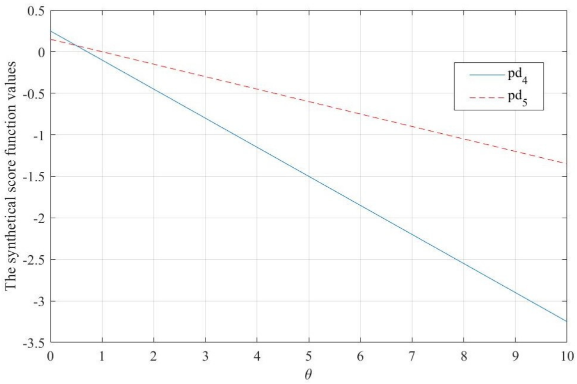

FIGURE 2. The synthetical score function values of pd_4 and pd_5 when $\theta \in [0, 10]$.

when $\theta < 1$, the priority relationship of the two PDHFEs may be changed. Thus, in this paper, we let $\theta > 1$, which indicates that the DM pay more attention to the non-membership (negative information).

The synthetical score function satisfies people's cognition and considers the uniformity (mean value) and stability (standard deviation) of the probabilistic dual hesitant fuzzy information. It is clear and accurate to distinguish two different PDHFEs using the synthetical score function. Thus, based on the synthetical score function, we show the new comparison method as follows:

Definition 4: Let pd_1 and pd_2 be two PDHFEs. Their synthetical score function values are denoted as $ss(pd_1)$, $ss(pd_2)$. If $ss(pd_1) < ss(pd_2)$, then pd_1 is inferior to pd_2, which can be denoted as $pd_1 \prec pd_2$. If $ss(pd_1) = ss(pd_2)$, then pd_1 is indifferent to pd_2, which can be denoted as $pd_1 \sim pd_2$.

In this paper, we use the new comparison method to rank probabilistic dual hesitant fuzzy information in the process of the VIKOR method.

B. THE DISTANCE MEASURE OF PROBABILISTIC DUAL HESITANT FUZZY INFORMATION

Distance measure is a necessary tool in information fusion and information mining for distinguishing different data and determining their real deviations. In order to determine the deviation of different probabilistic dual hesitant fuzzy information and complete the extended VIKOR method, we first give an axiomatic definition on the distance measure of the probabilistic dual hesitant fuzzy information and then, based on the synthetical score function and a proposed equiprobability distance measure, we define a specific distance measure.

The axiomatic definition can be shown as:

Definition 5: Let the distance measure of two PDHFEs pd_1 and pd_2 be $D_{pdhfe}(pd_1, pd_2)$, and pd_3 be another PDHFE. D_{pdhfe} satisfies the rules:

(1) $0 \leq D_{pdhfe}(pd_1, pd_2) \leq 1$;
(2) $D_{pdhfe}(pd_1, pd_2) = D_{pdhfe}(pd_2, pd_1)$;
(3) If the three PDHFEs satisfy $pd_1 \prec pd_2 \prec pd_3$, then $D_{pdhfe}(pd_1, pd_3) > D_{pdhfe}(pd_1, pd_2)$ and $D_{pdhfe}(pd_1, pd_3) > D_{pdhfe}(pd_2, pd_3)$;
(4) If $pd_1 \sim pd_2$, then $D_{pdhfe}(pd_1, pd_2) = 0$.

Before giving the concrete definition of the distance measure, we consider a normalized PDHFE, which the sum of probabilistic information in membership or non-membership is equal to one. Actually, any PDHFE could be transformed to a normalized PDHFE easily. The method is to replace each probabilistic value by the value of the probabilistic value divides the sum of all probabilistic values in membership degrees or non-membership degrees. Mathematically, it is to normalize the probabilistic information in membership degrees and non-membership degrees respectively for getting the normalized PDHFE. A simple example is used to illustrate the normalization method.

Example 6: Let consider a PDHFE pd_1 and it could be expressed as $pd_1 = \langle (0.5|0.3,\ 0.6|0.5), (0.2|0.5,\ 0.3|0.2) \rangle$. The corresponding normalized PDHFE can be got as $pd_1^N = \left\langle \left(0.5\left|\frac{3}{8},\ 0.6\right|\frac{5}{8}\right), \left(0.2\left|\frac{5}{7},\ 0.3\right|\frac{2}{7}\right)\right\rangle$.

Considering two normalized PDHFEs, we attempt to propose an equiprobability distance measure. In the process of the distance measure, we let two PDHFEs have same probability distributions firstly, and then get the distance value according to the corresponding membership and non-membership values. For completing the equiprobability distance measure, we use an algorithm to show the specific calculation. The algorithm is shown as follows:

Input: Any two PDHFEs pd_1 and pd_2.

Output: The value of the equiprobability distance measure between pd_1 and pd_2, denoted as $EPD(pd_1, pd_2)$.

Step 1: Normalize the two PDHFEs and arrange all elements of membership and non-membership as ascending sort according to the membership values and the non-membership values respectively. The two normalized PDHFEs can be shown as:

$$pd_1^N = \left\langle \left(\gamma^{h_1}_{\sigma(1)}\left|p^{h_1}_{\sigma(1)},\ \gamma^{h_1}_{\sigma(2)}\right|p^{h_1}_{\sigma(2)},\ \ldots, \gamma^{h_1}_{\sigma(\#h_1)}\left|p^{h_1}_{\sigma(\#h_1)}\right.\right), \left(\eta^{g_1}_{\sigma(1)}\left|p^{g_1}_{\sigma(1)},\ \eta^{g_1}_{\sigma(2)}\right|p^{g_1}_{\sigma(2)},\ \ldots, \eta^{g_1}_{\sigma(\#g_1)}\left|p^{g_1}_{\sigma(\#g_1)}\right.\right)\right\rangle$$
$$pd_2^N = \left\langle \left(\gamma^{h_2}_{\sigma(1)}\left|p^{h_2}_{\sigma(1)},\ \gamma^{h_2}_{\sigma(2)}\right|p^{h_2}_{\sigma(2)},\ \ldots, \gamma^{h_2}_{\sigma(\#h_2)}\left|p^{h_2}_{\sigma(\#h_2)}\right.\right), \left(\eta^{g_2}_{\sigma(1)}\left|p^{g_2}_{\sigma(1)},\ \eta^{g_2}_{\sigma(2)}\right|p^{g_2}_{\sigma(2)},\ \ldots, \eta^{g_2}_{\sigma(\#g_2)}\left|p^{g_2}_{\sigma(\#g_2)}\right.\right)\right\rangle$$

where $\gamma^{h_1}_{\sigma(j)} < \gamma^{h_1}_{\sigma(k)}$, $\eta^{g_1}_{\sigma(j)} < \eta^{g_1}_{\sigma(k)}$, $\gamma^{h_2}_{\sigma(j)} < \gamma^{h_2}_{\sigma(k)}$, $\eta^{g_2}_{\sigma(j)} < \eta^{g_2}_{\sigma(k)}$ if $j < k$. $\sum_{i=1}^{\#h_1} p^{h_1}_{\sigma(i)} = 1$, $\sum_{i=1}^{\#g_1} p^{g_1}_{\sigma(i)} = 1$, $\sum_{i=1}^{\#h_2} p^{h_2}_{\sigma(i)} = 1$, $\sum_{i=1}^{\#g_2} p^{g_2}_{\sigma(i)} = 1$.

Step 2: We consider the membership degrees and non-membership degrees respectively. In fact, the procedures are the same. Thus, for concise illustration, we just give the procedure of handling membership degrees. Let $H_1 = \left(\gamma^{h_1}_{\sigma(1)}\left|p^{h_1}_{\sigma(1)},\ \gamma^{h_1}_{\sigma(2)}\right|p^{h_1}_{\sigma(2)},\ \ldots, \gamma^{h_1}_{\sigma(\#h_1)}\left|p^{h_1}_{\sigma(\#h_1)}\right.\right)$,

$H_2 = \left(\gamma^{h_2}_{\sigma(1)} \left|p^{h_2}_{\sigma(1)}, \gamma^{h_2}_{\sigma(2)}\right| p^{h_2}_{\sigma(2)}, \ldots, \gamma^{h_2}_{\sigma(\#h_2)} \left|p^{h_2}_{\sigma(\#h_2)}\right.\right)$. Let $DH = \emptyset$, compare the first elements of H_1 and H_2,

- If $p^{h_1}_{\sigma(1)} = p^{h_2}_{\sigma(1)}$, then add the element $\left|\gamma^{h_1}_{\sigma(1)} - \gamma^{h_2}_{\sigma(1)}\right| \left|p^{h_1}_{\sigma(1)}\right.$ or $\left|\gamma^{h_1}_{\sigma(1)} - \gamma^{h_2}_{\sigma(1)}\right| \left|p^{h_2}_{\sigma(1)}\right.$ to the set DH as $DH = \left(\left|\gamma^{h_1}_{\sigma(1)} - \gamma^{h_2}_{\sigma(1)}\right| \left|p^{h_1}_{\sigma(1)}\right.\right)$ or $DH = \left(\left|\gamma^{h_1}_{\sigma(1)} - \gamma^{h_2}_{\sigma(1)}\right| \left|p^{h_2}_{\sigma(1)}\right.\right)$ and delete the first elements of H_1 and H_2.
- If $p^{h_1}_{\sigma(1)} < p^{h_2}_{\sigma(1)}$, then add the element $\left|\gamma^{h_1}_{\sigma(1)} - \gamma^{h_2}_{\sigma(1)}\right| \left|p^{h_1}_{\sigma(1)}\right.$ to the set DH as $DH = \left(\left|\gamma^{h_1}_{\sigma(1)} - \gamma^{h_2}_{\sigma(1)}\right| \left|p^{h_1}_{\sigma(1)}\right.\right)$ and delete the first element of H_1, replace the first element of H_2 with $\gamma^{h_2}_{\sigma(1)} \left|p^{h_2}_{\sigma(1)} - p^{h_1}_{\sigma(1)}\right.$.
- If $p^{h_1}_{\sigma(1)} > p^{h_2}_{\sigma(1)}$, then add the element $\left|\gamma^{h_1}_{\sigma(1)} - \gamma^{h_2}_{\sigma(1)}\right| \left|p^{h_2}_{\sigma(1)}\right.$ to the set DH as $DH = \left(\left|\gamma^{h_1}_{\sigma(1)} - \gamma^{h_2}_{\sigma(1)}\right| \left|p^{h_2}_{\sigma(1)}\right.\right)$ and delete the first element of H_2, replace the first element of H_1 with $\gamma^{h_1}_{\sigma(1)} \left|p^{h_1}_{\sigma(1)} - p^{h_2}_{\sigma(1)}\right.$.

Then, to compare the first elements of H_1 and H_2 until $H_1 = \emptyset$ and $H_2 = \emptyset$. We get the set $DH = (\gamma_1 |p_1, \gamma_2 |p_2, \ldots, \gamma_{\#DH} |p_{\#DH})$, $\#DH$ is the number of elements in DH. In a similar way, we get the set $DG = (\eta_1 |p_1, \eta_2 |p_2, \ldots, \eta_{\#DG} |p_{\#DG})$ ($\#DG$ is the number of elements in DG) according to the non-membership parts of pd_1^N and pd_2^N.

