IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received June 29, 2019, accepted July 11, 2019, date of publication July 18, 2019, date of current version August 2, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2929841

Adaptive Finite-Time Control Design for a Class of Uncertain Nonlinearly Parameterized Switched Systems

JIAN WU[1,2], XIAOLI YANG[3], CHUNSHENG ZHANG[2], AND JING LI[3]

[1]National Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China
[2]The University Key Laboratory of Intelligent Perception and Computing of Anhui Province, Anqing Normal University, Anqing 246011, China
[3]School of Mathematics and Statistics, Xidian University, Xi'an 710071, China

Corresponding author: Jing Li (xidianjing@126.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61603003, Grant 61673014, and Grant 61673308, in part by the Natural Science Foundation of Anhui Province under Grant 1608085QF131, in part by the China Postdoctoral Science Foundation under Grant 2017M620245, in part by the Anhui Province Department of Education University Scientific Research Fund under Grant KJ2018A0359, in part by the Foundation of the University Research and Innovation Platform Team for Intelligent Perception and Computing of Anhui Province, in part by the Program for Academic Top-Notch Talents of University Disciplines under Grant gxbjZD21, and in part by the Program for Innovative Research Team in Anqing Normal University.

ABSTRACT This paper focuses on the finite-time control issue for a class of uncertain nonlinearly parameterized systems under arbitrary switching. By using the adding a power integrator technique (APIT), a new adaptive controller with a tuning parameter is designed. Different from the previous results, we develop a new common Lyapunov function (CLF) with a tuning parameter to achieve the global finite-time stability (GFTS) for the closed-loop switched systems. It is proved that all the states of the considered switched systems converge to an equilibrium state in finite time. Some simulations are given to illustrate the feasibility and advantages of the suggested approach.

INDEX TERMS Switched nonlinear system, finite-time stability, power integrators, nonlinear parametrization, adaptive control.

I. INTRODUCTION

Switched system is a class of hybrid systems and it contains a logical switching law and several subsystem components. Many practical engineering systems can be described by the mathematical models of switched systems, such as robotic system, networked system and traffic surveillance & control system [1], [2]. Thus, in the past decades, lots of scholars focus on the control problems of the switched systems and a number of interesting and meaningful results have been obtained [3]–[18]. For instance, Yin *et al.* [3] discuss the fuzzy adaptive control issue and construct a fuzzy adaptive controller to ensure the stability of the constrained nonlinear switched stochastic pure-feedback systems. Under arbitrary switching, a necessary and sufficient condition of the asymptotic stability is developed for all subsystems of the switched systems in [4]. However, how to design a CLF for all subsystems is very difficult. Some CLFs with special structures have been developed for several classes of switched systems [5]–[12]. In [5], when the switching law is arbitrary, a new CLF is designed to warrant the asymptotic stability of the switched positive linear systems. Based on backstepping technique and variable separation method, a CLF with particular structure is constructed to achieve the convergence of the tracking error to the predefined range for the uncertain nonlinearly parameterized switching systems in [6]. For a class of high-order switched nonlinear parametric systems, an adaptive stabilization problem is studied when the solvability of subsystem adaptive stabilization is not required in [7], [8]. For the switched nonlinear systems with unknown system functions and unknown backlash-like hysteresis, some fuzzy/neural-approximation-based control schemes have been developed in [16] and [17]. In addition, for the switched systems with mode-dependent average dwell time, new stability and stabilization conditions have been established in [18]. It is noted that the foregoing references just focus on the asymptotic stability of uncertain switched systems.

However, some control performance (e.g., better disturbance rejection ability and faster convergence rates

The associate editor coordinating the review of this manuscript and approving it for publication was Engang Tian.

in [19], [20]) of actual industrial processes cannot be achieved by the asymptotic stability. Meanwhile, the finite-time stability theory is proposed to achieve these control performance and has been widely applied to engineering problems [21], [22]. During the last few decades, many references have reported the finite-time stability theory for the system analysis and many meaningful results have been proposed in [23]–[37]. For example, in [23], an output feedback controller is designed with the $n-1$ dimensional observer, where the GFTS is warranted for the continuous-time nonlinear systems. For the unknown switched nonlinear systems, an adaptive finite-time control strategy is presented by utilizing backstepping technique and neural network appropriate method in [24]. In [25], a new common coordinate transformation is constructed to handle the global finite-time control issue for switched high-order lower-triangular and high-order upper-triangular nonlinear systems. For the high-order non-triangular switched nonlinearly parameterized systems, the finite-time stabilization (FTS) problem has been tackled by the APIT and the MLFs method. By using the APIT and the MLFs method [26], an adaptive controller and a switching law are constructed to tackle the FTS problem for a class of high-order non-triangular switched nonlinearly parameterized systems. Fu *et al.* [27] use the APIT to develop an adaptive controller for the switched nonlinear systems with the positive odd rational numbers powers. In [28]–[31], several kinds of adaptive switching controllers are constructed to achieve the GFTS for the nonlinear systems. Very recently, for a class of time-varying systems with stochastic nonlinearities, a new finite-horizon control method has been proposed in [32].

Based on the foregoing observation, the finite-time control issue has been successfully analyzed and addressed with the APIT for the nonlinear systems without switching signal, but how to extend the APIT to the uncertain nonlinearly parameterized switched systems is needed to make further efforts. In the following, we briefly summarize the main contributions of our work.

- In contract to the previous results (e.g., see [26], [28]), our work has three distinguishing features. First of all, the CLF approach and APIT are borrowed to design a novel CLF, which can solve the global finite-time control issue of the considered switched systems. Secondly, to eliminate the common restriction of uncertain parameters in the switched system, the uncertain nonlinear parameterization problem is transformed into a linear parameterization problem. Thirdly, under arbitrary switching, a CLF with a tuning parameter for all subsystems is developed such that the proposed adaptive controller can ensure the GFTS of the closed-loop systems.
- For the uncertain nonlinearly parameterized switched systems under arbitrary switching, the finite-time stabilization problem is discussed in this paper. As we all know, no evidence shows that the existing methods can well address the finite-time control issue for the considered switched systems. That is, for the first time, the CLF approach and APIT are used to solve the global finite-time control problem for the uncertain nonlinearly parameterized switched systems under arbitrary switching.

The framework of this paper is composed of five sections. In section II, we give the system description, the definition of global finite-time stability and some lemmas, which are helpful for the control scheme design and analysis. Section III presents the proof of Lyapunov-based finite-time stability theorem. To prove the availability of the proposed strategy, one example is given in Section IV. Section V shows the conclusion of this paper V.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

The uncertain nonlinearly parameterized system under arbitrary switching is described by

$$\begin{cases} \dot{x}_j = x_{j+1} + \Phi_{j,\sigma(t)}(\bar{x}_j, \theta_j), \ j = 1, 2, \cdots, m-1 \\ \dot{x}_m = u + \Phi_{m,\sigma(t)}(\bar{x}_m, \theta_m) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_m = [x_1, x_2, \cdots, x_m]^T \in R^m$ is the state vector of the system with $\bar{x}_j = [x_1, x_2, \cdots, x_j]^T \in R^j, j = 1, 2, \cdots, m$. y and u are represented as the system output and control input, respectively. For any $j = 1, 2, \cdots, m$ and $l = 1, 2, \cdots, n$, θ_j is an unknown constant parameter and a known smooth nonlinear function is represented by $\Phi_{j,l}(\bar{x}_j, \theta_j)$. The switching signal can be expressed by $\sigma(t) : [0, +\infty) \to \mathcal{M} = \{1, 2, \cdots, n\}$. Assume that the system states do not jump under the moment of system (1) switching, which means that the solution $\bar{x}_m$ is continuous everywhere.

The control objective of this paper is to solve the global finite-time stabilization problem for the system (1). An adaptive control scheme is developed such that all the signals of the closed-loop control system can converge to zero in finite time.

To achieve this purpose, we first review some basic results, including a definition and seven lemmas. These results play a pivotal role in the subsequent theoretical derivation.

