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Deep Learning Detection in MIMO Decode-Forward Relay Channels

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ABSTRACT We consider a signal detection problem by using deep learning techniques in a multiple-input multiple-output (MIMO) decode-forward (DF) relay channel. There exist some suboptimal detectors such as the near maximum likelihood (NML) detector and the NML with two-level pair-wise error probability (NMLw2PEP) detector in the channel. However, the NML detectors require an exponentially increasing complexity as the number of transmit antennas increases. More seriously, without the channel state information (CSI) of the source-relay (SR) link, there is no detector that can achieve good performance even at high complexity. In this paper, we propose a deep learning approach to the NML (DL-NML) detector that achieves good performance with low complexity regardless of whether the CSI of the SR link is known or not at the destination. The DL-NML detector can detect signals in changing channels after a single training by using randomly generated channels. Furthermore, we propose a linear detector and a semidefinite relaxation approach to the NML detector to compare with the DL-NML detector in performance and complexity. The complexity analysis and simulation results validate the superiority of the proposed DL-NML detector.

INDEX TERMS Channel state information, decode–forward, machine learning, maximum likelihood, multiple-input multiple-output (MIMO), neural network, relay channel, TensorFlow.

I. INTRODUCTION

In wireless communications, deep fading often causes a failure in reliable data transmission. Relays help increase the transmission reliability between a source and a destination, and extend the network coverage by providing an additional link. The relay channel model, introduced by van der Meulen [1], is a basic channel model in network communications. This relay channel has been studied extensively in the literature [2]–[10]. Among various relaying operations, amplify-forward (AF) and decode-forward (DF) are the two most common methods [3]. Even though the AF relaying is simple, the transceivers require expensive radio frequency amplifiers [4]. Hence, we apply the DF relaying in this paper.

A. BACKGROUND ON SIGNAL DETECTION IN THE MIMO DF RELAY CHANNEL

Unlike in a multiple-input multiple-output (MIMO) system, a linear relationship does not exist between the input and the output in the DF relay channel due to the hard decision at the relay. As the received signal of the relay is not known at the destination, the maximum likelihood (ML) detection in the DF relay system requires more steps than that in the MIMO system [4]–[6]. Due to the complexity of the ML detection and the difficulty in analysis, a near-ML (NML) detector was proposed in [6], [7] under instantaneous channel state informations (CSIs) of the source-relay (SR), source-destination (SD), and relay-destination (RD) links. However, forwarding the instantaneous CSI of the SR channel from the relay to the destination requires additional work and reduces the data rate. With the statistical CSI of the SR link at the destination, an NML with two-level pair-wise error probability (PEP) (NMLw2PEP) detection was proposed in [8] that achieves good performance with relatively low complexity. Without any knowledge of the SR channel at the destination, the minimum distance (MD) detection[1] ignores detection error at the relay and shows poor performance [4], [6].

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[1]This was called a maximum ratio combining (MRC) in a single-antenna relay system in [4].

The above mentioned detection algorithms detect signals simultaneously by exhaustively searching all the possible signal sets, thus their complexities increase exponentially as the number of transmit antennas increases.

A method to reduce the detection complexity is to separate the signals by a linear operation and detect them individually. The typical linear detectors in the MIMO channel are the zero forcing (ZF) and minimum mean square estimation (MMSE) detectors [11]. Referring to the ZF detector, a linear detector of ZF with maximum ratio combining (MRC) (ZFwMRC) was proposed in the MIMO DF relay channel when the relay detects signals correctly [9]. However, this algorithm cannot achieve good performance for the relay with errors similar to the MD detection. A new detection method should be introduced, and a potential solution is to use the powerful tools in machine learning.

B. MACHINE LEARNING AND SIGNAL DETECTION

Machine learning is a subset of artificial intelligence that learns to solve a specific problem by themselves [12]. Supervised learning, a class of optimization problems, trains an approximation function of a target function f such that $x = f(y)$ using the known training data samples including the observation data y and reference data x. Meanwhile, traditional signal detection obtains an estimation of $\hat{x}$ directly from the observation y using a mathematical optimization method without reference signals (training data). However, it is not easy to find a detector with reasonable performance and complexity theoretically in complicated communication systems. Applying machine learning, a function g that approximates the existing detection algorithm is trained to minimize a loss function $l(\hat{x}, x)$ that measures the cost of estimating $\hat{x}$ when the actual answer is x. After training, the observation data y undergoes the final function g^*, and subsequently, the desired data $\hat{x}$ is detected and the detection process is completed in real time.

Advances in computer technology and big data processing have significantly reduced the cost and time of training deep learning algorithms. This has significantly improved the development of computer vision [13] and natural language processing [14]. In communication networks, deep learning has begun to receive much attention [15]. To reduce complexity, the detection and channel decoding problems have been investigated using powerful deep learning tools in the channel decoding related to belief propagation [16], [17], signal detection in MIMO systems [18], [19], and signal detection in chemical communications [20], [21]. A deep MIMO detection network in the MIMO channel is noteworthy [19]. This detection network applies a deep unfolding approach that transforms a computationally intractable probabilistic model into a deep neural network by unfolding iterative calculations into neural-network layers [22]. Embedding the existing mathematical methods into black-box-like deep neural networks improves the accuracy and reduces complexity.

C. CONTRIBUTIONS

In this paper, a deep learning detector in the MIMO DF relay channel is proposed by adopting the deep unfolding approach [19], [22], under three scenarios related to the knowledge of the SR channel at the destination: 1) with instantaneous SR channel; 2) with statistical SR channel; 3) without SR channel.

Unlike in the MIMO channel, there does not exist much research on the signal detection in the MIMO DF relay channel as introduced in Section I-A. To compare with the deep learning detector in performance and complexity, a linear detector and a semidefinite relaxation (SDR) detector are also proposed. The reason for choosing the linear detector is due to its low complexity, and choosing the SDR approach is due to its good performance and reasonable complexity. In detail, we list the three main reasons to choose the SDR algorithm.

- The SDR algorithm solves a convex optimization problem, thus this method does not suffer from local maxima [32], [31].
- The SDR algorithm guarantees a polynomial-time worst-case computational complexity.
- The SDR detector in the MIMO chanel achieves maximum possible diversity order and near ML performance in a wide SNR range [33], [31]. The derivation of the relay error probability of the SDR detector makes it possible to apply the SDR approach at both the relay and the destination.

In summary, the main contributions of this paper are as follows.

- A *deep learning approach to the near-ML (DL-NML) detector* is proposed in the MIMO DF relay channel under the three scenarios (Section III). A detailed training process for the DL-NML detector is introduced, which gives a hint to how to train machine learning detectors in network communications (Section VI). Unlike the MIMO channel, there exist some additional factors to be considered in network communications as well as DF relay channels. The advantages of the proposed DL-NML detector are summarized as follows.
  - Training process: The required training time of the DL-NML detector is not very long; and once being trained, the DL-NML detector can be implemented without retraining or with a little retraining for various channel environments such as the one with a different number of receive antennas or with different statistical conditions.
  - Detection process: The DL-NML detector with a fixed $L(< N)$ detection layers requires relatively low complexity that between $O(L(4N)^2)$ and $O((4N)^3)$ and takes short detection time by using batch signal detection and parallel computation in the MIMO DF relay channel with N transmit antennas.

- Performance: The DL-NML detector achieves excellent error performance in the three scenarios of the SR channel. In particular, in the scenario without SR channel, this deep learning detector achieves dramatically good performance through benefits of the learning process, while existing detectors could not obtain acceptable performance with technologies so far known.

- To evaluate the performance of the DL-NML detector, an *SDR approach to the NML (SDR-NML) detector* is proposed as the SDR version of the NML detectors. The SDR-NML detector exhibits admirable performance with polynomial complexity, and is thus a suitable choice for the MIMO DF relay channel without training.
- Additionally, a linear detector based on the zero gradient (ZG) of the metric of the NML detector is proposed (Definition 1). The ZG detector achieves much better performance than the existing linear ZFwMRC detector.
- Equivalent SR channel matrices used in detectors at the destination (DetD) are provided corresponding to various detectors at the relay (DetR) so that the proposed DetD such as the DL-NML, SDR-NML, and ZG detectors can be implemented for any DetR.
- Applying various DetR and DetD, we present some DetR:DetD methods according to the required error probability and complexity, which provides a basic idea and direction for the system configuration.

The remainder of the paper is organized as follows. In the next section, we formally introduce the MIMO DF relay channel and its equivalent real model. The main part of this paper is presented in Section III. The DL-NML detector is proposed by applying the deep unfolding approach under various conditions of the knowledge of the SR channel when the ML detection is applied at the relay. In Section IV, the SDR-NML detector is proposed to compare with the DL-NML detector in the case of the ML detector at the relay. Some representative detectors at the relay (DetR) are introduced and a deep learning approach to the ML detector in the MIMO channel is proposed in Section V. In Section VI, the training and detection details for the proposed DL-NML detector are explained. In Section VII, various DetR:DetD methods are presented by using TensorFlow and Matlab. Finally, the conclusions are given in Section VIII.

D. NOTATIONS

Throughout the paper, we use the following notations. The superscript $(\cdot)^T$ denotes the transpose of a matrix; $\text{tr}(\cdot)$ denotes the trace of a matrix; $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ denote the real and imaginary parts of a complex number, respectively; I_n denotes the $n \times n$ identity matrix (where the subscript n is omitted when it is irrelevant or clear from the context); $\mathbb{C}^{n\times m}$ and $\mathbb{R}^{n\times m}$ denote a set of $n \times m$ complex matrices and a set of $n \times m$ real matrices, respectively; $\mathbf{A} \sim \mathcal{CN}(0, \sigma_{\mathbf{A}}^2 I)$ denotes that the elements of $\mathbf{A}$ are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variance $\sigma_{\mathbf{A}}^2$, and $\mathbf{B} \sim \mathcal{N}(0, \sigma_{\mathbf{B}}^2 I)$ denotes that $\mathbf{B}$ is a real Gaussian random matrix with zero mean and covariance matrix $\sigma_{\mathbf{B}}^2 I$; $\text{diag}(\cdot)$ denotes a block diagonal matrix with the entries on its main diagonal; $[\cdot]_{i:j,k:l}$ means a matrix consisting of the entries from the ith row to the jth row, and from the kth column to the lth column in the original matrix.

II. SYSTEM MODEL

A half-duplex DF relay channel with one source, one destination, and one relay is considered. It is assumed that the relay knows the CSI of the SR channel, and the destination knows the CSIs of the SD and RD links. In the first phase, the source with N transmit antennas broadcasts N independently and uniformly distributed complex signals $\mathbf{x}^C = [x_1^C, \ldots, x_N^C]^T$ to the relay and the destination, where $\text{Re}(x_j^C) \in \mathcal{A}, \text{Im}(x_j^C) \in \mathcal{A}, j = 1, \ldots, N$, and $\mathcal{A} \in \{+1, -1\}$. Subsequently, the received signals at the relay with N receive antennas can be written as

$$\mathbf{y}_{\text{SR}}^C = H_{\text{SR}}^C \mathbf{x}^C + \mathbf{z}_{\text{SR}}^C \tag{1}$$

where $H_{\text{SR}}^C \in \mathbb{C}^{N\times N}$ is the channel coefficient matrix of the SR link and $\mathbf{z}_{\text{SR}}^C \sim \mathcal{CN}(0, \sigma^2 I)$ is the noise term at the relay. Simultaneously, the destination receives the signal transmitted from the source as

$$\mathbf{y}_{\text{SD}}^C = H_{\text{SD}}^C \mathbf{x}^C + \mathbf{z}_{\text{SD}}^C \tag{2}$$

where $H_{\text{SD}}^C \in \mathbb{C}^{N_{\text{D}}\times N}$ is the channel coefficient matrix of the SD link and $\mathbf{z}_{\text{SD}}^C \sim \mathcal{CN}(0, \sigma^2 I)$ is the noise term at the destination in the first phase. In the second phase, the relay decodes the received signal and forwards the decoded signal $\mathbf{x}_{\text{R}}^C = [x_{1\text{R}}^C, \ldots, x_{N\text{R}}^C]^T$, $\text{Re}(x_{j\text{R}}^C) \in \mathcal{A}, \text{Im}(x_{j\text{R}}^C) \in \mathcal{A}, j = 1, \ldots, N$ to the destination. The received signal at the destination with N_{D} receive antennas in the second phase is

$$\mathbf{y}_{\text{RD}}^C = H_{\text{RD}}^C \mathbf{x}_{\text{R}}^C + \mathbf{z}_{\text{RD}}^C \tag{3}$$

where $H_{\text{RD}}^C \in \mathbb{C}^{N_{\text{D}}\times N}$ is the channel coefficient matrix of the RD link and $\mathbf{z}_{\text{RD}}^C \sim \mathcal{CN}(0, \sigma^2 I)$ is the noise term at the destination in the second phase.