Step 3: Sum the values of multiplying the value and the corresponding probability in DH and DG as $\sum_{i=1}^{\#DH} \gamma_i \cdot p_i$ and $\sum_{i=1}^{\#DG} \eta_i \cdot p_i$. We get the equiprobability distance measure between pd_1 and pd_2, and it could be expressed as $EPD(pd_1, pd_2) = \frac{\sum_{i=1}^{\#DH} \gamma_i \cdot p_i + \sum_{i=1}^{\#DG} \eta_i \cdot p_i}{2}$.

Actually, in the process of computing equiprobability distance measure, the two PDHFEs are adjusted to be with the same probability distributions. It is easily to proof that the equiprobability distance measure satisfies the rules in Definition 5. Below, we use an example to illustrate the process of the equiprobability distance measure clearly.

Example 7: Let two different PDHFEs be $pd_1 = \langle (0.5 |0.3, 0.6 |0.5), (0.2 |0.5, 0.3 |0.2) \rangle$ and $pd_2 = \langle (0.4 |0.1, 0.5 |0.3, 0.6 |0.5), (0.2 |0.5, 0.3 |0.2, 0.4 |0.2) \rangle$. First, we need to normalize the two PDHFEs. The two normalized PDHFEs can be shown as:

$$pd_1^N = \left\langle \left(0.5 \left|\frac{3}{8}, 0.6\right| \frac{5}{8}\right), \left(0.2 \left|\frac{5}{7}, 0.3\right| \frac{2}{7}\right) \right\rangle$$

$$pd_2^N = \left\langle \left(0.4 \left|\frac{1}{9}, 0.5\right| \frac{1}{3}, 0.6 \left|\frac{5}{9}\right.\right), \left(0.2 \left|\frac{5}{9}, 0.3\right| \frac{2}{9}, 0.4 \left|\frac{2}{9}\right.\right) \right\rangle.$$

According to the algorithm above, we get two PDHFEs which have the same probability distributions.

$$pd_1' = \left\langle \left(0.5 \left|\frac{1}{9}, 0.5\right| \frac{19}{72}, 0.6 \left|\frac{5}{72}, 0.6\right| \frac{5}{9}\right), \left(0.2 \left|\frac{5}{9}, 0.2\right| \frac{10}{63}, 0.3 \left|\frac{4}{63}, 0.3\right| \frac{2}{9}\right) \right\rangle$$

$$pd_2' = \left\langle \left(0.4 \left|\frac{1}{9}, 0.5\right| \frac{19}{72}, 0.5 \left|\frac{5}{72}, 0.6\right| \frac{5}{9}\right), \left(0.2 \left|\frac{5}{9}, 0.3\right| \frac{10}{63}, 0.3 \left|\frac{4}{63}, 0.4\right| \frac{2}{9}\right) \right\rangle$$

They are the two PDHFEs. Obviously, the information is the same in pd_1^N and pd_1'. pd_2^N and pd_2' also have the same information. Then, we can get the value of the equiprobability distance measure:

$$\begin{aligned} &EPD(pd_1, pd_2) \\ &= \frac{1}{2}\left(|0.5-0.4| \frac{1}{9} + |0.5-0.5| \frac{19}{72} + |0.6-0.5| \frac{5}{72} \right. \\ &\quad \left. + |0.6-0.6| \frac{5}{9}\right) \\ &\quad + \frac{1}{2}\left(|0.2-0.2| \frac{5}{9} + |0.2-0.3| \frac{10}{63} \right. \\ &\quad \left. + |0.3-0.3| \frac{4}{63} + |0.3-0.4| \frac{2}{9}\right) = 0.0281 \end{aligned}$$

From Example 7, we can see that the equiprobability distance measure can be used to distinguish any two PDHFEs even if the numbers of the elements in membership degrees or non-membership degrees are not equal.

The equiprobability distance measure could cover some extreme situations. If the evaluation information in PDHFEs is $pd_1 = \langle (1 |1), (0 |1) \rangle$, $pd_2 = \langle (0 |1), (1 |1) \rangle$, $pd_3 = \langle (0.5 |1), (0.5 |1) \rangle$, $pd_4 = \langle (0.5 |1), (0 |1) \rangle$, $pd_5 = \langle (0 |1), (0.5 |1) \rangle$. Obviously, the rank order of the five PDHFEs is $pd_1 \succ pd_4 \succ pd_3 \succ pd_5 \succ pd_2$. According to the equiprobability distance measure, the distance values between them are $EPD(pd_1, pd_2) = 1$, $EPD(pd_1, pd_3) = 0.5$, $EPD(pd_1, pd_4) = 0.25$, $EPD(pd_1, pd_5) = 0.75$, $EPD(pd_2, pd_3) = 0.5$, $EPD(pd_2, pd_4) = 0.75$, $EPD(pd_2, pd_5) = 0.25$. Thus, the equiprobability distance measure satisfies people's intuition.

Let us consider the distance measure in a deeper level. DMs may pay more attention on the non-membership degrees just like the parameter θ is given for the non-membership degrees in Definition 2. We want to propose a general distance measure, which could contain the advantages of the equiprobability distance measure and the synthetical score function. The general distance measure will be used to handle the situation when DMs are more sensitive for the negative information (non-membership). The detailed definition of the general distance measure for PDHFEs are shown as follows:

Definition 8: Let pd_1 and pd_2 be any two PDHFEs, then the general distance measure between them can be shown as follows:

$$GD(pd_1, pd_2) = \nu EPD(pd_1, pd_2) + (1-\nu)\frac{|ss(pd_1) - ss(pd_2)|}{1+\theta} \quad (4)$$

where $\nu \in [0, 1]$ is the parameter that indicates the level of DMs' sensitivity to the negative information (i.e. non-membership degrees).

Because the value range of the synthetical score function is $[-\theta, 1]$, $\frac{|ss(pd_1)-ss(pd_2)|}{1+\theta} \in [0, 1]$. Thus, $GD(pd_1, pd_2) \in [0, 1]$. According to the function form of general distance measure, it is easily to proof that the general distance measure satisfies the axiomatic definition in Definition 5.

Obviously, the general distance measure is the convex combination of the equiprobability distance measure and the absolute value of the subtraction between two synthetical score function values. In this paper, considering the general case, we let $\nu = 0.5$.

Next, we use an example to show the detailed calculation and the sensitivity test of the general distance measure.

Example 9: Let two PDHFEs be $pd_1 = \langle (0.5|0.3,\ 0.6|0.5), (0.2|0.5,\ 0.3|0.2) \rangle$ and $pd_2 = \langle (0.4|0.1,\ 0.5|0.3,\ 0.6|0.5), (0.2|0.5,\ 0.3|0.2,\ 0.4|0.2) \rangle$. The equiprobability distance between them is 0.0281 according to Example 7. The synthetical score function values of them are $ss(pd_1) = 0.2935$ and $ss(pd_2) = 0.1892$ when θ=1.5. Then, the general distance measure of them is: $GD(pd_1, pd_2) = 0.5 \times 0.0281 + 0.5 \times \frac{|0.2935-0.1892|}{2.5} = 0.0349$. We get different distance values with the changing parameter ν and the variation tendency can be found in Figure 3.

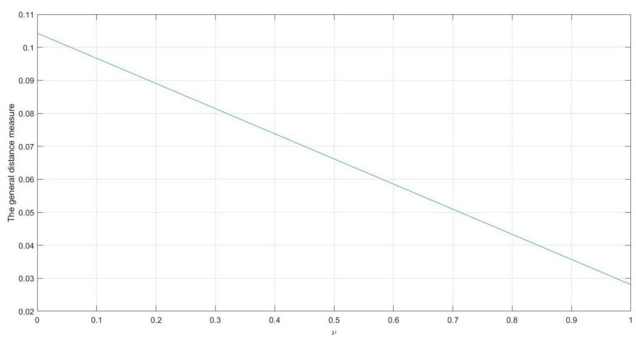

FIGURE 3. **The values of the general distance measure with respect to ν.**

Considering the parameter θ in the synthetical score function, we use a three-dimensional image Figure 4 to show the different distance values with the changing parameter ν and θ. Figure 4 shows that the parameter ν and θ could influence the final distance value simultaneously.

The general distance measure contains the advantages of the equiprobability distance measure and the synthetical score function. The parameter ν and θ provides more flexible options for DMs to show their attitudes in the decision-making method. Thus, the general distance measure could include more factors so that the result could be more accurate and satisfy DM's preference.

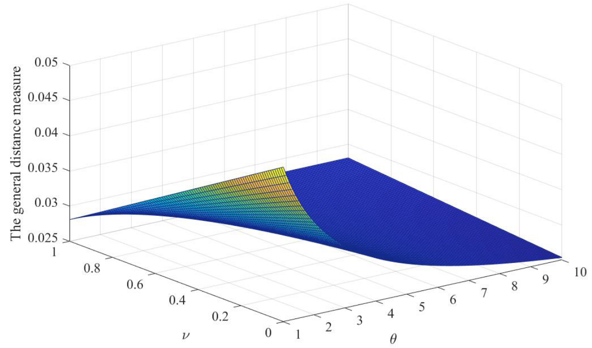

FIGURE 4. **The values of the general distance measure with respect to ν and θ.**

In this paper, we will use this general distance measure for extending the VIKOR method under probabilistic dual hesitant fuzzy information.

C. THE PROBABILISTIC DUAL HESITANT FUZZY COMPARISON MATRIX

In this subsection, we propose the concept of PDHFCM. In addition, we propose a method to calculate the consistency index of the PDHFCM and an approach for improving the consistency index if it could not satisfy the consistent requirement.

Before that, under probabilistic dual hesitant fuzzy information, we first consider the information format when two objects are equal or could not be compared. In this paper, we think that the undifferentiated information should be $\langle (0.5|1), (0.5|1) \rangle$ when two objects could not be distinguished.

Moreover, it is not appropriate to use the probabilistic dual hesitant fuzzy information in AHP method directly. An indirect mode is needed to transform the probabilistic dual hesitant fuzzy information to the 1-9 ratio scale in Table 1. Considering the symmetry of the 1-9 ratio scale and the interval [0, 1], for the purpose of using AHP under probabilistic dual hesitant fuzzy information, we give a specific transformation function according to the convert equations proposed by Yager [12].

Definition 10: Let φ_{pd} be a specific transformation function, which is used to transform [0, 1] to Saaty's 1-9 ratio scale. φ_{pd} satisfies that:

$$\varphi_{pd}(x) = \begin{cases} \dfrac{1}{9-16x}, & x \in [0, 0.5); \\ 1, & x = 0.5; \\ 16x - 7, & x \in (0.5, 1]; \end{cases}$$

In fact, the transformation function is a map between [0, 1] and $\left[\frac{1}{9}, 9\right]$.

In the next part, we use the transformation function to transform the PDHFCM to the probabilistic dual hesitant comparison matrix (PDHCM). First, we introduce the concepts of PDHFCM and the PDHCM.

