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# A Novel Disturbance Observer Design for a Larger Class of Nonlinear Strict-Feedback Systems via Improved DSC Technique

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**ABSTRACT** A novel scheme for disturbance observer is designed for an extended class of strict-feedback nonlinear systems with possibly unbounded, non-smooth, and state-independent compounded disturbance. To overcome these problems in disturbance observer design, the typical slide mode differentiators are improved by introducing hyperbolic tangent function to make the signals smooth, and then the improved slide mode differentiators are constructively used to estimate the errors of variables in the presence of disturbances. The unbounded, non-smooth or state-independent disturbances are therefore able to be eliminated by using the estimated variable errors. Thus, the bounded or differentiable conditions for disturbance observer design are removed. Furthermore, the convergence of the new disturbance observer is rigorously proved based on Lyapunov stability theorem, and the tracking error can be arbitrarily small. Finally, the simulation results are given to validate the feasibility and superiority of the proposed approach.

**INDEX TERMS** Disturbance observer, dynamic surface control, sliding mode differentiator.

## I. INTRODUCTION

As is well known, external disturbances, unmodeled dynamics and system uncertainties exist in a wide range of real control processes, which may cause the performance degradation and even the instability of the closed-loop control system. Thus, it is challenging to investigate disturbance estimation and rejection techniques in control systems societies [1]–[4]. Among numerous advanced disturbance estimations and attenuation techniques, disturbance observer-based control schemes have been extensively studied over the past years [5]–[8]. Since the promising properties of improving the control performance, disturbance observers have been widely used to estimate various disturbances and parametric uncertainties for many practical control systems, such as mechanical systems [9], optical disk drive systems [10], air vehicle systems [11], and so on [12], [13]. More precisely, under the assumption that the bounds of the disturbance

were unknown positive constants, a robust adaptive control scheme was presented by introducing a Nussbaum function in [14]. A disturbance observer-based dynamic surface control (DSC) approach was studied for the mobile wheeled inverted pendulum system with bounded lumped disturbance vector in [15]. To achieve output tracking for the saturated nonlinear systems with bounded external disturbance, a terminal sliding-mode-based disturbance observer is investigated in [16]. In [17], a disturbance observer combined with terminal sliding mode technique was proposed for the uncertain structural systems, the convergence of disturbance estimate error was guaranteed in finite time with differentiable disturbance. In [18], a disturbance observer-based robust backstepping control approach was developed for spacecraft attitude control systems in the presence of measurement uncertainties, while the time derivatives of the measurement uncertainties were assumed to be bounded. Furthermore, an output-feedback controller was designed based on the composite state observer and disturbance observer for nonlinear time-delay systems with input

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saturation, since the derivatives of the disturbances were required to be bounded for the disturbance estimators in the error dynamics [19]. The aforementioned control schemes have shown prominent disturbance rejection capability by introducing the designed disturbance observers. However, it has to be mentioned that for all the aforementioned strategies to work, the unknown disturbance is always assumed to be bounded and differentiable [20]–[23], which is very restrictive due to the fact that the compounded disturbances are usually unbounded or non-differentiable. This is because unmodeled dynamics, as same as some non-smooth nonlinearities such as dead zone and backlash, often occur in many physical systems. To the best of the authors' knowledge, no such disturbance observer designs that can handle both unbounded and non-differentiable compounded disturbances have been reported, which require new techniques go beyond the existing methods. This open issue is of great significance both in applicability and theory research.

On the other hand, adaptive control with disturbance observer has become an active area and attracted considerable attention. Different adaptive design approaches of disturbance observers have been developed by introducing fuzzy systems or neural networks (NNs) approximators [24]–[26]. In [27], a fuzzy nonlinear disturbance observer was designed based on the fuzzy approximation system in which the disturbance is observable. Similarly, combined with fuzzy approximator, a disturbance observer-based adaptive fuzzy control approach was investigated for a class of uncertain MIMO mechanical systems subject to unknown input nonlinearities in [28]. For nonlinear system with the states information being unavailable for the controller design, a novel fuzzy controller was presented by employing fuzzy logic systems (FLS) to construct the composite updating law in [29], [30], thus the adaptive compensation was given to minimize the effects of dynamic uncertainties to the control system. Moreover, neural networks as the universal approximator have been widely employed in control design. For instance, by using the powerful approximation ability of NNs, Chen *et al.* [31] studied an adaptive neural control method based on a disturbance observer for a class of MIMO nonlinear systems with control input saturation. In [32], a constrained adaptive neural controller was designed for the nonstrict-feedback system with the disturbance observer. In view of the unknown function term, the radial basis function neural networks (RBFNNs) were utilized to approximate the compounded disturbances in [33] and a nonlinear disturbance observer was proposed for control law design in the backstepping process. However, the performance of disturbance suppression is related to the approximation accuracy of neural networks, and the prior knowledge of the disturbance is required. For example, a common disturbance observer design approach using FLSs or NNs techniques is investigated under the condition that the input variable information of the disturbance term is known *a priori*. When the input variable information is insufficient, the methods based on NNs and FLSs would not work. It should be noted that this condition can be

commonly seen since the unmodeled dynamics included in the compounded disturbance may contain unknown variables. Moreover, the effect of disturbance rejection will heavily depend on the capability of the FLSs or NNs, which are not always robust when faced with strong disturbance. Therefore, it is urgent to propose a new method for disturbance estimation and attenuation.

Motivated by the above discussion, this paper first proposes a novel disturbance observer which, to the best of the authors' knowledge, successfully deals with the typical unbounded and non-smooth compounded disturbances. Combined with the designed disturbance observers, an adaptive tracking control scheme is presented for a class of nonlinear strict-feedback systems for the first time. The innovations are summarized as follows.

- 1) Unlike most of the existing control schemes, the restrictive assumptions that the compounded disturbance must be bounded, differentiable or slow time-varying have been removed and replaced by a possibly unbounded, non-differentiable and fast time-varying disturbances. To the best of our knowledge, this is the first work to design a disturbance observer relaxing all above restrictions simultaneously.
- 2) By combining first order sliding mode differentiator with improved DSC technique, the derivatives of the non-disturbance term are constructed. In what follows, a novel disturbance observer is designed, and the corresponding robust compensator is considered in adaptive control law in the meantime.
- 3) Considering that the prior knowledge of the compounded disturbance cannot be obtained precisely during the control design process, the stability and robustness of the closed loop system can be enhanced without involving FLSs or NNs approximators. Furthermore, it is analytically proved that the tracking error can be regulated to arbitrarily small in the absence of a compact set definition.

The organization of this paper is as follows. The problem description of the uncertain SISO strict-feedback nonlinear system is addressed in Section II. The disturbance observers and the corresponding adaptive controllers are designed by employing sliding mode differentiators and improved DSC techniques in Section III. In Section IV, the convergence of the new disturbance observer is rigorously proved based on Lyapunov stability theorem. Simulation examples are performed to demonstrate the effectiveness of the designed scheme in Section V. The concluding work is stated in Section VI.

## II. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider a class of nonlinear strict-feedback systems given by

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \delta_i(t) \\ \dot{x}_n = f_n(x) + g_n(x)u + \delta_n(t) \\ y = x_1 \end{cases} \quad (1)$$

where  $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$  and  $x = [x_1, x_2, \dots, x_n]^T \in R^n$  denote the state variables of the system,  $u \in R$  is system control input,  $y \in R$  is system output.  $f_i(\bar{x}_i)$  are known differentiable system functions,  $g_i(\bar{x}_i)$  represent the known differentiable control-gain functions. Particularly, the term  $\delta_i(t) = \Delta f_i(\bar{x}_i) + \Delta g_i(\bar{x}_i)x_{i+1} + d_i(t)$  are the compounded disturbance and continuous function,  $d_i(t)$ ,  $i = 1, 2, \dots, n$  are the external disturbance and system uncertainties,  $\Delta f_i(\bar{x}_i(t))$  are the uncertain parts of system functions  $f_i(\bar{x}_i)$ , and  $\Delta g_i(\bar{x}_i)$  are the uncertain parts of control-gain functions  $g_i(\bar{x}_i)$ .

The control objective is to design an adaptive tracking controller such that the system output  $y$  follows the desired trajectory  $y_d$  and the resulting tracking error can converge to a small neighborhood of the origin by appropriately choosing design parameters.

