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Fully Distributed AC Optimal Power Flow

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ABSTRACT Optimal generation dispatch is one of the challenging problems in the field of both market and system security due to its non-convexity and uncertainty. Many solutions have been proposed in recent years to solve it by convexification and linearization. Some of these methods fail to be practical, however, due to their specific assumptions. In this paper, a fully distributed algorithm is proposed for a complete optimal power flow without any convexification or linearization. A mathematical proof for its global optimality is provided for a simple version of the optimization problem; then it is extended to consider all constraints. Finally, two test power systems, a synthetic 37-bus case study and an IEEE 118-bus test feeder considering all equality and inequality constraints is simulated to evaluate the proposed algorithm's performance.

INDEX TERMS Distributed control method, consensus algorithm, generation dispatch.

I. INTRODUCTION

Optimal power flow (OPF), as a fundamental optimization problem in power systems, was introduced by J. L. Carpentier [1] to determine the power levels of all generators to support the requested demand in a system while minimizing the total cost and satisfying all local/global constraints at the same time [2], [3]. In OPF problem, an economic dispatch (ED) problem [4] with a power flow calculation is solved simultaneously. The coupling of these two problems and network/physical constraints make it one of the most insurmountable optimization problems. In sum, the reasons that make it difficult to be solved can be categorized as; 1) Non-linearity: there are nonlinear interrelations (*i.e.*, power flow nodal equations) among powers, voltages and system physical parameters [5]. 2) Non-convexity: the lower/upper bound on the voltage amplitude and the nonlinear power flow equations cause non-convexity [3], [6]. 3) Computational cost: an OPF not only must be run every year for system planning and every day for a day-ahead market, but also should be run every five minutes for real-time market and system security [7]. 4) Uncertainty: large-scale power systems involve uncertainty due to their integration of renewable energy resources, which make OPF more troublesome [8]. One of the immediate solutions for OPF is DCOPF, which uses DC power flow equations and assumptions in place of AC power flow. It provides us with a rough approximation of the AC power flow and is much faster and easier to solve. Although the power losses are ignored and its accuracy is very dependent on the system and case study, it could be useful for limited contingency analyses and economic studies [8]–[12]. However, such approximations that ignores system complexities may lead to unrealistic results and analysis. Apart from DCOPF, there are many methods proposed in previous years to solve OPF efficiently, accurately and as fast as possible. As we focus on a distributed ACOPF, this literature review categorizes OPF methods into two main groups: 1) Centralized methods, which need a central controller (*i.e.*, operator) to collect, share and coordinate data among power system components; and 2) Distributed methods, which use a specific algorithm to distributively coordinate information among components to reach an optimal point [13].

Centralized OPF methods have been studied since the early 1960s when a typical OPF was first formulated [7]. In a centralized method, all components directly communicate with a central operator *e.g.*, SCADA. This center should be able to monitor, gather and analyze real-time data and provide all components with appropriate control signals while it records events in a log file. Many methods, including interior point, quadratic programming, Lagrangian relaxation, gradient methods, mixed integer programming, Newton based methods, etc., have been reviewed and classified several times. We only mention four major literature reviews for readers' reference and do not review OPF optimization methods

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proposed prior to 2010 because these surveys cover all OPF methods. M. Huneault and F. Galiana provided one of the first literature reviews of OPF and investigated the evolution of various methods, such as successive approximation and Newton methods, from 1960 to 1991 [14]. J. A. Momoh *et al*. extensively analyzed the progress of OPF methods proposed during the 70s, 80s and 90s in two consecutive papers [15], [16]. In addition, the comprehensive literature survey by S. Frank *et al*. [17], [18], which reviewed various OPF formulations and solution techniques, their advantages, disadvantages, and computational characteristics, was applied to OPF for 40 years, from 1969 to 2010. Solving OPF using a convex relaxation draws many researchers' attention due to its powerful ability to ensure a solution is global or a lower bound on the global solution. Semidefinite programming (SDP) relaxations of OPF comprehensively evaluated up to 2014 in [19], [20]. S. Bruno *et al.* proposed an unbalanced three-phase optimal power flow (TOPF) based on a quasi-Newton method. TOPF as an extended real-time framework that provides control strategies for distribution radial networks [21]. J. Lavaei *et al*. proposed an SDP optimization method and guaranteed a zero duality gap limited to specific conditions for some IEEE test systems. Furthermore, the sufficiency of condition holds by adding a small resistance to transformers [6]. W. A. Bukhsh *et al.* cover an interesting and challenging topic, possible existence of local optima, in their paper. They show that standard local optimization techniques can converge to these local optima [22]. Due to the high penetration of renewable generation, the uncertainty level increases in the power system. Chance-constrained AC OPF, as one of the interesting methods, is able to deal with stochastic OPF [23]–[25].

Due to the high penetration of distributed energy sources/storage, dynamic topologies of power systems and the need for plug-and-play functionalities, centralized algorithms are no longer effective [26]–[28]. Consequently, distributed OPF methods have been taken into account by researchers as they are able to overcome these drawbacks. Thanks to the advanced technologies used in communication systems, distributed methods are rapidly maturing [29]. In distributed methods, agents are not required to communicate with a central controller. Instead, they only need to locally connect with their immediate neighbor(s); thus, it can be ensured that no private information is released by a third party. This advantage provides all agents with an opportunity to participate in a fair and competitive market, without any kind of monopoly or monopsony [4]. Apart from the privacy improvement, the single point of failure will naturally be resolved as there is no need for a center to supervise all agents [30], [31]. More ever, computational load will no longer affect a central controller as it is spread out over the entire network [32]. Dynamic topologies of power systems and plug-and-play functionalities will be requisite features of the future open-access power system, which can easily be supported by distributed algorithms. All of these features make it possible for distributed methods to support high scalability as an urgent need in future power systems.

