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LPV/PI Control for Nonlinear Aeroengine System Based on Guardian Maps Theory

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ABSTRACT The requirement of fighter's maneuverability puts forward higher indexes for the control performance of aeroengine. It is generally recognized that the proportion integration (PI) control is the most commonly used control method of aeroengine. However, due to the high nonlinearity and performance requirements of aeroengine, the selection of PI parameters needs more accurate methods to satisfy higher performance requirements. In this paper, the linear parameter varying (LPV) model is established according to a nonlinear mathematical model of a turbofan engine. Then, the LPV/PI control based on guardian maps theory is proposed to solve the highly nonlinear aeroengine control problem which improves the response characteristics of aeroengine without changing the control method in the range of the scheduling parameters. In the process of design, a set of controller parameters satisfying the performance requirements is automatically generated by a given initial controller parameter, so designing the controllers at multiple equilibrium points is avoided. Finally, the simulations are performed by the integral separation of PI control at different points in the flight envelope for the nonlinear model. The results illustrate that the control method based on guardian maps theory can contribute significantly to solving the nonlinear problems of aeroengine control system.

INDEX TERMS Aeroengine, guardian maps theory, linear parameter varying model, proportion integration control.

I. INTRODUCTION

The aeroengine is a complex system with highly nonlinearity, and the working state varies with the external changes when working in a wide range of flight envelope lines [1]. Therefore, in the process of aeroengine technology investigation, it is necessary to design an effective control system to ensure that it can work steadily and efficiently when external and internal conditions change. According to the characteristics of aeroengine in operation, the general linear system control method is not ideal for aeroengine control. Meanwhile, the control method of non-linear system is hardly to be used well in aeroengine control system. Aeroengine control aims at guaranteeing the desirable performance to be obtained. The design objectives include performance requirements (stability, transient performance and steady-state performance, etc.), reliability requirements, weight requirements and maintainability requirements. The existing control

methods for aeroengine have gain scheduling control [2], proportion integration differentiation (PID) control [3], linear quadratic Gaussian with loop transfer recovery (LQG/LTR) control [4], adaptive control [5], etc.

Generally, gain scheduling control is divided into two classes: traditional variable gain control and linear parameter varying (LPV) control. For traditional variable gain control, it is an idea which is widely used in the control of non-linear systems. Variable gain control method was put forward earlier, and it has a certain relationship with adaptive control theory. Its application in engineering is also increasing. However, the traditional variable gain control method has many defects. Firstly, the traditional variable gain control method is highly dependent on the number of selected equilibrium points. In order to make the linear model family composed of each state variable model better reflect the characteristics of the original non-linear system, the more equilibrium points selected, the better. A large number of equilibrium points have to be selected to enhance the control accuracy which leads to a large amount of computation for the design

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process of this controllers [6]. So the more equilibrium points, the more complicated the solution of the controller, and the higher the requirement of hardware configuration, the more difficult it is to be applied in aeroengine control system. Secondly, the traditional variable gain control requires that the parameters change slowly, while the modern aeroengine has a wide range of altitude and speed, and the parameters change rapidly. At the same time, the theoretical support is insufficient, the global performance of the overall system cannot be guaranteed theoretically. Therefore, the traditional variable gain controller has been difficult in application for aeroengine control system, which needs further exploration [7]. On the other hand, the idea of LPV control is to design a gain scheduling controller through the established LPV model transformed from the original nonlinear system. The LPV system is a class of linear time-varying system. Although the time-varying parameters have certain uncertainties, it can be measured in the process of system operation. There are many control methods which are implemented in aeroengine applications. However, PID control is the most widely used as it has the characteristics of simple principle, high reliability and easy implementation [8]. So the problem about how to improve the response characteristics of the aeroengine without changing the control method is worth studying.

Guardian maps theory is proposed by Saydy to analyze the generalized stability of parameterized matrix and polynomials families [9]. By using Guardian maps theory, the closed-loop poles are constrained in the target region, and the corresponding controller parameters can be obtained. The controller parameter adjustment method based on guardian maps theory does not need to design controllers at multiple design points, thus it can avoid the shortcomings of traditional gain scheduling control method. Guardian maps theory is first used in designing aircraft control system and the target stability region is determined to express the flight control quality index in [10]. A new control method is proposed by combining guardian maps theory, LQR technology and genetic algorithm (see [11] and the references therein). In addition, guardian maps theory is applied to the control system of hypersonic vehicles to maintain the system stability when operating in a wide range of envelopes [12].