To simplify the expressions, we convert the complex system model to a real system model. Let $\mathbf{x} = \begin{bmatrix}\text{Re}(\mathbf{x}^C)\\ \text{Im}(\mathbf{x}^C)\end{bmatrix}$, $\mathbf{x}_{\text{R}} = \begin{bmatrix}\text{Re}(\mathbf{x}_{\text{R}}^C)\\ \text{Im}(\mathbf{x}_{\text{R}}^C)\end{bmatrix}$, $\mathbf{y}_{ij} = \begin{bmatrix}\text{Re}(\mathbf{y}_{ij}^C)\\ \text{Im}(\mathbf{y}_{ij}^C)\end{bmatrix}$, $\mathbf{z}_{ij} = \begin{bmatrix}\text{Re}(\mathbf{z}_{ij}^C)\\ \text{Im}(\mathbf{z}_{ij}^C)\end{bmatrix}$, and $H_{ij} = \begin{bmatrix}\text{Re}(H_{ij}^C) & -\text{Im}(H_{ij}^C)\\ \text{Im}(H_{ij}^C) & \text{Re}(H_{ij}^C)\end{bmatrix}$ for $ij \in \{\text{SR}, \text{SD}, \text{RD}\}$. Then an equivalent real system model is written as

$$\mathbf{y}_{\text{SR}} = H_{\text{SR}}\mathbf{x} + \mathbf{z}_{\text{SR}} \tag{4}$$

$$\mathbf{y}_{\text{SD}} = H_{\text{SD}}\mathbf{x} + \mathbf{z}_{\text{SD}} \tag{5}$$

$$\mathbf{y}_{\text{RD}} = H_{\text{RD}}\mathbf{x}_{\text{R}} + \mathbf{z}_{\text{RD}} \tag{6}$$

where $\mathbf{x}, \mathbf{x}_{\text{R}} \in \mathcal{A}^{2N}$, and $\mathbf{z}_{ij} \sim \mathcal{N}(0, \frac{1}{2}\sigma^2 I)$ for $ij \in \{\text{SR}, \text{SD}, \text{RD}\}$. The signal-to-noise ratios (SNRs) at the relay and the destination are linearly proportional to $\rho = \frac{2N}{\sigma^2}$. The equivalent real system model is depicted in Fig. 1, where

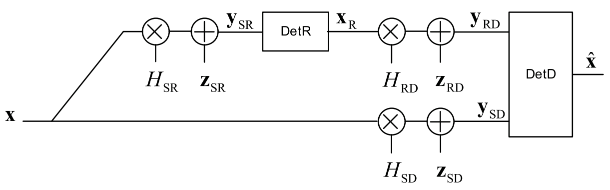


FIGURE 1. **The equivalent real MIMO DF relay system model.**

"DetR" and "DetD" represent the detectors at the relay and the destination, respectively. Various types of DetD and DetR will be introduced in the following sections.

III. DEEP LEARNING APPROACH TO THE NML DETECTOR AT DESTINATION

Consider the MIMO relay channel in Fig. 1. Due to the probabilistic distribution of the noise terms $\mathbf{z}_{\mathrm{SR}}$, $\mathbf{z}_{\mathrm{SD}}$, and $\mathbf{z}_{\mathrm{RD}}$, the optimal detection method to minimize the frame error probability is the ML detector that finds an $\mathbf{x} \in \mathcal{A}^{2N}$ that maximizes $p(\mathbf{y}_{\mathrm{SD}}, \mathbf{y}_{\mathrm{RD}}|\mathbf{x}, H_{\mathrm{SD}}, H_{\mathrm{RD}}, H_{\mathrm{SR}})$ for the uniformly distributed $\mathbf{x}$.

As detection errors may exist at the relay, the detection method for the DF relay channel is different from that of the MIMO channel. Both signals that are possibly to be transmitted from the source and from the relay should be considered in the DetD. This results in much higher detection complexity in the MIMO DF relay channel, approximately the square of the computational complexity of the point-to-point MIMO channel detection. The exhaustive search algorithms such as the ML and NML detectors cannot be applied in the MIMO DF relay channel with large numbers of antennas. Moreover, without the CSI of the SR link, there is no existing optimal or suboptimal detection algorithm that we can refer to.

To address these problems, this section explores a deep learning detection for three scenarios related to the knowledge of the SR channel at the destination: 1) with instantaneous SR channel; 2) with statistical SR channel; 3) without SR channel.

A. WITH INSTANTANEOUS SR CHANNEL

With the full CSIs of the SR, SD, and RD links, the ML detection that maximizes the likelihood function $p(\mathbf{y}_{\mathrm{SD}}, \mathbf{y}_{\mathrm{RD}}|\mathbf{x}, H_{\mathrm{SD}}, H_{\mathrm{RD}}, H_{\mathrm{SR}})$ for the real system model in (4)-(6) can be written as

$$\begin{aligned}\hat{\mathbf{x}} &= \arg\max_{\mathbf{x}\in\mathcal{A}^{2N}} p(\mathbf{y}_{\mathrm{SD}}, \mathbf{y}_{\mathrm{RD}}|\mathbf{x}, H_{\mathrm{SD}}, H_{\mathrm{RD}}, H_{\mathrm{SR}})\\ &= \arg\max_{\mathbf{x}\in\mathcal{A}^{2N}} p\left(\mathbf{y}_{\mathrm{SD}}|\mathbf{x}, H_{\mathrm{SD}}\right)\\ &\quad\cdot \sum_{\mathbf{x}_{\mathrm{R}}\in\mathcal{A}^{2N}} p\left(\mathbf{y}_{\mathrm{RD}}|\mathbf{x}_{\mathrm{R}}, H_{\mathrm{RD}}\right)P_{\mathrm{SR}}(\mathbf{x}_{\mathrm{R}}|\mathbf{x}, H_{\mathrm{SR}})\\ &= \arg\max_{\mathbf{x}\in\mathcal{A}^{2N}}\Big\{-\left\|\mathbf{y}_{\mathrm{SD}} - H_{\mathrm{SD}}\mathbf{x}\right\|^2\\ &\quad+\sigma^2\ln\sum_{\mathbf{x}_{\mathrm{R}}\in\mathcal{A}^{2N}}\exp\Big(-\frac{\|\mathbf{y}_{\mathrm{RD}}-H_{\mathrm{RD}}\mathbf{x}_{\mathrm{R}}\|^2-\sigma^2\ln P_{\mathrm{SR}}(\mathbf{x}_{\mathrm{R}}|\mathbf{x},H_{\mathrm{SR}})}{\sigma^2}\Big)\Big\}\end{aligned} \tag{7}$$

where $P_{\mathrm{SR}}(\mathbf{x}_{\mathrm{R}}|\mathbf{x}, H_{\mathrm{SR}})$ is the probability that the relay detects the received signal to $\mathbf{x}_{\mathrm{R}}$ when the source transmits $\mathbf{x}$. Since it is highly difficult to derive $P_{\mathrm{SR}}(\mathbf{x}_{\mathrm{R}}|\mathbf{x}, H_{\mathrm{SR}})$ in MIMO systems [23], the pair-wise error probability (PEP) between $\mathbf{x}$ and $\mathbf{x}_{\mathrm{R}}$, $P_{\mathrm{SR}}(\mathbf{x} \to \mathbf{x}_{\mathrm{R}}|H_{\mathrm{SR}})$ is used instead. Moreover, applying the widely-used max-log approximation $\ln\sum_i \exp(x_i) \approx \max_i x_i$ [24]- [26], the near-ML (NML) detector [6] is written as

$$\begin{aligned}\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathcal{A}^{2N}}\min_{\mathbf{x}_{\mathrm{R}}\in\mathcal{A}^{2N}}\Big\{&\left\|\mathbf{y}_{\mathrm{SD}}-H_{\mathrm{SD}}\mathbf{x}\right\|^2\\ &+\left\|\mathbf{y}_{\mathrm{RD}}-H_{\mathrm{RD}}\mathbf{x}_{\mathrm{R}}\right\|^2 - \sigma^2\ln P_{\mathrm{SR}}(\mathbf{x}\to\mathbf{x}_{\mathrm{R}}|H_{\mathrm{SR}})\Big\}\end{aligned} \tag{8}$$

where $P_{\mathrm{SR}}(\mathbf{x} \to \mathbf{x}_{\mathrm{R}}|H_{\mathrm{SR}}) = 1$ for $\mathbf{x}_{\mathrm{R}} = \mathbf{x}$; otherwise, $P_{\mathrm{SR}}(\mathbf{x} \to \mathbf{x}_{\mathrm{R}}|H_{\mathrm{SR}})$ is the PEP between $\mathbf{x}$ and $\mathbf{x}_{\mathrm{R}}$ for the ML detector at the relay (MLaR) [6]:

$$P_{\mathrm{SR}}(\mathbf{x}\to\mathbf{x}_{\mathrm{R}}|H_{\mathrm{SR}}) = Q\Big(\sqrt{\frac{1}{2\sigma^2}\|H_{\mathrm{SR}}(\mathbf{x}-\mathbf{x}_{\mathrm{R}})\|^2}\Big) \tag{9}$$

and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$.

To detect signals, the NML detector requires $|\mathcal{A}|^{4N}$ times of the calculation for the metric in (8). This exhaustive search detection algorithm cannot be used for large numbers of antennas. Hence, we reflect this mathematical relationship in deep learning networks to get both advantages of the mathematical method and training method simultaneously. By unfolding the iterations of the projected gradient descent method based on the NML detector, a deep learning algorithm is derived. This is described in detail in the following steps.