Definition 1 (GFTS) [38]: If, in a neighborhood of the equilibrium point $x(t_0) = 0$, there is a finite time $t' \geq t_0$, for all $t \geq t'$, such that $x(t; t_0, x_0) \equiv 0$ and $T(x_0, t_0) := \min\{t | x(t) = 0, t \geq t_0\}$ is called the settling time, then the equilibrium point of the system (1) is finite-time convergent. If the finite-time convergence and Lyapunov stability conditions are both satisfied, the equilibrium point 0 is said to be FTS. Note that when the domain is the whole state space, the equilibrium point of the system (1) is globally FTS.

Lemma 1 [38]: The equilibrium point of the system (1) is Lyapunov FTS, if and only if, in a neighborhood $\mathcal{S}$ of the equilibrium point $x = 0$, there exist a continuously differentiable, decrescent scalar function $V(t, x)$ and a $\mathcal{K}$ function $\alpha(\cdot)$ such that

$$\begin{aligned} V(t, 0) &= 0, \quad t = [0, \infty) \\ \alpha(\| x \|) &\leq V(t, x), \quad t = [0, \infty), \quad x \in \mathcal{S} \\ \dot{V}(t, x) &\leq -\rho V(t, x)^{\lambda}, \quad t = [0, \infty), \quad x \in \mathcal{S} \end{aligned} \tag{2}$$

where $0 < \lambda < 1$ and $\rho > 1$ are constants. Particularly, when $\mathcal{S} = R^n$, the zero solution $x(t) \equiv 0$ of the system (1) is globally FTS.

Remark 1: According to Lemma 1, there are two constants $0 < \lambda < 1$ and $\rho > 1$, if the Lyapunov function satisfies

$$\dot{V}(t,x) + \rho V(t,x)^{\lambda} \leq 0, \tag{3}$$

and then, the equilibrium point of the system (1) is FTS.

Remark 2: The above lemma implies that, if V is a non-negative function, it can be ensured that the system states converge to zero in finite-time from (3). Simultaneously, Definition 1 and Lemma 1 are very important because it will be used to prove Theorem 1. The next lemma has been commonly used in the references(e.g. see [37], [38], [41]–[43]), which is employed to separate unknown parameters from some complex nonlinear functions.

Lemma 2 [36]: There exist four smooth scalar functions $d(x) \geq 1$, $l(y) \geq 1$, $m(x) \geq 0$, $n(y) \geq 0$, and a real-valued continuous function $h(x,y)$, which can satisfy the following equalities

$$|h(x,y)| \leq m(x) + n(y),$$
$$|h(x,y)| \leq d(x)l(y),$$

where $x \in R^m, y \in R^n$.

Remark 3: To facilitate later discussion, the following lemma is extended by Lemma 2, which effectively simplifies the design difficulty of the desired controller.

Lemma 3 [36]: For $j = 1, 2, \cdots, m$, the two inequalities subsequently hold.

$$|\phi_j(\bar{\xi}_j,\theta)| \leq (|\xi_1| + \cdots + |\xi_j|)q_j(\bar{\xi}_j,\theta),$$
$$q_j(\bar{\xi}_j,\theta) \leq |\, q_j(\bar{\xi}_j,\theta)\,| \leq \kappa_j(\bar{\xi}_j)\eta_j(\theta),$$

where θ is a constant, $\eta_j(\theta) \geq 1$ is a constant function, and $\phi_j(\bar{\xi}_j,\theta) : R^j \times R \to R$ is a C^1 function (first order continuous derivable function) with $\phi_j(0,\cdots,0,\theta) = 0$. $q_j(\cdot)$ and $\kappa_j(\bar{\xi}_j) \geq 1$ are two nonnegative continuous functions.

A new unknown constant parameter can be represented by $\Theta := \sum_{j=1}^{m} \eta_j(\theta)$. Then,

$$|\phi_j(\xi_1,\xi_2,\cdots,\xi_j,\theta)| \leq (|\xi_1| + \cdots + |\xi_j|)\kappa_j(\xi_1,\xi_2,\cdots,\xi_j)\Theta.$$

Obviously, $\Theta \geq 1$.

Lemma 4 (Young's Inequality) [40]: For $\forall(d,e) \in R^2$, the following inequality holds

$$de \leq \frac{\nu^a}{a}|d|^a + \frac{1}{b\nu^b}|e|^b,$$

where $\nu > 0$, $a > 1$, $b > 1$, and $(a-1)(b-1) = 1$.

Lemma 5 [27]: For $d, e \in R$ and $1 \leq q \in Q_{odd} := \{x \in R | x = \vartheta/\upsilon, \vartheta$ and υ are positive odd integers$\}$, we have

$$|\, d + e \,| \leq 2^{1-1/q} |\, d^q + e^q \,|^{1/q}.$$

Lemma 6 [27]: The following inequality is true for any real number $s > 0$, $c > 0$ and any valuable function $\omega(d,e) > 0$, then we get

$$|\, d \,|^s |\, e \,|^c \leq \frac{s}{s+c}\omega(d,e) |\, d \,|^{s+c} + \frac{c}{s+c}\omega(d,e)^{-\frac{s}{c}} |\, e \,|^{s+c}.$$

Lemma 7 [27]: For $a, b, c \in R$, if $0 < a \leq b \leq c$, it concludes that

$$|\, d \,|^b \leq |\, d \,|^a + |\, e \,|^c = |\, d \,|^a(1 + |\, d \,|^{c-a}), d \in R.$$

III. CONVERGENCE ANALYSIS

Theorem 1: For the considered system (1), there exists a CLF satisfying the conditions of Lemma 1 such that the equilibrium point of the closed-loop system is globally FTS.

Proof: Inspired by [36], the nonlinear parameters will be tackled by using variable separation technique, which simplifies the difficulty of controller design. In this section, we will adopt the adaptive backstepping method and ADIT to design a CLF to achieve the proof of Theorem 1. First of all, we set a group of the virtual controllers, which will be designed later.

$$\begin{aligned}
x_1^* &= 0, & \xi_1 &= x_1^{\frac{1}{q_1}} - x_1^{*\frac{1}{q_1}}, \\
x_2^* &= -\xi_1^{q_2}\beta_1(\bar{x}_1), & \xi_2 &= x_2^{\frac{1}{q_2}} - x_2^{*\frac{1}{q_2}}, \\
&\vdots & &\vdots \\
x_m^* &= -\xi_1^{q_m}\beta_{m-1}(\bar{x}_{m-1}), & \xi_m &= x_m^{\frac{1}{q_m}} - x_m^{*\frac{1}{q_m}},
\end{aligned} \tag{4}$$

where $q_j = \frac{2m+3-2j}{2m+1}$, $j = 1, 2, \cdots, m$, and β_{i-1} are C^1 functions with $i = 2, \cdots, m$.

Next, we will divide into m steps to design a CLF for all subsystems of system (1), and then it can be proved that the system state is FTS.

Step 1: Select the following nonnegative Lyapunov function

$$V_1 = W_1(\bar{x}_1) + \frac{1}{2\lambda_1}\tilde{\Theta}_1^2, \tag{5}$$

where $W_1(\bar{x}_1)$ is a integral function, and $\lambda_1 > 0$ is a design parameter. For instance, here it can be selected as $W_1(\bar{x}_1) = \int_0^{x_1}(s^{\frac{1}{q_1}} - 0)^{2-q_1}ds$, and $\hat{\Theta}_1$ denotes the estimate of unknown constant Θ_1 and $\tilde{\Theta}_1 = \hat{\Theta}_1 - \Theta_1$. Combine the Lyapunov function (5) with the first subsystem in (1), and then the time derivative of V_1 is obtained by

$$\begin{aligned}
\frac{dV_1}{dt} &= \dot{x_1}x_1 + \frac{1}{\lambda_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1 \\
&= x_1(x_2 + \Phi_{1,\sigma(t)}(\bar{x}_1,\theta_1)) + \frac{1}{\lambda_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1.
\end{aligned} \tag{6}$$