Actually, the PDHFCM is a matrix consists of PDHFEs. The concept of the PDHFCM is shown as follows:

Definition 11: Let $X=\{x_1,x_2,\ldots,x_n\}$ be the fixed reference set, then a PDHFCM on X is defined as $PDHFCM_{pd}=\left(pd_{ij}\right)_{n\times n}$, where $pd_{ij}(i\leq j)$ is a probabilistic dual hesitant fuzzy element representing the comparison information of x_i over x_j. In the PDHFCM, pd_{ij} satisfies the following conditions: $\gamma_{ij}^{\sigma(l)}+\gamma_{ji}^{\sigma(l)}=1$, $\gamma_{ii}=0.5$, $\#\widetilde{hp}_{ij}=\#\widetilde{hp}_{ji}$, $\left(p^h\right)_{ij}^{\sigma(l)}=\left(p^h\right)_{ji}^{\sigma(l)}$, $\eta_{ij}^{\sigma(l)}+\eta_{ji}^{\sigma(l)}=1$, $\eta_{ii}=0.5$, $\#\widetilde{gp}_{ij}=\#\widetilde{gp}_{ji}$, $(p^g)_{ij}^{\sigma(l)}=(p^g)_{ji}^{\sigma(l)}$, $i,j=1,2,\cdots,n$, where $\sigma(l)$ denotes the lth minimum element in the set, $\#\widetilde{hp}$ and $\#\widetilde{gp}$ represent the element number in membership degrees and non-membership degrees respectively.

In many conditions, the element $pd_{ij}(i>j)$ may could not satisfy the strict information form of PDHFE. For instance, $pd_{12}=\langle(0.2|0.3,\ 0.3|0.6),(0.6|0.4,\ 0.7|0.3)\rangle$, but the corresponding element is $pd_{21}=\langle(0.8|0.3,\ 0.7|0.6),(0.4|0.4,\ 0.3|0.3)\rangle$, which is not a PDHFE because of the sum of the maximum value in membership degrees and the maximum value in non-membership degrees is bigger than one. We call such element patulous probabilistic dual hesitant fuzzy element (PPDHFE). In addition, the PPDHFEs in the PDHFCM could not influence the consistency of the PDHFCM due to the transformation function in Definition 10 and we consider the consistency of membership degrees and non-membership degrees separately.

The PDHCM is like a PDHFCM, but the membership values and the non-membership values are replaced by some values belong to $\left[\frac{1}{9},9\right]$.

Definition 12: Let $X=\{x_1,x_2,\ldots,x_n\}$ be the fixed reference set, then a PDHCM on X can be defined as $PDHCM_{\widetilde{pd}}=\left(\widetilde{pd}_{ij}\right)_{n\times n}$, where

$$\widetilde{pd}_{ij}=\left\langle\left(\tilde{\gamma}_{ij}^{\sigma(1)}\left|\left(p^h\right)_{ij}^{\sigma(1)},\tilde{\gamma}_{ij}^{\sigma(2)}\right|\left(p^h\right)_{ij}^{\sigma(2)},\ldots,\tilde{\gamma}_{ij}^{\sigma(\#h)}\left|\left(p^h\right)_{ij}^{\sigma(\#h)}\right.\right),\left(\tilde{\eta}_{ij}^{\sigma(1)}\left|\left(p^g\right)_{ij}^{\sigma(1)},\tilde{\eta}_{ij}^{\sigma(2)}\right|\left(p^g\right)_{ij}^{\sigma(2)},\ldots,\tilde{\eta}_{ij}^{\sigma(\#g)}\left|\left(p^g\right)_{ij}^{\sigma(\#g)}\right.\right)\right\rangle.$$

$\#h$ and $\#g$ indicate the number of the membership degrees and non-membership degrees respectively, $\sigma(i)$ denotes the ith minimum element in the set. $\tilde{\gamma}\in\left[\frac{1}{9},9\right]$, $\tilde{\eta}\in\left[\frac{1}{9},9\right]$, $\tilde{\gamma}$ and $\tilde{\eta}$ are transformed by corresponding γ and η in corresponding PDHFCM. Thus, $\widetilde{pd}_{ij}$ satisfies the conditions: $\tilde{\gamma}_{ij}^{\sigma(k)}\cdot\tilde{\gamma}_{ji}^{\sigma(k)}=1$, $\tilde{\gamma}_{ii}=1$, $\tilde{\eta}_{ij}^{\sigma(k)}\cdot\tilde{\eta}_{ji}^{\sigma(k)}=1$, $\tilde{\eta}_{ii}=1$ and the corresponding probabilistic information is same to the PDHFCM.

If we just consider the membership degrees or the non-membership degrees in a PDHCM, then we get a hesitant comparison matrix [7] (HCM). Thus, we get two HCMs by splitting a PDHCM according to its membership and non-membership parts. For distinguishing the two comparison matrices, the two HCMs comes from the membership part and the non-membership part are denoted as the membership hesitant comparison matrix (MHCM) and the non-membership hesitant comparison matrix (NHCM) respectively. Then, we use an example to show a PDHFCM, the transformation process and the corresponding PDHCM.

Example 13: For a set of objectives $X=\{x_1,x_2,x_3\}$, some DMs compare any two of them and provide their preferences. We use PDHFEs to collect the preferences. Then, a PDHFCM can be constructed as shown at the bottom of this page.

The normalized PDHFCM can be got according to the normalization method in Subsection III.B as shown at the top of the next page.

According to Definition 10, we only transform the membership values and the non-membership values of the normalized PDHFCM, the corresponding probabilistic information remains unchanged. After that, we can use the transformation function in Definition 10 to transform the normalized PDHFCM to PDHCM. We use one of the PDHFEs to illustrate the transformation process. In $\left\langle\left(0.2\left|\frac{1}{3},\ 0.3\right|\frac{2}{3}\right),\left(0.6\left|\frac{4}{7},\ 0.7\right|\frac{3}{7}\right)\right\rangle$, we can get $\varphi_{pd}(0.2)=\frac{5}{29}$, $\varphi_{pd}(0.3)=\frac{5}{21}$, $\varphi_{pd}(0.6)=\frac{13}{5}$, $\varphi_{pd}(0.7)=\frac{21}{5}$. Therefore, the result is $\left\langle\left(\frac{5}{29}\left|\frac{1}{3},\ \frac{5}{21}\right|\frac{2}{3}\right),\left(\frac{13}{5}\left|\frac{4}{7},\frac{21}{5}\right|\frac{3}{7}\right)\right\rangle$. Based on the similar calculation, separating the membership and the non-membership parts of the PDHCM, we can get the MHCM and the NHCM as shown at the top of the next page. Obviously, we can get the PDHCM by combining the MHCM and the NHCM.

Now, let us consider the consistency index of the PDHFCM. From Definition 10 and Example 13, we can

$$PDHFCM_{pd}=\begin{Bmatrix}\langle(0.5|1),(0.5|1)\rangle & \left\langle\begin{matrix}(0.2|0.3,\ 0.3|0.6),\\(0.6|0.4,\ 0.7|0.3)\end{matrix}\right\rangle & \left\langle\begin{matrix}(0.1|0.6,\ 0.2|0.2,\ 0.3|0.1),\\(0.7|0.4,\ 0.8|0.4)\end{matrix}\right\rangle\\ \left\langle\begin{matrix}(0.8|0.3,\ 0.7|0.6),\\(0.4|0.4,\ 0.3|0.3)\end{matrix}\right\rangle & \langle(0.5|1),(0.5|1)\rangle & \left\langle\begin{matrix}(0.5|0.6,\ 0.6|0.4),\\(0.2|0.3,\ 0.3|0.5,\ 0.4|0.1)\end{matrix}\right\rangle\\ \left\langle\begin{matrix}(0.9|0.6,\ 0.8|0.2,\ 0.7|0.1),\\(0.3|0.4,\ 0.2|0.4)\end{matrix}\right\rangle & \left\langle\begin{matrix}(0.3|0.6,\ 0.2|0.4),\\(0.8|0.3,\ 0.7|0.5,\ 0.6|0.1)\end{matrix}\right\rangle & \langle(0.5|1),(0.5|1)\rangle\end{Bmatrix}$$

$$PDHFCM_{pd}^{N}=\left\{\begin{matrix} \langle (0.5\,|1),(0.5\,|1)\rangle & \left\langle \begin{matrix}\left(0.2\left|\frac{1}{3},\ 0.3\right|\frac{2}{3}\right),\\ \left(0.6\left|\frac{4}{7},\ 0.7\right|\frac{3}{7}\right)\end{matrix}\right\rangle & \left\langle \begin{matrix}\left(0.1\left|\frac{2}{3},\ 0.2\right|\frac{2}{9}0.2\ ,0.3\left|\frac{1}{9}\right.\right),\\ \left(0.7\left|\frac{1}{2},\ 0.8\right|\frac{1}{2}\right)\end{matrix}\right\rangle \\ \left\langle \begin{matrix}\left(0.8\left|\frac{1}{3},\ 0.7\right|\frac{2}{3}\right),\\ \left(0.4\left|\frac{4}{7},\ 0.3\right|\frac{3}{7}\right)\end{matrix}\right\rangle & \langle (0.5\,|1),(0.5\,|1)\rangle & \left\langle \begin{matrix}\left(0.5\left|\frac{3}{5},\ 0.6\right|\frac{2}{5}\right),\\ \left(0.2\left|\frac{1}{3},\ 0.3\right|\frac{5}{9}\ ,0.4\left|\frac{1}{9}\right.\right)\end{matrix}\right\rangle \\ \left\langle \begin{matrix}\left(0.9\left|\frac{2}{3},\ 0.8\right|\frac{2}{9}\ ,0.7\left|\frac{1}{9}\right.\right),\\ \left(0.3\left|\frac{1}{2},\ 0.2\right|\frac{1}{2}\right)\end{matrix}\right\rangle & \left\langle \begin{matrix}\left(0.3\left|\frac{3}{5},\ 0.2\right|\frac{2}{5}\right),\\ \left(0.8\left|\frac{1}{3},\ 0.7\right|\frac{5}{9}\ ,0.6\left|\frac{1}{9}\right.\right)\end{matrix}\right\rangle & \langle (0.5\,|1),(0.5\,|1)\rangle \end{matrix}\right\}$$

$$MHCM=\left\{\begin{matrix} (1\,|1) & \left(\frac{5}{29}\left|\frac{1}{3},\ \frac{5}{21}\right|\frac{2}{3}\right) & \left(\frac{5}{37}\left|\frac{2}{3},\ \frac{5}{29}\right|\frac{2}{9},\ \frac{5}{21}\left|\frac{1}{9}\right.\right) \\ \left(\frac{29}{5}\left|\frac{1}{3},\ \frac{21}{5}\right|\frac{2}{3}\right) & (1\,|1) & \left(\frac{21}{5}\left|\frac{3}{5},\ \frac{29}{5}\right|\frac{2}{5}\right) \\ \left(\frac{37}{5}\left|\frac{2}{3},\ \frac{29}{5}\right|\frac{2}{9},\ \frac{21}{5}\left|\frac{1}{9}\right.\right) & \left(\frac{5}{21}\left|\frac{3}{5},\ \frac{5}{29}\right|\frac{2}{5}\right) & (1\,|1) \end{matrix}\right\}$$

$$NHCM=\left\{\begin{matrix} (1\,|1) & \left(\frac{13}{5}\left|\frac{4}{7},\ \frac{21}{5}\right|\frac{3}{7}\right) & \left(\frac{21}{5}\left|\frac{1}{2},\ \frac{29}{5}\right|\frac{1}{2}\right) \\ \left(\frac{5}{13}\left|\frac{4}{7},\ \frac{5}{21}\right|\frac{3}{7}\right) & (1\,|1) & \left(\frac{5}{29}\left|\frac{1}{3},\ \frac{5}{21}\right|\frac{5}{9},\ \frac{5}{13}\left|\frac{1}{9}\right.\right) \\ \left(\frac{5}{21}\left|\frac{1}{2},\ \frac{5}{29}\right|\frac{1}{2}\right) & \left(\frac{29}{5}\left|\frac{1}{3},\ \frac{21}{5}\right|\frac{5}{9},\ \frac{13}{5}\left|\frac{1}{9}\right.\right) & (1\,|1) \end{matrix}\right\}.$$

find that the transformation process is reversible. That is to say, the PDHFCM is equivalent to the transformed PDHCM. Thus, we just need to calculate the consistency index of the relative PDHCM. The PDHCM is divided into the MHCM and the NHCM according to the membership part and the non-membership part. Therefore, the PDHCM could satisfy the consistency requirement as long as the relative MHCM and NHCM satisfy the consistency requirement. We need to calculate the consistency indices of the two HCMs and improve the consistency indices if necessary.

