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# An Improved Blind Recognition Method of the Convolutional Interleaver Parameters in a Noisy Channel

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**ABSTRACT** In a digital communication system, forward error correction (FEC) codes and interleavers are implemented to code the data so as to improve the error performance, which is hindered by random and burst errors. In the context of noncooperative communication, interleaver parameter recognition, which is the prerequisite for frame synchronization, channel coding recognition subsequently, is of vital significance. Methods of blindly recognizing convolutional interleaver parameters have been proposed in published literature, but the accuracy of recognition decreases significantly when bit error rate (BER) is high. To improve the performance of the algorithm, the effect of error bits on Gauss-Jordan elimination through pivoting (GJTEP) algorithm is analyzed in this paper. The following conclusion is drawn: error bits on the principal diagonal of data storage matrix will exert a great impact on the recognition accuracy. Based on the conclusion, an improved blind recognition method with denoising, the core principle of which is reducing error bits on the principal diagonal of data storage matrix, is proposed in this paper. The simulation experiment results demonstrate that the performance on error resilience is markedly improved.

**INDEX TERMS** Convolutional interleavers, blind recognition, noncooperative communication, Gauss-Jordan elimination through pivoting (GJTEP) algorithm, denoising.

## I. INTRODUCTION

In a digital communication system, interleavers are usually implemented after FEC to resist burst errors. Interleavers can transform burst errors into independent random errors by exchanging the rows of the data matrix. The independent random errors are then corrected by FEC, contributing to the enhancement of the reliability of the communication system. Two types of interleavers are mainly considered, block interleavers and convolutional interleavers. For block interleavers, the data exchanging is limited in a complete interleaving block [1], while convolutional interleavers have a certain memory [2], [3].

In recent years, many scholars have focused on the analysis of the blind recognition of convolutional interleavers parameters in the context of noncooperative communication. Some methods have been proposed in published literatures. In [4], a classic algorithm based on principle component analysis (PCA) is proposed to solve the problem of block

interleaver coding in the noisy background, and more detailed explanation along with theoretical analysis are given in [5], which has laid a firm foundation for the recognition of convolutional interleaver parameters. Literature [6]–[9] have proposed effective solutions for the recognition of convolutional interleaver parameters with noise. Method in [6] includes a 4-dimension searching, but the algorithm is time-consuming and even difficult to realize. The algorithm is simplified in [8]. Algorithms based on rank criteria [10] are proposed in [7]–[9] to blindly estimate the convolutional interleaver parameters. However, the algorithms work only when the codeword length  $n$  equals the product of the interleaving depth  $M$  and the interleaving width  $B$ . There are also drawbacks on error performances. In [11], the algorithm is extended to  $GF(2^m)$  case assuming non-binary codes. In [12], [13], algorithms based on zero-to-mean-ratio are proposed to overcome the effect of error bits. The algorithm in [12] realizes the blind recognition of convolutional interleaver parameters when the codeword length is not equal to the product of the interleaving depth  $M$  and the interleaving width  $B$ . The error performance has a certain amount of

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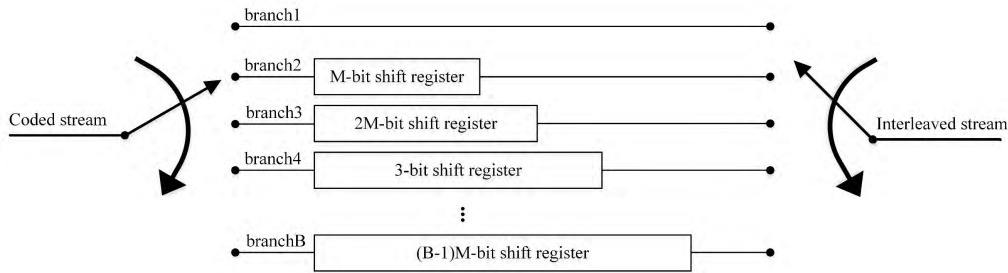


FIGURE 1. Structure of convolutional interleaver.

improvement, but when BER continues to increase, the accuracy of recognition is still relatively low. In [14], [15], a blind interleaver parameters estimation method enhanced by identifying relatively error-less partial symbols among intercepted streams is proposed. In [16], Swaminathan *et al.* propose a algorithms for the joint recognition of the type of FEC codes and interleaver parameters without knowing any information about the channel encoder. The proposed algorithm classifies the incoming data symbols among block coded, convolutional coded, and uncoded symbols. In [17], an efficient method is given to reconstruct the block interleaver and recover the convolutional code when several noisy interleaved codewords are given. In [18], algorithms blind estimation algorithms to identify RS code parameters are given. On top of that, estimate block interleaver parameters from RS coded and block interleaved data stream are proposed. In this paper, the mechanism error bits impacting recognition accuracy is analyzed and we find out that error bits on the principal diagonal of the data storage matrix play an important role in decreasing recognition accuracy. Then the denoising algorithm based on reducing error bits on the principal diagonal of data storage matrix is proposed to improve error performance. Simulation experiments are carried out and the results prove the effectiveness of the algorithm proposed in this paper.

The channel assumed in this paper is additive white Gaussian noise (AWGN) channel. The amplitude of additive white Gaussian noise follows Gaussian distribution. The spectrum components of AWGN follow uniform distribution in the whole bandwidth. The power spectral density (PSD) of AWGN is a constant in the range of entire bandwidth. These characteristics of AWGN channel determines that the errors are randomly distributed. Consequently, we can regard that the errors in the received bitstream are randomly distributed, which is one of the assumptions of the simulations. It is worth noting that the parameter SNR in AWGN channel simulation is converted into BER by calculating the ratio of errors and total bits.