1) PROJECTED GRADIENT DESCENT METHOD

The projected gradient descent method is based on the gradient of the metric in the original exhaustive search detection algorithm. However, the metric in (8) itself is unsuitable for the gradient descent method due to the complicated $Q(x)$ function. Instead, we use an approximation given in [6] as

$$-\lim_{\sigma^2\to 0}\sigma^2\ln Q\Big(\sqrt{\frac{1}{2\sigma^2}\|H_{\mathrm{SR}}(\mathbf{x}-\mathbf{x}_{\mathrm{R}})\|^2}\Big) = \frac{1}{4}\|H_{\mathrm{SR}}(\mathbf{x}-\mathbf{x}_{\mathrm{R}})\|^2. \tag{10}$$

Then the detection metric in (8) becomes a quadratic function of $\underline{\mathbf{x}} = \begin{bmatrix}\mathbf{x}\\ \mathbf{x}_{\mathrm{R}}\end{bmatrix}$ as

$$m(\underline{\mathbf{x}}) = \|\mathbf{y}_{\mathrm{SD}} - H_{\mathrm{SD}}\mathbf{x}\|^2 + \left\|\mathbf{y}_{\mathrm{RD}} - H_{\mathrm{RD}}\mathbf{x}_{\mathrm{R}}\right\|^2 + \frac{1}{4}\|H_{\mathrm{SR}}(\mathbf{x}-\mathbf{x}_{\mathrm{R}})\|^2 \tag{11}$$

$$= \|\underline{\mathbf{y}} - H_{\mathrm{D}}\underline{\mathbf{x}}\|^2 + \|H_{\mathrm{R}}\underline{\mathbf{x}}\|^2 \tag{12}$$

where $\underline{\mathbf{y}} = \begin{bmatrix}\mathbf{y}_{\mathrm{SD}}\\ \mathbf{y}_{\mathrm{RD}}\end{bmatrix}$, $H_{\mathrm{D}} = \mathrm{diag}\left(H_{\mathrm{SD}}, H_{\mathrm{RD}}\right)$, and $H_{\mathrm{R}} = \frac{1}{2}\left[H_{\mathrm{SR}}\ -H_{\mathrm{SR}}\right]$. An optimization problem is established as

$$\begin{aligned}&\text{minimize} \quad m(\underline{\mathbf{x}})\\ &\text{subject to} \quad \underline{\mathbf{x}} \in \mathcal{A}^{4N}.\end{aligned} \tag{13}$$

Applying the projected gradient descent method to the nonconvex optimization problem in (13), an update in the kth iteration is written as

$$\begin{aligned}\underline{\hat{\mathbf{x}}}_k &= \phi\big(\underline{\hat{\mathbf{x}}}_{k-1} - \delta_k \nabla m(\underline{\mathbf{x}})\big|_{\underline{\mathbf{x}}=\underline{\hat{\mathbf{x}}}_{k-1}}\big)\\ &= \phi\big(\underline{\hat{\mathbf{x}}}_{k-1} + \delta'_k H_{\mathrm{D}}^T \underline{\mathbf{y}} - \delta'_k H_{\mathrm{D}}^T H_{\mathrm{D}} \underline{\hat{\mathbf{x}}}_{k-1} - \delta'_k H_{\mathrm{R}}^T H_{\mathrm{R}} \underline{\hat{\mathbf{x}}}_{k-1}\big) \quad (14)\end{aligned}$$

where $\phi(\cdot)$ is a nonlinear projection operator, e.g., $\phi(x) = \mathrm{sgn}(x)$ for $x \in \mathcal{A} = \{+1, -1\}$, $\hat{\mathbf{x}}_{k-1}$ is the estimate in the $(k-1)$th iteration, δ'_k is a step size in the kth iteration for $k = 1, \ldots, L$,

$$\nabla m(\underline{\mathbf{x}}) = -2H_{\mathrm{D}}^T \underline{\mathbf{y}} + 2H_{\mathrm{D}}^T H_{\mathrm{D}} \underline{\mathbf{x}} + 2H_{\mathrm{R}}^T H_{\mathrm{R}} \underline{\mathbf{x}},$$

$$H_{\mathrm{D}}^T \underline{\mathbf{y}} = \begin{bmatrix} H_{\mathrm{SD}}^T \mathbf{y}_{\mathrm{SD}} \\ H_{\mathrm{RD}}^T \mathbf{y}_{\mathrm{RD}} \end{bmatrix}, \quad H_{\mathrm{D}}^T H_{\mathrm{D}} \underline{\hat{\mathbf{x}}}_{k-1} = \begin{bmatrix} H_{\mathrm{SD}}^T H_{\mathrm{SD}} \hat{\mathbf{x}}_{k-1} \\ H_{\mathrm{RD}}^T H_{\mathrm{RD}} \hat{\mathbf{x}}_{\mathrm{R},k-1} \end{bmatrix},$$

and

$$H_{\mathrm{R}}^T H_{\mathrm{R}} \underline{\hat{\mathbf{x}}}_{k-1} = \frac{1}{4} \begin{bmatrix} H_{\mathrm{SR}}^T H_{\mathrm{SR}} (\hat{\mathbf{x}}_{k-1} - \hat{\mathbf{x}}_{\mathrm{R},k-1}) \\ -H_{\mathrm{SR}}^T H_{\mathrm{SR}} (\hat{\mathbf{x}}_{k-1} - \hat{\mathbf{x}}_{\mathrm{R},k-1}) \end{bmatrix}.$$

The use of $H_{ij}^T H_{ij} \mathbf{x}$ and $H_{ij}^T \mathbf{y}_{ij}$, $ij \in \{\mathrm{SR}, \mathrm{SD}, \mathrm{RD}\}$ reduces the complexity since their sizes are only dependent on the number of transmit antennas N, not the number of receive antennas.

2) UNFOLDING ITERATIONS

The L iterations are unfolded to the L layers, i.e., a combination of the elements in (14) enters each single detection layer as the input, where $\underline{\mathbf{x}}_{k-1}$ is set to a separate input to improve performance. In addition to speed up training convergence, an auxiliary $\mathbf{v}_k$ can be added in the input and the output of each layer. By adding $\mathbf{v}_k$, the training time is shorten and the performance is improved.

Then the input vector of the kth layer is

$$\mathbf{i}_k = \begin{bmatrix} \mathbf{v}_{k-1} \\ \hat{\mathbf{x}}_{k-1} \\ \hat{\mathbf{x}}_{\mathrm{R},k-1} \\ H_{\mathrm{SD}}^T H_{\mathrm{SD}} \hat{\mathbf{x}}_{k-1} + \alpha_{1k} H_{\mathrm{SR}}^T H_{\mathrm{SR}} (\hat{\mathbf{x}}_{k-1} - \hat{\mathbf{x}}_{\mathrm{R},k-1}) - \alpha_{2k} H_{\mathrm{SD}}^T \mathbf{y}_{\mathrm{SD}} \\ H_{\mathrm{RD}}^T H_{\mathrm{RD}} \hat{\mathbf{x}}_{\mathrm{R},k-1} - \alpha_{1k} H_{\mathrm{SR}}^T H_{\mathrm{SR}} (\hat{\mathbf{x}}_{k-1} - \hat{\mathbf{x}}_{\mathrm{R},k-1}) - \alpha_{2k} H_{\mathrm{RD}}^T \mathbf{y}_{\mathrm{RD}} \end{bmatrix} \quad (15)$$

with the dimension of $10N$. In detail, the main parts in the kth layer include

$$\begin{aligned}\mathbf{v}_k &= \mathbf{W}_{1k} \mathbf{i}_k \\ \hat{\mathbf{x}}_k &= \psi_{t_k}(\mathbf{W}_{2k} \mathbf{i}_k) \\ \hat{\mathbf{x}}_{\mathrm{R},k} &= \psi_{t_k}(\mathbf{W}_{3k} \mathbf{i}_k)\end{aligned}$$

where $\psi_{t_k}(x) = -1 + \frac{\max(x+t_k,0)}{|t_k|} - \frac{\max(x-t_k,0)}{|t_k|} \in [-1, 1]$ is an element-wise soft decision operator [19] and acts as an activation function in this neural network. As shown in Fig. 3, the entire detection network includes L layers, where the outputs of the previous layer, $\mathbf{v}$, $\mathbf{x}$, and $\mathbf{x}_{\mathrm{R}}$, combined with the observation and side information $H_{\mathrm{SD}}^T \mathbf{y}_{\mathrm{SD}}$, $H_{\mathrm{RD}}^T \mathbf{y}_{\mathrm{RD}}$, $H_{\mathrm{SD}}^T H_{\mathrm{SD}}$, $H_{\mathrm{RD}}^T H_{\mathrm{RD}}$, and $H_{\mathrm{SR}}^T H_{\mathrm{SR}}$, enter the next layer. In the last layer, the final decision is made as $\hat{\mathbf{x}} = \phi(\hat{\mathbf{x}}_L)$. To improve performance, we adopt the residual learning [27], [19], i.e., setting a weighted average of the previous output and the current output as a new current output.

3) LEARNING FUNCTION

The learning function of the detection network at the destination in Fig. 3 is denoted as

$$\hat{\mathbf{x}} = g_\theta(\mathbf{y}_{\mathrm{SD}}, \mathbf{y}_{\mathrm{RD}}, H_{\mathrm{SD}}, H_{\mathrm{RD}}, H_{\mathrm{SR}}) \quad (16)$$

where

$$\theta = \{\alpha_{1k}, \alpha_{2k}, \mathbf{W}_{1k}, \mathbf{W}_{2k}, \mathbf{W}_{3k}, t_k, k = 1, \ldots, L\} \quad (17)$$

is a set of parameters that is trained during the training phase.

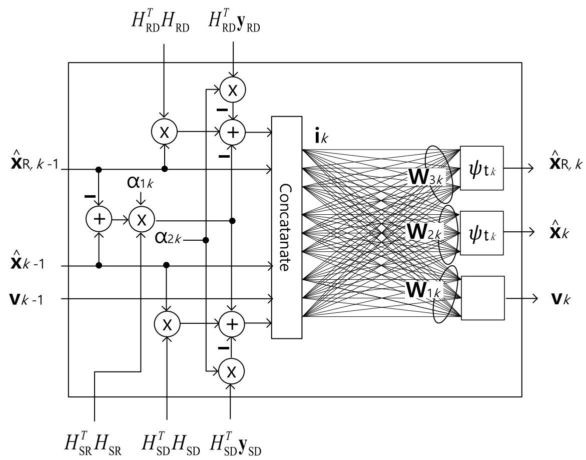


FIGURE 2. A single detection layer in the DL-NML detector.

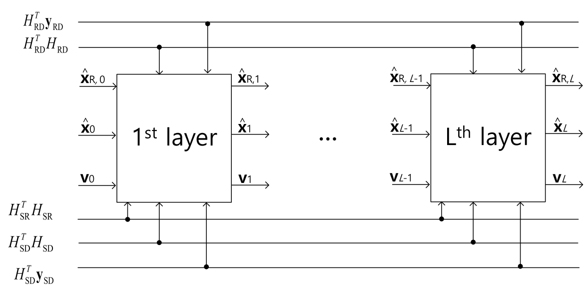


FIGURE 3. The DL-NML detector in the MIMO DF relay channel.

4) LOSS FUNCTION

To train g_θ, we can use the reference signal $\mathbf{x}$ and the observation $(\mathbf{y}_{\mathrm{SD}}, \mathbf{y}_{\mathrm{RD}})$ with the side information $(H_{\mathrm{SD}}, H_{\mathrm{RD}}, H_{\mathrm{SR}})$ as the training data. From the projected gradient descent method in (14) and the deep learning detection network in Fig. 2, we can find that $\mathbf{x}$ and $\mathbf{x}_{\mathrm{R}}$ affect each other in each iteration or layer. Thus, setting $\mathbf{x}_{\mathrm{R}}$ as another reference signal helps improve accuracy. Since the training phase is a preprocessing step, $\mathbf{x}_{\mathrm{R}}$ can be known as a reference signal before training. Combining outputs of L layers, two types of loss functions are proposed.

- When both $\mathbf{x}$ and $\mathbf{x}_R$ are known as the reference signal, the loss function l_1 can be employed as

$$l_1(\mathbf{x};\hat{\mathbf{x}}_\theta) = \sum_{k=1}^{L}\log(k+1)\left(\frac{\|\mathbf{x}-\hat{\mathbf{x}}_k\|^2}{\|\mathbf{x}-\tilde{\mathbf{x}}\|^2}+\frac{\|\mathbf{x}-\hat{\mathbf{x}}_{R,k}\|^2}{\|\mathbf{x}-\tilde{\mathbf{x}}_R\|^2}\right) \tag{18}$$

where $\tilde{\mathbf{x}} = [\tilde{\underline{\mathbf{x}}}]_{1:2N}$, $\tilde{\mathbf{x}}_R = [\tilde{\underline{\mathbf{x}}}]_{2N+1:4N}$, and $\tilde{\underline{\mathbf{x}}} = (H_D^T H_D + H_R^T H_R)^{-1} H_D^T \underline{\mathbf{y}}$, which will be defined in Definition 1.