*Assumption 1 [34]:* The desired trajectory  $y_d$  is a sufficiently smooth function of  $t$ , and  $y_d, \dot{y}_d$  and  $\ddot{y}_d$  are bounded, that is, there exists a positive constant  $B_0$  such that  $\Pi_0 := \{y_d, \dot{y}_d, \ddot{y}_d : (y_d)^2 + (\dot{y}_d)^2 + (\ddot{y}_d)^2 \leq B_0\}$ .

*Assumption 2:* For the known virtual control-gain functions  $g_i(\bar{x}_i)$ ,  $i = 1, 2, \dots, n - 1$ , there exist unknown positive constants  $g_m$  and  $g_M$  such that  $0 < g_m \leq g_i(\bar{x}_i) \leq g_M$ .

*Remark 1:* It is worth noting that, in most of the existing control schemes, the disturbance term  $\delta_i(t)$ ,  $i = 1, 2, \dots, n$  are assumed to satisfy  $|\delta_i(t)| \leq \delta_0^*$  or  $|\dot{\delta}_i(t)| \leq \delta_1^*$  with  $\delta_0^*$  and  $\delta_1^*$  being unknown positive constants. However, the disturbance may be possibly unbounded due to unmodeled dynamic and system uncertainties, and it may also be difficult to acquire prior knowledge of  $\delta_i(t)$  in practice. If taking no account of these factors, the system performance will be seriously degraded and even be unstable. Thus, the proposed scheme aims to remove these restrictive assumptions and to enlarge the application range of disturbance observer.

*Lemma 1 [35]:* Hyperbolic tangent function  $\tanh(\cdot)$  will be used in this paper, and it is well known that  $\tanh(\cdot)$  is continuous and differentiable, and it fulfills that for any  $q \in R$  and  $\forall v > 0$

$$\begin{cases} 0 \leq |q| - q \tanh\left(\frac{q}{v}\right) \leq 0.2785v \\ 0 \leq q \tanh\left(\frac{q}{v}\right) \end{cases} \quad (2)$$

*Lemma 2:* The first order sliding mode differentiator [36] is designed as

$$\begin{aligned} \dot{\rho}_0 &= \zeta_0 = -\tau_0 |\rho_0 - f(t)|^{\frac{1}{2}} \text{sign}(\rho_0 - f(t)) + \rho_1 \\ \dot{\rho}_1 &= -\tau_1 \text{sign}(\rho_1 - \zeta_0) \end{aligned} \quad (3)$$

where  $\rho_0, \rho_1$  and  $\zeta_0$  are the states of the system,  $\tau_0$  and  $\tau_1$  are the designed parameters of the first order sliding mode differentiator, and  $f(t)$  is a known function. Then,  $\zeta_0$  can approximate the differential term  $\dot{f}(t)$  to any arbitrary accuracy if the initial deviations  $\rho_0 - f(t_0)$  and  $\zeta_0 - \dot{f}(t_0)$  are bounded.

*Lemma 3:* For any  $x \in R$ , the following inequality holds

$$\left| |x|^{\frac{1}{2}} \text{sign}(x) - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \tanh\left(\frac{x}{\mu}\right) \right| \leq \gamma \quad (4)$$

where  $\mu$  is the designed parameter and  $\gamma$  is a unknown positive constant.

*Proof:* See the Appendix.

*Remark 2:* It has to be noticed that if a discontinuous tracking differentiator is constructed through the first order sliding mode differentiator in Lemma 2, as a consequence, the resulted dynamic system is discontinuous owing to the sign functions that are employed and certain issues on the uniqueness and existence of the solution of the closed loop system will raise. Such issues are very significant since they affect the closed loop performance severely, thus Lemma 3 is introduced by employing hyperbolic tangent function to ensure the feasibility in backstepping process.

*Lemma 4:* Let  $\tau \dot{\beta} + \beta = \alpha$ ,  $y = \beta - \alpha$ , where  $\alpha$  and  $\beta$  are the input and output of low pass filter respectively,  $y$  denotes the filtering error. Then, the filtering error  $y$  can be bounded and  $\beta$  can approximate  $\alpha$  to any arbitrary accuracy if  $\frac{1}{2\tau} = \hat{\alpha}^2 + \varepsilon_0$ , where  $\varepsilon_0$  is a positive constant and  $\hat{\alpha}$  is the estimate of the differential term  $\dot{\alpha}$ .

*Proof:* See the Appendix.

### III. ADAPTIVE TRACKING CONTROLLER DESIGN

In this section, backstepping technique is used to construct an adaptive controller for nonlinear system (1). To facilitate the readers' comprehension, the general block diagram of the proposed control scheme is given in Fig. 1.

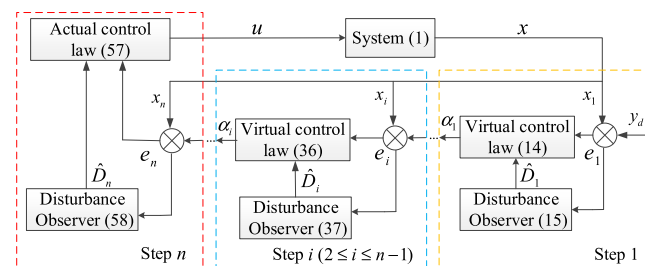


FIGURE 1. Block diagram of the proposed control scheme.

The design of adaptive control laws is based on the following change of coordinates:

$$\begin{cases} e_1 = x_1 - y_d \\ e_i = x_i - \alpha_{i-1} \end{cases}, \quad i = 2, 3, \dots, n \quad (5)$$

where  $e_1$  is the tracking error and  $\alpha_{i-1}$  is the virtual control input that will be designed later.

The recursive design procedure contains  $n$  steps. First, at each step of the backstepping design, the intermediate control  $\alpha_{i-1}$  is designed to make the corresponding subsystem toward equilibrium position. And at the final step, the stabilization of system (1) can be achieved with the actual control input  $u$  being designed.

Step 1: To start, considering the following subsystem of (1) and noting  $e_1 = x_1 - y_d$ ,

$$\dot{e}_1 = f_1(x_1) + g_1(x_1)x_2 + \delta_1 - \dot{y}_d \quad (6)$$

where  $x_2$  is regarded as a virtual control input.

Consider the following quadratic Lyapunov function candidate:

$$V_{e1} = \frac{1}{2}e_1^2 \quad (7)$$

The time derivative of  $V_{e1}$  along (6) is

$$\dot{V}_{e1} = e_1[f_1(x_1) + g_1(x_1)x_2 + \delta_1 - \dot{y}_d] \quad (8)$$

Invoking (6), we obtain

$$\delta_1 = \dot{e}_1 - (f_1(x_1) + g_1(x_1)x_2 - \dot{y}_d) \quad (9)$$

Since  $\dot{e}_1$  is unavailable, the following first order sliding mode differentiator is adopted so as to produce an auxiliary variable to estimate  $\dot{e}_1$ .

$$\begin{aligned} \dot{\rho}_{1,0} &= \zeta_{1,0} = -\varepsilon_{1,0} |\rho_{1,0} - e_1(t)|^{\frac{1}{2}} \text{sign}(\rho_{1,0} - e_1(t)) + \rho_{1,1} \\ \dot{\rho}_{1,1} &= -\varepsilon_{1,1} \text{sign}(\rho_{1,1} - \zeta_{1,0}) \end{aligned} \quad (10)$$

where  $\rho_{1,0}$ ,  $\rho_{1,1}$  and  $\zeta_{1,0}$  are the states of the system,  $\varepsilon_{1,0}$  and  $\varepsilon_{1,1}$  are positive design constants.

*Remark 3:* In view of Eq. (6), it can be seen that there need assumptions for the signal  $e_1$  [37], [38] and meanwhile, derivative term  $\dot{e}_1$  involves the disturbance term  $\delta_1$ . We would emphasize that if the disturbance term  $\delta_1$  is unbounded, the computation of the derivative term  $\dot{e}_1$  is therefore complicated. To efficiently handle this problem, the first order sliding mode differentiator according to Lemma 2 can be used to approach the value of  $\dot{e}_1$  in disturbance observer design to reduce the computational burden, and this method will show capable of preserving the closed-loop system tracking performance later.

According to Lemma 2, we have

$$|\zeta_{1,0} - \dot{e}_1(t)| \leq v_{1,0} \quad (11)$$

where  $v_{1,0}$  is a positive constant due to the approximation property of the first order sliding mode differentiator.