A growing interest in new distributed algorithms, particularly applicable to OPF problems, can be found in recent research. As we discussed earlier, the non-convexity is a major barrier against finding the global optimality. There is a chain of research trying to resolve this issue. Semi-definite programming, known as the SDP relaxation technique, transforms a non-convex problem in the equivalent convex one. E. Dall'Anese *et al.* built a distributed OPF based on SDP relaxation for an unbalanced distribution system by decomposing a main SDP problem into multiple convex subprograms [5]. A. Lam *et al.* also offered a distributed algorithm by decomposing a main optimization problem into smaller sub-problems tha can be solved by SDP [33]. T. Erseghe proposed a distributed OPF using the alternating direction multiplier method (ADMM). His method is designed based on local optimization, where information only exchanged inside of a region [34]. Another interesting topic has been covered in [35]. The authors discuss synchronization of regions for a distributed OPF problem based on an algorithmic framework that allows each region to perform local updates in an asynchronous fashion. A distributed optimal gas-power flow (OGPF) based on the ADMM, proposed in [36]. At both the power and gas distribution sides, a convex relaxation has been performed and then two problems are coordinated by the ADMM. In [37], both SDP relaxation and the ADMM are used together to build a scheduled-asynchronous algorithm for solving OPF problems in a distributed fashion. SDP relaxation and the ADMM are used to convexify formulated sub-problems and help agents to update their local variables, respectively. The authors of [38] designed an ADMM-based distributed AC-OPF using a linear approximation of power flow equations. They considered two control methods to balance convergence and computational load. A distributed cooperative real-time OPF has been proposed in [39]. This method is able to coordinate the active power of synchronous generators and virtual power plants to cover the nominal frequency while minimizing the generation cost in real time under optimization constraints.

A fully consensus-based distributed OPF is proposed in this paper. We show that the capacity of line and voltage amplitude limits are no longer areas of non-convexity's concern given a new approach. Power flow constraints, as another reason for non-convexity, are replaced with a local convex optimization problem without loss of generality. The main contributions of our work are as follows:

- Neither convex relaxation nor linear approximation is used; therefore, easy implementation and accurate results, respectively, can be ensured.
- The privacy of each component and, subsequently, the privacy of the entire system, is improved due to very limited shared information.
- In addition, there is no need for any kind of aggregator and/or coordinator.

- No specific assumption is considered for the system topology, e.g., mesh/radial grid, and the proposed method can be applied to both the transmission and distributed level.

The structure for the rest of this paper is organized as follows: Section II formulates a comprehensive OPF problem as a global objective function, considering cost functions and constraints. Section III introduces a distributed consensus-based algorithm with a brief review of graph theory. Section IV provides a mathematical proof for a limited version of OPF and simulation results. This solution extended to a full version of OPF without loss of generality. Section V demonstrates simulation results for a synthetic 37-bus case study and an IEEE 118-bus test feeder. Finally, section VI summarizes this paper and presents the concluding remarks.

II. SYSTEM MODELING AND PROBLEM DESCRIPTION

In this section, the optimization power flow problem, including cost functions and all equality and inequality constraints, are elaborated.

A. GENERATORS' COST FUNCTION

Generators need to mathematically represent their costs to participate in an electricity market. Estimated generation cost can be represented by various cost functions, such as a multiple piecewise linear, quadratic or cubic functions. For this study, a known quadratic fuel cost function, shown in (1), is considered for all generators.

$$\mathcal{C}_k(P_{\mathcal{G}_k}) = \alpha_k P_{\mathcal{G}_k}^2 + \beta_k P_{\mathcal{G}_k} + \gamma_k, \quad \forall k \in \mathbb{N}_\mathcal{G} \tag{1}$$

where $\alpha^{\$}/(\mathrm{kW})^2\,\mathrm{h}$, $\beta^{\$}/\mathrm{kWh}$ and $\gamma^{\$}/\mathrm{h}$ are coefficients that customize the cost function for each generator. $\mathrm{P}_{\mathcal{G}_k}$ kW is the amount of power generated by the $k-th$ generator and $\mathbb{N}_\mathcal{G}$ shows a set of buses associated with a generator.

B. SYSTEM EQUALITY AND INEQUALITY CONSTRAINTS

As other optimization problems, OPF has some important constraints due to the topology of the power system, flow capacity of transmission lines, some restrictions on the power generation capacity, etc. Power flow equations, *i.e.*, nodal KCL, and load balance are the equality constraints. Voltage amplitude and generation capacity, along with transmission line flow, constitute the inequality constraints. We also need to consider slack bus voltage amplitude and phase angle constraints.

C. STATEMENT OF THE GLOBAL OPTIMIZATION POWER FLOW PROBLEM

As shown in (2), the total cost function ($\mathfrak{F}_\mathfrak{T}$, where $\overrightarrow{P}_\mathcal{G} = \left[P_{\mathcal{G}_1}, P_{\mathcal{G}_2}, \cdots, P_{\mathcal{G}_N}\right]^T$) is a summation of all generators' cost functions. Equation (3) indicates a box constraint for the power capacity of each generator, where $P_{\mathcal{G}_k}^{max}$ and $P_{\mathcal{G}_k}^{min}$ indicates the maximum and minimum possible generation level. We consider $P_{\mathcal{G}_k}^{min} = 0, \quad \forall k \in \mathbb{N}_\mathcal{G}$. The voltage magnitude boundary is shown by (4), which makes the optimization problem a non-convex problem. Pre-set values of the slack bus are defined by (5). Equation (6) shows the power balance between generation and consumption. The first statement on the right side of (6) shows the total load installed on the generator and load buses, where $\mathbb{N}_\mathcal{L}$ indicates the load-only buses. The second statement on right side of (6) represents the summation of the estimated active power loss of each bus, where $\mathbb{N}_\mathcal{S}$ defines the set of all buses, including generator, load, slack and connection buses.