According to Lemmas 2 and 3, we have

$$\Phi_{1,\sigma(t)}(\bar{x}_1,\theta_1) \leq |\, x_1 \,| \,(\frac{1}{2}\sum_{l=1}^{n}\varphi_{1,l}^2(\bar{x}_1) + \frac{1}{2}\theta_1^2). \tag{7}$$

Substituting (7) into (6) gains

$$\begin{aligned}
\frac{dV_1}{dt} &\leq x_1x_2 + x_1 \,|\, x_1 \,|\, (\frac{1}{2}\sum_{l=1}^{n}\varphi_{1,l}^2(\bar{x}_1) + \frac{1}{2}\theta_1^2) + \frac{1}{\lambda_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1 \\
&= x_1(x_2 - x_2^*) + x_1x_2^* + x_1^2(Y_1 + \Theta_1) + \frac{1}{\lambda_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1 \\
&= -m\xi_1^d + x_1(x_2 - x_2^*) + x_1x_2^* + m\xi_1^d
\end{aligned}$$

$$+x_1^2(Y_1+\hat{\Theta}_1)+\frac{1}{\lambda_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1-x_1^2\tilde{\Theta}_1$$
$$=-m\xi_1^d+x_1(x_2-x_2^*)+\frac{1}{\lambda_1}\tilde{\Theta}_1(\dot{\hat{\Theta}}_1-\lambda_1x_1^2)$$
$$+\xi_1(x_2^*+m\xi_1^{d-1}+\xi_1(Y_1+\hat{\Theta}_1)), \quad (8)$$

where $d=\frac{4\,m}{2m+1}$, $Y_1=\frac{1}{2}\sum_{l=1}^{n}\varphi_{1,l}^2(\bar{x}_1)$, and $\Theta_1=\frac{1}{2}\theta_1^2$.

Choose a virtual controller x_2^* with the following form

$$x_2^*=-\xi_1^{d-1}(m+\xi_1^{2-d}(Y_1+\hat{\Theta}_1))=-\xi_1^{d-1}\beta_1, \quad (9)$$

where $\beta_1=m+\xi_1^{2-d}(Y_1+\hat{\Theta}_1)$.

Based on the above information, substituting (9) into (8) yields

$$\dot{V}_1\leq-m\xi_1^d+x_1(x_2-x_2^*)+\frac{1}{\lambda_1}\tilde{\Theta}_1(\dot{\hat{\Theta}}_1-\lambda_1x_1^2), \quad (10)$$

and the adaptive law is designed as

$$\dot{\hat{\Theta}}_1=\lambda_1\,x_1^2.$$

Step **j** ($2\leq j\leq m-1$)**:** Recursion method is adopted in each step when the value of i increases from 2 to $m-1$. Select the Lyapunov function below

$$V_j=V_{j-1}+W_j(\bar{x}_j)+\frac{1}{2\lambda_j}\tilde{\Theta}_j^2, \quad (11)$$

where $W_j(\bar{x}_j)=\int_{x_j^*}^{x_j}(s^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})^{2-q_j}ds$, and $\lambda_j>0$ is a design parameter. Then, we get the time derivative of V_j

$$\frac{dV_j}{dt}=\frac{dV_{j-1}}{dt}+\frac{dW_j(\bar{x}_j)}{dt}+\frac{1}{\lambda_j}\tilde{\Theta}_j\dot{\hat{\Theta}}_j. \quad (12)$$

Next, in order to complete the derivation of (12), some useful Propositions will be presented. As we all know, according to Proposition 1 in [27], it is true that $W_j(\bar{x}_j), j=1,2,\cdots,m-1$ satisfy the following proposition:

Proposition 1: For the C^1 function $W_j(\bar{x}_j)$, the following equations hold.

$$\frac{\partial W_j(\bar{x}_j)}{\partial x_j}=(x_j^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})^{2-q_j}=\xi_j^{2-q_j},$$
$$\frac{\partial W_j(\bar{x}_j)}{\partial x_i}=-\int_{x_j^*}^{x_j}(2-q_j)(s^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})^{1-q_j}ds\frac{\partial x_j^{*q_j}}{\partial x_i},$$
$$i=1,2,\cdots,j-1.$$

According to Proposition 1, we can obtain

$$\frac{dV_j}{dt}=\frac{dV_{j-1}}{dt}+\frac{dW_j(\bar{x}_j)}{dt}+\frac{1}{\lambda_j}\tilde{\Theta}_j\dot{\hat{\Theta}}_j$$
$$\leq\frac{dV_{j-1}}{dt}+(x_j^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})^{2-q_j}\dot{x}_j+\frac{1}{\lambda_j}\tilde{\Theta}_j\dot{\hat{\Theta}}_j$$
$$+\sum_{i=1}^{j-1}\int_{x_j^*}^{x_j}(2-q_j)(s^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})^{1-q_j}ds\frac{\partial x_j^{*q_j}}{\partial x_i}\dot{x}_i$$
$$\leq\frac{dV_{j-1}}{dt}+\xi_j^{2-q_j}\big(x_{j+1}+\Phi_{j,\sigma(t)}(\bar{x}_j,\theta_j)\big)$$
$$+\frac{1}{\lambda_j}\tilde{\Theta}_j\dot{\hat{\Theta}}_j+\sum_{i=1}^{j-1}\int_{x_j^*}^{x_j}(2-q_j)(s^{\frac{1}{q_j}}+x_j^{*\frac{1}{q_j}})^{1-q_j}ds$$
$$\times\frac{\partial x_j^{*q_j}}{\partial x_i}\big(x_{i+1}+\Phi_{i,\sigma(t)}(\bar{x}_i,\theta_i)\big)$$
$$\leq-(m-j+2)\sum_{i=1}^{j-1}\xi_i^d+\xi_{j-1}^{2-q_{j-1}}(x_j^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})$$
$$+\sum_{i=1}^{j-1}\int_{x_j^*}^{x_j}(2-q_j)(s^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})^{1-q_j}ds\frac{\partial x_j^{*q_j}}{\partial x_i}$$
$$\times\big(x_{i+1}+\Phi_{i,\sigma(t)}(\bar{x}_i,\theta_i)\big)+\frac{1}{\lambda_j}\tilde{\Theta}_j\dot{\hat{\Theta}}_j$$
$$+\xi_j^{2-q_j}\big(x_{j+1}+\Phi_{j,\sigma(t)}(\bar{x}_j,\theta_j)\big). \quad (13)$$

Proposition 2: There exist a set of known functions $L_{j,1}(\bar{x}_j)$, such that

$$\xi_{j-1}^{2-q_{j-1}}(x_j^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})\leq\frac{1}{3}\sum_{i=1}^{j-1}\xi_i^d+L_{j,1}(\bar{x}_j)\mid\xi_j\mid^d. \quad (14)$$

The above conclusion is very similar to Proposition 2 in [27], and thus the Proposition 2 is not proved in detail here.

Proposition 3: There exist a set of continuous functions $L_{j,2}(\bar{x}_j)$ and $L_{j,3}(\bar{x}_j,\hat{\Theta}_j)$, such that

$$\xi_j^{2-q_j}\Phi_{j,\sigma(t)}(\bar{x}_j,\theta_j)$$
$$\leq\frac{1}{3}\sum_{i=1}^{j-1}\xi_i^d+(L_{j,2}(\bar{x}_j)+L_{j,3}(\bar{x}_j,\hat{\Theta}_j))\mid\xi_j\mid^d$$
$$-(\mid x_1\mid+\mid x_2\mid+\cdots+\mid x_j\mid)\tilde{\Theta}_j\xi_j^{2-q_j}. \quad (15)$$

Proposition 4: There exist a group of continuous functions $L_{j,4}(\bar{x}_j)$ and $L_{j,5}(\bar{x}_j,\hat{\Theta}_j)$. By combining (17) with (18), it concludes that

$$\sum_{i=1}^{j-1}\int_{x_j^*}^{x_j}(2-q_j)(s^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})^{1-q_j}ds\frac{\partial x_j^{*q_j}}{\partial x_i}\big(x_{i+1}+\Phi_{i,\sigma(t)}(\bar{x}_i,\theta_i)\big)$$
$$\leq-\sum_{i=1}^{j-1}(2-q_j)2^{2-q_j}\mid\xi_j\mid\frac{\partial x_j^{*q_j}}{\partial x_i}(\mid x_1\mid+\mid x_2\mid+\cdots+\mid x_i\mid)\tilde{\Theta}_i$$
$$+\frac{1}{3}\sum_{i=1}^{j-1}\xi_i^d+L_{j,4}(\bar{x}_j)\xi_j^d+L_{j,5}(\bar{x}_j,\hat{\Theta}_j)\mid\xi_j\mid^d. \quad (16)$$