The HCM has the same data form as the MHCM and the NHCM does. Moreover, the consistency checking algorithm could provide the priorities of relative objects which is very important for completing our decision-making method. Hence, we propose the consistency checking algorithm and consistency improving algorithm for calculating and improving the consistency indices of the MHCM and the NHCM based on the idea of [7]. In order to simplify the calculation, we give relatively accurate iterations in the consistency algorithm.

Actually, DMs consider problems from two different viewpoints under probabilistic dual hesitant fuzzy information directly. One problem should be noted that, the evaluations in non-membership degrees comes from a contrary viewpoint against membership degrees. Therefore, we should consider the consistency index of the transpose of the NHCM. Then, we give the detailed algorithm steps in Algorithm 1. Besides, due to the symmetry property of the comparison matrix, we just need to consider the upper triangular matrix in the process of consistency checking and improving.

In Algorithm 1, an expected geometric consistency index (EGCI) indicates the consistency index of the MHCM or the NHCM. Let a HCM be $(hc_{ij})_{n\times n}$, $\#hc_{ij}$ is the number of elements in hc_{ij}.

If EGCI<GCI$^{(n)}$ (the concrete value can be found in Table 3), then the HCM is consistency; otherwise, the HCM is inconsistency. The consistency improving is unescapable for the inconsistent comparison matrix. We propose a consistency improving algorithm for the inconsistent comparison matrix. The specific procedure can be found in Algorithm 2.

According to Algorithm 1 and Algorithm 2, we get the consistent MHCM and the transpose of NHCM. In addition, the corresponding priorities (weight values) of objects based on the consistent MHCM and the consistent transpose of the NHCM can be got as per Algorithm 3 and they are denoted as $\omega^M=(\omega_1^M,\omega_2^M,\ldots,\omega_n^M)$ and $\omega^N=(\omega_1^N,\omega_2^N,\ldots,\omega_n^N)$. We call ω^M membership weighting vector and ω^N non-membership weighting vector.

In real decision-making, DMs may pay different attentions on membership degrees and non-membership degrees.

Algorithm 1 Checking the Consistency Index

Input: An HCM, which is constructed from the membership (MHCM) or the transpose of the non-membership (NMHCM) of a PDHFCM.

Output: The EGCI.

Step 1. The upper limit value of iteration is $P \geq 3\prod_{i=1}^{n}\prod_{j=1}^{n}\#hc_{ij}$ (the bigger the value of P, the more accurate the value of EGCI). The initial values of iteration $\rho = 1$ and EGCI=0.

Step 2. If $\rho \leq$ P, a stochastically comparison matrix $C = \left(c_{ij}\right)_{n\times n}$ can be constructed according the probability distribution in the HCM $\left(hc_{ij}\right)_{n\times n}$. Otherwise, go to Step 5.

Step 3. The priorities of objectives ω_i $(i = 1, 2, \cdots, n)$ can be calculated using the row geometric mean method [27]. Then, we can get the GCI:

$$\omega_i = \frac{\left(\prod_{j=1}^{n} c_{ij}\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} c_{ij}\right)^{\frac{1}{n}}},$$

$$GCI_c = \frac{2}{(n-1)(n-2)}\sum_{i<j}\log^2 \varepsilon_{ij},$$

$$\varepsilon_{ij} = c_{ij}\frac{\omega_j}{\omega_i}$$

Step 4. Let EGCI=EGCI+GCI, $\rho = \rho + 1$. Go to Step 2.

Step 5. Let EGCI=EGCI/P.

Let $\phi \in [0, 1]$ be the attention level of DMs to membership degrees and $1 - \phi$ be the attention level of DMs to non-membership degrees, then the synthetical weighting vector of objects are $\omega^S = \phi\omega^M + (1-\phi)\,\omega^N$. In this paper, ω^S is regarded as the synthetical weighting vector of relative objects. For the purpose of illustration, we let $\phi = 0.5$ in numerical examples.

Next, an example is used to show the concrete calculation process.

Example 14: We use the MHCM and the NHCM in Example 13. Based on the probabilistic information in MHCM and NHCM, we can get some stochastic comparison matrices. Two stochastic comparison matrices from the MHCM and the NHCM might be as follows:

$$CM_{MHCM} = \begin{Bmatrix} 1 & \frac{5}{21} & \frac{5}{29} \\ \frac{21}{5} & 1 & \frac{21}{5} \\ \frac{29}{5} & \frac{5}{21} & 1 \end{Bmatrix},$$

$$CM_{NHCM} = \begin{Bmatrix} 1 & \frac{13}{5} & \frac{29}{5} \\ \frac{5}{13} & 1 & \frac{5}{21} \\ \frac{5}{29} & \frac{21}{5} & 1 \end{Bmatrix}.$$

Algorithm 2 Improving the Consistency Index

Input: The inconsistent comparison matrix HCM $\left(hc_{ij}\right)_{n\times n}$. The HCM is the inconsistent MHCM or the transpose of the inconsistent NHCM.

Output: An HCM satisfies the consistency requirement.

Step 1: According to Algorithm 1, we can get the EGCI of the HCM.

Step 2: If EGCI>GCI$^{(n)}$, then a stochastically comparison matrix $C = \left(c_{ij}\right)_{n\times n}$ can be constructed according the probability distribution in the HCM $\left(hc_{ij}\right)_{n\times n}$; otherwise, go to Step 10.

Step 3: Calculate the maximum eigenvalue $\lambda_{\max}$ of C and the corresponding normalized eigenvector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ according to the equation:

$$\omega C = \omega\lambda_{\max}, \quad \sum_{i=1}^{n}\omega_i = 1.$$

Step 4: Calculate the consistency ratio $CR_C = \frac{(\lambda_{\max}-n)}{(n-1)RI^{(n)}}$, where $RI^{(n)}$ is the relative RI in Table 2.

Step 5: If $CR_C < 0.1$, then go to Step 8; otherwise, go to Step 6.

Step 6: We can get $C' = \left(c'_{ij}\right)_{n\times n}$ according to $c'_{ij} = c_{ij}^{\vartheta}\left(\frac{\omega_i}{\omega_j}\right)^{1-\vartheta}$, $\vartheta \in [0, 1]$. In this paper, we let $\vartheta = 0.5$.

Step 7: Let $C = C'$, then go to Step 3.

Step 8: We get the comparison matrix $C'' = C$, which satisfies the consistent requirement.

Step 9: Use the values c''_{ij} replaces the corresponding value in hc_{ij}, then we get a new HCM $\left(hc'_{ij}\right)_{n\times n}$. Let $\left(hc_{ij}\right)_{n\times n} = \left(hc'_{ij}\right)_{n\times n}$, then go to Step 1.

Step 10: Output $\left(hc_{ij}\right)_{n\times n}$.

According to Algorithm 1, the consistency index could be calculated, $EGCI\,(MHCM) = 0.9204$ and $EGCI\,(NHCM) = 0.9114$. We can see that $0.9204 > 0.3147$ and $0.9114 > 0.3147$, where 0.3147 is the threshold value in Table 3 when $m = 3$. Thus, the consistency improving process is necessary. According to Algorithm 2, we get the consistent MHCM and NHCM, which can be shown as shown at the top of the next page. and their consistency indexes are 0.1937 and 0.2248 respectively. According to Algorithm 3, the membership weight vector and the non-membership weight vector could be calculated. $\omega^M = (0.0746, 0.6737, 0.2517)$ and $\omega^N = (0.1001, 0.6431, 0.2568)$. If we let $\phi = 0.5$ (the attention levels of DMs to membership degrees and non-membership degrees are equal), we get the synthetical weighting vector $\omega^S = (0.0873, 0.6584, 0.2542)$. On account of the probability distribution of original information, we may get different final results in different calculation processes. In spite of this, the final weighting vector can reflect the real priority steadily.

$$MHCM_{consistent} = \begin{Bmatrix} (1\,|1) & \left(\frac{635}{6233}\Big|\frac{1}{3}, \frac{164}{1091}\Big|\frac{2}{3}\right) & \left(\frac{677}{2391}\Big|\frac{2}{3}, \frac{268}{1215}\Big|\frac{2}{9}, \frac{5}{21}\Big|\frac{1}{9}\right) \\ \left(\frac{6233}{635}\Big|\frac{1}{3}, \frac{1091}{164}\Big|\frac{2}{3}\right) & (1\,|1) & \left(\frac{5725}{2293}\Big|\frac{3}{5}, \frac{768}{211}\Big|\frac{2}{5}\right) \\ \left(\frac{2391}{677}\Big|\frac{2}{3}, \frac{1215}{268}\Big|\frac{2}{9}, \frac{21}{5}\Big|\frac{1}{9}\right) & \left(\frac{2293}{5725}\Big|\frac{3}{5}, \frac{211}{768}\Big|\frac{2}{5}\right) & (1\,|1) \end{Bmatrix}$$

$$NHCM_{consistent} = \begin{Bmatrix} (1\,|1) & \left(\frac{2678}{625}\Big|\frac{4}{7}, \frac{1261}{189}\Big|\frac{3}{7}\right) & \left(\frac{1111}{336}\Big|\frac{1}{2}, \frac{1635}{581}\Big|\frac{1}{2}\right) \\ \left(\frac{625}{2678}\Big|\frac{4}{7}, \frac{189}{1261}\Big|\frac{3}{7}\right) & (1\,|1) & \left(\frac{701}{3251}\Big|\frac{1}{3}, \frac{496}{941}\Big|\frac{5}{9}, \frac{5}{13}\Big|\frac{1}{9}\right) \\ \left(\frac{336}{1111}\Big|\frac{1}{2}, \frac{581}{1635}\Big|\frac{1}{2}\right) & \left(\frac{3251}{701}\Big|\frac{1}{3}, \frac{941}{469}\Big|\frac{5}{9}, \frac{13}{5}\Big|\frac{1}{9}\right) & (1\,|1) \end{Bmatrix}$$

Algorithm 3 Calculating the Weight Vectors

Input: A consistent HCM $\left(hc_{ij}\right)_{n\times n}$.