Define

$$\begin{aligned} \hat{\zeta}_{1,0} &= -\varepsilon_{1,0} \left( (\rho_{1,0} - e_1(t)) \tanh \left( \frac{\rho_{1,0} - e_1(t)}{\mu_{1,0}} \right) \right)^{\frac{1}{2}} \\ &\quad \times \tanh \left( \frac{\rho_{1,0} - e_1(t)}{\mu_{1,0}} \right) + \rho_{1,1} \end{aligned} \quad (12)$$

where  $\hat{\zeta}_{1,0}$  is the estimate of the auxiliary variable  $\zeta_{1,0}$ .

According to Lemma 3 and replace  $x$  with  $\rho_{1,0} - e_1(t)$ , then using (10) and (12), one has

$$|\zeta_{1,0} - \hat{\zeta}_{1,0}| \leq \gamma_1 \quad (13)$$

where  $\gamma_1$  is a positive constant that can converge to arbitrarily small by appropriately selecting design parameters.

*Remark 4:* It can be seen that an auxiliary variable  $\zeta_{1,0}$  is designed to estimate  $\dot{e}_1$  by a first order sliding mode

differentiator and then,  $\hat{\zeta}_{1,0}$  can be regarded as the approximator of  $\dot{e}_1$  similarly, which can be utilized to design the disturbance observer with the help of (6) and the estimation error can converge to arbitrarily small by appropriately adjusting design parameters. Both variables have well estimation performance for  $\dot{e}_1$ , but to avoid the discontinuity of the sign functions,  $\hat{\zeta}_{1,0}$  is presented necessarily according to Lemma 3 by employing hyperbolic tangent function to ensure the feasibility in backstepping process.

Invoking (5), we obtain  $x_2 = e_2 + \alpha_1$ .

Now, we construct a virtual control law  $\alpha_1$  and the adaptation function  $\hat{\delta}_1$  as follows

$$\alpha_1 = g_1^{-1}(x_1) \left( -k_1 e_1 - f_1(x_1) + \dot{y}_d - \lambda_1 \hat{D}_1^2 e_1 \right) \quad (14)$$

$$\hat{\delta}_1 = \hat{\zeta}_{1,0} - (f_1(x_1) + g_1(x_1)x_2 - \dot{y}_d) \quad (15)$$

where  $\tau_1 \hat{D}_1 + \hat{D}_1 = \hat{\delta}_1$ ,  $y_1 = \hat{D}_1 - \hat{\delta}_1$  and  $\lambda_1$  is a design constant. According to Lemma 4, we know that the filtering error  $y_1$  can be a positive constant by appropriately tuning the design parameters  $\tau_1$ .

Then, substituting (14) into (8) gives

$$\dot{V}_{e1} = g_1(x_1)e_1e_2 - k_1e_1^2 - \lambda_1\hat{D}_1^2e_1^2 + \delta_1e_1 \quad (16)$$

In view of (6) and (15), one has

$$\begin{aligned} -\hat{D}_1 + \delta_1 &= \dot{e}_1 - (f_1(x_1) + g_1(x_1)x_2 - \dot{y}_d) \\ &\quad - \left( \hat{\zeta}_{1,0} - (f_1(x_1) + g_1(x_1)x_2 - \dot{y}_d) + y_1 \right) \\ &= \dot{e}_1 - \hat{\zeta}_{1,0} - y_1 \end{aligned} \quad (17)$$

With the aid of (11) and (13), it yields

$$\begin{aligned} |-\hat{D}_1 + \delta_1| &= |\dot{e}_1 - \hat{\zeta}_{1,0} - y_1| \\ &\leq |\dot{e}_1 - \zeta_{1,0}| + |\zeta_{1,0} - \hat{\zeta}_{1,0}| + |y_1| \\ &\leq v_{1,0} + \gamma_1 + |y_1| = \gamma_1^* \end{aligned} \quad (18)$$

It further gives rise to

$$\begin{aligned} \delta_1^2 - \hat{D}_1^2 &\leq |\delta_1^2 - \hat{D}_1^2| = |\delta_1 - \hat{D}_1| |\delta_1 + \hat{D}_1| \\ &\leq \gamma_1^* (2|\delta_1| + \gamma_1^*) \leq 3\gamma_1^{*2} + \frac{\delta_1^2}{2} \end{aligned} \quad (19)$$

which implies

$$\frac{\delta_1^2}{2} \leq 3\gamma_1^{*2} + \hat{D}_1^2 \quad (20)$$

Thus, we can rewrite (16) as

$$\begin{aligned} \dot{V}_{e1} &\leq g_1(x_1)e_1e_2 - k_1e_1^2 - \lambda_1\hat{D}_1^2e_1^2 + \frac{\lambda_1\delta_1^2e_1^2}{2} + \frac{1}{2\lambda_1} \\ &\leq g_1(x_1)e_1e_2 - \left( k_1 - 3\lambda_1\gamma_1^{*2} \right) e_1^2 + \frac{1}{2\lambda_1} \end{aligned} \quad (21)$$

Let  $k_1 = 3\lambda_1\gamma_1^{*2} + k_{10}$ , where  $k_{10} > 0$

$$\dot{V}_{e1} \leq g_1(x_1)e_1e_2 - k_{10}e_1^2 + \frac{1}{2\lambda_1} \quad (22)$$

As  $e_2$  is presented in (21), therefore, the regulation of  $e_2$  will be investigated in the next step as follows.

Step  $i$  ( $2 \leq i \leq n - 1$ ): A similar procedure is employed recursively for each step  $i$  ( $2 \leq i \leq n - 1$ ).

Noting  $e_i = x_i - \alpha_{i-1}$ , the dynamics of  $e_i$ -subsystem can be described as follows

$$\dot{e}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \delta_i - \dot{\alpha}_{i-1} \quad (23)$$

Consider the following quadratic Lyapunov function candidate:

$$V_{ei} = \frac{1}{2}e_i^2 \quad (24)$$

The time derivative of  $V_{ei}$  along (23) is

$$\dot{V}_{ei} = e_i[f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \delta_i - \dot{\alpha}_{i-1}] \quad (25)$$

To estimate the differential term  $\dot{\alpha}_{i-1}$ , an auxiliary variable  $\Theta_{i,0}$  is designed as

$$\begin{aligned} \dot{\vartheta}_{i-1,0} &= \Theta_{i,0} \\ &= -\sigma_{i,0} \left| \vartheta_{i-1,0} - \alpha_{i-1} \right|^{\frac{1}{2}} \text{sign}(\vartheta_{i-1,0} - \alpha_{i-1}) + \vartheta_{i-1,1} \\ \dot{\vartheta}_{i-1,1} &= -\sigma_{i,1} \text{sign}(\vartheta_{i-1,1} - \Theta_{i,0}) \end{aligned} \quad (26)$$

where  $\vartheta_{i-1,0}$ ,  $\vartheta_{i-1,1}$  and  $\Theta_{i,0}$  are the states of the system,  $\sigma_{i,0}$  and  $\sigma_{i,1}$  are positive design constants.

By virtue of the approximation property of the first order sliding mode differentiator, we arrive

$$\left| \Theta_{i,0} - \dot{\alpha}_{i-1} \right| \leq \nu_{i,1} \quad (27)$$

where  $\nu_{i,1}$  is a positive constant.

According to (26), the estimate of the differential term  $\dot{\alpha}_{i-1}$  is defined as follows:

$$\begin{aligned} \hat{\alpha}_{i-1} &= -\sigma_{i,0} \left( (\vartheta_{i-1,0} - \alpha_{i-1}) \tanh \left( \frac{\vartheta_{i-1,0} - \alpha_{i-1}}{\mu_{i,1}} \right) \right)^{\frac{1}{2}} \\ &\quad \times \tanh \left( \frac{\vartheta_{i-1,0} - \alpha_{i-1}}{\mu_{i,1}} \right) + \vartheta_{i-1,1} \end{aligned} \quad (28)$$

Noting (26), (28) and Lemma 3, one reaches

$$\left| \hat{\alpha}_{i-1} - \Theta_{i,0} \right| \leq \varsigma_i \quad (29)$$

where  $\varsigma_i$  is a positive constant.