Remark 4: With the help of Lemma 5, we get

$$\left|\int_{x_j^*}^{x_j}(2-q_j)(s^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}})^{1-q_j}ds\right|$$
$$\leq(2-q_j)\mid x_j-x_j^*\mid\left|x_j^{\frac{1}{q_j}}-x_j^{*\frac{1}{q_j}}\right|^{1-q_j}$$
$$\leq(2-q_j)2^{2-q_j}\mid\xi_j\mid. \quad (17)$$

By combining with Lemmas 2 and 3, the following fact is obtained

$$\begin{aligned}&\frac{\partial x_j^{*q_j}}{\partial x_i}\big(x_{i+1}+\Phi_{i,\sigma(t)}(\bar{x}_i,\theta_i)\big)\\&\leq \frac{\partial x_j^{*q_j}}{\partial x_i}\big[x_{i+1}+(|x_1|+|x_2|+\cdots+|x_i|)\\&\quad\times\big(\frac{1}{2}\sum_{l=1}^{n}\varphi_{i,l}^2(\bar{x}_i)+\frac{1}{2}\theta_i^2\big)\big]\\&= \frac{\partial x_j^{*q_j}}{\partial x_i}\big[x_{i+1}+(|x_1|+|x_2|+\cdots+|x_i|)\big(Y_i+\Theta_i\big)\big]\\&= \frac{\partial x_j^{*q_j}}{\partial x_i}\big[x_{i+1}+(|x_1|+|x_2|+\cdots+|x_i|)\big(Y_i+\hat{\Theta}_i\big)\big]\\&\quad-\frac{\partial x_j^{*q_j}}{\partial x_i}(|x_1|+|x_2|+\cdots+|x_i|)\tilde{\Theta}_i, \qquad (18)\end{aligned}$$

where $Y_i=\frac{1}{2}\sum_{l=1}^{n}\varphi_{i,l}^2(\bar{x}_i)$ and $\Theta_i=\frac{1}{2}\theta_i^2$.

The detailed proof of Proposition 3 will be given in Appendix. Note that the proof process of the Proposition 4 is similar to the proof of Proposition 3. Hence, the proof of Proposition 4 is omitted.

According to Propositions 2-4, (12) can be rewritten as

$$\begin{aligned}\frac{dV_j}{dt}&\leq -(m-j+1)\sum_{i=1}^{j-1}\xi_i^d+(L_{j,1}+L_{j,2}+L_{j,3}+L_{j,4}\\&\quad+L_{j,5})|\xi_j|^d+\xi_j^{2-q_j}x_{j+1}^*+\xi_j^{2-q_j}(x_{j+1}-x_{j+1}^*)\\&\quad+\frac{1}{\lambda_j}\tilde{\Theta}_j\dot{\hat{\Theta}}_j-|\xi_j|^{2-q_j}\sum_{i=1}^{j}|x_i|\tilde{\Theta}_i-(2-q_j)2^{2-q_j}\\&\quad\times|\xi_j|\sum_{i=1}^{j-1}\frac{\partial x_j^{*q_j}}{\partial x_i}(|x_1|+|x_2|+\cdots+|x_i|)\tilde{\Theta}_i\\&\leq -(m-j+1)\sum_{i=1}^{j-1}\xi_i^d+(L_{j,1}+L_{j,2}+L_{j,3}+L_{j,4}\\&\quad+L_{j,5})|\xi_j|^d+\xi_j^{2-q_j}x_{j+1}^*+\xi_j^{2-q_j}(x_{j+1}-x_{j+1}^*)\\&\quad+\frac{1}{\lambda_j}\tilde{\Theta}_j\big[\dot{\hat{\Theta}}_j-\lambda_j\big(|\xi_j|^{2-q_j}\sum_{i=1}^{j}|x_i|-(2-q_j)2^{2-q_j}\\&\quad\times|\xi_j|\sum_{i=1}^{j-1}\frac{\partial x_j^{*q_j}}{\partial x_i}(|x_1|+|x_2|+\cdots+|x_i|)\big)\big].\end{aligned} \qquad (19)$$

Clearly, the virtual controller is designed from the above information

$$\begin{aligned}x_{j+1}^*&=-\xi_j^{\frac{d}{2-q_j}}(m-j+1+L_{j,1}+L_{j,2}+L_{j,3}+L_{j,4}+L_{j,5})\\&=-\xi_j^{\frac{d}{2-q_j}}\beta_j,\end{aligned} \qquad (20)$$

where $\beta_j=(m-j+1+L_{j,1}+L_{j,2}+L_{j,3}+L_{j,4}+L_{j,5})$. Then, the adaptive law is chosen as

$$\begin{aligned}\dot{\hat{\Theta}}_j&=\lambda_j\big(|\xi_j|^{2-q_j}\sum_{i=1}^{j}|x_i|-(2-q_j)2^{2-q_j}|\xi_j|\\&\quad\times\sum_{i=1}^{j-1}\frac{\partial x_j^{*q_j}}{\partial x_i}(|x_1|+|x_2|+\cdots+|x_i|)\big).\end{aligned} \qquad (21)$$

Step m. According to the inductive method, we consider the case $j=m$. Consider $V_m=V_{m-1}+W_m(\bar{x}_m)+\frac{1}{\lambda_m}\tilde{\Theta}_m^2$, where $\tilde{\Theta}_m=\hat{\Theta}_m-\Theta_m$, $\Theta_m=\frac{1}{2}\theta_m^2$, and $\lambda_m>0$ is a design parameter.

By observing the computation process of Step j, the actual control input can be selected as

$$u=-\xi_m^{\frac{d}{2-q_m}}(1+L_{m,1}+L_{m,2}+L_{m,3}+L_{m,4}+L_{m,5}), \qquad (22)$$

and the following adaptive law is chosen

$$\begin{aligned}\dot{\hat{\Theta}}_m&=\lambda_m\big(|\xi_m|^{2-q_m}\sum_{i=1}^{m}|x_i|-(2-q_m)2^{2-q_m}|\xi_m|\\&\quad\times\sum_{i=1}^{m-1}\frac{\partial x_m^{*q_m}}{\partial x_i}(|x_1|+|x_2|+\cdots+|x_i|)\big).\end{aligned} \qquad (23)$$

By combining (22) with (23), it is easy to see that

$$\begin{aligned}\dot{V}_m&\leq -\sum_{m=1}^{j}|\xi_j|^d=-\sum_{j=1}^{m}|\xi_j^2|^{\frac{d}{2}}\leq -\big(\sum_{j=1}^{m}|\xi_j^2|\big)^{\frac{d}{2}}\\&\leq -\big(\frac{1}{2}V_m\big)^{\frac{d}{2}}=-aV_m^{\frac{d}{2}},\end{aligned} \qquad (24)$$

where $a=2^{-\frac{d}{2}}$ and $\frac{d}{2}=\frac{2m}{2m+1}<1$. That is,

$$\dot{V}_m+aV_m^{\frac{d}{2}}\leq 0. \qquad (25)$$

It is easily seen from (25) that the conditions of Lemma 1 are satisfied. That is, global finite time stability is true for the nonlinear switched systems with uncertain nonlinear parameters, and all the signals of system (1) would fall into a sufficiently small neighborhood of zero in finite time. In addition, based on the inequality (25), it can be concluded that the settling time T is

$$T=t_0+\frac{(V_m(t_0))^{1-\frac{d}{2}}}{a(1-\frac{d}{2})},$$

where t_0 is the initial time of the control system.

Thus, the proof of Theorem 1 is completed.