$$\min\ \mathfrak{F}_\mathfrak{T}\left(\overrightarrow{P}_\mathcal{G}\right) = \sum_{\forall k \in \mathbb{N}_\mathcal{G}}^{N} \mathcal{C}_k(P_{\mathcal{G}_k}), \quad \forall k \in \mathbb{N}_\mathcal{G} \tag{2}$$

$$P_{\mathcal{G}_k}^{min} \le P_{\mathcal{G}_k} \le P_{\mathcal{G}_k}^{max}, \quad \forall k \in \mathbb{N}_\mathcal{G} \tag{3}$$

$$V_k^{min} \le \left|V_k\right| \le V_k^{max}, \quad \forall k \in \mathbb{N}_\mathcal{G} \tag{4}$$

$$\left|V_{k_s}\right| = 1,\ \delta_{k_s} = 0, \quad k_s \in \mathbb{N}_\mathcal{G} \tag{5}$$

$$\sum_{\forall k \in \mathbb{N}_\mathcal{G}}^{N} P_{\mathcal{G}_k} = \sum_{\forall k \in (\mathbb{N}_\mathcal{G} + \mathbb{N}_\mathcal{L})}^{N} P_{\mathcal{L}_k} + \sum_{\forall k \in \mathbb{N}_\mathcal{S}}^{N} \mathcal{P}_{Loss_k} \tag{6}$$

$$P_{net,k} = \begin{cases} P_{\mathcal{G}_k} - P_{\mathcal{L}_k}, & \forall k \in \left(\mathbb{N}_\mathcal{G} + \mathbb{N}_\mathcal{L}\right) \\ 0, & \forall k \notin \left(\mathbb{N}_\mathcal{G} + \mathbb{N}_\mathcal{L}\right) \end{cases} \tag{7}$$

$$Q_{net,k} = \begin{cases} Q_{\mathcal{G}_k} - Q_{\mathcal{L}_k}, & \forall k \in \left(\mathbb{N}_\mathcal{G} + \mathbb{N}_\mathcal{L}\right) \\ 0, & \forall k \notin \left(\mathbb{N}_\mathcal{G} + \mathbb{N}_\mathcal{L}\right) \end{cases} \tag{8}$$

$$\mathcal{S}_{net,k} = P_{net,k} + jQ_{net,k} = V_k \left[\sum_{\forall n \in \mathbb{N}_\mathcal{S}}^{N} Y_{kn} V_n\right]^* \tag{9}$$

$$\mathcal{S}_{kn} = V_k \left[(V_k - V_n)\, Y_{kn}\right]^* \le \mathcal{S}_{kn}^{max}, \quad \forall k\ \&\ \forall n \in \mathbb{N}_\mathcal{S} \tag{10}$$

The injected active and reactive power are shown by $P_{net,k}$ and $Q_{net,k}$, respectively. $\mathcal{S}_{net,k}$ in (9) shows the power flow nodal equation for the $k-th$ bus. The line flow constraint is formulated by (10), where $\mathcal{S}_{kn}^{max}$ indicates the maximum complex power flowing through the line connected the $k-th$ and $n-th$ buses.

III. DISTRIBUTED ALGORITHM FOR OPTIMAL POWER FLOW

As mentioned earlier, a consensus-based distributed algorithm for finding the globally optimal solution is proposed in this paper. A brief graph theory shows our assumptions and justifies our model for a communication network. Then we show how to apply an average-consensus distributed protocol [40] to an OPF problem without network constraints *i.e.*, the economic dispatch (ED) problem. After optimality analysis, power flow nodal equations, and line flow constraints will be added without loss of generality.

$\mathcal{N}(\mathcal{K}, \xi)$ denotes communication network (undirected graph) with N connected buses, designated by $\mathcal{K} = \{1, 2, \ldots N\}$ and $\xi \subseteq \mathcal{K} \times \mathcal{K}$, which represents a set of edges. The physical power system is mapped onto the communication network. This means that there is a communication link between two buses connected by a transmission line. The undirected edge $e_{kn} = (k, n)$ indicates that bus k and n can share information with each other. Two matrices will commonly be used to represent the communication topology of a multiple-agent network. The adjacency matrix denoted by $\mathcal{A} = \{[a_{kn}] \,|a_{kn} \in \mathcal{R}^{\mathcal{P}\times\mathcal{P}}\}$ of an undirected network $\mathcal{N}$ is symmetric. The entries of the adjacency matrix is defined by (11).

$$\mathcal{A} = \begin{cases} a_{kn} \neq 0, & \forall e_{kn} \in \xi \\ a_{kn} = 0, & \forall e_{kn} \notin \xi \\ a_{kk} = 0, & \forall k \in \mathbb{N}_{\mathcal{S}} \end{cases} \tag{11}$$

The second matrix is Laplacian matrix $L = D - \mathcal{A} = \{[l_{kn}] \,|l_{kn} \in \mathcal{R}^{\mathcal{P}\times\mathcal{P}}\}$, where D is a network degree matrix. As can be inferred, the definition of a Laplacian matrix is very similar to admittance matrix. The entries of the Laplacian matrix defined by (12).

$$L = \begin{cases} l_{kk} = \sum\limits_{\forall n \in \mathbb{N}_{\mathcal{S}}} a_{kn}, & \forall k \in \mathbb{N}_{\mathcal{S}} \\ l_{kn} = -a_{kn}, & \forall e_{kn} \in \xi \\ l_{kn} = 0, & otherwise \end{cases} \tag{12}$$

A consensus-based distributed protocol helps agents share information with their immediate neighbors to reach a consensus at an optimal point. A consensus is defined as an equal value of the state of the $k - th$ and $n - th$ agents [40], [41]. In other words, $k - th$ and $n - th$ agent will have reached a consensus if and only if the value of the state of the $k - th$ agent (x_k) and the state of the $n - th$ agent (x_n) are equal. If we consider that each bus shares its information with its neighbors, a standard linear distributed protocol [40] can be defined in (13), where $\vec{x} = [x_1, x_2, \ldots, x_N]^T$.