Therefore, the following inequality satisfies

$$\begin{aligned} \left| \hat{\alpha}_{i-1} - \dot{\alpha}_{i-1} \right| &\leq \left| \Theta_{i,0} - \dot{\alpha}_{i-1} \right| + \left| \hat{\alpha}_{i-1} - \Theta_{i,0} \right| \\ &\leq \nu_{i,1} + \varsigma_i \end{aligned} \quad (30)$$

Invoking(23), we obtain

$$\delta_i = \dot{e}_i - (f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} - \dot{\alpha}_{i-1}) \quad (31)$$

Similar to Step 1, since  $\dot{e}_i$  is unavailable, the following first order sliding mode differentiator is adopted as follows:

$$\begin{aligned} \dot{\rho}_{i,0} &= \zeta_{i,0} = -\varepsilon_{i,0} \left| \rho_{i,0} - e_i(t) \right|^{\frac{1}{2}} \text{sign}(\rho_{i,0} - e_i(t)) + \rho_{i,1} \\ \dot{\rho}_{i,1} &= -\varepsilon_{i,1} \text{sign}(\rho_{i,1} - \zeta_{i,0}) \end{aligned} \quad (32)$$

where  $\rho_{i,0}$ ,  $\rho_{i,1}$  and  $\zeta_{i,0}$  are the states of the system,  $\varepsilon_{i,0}$  and  $\varepsilon_{i,1}$  are positive design constants.

According to Lemma 2, it holds that

$$\left| \zeta_{i,0} - \dot{e}_i(t) \right| \leq \nu_{i,0} \quad (33)$$

where  $\nu_{i,0}$  is any positive constant due to the approximation property of the first order sliding mode differentiator.

Similarly, define functions  $\hat{\zeta}_{i,0}$  as follows

$$\begin{aligned} \hat{\zeta}_{i,0} &= -\varepsilon_{i,0} \left( (\rho_{i,0} - e_i(t)) \tanh \left( \frac{\rho_{i,0} - e_i(t)}{\mu_{i,0}} \right) \right)^{\frac{1}{2}} \\ &\quad \times \tanh \left( \frac{\rho_{i,0} - e_i(t)}{\mu_{i,0}} \right) + \rho_{i,1} \end{aligned} \quad (34)$$

where  $\hat{\zeta}_{i,0}$  is the estimate of the auxiliary variable  $\zeta_{i,0}$ .

According to (32), (34) and Lemma 3, it follows that

$$\left| \zeta_{i,0} - \hat{\zeta}_{i,0} \right| \leq \gamma_i \quad (35)$$

where  $\gamma_i$  is a positive constant.

Then, we construct the virtual control law  $\alpha_i$  and the adaptation function  $\hat{\delta}_i$  as follows

$$\alpha_i = g_i^{-1}(\bar{x}_i) \left( -k_i e_i - f_i(\bar{x}_i) + \hat{\alpha}_{i-1} - \lambda_i \hat{D}_i^2 e_i \right) \quad (36)$$

$$\hat{\delta}_i = \hat{\zeta}_{i,0} - \left( f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} - \hat{\alpha}_{i-1} \right) \quad (37)$$

where  $\tau_i \hat{D}_i + \hat{D}_i = \hat{\delta}_i$ ,  $y_i = \hat{D}_i - \hat{\delta}_i$  and  $\lambda_i$  is a design constant.

Noting that  $e_i = x_i - \alpha_{i-1}$ , one has  $x_{i+1} = e_{i+1} + \alpha_i$ .

Substituting (36) into (25) and following a similar way as Step 1 lead to

$$\begin{aligned} \dot{V}_{ei} &= g_i(\bar{x}_i)e_i e_{i+1} - k_i e_i^2 + e_i \left( \hat{\alpha}_{i-1} - \dot{\alpha}_{i-1} \right) \\ &\quad + e_i (-\lambda_i \hat{D}_i^2 e_i + \delta_i) \end{aligned} \quad (38)$$

In view of (23) and (37), it immediately gets

$$\begin{aligned} -\hat{D}_i + \delta_i &= \dot{e}_i - (f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} - \dot{\alpha}_{i-1}) \\ &\quad - \left( \hat{\zeta}_{i,0} - (f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} - \hat{\alpha}_{i-1}) + y_i \right) \\ &= \dot{e}_i - \hat{\zeta}_{i,0} + \dot{\alpha}_{i-1} - \hat{\alpha}_{i-1} - y_i \end{aligned} \quad (39)$$

Utilizing (30), (33) and (35) gives

$$\begin{aligned} \left| -\hat{D}_i + \delta_i \right| &= \left| \dot{e}_i - \zeta_{i,0} \right| + \left| \zeta_{i,0} - \hat{\zeta}_{i,0} \right| + \left| \dot{\alpha}_{i-1} - \hat{\alpha}_{i-1} \right| + y_i \\ &\leq \nu_{i,0} + \gamma_i + \nu_{i,1} + \varsigma_i + y_i = \gamma_i^* \end{aligned} \quad (40)$$

It further gives rise to

$$\begin{aligned} \delta_i^2 - \hat{D}_i^2 &\leq \left| \delta_i^2 - \hat{D}_i^2 \right| = \left| \delta_i - \hat{D}_i \right| \left| \delta_i + \hat{D}_i \right| \\ &\leq \gamma_i^* (2|\delta_i| + \gamma_i^*) \leq 3\gamma_i^{*2} + \frac{\delta_i^2}{2} \end{aligned} \quad (41)$$

which suggests

$$\frac{\delta_i^2}{2} \leq 3\gamma_i^{*2} + \hat{D}_i^2 \quad (42)$$

Thus, we can rewrite (38) as

$$\begin{aligned} \dot{V}_{ei} &\leq g_i(\bar{x}_i)e_i e_{i+1} - k_i e_i^2 + (v_{i,1} + \varsigma_i) e_i - \lambda_i \hat{D}_i^2 e_i^2 \\ &\quad + \frac{\lambda_i \delta_i^2 e_i^2}{2} + \frac{1}{2\lambda_i} \\ &\leq g_i(\bar{x}_i)e_i e_{i+1} + (v_{i,1} + \varsigma_i) |e_i| - \left(k_i - 3\lambda_i \gamma_i^{*2}\right) e_i^2 \\ &\quad + \frac{1}{2\lambda_i} \end{aligned} \quad (43)$$

Let  $k_i = 3\lambda_i \gamma_i^{*2} + k_{i0}$ , where  $k_{i0} > 0$

$$\dot{V}_{ei} \leq g_i(\bar{x}_i)e_i e_{i+1} + (v_{i,1} + \varsigma_i) |e_i| - k_{i0} e_i^2 + \frac{1}{2\lambda_i} \quad (44)$$

Step  $n$ : Noting  $e_n = x_n - \alpha_{n-1}$ , the dynamics of  $e_n$ -subsystem can be written as

$$\dot{e}_n = f_n(x) + g_n(x)u + \delta_n - \dot{\alpha}_{n-1} \quad (45)$$

Similarly, consider the following quadratic Lyapunov function candidate:

$$V_{en} = \frac{1}{2} e_n^2 \quad (46)$$

The time derivative of  $V_{en}$  along (45) is

$$\dot{V}_{en} = e_n [f_n(x) + g_n(x)u + \delta_n - \dot{\alpha}_{n-1}] \quad (47)$$

Similarly, utilizing the first order sliding mode differentiator to estimate  $\dot{\alpha}_{n-1}$ .

$$\begin{aligned} \dot{\vartheta}_{n-1,0} &= \Theta_{n,0} = -\sigma_{n,0} |\vartheta_{n-1,0} - \alpha_{n-1}|^{\frac{1}{2}} \\ &\quad \times \text{sign}(\vartheta_{n-1,0} - \alpha_{n-1}) + \vartheta_{n-1,1} \\ \dot{\vartheta}_{n-1,1} &= -\sigma_{n,1} \text{sign}(\vartheta_{n-1,1} - \Theta_{n,0}) \end{aligned} \quad (48)$$

where  $\vartheta_{n-1,0}$ ,  $\vartheta_{n-1,1}$  and  $\Theta_{n,0}$  are the states of the system,  $\sigma_{n,0}$  and  $\sigma_{n,1}$  are positive design constants.

In view of (48) and Lemma 2, one has

$$|\Theta_{n,0} - \dot{\alpha}_{n-1}| \leq \nu_{n,1} \quad (49)$$

where  $\nu_{n,1}$  is any positive constant.

Similar to Step  $i$ , the estimate of the differential term  $\dot{\alpha}_{n-1}$  is defined as:

$$\begin{aligned} \hat{\alpha}_{n-1} &= -\sigma_{n,0} \left( (\vartheta_{n-1,0} - \alpha_{n-1}) \tanh \left( \frac{\vartheta_{n-1,0} - \alpha_{n-1}}{\mu_{n,1}} \right) \right)^{\frac{1}{2}} \\ &\quad \times \tanh \left( \frac{\vartheta_{n-1,0} - \alpha_{n-1}}{\mu_{n,1}} \right) + \vartheta_{n-1,1} \end{aligned} \quad (50)$$

Noting (48), (50) and Lemma 3, one gets

$$|\hat{\alpha}_{n-1} - \Theta_{n,0}| \leq \varsigma_n \quad (51)$$

where  $\varsigma_n$  is a positive constant.