IV. NUMERICAL SIMULATION

Consider the nonlinear switched system

$$\begin{cases}\dot{x}_1=x_2,\\ \dot{x}_2=u+\Phi_{2,\sigma(t)}(\bar{x}_2,\theta_2),\ \sigma(t)=1,2,\\ y=x_1,\end{cases} \qquad (26)$$

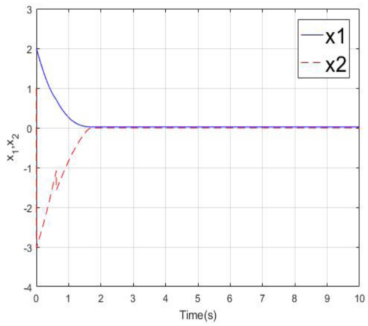

FIGURE 1. The state signals $x_1(t)$ and $x_2(t)$.

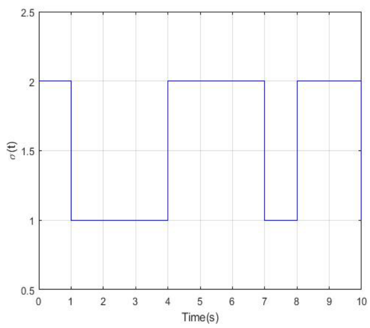

FIGURE 2. The switching signal $\sigma(t)$.

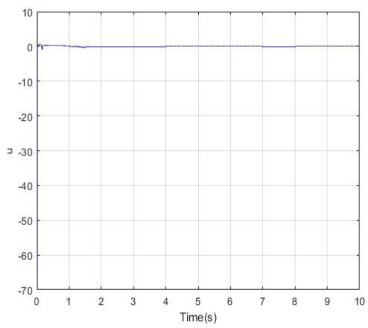

FIGURE 3. The control input signal $u(t)$.

where $\bar{x}_2 = [x_1, x_2]^T$ is the system state vector, u denotes the control input of the system, $\Phi_{2,1}(\bar{x}_2, \theta_2) = 0$, and $\Phi_{2,2}(\bar{x}_2, \theta_2) = \sin(x_1 \ x_2\theta_2) + 2$. Let the initial conditions be $x_1(0) = 2, x_2(0) = 1$, and $\hat{\Theta}_2(0) = 0$.

Based on Lemmas 2 and 3, two subsequent functions are easy to get $\varphi_{2,2}(\bar{x}_1) = |x_1 \ x_2| + 2$ and $\Psi_2(\bar{x}_1) = (|x_1 \ x_2| + 2)^2/2$, where $\Psi_j(\bar{x}_j) = \frac{1}{2}\sum_{l=1}^{j} \varphi_{j,l}^2(\bar{x}_j), (j = 1, 2)$. The detailed theoretical derivation is presented in Appendix 2.

Based on the above information, the actual controller is formulated as

$$u = -(1 + L_{2,1} + L_{2,2} + L_{2,3} + L_{2,4}) | \xi_2 |^{\frac{1}{5}}, \tag{27}$$

and the parameter learning law

$$\dot{\hat{\Theta}}_2 = \lambda_2 | \xi_2 |^{7/5}(| x_1 | + | x_2 |). \tag{28}$$

Figures 1-2 show the simulation results. For the uncertain nonlinearly parameterized switched systems, all states of the closed-loop system converge to zero after 2 seconds, which can confirm the validity of the proposed scheme, as shown in Figure 1. Figure 2 and Figure 3 show the switching signal and control input signal, respectively.

V. CONCLUSION

For the uncertain nonlinearly parameterized switched systems, the finite-time control issue under arbitrary switching is addressed in this paper. Firstly, a CLF with power function terms is designed by using the APIT and the Lyapunov stability theory. Then, the variable separation method is borrowed to deal with nonlinear parameter terms, which simplifies the difficulty of the controller design. Finally, an adaptive finite-time control scheme is proposed, and all signals of the considered switched system converge to equilibrium in finite time. In the future, there are still many interesting problems suggested to be considered, such as the finite time control for system (1) with the white noise or input delay, the finite time stabilization of system (1) under event-triggered communication scheme proposed in [44] to reduce the communication traffic. In addition, it is a good topic to solve the practical control problem by using the proposed control method, and it will be considered in the further research.

APPENDIX

1. The proof of Proposition 3.

Proof: Using Lemmas 2 and 3, we have

$$\begin{aligned}
&\xi_j^{2-q_j}\Phi_{j,\sigma(t)}(\bar{x}_j, \theta_j) \\
&\le | \xi_j^{2-q_j} || \Phi_{j,\sigma(t)}(\bar{x}_j, \theta_j) | \\
&\le | \xi_j^{2-q_j} | (| x_1 | + | x_2 | + \cdots + | x_j |)\Big(\frac{1}{2}\sum_{l=1}^{n}\varphi_{j,l}^2(\bar{x}_j) + \frac{1}{2}\theta_j^2\Big) \\
&\le | \xi_j^{2-q_j} | (| x_1 | + | x_2 | + \cdots + | x_j |)(Y_j + \Theta_j) \\
&\le | \xi_j^{2-q_j} | (| x_1 | + | x_2 | + \cdots + | x_j |)(Y_j + \hat{\Theta}_j) \\
&\quad -(| x_1 | + | x_2 | + \cdots + | x_j |)\tilde{\Theta}_j,
\end{aligned} \tag{29}$$

where $Y_j = \frac{1}{2}\sum_{l=1}^{n}\varphi_{j,l}^2(\bar{x}_j)$, $\Theta_j = \frac{1}{2}\theta_j^2$ and $\tilde{\Theta}_j = \hat{\Theta}_j - \Theta_j$.

Using (4) and Lemma 5, we can obtain from (29),

$$\begin{aligned}
&| \xi_j^{2-q_j} | (| x_1 | + | x_2 | + \cdots + | x_j |)(Y_j + \hat{\Theta}_j) \\
&\quad \le | \xi_j |^{2-q_j}\Big(\sum_{i=1}^{j} | x_i - x_i^* | + \sum_{i=2}^{j} | x_i^* |\Big)(Y_j + \hat{\Theta}_j) \\
&\quad \le | \xi_j |^{2-q_j}\Big(\sum_{i=1}^{j} 2^{1-q_i} | \xi_i |^{q_i} \\
&\qquad + \sum_{i=2}^{j} | \xi_{i-1} |^{q_i} \beta_{i-1}\Big)(Y_j + \hat{\Theta}_j).
\end{aligned} \tag{30}$$

Based on Lemma 7, we can obtain

$$\begin{aligned}
| \xi_i |^{q_i} &\le | \xi_i |^{d-2+q_j} + | \xi_i | \\
&= | \xi_i |^{d-2+q_j} (1 + | \xi_i |^{1-(d-2+q_{j-1})}), \\
&\quad i = 1, 2, \cdots j, \\
| \xi_{i-1} |^{q_i} &\le | \xi_i |^{d-2+q_{j-1}} + | \xi_i |
\end{aligned}$$

$$= |\xi_i|^{d-2+q_{j-1}}(1+|\xi_i|^{1-(d-2+q_{j-1})}), \quad i=2,3,\cdots,j. \tag{31}$$

Remark 5: From $0 < q_j = \frac{2m+3-2j}{2m+1} \leq 1$ and $1 < d = \frac{4m}{2m+1} < 2$, it is easy to get $d-2+q_j < q_i < 1$, $i = 1,2,\cdots,j$.

$$\begin{aligned} q_i-(d-2+q_j) &= \frac{2m+3-2i}{2m+1} - \frac{4m}{2m+1} + 2 - \frac{2m+3-2j}{2m+1} \\ &= \frac{2-2(i-j)}{2m+1} > 0. \end{aligned}$$