Output: The mean weight values of objects.

Step 1: The upper limit value of iteration $P \geq 3\prod_{i=1}^{n}\prod_{j=1}^{n}\#hc_{ij}$. The initial values of iteration $\rho = 1$. The initial priorities (weight values) of objects $(\omega_1 = 0, \omega_2 = 0, \ldots, \omega_n = 0)$.

Step 2: If $\rho \leq P$, a stochastically comparison matrix $C = \left(c_{ij}\right)_{n\times n}$ can be constructed according the probability distribution in the HCM $\left(hc_{ij}\right)_{n\times n}$. Otherwise, go to step 4.

Step 3: The priorities of objectives $\omega_i (i = 1, 2, \cdots, n)$ can be calculated using the row geometric mean method. $\omega_i' = \frac{\left(\prod_{j=1}^{n} c_{ij}\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} c_{ij}\right)^{\frac{1}{n}}}$. Then, $\omega_i = \omega_i + \omega_i' \quad i = 1, 2, \ldots, n$. Go to step 2.

Step 4: The mean weight values can be got as per $\omega_i = \frac{\omega_i}{P} i = 1, 2, \ldots, n$.

To get a bird's-eye view of this subsection, we use Figure 5 to show the outline of all the procedure about the PDHFCM and its consistency problem.

IV. AN INTERGRATED VIKOR AND AHP METHOD

For solving MCGDM, based on the proposed distance measure and the comparison matrix of probabilistic dual hesitant fuzzy information, we propose an integrated VIKOR and AHP method in this section. The AHP is mainly used to determine the weights of DMs and criteria. The VIKOR method is mainly used to rank alternatives. Next, we give the detailed procedures of the proposed method.

Let us specify the considered MCGDM problem. K relative experts are invited as the DMs. The m alternatives are $a_1, a_2, \cdots, a_m$. They will be evaluated under n conflicting criteria $(c_1, c_2, \cdots, c_n)$. The weights of the DMs and the criteria are completely unknown. Thus, the AHP will be used to determine the weights of the DMs and the importance of the criteria.

Due to the characteristic of group decision-making and the inevitable uncertainty of DMs to support and oppose in the evaluating process, the PDHFSs are used to collect all subjective judgments. Many necessary comparison matrices will be got for calculating the weights of the DMs and the importance of the criteria.

For each two DMs, the administrator, who will make the final decision, could give his/her preferences among all DMs according to their professional degrees. Then, the comparison matrix can be constructed by collecting all preferences as follows:

$$PI^{DM} = \begin{Bmatrix} pd_{11}^{DM} & pd_{12}^{DM} & \cdots & pd_{1k}^{DM} \\ pd_{21}^{DM} & pd_{22}^{DM} & \cdots & pd_{2k}^{DM} \\ \vdots & \vdots & & \vdots \\ pd_{k1}^{DM} & pd_{k2}^{DM} & \cdots & pd_{kk}^{DM} \end{Bmatrix}$$

For each two criteria, DMs will give their preferences which could be collected in a comparison matrix as follows:

$$PI^{C} = \begin{Bmatrix} pd_{11}^{C} & pd_{11}^{C} & \cdots & pd_{1n}^{C} \\ pd_{21}^{C} & pd_{22}^{C} & \cdots & pd_{2n}^{C} \\ \vdots & \vdots & & \vdots \\ pd_{n1}^{C} & pd_{n2}^{C} & \cdots & pd_{nn}^{C} \end{Bmatrix}$$

Using Algorithm 1, Algorithm 2 (if necessary) and Algorithm 3, the weights of DMs $(\zeta_1, \zeta_2, \cdots, \zeta_K)$ and the importance of criteria $(\omega_1, \omega_2, \cdots, \omega_n)$ could be determined based on the two comparison matrices.

Collecting all evaluation information of each DM to all alternatives with respect to the criteria, the decision matrix could be constructed $(k = 1, 2, \cdots, K)$.

$$M^{DM_k} = \begin{Bmatrix} pd_{11}^{DM_k} & pd_{12}^{DM_k} & \cdots & pd_{1n}^{DM_k} \\ pd_{21}^{DM_k} & pd_{22}^{DM_k} & \cdots & pd_{2n}^{DM_k} \\ \vdots & \vdots & & \vdots \\ pd_{m1}^{DM_k} & pd_{m2}^{DM_k} & \cdots & pd_{mn}^{DM_k} \end{Bmatrix}$$

Thus, we get K decision matrices and then use the basic aggregation operator [1] to aggregate all matrices.

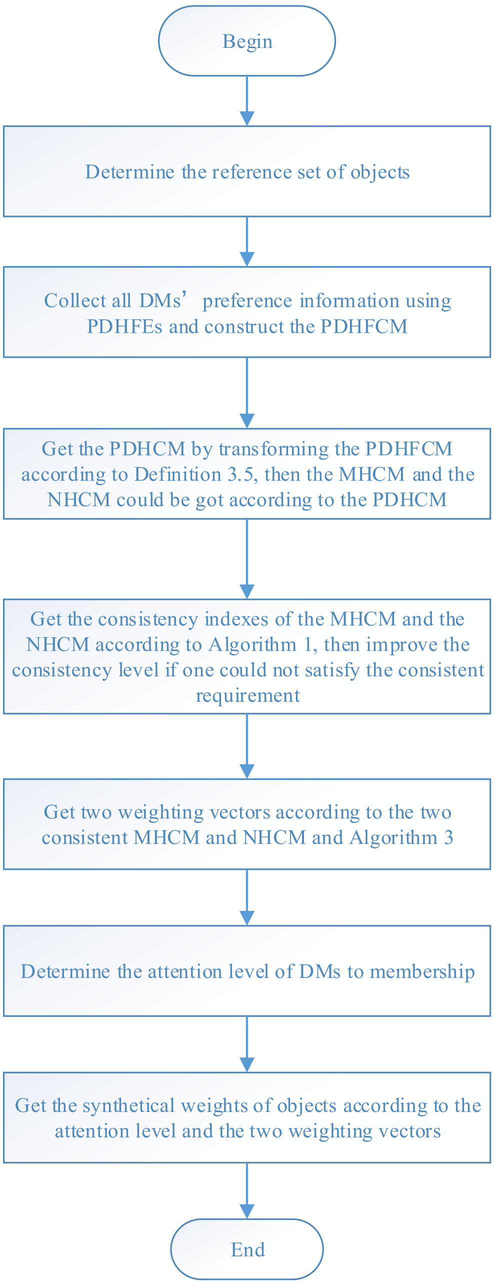

FIGURE 5. The main procedures of the subsection III.C.

The weights of DMs are used in the basic aggregation operator. The group decision matrix is:

$$M^{DM} = \begin{Bmatrix} pd_{11}^{DM} & pd_{12}^{DM} & \cdots & pd_{1n}^{DM} \\ pd_{21}^{DM} & pd_{22}^{DM} & \cdots & pd_{2n}^{DM} \\ \vdots & \vdots & & \vdots \\ pd_{m1}^{DM} & pd_{m2}^{DM} & \cdots & pd_{mn}^{DM} \end{Bmatrix}$$

where

$$\begin{aligned} pd_{ij}^{DM} &= PDHFWA\left(pd_{ij}^{DM_1}, pd_{ij}^{DM_2}, \cdots, pd_{ij}^{DM_K}\right) \\ &= \mathop{\oplus}_{k=1}^{K} \zeta_k pd_{ij}^{DM_k} \\ &= \bigcup_{\gamma_k \in \widetilde{hp}_k, \eta_k \in \widetilde{hg}_k} \left\{ \left\langle \left(1 - \prod_{k=1}^{K} (1-\gamma_k)^{\zeta_k}\right) \middle| \prod_{k=1}^{K} p_{\gamma_k}^{h} \right\rangle, \right. \\ &\quad \left. \left\langle \prod_{k=1}^{K} \eta_k^{\zeta_k} \middle| \prod_{k=1}^{K} p_{\eta_k}^{g} \right\rangle \right\}, \quad pd_{ij}^{DM_k} = \langle \widetilde{hp}_k, \widetilde{hg}_k \rangle. \\ & i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n. \end{aligned}$$

Then, according to the traditional VIKOR method, we need to determine the best values pd_j^* and the worst values pd_j^- of criteria $(j = 1, 2, \cdots, n)$. $pd_j^* = \max_i pd_{ij}^{DM}$, $pd_j^- = \min_i pd_{ij}^{DM}$. In the comparison process, we will use the synthetical score function (Definition 2) of PDHFE to get the best and worst values.

Using the importance of criteria $\omega_1, \omega_2, \cdots, \omega_n$, the other two values can be calculated as:

$$\tilde{S}_i = \sum_{j=1}^{n} \frac{\omega_j \cdot GD\left(pd_j^*, pd_{ij}^{DM}\right)}{GD\left(pd_j^*, pd_j^-\right)},$$

$$\widetilde{R}_i = \max_j \frac{\omega_j \cdot GD\left(pd_j^*, pd_{ij}^{DM}\right)}{GD\left(pd_j^*, pd_j^-\right)},$$

where $GD(\cdot)$ is the general distance measure of PDHFEs defined in Definition 8, $\tilde{S}_i$ is the deviation degree of a_i to the ideal alternative and indicates the majority rule, $\tilde{R}_i$ is the maximum deviation degree of a_i to the ideal alternative and indicates the maximum regret rule. Next, the third value $\tilde{Q}_i$ can be calculated as $\tilde{Q}_i = \nu\left(\tilde{S}_i - \tilde{S}^-\right)/\left(\tilde{S}^* - \tilde{S}^-\right) + (1-\nu)\left(\tilde{R}_i - \tilde{R}^-\right)/\left(\tilde{R}^* - \tilde{R}^-\right)$, where ν is the weight of the strategy of the maximum group utility and $\tilde{S}^- = \min_i \tilde{S}_i$, $\tilde{S}^* = \max_i \tilde{S}_i$, $\tilde{R}^- = \min_i \tilde{R}_i$ and $\tilde{R}^* = \max_i \tilde{R}_i$.

Now, we can give the rank of all alternatives according to the sorting principles in Subsection II.D.2) (Step 5).

To summarize the methodology simply, the steps of the integrated VIKOR and AHP method are listed in Figure 6.

It is necessary to give some detailed explanation about the final step in Figure 6.

A compromise solution alternative $a^{(1)}$, which has the minimum value in $\widetilde{Q}$, can be determined if it satisfies two conditions ($a^{(i)}$ is the *ith* optimal alternative):

Condition1: $\widetilde{Q}\left(a^{(2)}\right) - \widetilde{Q}\left(a^{(1)}\right) \geq DQ$, where $a^{(2)}$ has the second minimum value in $\widetilde{Q}$ and $\widetilde{Q}(\cdot)$ indicates the real value of alternative in $\widetilde{Q}$. $DQ = 1/(m-1)$.

Condition2: $a^{(1)}$ is also the best alternative according to $\widetilde{S}$ or $\widetilde{R}$.