Therefore, the following inequality satisfies

$$\begin{aligned} |\hat{\alpha}_{n-1} - \dot{\alpha}_{n-1}| &= |\Theta_{n,0} - \dot{\alpha}_{n-1}(t)| + |\hat{\alpha}_{n-1} - \Theta_{n,0}| \\ &\leq \nu_{n,1} + \varsigma_n \end{aligned} \quad (52)$$

Invoking (45), we can get

$$\delta_n = \dot{e}_n - (f_n(x) + g_n(x)u - \dot{\alpha}_{n-1})$$

Similar to Step 1, the first order sliding mode differentiator is adopted as follows:

$$\begin{aligned} \dot{\rho}_{n,0} &= \zeta_{n,0} = -\varepsilon_{n,0} |\rho_{n,0} - e_n(t)|^{\frac{1}{2}} \text{sign}(\rho_{n,0} - e_n(t)) + \rho_{n,1} \\ \dot{\rho}_{n,1} &= -\varepsilon_{n,1} \text{sign}(\rho_{n,1} - \zeta_{n,0}) \end{aligned} \quad (53)$$

where  $\rho_{n,0}$ ,  $\rho_{n,1}$  and  $\zeta_{n,0}$  are the states of the system,  $\varepsilon_{n,0}$  and  $\varepsilon_{n,1}$  are positive design constants.

According to Lemma 2, one reaches

$$|\zeta_{n,0} - \dot{e}_n(t)| \leq \nu_{n,0} \quad (54)$$

where  $\nu_{n,0}$  is any positive constant.

Similarly, define functions  $\hat{\zeta}_{n,0}$  as follows

$$\begin{aligned} \hat{\zeta}_{n,0} &= -\varepsilon_{n,0} \left( (\rho_{n,0} - e_n(t)) \tanh \left( \frac{\rho_{n,0} - e_n(t)}{\mu_{n,0}} \right) \right)^{\frac{1}{2}} \\ &\quad \times \tanh \left( \frac{\rho_{n,0} - e_n(t)}{\mu_{n,0}} \right) + \rho_{n,1} \end{aligned} \quad (55)$$

where  $\hat{\zeta}_{n,0}$  is the estimate of the auxiliary variable  $\zeta_{n,0}$ .

According to (53), (55) and Lemma 3, we can know that

$$|\zeta_{n,0} - \hat{\zeta}_{n,0}| \leq \gamma_n \quad (56)$$

where  $\gamma_n$  is a positive constant.

Then, we construct the actual control law  $u$  and the adaptation function  $\hat{\delta}_n$  as follows

$$u = g_n^{-1}(x) \left( -k_n e_n - f_n(x) + \hat{\alpha}_{n-1} - \lambda_n \hat{D}_n^2 e_n \right) \quad (57)$$

$$\hat{\delta}_n = \hat{\zeta}_{n,0} - \left( f_n(x) + g_n(x)u - \hat{\alpha}_{n-1} \right) \quad (58)$$

where  $\tau_n \hat{D}_n + \hat{D}_n = \hat{\delta}_n$ ,  $y_n = \hat{D}_n - \hat{\delta}_n$  and  $\lambda_n$  is a design constant.

Similarly, substituting (57) into (47) yields, it holds that

$$\dot{V}_{en} = -k_n e_n^2 + e_n (-\lambda_n \hat{D}_n^2 e_n + \delta_n) + e_n (\hat{\alpha}_{n-1} - \dot{\alpha}_{n-1}) \quad (59)$$

In view of (45) and (58), we arrive

$$\begin{aligned} -\hat{D}_n + \delta_n &= \dot{e}_n - (f_n(x) + g_n(x)u - \dot{\alpha}_{n-1}) \\ &\quad - \left( \hat{\zeta}_{n,0} - \left( f_n(x) + g_n(x)u - \hat{\alpha}_{n-1} \right) + y_n \right) \\ &= \dot{e}_n - \hat{\zeta}_{n,0} + \dot{\alpha}_{n-1} - \hat{\alpha}_{n-1} - y_n \end{aligned} \quad (60)$$

Utilizing (52), (54) and (56) leads to

$$\begin{aligned} |-\hat{D}_n + \delta_n| &= |\dot{e}_n - \zeta_{n,0}| + |\zeta_{n,0} - \hat{\zeta}_{n,0}| \\ &\quad + |\hat{\alpha}_{n-1} - \dot{\alpha}_{n-1}| + y_n \\ &\leq \nu_{n,0} + \gamma_n + \nu_{n,1} + \varsigma_n + y_n = \gamma_n^* \end{aligned} \quad (61)$$

It further gives rise to

$$\begin{aligned} \delta_n^2 - \hat{D}_n^2 &\leq |\delta_n^2 - \hat{D}_n^2| = |\delta_n - \hat{D}_n| |\delta_n + \hat{D}_n| \\ &\leq \gamma_n^* (2|\delta_n| + \gamma_n^*) \leq 3\gamma_n^{*2} + \frac{\delta_n^2}{2} \end{aligned} \quad (62)$$

which indicates

$$\frac{\delta_n^2}{2} \leq 3\gamma_n^{*2} + \hat{D}_n^2 \quad (63)$$

Thus, we can rewrite (59) as

$$\dot{V}_{en} \leq (v_{n,1} + \varsigma_n) |e_n| - (k_n - 3\lambda_n \gamma_n^{*2}) e_n^2 + \frac{1}{2\lambda_n} \quad (64)$$

Let  $k_n = 3\lambda_n \gamma_n^{*2} + k_{n0}$ , where  $k_{n0} > 0$

$$\dot{V}_{en} \leq (v_{n,1} + \varsigma_n) |e_n| - k_{n0} e_n^2 + \frac{1}{2\lambda_n} \quad (65)$$

The design process of adaptive tracking controller has been completed.

#### IV. STABILITY ANALYSIS

In this section, the main results will be stated and the convergence of the disturbance observer will be proven.

*Theorem 1:* Consider the nonlinear system (1) under Assumptions 1-2. The virtual control laws are constructed as (14) and (36). The sliding mode disturbance observers without any bounded conditions are designed according to (15), (37) and (58). Based on the designed disturbance observers, the actual control law is proposed as (57). The tracking error  $e_1 = x_1 - y_d$  converges to a small neighborhood of the origin by appropriately choosing designed parameters.

*Proof:* To analyze the stability of the closed-loop system, we consider the following Lyapunov function candidate:

$$V = \sum_{i=1}^n V_{ei} \quad (66)$$

It follows from (22), (44) and (65) that the time derivative of  $V$  is:

$$\begin{aligned} \dot{V} &\leq - \sum_{i=1}^n k_{i0} e_i^2 + \sum_{i=2}^n (v_{i,1} + \varsigma_i) |e_i| \\ &\quad + \sum_{i=1}^{n-1} g_i(x_i) e_i e_{i+1} + \sum_{i=1}^n \frac{1}{2\lambda_i} \\ &\leq - \sum_{i=1}^n k_{i0} e_i^2 + \sum_{i=2}^n v_{i,2} |e_i| \\ &\quad + \sum_{i=1}^{n-1} g_i(x_i) e_i e_{i+1} + \sum_{i=1}^n \frac{1}{2\lambda_i} \end{aligned} \quad (67)$$

where  $v_{i,2} = v_{i,1} + \varsigma_i$ .

In combination with Assumption 2 and the following inequalities:

$$v_{i,2} |e_i| \leq \frac{1}{2} \left( \frac{v_{i,2}^2 e_i^2}{c_1} + c_1 \right) \quad (68)$$

$$g_i(x_i) e_i e_{i+1} \leq \frac{1}{2} g_M (e_i^2 + e_{i+1}^2) \quad (69)$$

Then, we can rewrite (67) as

$$\begin{aligned} \dot{V} &\leq - \sum_{i=1}^n k_{i0} e_i^2 + \sum_{i=2}^n \frac{1}{2} \left( \frac{v_{i,2}^2 e_i^2}{c_1} + c_1 \right) + \sum_{i=1}^n \frac{1}{2\lambda_i} \\ &\quad + \sum_{i=1}^{n-1} \frac{1}{2} g_M (e_i^2 + e_{i+1}^2) \end{aligned} \quad (70)$$

Noting that  $\sum_{i=1}^{n-1} \frac{1}{2} g_M (e_i^2 + e_{i+1}^2) \leq \sum_{i=1}^n g_M e_i^2$ , one further has

$$\begin{aligned} \dot{V} &\leq - \sum_{i=1}^n k_{i0} e_i^2 + \sum_{i=1}^n \frac{1}{2} \frac{v_{i,2}^2}{c_1} e_i^2 + \sum_{i=1}^n g_M e_i^2 \\ &\quad + \frac{1}{2} n c_1 + \sum_{i=1}^n \frac{1}{2\lambda_i} \end{aligned} \quad (71)$$

where  $c_1, v_{i,2}$  and  $g_M$  are positive constants.