The remainder of this paper is organized as follows. Section II briefly introduces the structure of convolutional interleavers. In section III, the blind recognition algorithm of convolutional interleaver parameters is summarized. Section IV analyzes the influence of error bits on the principal

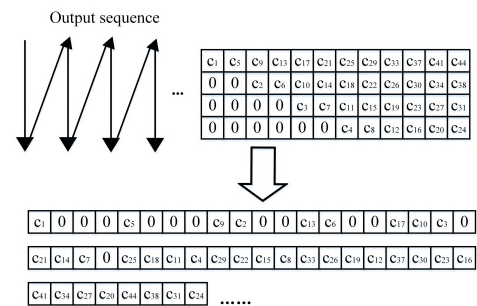


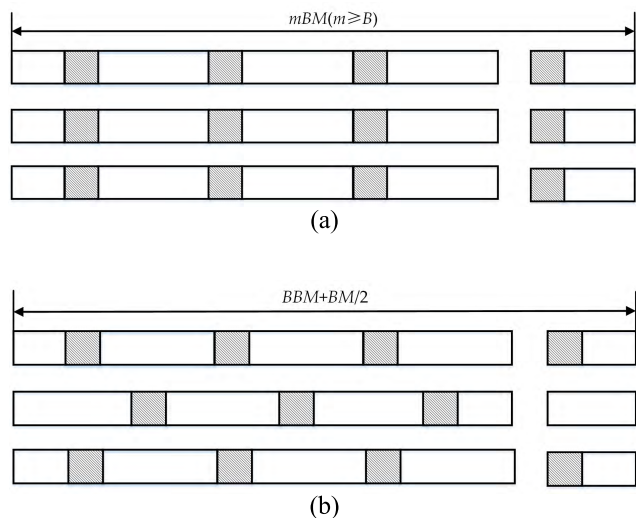
FIGURE 2. Output of CI (4,2).

diagonal of data storage matrix on the recognition accuracy. Based on the conclusion drawn in section IV, a novel denoising algorithm is proposed in section V to enhance error performance by reducing error bits on the principal diagonal of data storage matrix. In section VI, the simulation results are presented and finally the conclusion is given in VII.

## II. STRUCTURE OF CONVOLUTIONAL INTERLEAVERS

The convolutional interleaver was proposed in 1970 and 1971 by Ramsey [3] and Forney [2]. The structure is demonstrated in Figure 1. The convolutional interleaving encoder includes  $B$  shift registers with different length, realizing the delay of input data in each branch. The bitstream before entering the interleaver is a FEC codeword sequence.  $B$  bits are a group and entered in  $B$  branches. Then the output of the shift registers is regarded as the output of the convolutional interleaver. The delay of each branch is different. The delay of the first branch is zero, the delay of the second branch is  $M$  bits, the delay of the third branch is  $2M$  bits, ..., the delay of the  $i$ th branch is  $(i-1)M$  bits. In this paper, the convolutional interleaver is written as CI  $(B, M)$ . The structure of convolutional interleaver is shown in Figure 1.

Here we take CI (4,2) interleaver as an example. Assume that the input bitstream is  $c_1, c_2, c_3, \dots$ . As can be seen in Figure 2, the output of the interleaver is  $c_1, 0, 0, 0, c_5, 0, 0, 0, c_9, c_2, 0, 0, c_{13}, c_6, 0, 0, c_{17}, c_{10}, c_3, 0, c_{21}, c_{14}, c_7, 0, c_{25}, c_{18}, c_{11}, c_4, c_{29}, c_{22}, c_{15}, c_8, \dots$



**FIGURE 3.** Illustration of the storage matrix. The shaded boxes represent frame synchronization symbols. (a)  $mBM$ , ( $m \geq B$ ) columns; (b)  $BBM + BM/2$  columns.

### III. BLIND RECOGNITION ALGORITHM OF CONVOLUTIONAL INTERLEAVERS PARAMETERS

The assumption  $n = BM$  is proposed in [4],  $n$  is the length of codeword before entering the interleaver. But this assumption is not necessarily true in practical applications. We assume that the received bit sequence is  $Y$ :

$$Y = [y_1 \ y_2 \ y_3 \ \dots]. \quad (1)$$

The data storage matrix  $R_l$  is defined as:

$$R_l = \begin{bmatrix} y_1 & y_2 & \dots & y_1 \\ y_{1+1} & y_{1+2} & \dots & y_{21} \\ \vdots & \vdots & \ddots & \vdots \\ y_{(\mu-1)l+1} & y_{(\mu-1)l+2} & \dots & y_{\mu l} \end{bmatrix}, \quad (2)$$

where  $\mu$  and  $l$  are the numbers of rows and columns of  $R_l$  respectively ( $\mu \gg l$ ).

The following conclusion has been given in [8]. There are several complete FEC coding codewords in each row of  $R_l$  and the position of the codeword is fixed in each row when  $l \geq BBM$  and  $l$  is the integer multiple of  $BM$  (shown in Figure 3). So, Gauss-Jordan elimination through pivoting (GJETP) [19] algorithm can be applied to find dependent columns [5], [8] in  $R_l$ . When  $l < BBM$  or  $l$  is not the integer multiple of  $BM$ , dependent columns will not be found in  $R_l$ .

But in actual situation, the assumption  $n = BM$  is not always true. When  $n \neq BM$ , the conclusion above will be incorrect. The least common multiple of  $x$  and  $y$  is represented by  $\text{lcm}(x, y)$ . When the number of columns of  $R_l$  is  $l \geq \lambda \text{lcm}(n, B)$  and  $l$  is an integer multiple of  $\text{lcm}(n, B)$ , there are several complete FEC codewords in each row of  $R_l$ , the positions of which are fixed in each row. The parameter  $\lambda$  is a positive integer, depending on the structure of the interleaving parameters and the parity

matrix of the FEC coding before interleaving. So GJETP algorithm can be applied to find dependent columns. When  $l < nB$  or  $l$  is not an integer multiple of  $nB$ , there are no dependent columns in  $R_l$ . Therefore, the following steps can be taken to estimate the convolutional interleaving parameters.

#### A. ESTIMATE THE LEAST COMMON MULTIPLE $\text{lcm}(n, B)$ OF $n$ AND $B$

Traverse all values of  $l$  in the preset searching range  $l_{\min}$  to  $l_{\max}$ , filling the received bit sequence in  $Y$  into the storage matrix  $R_l$  with  $l$  columns. For each value of  $l$ , apply GJETP algorithm to the corresponding  $R_l$  to acquire the lower triangular matrix  $R'_l$ . Then the normalized rank of  $R'_l$  is calculated. According to the rank criteria in [10], the dependent columns can be detected. During the traverse, record the value of  $l$  as  $N_{l1}$  for the first time finding the dependent column and record the value of  $l$  as  $N_{l2}$  for the second time. Then the least common multiple  $\text{lcm}(n, B)$  can be estimated as  $\hat{N} = \text{lcm}(n, B) = N_{l2} - N_{l1}$ .