Motivated by these considerations, this paper addresses an issue of LPV/PI control based on guardian maps theory for nonlinear aero-engine system. It overcomes the difficulty of guaranteeing global control performance compared to traditional variable gain control. The main contributions of this paper are in three aspects: Firstly, according to a nonlinear model of a turbofan engine, the state variable models of different design points in the range of scheduling parameters are obtained. Then the corresponding LPV model is established by Jacobian linear modeling method [13]. Secondly, a detailed PI control algorithm based on the guardian maps theory is proposed, and PI controllers with different scheduling parameters can be obtained offline by the algorithm which avoid the real-time problem compared with traditional

adaptive algorithms. Finally, simulations which contain verification of robustness, consideration of saturation and addition of interference are performed on the nonlinear model of the certain turbofan engine by integral separation LPV/PI control method to verify the above theoretical results [14].

This paper is organized as follows. Section II provides the modeling process of aeroengine, section III explains guardian maps theory, parameter tuning algorithm is proposed in section IV. Finally, controller design and the simulations are presented in section V. In section VI, we draw the conclusions briefly.

II. AEROENGINE MODEL

Modeling methods of aeroengine generally include experimental and analytical methods. The principle of the experimental method is to get the operation characteristics of aeroengine by processing the data obtained from the experience, and finally get the mathematical model of the engine. The mathematical model established by the test method, because the model does not have physical characteristics, when the characteristics of any component change, all the coefficients of the model need to be adjusted, it is only suitable for existing aeroengines. In the process of analytic modeling, considering the changing rules of aeroengine characteristics under various operation conditions, when the characteristics of a certain part of the engine need to be changed, only the equation of the part is needed to be adjusted, and the analytic method can be used to predict the operation, which has strong versatility.

The non-linear model of a certain turbofan engine investigated in this paper is a component-level non-linear model established by analytical method. In the process of modeling, firstly, the aerothermodynamic equations of each component are established, that is, the operation process of each component is described by mathematical expression according to physical mechanism, then the operation equation of each component is established according to the principles of flow continuity, rotor dynamics and power balance. Finally, the variation of parameters is solved by numerical iteration method of non-linear equation. The component level non-linear model is programmed by C++ programming language. The model has the advantages of clear structure, easy to understand and easy to transplant. It can calculate different operation processes of aeroengine and meet the requirements of precision and real-time for aeroengine modeling.

A. STATE VARIABLE MODEL

The non-linear mathematical model of a turbofan engine under certain flight conditions is as follows:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= \mathbf{g}(\mathbf{x}, \mathbf{u})\end{aligned}\quad (1)$$

where state vector $\mathbf{x} \in \mathbf{R}^n$, control vector $\mathbf{u} \in \mathbf{R}^p$, output vector $\mathbf{y} \in \mathbf{R}^m$. When the aeroengine operates at a

steady-state point (x_0, u_0, y_0) , there is

$$\begin{aligned} \dot{x}_0 &= f(x_0, u_0) = 0 \\ y_0 &= g(x_0, u_0) \end{aligned} \quad (2)$$

At this steady-state point, Taylor series expansion is carried out, and the higher-order infinitesimal terms in the formula are removed, we have

$$\begin{cases} \dot{x} = \frac{\partial \dot{x}}{\partial x} \Big|_{x=x_0, u=u_0} (x - x_0) + \frac{\partial \dot{x}}{\partial u} \Big|_{x=x_0, u=u_0} (u - u_0) \\ y = \frac{\partial y}{\partial x} \Big|_{x=x_0, u=u_0} (x - x_0) + \frac{\partial y}{\partial u} \Big|_{x=x_0, u=u_0} (u - u_0) \end{cases} \quad (3)$$

Then the state variable model can be expressed as

$$\begin{cases} \Delta \dot{x} = A \Delta x + B \Delta u \\ \Delta y = C \Delta x + D \Delta u \end{cases} \quad (4)$$

where $\Delta \dot{x} = \dot{x} - \dot{x}_0$, $\Delta x = x - x_0$, $\Delta u = u - u_0$, $\Delta y = y - y_0$, matrix vector $A = \frac{\partial \dot{x}}{\partial x} \Big|_{x=x_0, u=u_0}$, $B = \frac{\partial \dot{x}}{\partial u} \Big|_{x=x_0, u=u_0}$, $C = \frac{\partial y}{\partial x} \Big|_{x=x_0, u=u_0}$, $D = \frac{\partial y}{\partial u} \Big|_{x=x_0, u=u_0}$.