- When only $\mathbf{x}$ is known as the reference signal, the transmitted signal from the relay, $\mathbf{x}_R$, cannot be used for training, and the loss function l_2 is performed as

$$l_2(\mathbf{x};\hat{\mathbf{x}}_\theta) = \sum_{k=1}^{L}\log(k+1)\frac{\|\mathbf{x}-\hat{\mathbf{x}}_k\|^2}{\|\mathbf{x}-\tilde{\mathbf{x}}\|^2} \tag{19}$$

where $\tilde{\mathbf{x}} = [\tilde{\underline{\mathbf{x}}}]_{1:2N}$.

This detection method is called a *deep learning approach to the NML (DL-NML)* detector.

Definition 1 (ZG Detector): Deriving the zero gradient point of the convex function in (12)*, i.e., the solution of*

$$\nabla m(\underline{\mathbf{x}}) = -2H_D^T\underline{\mathbf{y}} + 2H_D^T H_D\underline{\mathbf{x}} + 2H_R^T H_R\underline{\mathbf{x}} = 0,$$

a linear receiver can be obtained as

$$\tilde{\underline{\mathbf{x}}} = (H_D^T H_D + H_R^T H_R)^{-1}\begin{bmatrix} H_{SD}^T\mathbf{y}_{SD} \\ H_{RD}^T\mathbf{y}_{RD}\end{bmatrix} \tag{20}$$

where $H_D^T H_D = \operatorname{diag}\left(H_{SD}^T H_{SD}, H_{RD}^T H_{RD}\right)$ *and* $H_R^T H_R = \frac{1}{4}\begin{bmatrix} H_{SR}^T H_{SR} & -H_{SR}^T H_{SR} \\ -H_{SR}^T H_{SR} & H_{SR}^T H_{SR}\end{bmatrix}$*. Then a new linear detector in the MIMO DF relay channel is obtained as*

$$\hat{\mathbf{x}} = \phi\left([\tilde{\underline{\mathbf{x}}}]_{1:2N}\right). \tag{21}$$

This is called a zero gradient (ZG) detector.

B. WITH STATISTICAL SR CHANNEL

In this section, we handle the case where only the statistical CSI of the SR link is known at the destination. Since the instantaneous CSI of the SR link, H_{SR}, is unknown at the destination, the exact PEP, $P_{SR}(\mathbf{x} \to \mathbf{x}_R|H_{SR})$, could not be applied in (8). Instead, the average PEP, $\bar{P}_{SR}$, can be used. Subsequently, an NML with two-level-PEP (NMLw2PEP) detector [8] is written as

$$\begin{aligned}\hat{\mathbf{x}} &= \arg\min_{\mathbf{x}\in\mathcal{A}^{2N}}\min_{\mathbf{x}_R\in\mathcal{A}^{2N}}\Big\{\left\|\mathbf{y}_{SD}-H_{SD}\mathbf{x}\right\|^2 \\ &\qquad + \left\|\mathbf{y}_{RD}-H_{RD}\mathbf{x}_R\right\|^2 - \sigma^2\ln\bar{P}_{SR}\Big\} \\ &= \arg\min_{\mathbf{x}\in\mathcal{A}^{2N}}\min_{\mathbf{x}_R\in\mathcal{A}^{2N}}\Big\{\left\|\mathbf{y}_{SD}-H_{SD}\mathbf{x}\right\|^2 \\ &\qquad + \left\|\mathbf{y}_{RD}-H_{RD}\mathbf{x}_R\right\|^2 + \sigma^2\ln P_e^{-1}\cdot 1_{\mathbf{x}_R\neq\mathbf{x}}\Big\}\end{aligned} \tag{22}$$

where

$$\bar{P}_{SR} = \begin{cases}1 & \text{for } \mathbf{x}_R = \mathbf{x} \\ P_e & \text{for } \mathbf{x}_R \neq \mathbf{x},\end{cases}$$

$$1_{\mathbf{x}_R\neq\mathbf{x}} = \begin{cases}0 & \text{for } \mathbf{x}_R = \mathbf{x} \\ 1 & \text{for } \mathbf{x}_R \neq \mathbf{x},\end{cases}$$

$P_e \in (0, 1]$ can be expressed as $P_e = \gamma_{SR}^{-d_R}$, $\gamma_{SR} = \frac{2N}{\sigma^2}$ is the average SNR, and $d_R = N$ is the diversity order when the ML is used at the relay in the Rayleigh fading SR channel $H_{SR}^C \sim \mathcal{CN}(0, I)$ [8, Lemma 1].

To apply the projected gradient descent method, we need to take a gradient for the metric in (22), but $1_{\mathbf{x}_R\neq\mathbf{x}}$ is inappropriate for taking the gradient. When only two nearest points $\mathbf{x}$ and $\mathbf{x}'$ (satisfying $\|\mathbf{x}-\mathbf{x}'\| = 2$) exist, i.e., $\mathbf{x}, \mathbf{x}_R \in \{\mathbf{x}, \mathbf{x}'\}$, we have $1_{\mathbf{x}_R\neq\mathbf{x}} = \frac{1}{4}\|\mathbf{x}-\mathbf{x}_R\|^2$. Hence, we apply $\frac{1}{4}\|\mathbf{x}-\mathbf{x}_R\|^2$ instead of $1_{\mathbf{x}_R\neq\mathbf{x}}$ as an approximation. Then the metric for the NMLw2PEP can be rewritten as a similar form of (11):

$$\begin{aligned} m(\underline{\mathbf{x}}) = \left\|\mathbf{y}_{SD}-H_{SD}\mathbf{x}\right\|^2 + \left\|\mathbf{y}_{RD}-H_{RD}\mathbf{x}_R\right\|^2 \\ + \frac{1}{4}\sigma^2\ln P_e^{-1}\cdot\|\mathbf{x}-\mathbf{x}_R\|^2\end{aligned} \tag{23}$$

where $\underline{\mathbf{x}} = \left[\mathbf{x}^T\ \mathbf{x}_R^T\right]^T$.

Substituting $H_{SR}^T H_{SR} = \sigma^2\ln P_e^{-1}\cdot I$ into Fig. 2 and training the detection network in Fig. 3 with the loss function in (18) or (19), the DL-NML detector with statistical SR channel is achieved. The loss functions are normalized using the ZG receiver in Definition 1 by plugging $H_{SR}^T H_{SR} = \sigma^2\ln P_e^{-1}\cdot I$ into Definition 1.

C. WITHOUT SR CHANNEL

Without the CSI of the SR link, the minimum distance (MD) detector [6] can be considered first. The MD detector ignores the detection errors at the relay although the error may occur. Setting $\mathbf{x}_R = \mathbf{x}$ in (8), the MD detection algorithm is written as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathcal{A}^{2N}}\left\{\left\|\mathbf{y}_{SD}-H_{SD}\mathbf{x}\right\|^2 + \left\|\mathbf{y}_{RD}-H_{RD}\mathbf{x}\right\|^2\right\} \tag{24}$$

and its linear detection version is

$$\hat{\mathbf{x}} = \phi\left(\left(H_{SD}^T H_{SD} + H_{RD}^T H_{RD}\right)^{-1}\left(H_{SD}^T\mathbf{y}_{SD} + H_{RD}^T\mathbf{y}_{RD}\right)\right) \tag{25}$$

which was called a ZF with MRC (ZFwMRC) detector [9].

Since the error probability at the relay is not considered in the MD and the ZFwMRC detectors, they exhibit poor performance [6]. From the metrics in (11) and (23), we can find that the influence of the SR link in the metrics exists only when $\mathbf{x}_R \neq \mathbf{x}$. Due to the similar reason explained in Section III-B, $\frac{1}{4}\|\mathbf{x}-\mathbf{x}_R\|^2$ can represent the influence of the SR link well regardless of the CSI of the SR link. Meanwhile, adding $\frac{1}{4}\|\mathbf{x}-\mathbf{x}_R\|^2$ can be regarded as a regularization method in convex optimization problems [28]. Subsequently, we propose a suboptimal detection algorithm in the following definition.

Definition 2 (NMLwoSRC Detector): Without the SR channel, a new suboptimal detector called an NML without SR channel (NMLwoSRC) detector is defined as

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}\in\mathcal{A}^{2N}} \min_{\mathbf{x}_{\mathrm{R}}\in\mathcal{A}^{2N}} \left\{ \left\| \mathbf{y}_{\mathrm{SD}} - H_{\mathrm{SD}}\mathbf{x} \right\|^2 + \left\| \mathbf{y}_{\mathrm{RD}} - H_{\mathrm{RD}}\mathbf{x}_{\mathrm{R}} \right\|^2 + \frac{1}{4}\|\mathbf{x}-\mathbf{x}_{\mathrm{R}}\|^2 \right\}. \quad (26)$$

This NMLwoSRC detector will achieve improved performance by considering the influence of the SR channel, however, it requires exhaustive search for all possible signal sets.

Substituting $H_{\mathrm{SR}}^T H_{\mathrm{SR}} = I$ into Fig. 3 and Definition 1, we have the DL-NML detector and ZG detector, respectively, without SR channel.

IV. SDR APPROACH TO THE NML DETECTOR AT THE DESTINATION

In this section, the SDR approach [32]–[34] with polynomial complexity is applied in the MIMO DF relay channels as a performance comparison to the proposed DL-NML detector due to the reasons in Section I-C.

We start from the optimization problem in (13) that is a revised version of the NML detector in (8). The optimization problem for $\mathcal{A} = \{1, -1\}$ can equivalently be rewritten as

$$\begin{aligned} &\text{minimize} \quad \mathrm{tr}(\mathbf{LX}) \\ &\text{subject to} \quad [\mathbf{X}]_{ii} = 1, \quad i = 1, \ldots, 4N+1 \\ &\qquad\qquad \mathbf{X} = \mathbf{s}\mathbf{s}^T \end{aligned} \quad (27)$$

where

$$\mathbf{L} = \begin{bmatrix} H_{\mathrm{D}}^T H_{\mathrm{D}} + H_{\mathrm{R}}^T H_{\mathrm{R}} & -H_{\mathrm{D}}^T \underline{\mathbf{y}} \\ -\underline{\mathbf{y}}^T H_{\mathrm{D}} & \underline{\mathbf{y}}^T \underline{\mathbf{y}} \end{bmatrix}, \quad (28)$$

$\mathbf{s} = \begin{bmatrix} \hat{\underline{\mathbf{x}}} \\ 1 \end{bmatrix}$, $\hat{\underline{\mathbf{x}}} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{x}}_{\mathrm{R}} \end{bmatrix}$, $\underline{\mathbf{y}} = \begin{bmatrix} \mathbf{y}_{\mathrm{SD}} \\ \mathbf{y}_{\mathrm{RD}} \end{bmatrix}$, $H_{\mathrm{D}} = \mathrm{diag}\left(H_{\mathrm{SD}}, H_{\mathrm{RD}}\right)$, and $H_{\mathrm{R}}^T H_{\mathrm{R}} = \frac{1}{4}\begin{bmatrix} H_{\mathrm{SR}}^T H_{\mathrm{SR}} & -H_{\mathrm{SR}}^T H_{\mathrm{SR}} \\ -H_{\mathrm{SR}}^T H_{\mathrm{SR}} & H_{\mathrm{SR}}^T H_{\mathrm{SR}} \end{bmatrix}$. By replacing the last constraint $\mathbf{X} = \mathbf{s}\mathbf{s}^T$ with $\mathbf{X} \succeq 0$ in the nonconvex optimization problem in (27), an semidefinite programming is obtained as

$$\begin{aligned} &\text{minimize} \quad \mathrm{tr}(\mathbf{LX}) \\ &\text{subject to} \quad [\mathbf{X}]_{ii} = 1, \quad i = 1, \ldots, 4N \\ &\qquad\qquad \mathbf{X} \succeq 0 \end{aligned} \quad (29)$$

which can be solved by standard convex optimization techniques [28] or the CVX packages in MATLAB, e.g., [35]. Subsequently, the desired signal can be detected as

$$\hat{\mathbf{x}} = \mathrm{sgn}([\mathbf{X}]_{1:2N,4N+1}). \quad (30)$$

Replacing the rank-1 constraint $\mathbf{X} = \mathbf{s}\mathbf{s}^T$ with a positive semi-definite constraint $\mathbf{X} \succeq 0$, SDR leads to an increase in problem dimension. Hence, a randomization method has been introduced to this SDR algorithm [32]. Doing the Cholesky factorization to $\mathbf{X} = \mathbf{V}^T\mathbf{V}$ and generating K i.i.d. zero-mean unit-variance Gaussian random vectors $\mathbf{u}_1, \ldots, \mathbf{u}_K$ of dimension $4N+1$, we have the candidate solutions as

$$\hat{\underline{\mathbf{x}}}_k = \mathrm{sgn}\left(\frac{[\mathbf{V}^T]_{1:4N,:}\,\mathbf{u}_k}{[\mathbf{V}^T]_{4N+1,:}\,\mathbf{u}_k}\right), \quad k = 1, 2, \ldots, K.$$