#### B. ESTIMATE $B, M$ AND SYNCHRONIZATION POSITION $d_0$

According to the structural characteristics of the convolutional interleavers, algorithm 1 is proposed to estimate the parameters  $B, M$  and  $d_0$ .

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#### Algorithm 1 Estimation of Parameters $B, M$ and $d_0$

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**Notations:**  $M_{\max}$  denotes the maximum value of interleaver width.  $\Phi$  is the set of all the factors of  $\hat{N} = \text{lcm}(n, B)$  (except 1).  $R$  is data storage matrix;

**Assumptions:** The size of  $\Phi$  is  $1 \times \eta$ . The received data stream is assumed to have bit errors;

**Input:** The least common multiple  $\hat{N} = \text{lcm}(n, B)$ . The received bit sequence  $Y$ ;

**Output:**  $\hat{B}, \hat{M}$  and  $\hat{d}_0$ ;

1: Take all the factors of  $\text{lcm}(n, B)$  (except 1) as candidates of  $B$ , recorded as  $\Phi = \{B_1 B_2 \dots B_\eta\}$ ;

2: **for**  $i \leq \eta$  **do**

**for**  $M \leq M_{\max}$  **do**

**for**  $d < \hat{N}$  **do**

Deinterleave the bit sequence  $Y$  starting from the  $d$ th bit of the received bit sequence with parameters  $B_i$  and  $M$ . Then fill the deinterleaved data into the matrix  $R$  with  $\hat{N}$  bits per row;

Apply GJETP algorithm on  $R$  to obtain the lower triangular matrix  $R'$  and calculate the normalized rank of  $R'$ , recorded as  $\rho_{i,M,d}$ ;

**end**

**end**

**end**

3:  $[\hat{B}, \hat{M}, \hat{d}_0] = \underset{B_i, M, d}{\text{argmin}}(\rho_{i,M,d})$ ;

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**IV. ANALYSIS ON THE IMPACT OF ERROR BITS ON THE PRINCIPAL DIAGONAL OF DATA STORAGE MATRIX**

According to the previous sections, the key steps of blind recognition algorithm of convolutional interleaver parameters are listed as follows:

1. The construction of data storage matrix.
2. Acquiring the lower triangular matrix by GJETP algorithm.
3. Calculating the normalized rank of the lower triangular matrix and observing whether there is rank-deficiency [5], [8], [12].
4. Estimating the lcm ( $n, B$ ).

Once the value of lcm ( $n, B$ ) is obtained correctly, the parameters  $B, M, d$  can be recognized by the algorithm described in section III (2) easily.

By analyzing the principle of the proposed blind recognition algorithms, it can be concluded that the core of the algorithms is finding the rank-deficiency matrices. However, when BER is comparatively high, error bits in the data storage matrix will impact the result of GJETP algorithm, which will probably lead to the miscalculation of the normalized rank. To overcome the influence of error bits, Hamming weight of the column and probability threshold are introduced to determine dependent columns [5]. In [12], the denoising algorithm based on zero-mean-ratio is proposed. Compared with the algorithm in [5], the error performance of the algorithm in [12] is improved. To achieve better error performance, the impact of error bits on the principal diagonal of data storage matrix is analyzed, and then a novel denoising algorithm is proposed.

GJETP algorithm is essentially a kind of linear transformation method. We briefly recall the GJETP for the binary field. A storage matrix  $X$  with size  $\mu \times n$  is given. Let  $L = X$ , and then we initialize two identity matrices  $A$  and  $B$  with size  $\mu \times \mu$  and  $n \times n$  respectively. We denote  $z^i$  as the  $i$ th column of a given matrix  $Z$ . Three steps of this algorithm are as follows:

For  $i = 1$  to  $i = n$  do

1. If the  $i$ th element of  $l^i$  is equal to zero, exchange  $l^i$  with the first  $l^{i'}$  ( $i' > i$ ) that has a one on its  $i$ th element. Exchange  $b^i$  and  $b^{i'}$ .
2. If the  $i$ th element of  $l^i$  is equal to zero, exchange the  $i$ th row of  $L$  with its first row  $i'$  ( $i' > i$ ) that has a one on its  $i$ th element. Exchange the  $i$ th row of  $A$  with its  $i'$ .
3. If the  $i$ th element of  $l^i$  is equal to one, xor  $l^i$  to any  $l^{i'}$  ( $i' > i$ ) that has a one on its  $i$ th row and xor  $b^i$  and  $b^{i'}$ .

End for, output  $L, A$  and  $B$ .

Based on this algorithm, the relationship between  $L, A$  and  $B$  is:

$$L = AXB \tag{3}$$

$L$  is a lower triangular matrix.

Two conjectures are raised according to the principle of GJETP algorithm.

*Conjecture 1:* Error bits on the principal diagonal of data storage matrix will cause a more serious error propagation

**TABLE 1. The simulation conditions.**

FEC coding	Convolutional interleaver	Columns of data storage matrix	Rows of data storage matrix	Number of error bits (BER)
BCH (7,4)	CI (3,2)	42	500	210(0.01)
BCH (7,4)	CI (4,2)	56	375	210(0.01)
BCH (15,11)	CI (5,3)	75	600	450(0.01)

after GJETP and the recognition accuracy of rank deficient matrix will be decreased.

*Conjecture 2:* Assume that the number of error bits on the principal diagonal of data storage matrix is  $\alpha$ , the total number of bits on the principal diagonal of data storage matrix is  $\beta$ . When  $E = \alpha/\beta$  is a constant value, the recognition accuracy varies inversely with the number of data storage matrix columns.