Aeroengine control system is a multivariable control system. In this paper, the main control channel is taken as an example to establish the linear model of the engine and design the control system. Therefore, the state variables in the aeroengine state variable model established in this section are low-pressure rotor speed N_l and high-pressure rotor speed N_h . The control variable is the fuel supply of the main combustion chamber W_f and the output is the high-pressure rotor speed N_h .

When establishing the aeroengine state variable model, if the actual value of each physical quantity is used directly, not only the amount of data calculation and storage will be increased, but also magnitude of elements in coefficient matrices A , B , C and D will vary dramatically because of the great difference of magnitude for each physical quantity, which makes the coefficient matrices malformed and affect the accuracy of the model. Therefore, before establishing the state variable model, the values of each physical quantity are normalized by similarity, and then the state variable model is established by using the parameters after similarity normalization. The expression of similarity normalization for each parameter is as follows:

$$\begin{aligned} n_l &= \left(\frac{N_l}{\sqrt{T}} \right) / \left(\frac{N_l}{\sqrt{T}} \right)_d \\ n_h &= \left(\frac{N_h}{\sqrt{T}} \right) / \left(\frac{N_h}{\sqrt{T}} \right)_d \\ PW_f &= \left(\frac{W_f}{P\sqrt{T}} \right) / \left(\frac{W_f}{P_2\sqrt{T}} \right)_d \end{aligned} \quad (5)$$

where n_l , n_h , PW_f stand for the normalized parameters of N_l , N_h and W_f , respectively. T and P represent the total temperature and pressure of the fan inlet respectively, and the subscript d represents the corresponding parameters of

aeroengine at the design point. Finally, the state variable model of the engine is expressed as

$$\begin{aligned} \begin{bmatrix} \Delta \dot{n}_l \\ \Delta \dot{n}_h \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta n_l \\ \Delta n_h \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \Delta PW_f \\ \Delta n_h &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta n_l \\ \Delta n_h \end{bmatrix} \end{aligned} \quad (6)$$

There are many methods to solve the coefficient matrix of aeroengine state variable model, and the small perturbation method (partial derivative method) is the most widely used. Its basic principle is to give a small perturbation at a steady point of an aeroengine based on the non-linear model, and then calculate dynamically until the model reaches a stable state. The variation of output and derivatives of state variables can be obtained by calculation, the ratio between them and the given small perturbation value is calculated, then the coefficient matrix values in the state variable model are obtained. Although the small perturbation method is easy to implement in the solution of coefficient matrix, the accuracy is not ideal.

The least square fitting method is also used to solve the coefficient matrix of the state variable model. Its idea is to construct the analytical expression between the coefficient matrix of the state variable model and its dynamic response, and then to fit the dynamic response data obtained after giving a small perturbation to the non-linear model, so as to obtain the value of the coefficient matrix and make the modeling error smallest under the least square meaning. The precision of coefficient matrix obtained by least square fitting method is greatly improved, but it is not suitable for high-order object modeling.

The solution of coefficient matrices used in this section is a combination of small perturbation method and least square fitting method. The specific steps are as follows:

- 1) The initial value of coefficient matrix A is obtained by small perturbation method. For example, if a small disturbance is applied to the low-pressure rotor speed of the engine at a certain time and the values of other physical quantities remain unchanged, then $a_{11} = \Delta \dot{n}_l / \Delta n_l$, $a_{21} = \Delta \dot{n}_h / \Delta n_l$. Similarly, a_{12} and a_{22} can be obtained;
- 2) The dynamic sequential response of non-linear model $\{\Delta x(t), \Delta y(t)\}$ is obtained by stepping the control value PW_f .
- 3) The initial value of coefficient matrix B is obtained by calculating the final steady-state value of the dynamic response in above. The formula is as follows:

$$\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta n_l \\ \Delta n_h \end{bmatrix} \Delta PW_f^{-1} \quad (7)$$

- 4) The initial state variable model is established by using the values of A and B above. And its dynamic response sequence is obtained by stepping the control value PW_f , which is the same as that in step 2.
- 5) The objective function (8) is optimized by least square method, and the optimal solution of A is obtained. Then the final solution of B is calculated by formula (7)

according to the optimal solution of A . The state variable model can be obtained by the values of coefficient matrices A and B .

$$\min J = \sum [\Delta y(t) - \Delta \bar{y}(t)]^T [\Delta y(t) - \Delta \bar{y}(t)] \quad (8)$$