$$\dot{\vec{x}} = -\nabla\left(2\vec{x}^T L\vec{x}\right) = -L\vec{x} \tag{13}$$

Let us consider the total cost function (2), box constraints (3) and power balance (6) as a simple optimization problem. The rest of the constraints will be added later during the analysis of optimality in section IV. We assume that $k - th$ bus *only* accesses its own private information, such as its generator's cost function ($\mathcal{C}_k(P_{\mathcal{G}_k})$), generated active power ($P_{\mathcal{G}_k}$) and its local load ($P_{\mathcal{L}_k}$). Any of these pieces of information could be zero depending on the type of bus, such as PV, PQ or connection buses. They also can pick an arbitrary value for their particular incremental cost (λ_k). Incremental costs are the only piece of information shared with immediate neighbors through the average-consensus distributed protocol in (13). Eventually, all of the buses reach an identical value of λ_c as a consensus. Then, each bus estimates its active power generation using (15); a bus may have no active power generation. An individual incremental cost (λ_k^{i+1}) for the next iteration is calculate by (16), based on the private information and consensus. The whole procedure is shown by (14), (15) and (16), where $\vec{\lambda} = \left\{[\lambda_1, \lambda_2, \ldots, \lambda_N]^T \,|\lambda_k \in \mathcal{K}\right\}$ and i is the iteration number.

$$\dot{\vec{\lambda}}^i = -L\vec{\lambda}^i \tag{14}$$

$$P^i_{\mathcal{G}_k} = \frac{\lambda^i_{c,k} - \beta_k}{2\alpha_k}, \quad \forall k \in \mathbb{N}_{\mathcal{S}} \tag{15}$$

$$\lambda_k^{i+1} = \lambda^i_{c,k} + \rho(P^i_{\mathcal{G}_k} - P_{\mathcal{L}_k}), \quad \forall k \in \mathbb{N}_{\mathcal{S}} \tag{16}$$

Finally, all λ_k will be identical, due to the nature of the distributed algorithm in (13), $\sum\limits_{\forall k\in\mathbb{N}_{\mathcal{G}}}^{N} P_{\mathcal{G}_k}$ will cover the total demand, and the power balance constraint will be satisfied It is worth mentioning that

- None of the private information, such as $P_{\mathcal{G}_k}$, α_k, β_k, $P_{\mathcal{L}_k}$ are shared.
- The only information shared with immediate neighbors is estimated λ_k.

In the next section, it is shown that this protocol converges to the global optimal point of the optimization problem.

IV. ANALYSIS OF OPTIMALITY

This section is organized as follows: Subsection IV-A provides the optimality analysis for a simple optimization problem (similar to the economic dispatch problem), including the total cost function (2), box constraints (3) and power balance (6). Then, subsection IV-B adds two important constraints, including the voltage amplitude limitation (4) and line flow constraint (10), in subsections IV-B1 and IV-B2.

A. SIMPLE OPTIMIZATION PROBLEM

Remark 1: All local constraints (1) and the total cost function (2) are strictly convex.

Remark 2: As the box constraints and power balance equation are considered in this step, all constraints of the ED problem are affine.

Remark 3: The incremental cost of each bus should be equal at the optimal point, i.e, $\lambda_1 = \lambda_2 =, \ldots, = \lambda_N$, because of the dual variable of the power balance constraint [42].

Lemma: Based on *Remark* (1) and *Remark (2),* the ED optimization problem that satisfies Slater's condition and KKT conditions can provide the necessary and sufficient conditions for optimality.

Proposition : Combining *Remark* (1), *Remark* (2), and the lemma, a dual gradient method (18-21) will converge to the global optimal given the Lagrangian function provided in (17).

$$\begin{aligned} L(\vec{P}_{\mathcal{G}}, \lambda, \mu, \zeta) &= \sum_{\forall k\in\mathbb{N}_{\mathcal{G}}}^{N} \mathcal{C}_k(P_{\mathcal{G}_k}) \\ &\quad + \lambda\left(\sum_{\forall k\in\mathbb{N}_{\mathcal{S}}}^{N} \mathcal{P}_{Loss_k} + \sum_{\forall k\in(\mathbb{N}_{\mathcal{G}}+\mathbb{N}_{\mathcal{L}})}^{N} P_{\mathcal{L}_k} - \sum_{\forall k\in\mathbb{N}_{\mathcal{G}}}^{N} P_{\mathcal{G}_k}\right) \\ &\quad + \sum_{\forall k\in\mathbb{N}_{\mathcal{G}}}^{N} \mu_k\left(P_{\mathcal{G}_k} - P^{max}_{\mathcal{G}_k}\right) + \sum_{\forall k\in\mathbb{N}_{\mathcal{G}}}^{N} \zeta_k\left(-P_{\mathcal{G}_k}\right) \end{aligned} \tag{17}$$

$$\vec{P}_{\mathcal{G}}^{i+1} = \underset{P_{\mathcal{G}_k}}{argmin}\; L(\vec{P}_{\mathcal{G}}, \lambda^i, \mu^i, \zeta^i) \Rightarrow$$

$$P_{\mathcal{G}_k}^{i+1} = \frac{\lambda^{i+1} - \beta_k - \mu_k^{i+1} + \zeta_k^{i+1}}{2\alpha_k} \quad \forall k \in \mathbb{N}_{\mathcal{S}} \tag{18}$$

$$\lambda^{i+1} = \lambda^i + \epsilon(\sum_{\forall k \in \mathbb{N}_{\mathcal{S}}}^{N} \mathcal{P}_{Loss_k} + \sum_{\forall k \in \mathbb{N}_{\mathcal{G}}}^{N} P_{\mathcal{G}_k} - \sum_{\forall k \in (\mathbb{N}_{\mathcal{G}} + \mathbb{N}_{\mathcal{L}})}^{N} P_{\mathcal{L}_k}) \tag{19}$$

$$\mu_k^{i+1} = \left[\mu_k^i + \rho(P_{\mathcal{G}_k} - P_{\mathcal{G}_k}^{max})\right]^+, \quad \forall k \in \mathbb{N}_{\mathcal{S}} \tag{20}$$

$$\zeta_k^{i+1} = \left[\zeta_k^i + \rho(-P_{\mathcal{G}_k})\right]^+, \quad \forall k \in \mathbb{N}_{\mathcal{S}} \tag{21}$$

Theorem: If the optimization problem presented in (2), constrained by box and power balance constraints, has a feasible globally optimal point, then the consensus-based distributed algorithm proposed in section (III) will converge to that globally optimal point.