Substituting (31) into (30) yields

$$\begin{aligned} &|\xi_j^{2-q_j}|(|x_1|+|x_2|+\cdots+|x_j|)(Y_j+\hat{\Theta}_j) \\ &\leq |\xi_j|^{2-q_j}\Big(\sum_{i=1}^{j} 2^{1-q_i}|\xi_i|^{q_i} + \sum_{i=2}^{j}|\xi_{i-1}|^{q_i}\beta_{i-1}\Big)(Y_j+\hat{\Theta}_j) \\ &\leq |\xi_j|^{2-q_j}\Big(\sum_{i=1}^{j} 2^{1-q_i}|\xi_i|^{d-2+q_j}(1+|\xi_i|^{1-(d-2+q_{j-1})}) \\ &\quad + \sum_{i=2}^{j}|\xi_i|^{d-2+q_{j-1}}(1+|\xi_i|^{1-(d-2+q_{j-1})})\beta_{i-1}\Big)(Y_j+\hat{\Theta}_j). \end{aligned} \tag{32}$$

Since $\tilde{Y}_j(\bar{x}_j) > 0$ and $h_j(\bar{x}_j, \hat{\Theta}_j) > 0$ are C^1 functions, we have

$$\begin{aligned} \tilde{Y}_j(\bar{x}_j) &\geq (1+|\xi_j|^{1-(d-2+q_{j-1})})Y_j, \\ \tilde{Y}_j(\bar{x}_j) &\geq (1+|\xi_j|^{1-(d-2+q_{j-1})})\beta_{j-1}Y_j, \\ h_j(\bar{x}_j,\hat{\Theta}_j) &\geq (1+|\xi_j|^{1-(d-2+q_{j-1})})\hat{\Theta}_j, \\ h_j(\bar{x}_j,\hat{\Theta}_j) &\geq (1+|\xi_j|^{1-(d-2+q_{j-1})})\beta_{j-1}\hat{\Theta}_j. \end{aligned} \tag{33}$$

For $j = 1,2,\cdots,m$, a C^1 functions is chosen as

$$\begin{aligned} \tilde{Y}_j(\bar{x}_j) &= 4[1+\frac{1}{2}(1+(d-2+q_j)\cdot 1^2) \\ &\quad +\frac{1}{2}(1-(d-2+q_j)\cdot|\xi_j|^2) \\ &\quad +\big(1+\frac{1}{2}(1+(d-2+q_j)\cdot 1^2) \\ &\quad +\frac{1}{2}(1-(d-2+q_j)\cdot|\xi_{j-1}|^2)\big)\beta_{j-1}]Y_j, \\ h_j(\bar{x}_j,\hat{\Theta}_j) &= 4[1+\frac{1}{2}(1+(d-2+q_j)\cdot 1^2) \\ &\quad +\frac{1}{2}(1-(d-2+q_j)\cdot|\xi_j|^2) \\ &\quad +\big(1+\frac{1}{2}(1+(d-2+q_j)\cdot 1^2) \\ &\quad +\frac{1}{2}(1-(d-2+q_j)\cdot|\xi_{j-1}|^2)\big)\beta_{j-1}]\hat{\Theta}_j. \end{aligned}$$

Subsequently, we have

$$\begin{aligned} &(|\xi_j|^{2-q_j}|\xi_i|^{d-2+q_j}+|\xi_j|^{2-q_j}|\xi_i|^{d-2+q_j})\tilde{Y}_j \\ &\quad \leq \frac{1}{6}\sum_{i=1}^{j-1}\xi_i^d + L_{j,2}(\bar{x}_j)|\xi_j|^d, \end{aligned}$$

$$\begin{aligned} &(|\xi_j|^{2-q_j}|\xi_i|^{d-2+q_j}+|\xi_j|^{2-q_j}|\xi_i|^{d-2+q_j})h_j \\ &\quad \leq \frac{1}{6}\sum_{i=1}^{j-1}\xi_i^d + +L_{j,3}(\bar{x}_j,\hat{\Theta}_j)|\xi_j|^d. \end{aligned} \tag{34}$$

Consequently, substituting (34) into (32), it is easy to see that

$$\begin{aligned} &\xi_j^{2-q_j}\Phi_{j,\sigma(t)}(\bar{x}_j,\theta_j) \\ &\leq \sum_{i=1}^{j}2^{1-q_i}|\xi_i|^{d-2+q_j}\tilde{Y}_j + \sum_{i=2}^{j}2^{1-q_i}|\xi_i|^{d-2+q_j}\tilde{Y}_j \\ &\quad + \sum_{i=1}^{j}2^{1-q_i}|\xi_i|^{d-2+q_j}h_j + \sum_{i=2}^{j}2^{1-q_i}|\xi_i|^{d-2+q_j}h_j \\ &\quad -(|x_1|+|x_2|+\cdots+|x_j|)\tilde{\Theta}_j|\xi_j|^{2-q_j} \\ &\leq \frac{1}{3}\sum_{i=1}^{j-1}\xi_i^d + (L_{j,2}(\bar{x}_j)+L_{j,3}(\bar{x}_j,\hat{\Theta}_j))|\xi_j|^d \\ &\quad -(|x_1|+|x_2|+\cdots+|x_j|)\tilde{\Theta}_j|\xi_j|^{2-q_j}, \end{aligned} \tag{35}$$

where $L_{j,2}(\bar{x}_j) > 0$ and $L_{j,3}(\bar{x}_j,\hat{\Theta}_j) > 0$ are appropriate continuous functions.

2.According to the addressed control method, let $\xi_1 = x_1$, and consider $V_1(x_1) = \frac{1}{2}x_1^2$, and then the time derivative is described by

$$\begin{aligned} \dot{V}_1 &= x_1\dot{x}_1 = \xi_1(x_2-x_2^*) + x_1x_2^* \\ &= -2\xi_1^d + \xi_1(x_2-x_2^*) + \xi_1x_2^* + 2\xi_1^d \\ &= -2\xi_1^d + \xi_1(x_2-x_2^*) + \xi_1^d(\xi_1^{1-d}x_2^*+2), \end{aligned} \tag{36}$$

where the virtual controller $x_2^* = -\beta_1\xi_1^{q_2} = -2\xi_1^{-\frac{3}{5}}$ and $d = \frac{4\times 2}{2\times 2+1} = \frac{8}{5}$.

Then, the overall Lyapunov function is designed as

$$V_2 = V_1 + \int_{x_2^*}^{x_2}(s^{\frac{1}{q_2}} - x_2^{*\frac{1}{q_2}})^{2-q_2}ds + \frac{1}{2\lambda_2}\tilde{\Theta}_2^2. \tag{37}$$

Its derivative is

$$\begin{aligned} \frac{dV_2}{dt} &= \frac{dV_1}{dt} - \int_{x_2^*}^{x_2}\frac{7}{5}(s^{\frac{5}{3}} - x_2^{*\frac{5}{3}})^{\frac{2}{5}}ds\frac{\partial x_2^{*\frac{5}{3}}}{\partial x_1}\dot{x}_1 \\ &\quad +(x_2^{\frac{5}{3}} - x_2^{*\frac{5}{3}})^{\frac{7}{5}}\dot{x}_2 + \frac{1}{\lambda_2}\tilde{\Theta}_2\dot{\hat{\Theta}}_2 \\ &= -2\xi_1^{\frac{8}{5}} + \xi_1(x_2-x_2^*) + (x_2^{\frac{5}{3}} - x_2^{*\frac{5}{3}})^{\frac{7}{5}} \\ &\quad \times(\Phi_{2,\varrho(t)}(\bar{x}_2,\theta_2)+u) + \frac{1}{\lambda_2}\tilde{\Theta}_2\dot{\hat{\Theta}}_2 \\ &\quad - \int_{x_2^*}^{x_2}\frac{7}{5}(s^{\frac{5}{3}} - x_2^{*\frac{5}{3}})^{\frac{2}{5}}ds\frac{\partial x_2^{*\frac{5}{3}}}{\partial x_1}x_2 \\ &\leq -2\xi_1^{\frac{8}{5}} + \xi_1(x_2-x_2^*) + \xi_2^{\frac{7}{5}}u + \xi_2^{\frac{7}{5}}(Y_2+\Theta_2) \\ &\quad +\frac{1}{\lambda_2}\tilde{\Theta}_2\dot{\hat{\Theta}}_2 - \int_{x_2^*}^{x_2}\frac{7}{5}(s^{\frac{5}{3}} - x_2^{*\frac{5}{3}})^{\frac{2}{5}}ds\frac{\partial x_2^{*\frac{5}{3}}}{\partial x_1}x_2, \end{aligned} \tag{38}$$

where $Y_2 = \frac{1}{2}\sum_{l=1}^{2}\varphi_{2,l}^2(\bar{x}_2)$ and $\Theta_2 = \frac{1}{2}\theta_2^2$.