Numerical simulations are carried out to check the conjecture 1. Firstly, the impact of error bits on the principal diagonal on the lower triangular matrix obtained by GJETP algorithm is analyzed. Keep the total number of error bits fixed and vary the number of error bits on the principal diagonal. In order to measure the level of error propagation after GJETP algorithm, the concept of error-propagation-index (EPI) is introduced. Assume that there are no errors in the received bitstream  $Y$ , then  $Y$  is filled into the data storage matrix  $R$ . A lower triangular matrix  $R_1$  can be obtained by applying GJETP algorithm on  $R$ . When there are errors in  $Y$ , the lower triangular matrix is denoted as  $R_2$ . Take  $R_1$  and  $R_2$  into consideration, count the number of bit-flipping cases in the same position of the matrices  $R_1$  and  $R_2$ , denoted as  $\theta$ . The total number of elements of the matrix is  $\lambda$ , EPI is defined as:

$$EPI = \theta/\lambda \tag{4}$$

EPI describes the influence that error bits exert on the lower triangular matrix after GJETP algorithm. When EPI is high, it is indicated that the error propagation is severe. Obviously, the range of EPI is (0,1).

The conditions of the simulation are listed in TABLE 1. Each EPI value is the mean value of 100 simulation results. The trend of EPI variation is demonstrated in FIGURE 4.

From FIGURE 4 we can observe that EPI is near zero when there are no error bits on the principal diagonal. While EPI rockets up as soon as the number of error bits on the principal diagonal increases. Finally, the curve of EPI flattens out with the growth of abscissa value. So, the conclusion can be drawn that error bits on the principal diagonal will cause fateful error propagation when applying GJETP algorithm. The effect of error propagation on the recognition accuracy will be analyzed in the following part.

For the sake of exploring the relationship between the number of error bits on the principal diagonal and recognition accuracy of rank deficient matrix, another simulation experiment is carried out. The simulation conditions are listed in TABLE 2. Keep the total number of error bits fixed and

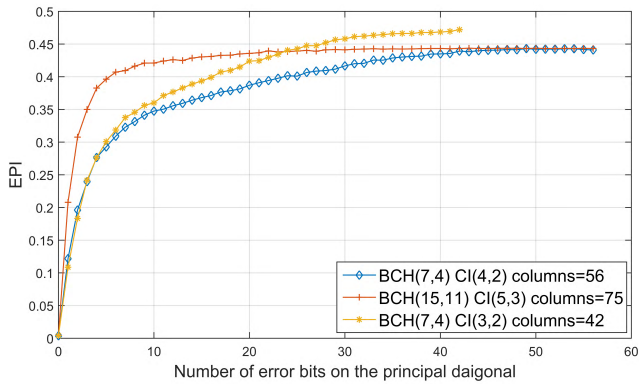


FIGURE 4. Trend of EPI variation with the increasing of number of error bits on the principal diagonal.

TABLE 2. The simulation conditions.

FEC coding	Convolutional interleaver	Columns of data storage matrix	Rows of data storage matrix	BER
BCH (7,4)	CI (4,2)	56	375	0.0044
BCH (7,4)	CI (4,2)	56	375	0.0061
BCH (7,4)	CI (4,2)	56	375	0.0108
BCH (7,4)	CI (4,2)	56	375	0.0134
BCH (7,4)	CI (4,2)	56	375	0.0163
BCH (7,4)	CI (4,2)	56	375	0.0213

vary the number of error bits on the principal diagonal. FIGURE 5 shows the recognition accuracy of rank deficient matrix at different BER.

As can be seen in FIGURE 5, the recognition accuracy keeps a high level when there are no error bits on the principal diagonal. However, if the number of error bits on the principal diagonal exceeds a certain threshold, the recognition accuracy will decrease dramatically. When the number of data storage matrix columns is fixed, the threshold depends on BER. Overall, the higher BER is, the lower the threshold is. Take BER = 0.0213 (the green curve in FIGURE 5) as a case. The recognition accuracy is near 0.9 when there are no error bits on the principal diagonal, but the recognition accuracy drops to 0.6 as soon as the number of error bits on the principal diagonal increases to 15. While the number of error bits on the principal diagonal makes up only 0.033 of the total number of error bits.

According to the results of numerical simulations above, we can conclude that error bits on the principal diagonal of data storage matrix will cause a more serious error propagation after GJETP algorithm and the recognition accuracy of rank deficient matrix will be decreased accordingly.

At this point, conjecture 1 has been checked. Then we will check conjecture 2. The simulation experiment is carried out, the conditions of the simulation are listed in TABLE 3. According to the conclusion in [5], [8], when the number of columns is 42, 56, 84 and 105 respectively, the lower triangular matrices after GJETP algorithm are supposed to

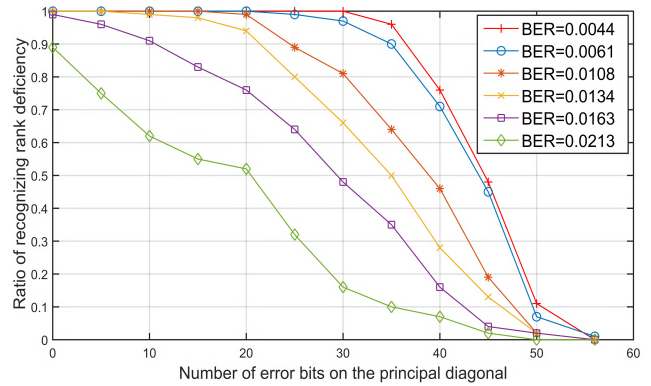


FIGURE 5. The ratio of recognizing rank-deficiency matrix correctly at different BER.

TABLE 3. The simulation conditions.

FEC coding	Convolutional interleaver	Columns of data storage matrix	Rows of data storage matrix	BER
BCH (7,4)	CI (3,2)	42	500	0.01
BCH (7,4)	CI (4,2)	56	375	0.01
BCH (7,4)	CI (6,2)	84	250	0.01
BCH (7,4)	CI (7,2)	105	200	0.01

be rank deficient matrices. In FIGURE 6, X-axis represents the ratio of the number of error bits on the principal diagonal to the total number of error bits ( $E$  in conjecture 2), Y-axis represents the ratio of recognizing rank deficient matrix correctly.

In general, when  $E$  in conjecture 2 is fixed, the larger the number of columns is, the lower the ratio of recognizing rank deficient matrix correctly is. That means, when the value of columns rises in the traversal search algorithm, the recognition accuracy will descend significantly. So far, conjecture 2 has been checked.

To sum up, the following conclusion can be drawn. In order to improve the capacity of error resistance, it is critical to improve the accuracy of recognizing rank deficient matrix correctly. Error bits on the principal diagonal of data storage matrix have great impacts on the accuracy of recognizing rank deficient matrix, so priority should be given to reducing the number of error bits on the principal diagonal.