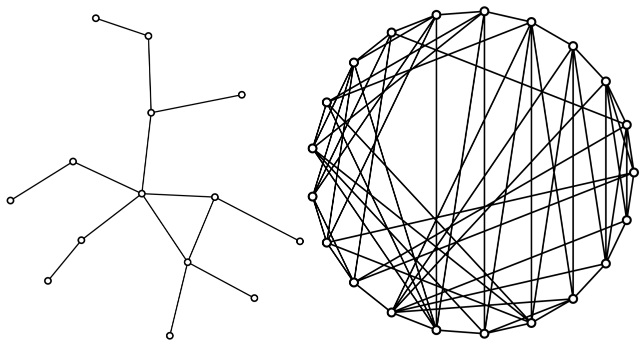
Proof: The decomposition approach of the dual gradient helps us to provide a set of separate equations for each bus. Equations (17), (20) and (21) can easily be calculated, in a parallel fashion, with only private information (no need for shared information among immediate neighbors). However, equation (19) needs a kind of coordinator to gather and send bus information and λ (see Remark (3)). This approach obviously violate the privacy and puts extra computational load on a coordinator. We can consider λ^i as a summation of N arbitrary real values $\lambda^i = \lambda_1^i + \lambda_2^i + \cdots + \lambda_N^i = \sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i$, where the N is number of buses. Then, we can rewrite the right side of this statement as (22). Therefore, equation (19) can be re-written as (23) without loss of generality.

$$\lambda^i = \underbrace{\frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N} + \frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N} + \cdots + \frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N}}_{N-times} \tag{22}$$

$$\lambda^{i+1} = \underbrace{\frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N} + \frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N} + \cdots + \frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N}}_{N-times} + \epsilon(\sum_{\forall k \in \mathbb{N}_{\mathcal{S}}}^{N} \mathcal{P}_{Loss_k} + \sum_{\forall k \in \mathbb{N}_{\mathcal{G}}}^{N} P_{\mathcal{G}_k} - \sum_{\forall k \in (\mathbb{N}_{\mathcal{G}} + \mathbb{N}_{\mathcal{L}})}^{N} P_{\mathcal{L}_k}) \tag{23}$$

Now, we are able to write (23) in a decomposition fashion as (24),

$$\lambda_1^{i+1} + \lambda_2^{i+1} + \cdots + \lambda_N^{i+1} = \frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N} + \epsilon(\mathcal{P}_{Loss_1} + P_{\mathcal{G}_1} - P_{\mathcal{L}_1}) + \frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N} + \epsilon(\mathcal{P}_{Loss_2} + P_{\mathcal{G}_2} - P_{\mathcal{L}_2}) + \cdots + \frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N} + \epsilon(\mathcal{P}_{Loss_N} + P_{\mathcal{G}_N} - P_{\mathcal{L}_N}) \tag{24}$$



(a) Network A) Erdos–Renyi model (b) Network B) Watts–Strogatz model

FIGURE 1. **Test graph randomly generated by two different models, a) $G_{ER}(n, p): n = 14, p = 0.1$, b) $G_{WS}(n, k', \beta'): n = 21, k' = 6, \beta' = 0.1$.**

Based on (25), each bus can take its own dual gradient equation. The first righ-side statement is the average of all the estimated λ_ks and can be distributively calculated by introducing the protocol in (13) or (14) because this protocol always calculate the average.

$$\lambda_k^{i+1} = \frac{\sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i}{N} + \epsilon(\mathcal{P}_{Loss_k}^i + P_{\mathcal{G}_k}^i - P_{\mathcal{L}_k}) \tag{25}$$

Equation (25) is the one introduced in (16), where $\lambda_{c,k}^i = \sum_{\forall k \in \mathbb{N}_S}^{N} \lambda_k^i / N$, $\rho = N\epsilon$. ■

For the sake of accuracy assessment, two test networks, network A (14 generators) and network B (21 generators), are randomly generated by the Erdos–Renyi model and the Watts–Strogatz model, respectively. Figure. (1a) and Figure. (1b) show their visual topology and related model parameters. All random networks are, in this section, generated by Cytoscape [43]. The demanded load on network A and network B are 2602 kW and 4832 kW, accordingly.

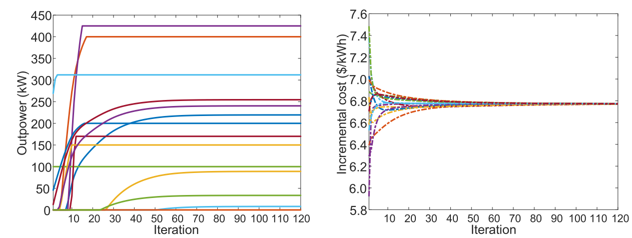
Table (1) presents the results of the optimization problem given the proposed algorithm and a centralized method (YALMIP is applied [44]) to provide an accuracy evaluation following our proof. As shown in Table (1), the solution mismatch between the distributed and centralized methods is less than 0.008% of the average. As can be seen, this value is almost zero and demonstrates our proposed method can find the same optimal point as a centralized method.

Figures. (2a) and (3a) show a visual convergence of dispatched power among the generators for both systems. As shown in Figures. (2b) and (3b), the incremental cost (λ) finally converges to $6.77^{\$}$/kWh for network A and $7.13^{\$}$/kWh for network B.

A bigger system, shown in Figure. (4), is used for the sake of scalability study. This network has 1000 nodes, which are randomly generated by the Watts–Strogatz model. In addition, the mean parameters of the coefficients of the generators' cost functions are selected to be $0.0085^{\$}/(\text{kW})^2$ h and $4.21^{\$}$/kWh. The demand is also normally distributed among all buses. The total generation (450234.1 kW), which

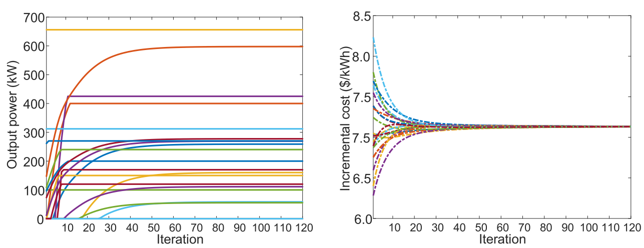
TABLE 1. **Numerical comparison of proposed distributed and centralized methods.**