However, if one of the two conditions could not be satisfied, the compromise solutions have to be proposed. The followings are two kinds of compromise solutions.

- The compromise solutions are $a^{(1)}$ and $a^{(2)}$ if only the condition2 is not satisfied.
- The compromise solutions are $a^{(1)}, a^{(2)}, \ldots, a^{(M)}$ if the condition1 is not satisfied. $a^{(M)}$ satisfies $\widetilde{Q}\left(a^{(M)}\right) - \widetilde{Q}\left(a^{(1)}\right) < DQ$, $\widetilde{Q}\left(a^{(M+1)}\right) - \widetilde{Q}\left(a^{(1)}\right) \geq DQ$.

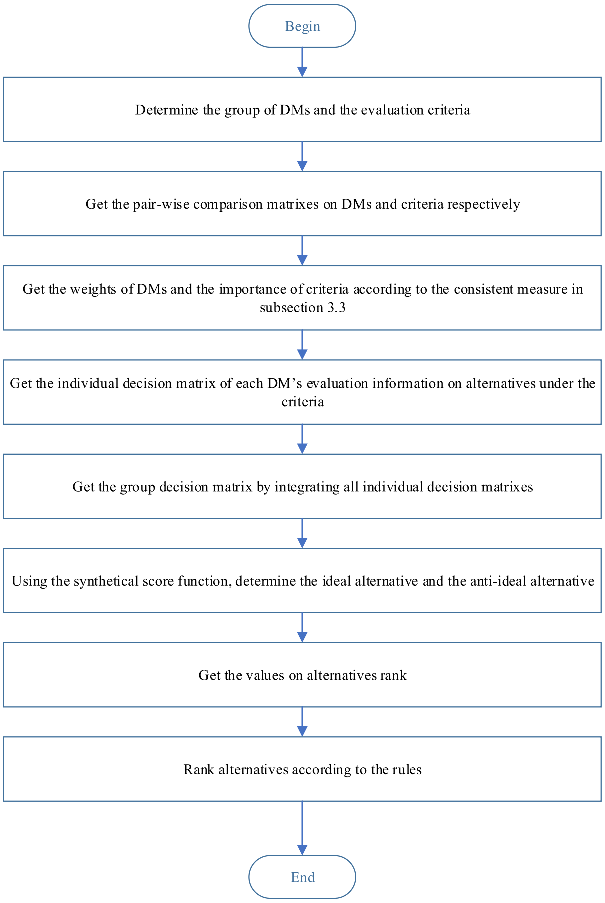

FIGURE 6. The main steps of the integrated VIKOR and AHP method.

The alternatives $a^{(1)}, a^{(2)}, \ldots, a^{(M)}$ are denoted as indistinguishable.

The integrated VIKOR and AHP could determine the weights of DMs and the importance of criteria by using people's experience ability efficiently. In addition, the proposed synthetical score function and the general distance measure could reflect the characteristics of probabilistic dual hesitant fuzzy information so that the extended VIKOR could handle the decision-making problems under probabilistic dual hesitant fuzzy information more rationally.

V. A CASE STUDY ON AI STRATEGY SELECTION AND THE RELATIVE COMPARISON AND ANALYSIS

We use a practical example on AI strategy selection in a company to show the availability and the effectiveness of the proposed decision-making method. Then, some comparisons between the proposed method and some existing methods are provided and some necessary analyses are used to show the advantages and weak points of the proposed method.

A. NECESSARY DESCRIPTION ON AI

In 2016, AlphaGo won the match of go and the total score is AlphaGo 4: Lee Se-dol 1. One year later, the upgraded AlphaGo won Ke Jie who was the top of the human being's go. From then on, almost everyone prefers to belief that the AI will be able to unveil the real intelligence and could change the world before long. More than that, 2017 is evaluated by some professional visitors as the first year of the AI. The tide of the AI has already unstoppable [13]. AI has no longer a theoretical research in an academic institution or a laboratory room. On the contrary, AI is a basic technology which could bring decades of innovation and subvert the whole society in nature. From the way of people's work and the way of doctors diagnose and treat, AI could provide infinite possibility in the future. Almost everyone would not be free from the influence of AI, let along the involved companies. Including intelligent manufacturing, intelligent research and development, intelligent management and so on, AI could change the operation of companies in all industries fundamentally. The development of AI has provided companies with automated business processes and the opportunity of optimizing customer experience and product differentiation. Some internet behemoth companies have gained the increasing competitive edge by using the AI technologies. Such as Google and Amazon in USA, Alibaba and Baidu in CN. Amazon is using AI to improve personalized recommendations and optimize inventory management. Google has reduced 40 percent cooling cost by using the DeepMind technology to manage the power of data center. Facebook devotes himself to building the basic technology of AI. Microsoft focuses on pushing AI into the company's products based on an AI business department which contains more than 5000 computer scientists and engineers. The database administration server of Intel has been updated for more computation used to training the AI system. Baidu is investing heavily in AI on some significant aspects such as establishing image recognition technology, promoting autonomous driving, launching digital assistant and developing augmented reality tools. It is obviously to find their advantages from AI strategies. Facing the reality, we should admit a situation that it is still in its infancy to apply AI at the enterprise level. However, some pre-works will be significative and provide some differentiated competitive advantages for today's enterprises. For instance, leaders should understand in depth the concept of AI and its ecosystem, learning the AI strategies of industry giants.

Now, AI has been risen to national strategy and it will become a strategic capability of enterprises [14]–[16]. In terms of enterprises, the main influence of AI could be divided into five aspects, which can be found in Table 4. Thus, almost every company will be flooded by the tide of AI, let along internet companies.

B. THE CASE ON AI STRATEGY DEPLOMENT

One company called INIM wants to develop its own AI strategy. But, the limited financial resource and scale of the company result in that they could focus on developing AI in just one department. The main departments are product department (a_1), design department (a_2), test department (a_3), research and development department (a_4), customer service department (a_5), marketing department (a_6) and operational department (a_7). The CEO of INIM invites the group of

TABLE 4. The main influence of AI to enterprises.

The main influence of AI	Brief description
Increasing revenue	The combination of IoT (Internet of Things), intelligent sensors and AI will increase the revenue of three business areas. IoT device manufacturers, IoT data and information providers/integrators and enterprises that provides application services based on intelligent sensors.
Enhancing security	Real-time monitoring can prevent disasters and improve overall safety.
Reducing losses	Real-time monitoring can reduce casualties or property damage.
Reducing cost	Smart monitoring devices will reduce operating costs for households and businesses.
Improving customer experience	Intelligent sensors can adapt customers' behavioral preferences by understanding their preferences.

TABLE 5. The comparison matrix of DMs.

	DM_1	DM_2	DM_3
DM_1	<(0.5|1),(0.5|1)>	<(0.4|0.7,0.5|0.3),(0.4|0.5,0.5|0.2)>	<(0.1|0.2,0.2|0.7),(0.2|0.2,0.4|0.7)>
DM_2	<(0.6|0.7,0.5|0.3),(0.6|0.5,0.5|0.2)>	<(0.5|1),(0.5|1)>	<(0.3|0.4,0.4|0.5),(0.2|0.7,0.4|0.3)>
DM_3	<(0.9|0.2,0.8|0.7),(0.8|0.2,0.6|0.7)>	<(0.7|0.4,0.6|0.5),(0.8|0.7,0.6|0.3)>	<(0.5|1),(0.5|1)>

DMs, which consists of the seven department managers. It is difficult to determine the weights of the seven DMs. Nevertheless, the CEO could give the preferences between each two DMs. We use the PDHFEs to collect all preference information. The PDHFCM can be shown in Table 5 (Due to the space limitation, only the comparative information of the first three DMs is displayed). After the normalization process, the consistency checking and improving with Algorithm 1 and Algorithm 2, the weighting vector calculation process with Algorithm 3, we get the membership weighting vector (0.3031, 0.2305, 0.1785, 0.1183, 0.0777, 0.0552, 0.0367) and the non-membership weighting vector (0.2854, 0.2779, 0.1626, 0.1064, 0.0689, 0.0519, 0.0469). Then, the synthetical weighting vector is $\omega^{DMs} = (0.2943, 0.2542, 0.1705, 0.1124, 0.0733, 0.0535, 0.0418)$ when the attention level of DMs to the membership degrees is 0.5. On account of the limitation of the paper and it is inefficient to show all data in this paper. We just give the final results of the consistency checking and improving. It's the same process for getting the DMs' weights and the criteria weights.

The seven department managers need also to determine the criteria on the AI strategy problem. According to their discussion and research, six necessary criteria are necessary to develop AI strategy in a department. The criteria are shown and illuminated clearly in Table 6.

TABLE 6. Criteria and the explanations.

Criterion	Explanation
C_1 : clear business model	The business goal should be clear and avoid products and services being reduced to technology demonstrations. Good business model could provide moderate value for more users and obtain the value far beyond the input costs.
C_2 : boundaries clear	Clear problem definition and domain boundaries. Some open questions and some fuzzy boundaries could result in the failure of AI.
C_3 : high quality big data	It should be easy to get and store a large amount of relevant data. Big data is the preliminary basic of AI.
C_4 : user feedback	Users and application scenarios are necessary. Users are both the consumers and the providers of data.
C_5 : computational resource	The computational capabilities and complete architecture are the key of producing AI.
C_6 : professionals	AI experts who are good at field business and industrial application. they can well define commercial business problems as artificial intelligence problems, and then successfully apply them to the actual product scenarios and continuously optimize them in the application.

Then, they need to determine the weights of the criteria. However, it is not easy to use precise values to show the weights of the criteria directly. In order to get more accurate weight information, we use PDHFEs to collect the seven department managers' comparison information between any two criteria. Then, we can get the comparison matrix of criteria and Table 7 shows the comparison matrix (Due to the space limitation, only the comparative information of the first three criteria is displayed). After that, to normalize the matrix and use Algorithm 1, Algorithm 2 to check and improve the consistency of the normalized matrix. Using Algorithm 3 to get the membership weighting vector (0.309, 0.2574, 0.1644, 0.109, 0.0821, 0.0781) and the non-membership weighting vector (0.2981, 0.2954, 0.1509, 0.0683, 0.1193, 0.068). Then, the synthetical weighting vector can be calculated as $\omega^{criteria} = (0.3036, 0.2764, 0.1576, 0.0886, 0.1007, 0.0731)$.