### V. THE DENOISING ALGORITHM TO REDUCE NUMBER OF ERROR BITS ON THE PRINCIPAL DIAGONAL OF DATA STORAGE MATRIX

Based on the conclusion in section V, a novel denoising algorithm is proposed in this section. The core principle of the denoising algorithm is reducing error bits on the principal diagonal so as to restrain error propagation after GJETP algorithm, and finally the goal of improving error performance is achieved.

Lemma 1 is proposed.

Lemma 1: Assume that there are linearly dependent column vectors in matrix  $A$ . The correlation of linearly

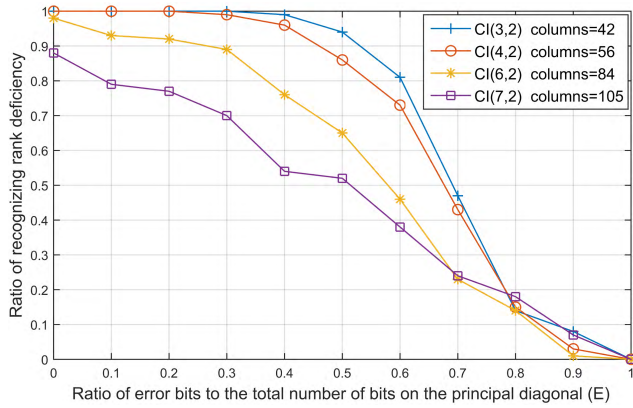


FIGURE 6. The ratio of recognizing rank-deficiency matrix correctly for different columns.

dependent column vectors will not be altered after the random exchange of columns or rows.

Then we will prove lemma 1. In matrix  $A_{\mu \times l} = [\alpha_1 \alpha_2 \dots \alpha_l]$ , there are  $k$  ( $0 < k \leq l$ ) linearly dependent column vectors. According to the definition of linearly dependent columns, there must be a nonzero vector  $\lambda = [\lambda_1 \lambda_2 \dots \lambda_k]$  that makes (6) come into existence:

$$B \times \lambda^T = \mathbf{0}. \tag{5}$$

$B$  is the set of linearly dependent column vectors in  $A$ :

$$B = [\alpha_{m_1} \alpha_{m_2} \dots \alpha_{m_k}], \tag{6}$$

where  $0 < m_i \leq k, i = 1, 2, \dots, k$ .

We firstly prove that the correlation of linearly dependent column vectors will not be altered after the random exchange of rows.

The random row exchanges in matrix  $A$  are equivalent to the random row exchanges in matrix  $B$ . So, we randomly exchange the rows of  $B$  and denote the result as  $B_1$ :

$$B_1 = X \times B. \tag{7}$$

$X$  is a nonzero invertible square matrix, representing the random row exchanges in  $B$ .

Then left multiply both sides of (6) by matrix  $X$ :

$$X \times B \times \lambda^T = \mathbf{0}. \tag{8}$$

By putting (8) in (9) we can get:

$$B_1 \times \lambda^T = \mathbf{0}, \tag{9}$$

which means that the column vectors in  $B_1$  are linearly dependent. The proposition: the correlation of linearly dependent column vectors will not be altered after the random exchange of rows is proved.

And then we will prove that the correlation of linearly dependent column vectors will not be altered after the random exchange of columns.

The random column exchanges in matrix  $A$  are equivalent to the random column exchanges in matrix  $B$ . So, we randomly exchange the columns of  $B$  and denote the result as  $B_2$ :

$$B_2 = B \times Y. \tag{10}$$

$Y$  is a nonzero invertible square matrix, representing the random column exchanges in  $B$ .

By using absurdity, the assumption can be made that the correlation of linearly dependent column vectors will be altered after the random exchange of columns, which means there is no longer a nonzero vector  $\lambda_2$  that makes (12) come into existence:

$$B_2 \times \lambda^T = \mathbf{0}. \tag{11}$$

Equation (6) can be transformed as:

$$B \times Y \times Y^{-1} \times \lambda^T = \mathbf{0}. \tag{12}$$

By putting (11) in (13) we can get:

$$B_2 \times Y^{-1} \times \lambda^T = \mathbf{0}. \tag{13}$$

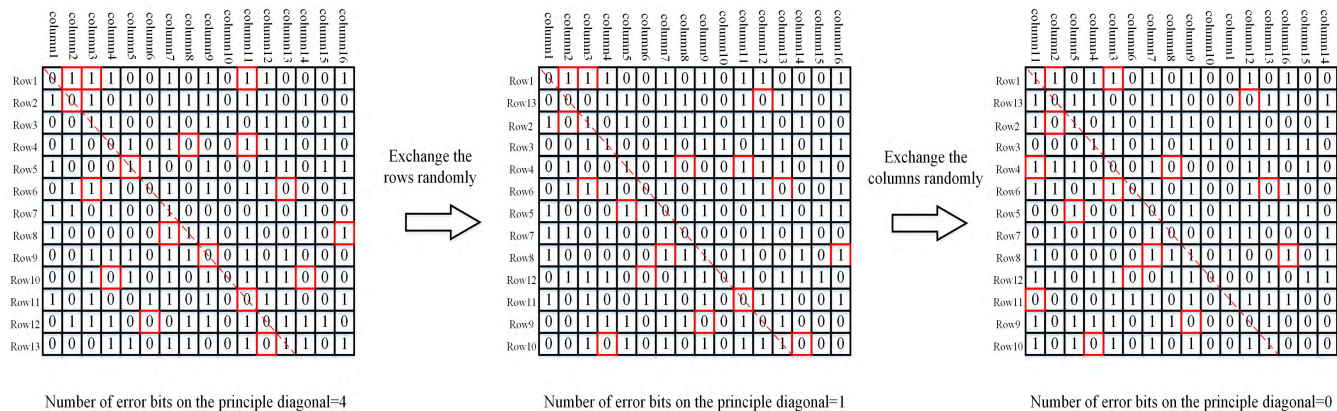
$Y^{-1}$  is a nonzero square matrix and  $\lambda^T$  is a nonzero vector. Let  $\lambda_2$ :

$$\lambda_2 = Y^{-1} \times \lambda^T (\lambda_2 \neq \mathbf{0}) \tag{14}$$

Therefore, there is a nonzero vector  $\lambda_2$  that makes (12) come into existence, which is contradictory to the assumption in the absurdity. Given the above, the proposition: the correlation of linearly dependent column vectors will not be altered after the random exchange of rows is proved.