Setting  $k_{i0} = \frac{1}{2} \frac{v_{i,2}^2}{c_1} + g_M + k_{i1}$ , with  $k_{i1}$  being a positive constant, then (71) can be expressed by

$$\begin{aligned} \dot{V} &\leq - \sum_{i=1}^n k_{i1} e_i^2 + \frac{1}{2} n c_1 + \sum_{i=1}^n \frac{1}{2\lambda_i} \\ &\leq -C_1 V + C_2 \end{aligned} \quad (72)$$

where  $C_1 = 2k_{i1}$  and  $C_2 = \frac{1}{2} n c_1 + \sum_{i=1}^n \frac{1}{2\lambda_i}$ .

Using (72), we can obtain

$$\begin{aligned} V(t) &\leq (V(0) - C_3) e^{-C_1 t} + C_3 \\ &\leq V(0) + C_3 \end{aligned} \quad (73)$$

According to (66) and (73), the following inequality holds:

$$\lim_{t \rightarrow \infty} |e_1| \leq \sqrt{2C_3} \quad (74)$$

It can be observed from the definition that  $C_3 = C_2/C_1$  can be adjusted to arbitrarily small by increasing  $\lambda_i$ . Therefore, by appropriately online-tuning the design parameters, the tracking error  $e_1$  converges to an arbitrarily small neighborhood of the origin.

*Remark 5:* Specifically, two cases on the compounded disturbance  $\delta_i(x, t)$  (i.e.  $\delta_i(x, t)$  is bounded and unbounded) are considered: 1) As for the case of  $\delta_i(x, t)$  being bounded, one sees that  $V$  and the tracking error  $e_i, i = 1, 2, \dots, n$  are bounded from (73). So for  $e_1 = x_1 - y_d$  and  $y_d$  being bounded,  $x_1$  is certainly bounded. Taking (18) into account, the estimate of the compounded disturbance  $\hat{D}_1$  is bounded under this case. Since  $\alpha_1$  is a function of bounded signals  $x_1, e_1, \dot{y}_d$  and  $\hat{D}_1$ , the virtual control law  $\alpha_1$  is also bounded. Noting  $x_i = e_i + \alpha_{i-1}$ , it can be seen that  $\alpha_{i-1}$  and state variables  $x_{i,i} = 2, 3, \dots, n$  are bounded, and similarly, the actual control law  $u$  is bounded. Therefore, all the signals of the closed-loop system are bounded; 2) As for the case of  $\delta_i(x, t)$  being unbounded, the estimates of  $\delta_i(x, t)$ , namely  $\hat{D}_i$ , are unbounded according to (40), which results in that the virtual control laws  $\alpha_i$  and the control input  $u$  are unbounded since  $\hat{D}_i$  are included in them as seen in (36) and (57). This means that an unbounded control effort is required so as to circumvent the influence brought by unbounded disturbances. However, it should be pointed out that the boundedness of the tracking error  $e_1$  and the tracking performance can be still guaranteed in this case.

This completes the proof. ■

V. SIMULATION RESULTS

In this section, two simulation examples are given to demonstrate the effectiveness of designed method.

Example 1: To illustrate the validity of the proposed control scheme, consider the following second-order nonlinear system with disturbance and its derivative being unbounded as follows:

$$\begin{cases} \dot{x}_1 = x_1 e^{-0.5x_1} + (1 + e^{-0.1x_1^2})x_2 \\ \dot{x}_2 = -p_1 x_1 - p_2 x_2 - x_1^3 + q \cos(\omega t) + u + \delta(x, t) \\ y = x_1 \end{cases} \quad (75)$$

where the compounded disturbance is given by  $\delta(x, t) = x_1^2 + x_1 + t \sin(t)$ . For the purpose of simulation, we suppose that  $p_1 = 0.3 + 0.2 \sin(10t)$ ,  $p_2 = 0.2 + 0.2 \cos(5t)$ ,  $q_0 = 5 + 0.1 \cos(t)$  and  $\omega = 0.5 + 0.1 \sin(t)$ . Let the desired trajectory be  $y_d = \sin(0.5t)$ .

Remark 6: Differently from the state-of-the-art, it can be seen that the compounded disturbance  $\delta(x, t)$  grows with time  $t$ , so it can be easily verified that the compounded disturbance  $\delta(x, t)$  is not bounded by upper and lower bounds, and moreover, its derivative is also unbounded. This specific example breaks the conventional bound assumptions and makes the control design extremely challenging, so in authors' opinion, the existing works cannot be applied. To overcome this difficulty, we firstly propose a disturbance observer design method without bounded assumptions, which basically distinguishes our work from all available methods.

In accordance with Theorem 1, the adaptive tracking controller is proposed as (57) and the disturbance observers are given as (15) and (58). For the compounded disturbance  $\delta(x, t)$ , the design parameters are set as:  $\varepsilon_{1,0} = 10$ ,  $\varepsilon_{1,1} = 1$ ,  $\mu_{1,0} = 0.1$ ;  $\sigma_{2,0} = 30$ ,  $\sigma_{2,1} = 1$ ,  $\mu_{2,1} = 0.5$ ;  $\varepsilon_{2,0} = 20$ ,  $\varepsilon_{2,1} = 1$ ,  $\mu_{2,0} = 0.1$ . The other design parameters are taken as  $k_1 = 6$ ,  $\lambda_1 = 1$  and  $k_2 = 4$ ,  $\lambda_2 = 1$ . Let the initial conditions for  $[x_1(0), x_2(0)]^T = [0, 0]^T$ ,  $\rho_{1,1}(0) = \rho_{2,1}(0) = \vartheta_{1,1}(0) = 0$ ,  $\hat{\zeta}_{1,0}(0) = \hat{\zeta}_{2,0}(0) = \hat{a}_1(0) = 0$ . The simulation results are shown as Figs. 2-6.

It can be obviously observed from Fig. 2 that the system output  $y$  can follow the desired trajectory  $y_d$  and fairly good tracking performance is obtained. Fig. 3 shows the tracking

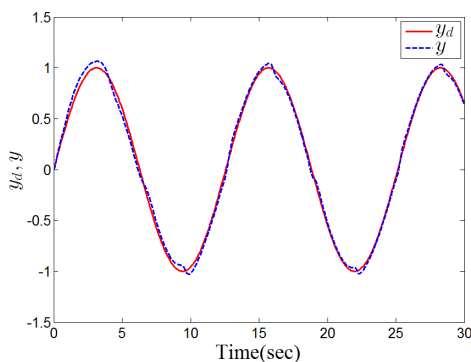


FIGURE 2. System output  $y$  and desired trajectory  $y_d$ .

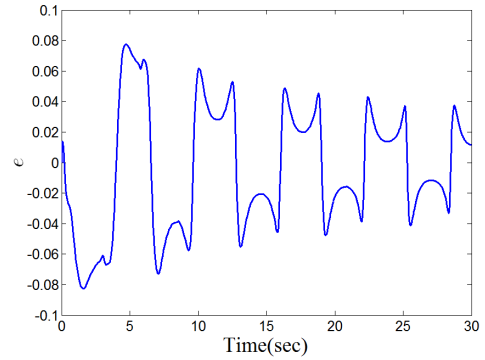


FIGURE 3. Tracking error  $e$ .

error  $e$  converges to an allowable range. The response curve of the system state  $x_2$  is depicted in Fig. 4. And the control input  $u$  is shown in Fig. 5. Specially, it can be seen from Fig. 6 that the disturbance observer  $\hat{D}$  can approach the growing compounded disturbance  $\delta$  effectively. It should be noted that excellent tracking performance has been achieved even though the compounded disturbance is unbounded by utilizing the first order sliding mode differentiator in our paper, which enhances the robustness and reliability of the system drastically.