	Network A		Network B	
Power	Distributed	Centralized	Distributed	Centralized
P_1	219.43	219.45	258.97	258.97
P_2	400.00	400.00	400.00	400.00
P_3	88.98	89.014	160.24	160.26
P_4	425.00	425.00	425.00	425.00
P_5	100.00	100.00	100.00	100.00
P_6	8.016	8.034	58.516	58.533
P_7	170.00	170.00	170.00	170.00
P_8	200.00	200.00	200.00	200.00
P_9	0.00	0.00	0.00	0.00
P_{10}	150.00	150.00	150.00	150.00
P_{11}	240.37	240.39	269.36	269.37
P_{12}	33.55	33.56	55.54	55.55
P_{13}	312.00	312.00	312.00	312.00
P_{14}	254.54	254.55	277.50	277.51
P_{15}	-	-	270.00	270.00
P_{16}	-	-	597.59	597.60
P_{17}	-	-	656.00	656.00
P_{18}	-	-	111.19	111.20
P_{19}	-	-	240.00	240.00
P_{20}	-	-	0.00	0.00
P_{21}	-	-	120.00	120.00
Total	2601.89	2601.998	4831.906	4831.993



(a) Output powers of generators (kW) (b) Consensus on λ $^{\$}/_{kWh}$

FIGURE 2. **Network A: Convergence of parameters.**



(a) Output powers of generators (kW) (b) Consensus on λ $^{\$}/_{kWh}$

FIGURE 3. **Network B: Convergence of parameters.**

is achieved after 350 iterations, supports the total demand (450238 kW). It indicates that although the number of nodes is 50 times bigger than the first two networks, the iteration is only (almost) doubled. The incremental cost of all 1000 agents converges to $13.01^{\$}$/kWh at the optimal point.

The solution mismatch between the distributed and centralized methods is about 0.0062% of the average. Again,

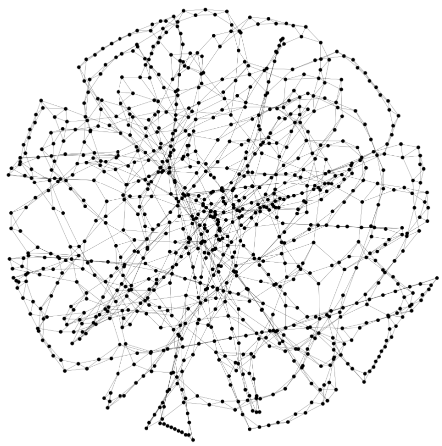


FIGURE 4. **1000 node network by the Watts–Strogatz model, $G_{WS}(n, k', \beta')$: $n = 1000$, $k' = 4$, $\beta' = 0.05$.**

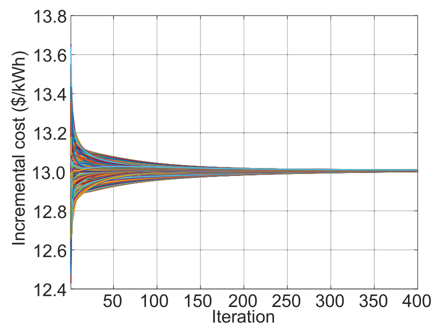


FIGURE 5. **Consensus on incremental cost for random network with 1000 nodes.**

this value confirms the precision of the proposed method for large-size networks.

B. VOLTAGE AMPLITUDE AND LINE FLOW CONSTRAINTS

As known, the OPF problem is more complex than what has been discussed here. One of the important groups of constraints is the power flow nodal equations and voltage amplitude limitation, as expressed in (9) and (4). It may perhaps be observed that without having limitation on voltage amplitude (4), nodal equations (9) are not effective because voltage of buses can choose any values, satisfying nodal equations. This can be calculated by power flow easily.

1) VOLTAGE AMPLITUDE CONSTRAINTS

As it is known, in a transmission network, the flow of active power depends on the voltage angle, and the flow of reactive power depends on the voltage amplitude. Hence, active power moves from a larger voltage angle bus to a smaller voltage angle bus. On the other hand, reactive power flows from a higher voltage amplitude bus to a smaller voltage magnitude bus.

There are two general approaches for controlling the voltage amplitude at a bus, a local voltage controller and power re-dispatching, to make sure that the voltage amplitude satisfies the constraints provided by equation (4). Preferably, the voltage amplitude is controlled locally by changing the reactive power of a given bus as is the case with PV buses. However, it is not possible to have a voltage controller on all buses, like PQ buses and connection buses. Therefore, re-dispatching active and reactive power could be a potential alternative solution. Any re-dispatching of active power, other than the solution found by ED, increases the price of total generation. Additionally, the re-dispatching of active power cannot effectively change the voltage amplitude at the target bus because, as has been discussed, voltage amplitudes are more sensitive to reactive power than active power. This fact can also be investigated by calculation of the sensitivity of the voltage amplitude to active and reactive power changes. On the other hand, re-dispatching of reactive power can be very effective in comparison with re-dispatching of active power because it does not change the optimal point calculated by ED. In this part, the set point voltage of the generators is used to compensate for the violation of voltage amplitude in other buses. We assume that there is enough amount of the reactive power to control voltage. When the voltage amplitude in a bus violates its limitation, the amount of violation can be used as a factor to change the set point voltage of the generators and consequently change the reactive power injected by each generator. Equation (26) shows the simple relation used to compensate for violated voltage amplitude, where α parameters can be any number with absolute value less than unity. Using a sensitivity index for α, which measures the sensitivity of bus voltages to changes in generator voltage set points could be very effective. $\left|V_n^{limit}\right|$ indicates the $\left|V_n^{min}\right|$ or $\left|V_n^{max}\right|$.

$$V_k^{Gen,\,i+1} = V_k^{Gen,\,i} + \sum_{i=1}^{i} \sum_{\forall n \in \mathbb{N}_S}^{N} \alpha_{n,k} \left| \left|V_n^i\right| - \left|V_n^{limit}\right| \right|, \quad \forall k \tag{26}$$

The new voltage set point of a generator installed on k^{th} bus will be supported by the reactive power injected at the respective bus.