In conclusion, lemma 1 is proved.

According to lemma 1, if there are dependent columns in data storage matrix, the random exchanges of rows or columns will not alter the correlation of the dependent columns, having no effect on the recognition of rank deficient matrices. In order to reduce error bits on the principal diagonal of data storage matrix, the rows and columns are randomly exchanged. As shown in FIGURE 7, the red boxes represent error bits and the black ones represent correct bits. The number of error bits on the principal diagonal decreases after the random exchanges of rows and columns. However, FIGURE 7 just shows an ideal situation. In fact, there is a serious risk that the number of error bits will not be reduced after the random exchanges of rows and columns, hence the loops of exchanging rows and columns randomly are adopted to find the case that the number of error bits on the principal diagonal is minimized. Theoretically, the larger the number of loops is, the stronger the error tolerance is. However, the computational cost will be improved. Denote the number of loops as *excnt*, the recommended value of *excnt* is 20. The normalized rank obtained in each loop is recorded, and the minimum value is selected as the temporary normalized rank  $\xi_{temp}$ . We call the method Exchanging-the-Rows-and-Columns-Randomly- GJETP (ERCR-GJETP) algorithm (algorithm 2), which is listed below.



**FIGURE 7.** The rows and columns are exchanged randomly and the number of error bits on the principle diagonal decreases. The red boxes represent error bits and the dotted line represents the principal diagonal. This figure shows an ideal result. In fact, it is hard to reduce the number of error bits on the principal diagonal to 0 by exchanging the rows and columns randomly.

**Algorithm 2** ERRCR-GJETP Algorithm

**Notations:**  $\xi_{temp}$  denotes the temporary normalized rank.  $excnt$  denotes the number of loops;  $\Gamma$  is the set of the recorded normalized ranks.

**Assumptions:** The size of data storage matrix is  $\mu \times l$  and  $\mu \geq 2l$ . The received data stream is assumed to have bit errors;

**Input:** The data storage matrix  $R_l$ ;

**Output:**  $\xi_{temp}$ ;

```

1: while  $i \leq excnt$  do
    Exchange the rows of  $R_l$  randomly;
    Exchange the columns of  $R_l$  randomly;
    Apply GJETP algorithm on  $R_l$  to get the lower
    triangular matrix;
    Calculate the normalized rank of the lower triangular
    matrix and record it in  $\Gamma$ ;
end
2:  $\xi_{temp} = \min(\Gamma)$ ;
    
```

However, the goal of reducing error bits on the principal diagonal may not be achieved by ERRCR-GJETP algorithm when BER is high, which means that the value of temporary normalized rank  $\xi_{temp}$  is not estimated correctly. If  $\xi_{temp}$  is 1, the following steps are considered.

The first  $a$  rows of the data storage matrix  $R_l$  are deleted and the rest part of  $R_l$  is denoted as  $R_{delete}$ , where  $a$  is an adjustable parameter and the recommended value of  $a$  is 5. Then ERRCR-GJETP algorithm is applied on matrix  $R_{delete}$  to get the lower triangular matrix. The normalized rank of the lower triangular matrix is calculated and recorded.

Loop the steps described in the previous paragraph. Note that the  $R_l$  in the current loop is the  $R_{delete}$  in the former one. The number of loops  $D$  is determined by (16):

$$D = \lceil \mu - 2l \rceil / a, \tag{15}$$

where  $\mu$  and  $l$  are the numbers of rows and columns of the data storage matrix respectively. Select the minimum of the recorded normalized ranks as the final normalized rank  $\xi_{final}$ . If the value of  $\xi_{final}$  is still 1, the number of columns is added by one for the next round of search.

Two reasons for the deletions of the rows are listed below. Firstly, if there are quite a number of error bits in the deleted rows, the error propagation after ERRCR-GJETP will be decreased. Secondly, the principal diagonal will be updated after each deletion, which means the number of error bits on the principal diagonal is possible to be reduced.

With the benefit of denoising algorithm proposed in this section,  $\hat{N} = \widehat{lcm}(n, B)$  can be estimated more accurately for erroneous case. The improved  $\hat{N}$  estimation algorithm is given in algorithm 3. The estimation of  $B$ ,  $M$  and synchronization position  $d_0$  follows the steps described in section III (2).

**VI. SIMULATION RESULTS**

In this section, the performance of the proposed algorithm is analyzed by simulation experiments.

FIGURE 8 shows the rank deficiency when BER is 0. The simulation conditions are listed in TABLE 4.

As shown in FIGURE 8, for the first time and the second time rank deficiency are detected, the corresponding numbers of columns are 120 and 150 respectively. The difference of them is 30, which is the least common multiple of the interleaving width  $B = 6$  and the codeword length  $n = 15$ .

FIGURE 9 plots the value of normalized rank when  $d$  and  $M$  varies.  $\rho$  represents the normalized rank. For clear display, we replace  $\rho$  with  $1 - \rho$ . The simulation conditions are listed in TABLE 5. According to FIGURE 10, when  $\rho$  is the minimum ( $1 - \rho$  reaches the maximum, correspondingly), the estimation of  $d$  is 3 and  $M$  is 5, the corresponding  $B$  is 3 ( $B$  is not shown in FIGURE 10). The convolutional interleaver parameters are blindly recognized successfully

**Algorithm 3**  $\hat{N}$  Estimation Algorithm With Denoising

**Notations:**  $\mu$  and  $l$  indicate the number of columns and rows of the data storage matrix  $R_l$ , respectively.  $R_{delete}$  is the rest part of  $R_l$  after deletions of rows.  $a$  is the number of rows deleted in each loop,  $D$  is the number of loops.  $\xi_{temp}$  denotes the temporary normalized rank.  $\xi_{final}$  denotes the final normalized rank.  $\Theta$  is the set of the recorded normalized ranks;

**Assumptions:**  $\mu \geq 2l$ .  $l \in (l_{min}, l_{max})$ .  $t$  is the while loop count. The received data stream is assumed to have bit errors;