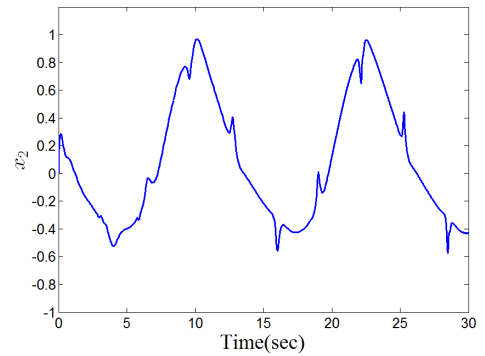


FIGURE 4. System state  $x_2$ .

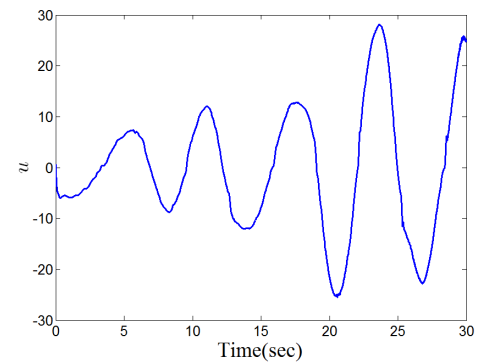


FIGURE 5. Control input  $u$ .

Remark 7: In most existing traditional disturbance observer design approaches, to the best of authors' knowledge, the disturbance observer is always designed using a constant.



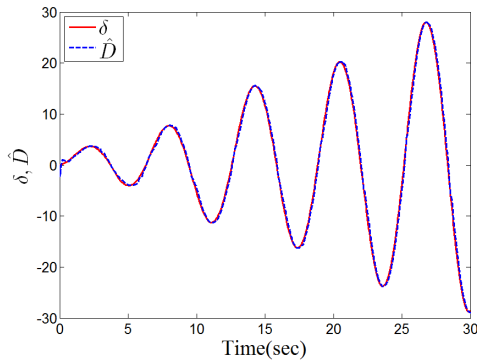


FIGURE 6. Compounded disturbance  $\delta$  and its estimate  $\hat{D}$ .

It is very restrictive arising from the fact that the bound for disturbance may not exist with the system state and time in practice, implies that the effect of disturbance cannot be assumed to be bounded before obtaining stability. Thanks to the slide mode differentiators and hyperbolic tangent function, the error dynamics in our paper have a clear advantage over it in standard disturbance observer-based design, even under the case that the disturbance is unbounded and non-smooth. It is noticed that the proposed disturbance observer can come across these difficulties for the reason that the restrictive assumptions on the disturbance terms are removed. Therefore, compared with conventional disturbance observer designs, our scheme has more relaxed condition and a wider range of application.

*Example 2:* To further verify the effectiveness of the proposed scheme, consider the following tracking control problem for a pole-balancing of an inverted pendulum [39]. The system is represented by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{mlx_2^2 \sin x_1 \cos x_1 - (M + m)g \sin x_1}{ml \cos^2 x_1 - 4/3l(M + m)} \\ \quad + \frac{-\cos x_1}{ml \cos^2 x_1 - 4/3l(M + m)}u + \delta(x, t) \\ y = x_1 \end{cases} \quad (76)$$

where  $x_1$  represents the angle  $\theta$  (in radians) of the pendulum from the vertical,  $M$  is the mass of the cart and  $m$  is the mass of the pole,  $g$  is gravitational constant,  $l$  is the half length of the pole,  $u$  means force applied to the cart and  $\delta(x, t)$  is the compounded disturbance.

The parameters employed in this simulation are given as follows:  $M = 1$  Kg,  $m = 0.1$  Kg,  $l = 0.5$  m,  $g = 9.8$  m/s<sup>2</sup>. The disturbance term  $\delta(x_2, t)$  is a dead-zone model in the presence of non-smooth nonlinearity, which can be given as follows

$$\delta(x, t) = \begin{cases} 10(x_2 - 0.3) + \frac{(x_2 - 0.3)^2}{7}, & x_2 \geq 0.3 \\ 0, & -0.3 < x_2 < 0.3 \\ 10(x_2 + 0.3) + \frac{(x_2 + 0.3)^2}{7}, & x_2 \leq -0.3 \end{cases} \quad (77)$$

It can be seen that the compounded disturbance  $\delta(x_2, t)$  is not partial differentiable with respect to  $x_2$  as non-smooth nonlinearity is present in it. Furthermore, we assume the desired trajectory  $y_d = 0.5(\sin(t) + \sin(0.5t))$ .

In this simulation, the designed parameters are taken as  $\varepsilon_{1,0} = 10$ ,  $\varepsilon_{1,1} = 1$ ,  $\mu_{1,0} = 1$ ;  $\sigma_{2,0} = 10$ ,  $\sigma_{2,1} = 1$ ,  $\mu_{2,1} = 0.1$ ;  $\varepsilon_{2,0} = 10$ ,  $\varepsilon_{2,1} = 1$ ,  $\mu_{2,0} = 0.1$  and  $k_1 = 4$ ,  $\lambda_1 = 1$ ;  $k_2 = 4$ ,  $\lambda_1 = 1$ . Set the initial conditions as  $[x_1(0), x_2(0)]^T = [0.2, 0]^T$ ,  $\rho_{1,1}(0) = \rho_{2,1}(0) = \vartheta_{1,1}(0) = 0$  and  $\hat{\zeta}_{1,0}(0) = \hat{\zeta}_{2,0}(0) = \hat{\alpha}_1(0) = 0$ . The simulation results are shown in Figs. 7-10.

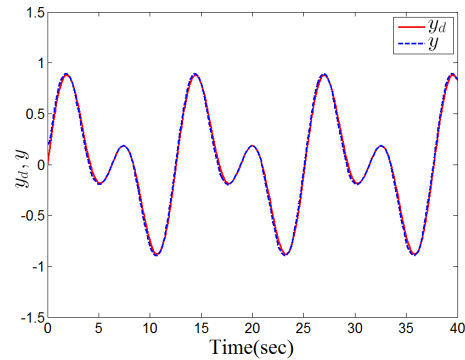


FIGURE 7. System output  $y$  and desired trajectory  $y_d$ .

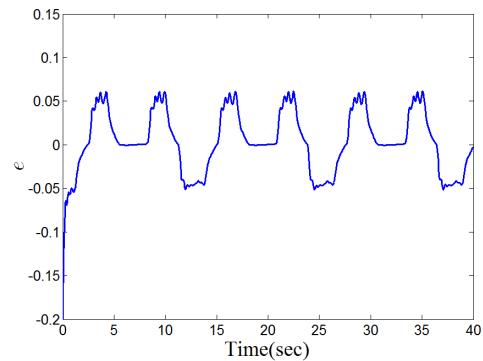


FIGURE 8. Tracking error  $e$ .

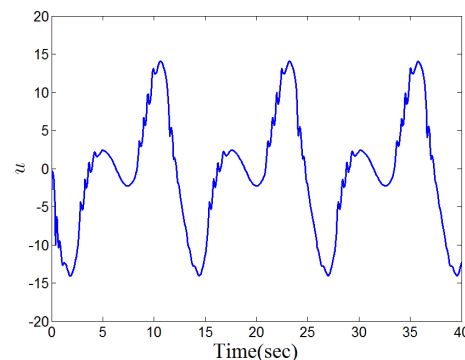


FIGURE 9. Control input  $u$ .

To show the good compensation effect of the disturbance observers on the dynamic response, the output response curves are depicted in Fig. 7 and it can be observed clearly

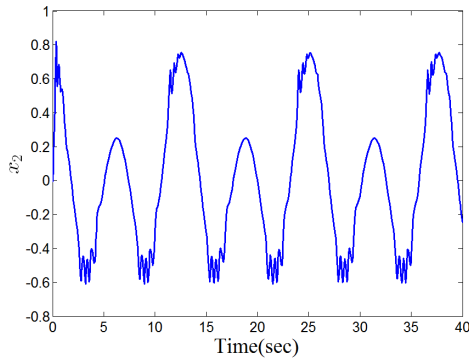


FIGURE 10. System state  $x_2$ .

that system output  $y$  can converge rapidly to the desired trajectory  $y_d$ . The tracking error is acceptable from Fig. 8. And it can be seen from Fig. 9 and Fig. 10 that the control input  $u$  and system state  $x_2$  are bounded. The simulation results of physical system model indicate that the proposed controller based on disturbance observer can achieve excellent tracking performance even though the compounded disturbance is non-differentiable. Particularly, the input variable information of the compounded disturbance is not mentioned in the controller design, thus the prior knowledge of the compounded disturbance can be unknown, which has wide application prospects in practical control systems.