As discussed earlier, re-dispatching of reactive power would not affect the ED solution, which is a global optimal point. However, it may change the total loss because it is changing the voltage magnitude. It is worth mentioning that we are not minimizing the active loss function as it is not in the scope of this paper.

2) LINE FLOW CONSTRAINTS

Now, the line flow constraints shown in (10) are the only group of constraints causing non-convexity. There is no easy way to convexify these constraints without linearization. In this section, an intuitive method is used to easily replace these constraints. We define another optimization problem, whose solution satisfies constraint (10). This optimization problem only replaces the line flow constraints and does not replace whole optimization problem. To alleviate overloading, the output power of generators connected to the network must be re-dispatched in a cost-effective way. This means each generator should change its output by $\triangle P_{\mathcal{G}_k}$, compared to its output in the solution of the unconstrained problem (*i.e.* the ED problem), to reduce the power flow through the congested lines.

Assumption: the global optimal solution of the unconstrained problem (without constraint (10)) is $\{P_{\mathcal{G}_1}, P_{\mathcal{G}_2}, \cdots, : P_{\mathcal{G}_n}\}$, whose total minimum cost is $\mathfrak{F}_{\mathfrak{T}} = \mathcal{C}_1 + \mathcal{C}_2 + \cdots + \mathcal{C}_n$.

Theorem: in a constrained problem (with constraint (10)), a set of re-dispatching generators *i.e.*, $\{\triangle P_{\mathcal{G}_1}, \triangle P_{\mathcal{G}_2}, \cdots, : \triangle P_{\mathcal{G}_n}\}$ which has minimal price changes from the unconstrained problem, among feasible solutions, is global optimal point. This re-dispatching should not violate constraint (6); therefore, $\sum_{\forall k \in \mathbb{N}_{\mathcal{G}}} \triangle P_{\mathcal{G}_k} = 0$.

Each $\triangle P_{\mathcal{G}_k}$ causes a price change ($\triangle \mathcal{C}_k$) in the respective generator. The summation of all $\triangle \mathcal{C}_k$ should be minimized to guarantee the closest feasible solution to the solution of the unconstrained problem, which is the global optimal point. The line flow constraint (10) is replaced by (27), subject to constraint (28) and other local constraints from the main problem, where $\triangle \mathcal{C}_k = \mathcal{C}_k(\triangle P_{\mathcal{G}_k}) = \alpha_k(\triangle P_{\mathcal{G}_k})^2 + \beta_k(\triangle P_{\mathcal{G}_k}) + \gamma_k, \quad \forall k \in \mathbb{N}_{\mathcal{G}}$.

$$\min \sum_{\forall k \in \mathbb{N}_{\mathcal{G}}}^{N} \mathcal{C}_k(\triangle P_{\mathcal{G}_k}), \quad \forall k \in \mathbb{N}_{\mathcal{G}} \tag{27}$$

$$\triangle P_{\mathcal{G}_1} + \triangle P_{\mathcal{G}_2} + \cdots + \triangle P_{\mathcal{G}_n} = 0 \tag{28}$$

Equation (27) shows that the cost of the power changes should be minimal and equation (28) indicates the power changes should not violate power balance constraints. The dual ascent solution for this convex optimization problem is (29), where χ is the dual variable related to the equality constraint in (28).

$$\triangle P_{\mathcal{G}_k}^i = \frac{\chi^i - \beta_k}{2\alpha_k}, \quad \forall k \in \mathbb{N}_{\mathcal{G}} \tag{29}$$

Overloading is the only reason that (29) comes into play; thus, $\chi = P_{kn} - P_{kn}^{max}$ to satisfy line maximum flow constraints. It is worth mentioning that χ is calculated by sending and receiving end buses and distributed among their neighbors. Thus, privacy is not violated. Furthermore, the whole problem (27) is still convex. The value calculated in (29) is added to (15).

V. PERFORMANCE ASSESSMENT

In this section, we apply the proposed method to two test models of a power system, a synthetic 37-bus case study [45] and an IEEE 118-bus test feeder [47], to evaluate the recently introduced constraints in section (IV-B).

In the first case study, the set of generators, load and connection buses are, $\mathbb{N}_{\mathcal{G}} = \{7, 17, 20, 30, 32, 33, 34, 35\}$, $\mathbb{N}_{\mathcal{L}} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 22, 23, 25, 30, 32, 33, 34, 35, 36, 37\}$ and $\mathbb{N}_{\mathcal{S}} - (\mathbb{N}_{\mathcal{G}} \cup \mathbb{N}_{\mathcal{L}}) = \{1, 18, 21, 24, 26, 27, 28, 29, 31\}$, respectively. In addition, bus 20 is selected as a slack bus, *i.e.*, $\delta_{k_s} = 0$, $k_s = 20$. Figure. (6) shows the network configuration. There is no assumption

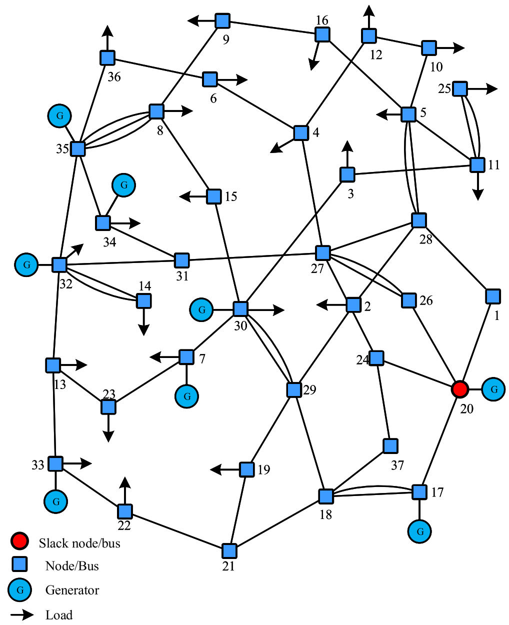
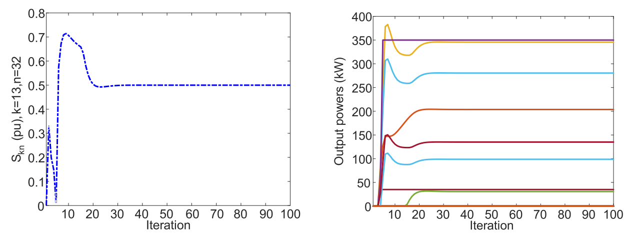


FIGURE 6. 37-bus network [45] for OPF performance evaluation.