By using the aforementioned lemmas and propositions, we obtain

$$\begin{aligned}
&\xi_1(x_2 - x_2^*) \le \frac{1}{3}|\ \xi_1\ |^{\frac{8}{5}} + L_{2,1}|\ \xi_2\ |^{\frac{8}{5}},\\
&\quad \xi_2^{\frac{7}{5}}(Y_2 + \Theta_2) \le \frac{1}{3}|\ \xi_1\ |^{\frac{8}{5}} + (L_{2,2} + L_{2,3})|\ \xi_2\ |^{\frac{8}{5}}\\
&\quad -|\ \xi_2\ |^{\frac{7}{5}}(|\ x_1\ | + |\ x_2\ |)\tilde{\Theta}_2,\\
&\quad \int_{x_2^*}^{x_2} \frac{7}{5}(s^{\frac{5}{3}} - x_2^{*\frac{5}{3}})^{\frac{2}{5}} ds \frac{\partial x_2^{*\frac{5}{3}}}{\partial x_1} x_2\\
&\ \le \frac{1}{3}|\ \xi_1\ |^{\frac{8}{5}} + (L_{2,4} + L_{2,5})|\ \xi_2\ |^{\frac{8}{5}},
\end{aligned} \tag{39}$$

where $L_{2,1} = \frac{3}{8} \cdot 2^{\frac{16}{15}} \cdot \frac{15}{8}^{\frac{5}{3}}$; $L_{2,2} = 2 \cdot \frac{7}{8} \cdot \frac{3}{2}^{\frac{1}{7}} \cdot \tilde{Y}_2^{\frac{8}{7}} + \tilde{Y}_2$; $L_{2,3} = 2 \cdot \frac{7}{8} \cdot \frac{3}{2}^{\frac{1}{7}} \cdot h^{\frac{8}{7}} + h$; $L_{2,4} = \frac{3}{8} \cdot \frac{15}{4}^{\frac{5}{3}} \cdot (\frac{7}{5} \cdot 2^{\frac{4}{5}} \cdot (-2)^{\frac{5}{3}} \cdot |\ \xi_2\ |)^{\frac{8}{3}} + \frac{5}{8} \cdot \frac{9}{4}^{\frac{3}{5}} \cdot (\frac{7}{5} \cdot 2^{\frac{2}{5}} \cdot (-2)^{\frac{8}{3}} \cdot |\ \xi_1\ |)^{\frac{8}{5}}$; $L_{2,5} = 0$ with $\tilde{Y}_2 = 4[\frac{8}{5} + \frac{2}{5}|\ \xi_2\ |^2 + \big(\frac{8}{5} + \frac{4}{5}|\ \xi_1\ |^2\big)\beta_1]Y_2$ and $h = 4[\frac{8}{5} + \frac{2}{5}|\ \xi_2\ |^2 + \big(\frac{8}{5} + \frac{4}{5}|\ \xi_1\ |^2\big)\beta_1]\hat{\Theta}_2$.

Combing the above inequations, we have

$$\begin{aligned}
\frac{dV_2}{dt} &\le -2|\ \xi_1\ |^{\frac{8}{5}} + \frac{1}{3}|\ \xi_1\ |^{\frac{8}{5}} + L_{2,1}|\ \xi_2\ |^{\frac{8}{5}} + \frac{1}{3}|\ \xi_1\ |^{\frac{8}{5}}\\
&\quad +(L_{2,2} + L_{2,3})|\ \xi_2\ |^{\frac{8}{5}} + \frac{1}{3}|\ \xi_1\ |^{\frac{8}{5}}\\
&\quad +(L_{2,4} + L_{2,5})|\ \xi_2\ |^{\frac{8}{5}} + \frac{1}{\lambda_2}\tilde{\Theta}_2\dot{\hat{\Theta}}_2\\
&\quad -|\ \xi_2\ |^{\frac{7}{5}}(|\ x_1\ | + |\ x_2\ |)\tilde{\Theta}_2 + |\ \xi_2\ |^{\frac{7}{5}} u\\
&\le -|\ \xi_1\ |^{\frac{8}{5}} - |\ \xi_2\ |^{\frac{8}{5}} + \frac{1}{\lambda_2}\tilde{\Theta}_2\big[\dot{\hat{\Theta}}_2 - \lambda_2|\ \xi_2\ |^{\frac{7}{5}}\\
&\quad \times(|\ x_1\ | + |\ x_2\ |)\big] + |\ \xi_2\ |^{\frac{7}{5}} u + (1 + L_{2,1}\\
&\quad +L_{2,2} + L_{2,3} + L_{2,4})|\ \xi_2\ |^{\frac{8}{5}}.
\end{aligned} \tag{40}$$

REFERENCES

[1] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Syst.*, vol. 19, no. 5, pp. 59–70, Oct. 1999.

[2] D. Liberzion, *Switching in Systems and Control*. Boston, MA, USA: Brikhäuser, 2003.

[3] S. Yin, H. Yu, R. Shahnazi, and A. Haghani, "Fuzzy adaptive tracking control of constrained nonlinear switched stochastic pure-feedback systems," *IEEE Trans. Cybern.*, vol. 47, no. 3, pp. 579–588, Mar. 2017.

[4] L. Vu and D. Liberzon, "Common Lyapunov functions for families of commuting nonlinear systems," *Syst. Control Lett.*, vol. 54, pp. 405–416, May 2005.

[5] X. Zhao, X. Liu, S. Yin, and H. Li, "Improved results on stability of continuous-time switched positive linear systems," *Automatica*, vol. 50, no. 2, pp. 614–621, 2014.

[6] J. Li, X. Yang, and J. Wu, "Adaptive tracking control approach with prespecified accuracy for uncertain nonlinearly parameterized switching systems," *IEEE Access*, vol. 6, no. 3, pp. 3786–3793, Jan. 2018.

[7] L. Long, Z. Wang, and J. Zhao, "Switched adaptive control of switched nonlinearly parameterized systems with unstable subsystems," *Automatica*, vol. 54, no. 4, pp. 217–228, Apr. 2015.

[8] L. Long and J. Zhao, "Adaptive control for a class of high-order switched nonlinearly parameterized systems," *Int. J. Robust Nonlinear Control*, vol. 27, no. 4, pp. 547–565, Mar. 2017.

[9] C. Bria and A. Seuret, "Affine minimal and mode-dependent dwell-time characterization for uncertain switched linear systems," *IEEE Trans. Autom. Control*, vol. 58, no. 5, pp. 1304–1310, May 2013.

[10] L. Jie and J. Zhao, "Adaptive variable structure control for uncertain switched delay systems," *Circuits Syst. Signal Process.*, vol. 29, no. 6, pp. 1089–1102, Apr. 2010.

[11] R. Ma and J. Zhao, "Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings," *Automatica*, vol. 46, no. 11, pp. 1819–1823, 2010.

[12] W. Xiang and J. Xiao, "Stabilization of switched continuous-time systems with all modes unstable via dwell time switching," *Automatica*, vol. 50, no. 3, pp. 940–945, 2014.

[13] J. Zhang, F. Zhu, X. Zhao, and F. Wang, "Robust impulsive reset observers of a class of switched nonlinear systems with unknown inputs," *J. Franklin Inst.*, vol. 354, no. 7, pp. 2924–2943, Feb. 2017.

[14] Y. Mao, H. Zhang, and S. Xu, "The exponential stability and asynchronous stabilization of a class of switched nonlinear system via the T–S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 4, pp. 817–828, Aug. 2014.

[15] C. A. Ibanez, M. S. Suarez-Castanon, and O. O. Gutierrez-Frias, "A switching controller for the stabilization of the damping inverted pendulum cart system," *Int. J. Innov. Comput., Inf., Control*, vol. 9, no. 9, pp. 3585–3597, Sep. 2013.