**Input:** The received data stream;

**Output:** The least common multiple of  $n$  and  $B$ :  $\hat{N}$ ;

1:  $t = 0$ ;

2:  $l = l_{min}$ ;

3: **while**  $t < 2$  **do**

The received data stream is reshaped into the data storage matrix  $R_l$  of size  $\mu \times l$ ;

Apply ERCR-GJETP algorithm on  $R_l$  to get  $\xi_{temp}$ ;

**if**  $\xi_{temp} = 1$  **then**

$D =$

$\lceil (\mu - 2l) / a \rceil$ ;

**for**  $i = D$  **do**

Delete the first  $a$  rows of  $R_l$  to get  $R_{delete}$ ;

Apply ERCR-GJETP algorithm on  $R_{delete}$  to obtain the lower triangular matrix;

the normalized rank of the lower triangular matrix is calculated and stored in  $\Theta$ ;

Assign  $R_{delete}$  to  $R_l$ ;

**end**

$\xi_{final} = \min(\Theta)$ ;

**end**

**if**  $\xi_{final} < 1$  **then**

$\hat{N} = l - \hat{N}$ ;

$t = t + 1$ ;

**end**

$l = l + 1$ ;

**end**

4: **return**  $\hat{N}$ ;

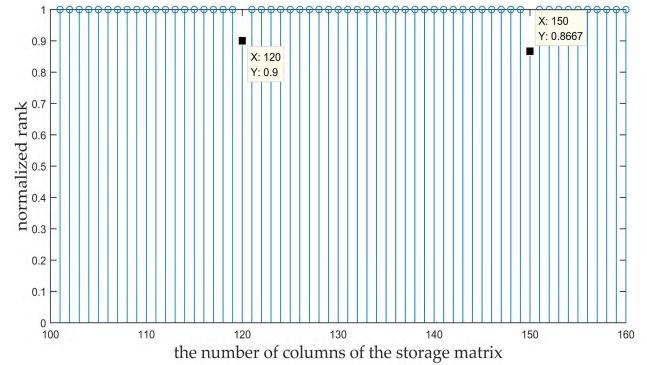
**TABLE 4.** The simulation conditions.

FEC coding	Convolutional interleaver	Synchronization position $d_0$	Modulation format	BER
BCH (15,11)	CI (6,3)	1	QPSK	0

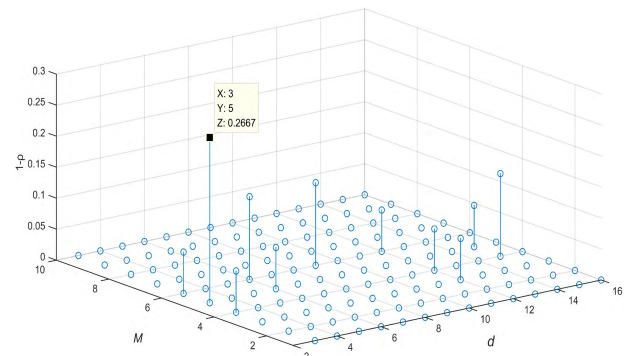
**TABLE 5.** The simulation conditions.

FEC coding	Convolutional interleaver	Synchronization position $d_0$	Modulation format	BER
BCH (15,11)	CI (3,5)	3	QPSK	0.01

The error performance of algorithms proposed in [8], [12] and this paper is compared and the result is shown in FIGURE 10. The simulation conditions are listed in TABLE 6.



**FIGURE 8.** The rank deficiency difference can be estimated correctly when BER = 0.



**FIGURE 9.** Result of the searching. The maximum of  $1 - \rho$  corresponds the estimated  $d$  and  $M$  (B).

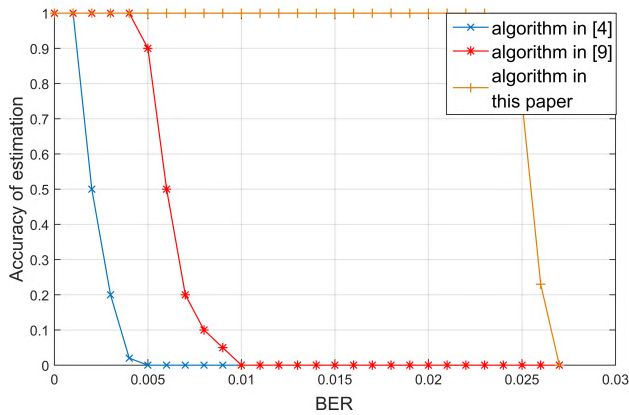
**TABLE 6.** The simulation conditions.

FEC coding	Convolutional interleaver	<i>exccnt</i>	<i>a</i>	Algorithm	BER
BCH (15,11)	CI (5,3)	/	/	in [8]	[0,0.027]
BCH (15,11)	CI (5,3)	/	/	in [12]	[0,0.027]
BCH (15,11)	CI (5,3)	20	5	in this paper	[0,0.027]

It can be concluded from FIGURE 10 that the algorithm proposed in this paper has advanced the capacity of error resistance. Algorithm in [12] has better error performance than algorithm in [8] does. However, the denoising part of algorithm in [12] just calculates the number of '0's in the matrix. When BER is high, the denoising algorithm is incapable of overcoming the effect of error propagation after GJETP algorithm. The algorithm proposed in this paper focuses on reducing error bits on the principal diagonal of data storage matrix and shows better performance.

The error performance of the proposed algorithm is improved at the expense of the sharp increase of computational complexity. TABLE 7 shows the time consumption of the three algorithms on the same computing platform with the simulation conditions in TABLE 6. In TABLE 7, the time consumption of the algorithm in this paper is far more than that of the algorithms in [8] and [11]. The large number of loops in the proposed algorithm should account for this.





**FIGURE 10.** Estimation accuracy of the 3 algorithms at different BER. The error performance of the algorithm proposed in this paper is greatly improved.