The seven DMs give their evaluation information of the seven departments with respect to the six criteria. We also use PDHFSs to collect their evaluation information and construct seven decision-making matrices (Appendix). Then, we get the group decision-making matrix by aggregating the seven decision-making matrixes with DMs' weights ω^{DMs} using the basic aggregation operator in [1]. The group decision-making matrix can be shown in Table 8 (Due to the space limitation,

TABLE 7. **The comparison matrix of criteria.**

<table>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>C_1</td><td><(0.5|1),(0.5|1)></td><td><(0.2|0.2,0.3|0.6,0.4|0.2),(0.4|0.2,0.5|0.5,0.6|0.3)></td><td><(0.3|0.2,0.4|0.2,0.6|0.5),(0.2|0.5,0.3|0.2,0.4|0.2)></td></tr>
<tr><td>C_2</td><td><(0.8|0.2,0.7|0.6,0.6|0.2),(0.6|0.2,0.5|0.5,0.4|0.3)></td><td><(0.5|1),(0.5|1)></td><td><(0.2|0.2,0.3|0.4,0.5|0.3),(0.1|0.3,0.3|0.3,0.4|0.1)></td></tr>
<tr><td>C_3</td><td><(0.7|0.2,0.6|0.2,0.4|0.5),(0.8|0.5,0.7|0.2,0.6|0.2)></td><td><(0.8|0.2,0.7|0.4,0.5|0.3),(0.9|0.3,0.7|0.3,0.6|0.1)></td><td><(0.5|1),(0.5|1)></td></tr>
</table>

TABLE 8. **The group decision-making matrix (only the first elements of each membership and non-membership degrees is shown).**

<table>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>a_1</td><td><(0.261|6e-6,…),(0.2726|7.2e-6,…)></td><td><(0.2381|8.64e-5,…),(0.2118|4.41e-4,…)></td><td><(0.1907|8.96e-05,...),(0.2389|5.76e-05,...)></td></tr>
<tr><td>a_2</td><td><(0.2365|1.12e-4,...),(0.1764|8.64e-4,...)></td><td><(0.2535|6.4e-05,...),(0.1979|1.536e-3,...)></td><td><(0.2535|4.32e-4,...),(0.2166|2.592e-4,...)></td></tr>
<tr><td>a_3</td><td><(0.2044|2.16e-4,...),(0.2251|7.168e-4,...)></td><td><(0.2506|3.15e-4,...),(0.2038|8.1e-05,...)></td><td><(0.2237|2.4e-05,...),(0.3026|3.78e-4,...)></td></tr>
</table>

TABLE 9. **The synthetical score values of the group decision-making matrix.**

	C_1	C_2	C_3	C_4	C_5	C_6
a_1	-0.0906	-0.0536	-0.1847	0.0384	-0.0621	-0.1395
a_2	0.0016	-0.0309	-0.0621	-0.0915	-0.0144	-0.0321
a_3	-0.0704	-0.0653	-0.1539	-0.1221	-0.0905	0.0127
a_4	-0.0359	0.0199	-0.0769	-0.0453	-0.2533	-0.0899
a_5	-0.1689	-0.0318	-0.0186	0.0296	-0.1059	-0.1822
a_6	-0.1146	-0.0373	-0.0859	0.0497	-0.0938	-0.0596
a_7	-0.0662	0.072	-0.1057	0.0265	-0.036	0.0778

only the evaluation information of the first three alternatives under the first three criteria is displayed). Because the data scale is too large, we just list the first element of the membership degrees and the non-membership degrees of each entry for illustration.

Besides, we use the score values to show the group decision-making matrix in Table 9. We get the score value of each PDHFE in the group decision-making matrix based on Definition 2. From Table 9, we find the ideal alternative and the anti-ideal alternative easily. In Table 9, the red font represents the maximum values and the green font indicates the minimum values under each criterion.

According to Table 8 and Table 9, the ideal alternative pd^* and the anti-ideal alternative pd^- can be shown as: $pd^* = \{\langle(0.2365|1.12e-4,\ldots),(0.1764|8.64e-4,\ldots)\rangle, \langle(0.2560|3.24e-05,\ldots),(0.1328|4.8384e-3,\ldots)\rangle, \langle(0.3119|2.592e-4,\ldots),(0.2138|2.1e-4,\ldots)\rangle, \langle(0.2664|3.84e-4,\ldots),(0.1661|1.296e-4,\ldots)\rangle, \langle(0.2534|2.16e-4,\ldots),(0.2111|4.32e-4,\ldots)\rangle, \langle(0.3643|8e-05,\ldots),(0.1713|5.76e-05,\ldots)\rangle\}$. $pd^- = \{\langle(0.1679|6.4e-4,\ldots),(0.2849|2.7e-05,\ldots)\rangle, \langle(0.2506|3.15e-4,\ldots),(0.2038|8.1e-05,\ldots)\rangle, \langle(0.1907|8.96e-05,\ldots),(0.2389|5.76e-05,\ldots)\rangle, \langle(0.2549|3.84e-05,\ldots),(0.3332|2.7e-4,\ldots)\rangle, \langle(0.1618|7.68e-05,\ldots),(0.2312|1.728e-3,\ldots)\rangle, \langle(0.2753|9.72e-05,\ldots),(0.2954|2.56e-05,\ldots)\rangle\}$.

Then, we can complete the method with the remaining steps of VIKOR method. Using the synthetical weighting vector $\omega^{criteria} = (0.3036, 0.2764, 0.1576, 0.0886, 0.1007, 0.0731)$ and the general distance measure $GD(\cdot)$ in Definition 8, according to the two equations $\widetilde{S}_i = \sum_{j=1}^{n} \frac{\omega_j^{criteria} \cdot GD\left(pd_j^*, pd_{ij}^{DM}\right)}{GD\left(pd_j^*, pd_j^-\right)}$ and $\widetilde{R}_i = \max_j \frac{\omega_j^{criteria} \cdot GD\left(pd_j^*, pd_{ij}^{DM}\right)}{GD\left(pd_j^*, pd_j^-\right)}$, we get $\widetilde{S}_1 = 0.5466$, $\widetilde{S}_2 = 0.6874$, $\widetilde{S}_3 = 0.4421$, $\widetilde{S}_4 = 0.6297$, $\widetilde{S}_5 = 0.4904$, $\widetilde{S}_6 = 0.6734$, $\widetilde{S}_7 = 0.9204$ and $\widetilde{R}_1 = 0.1888$, $\widetilde{R}_2 = 0.3036$, $\widetilde{R}_3 = 0.216$, $\widetilde{R}_4 = 0.2549$, $\widetilde{R}_5 = 0.1707$, $\widetilde{R}_6 = 0.237$, $\widetilde{R}_7 = 0.2764$. After that, according to $\widetilde{Q}_i = \nu\left(\widetilde{S}_i - \widetilde{S}^-\right)/\left(\widetilde{S}^* - \widetilde{S}^-\right) + (1-\nu)\left(\widetilde{R}_i - \widetilde{R}^-\right)/\left(\widetilde{R}^* - \widetilde{R}^-\right)$, $\widetilde{S}^* = \max\left(\widetilde{S}_i\right)$, $\widetilde{S}^- = \min\left(\widetilde{S}_i\right)$, $\widetilde{R}^* = \max\left(\widetilde{R}_i\right)$, $\widetilde{R}^- = \min\left(\widetilde{R}_i\right)$, when $\nu = 0.5$, we get $\widetilde{Q}_1 = 0.1776$, $\widetilde{Q}_2 = 0.7565$, $\widetilde{Q}_3 = 0.1703$, $\widetilde{Q}_4 = 0.5129$, $\widetilde{Q}_5 = 0.0505$, $\widetilde{Q}_6 = 0.4911$, $\widetilde{Q}_7 = 0.8977$. Then, we get three kinds of sort about the alternatives (the smaller the value, the better the alternative):

$$\widetilde{S}:\ a_3 \succ a_5 \succ a_1 \succ a_4 \succ a_6 \succ a_2 \succ a_7$$
$$\widetilde{R}:\ a_5 \succ a_1 \succ a_3 \succ a_6 \succ a_4 \succ a_7 \succ a_2$$
$$\widetilde{Q}:\ a_5 \succ a_3 \succ a_1 \succ a_6 \succ a_4 \succ a_2 \succ a_7$$

According to the ordering rule of VIKOR method. a_5 is the optimal alternative in $\widetilde{Q}$ and $\widetilde{R}$. But, $\widetilde{Q}(a_3) - \widetilde{Q}(a_5) = 0.1198 < \frac{1}{6}$. Thus, the compromise solutions should be (a_5, a_3, a_1) because $\widetilde{Q}(a_1) - \widetilde{Q}(a_5) = 0.1271 < \frac{1}{6}$, $\widetilde{Q}(a_6) - \widetilde{Q}(a_5) = 0.4406 > \frac{1}{6}$. In addition, a_5, a_3 and a_1 are both the top three in the three ranks.

INIM company should pay all attention to develop the AI strategy in customer service department (a_5), product department (a_1) and test department (a_3). If the manpower and financial resources could not support the three departments' AI strategy, the company may focus all resources on the AI strategy in the customer service department (a_5). From the angle of reality, customers could provide vast data in the process of using the relative products, which is closer to real life. The data will be used to train the AI framework, which has been constructed by some experts in related fields, so that the company could provide better customer service automatically. In addition, the work procedures of the product department (a_1) and test department (a_3) are easier to be programmed relatively. The two departments are also suitable for the development of AI strategy. Therefore, the compromise solutions are reasonable and tally with the actual situation. That means that our proposed method is effective and feasible.

C. COMPARISON AND ANALYSIS

In this subsection, we give a comparison between the proposed method and a mature decision-making method which includes a mature decision-making method based on the basic aggregation operator for PDHFSs and a comparison method of PDHFEs in [1].

We use the group decision-making matrix in Table 7 and the synthetical weighting vector $\omega^{criteria} = (0.3036, 0.2764, 0.1576, 0.0886, 0.1007, 0.0731)$ to calculate the PDHFWA evaluation value of each alternative. Then, we could get the score function values (SVs) and the deviation values (DVs). The results can be shown in Table 10. We omit a mass of data for limiting the length of the paper. The evaluation values are replaced by the first elements

From Table 10, we can get the rank of alternatives $a_6 \succ a_1 \succ a_5 \succ a_4 \succ a_2 \succ a_3 \succ a_7$ just according to the SVs (the bigger the value, the better the alternative). The optimal alternative is a_6 and the worst alternative is a_7. The result is different to the solution in Subsection V.A. One of the reasons could be that the DVs are neglected because the SVs are totally different and some important ordering information can be reflected by the DVs. Besides, the rationality of the deviation degree also needs to be considered. It reflects the drawback of the comparison method in [1].

TABLE 10. The alternatives' evaluation values, score values and deviation values.

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
SVs	0.0177	-0.0175	-0.028	-0.0083	-0.0066	0.0295	-0.0426
DVs	0.1949	0.2321	0.2006	0.2366	0.2076	0.2008	0.2528

Even so, the alternatives a_1 and a_5 are the second and the third in the order of alternatives. The comparison solution in Subsection V.A is (a_5, a_3, a_1). It reflects that the proposed decision-making method is feasible and effective in some ways. Additionally, the main advantages of the proposed method in this paper could be listed as follows:

(1) The synthetical score function could consider not only the mean value and the stability of information simultaneously, but also people's data tendency and sensibility to negative information. Thus, the synthetical score function could be used to compare different probabilistic dual hesitant fuzzy information more accurately and reasonably.

(2) The general distance measure of probabilistic dual hesitant fuzzy information consists of the synthetical score function and the equiprobability distance measure. The equiprobability distance measure could get the real distance between different probabilistic dual hesitant fuzzy information simply and conveniently. The synthetical score function is used to modify the distance value. Therefore, the general distance measure could depict the distance of different probabilistic dual hesitant fuzzy information more elaborately.