TABLE 2. Numerical comparison of the proposed distributed and centralized methods.

Power (kW)	Distributed Method	Centralized Method
P_7	35	35
P_{17}	343.9884	348.75
P_{20}	279.2432	284.09
P_{30}	205.5574	195.01
P_{32}	350	350
P_{33}	34.23851	35.74
P_{34}	98.21342	87.52
P_{35}	134.5003	144.78
Total loss	33.641	33.817
Total generation	1480.7412	1480.89



(a) Complex power flow through the line between buses 13 and 32 (pu) (b) Output powers of generators kW

FIGURE 7. OPF simulaion results for a synthetic 37-bus system system.

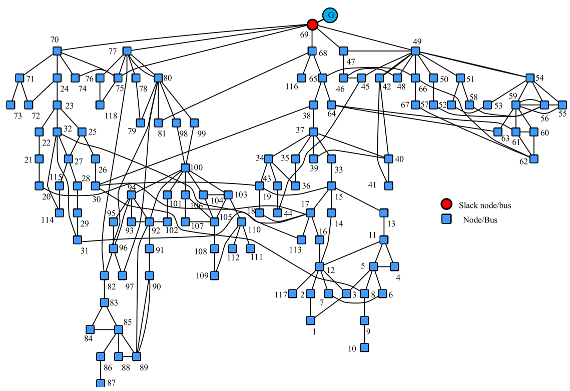


FIGURE 8. 118-bus network [47] for OPF performance evaluation.

regarding transmission line resistance or inductance. Hence, the algorithm can be applied to both transmission and distribution systems. There are some parallel lines in the system, such as the lines between buses 8 and 35, which are considered as a single line.

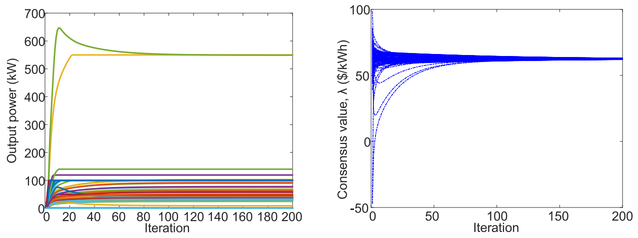
It is worth mentioning that active power loss, which is estimated by each bus, is considered as an amount of load reported by each bus, as shown by (30), for the sake of simplicity. The reason is that the total losses of the power system are inherently part of the power flow calculation and there is no need for any specific calculation.

$$\mathcal{P}_{Loss_k} = \sum_{n \neq k} (1/2)\, G_{kn} \left|V_k - V_n\right|^2, \quad \forall k \,\&\, n \in \mathbb{N}_{\mathcal{S}} \tag{30}$$

Due to the page limitation, voltage results are ignored in this paper. Table (2) compares the generators' output calculated by the proposed method and those of the centralized method, obtained from MATPOWER [46] as benchmark results. The solution mismatch between the distributed and centralized methods is almost less than 3.9% of the average. The total cost found by the proposed method (26090.7195$^{\$}$/hr) is close to that of centralized methods (26090.60$^{\$}$/hr). The total generation is about 1480.7412 kW, which covers the total demand (1447.1 kW) plus total loss (33.641 kW). The mismatch between the total estimated power loss between the distributed methods and the benchmark is 0.52%.

The overloaded transmission line between buses 13 and 32 is detected by the proposed method and loaded up to its maximum capacity. Figure. (9) shows the evolution of the line flow congestion and output power of the all generators. As shown in Figure. (7a), the algorithm detects the line overloading and tries to shift power in both sides of the line. As can be seen, the power flow is decreased around the 10th iteration and reaches maximum line capacity within 20 iteration. Figure. (7b) demonstrates how generators change their output to avoid line overloading. The final consensus value for the entire system is $\lambda = 22.6^{\$}$/kWh.

In the second case study, we test the proposed algorithm on a larger and more realistic power system model, *i.e.*, the IEEE 118-bus, and compare our results with those obtained by the MATPOWER toolbox as a benchmark. The overall system configuration is shown in figure. (8). The financial information of the generators' cost function (*i.e.*, $\alpha^{\$}/(\text{kW})^2\,\text{h}$, $\beta^{\$}/\text{kWh}$) are randomly generated. More information about the system data including bus types, line electrical impedance and other required information regarding, topology can



(a) The consensus value of all buses (b) Output powers of generators kW

FIGURE 9. OPF simulaion results for IEEE 118-bus system system.

be found in [47]. The comparison between the results obtained by the proposed algorithm and those obtained by MATPOWER shows the accuracy and efficiency of our algorithm. The solution mismatch of the generators' output calculated by the distributed methods and those of the benchmark is almost less than 6% of the average. The total cost found by the distributed methods is different from the benchmark results by 0.26%. The total generation is about 4380.55 kW, which covers the total demand (4242.00 kW) plus total loss (138.55 kW). We should note here that our algorithm is only minimizing the total cost of generation and does not consider minimizing power loss in systems.

As can be seen, the number of iterations required for converging to the final solution does not significantly change with the system size. The number of iterations required for convergence is about 30 in the synthetic 37-bus system. This number is about 42 in the IEEE 118-bus system, while the size of this system is almost 3 times bigger than the first case study. This means the proposed algorithm can efficiently work for bigger systems.

VI. CONCLUSION

This work shows that an OPF problem can be solved without linearization and convexification, which in turn does not limit OPF to some specific network and assumptions. As voltage amplitudes are more sensitive to reactive power than active power, the set point voltage of the generators is used to compensate for the violation of voltage amplitude in other buses assuming there is enough amount of the reactive power to control voltage. In addition, line flow constraints are replaced with a local convex problem to pave the way for generalizing the mathematical proof. Simulation results of a synthetic 37-bus case study and an IEEE 118-bus test feeder confirm the optimality analysis.

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