Finally, the optimal one which minimizes the metric in (12) is selected as

$$\hat{\underline{\mathbf{x}}} = \arg\min_k m(\hat{\underline{\mathbf{x}}}_k). \quad (31)$$

Taking the desired part $\hat{\mathbf{x}} = [\hat{\underline{\mathbf{x}}}]_{1:2N}$, we have an *SDR approach to the NML detector (SDR-NML) detector*. Although this SDR-NML detector cannot achieve the same performance as the one in (13) and (27), we can expect a fine performance similar to the case of the MIMO channel [33], [34], where the SDR algorithm achieves the same diversity order with the ML detector. Replacing $H_{\mathrm{SR}}^T H_{\mathrm{SR}} = \sigma^2 \ln P_e^{-1} \cdot I$ and $H_{\mathrm{SR}}^T H_{\mathrm{SR}} = I$ instead of the exact value, we achieve the SDR-NML detector with statistical SR channel and without SR channel, respectively.

V. DETECTORS AT THE RELAY

In previous sections, we proposed several detectors at the destination (DetD) when the ML detector is applied at the relay. In practical systems, various detectors could be used at the relay. In this section, we briefly introduce some representative detectors at the relay (DetR) such as the ML, ZF, and SDR approach to the ML (SDR-ML) [32] detectors, propose a deep learning approach to the ML (DL-ML) detector, and handle the corresponding equivalent SR channel H_{SR} applied for the DetD in Section III and IV. While the ML detector achieves optimal performance, the ZF detector has the simplest complexity, and the SDR-ML detector is a suboptimal one with near-ML performance and polynomial complexity. The three detectors have been theoretically well analyzed.

A. THE ML DETECTOR AT THE RELAY

The ML detector at the relay (MLaR) is written as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathcal{A}^{2N}} \left\| \mathbf{y}_{\mathrm{SR}} - H_{\mathrm{SR}}\mathbf{x} \right\|^2. \quad (32)$$

The original channel coefficient H_{SR} and the corresponding relay error probability P_e are used in the DetD for the scenarios with instantaneous SR channel and with statistical SR channel, respectively, in Section III and IV.

B. THE ZF DETECTOR AT THE RELAY

The ZF detector at the relay (ZFaR)[2] [11] is written as

$$\hat{\mathbf{x}} = \phi(\tilde{\mathbf{x}})$$

where

$$\tilde{\mathbf{x}} = (H_{\mathrm{SR}}^T H_{\mathrm{SR}})^{-1} H_{\mathrm{SR}}^T \mathbf{y}_{\mathrm{SR}}. \quad (33)$$

The SNR for x_i is $SNR_i = \frac{1}{\frac{1}{2}\sigma^2[(H_{\mathrm{SR}}^T H_{\mathrm{SR}})^{-1}]_{ii}}$, $i = 1, \ldots, N$. Thus, an equivalent channel model can be written as

$$\tilde{\mathbf{y}} = H_{\mathrm{SR}}^{\mathrm{ZF}}\mathbf{x} + \tilde{\mathbf{z}} \quad (34)$$

[2]The MMSE detector is also widely used in the MIMO channel. When $\sigma^2 \to 0$, the MMSE detector is the same as the ZF detector.

where $\tilde{\mathbf{z}} \sim \mathcal{N}(0, \frac{1}{2}\sigma^2 I)$ and $H_{\rm SR}^{\rm ZF} = \mathrm{diag}\left(\frac{1}{\sqrt{[(H_{\rm SR}^T H_{\rm SR})^{-1}]_{11}}}, \ldots, \frac{1}{\sqrt{[(H_{\rm SR}^T H_{\rm SR})^{-1}]_{2N,2N}}}\right)$. Applying $H_{\rm SR}^{\rm ZF}$ instead of $H_{\rm SR}$ in (9), we obtain the PEP for the ZFaR. Thus, we use $H_{\rm SR} = H_{\rm SR}^{\rm ZF}$ for the NML, DL-NML, ZG and SDR-NML detectors with instantaneous SR channel. Since $\frac{2}{[(H_{\rm SR}^T H_{\rm SR})^{-1}]_{ii}} \sim \chi_2^2$, i.e., a chi-squared distribution with 2 degree of freedom, the diversity order of the ZFaR is $d_{\rm R} = 1$ [36]. Subsequently, $d_{\rm R} = 1$ is used to determine the average error probability, P_e, for the NMLw2PEP, DL-NML, ZG, and SDR-NML detectors with statistical SR channel.

C. THE SDR APPROACH TO THE ML DETECTOR AT THE RELAY

The SDR problem [32] - [34] for the ML detector in (32) can be written as

$$\begin{aligned} &\text{minimize} \quad \mathrm{tr}(\mathbf{LX}) \\ &\text{subject to} \quad [\mathbf{X}]_{ii} = 1, \quad i = 1, \ldots, 2N+1 \\ &\qquad\qquad\quad \mathbf{X} \succeq 0, \end{aligned} \tag{35}$$

where

$$\mathbf{L} = \begin{bmatrix} H_{\rm SR}^T H_{\rm SR} & -H_{\rm SR}^T \mathbf{y}_{\rm SR} \\ -\mathbf{y}_{\rm SR}^T H_{\rm SR} & \mathbf{y}_{\rm SR}^T \mathbf{y}_{\rm SR} \end{bmatrix}.$$

Taking the desired signal, we have the SDR detector as

$$\hat{\mathbf{x}} = \mathrm{sgn}([\mathbf{X}]_{1:2N,2N+1}). \tag{36}$$

Similar to Section IV, the SDR approach to the ML (SDR-ML) detector [32] is written as

$$\hat{\mathbf{x}} = \arg\min_k \left\| \mathbf{y}_{\rm SR} - H_{\rm SR}\hat{\mathbf{x}}_k \right\|^2 \tag{37}$$

where $\hat{\mathbf{x}}_k = \mathrm{sgn}\left(\frac{[\mathbf{V}^T]_{1:2N,:}\mathbf{u}_k}{[\mathbf{V}^T]_{2N+1,:}\mathbf{u}_k}\right)$, $\mathbf{V}$ is the result of Cholesky factorization $\mathbf{X} = \mathbf{V}^T\mathbf{V}$, and $\mathbf{u}_1, \ldots, \mathbf{u}_K$ are the i.i.d. zero-mean unit-variance Gaussian random vectors of dimension $2N+1$. Since the SDR detector achieves the same diversity order as the ML detector [33], we use the original parameter $H_{\rm SR}$ and the corresponding relay error probability P_e in the DetD for the scenarios with instantaneous SR channel and with statistical SR channel, respectively, when the SDR-ML detector is used at the relay.

D. THE DEEP LEARNING APPROACH TO THE ML DETECTOR AT THE RELAY

We design a new deep learning detector called a deep learning approach to the ML (DL-ML) detector in MIMO channels. The DL-ML detector contains L layers, and each single layer is similar to the one in Fig. 2 except that the part related to the helping signal $\hat{\mathbf{x}}_{{\rm R},k}$ does not exist. The input vector in the kth layer is

$$\mathbf{i}_k = \begin{bmatrix} \mathbf{v}_{k-1} \\ \hat{\mathbf{x}}_{k-1} \\ H_{\rm SR}^T H_{\rm SR}\hat{\mathbf{x}}_{k-1} - \alpha_k H_{\rm SR}^T \mathbf{y}_{\rm SR} \end{bmatrix} \tag{38}$$

and the parameters

$$\theta = \{\alpha_k, \mathbf{W}_{1k}, \mathbf{W}_{2k}, t_k, k = 1, \ldots, L\}$$

are trained to minimize a loss function

$$l(\mathbf{x}; \hat{\mathbf{x}}_\theta) = \sum_{k=1}^{L} \log(k+1)\frac{\|\mathbf{x} - \hat{\mathbf{x}}_k\|^2}{\|\mathbf{x} - \tilde{\mathbf{x}}\|^2} \tag{39}$$

where $\tilde{\mathbf{x}} = (H_{\rm SR}^T H_{\rm SR})^{-1} H_{\rm SR}^T \mathbf{y}_{\rm SR}$ is the ZF receiver in (33). Note that each single layer includes only one neural network layer unlike the deep MIMO detector in [19].

As shown in Fig. 4 of Section VII-C, this DL-ML detector shows similar performance compared to the SDR-ML detector in the MIMO channel with $N = 20, 30, 50$; thus we use the original parameter $H_{\rm SR}$ and its corresponding error probability P_e in the NML, DL-NML, ZG, and SDR-NML detectors for the case of DL-ML detector at the relay (DL-MLaR). The detailed comparison will be dealt with in Section VII-C.

VI. TRAINING AND DETECTION DETAILS OF THE DL-NML DETECTOR

The training is implemented on the TensorFlow frameworks [29] by applying the Adam optimizer, a variation of the stochastic gradient descent method [30].

To train the DL-NML detector, data samples of $\mathbf{x}, \mathbf{x}_{\rm R}, \mathbf{y}_{\rm SD}, \mathbf{y}_{\rm RD}, H_{\rm SR}, H_{\rm SD}$, and $H_{\rm RD}$ are required at the destination. After randomly generating $\mathbf{x}, H_{\rm SD}, \mathbf{z}_{\rm SD}, H_{\rm RD}$, and $\mathbf{z}_{\rm RD}$, $\mathbf{y}_{\rm SD}$ can be directly derived from (5). Alternatively, $\mathbf{y}_{\rm RD}$ requires $\mathbf{x}_{\rm R}$, and $\mathbf{x}_{\rm R}$ is an output of a specific detector at the relay as show in Fig. 1. As explained in Section V, there are various candidates for the detector at the relay. Among them, we apply the DL-MLaR in Section V-D during training since the DL-MLaR is trained on TensorFlow frameworks which leads to two advantages: 1) being easy to be embedded in the training model and 2) shortening the training time due to much less time of parallel computation and batch signal detection. After training, the deep learning detector can also be applied for the MLaR, SDR-MLaR, and ZFaR with their corresponding equivalent $H_{\rm SR}$ explained in Section V.