### VI. CONCLUSION

A novel adaptive tracking control scheme based on disturbance observer has been proposed for a class of strict-feedback nonlinear systems under loose disturbance constraint conditions. Compared with the existing approaches, the restrictive assumptions that the compounded disturbance must be bounded, differentiable or slow time-varying are relaxed by only assuming that the disturbance functions are continuous. Without NNs or FLSs techniques, the sliding mode differentiator and the backstepping method have been utilized to estimate the compounded disturbance and design adaptive control laws in proposed control scheme. Moreover, the influences of unknown disturbance and system uncertainties are eliminated without knowing any prior knowledge of the compounded disturbance.

### APPENDIX

*Proof of Lemma 3:* To obtain the conclusion, two cases are discussed as follows:

*Case 1:* For any  $x \in \mathbb{R}$ ,  $|x|^{\frac{1}{2}} + \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \geq 1$

$$\begin{aligned} & \left| |x|^{\frac{1}{2}} - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \right| \\ & \leq \left| |x|^{\frac{1}{2}} - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \right| \cdot \left| |x|^{\frac{1}{2}} + \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \right| \\ & = \left| |x| - x \tanh\left(\frac{x}{\mu}\right) \right| \end{aligned} \quad (78)$$

According to Lemma 1, one has

$$\left| |x|^{\frac{1}{2}} - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \right| \leq |x| - x \tanh\left(\frac{x}{\mu}\right) \leq 0.2785\mu \quad (79)$$

Consider the property of the sign function, we know that

$$\begin{aligned} & \left| |x|^{\frac{1}{2}} \text{sign}(x) - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \text{sign}(x) \right| \\ & = \left| |x|^{\frac{1}{2}} - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \right| \\ & \leq 0.2785\mu \end{aligned} \quad (80)$$

$$\begin{aligned} & \left| \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \text{sign}(x) \right. \\ & \left. - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \text{sign}\left(\frac{x}{\mu}\right) \right| = 0 \end{aligned} \quad (81)$$

Noting that  $\mu$  is an unknown positive constant, it holds that

$$\begin{aligned} & \left| \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \text{sign}\left(\frac{x}{\mu}\right) - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \tanh\left(\frac{x}{\mu}\right) \right| \\ & \leq \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \left| \text{sign}\left(\frac{x}{\mu}\right) - \tanh\left(\frac{x}{\mu}\right) \right| \end{aligned} \quad (82)$$

When  $x > 0$ ,

$$\begin{aligned} & \left| \text{sign}\left(\frac{x}{\mu}\right) - \tanh\left(\frac{x}{\mu}\right) \right| \\ & = \left| 1 - \frac{e^{(x/\mu)} - e^{-(x/\mu)}}{e^{(x/\mu)} + e^{-(x/\mu)}} \right| \\ & = \frac{2e^{-(x/\mu)}}{e^{(x/\mu)} + e^{-(x/\mu)}} \leq e^{-(x/\mu)} \end{aligned} \quad (83)$$

When  $x < 0$ ,

$$\begin{aligned} & \left| \text{sign}\left(\frac{x}{\mu}\right) - \tanh\left(\frac{x}{\mu}\right) \right| \\ & = \left| -1 - \frac{e^{(x/\mu)} - e^{-(x/\mu)}}{e^{(x/\mu)} + e^{-(x/\mu)}} \right| \\ & = \frac{2e^{(x/\mu)}}{e^{(x/\mu)} + e^{-(x/\mu)}} \leq e^{(x/\mu)} \end{aligned} \quad (84)$$

In view that the difference between the sign function and the hyperbolic tangent function shows exponential growth, one reaches

$$\begin{aligned} & \left| \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \text{sign}\left(\frac{x}{\mu}\right) - \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \tanh\left(\frac{x}{\mu}\right) \right| \\ & \leq \left(x \tanh\left(\frac{x}{\mu}\right)\right)^{\frac{1}{2}} \left(e^{-|x/\mu|}\right) \leq \gamma^* \end{aligned} \quad (85)$$

where  $\gamma^*$  is a positive constant.

Therefore, we can obtain that

$$\begin{aligned} & \left| |x|^{\frac{1}{2}} \operatorname{sign}(x) - \left( x \tanh \left( \frac{x}{\mu} \right) \right)^{\frac{1}{2}} \tanh \left( \frac{x}{\mu} \right) \right| \\ & \leq \left| |x|^{\frac{1}{2}} \operatorname{sign}(x) - \left( x \tanh \left( \frac{x}{\mu} \right) \right)^{\frac{1}{2}} \operatorname{sign}(x) \right| \\ & \quad + \left| \left( x \tanh \left( \frac{x}{\mu} \right) \right)^{\frac{1}{2}} \operatorname{sign}(x) \right. \\ & \quad \left. - \left( x \tanh \left( \frac{x}{\mu} \right) \right)^{\frac{1}{2}} \operatorname{sign} \left( \frac{x}{\mu} \right) \right| \\ & \quad + \left| \left( x \tanh \left( \frac{x}{\mu} \right) \right)^{\frac{1}{2}} \operatorname{sign} \left( \frac{x}{\mu} \right) \right. \\ & \quad \left. - \left( x \tanh \left( \frac{x}{\mu} \right) \right)^{\frac{1}{2}} \tanh \left( \frac{x}{\mu} \right) \right| \\ & \leq 0.2785\mu + \gamma^* = \gamma \end{aligned} \quad (86)$$

Case 2: For any  $x \in \mathbb{R}$ ,  $|x|^{\frac{1}{2}} + \left( x \tanh \left( \frac{x}{\mu} \right) \right)^{\frac{1}{2}} < 1$

It is easy to know that  $|x| < 1$ , and then we further have

$$\left| |x|^{\frac{1}{2}} \operatorname{sign}(x) - \left( x \tanh \left( \frac{x}{\mu} \right) \right)^{\frac{1}{2}} \tanh \left( \frac{x}{\mu} \right) \right| \leq \gamma \quad (87)$$

This completes the proof. ■

*Proof of Lemma 4:* By noting that  $\tau\dot{\beta} + \beta = \alpha$  and  $y = \beta - \alpha$ , one has  $\dot{\beta} = -\frac{y}{\tau}$

Choose the following quadratic function as

$$V_F = \frac{1}{2}y^2 \quad (88)$$

The time derivative of  $V_F$  is

$$\dot{V}_F = y\dot{y} = y\left(-\frac{y}{\tau} - \dot{\alpha}\right) = -\frac{y^2}{\tau} - y\dot{\alpha} \quad (89)$$

Applying a first order sliding mode differentiator in Lemma 2 yields to get  $\hat{\alpha}$ , it follows that

$$\left| \dot{\alpha} - \hat{\alpha} \right| \leq \varepsilon \quad (90)$$

where  $\varepsilon$  is a positive constant.

Choose  $\frac{1}{2\tau} = \hat{\alpha}^2 + \varepsilon_0$  where  $\varepsilon_0 > 0$  being the designed parameter, and then we can obtain

$$\begin{aligned} -\frac{y^2}{2\tau} - y\dot{\alpha} &= -(\hat{\alpha}^2 + \varepsilon_0)y^2 - y\dot{\alpha} \\ &\leq \frac{1}{4} - \varepsilon_0y^2 - \left| \hat{\alpha} \right| |y| - y\dot{\alpha} \\ &\leq \frac{1}{4} - \varepsilon_0y^2 + |\varepsilon y| \end{aligned} \quad (91)$$

Substituting (91) into (89) yields, it holds that

$$\begin{aligned} \dot{V}_F &\leq -\frac{y^2}{2\tau} + \frac{1}{4} - \varepsilon_0y^2 + |\varepsilon y| \\ &\leq -\left(\varepsilon_0 - \frac{1}{2} + \frac{1}{2\tau}\right)y^2 + \frac{1}{4} + \frac{1}{2}\varepsilon^2 \\ &\leq -c_1V_F + c_2 \end{aligned} \quad (92)$$

where  $c_1 = 2\varepsilon_0 - 1 + \frac{1}{\tau}$  and  $c_2 = \frac{1}{4} + \frac{1}{2}\varepsilon^2$ .

Therefore, we can know that the filtering error  $y$  can be regulated to arbitrarily small by appropriately online-tuning the design parameters.

This completes the proof. ■

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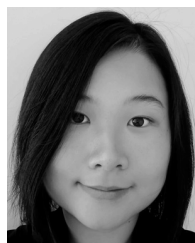
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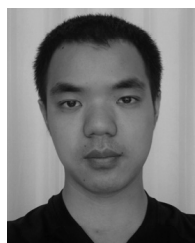
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