[16] X. Huo, L. Ma, X. Zhao, B. Niu, and G. Zong, "Observer-based adaptive fuzzy tracking control of MIMO switched nonlinear systems preceded by unknown backlash-like hysteresis," *Inf. Sci.*, vol. 490, pp. 369–386, Jul. 2019.

[17] L. Ma, X. Huo, X. Zhao, B. Niu, and G. Zong, "Adaptive neural control for switched nonlinear systems with unknown backlash-like hysteresis and output dead-zone," *Neurocomputing*, vol. 357, pp. 203–214, Sep. 2019.

[18] Y. Yin, X. Zhao, and X. Zheng, "New stability and stabilization conditions of switched systems with mode-dependent average dwell time," *Circuits, Syst. Signal Process.*, vol. 36, no. 1, pp. 82–98, 2017.

[19] X. Huang, W. Lin, and B. Yang, "Global finite-time stabilization of a class of uncertain nonlinear systems," *Automatica*, vol. 41, no. 5, pp. 881–888, May 2005.

[20] S. Li, H. Du, and X. Lin, "Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics," *Automatica*, vol. 47, no. 8, pp. 1706–1712, Aug. 2011.

[21] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, Jan. 2000.

[22] X. Zhang, G. Feng, and Y. Sun, "Finite-time stabilization by state feedback control for a class of time-varying nonlinear systems," *Automatica*, vol. 48, no. 3, pp. 499–504, Mar. 2012.

[23] J. Li and C. Qian, "Global finite-time stabilization by dynamic output feedback for a class of continuous nonlinear systems," *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 879–884, May 2006.

[24] S. Huang and Z. Xiang, "Adaptive finite-time stabilization of a class of switched nonlinear systems using neural networks," *Neurocomputing*, vol. 173, pp. 2055–2061, Jan. 2016.

[25] J. Zhai, Z. Song, and H. R. Karimi, "Global finite-time control for a class of switched nonlinear systems with different powers via output feedback," *Int. J. Syst. Sci.*, vol. 49, no. 13, pp. 1464–5319, Aug. 2018.

[26] H. Ye, Y. Jiang, and J. Zhai, "Adaptive finite-time stabilization for a class of high-order switched nonlinear parameterized systems," in *Proc. 37th Chin. Control Conf.*, Jul. 2018, pp. 1976–1981.

[27] J. Fu, R. C. Ma, and T. Y. Chai, "Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers," *Automatica*, vol. 54, no. 4, pp. 360–373, Apr. 2015.

[28] J. Fu, R. Ma, and T. Chai, "Adaptive finite-time stabilization of a class of uncertain nonlinear systems via logic-based switchings," *IEEE Trans. Autom. Control*, vol. 62, no. 11, pp. 5998–6003, Nov. 2017.

[29] W. Chen, C. Wen, and J. Wu, "Global exponential/finite-time stability of nonlinear adaptive switching systems with applications in controlling systems with unknown control direction," *IEEE Trans. Autom. Control*, vol. 63, no. 8, pp. 2738–2744, Aug. 2018.

[30] J. Wu, W. S. Chen, and J. Li, "Global finite-time adaptive stabilization for nonlinear systems with multiple unknown control directions," *Automatica*, vol. 69, no. 1, pp. 298–307, 2016.

[31] J. Wu, J. Li, G. Zong, and W. Chen, "Global finite-time adaptive stabilization of nonlinearly parametrized systems with multiple unknown control directions," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 7, pp. 1405–1414, Jul. 2017.

[32] E. Tian, Z. Wang, L. Zou, and D. Yue, "Chance-constrained H_∞ control for a class of time-varying systems with stochastic nonlinearities: The finite-horizon case," *Automatica*, vol. 107, pp. 296–305, Sep. 2019.

[33] J. Mao, S. Huang, and Z. Xiang, "Adaptive practical finite-time stabilization for switched nonlinear systems in pure-feedback form," *J. Franklin Inst.*, vol. 354, no. 10, pp. 3971–3994, Jul. 2017.
[34] Y. Hong, J. Wang, and D. Cheng, "Adaptive finite-time control of nonlinear systems with parametric uncertainty," *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 858–862, May 2006.
[35] Z.-Y. Sun, L.-R. Xue, and K. Zhang, "A new approach to finite-time adaptive stabilization of high-order uncertain nonlinear system," *Automatica*, vol. 58, no. 8, pp. 60–66, Aug. 2015.
[36] W. Lin and C. Qian, "Adaptive control of nonlinearly parameterized systems: The smooth feedback case," *IEEE Trans. Autom. Control*, vol. 47, no. 8, pp. 1249–1266, Aug. 2002.
[37] C. Y. Wang and X. H. Jiao, "Adaptive control under arbitrary switching for a class of switched nonlinear systems with nonlinear parameterisation," *Int. J. Control*, vol. 88, no. 10, pp. 2044–2054, Apr. 2015.
[38] W. M. Haddad, S. G. Nersesov, and L. Du, "Finite-time stability for time-varying nonlinear dynamical systems," in *Proc. Amer. Control Conf.*, Jun. 2008, pp. 4135–4139.
[39] X. Lin, S. Huang, C. Qian, and S. Li, "Smooth state feedback stabilization for a class of planar switched nonlinear systems under arbitrary switching," in *Proc. 12th World Congr. Intell. Control Automat.*, Jun. 2016, pp. 1454–1458.
[40] W. Chen, S. S. Ge, J. Wu, and M. Gong, "Globally stable adaptive backstepping neural network control for uncertain strict-feedback systems with tracking accuracy known *a priori*," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 9, pp. 1842–1854, Sep. 2015.
[41] M. Cai, Z. Xiang, and J. Guo, "Adaptive finite-time control for a class of switched nonlinear systems using multiple Lyapunov functions," *Int. J. Syst. Sci.*, vol. 48, no. 2, pp. 324–336, 2017.
[42] L. J. Long and J. Zhao, "Adaptive fuzzy tracking control of switched uncertain nonlinear systems with unstable subsystems," *Fuzzy Sets Syst.*, vol. 273, no. 15, pp. 49–67, Aug. 2015.
[43] L. Long and J. Zhao, "Global stabilization for a class of switched nonlinear feedforward systems," *Syst. Control Lett.*, vol. 60, no. 9, pp. 734–738, Sep. 2011.
[44] E. Tian, Z. Wang, L. Zou, and D. Yue, "Probabilistic-constrained filtering for a class of nonlinear systems with improved static event-triggered communication," *Int. J. Robust Nonlinear Control*, vol. 29, no. 5, pp. 1484–1498, 2019.

JIAN WU received the Ph.D. degree in applied mathematics from Xidian University, Xi'an, China, in 2015. He is currently an Associate Professor with the School of Computer and Information, Anqing Normal University. His current research interests include intelligent control, adaptive control, and adaptive switching control.

XIAOLI YANG is currently pursuing the master's degree with the School of Mathematics and Statistics, Xidian University, Xi'an, China. Her research interests include adaptive backstepping control and switching control.

CHUNSHENG ZHANG received the master's degree in software engineering from Yunnan University, Kunming, China, in 2005. He is currently an Associate Professor with the School of Compute and Information, Anqing Normal University. His current research interests include network and information security, and multi-party confidential calculations.

JING LI received the B.S. degree in mathematics from Henan University, Kaifeng, China, in 2001, and the M.Sc. degree in operational research and cybernetics and the Ph.D. degree in applied mathematics from Xidian University, Xi'an, China, in 2004 and 2010, respectively.

From September 2009 to July 2010, she was a Visiting Scholar with the School of Control Science and Engineering, Shandong University, Jinan, China. She is currently an Associate Professor with the School of Mathematics and Statistics, Xidian University. From October 2011 to October 2012, she was an Academic Visitor in robotics with Plymouth University, Plymouth, U.K., and an Academic Visitor in human robot interaction with Imperial College London, London, U.K. Her current research interests include adaptive control, neural network control, switching control, robotics, and human–robot interaction.