Although deep learning detectors have many advantages, such as low dependence on mathematical methods, relatively low complexity, and batch signal detection, it is not easy to be applied in practical communication systems due to requirement of large training data and long training time caused by pre-training. Our goal is to train a DL-NML detector offline and apply it in various channel environments directly without retraining or with a little retraining. To accomplish the purpose, we apply following three methods in the training.

i) Training the detector by using samples of uniformly distributed signal $\mathbf{x}$ and channel matrices $H_{ij}^C \sim \mathcal{CN}(0, I)$, $ij \in \{\rm SR, SD, RD\}$. The reason for choosing the channel model is that Rayleigh fading channel is a reasonable channel model in urban environments and its randomness makes it possible to apply the trained DL-NML detector in a wide variety of channels rather than some

Algorithm 1 The Training Process of the DL-NML Detector Using the Loss Function l_1 in (18)

```
Initialize x̂_0, x̂_{R,0}, v_0, the parameter set θ_0 in (17), and P_min
for iteration n = 1, 2, ..., N_iteration do
    Generate N_batch data samples of x, y_SR, y_SD, y_RD, H_SD, H_RD, H_SR randomly according to (1)-(6) and obtain x_R from y_SR and H_SR applying the DL-MLaR
    Compute H_SD^T y_SD, H_RD^T y_RD, H_SD^T H_SD, H_RD^T H_RD, and H_SR^T H_SR
    for Layer k = 1, 2, ..., L do
        Compute i_k using x̂_{k-1}, x̂_{R,k-1}, v_{k-1}, H_SD^T y_SD, H_RD^T y_RD, H_SD^T H_SD, H_RD^T H_RD, H_SR^T H_SR and parameters α_{1k}, α_{2k} in θ_{n-1}
        Compute x̂_k, x̂_{R,k}, and v_k using i_k and parameters W_{1k}, W_{2k}, W_{3k}, t_k in θ_{n-1}
    end for
    Update θ_{n-1} to θ_n with Adam optimizer [30] to minimize l_ave = (1/N_batch) Σ_{m=1}^{N_batch} l_1(x^m; x̂_θ^m) where l_1(x^m; x̂_θ^m) is the one in (18) for mth batch
    Determine x̂^m = [x̂_1^m, ..., x̂_{2N}^m]^T = φ(x̂_L^m) for m = 1, ..., N_batch
    Calculate P_b = 1/(2N N_batch) Σ_{m=1}^{N_batch} Σ_{i=1}^{2N} 1_{x_i^m ≠ x̂_i^m}
    if P_b < P_min then
        P_min = P_b
        Save parameters θ_n to θ*
    end if
end for
return θ*
```

deterministic channels. The DL-NML detector trained under this channel model achieves acceptable performance under other channel environments.

ii) Training the detector under uniformly distributed SNR in a reasonable range so that the detector can be applied to various SNRs.

iii) Training the detector using normalized input, i.e., applying $\frac{H_{ij}^T H_{ij}}{\frac{1}{N}\,\mathrm{tr}(H_{ij}^T H_{ij})}$ and $\frac{H_{ij}^T \mathbf{y}_{ij}}{\frac{1}{N}\,\mathrm{tr}(H_{ij}^T H_{ij})}$, $ij \in \{\mathtt{SR}, \mathtt{SD}, \mathtt{RD}\}$ in Figs 2 and 3. By doing so, pretty good performance can be obtained without retraining or with a little retraining when the channel conditions change. For example, in an unbalanced relay channel of $H_{ij}^C \sim \mathcal{CN}(0, \sigma_{ij}^2 I)$, $ij \in \{\mathtt{SR}, \mathtt{SD}, \mathtt{RD}\}$, the normalization makes it possible to apply the deep learning detector without modification.

In the training phase, N_{batch} data samples of $\mathbf{x}, \mathbf{x}_{\mathrm{R}}, \mathbf{y}_{\mathtt{SD}}, \mathbf{y}_{\mathtt{RD}}, H_{\mathtt{SR}}, H_{\mathtt{SD}}$, and $H_{\mathtt{RD}}$ are generated according to their relationship in each iteration, and $N_{\text{iteration}}$ iterations are implemented to $L = 10$ ($L < N$) layers. The training process of the detection networks with the loss function l_1 in (18) is shown in Algorithm 1, where $\mathbf{v}_k$ is a $2N \times 1$ vector, $\mathbf{v}_0 = \mathbf{1}$, $\hat{\mathbf{x}}_0 = \mathbf{0}$, $\hat{\mathbf{x}}_{\mathrm{R},0} = \mathbf{0}$, $H_{\mathtt{SR}}$ is the equivalent SR channel matrix depending on the scenarios described in Section III, and $P_b = \frac{1}{2NN_{\text{batch}}}\sum_{m=1}^{N_{\text{batch}}}\sum_{i=1}^{2N} 1_{x_i^m \neq \hat{x}_i^m}$ is the average bit error

Algorithm 2 The Detection Process of the DL-NML Detector

```
Set x̂_0, x̂_{R,0}, and v_0 as the same as the ones in Algorithm 1
Compute H_SD^T y_SD, H_RD^T y_RD, H_SD^T H_SD, H_RD^T H_RD, and H_SR^T H_SR
for Layer k = 1, 2, ..., L do
    Compute i_k using x̂_{k-1}, x̂_{R,k-1}, v_{k-1}, H_SD^T y_SD, H_RD^T y_RD, H_SD^T H_SD, H_RD^T H_RD, H_SR^T H_SR and parameters α_{1k}, α_{2k} in the final parameter set θ* trained in Algorithm 1
    Compute x̂_k, x̂_{R,k}, and v_k using i_k and parameters W_{1k}, W_{2k}, W_{3k}, t_k in θ*
end for
return x̂ = φ(x̂_L)
```

rate (BER) for one batch data. If $P_b < P_{\min}$, the parameters in θ_n are saved in θ^* and $P_{\min}$ is set to P_b. After $N_{\text{iteration}}$ iterations, we have the final parameter set θ^*. The training for the loss function l_2 in (19) can be done similarly.

Once θ^* is determined, the transmitted signal $\mathbf{x}$ can be detected in real time using Algorithm 2.

VII. PERFORMANCE EVALUATION

In the previous sections, we proposed various detectors at the destination (DetD) such as the DL-NML, SDR-NML, ZG and introduced some DetR. In this section, we discuss complexities of these DetD and DetR, present various DetR:DetD methods according to the error performance and detection complexity, and finally compare their simulation results.

A. DETECTION COMPLEXITY

For the complexity measure of the detection algorithms, we apply the naive calculation method, i.e., the complexity is $O(nmp)$ for the multiplication of matrices of $n \times m$ and $m \times p$, and $O(n^3)$ for the $n \times n$ matrix inversion. The detection complexities for the DetR and DetD are discussed for a constant $\beta_{\mathrm{D}} = \frac{N_{\mathrm{D}}}{N}$.

Based on the above rules of computational complexity, the following results are obtained.

- The complexities of the NML detectors are at least $O((4N)^2 \cdot |\mathcal{A}|^{4N})$ since $\|\mathbf{y}_{\mathtt{SD}} - H_{\mathtt{SD}}\mathbf{x}\|^2$, $\|\mathbf{y}_{\mathtt{RD}} - H_{\mathtt{RD}}\mathbf{x}_{\mathrm{R}}\|^2$, and $\|H_{\mathtt{SR}}(\mathbf{x} - \mathbf{x}_{\mathrm{R}})\|^2$ require the complexity of $O((4N)^2)$ for each $\underline{\mathbf{x}} = \left[\mathbf{x}^T \ \mathbf{x}_{\mathrm{R}}^T\right]^T \in \mathcal{A}^{4N}$.
- The ZG detector requires the complexity of $O((4N)^3)$ due to the matrix multiplication and the matrix inversion. For a quasi-static fading channel (fixed CSI), the complexity is $O((4N)^2)$ since the ZG detector only needs to do a multiplication of a $4N \times 4N_{\mathrm{D}}$ matrix, $(H_{\mathrm{D}}^T H_{\mathrm{D}} + H_{\mathrm{R}}^T H_{\mathrm{R}})^{-1} H_{\mathrm{D}}^T$, and a $4N_{\mathrm{D}} \times 1$ vector, $\underline{\mathbf{y}}$.
- For the DL-NML detector, the complexity order is $O\big((4N)^3\big)$. In detail, the computation of $H_{\mathtt{SD}}^T H_{\mathtt{SD}}$, $H_{\mathtt{RD}}^T H_{\mathtt{RD}}$, and $H_{\mathtt{SR}}^T H_{\mathtt{SR}}$ requires $(\frac{\beta_{\mathrm{D}}}{4} + \frac{1}{8})(4N)^3$ times of multiplications, and calculation of $H_{\mathtt{SD}}^T \mathbf{y}_{\mathtt{SD}}$ and $H_{\mathtt{RD}}^T \mathbf{y}_{\mathtt{RD}}$ requires $\frac{1}{2}(4N)^2$ multiplications. Calculation of $\mathbf{i}_k$ in (15) requires $8N$ multiplications, and the multiplication of $\mathbf{i}_k$ and $\mathbf{W}_{1k}$, $\mathbf{W}_{2k}$, $\mathbf{W}_{3k}$ requires $\frac{15}{4}(4N)^2$

multiplications in each layer (see Fig. 2), thus we have a complexity of $O(L(4N)^2)$ in $L\ (< N)$ detection layers. Hence, the overall detection complexity of the DL-NML detector is $O((4N)^3)$. For quasi-static fading channels, the channel terms of $H_{\mathrm{SD}}^T H_{\mathrm{SD}}$, $H_{\mathrm{RD}}^T H_{\mathrm{RD}}$, and $H_{\mathrm{SR}}^T H_{\mathrm{SR}}$ do not need to be computed again; therefore, the complexity of $O(L(4N)^2)$ is required. The complexity of the DL-NML detector can be lowered by reducing L appropriately according to the required BER.

- The SDR-NML detector has the complexity of $O((4N)^{3.5}\log(1/\epsilon))$ given a solution accuracy $\epsilon > 0$, where $4N$ is from the dimension in (29) [34], [37].

We summarize the detection complexity for the DetD as follows.

- NML: $O((4N)^2 \cdot |\mathcal{A}|^{4N})$
- ZG: $O((4N)^3) \rightarrow O((4N)^2)$ quasi-static fading
- DL-NML: $O((4N)^3) \rightarrow O(L(4N)^2)$ quasi-static fading
- SDR-NML: $O((4N)^{3.5}\log(1/\epsilon))$

As can be seen, the ZG detector has lowest complexity and the DL-NML detector has second lowest complexity for a large number of N. Similarly, the detection complexity for the DetR mentioned in Section V is also derived.

- MLaR: $O((2N)^2 \cdot |\mathcal{A}|^{2N})$
- ZFaR: $O((2N)^3) \rightarrow O((2N)^2)$ quasi-static fading
- DL-MLaR: $O((2N)^3) \rightarrow O(L(2N)^2)$ quasi-static fading
- SDR-MLaR: $O((2N)^{3.5}\log(1/\epsilon))$

B. SYSTEM CONFIGURATION

Using various DetR and DetD, the DF relay channel in Fig. 1 exhibits different characteristics in error performance and detection complexity. We introduce and compare several types of system methods depending on the applied DetR and DetD. The complexity is based on the discussion in Section VII-A, and the error performance will be demonstrated in Section VII-C.

- MLaR:NML
  Exhaustive search detection algorithms are implemented at the relay and the destination, i.e., the ML detector is used at the relay, and the suboptimal NML detectors including the NML, NMLw2PEP, and NMLwoSRC are applied at the destination. This system achieves excellent performance, but cannot be used in large-scale antenna systems due to their high complexity.
- ZFaR:ZG
  A ZF detector is used at the relay, and the ZG detector in Definition 1 is applied at the destination under various knowledge of SR channel. This method has a simple detection complexity at both the relay and the destination, but exhibits poor performance.
- SDR-MLaR:SDR-NML
  The SDR-ML and SDR-NML detectors in Section V-C and IV are implemented at the relay and the destination, respectively. This method exhibits a fair performance with a polynomial complexity.
- DL-MLaR:DL-NML
  The deep learning algorithms, DL-ML and DL-NML, are employed at the relay and the destination, respectively. This method achieves nice performance with low complexity through a pre-process of training. Especially, in the scenario without SR channel at the destination, this method obtains excellent error performance compared to other methods.