(3) The proposed transformation function could transform the PDHFCM to PDHCM so that the probabilistic dual hesitant fuzzy information can be used in the AHP method. In addition, we consider the membership degrees and non-membership degrees respectively in the process of AHP. The synthetical weights are the fusion of the membership weights and the non-membership weights.

(4) We propose a consistency measure checking approach and an appropriate information-improved approach in the process of AHP. According to the process of consistency checking or improving, we get the accurate and rational weight information of relative criteria.

(5) We combine the advantages of the AHP and VIKOR method under probabilistic dual hesitant fuzzy information.

The proposed integrated VIKOR and AHP method could make full use of the characteristics of probabilistic dual hesitant fuzzy information and be applied in many other decision-making problems.

VI. CONCLUSIONS

AI is sweeping the world like a giant wave. In this paper, an integrated VIKOR and AHP method has been proposed to solve MCGDM under probabilistic dual hesitant fuzzy information and applied in the AI strategy selection problem. For completing the method, we have studied some necessary properties of the PDHFSs. First, in order to compare different PDHFEs, we have proposed a synthetical score function and a new comparison method. The synthetical score function could consider the mean and stability of information and people's data tendency and sensibility to negative information simultaneously. Second, we have proposed an equiprobability distance measure. Based on the equiprobability distance measure and the synthetical score function, we have proposed a general distance measure for distinguishing the different probability dual hesitant fuzzy information. Besides, with the view of applying probabilistic dual hesitant fuzzy information in AHP, we have proposed a transformation function. Third,

TABLE 11. The individual decision making matrix from DM_1.

	C_1	C_2	C_3
a_1	<(0.2|0.5,0.3|0.4),(0.5|0.1,0.6|0.7)>	<(0.1|0.3,0.3|0.5),(0.2|0.6,0.3|0.1,0.4|0.3)>	<(0.1|0.8,0.2|0.2),(0.2|0.2,0.4|0.2,0.5|0.2)>
a_2	<(0.2|0.7,0.4|0.1),(0.2|0.2,0.4|0.8)>	<(0.2|0.4,0.4|0.3),(0.2|0.8,0.5|0.2)>	<(0.2|0.3,0.3|0.4,0.4|0.3),(0.2|0.6,0.4|0.1,0.5|0.3)>
a_3	<(0.2|0.2,0.4|0.1,0.5|0.4),(0.2|0.2,0.4|0.1)>	<(0.3|0.2,0.4|0.2),(0.1|0.3,0.4|0.1)>	<(0.2|0.2,0.3|0.2,0.5|0.2),(0.3|0.3,0.4|0.2)>

TABLE 12. The individual decision making matrix from DM_2.

<table>
<tr><th></th><th colspan="3">DM_2</th></tr>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>a_1</td><td><(0.2|0.1,0.4|0.2,0.5|0.3),(0.4|0.1,0.5|0.6)></td><td><(0.4|0.2,0.5|0.1,0.6|0.3),(0.2|0.1,0.3|0.7)></td><td><(0.2|0.2,0.3|0.1,0.4|0.3),(0.2|0.2,0.3|0.2,0.5|0.3)></td></tr>
<tr><td>a_2</td><td><(0.4|0.4,0.6|0.5,0.7|0.1),(0.1|0.6,0.2|0.2,0.3|0.2)></td><td><(0.3|0.1,0.4|0.3),(0.2|0.4,0.3|0.1,0.5|0.3)></td><td><(0.2|0.2,0.3|0.4),(0.4|0.1,0.5|0.2,0.6|0.6)></td></tr>
<tr><td>a_3</td><td><(0.1|0.1,0.3|0.2),(0.3|0.8,0.4|0.1,0.6|0.1)></td><td><(0.2|0.9,0.3|0.1),(0.4|0.1,0.6|0.1,0.7|0.8)></td><td><(0.1|0.2,0.3|0.3),(0.5|0.1,0.6|0.3,0.7|0.4)></td></tr>
</table>

TABLE 13. The individual decision making matrix from DM_3.

<table>
<tr><th></th><th colspan="3">DM_3</th></tr>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>a_1</td><td><(0.3|0.1,0.4|0.3,0.5|0.5),(0.2|0.3,0.3|0.4,0.5|0.1)></td><td><(0.1|0.3,0.2|0.4,0.4|0.3),(0.4|0.5,0.6|0.5)></td><td><(0.1|0.1,0.2|0.7),(0.5|0.3,0.6|0.4)></td></tr>
<tr><td>a_2</td><td><(0.1|0.1,0.3|0.2,0.4|0.5),(0.2|0.4,0.4|0.2)></td><td><(0.2|0.2,0.3|0.2,0.5|0.3),(0.3|0.5,0.5|0.2)></td><td><(0.2|0.4,0.3|0.3),(0.1|0.1,0.2|0.5,0.4|0.2)></td></tr>
<tr><td>a_3</td><td><(0.4|0.2,0.5|0.6),(0.2|0.7,0.5|0.3)></td><td><(0.3|0.1,0.4|0.1,0.5|0.4),(0.2|0.3,0.3|0.2,0.4|0.5)></td><td><(0.3|0.5,0.4|0.3,0.5|0.2),(0.3|0.7,0.5|0.3)></td></tr>
</table>

TABLE 14. The individual decision making matrix from DM_4.

<table>
<tr><th></th><th colspan="3">DM_4</th></tr>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>a_1</td><td><(0.3|0.3,0.4|0.4,0.6|0.2),(0.1|0.2,0.2|0.3,0.3|0.2)></td><td><(0.3|0.1,0.5|0.3,0.6|0.2),(0.1|0.3,0.2|0.3)></td><td><(0.3|0.7,0.4|0.2),(0.2|0.2,0.4|0.3,0.5|0.1)></td></tr>
<tr><td>a_2</td><td><(0.2|0.4,0.4|0.2,0.5|0.1),(0.2|0.3,0.4|0.2,0.5|0.4)></td><td><(0.1|0.2,0.3|0.2,0.4|0.6),(0.2|0.3,0.3|0.3,0.4|0.2)></td><td><(0.6|0.5,0.7|0.2),(0.2|0.8,0.3|0.2)></td></tr>
<tr><td>a_3</td><td><(0.2|0.6,0.3|0.1),(0.2|0.4,0.3|0.2,0.4|0.4)></td><td><(0.1|0.5,0.2|0.1,0.3|0.4),(0.2|0.2,0.4|0.4,0.5|0.3)></td><td><(0.4|0.1,0.5|0.2),(0.2|0.3,0.4|0.3,0.5|0.4)></td></tr>
</table>

TABLE 15. The individual decision making matrix from DM_5.

<table>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>a_1</td><td><(0.5|0.2,0.7|0.2),(0.1|0.2,0.2|0.3,0.3|0.3)></td><td><(0.3|0.3,0.4|0.4,0.5|0.3),(0.2|0.7,0.4|0.1,0.5|0.2)></td><td><(0.4|0.2,0.5|0.5,0.6|0.3),(0.3|0.2,0.4|0.4)></td></tr>
<tr><td>a_2</td><td><(0.1|0.2,0.2|0.5,0.3|0.3),(0.2|0.6,0.3|0.1,0.5|0.3)></td><td><(0.4|0.4,0.6|0.3),(0.1|0.2,0.3|0.2,0.4|0.3)></td><td><(0.1|0.4,0.2|0.5,0.4|0.1),(0.4|0.3,0.6|0.7)></td></tr>
<tr><td>a_3</td><td><(0.1|0.5,0.3|0.3),(0.1|0.1,0.2|0.4,0.3|0.2)></td><td><(0.1|0.1,0.3|0.3,0.4|0.4),(0.3|0.3,0.5|0.4,0.6|0.1)></td><td><(0.3|0.2,0.5|0.5,0.6|0.2),(0.3|0.3,0.4|0.2)></td></tr>
</table>

we have given a new consistency measure and propose an appropriate information-improved approach in the process of AHP. The membership part and the non-membership part have been considered separately. Then, we proposed the integrated VIKOR and AHP method for solving MCGDM problems. We have used the AI strategy selection case and

TABLE 16. The individual decision making matrix from DM_6.

<table>
<tr><th></th><th colspan="3">DM_6</th></tr>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>a_1</td><td><(0.3|0.2,0.4|0.5),(0.3|0.3,0.4|0.4)></td><td><(0.2|0.4,0.3|0.2,0.4|0.2),(0.2|0.1,0.5|0.3)></td><td><(0.3|0.1,0.5|0.2,0.6|0.2),(0.1|0.6,0.2|0.2)></td></tr>
<tr><td>a_2</td><td><(0.3|0.5,0.4|0.1,0.5|0.4),(0.3|0.5,0.4|0.2,0.5|0.2)></td><td><(0.4|0.2,0.6|0.6),(0.2|0.4,0.3|0.4,0.4|0.1)></td><td><(0.2|0.3,0.4|0.2),(0.2|0.6,0.4|0.3,0.5|0.1)></td></tr>
<tr><td>a_3</td><td><(0.2|0.6,0.3|0.2,0.5|0.2),(0.4|0.8,0.5|0.1)></td><td><(0.5|0.7,0.6|0.3),(0.2|0.3,0.3|0.5,0.4|0.1)></td><td><(0.2|0.3,0.4|0.7),(0.1|0.5,0.3|0.3,0.4|0.2)></td></tr>
</table>

TABLE 17. The individual decision making matrix from DM_7.

<table>
<tr><th></th><th colspan="3">DM_7</th></tr>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>a_1</td><td><(0.2|0.1,0.4|0.8,0.5|0.1),(0.1|0.2,0.2|0.2,0.3|0.6)></td><td><(0.3|0.4,0.4|0.4),(0.3|0.7,0.4|0.2)></td><td><(0.2|0.4,0.4|0.5),(0.4|0.2,0.5|0.5,0.6|0.3)></td></tr>
<tr><td>a_2</td><td><(0.1|0.1,0.2|0.4,0.3|0.3),(0.4|0.2,0.6|0.4,0.7|0.1)></td><td><(0.4|0.5,0.5|0.3,0.6|0.1),(0.1|0.4,0.3|0.3,0.4|0.2)></td><td><(0.2|0.3,0.4|0.1,0.5|0.5),(0.1|0.3,0.3|0.3)></td></tr>
<tr><td>a_3</td><td><(0.1|0.3,0.4|0.1),(0.4|0.2,0.6|0.7)></td><td><(0.2|0.5,0.3|0.2,0.4|0.3),(0.3|0.5,0.5|0.3)></td><td><(0.1|0.2,0.2|0.2,0.3|0.3),(0.2|0.4,0.3|0.2)></td></tr>
</table>

the comparison with other decision-making method to show the availability and effectiveness of the proposed method.

In the future work, we want to study deep psychological behavior of people under the probabilistic dual hesitant fuzzy information so that the comparison or fusion of probabilistic dual hesitant fuzzy information could be more accurate. Additionally, the proposed method could be use in some necessary fields such as portfolio selection, investment choice, urban planning and so on.

APPENDIX

See Tables 11–17.

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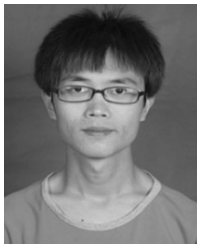

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