**TABLE 7.** Time consumption of three algorithms.

Algorithm	Time consumption
In [8]	8s
In [12]	13s
In this paper	525s

**TABLE 8.** The simulation conditions.

FEC coding	Convolutional interleaver	<i>excnt</i>	<i>a</i>	BER
BCH (7,4)	CI (4,2)	10	5	[0,0.08]
BCH (7,4)	CI (4,2)	20	5	[0,0.08]
BCH (7,4)	CI (4,2)	30	5	[0,0.08]

**TABLE 9.** The simulation conditions.

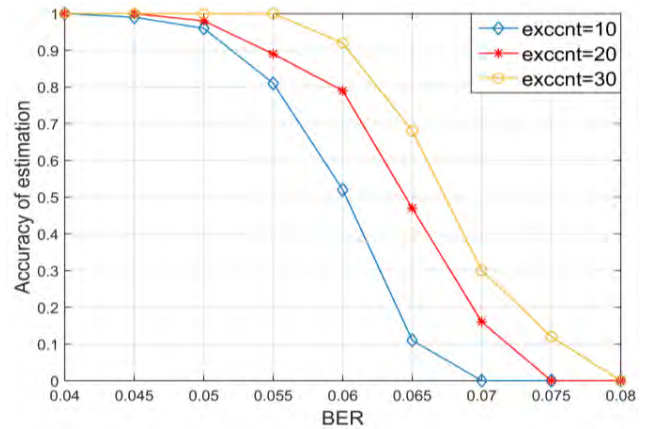
FEC coding	Convolutional interleaver	<i>excnt</i>	<i>a</i>	BER
BCH (7,4)	CI (3,2)	20	5	[0,0.13]
BCH (7,4)	CI (4,2)	20	5	[0,0.13]
BCH (7,4)	CI (6,2)	20	5	[0,0.13]

The parameter *excnt* affects the performance of the algorithm. FIGURE 11 shows the recognition accuracy when *excnt* is 10, 20 and 30 respectively. The conditions of the simulation are listed in TABLE 8.

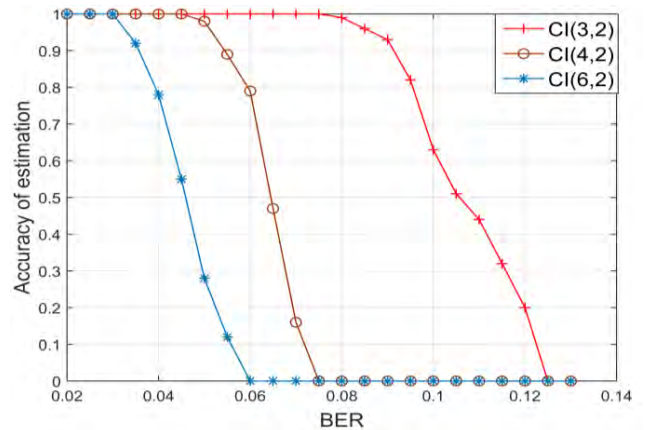
Obviously, with the increase of *excnt*, the accuracy of estimation rises. To balance the computational cost and performance, we usually choose *excnt* = 20.

Different convolutional interleavers are considered. FIGURE 12 demonstrates the recognition accuracy of the algorithm for different convolutional interleavers. The conditions of the simulation are listed in TABLE 9.

The performance plot in FIGURE 12 indicates that the parameter of convolutional interleavers influences the recognition accuracy. For CI (3,2), the accuracy is near 1 when BER = 0.08. However, the corresponding BER is



**FIGURE 11.** Accuracy of estimation with different *excnt*.



**FIGURE 12.** Accuracy of estimation for different convolutional interleavers.

**TABLE 10.** Test code set.

No.	Code rate	<i>k</i>	<i>n</i>	<i>K</i>	$\mathcal{S}_i^j$
Code 1	1/2	2	1	10	[1167,1545]
Code 2	1/2	2	1	7	[171,133]
Code 3	1/3	3	1	4	[13,15,17]
Code 4	1/4	4	1	7	[133,171,117,165]

merely 0.03 for CI (6,2). Essentially, when the least common multiple of parameter *B* and the codeword length *n* is large, the number of data storage matrix columns in the searching algorithm will rise up in turn. According to the conjecture 2 in section IV, the accuracy of recognition will fall down.

The performance of the proposed algorithm in estimating the convolutional interleaver parameters from erroneous convolutional coded data is demonstrated in FIGURE 13. The set of convolutional codes for simulation is listed in TABLE 10. The simulation conditions are listed in TABLE 11.

According to the simulation results, the convolutional interleaver parameters of convolution coded data can also be estimated by the algorithm when BER is high. Similar to the block code cases, the error performance is related to the

TABLE 11. The simulation conditions.

FEC coding	Convolutional interleaver	exccnt	a	BER
Convolutional codes	CI (3,5)	20	5	[0.017,0.08]

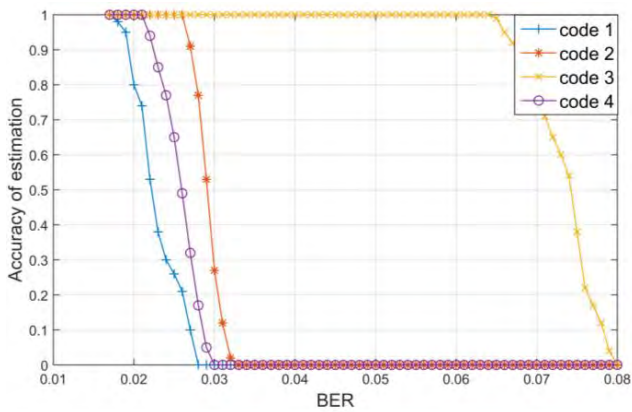


FIGURE 13. Accuracy of estimation for convolutional coded data.

least common multiple of the codelength and the product of  $B$  and  $M$ .

VII. CONCLUSION

In this paper, the blind recognition of convolutional interleaver parameters in a noisy channel is investigated. The structure of convolutional interleaver is briefly introduced. The proposed methods in published literatures are summarized, but the error performances of the methods are generally in need of improvement. The effect of error bits on the principal diagonal of data storage matrix is analyzed and a denoising algorithm focusing on reducing error bits on the principal diagonal of data storage matrix is then proposed. The noisy channel is assumed to be AWGN channel in our work. Simulation results show that the performance of the algorithm proposed in this paper makes great progress in AWGN channel. The algorithm is proved to be applicable to both block codes and convolution codes. It is noteworthy that the improvement of the algorithm error performance is at expense of increasing computational complexity. The algorithm needs to be optimized in our future work.

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