C. SIMULATION RESULTS

We evaluate the error performance of various system methods and compare the proposed detection algorithms. Since the MLaR:NML method could not be implemented in real time due to its high complexity, we compare the DL-MLaR : DL-NML and SDR-MLaR:SDR-NML methods for large numbers of antennas. For the best-performance NML detectors and the poorest-performance ZG detector, we evaluate later with a smaller number of antennas.

We compare the BERs of the DL-MLaR:DL-NML and SDR-MLaR:SDR-NML methods under the independent and identically distributed (i.i.d.) Rayleigh fading channel of $N_{\mathrm{D}} = N$, i.e., $H_{ij}^C \sim \mathcal{CN}(0, I)$, $ij \in \{\mathtt{SR}, \mathtt{SD}, \mathtt{RD}\}$. As mentioned in Section VI, we implement the DL-ML detector at the relay and the DL-NML detector at the destination, both of which are trained on the channel of $H_{ij}^C \sim \mathcal{CN}(0, I)$, $ij \in \{\mathtt{SR}, \mathtt{SD}, \mathtt{RD}\}$.

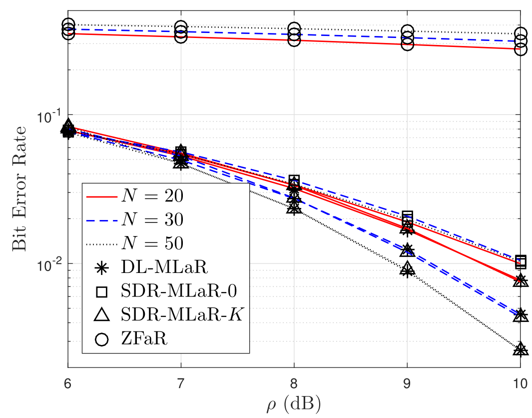


FIGURE 4. BER comparisons of various DetR over the Rayleigh fading MIMO channel. The DL-MLaR applies $L = 20$ layers for all of $N = 20, 30, 50$, and SDR-MLaR-K applies $K = 20$ for $N = 20$, $K = 60$ for $N = 30$, and $K = 300$ for $N = 50$.

We begin with the BERs of the DetR in Section V in the MIMO channel with $N = 20, 30, 50$. The DL-MLaR in Fig. 4 has undergone the structure in Session V-D with $L = 20$ layers and has been trained by $N_{\mathrm{iteration}} = 50000$ iterations with $N_{\mathrm{batch}} = 1000$ batches in each iteration. As shown in Fig. 4, the ZFaR exhibits the worst performance and SDR-MLaR-0 (the original SDR detector in (36)) follows it. The DL-MLaR and the SDR-MLaR-K (the K-randomization method in (37)) achieve the similar performance in the MIMO channel for all cases of $N = 20, 30, 50$. This makes it possible to make a fair comparison between the DL-NML detector and the

SDR-NML detector in the MIMO DF relay channel and also supports the explanation in Session V-D well. The DL-MLaR and SDR-MLaR detectors with the parameters mentioned above and the performance in Fig. 4 will be used in the following comparisons of the DL-MLaR:DL-NML and SDR-MLaR:SDR-NML methods.

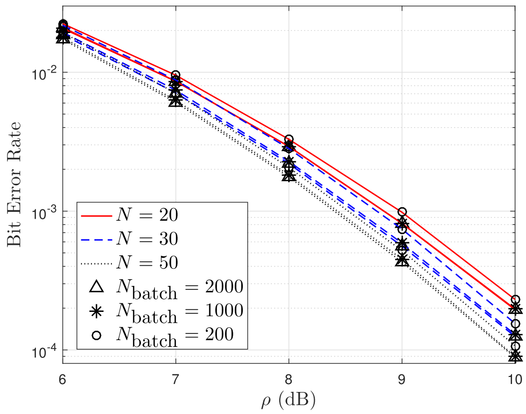


FIGURE 5. BER comparisons of the DL-MLaR:DL-NML method with loss function in (18) trained by various batch sizes over the Rayleigh fading MIMO DF relay channel of $N_D = N$.

TABLE 1. Training time of the DL-NML detector for various batch sizes when $N_D = N$. The units of time are hour and minute denoted by 'h' and 'm', respectively.

Training time	$N_{\text{batch}} = 200$	1000	2000
$N = 20$	18m	52m	1h 32m
$N = 30$	28m	1h 40m	3h 10m
$N = 50$	1h 6m	4h 50m	9h 40m

In oder to determine an appropriate batch size, we compare the performance and the training time of the DL-NML detector with $L = 10$ for various batch sizes in $N_{\text{iteration}} = 50000$ iterations when $N_D = N$ in the computer environment with Intel Core i7-8700K CPU 3.7GHz and NVIDIA GeForce GTX 1080 Ti. As shown in Fig. 5 and Table 1, the BER trained with $N_{\text{batch}} = 1000$ is very close to that trained with $N_{\text{batch}} = 2000$, while the required training time is only half the time for all the cases of $N = 20, 30, 50$. Hence, we will train the DL-NML detector with the batch size of $N_{\text{batch}} = 1000$ in the following performance comparisons. Obviously, we can use less batch sizes if we want shorter training time with reasonable performance. On the other hand, we are wondering how long it will take to detect signals by using this deep learning detector. Table 2 shows the required detection time for a single signal vector $\mathbf{x}$ depending on the size of N_{batch} and computation with GPU or with CPU only. The results show that the required training time is not very long, the detection time is short, and the application of larger batch size can further shorten the detection time.

Now we are ready to compare the performance of the DL-MLaR:DL-NML and SDR-MLaR:SDR-NML methods.

TABLE 2. Detection time for a single signal vector, $\mathbf{x}$, in the DL-NML detector with various N_{batch} for $N_D = N$. The unit is second.

Detection time		$N_{\text{batch}} = 1$	10	100	1000
$N = 20$	GPU	3.1E-3	3.4E-4	4.2E-5	1.5E-5
	CPU	1.2E-3	2.6E-4	1.8E-4	1.8E-4
$N = 30$	GPU	3.2E-3	3.8E-4	5.1E-5	2.3E-5
	CPU	1.6E-3	4.9E-4	4.1E-4	4.1E-4
$N = 50$	GPU	3.3E-3	3.9E-4	7.9E-5	5.7E-5
	CPU	2.4E-3	1.3E-3	1.2E-3	1.2E-3

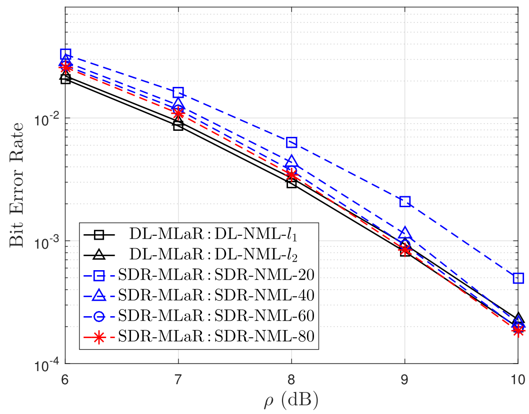
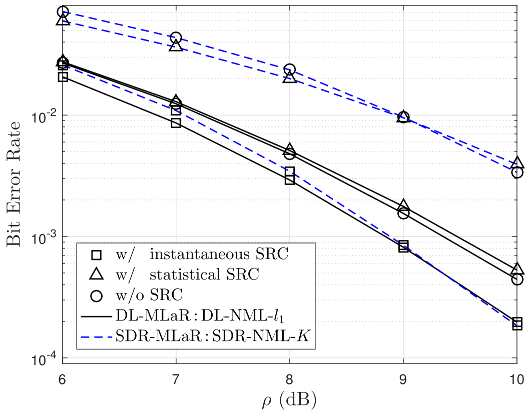


FIGURE 6. BER comparison of the DL-MLaR:DL-NML and SDR-MLaR:SDR-NML methods in the MIMO DF relay channel of $N_D = N = 20$ in the scenario with instantaneous SR channel. Although the SDRaR-NMLSDR method will achieve better performance by comparing more candidate solutions in (31), we stop comparisons at a moderate K due to high complexity.

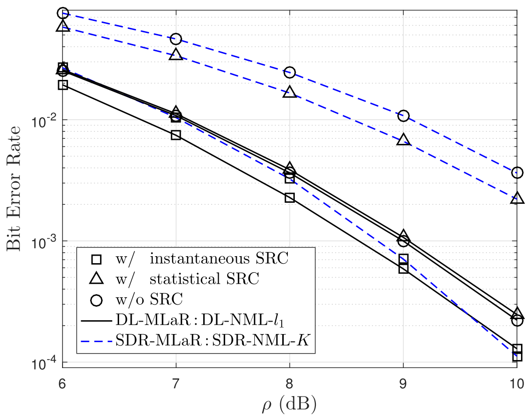
Figs. 6 and 7 show the BERs in the Rayleigh fading MIMO DF relay channel of $N_D = N = 20, 30, 50$ under various scenarios. Fig. 6 compares BERs of both methods in the case of $N_D = N = 20$ with instantaneous SR channel (SRC) and shows the convergence of SDR-NML depending on the numbers of candidate K (indicated by SDR-NML-K). In Fig. 7, the results with only one K of moderate size are evaluated for $N_D = N = 20, 30, 50$. From the curves, one can observe as follows.

- The DL-MLaR:DL-NML method that is trained using the loss function l_1 achieves better performance than that trained by the loss function l_2.
- The DL-MLaR:DL-NML method achieves better or similar performance compared to the SDR-MLaR:SDR-NML method with the instantaneous SRC.
- The DL-MLaR:DL-NML method achieves much better performance (about 1.5~2.3 dB SNR improvement at BER $= 10^{-2}$) than the SDR-MLaR:SDR-NML method with statistical SRC or without SRC at the destination.
- The DL-MLaR:DL-NML method is relatively less impacted by the knowledge of the CSI of the SR link than the SDR-MLaR:SDR-NML method on performance.

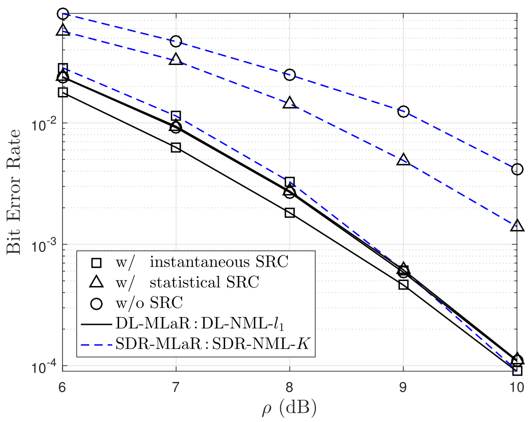
From above simulations, it can be seen that the DL-MLaR:DL-NML method works well in the channel when being trained under the same channel environments. We apply this method with parameters in θ^* which is trained in i.i.d. Rayleigh fading channels with $N_D = N = 20$



(a) $N_D = N = 20$

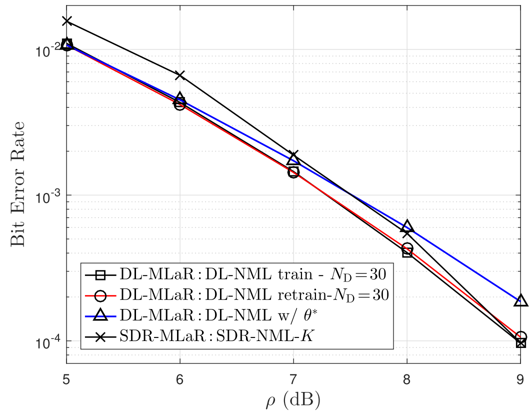


(b) $N_D = N = 30$

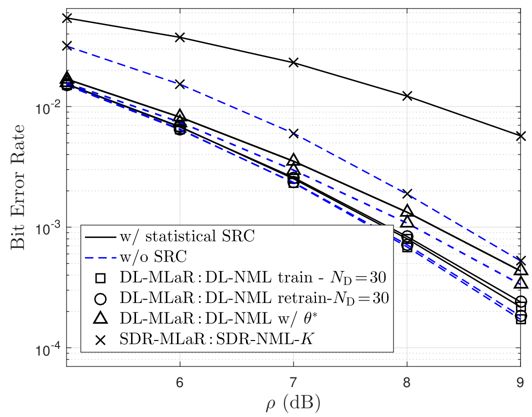


(c) $N_D = N = 50$

FIGURE 7. BER comparison of the DL-MLaR:DL-NML and SDR-MLaR:SDR-NML methods over the Rayleigh fading MIMO DF relay channel. $K = 80$, $K = 120$, and $K = 200$ are applied for the SDR-NML detector in the cases of $N = 20$, $N = 30$, and $N = 50$, respectively.



(a) With instantaneous SRC



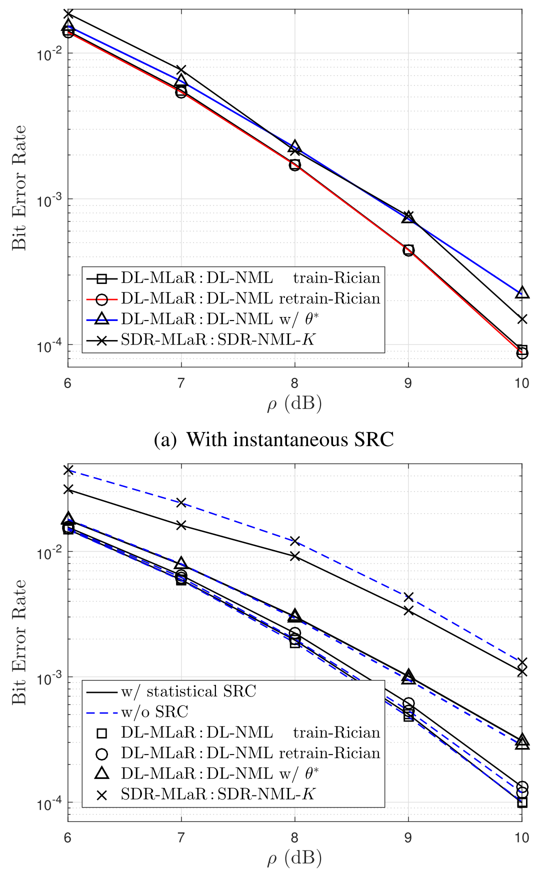
(b) With statistical SRC and without SRC

FIGURE 8. BER comparison of the DL-MLaR:DL-NML and SDR-MLaR:SDR-NML methods over Rayleigh fading MIMO DF relay channels of $N_D = 30$ and $N = 20$. θ^* means the parameter set trained for $N_D = N = 20$.

to other channel environments to see how it works in a different channel. As introduced in Section III-A, the input and output dimensions of each detection layer in Fig. 2 only depend on the number of the transmit antennas. The trained DL-NML detector can also be applied for different numbers of receive antennas when the same number of transmit antennas is used. Hence, the first channel we would like to consider is an i.i.d. Rayleigh fading channel with $N_D = 30$ and $N = 20$ shown in Fig. 8. The $\square$ markers denote the BERs for the DL-MLaR:DL-NML method trained for $N_D = 30$, the $\triangle$ markers denote the one with θ^* (trained for $N_D = 20$); the $\bigcirc$ markers represent the method initialized with θ^* and retrained for $N_D = 30$ in $N_{\text{iteration}} = 1000$ iterations; and the **x** markers denote the results of the SDR-MLaR:SDR-NML method with $K = 20$ at the relay and $K = 80$ at the destination. The second one is a Rician fading channel of $H_{ij}^C = \sqrt{\frac{\mathbf{K}}{1+\mathbf{K}}} H_{ij}^{\text{LOS}} + \sqrt{\frac{1}{1+\mathbf{K}}} \tilde{H}_{ij}^C$, $\tilde{H}_{ij}^C \sim \mathcal{CN}(0, I)$, $ij \in \{\mathtt{SR}, \mathtt{SD}, \mathtt{RD}\}$ with a strong line-of-sight (LOS) component of $\mathbf{K} = 10$ [39] in Fig. 9. The $\square$ markers denote the BERs for the DL-MLaR:DL-NML method trained under the Rician fading; the $\triangle$ ones are with the parameter set θ^* (trained under Rayleigh fading); the $\bigcirc$ markers represent the method that is initialized with θ^* and retrained under the Rician fading in $N_{\text{iteration}} = 1000$ iterations; and the **x** markers denote the results of the SDR-MLaR:SDR-NML method with $K = 20$ at the relay and $K = 80$ at the destination. From curves, one can observe that

1) the DL-NML detector with θ^* achieves acceptable performance in both channel conditions ($\triangle$ markers in Figs 8 and 9);

2) the DL-NML detectors that are retrained with $N_{\text{iteration}} = 1000$ iterations[3] under the i.i.d. Rayleigh fading channel of $N_{\text{D}} = 30$ (◯ markers in Fig. 8) and under Rician fading channel (◯ markers in Fig. 9) achieve excellent performance compared to the SDR-NML detector.



(a) With instantaneous SRC

(b) With statistical SRC and without SRC

FIGURE 9. BER comparison of the DL-MLaR:DL-NML and SDR-MLaR:SDR-NML methods over Rician fading MIMO DF relay channels of $N_{\text{D}} = N = 20$.

Additionally, we evaluate the MLaR:NML, SDR-MLaR: SDR-NML, and ZFaR:ZG methods in the i.i.d. Rayleigh fading MIMO DF relay channel with $N_{\text{D}} = N = 2$. The DL-MLaR:DL-NML method is not compared in this case since the deep learning detection does not have any advantages in both performance and complexity for small-number-antenna systems. Fig. 10 shows that the MLaR:NML method obtains the best performance, and the ZFaR:ZG method exhibits the worst performance, while the SDR-MLaR: SDR-NML method shows a nice performance with slopes similar to the corresponding MLaR:NML method. Here, we would like to mention that the proposed NMLwoSRC detector in Definition 2 (□ markers with dotted line) exhibits much better performance than the existing MD detector (**x** markers with dotted line) when the CSI of the SR link is

[3]With $N_{\text{iteration}} = 1000$, the retraining takes about 1.5 minutes.

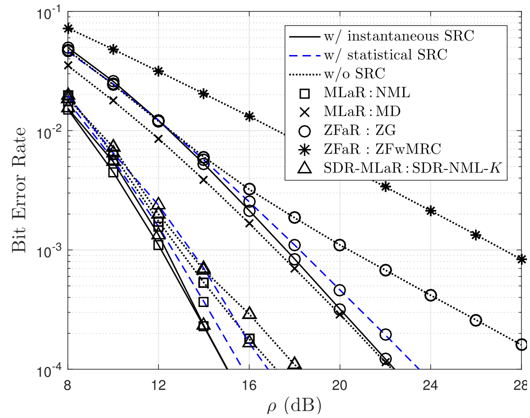


FIGURE 10. BER comparison of various DetR and DetD methods over the MIMO DF relay channel with $N_{\text{D}} = N = 2$. The SDR-MLaR:SDR-NML method is applied with $K = 6$ at the relay and $K = 8$ at the destination.

unknown at the destination. This makes it possible for the DL-NML detector to achieve excellent performance without SR channel. Moreover, the ZG detector in Definition 1 exhibits good performance compared to the existing ZFwMRC detector under various scenarios related to the knowledge of SR channel. In detail, the NMLwoSRC detector and its semi-definite relaxation approach, SDR-NML detector, obtain approximately 4.1 dB and 3.5 dB SNR improvements compared to the MD method at BER $= 10^{-3}$ without CSI of the SR link. The ZG detector yields approximately 9.6 dB, 9 dB, and 6.8 dB SNR improvements under three different scenarios compared to the ZFwMRC detector at BER $= 10^{-3}$ when the ZF detector is used at the relay.

D. A BRIEF INTRODUCTION TO GENERAL M^2-QAM

For M^2 quadrature amplitude modulation (QAM), we have the signal set $\mathcal{A} = \{-(2^m - 1), -(2^m - 3), \ldots, -1, 1, \ldots, (2^m - 3), (2^m - 1)\}$, where $m = \log_2 M$. The equivalent real signals $\mathbf{x}$ and $\mathbf{x}_{\text{R}}$ are expressed as

$$\mathbf{x} = \sum_{i=1}^{m} 2^{i-1}\mathbf{s}_i = B\mathbf{s}$$
$$\mathbf{x}_{\text{R}} = \sum_{i=1}^{m} 2^{i-1}\mathbf{s}_{i\text{R}} = B\mathbf{s}_{\text{R}} \quad (40)$$

where $B = \left[I,\ 2I,\ \ldots,\ 2^{m-1}I\right]$, $\mathbf{s} = [\mathbf{s}_1^T,\ \ldots,\ \mathbf{s}_m^T]^T$, $\mathbf{s}_{\text{R}} = [\mathbf{s}_{1\text{R}}^T,\ \ldots,\ \mathbf{s}_{m\text{R}}^T]^T$, and $\mathbf{s}_i, \mathbf{s}_{i\text{R}} \in \{-1, 1\}^{2N}$ for $i = 1, \ldots, m$.

We use $\hat{\mathbf{s}}$ and $\hat{\mathbf{s}}_{\text{R}}$ instead of $\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}_{\text{R}}$, respectively, in both the input and the output of each layer in Fig. 3. Using the relationship in (40), the input vector for the kth layer becomes

$$\mathbf{i}_k = \left[\mathbf{v}_{k-1}^T\ \ \hat{\mathbf{s}}_{k-1}^T\ \ \hat{\mathbf{s}}_{\text{R},k-1}^T\ \ A_{k-1}^T\ \ B_{k-1}^T\right]^T$$

where $A_{k-1} = H_{\text{SD}}^T H_{\text{SD}} B\hat{\mathbf{s}}_{k-1} + \alpha_{1k} H_{\text{SR}}^T H_{\text{SR}} B(\hat{\mathbf{s}}_{k-1} - \hat{\mathbf{s}}_{\text{R},k-1}) - \alpha_{2k} H_{\text{SD}}^T \mathbf{y}_{\text{SD}}$ and $B_{k-1} = H_{\text{RD}}^T H_{\text{RD}} B\hat{\mathbf{s}}_{\text{R},k-1} - \alpha_{1k} H_{\text{SR}}^T H_{\text{SR}} B(\hat{\mathbf{s}}_{k-1} - \hat{\mathbf{s}}_{\text{R},k-1}) - \alpha_{2k} H_{\text{RD}}^T \mathbf{y}_{\text{RD}}$. Similarly, the DL-MLaR for M^2-QAM can also be obtained. For the SDR-MLaR and SDR-NML detectors, three equivalent SDR based methods such as a polynomial-inspired SDR (PI-SDR), a bound-constrained SDR (BC-SDR), and a virtually-antipodal SDR (VA-SDR) can be applied [38].

VIII. CONCLUSION

In this paper, we proposed a deep learning detector so called DL-NML in the MIMO DF relay channel. This DL-NML detector achieves excellent error performance in three scenarios of the knowledge of the SR channel. To evaluate the deep learning detector, we also proposed the high-performance polynomial-complexity SDR-NML detector and the low-complexity ZG detector. Furthermore, we presented and compared various DetR:DetD methods according to the error performance and detection complexity, which provides a basic idea and direction for the system configuration.

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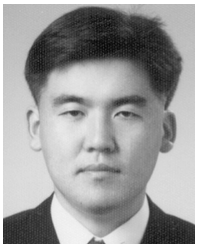

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