Thus the state variable model is established at the design point $H = 3.5\text{km}$, $Ma = 0.5$ according to the above method. For example, at the operating point $H = 3.5\text{km}$, $Ma = 0.5$, $n_h = 0.8563$, the state variable model is:

$$\begin{aligned} \begin{bmatrix} \Delta \dot{n}_1 \\ \Delta \dot{n}_h \end{bmatrix} &= \begin{bmatrix} -4.3104 & 0.3324 \\ 1.4568 & -3.0464 \end{bmatrix} \begin{bmatrix} \Delta n_1 \\ \Delta n_h \end{bmatrix} \\ &+ \begin{bmatrix} 1.7254 \\ 0.3080 \end{bmatrix} \Delta PW_f \\ \Delta n_h &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta n_1 \\ \Delta n_h \end{bmatrix} \end{aligned} \quad (9)$$

In order to verify the precision of the established state variable model, 1% step simulation of the control value PW_f of the non-linear model and the state variable model of component-level aeroengine is carried out respectively. The simulation results are shown in Figure 1. It can be seen that the step response of the state variable model can track the step response of the non-linear model well, the precision of the established state variable model is good.

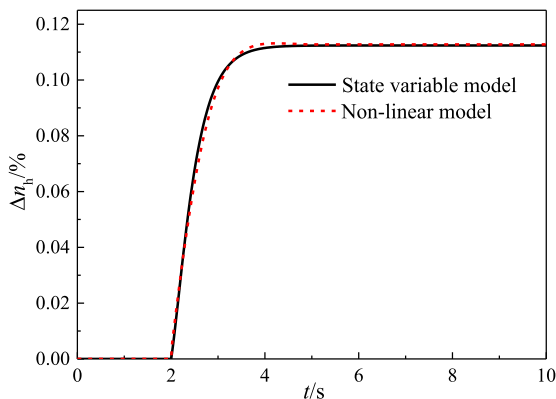


FIGURE 1. Simulation with $H = 3.5\text{km}$, $Ma = 0.5$, $n_h = 0.8563$.

B. LPV MODEL

We consider a nonlinear mathematical model for a certain military turbofan engine under certain flight conditions

$$\begin{aligned} \dot{x} &= f(x, u, \rho) \\ y &= g(x, u, \rho) \end{aligned} \quad (10)$$

$f(\cdot)$ and $g(\cdot)$ are continuously differentiable nonlinearity functions, and the corresponding LPV model is

$$\begin{aligned} \dot{x} &= A(\rho)x + B(\rho)u \\ y &= C(\rho)x + D(\rho)u \end{aligned} \quad (11)$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^p$ and $y \in \mathbf{R}^m$ represent the measurable state, control input and output vectors respectively.

$\rho \in \mathbf{R}^j$ is scheduling parameter vector which is time-varying. The coefficient matrix $A(\rho)$, $B(\rho)$, $C(\rho)$ and $D(\rho)$ are all matrix functions of ρ with appropriate dimensions [15].

This article considers a certain turbofan engine with a dual rotor. The parameters of the engine are similarly normalized to avoid the reduction of the modeling accuracy caused by the large magnitude differences between various physical quantities. Aero-engine is a multi-variable system which has many channels in its control system. Due to the limitation of space, only the oil supply increment of the main combustion chamber ΔW_f is selected as the input, high pressure rotor speed increment Δn_h as the output, low pressure rotor speed Δn_1 and high pressure rotor speed increment Δn_h as the states of the engine.

The main acceleration and deceleration simulation of aero-engine mainly includes idle rating to the intermediate state of acceleration and deceleration, 85% speed to the intermediate state of acceleration and deceleration, intermediate state to idle rating and intermediate state of acceleration [16]. In this paper, the state of the engine considered is 85% speed to the intermediate state of acceleration and deceleration.

The LPV model is established through Jacobian linear modeling approach. Firstly, nominal points $H = 3.5\text{km}$, $Ma = 0.5$ is selected as the design point where H stands for height and Ma represents Mach number. Simultaneously, high pressure rotor speed n_h ($n_h \in [0.8500, 1.0500]$) is set as the scheduling parameters of LPV model. Moreover, linearization technique [17] is used to obtain a state variable model of different rotating speed points of high pressure rotor at different design points. Then n_h is normalized to the range of $[0, 1]$ to improve the accuracy of the model, i.e., $\tilde{n}_h \in [0, 1]$. Finally, polynomial fitting is performed on all coefficient matrices of the state variable model. After fitting, the model is

$$\begin{aligned} \dot{x} &= \begin{bmatrix} a_{11}(\tilde{n}_h) & a_{12}(\tilde{n}_h) \\ a_{21}(\tilde{n}_h) & a_{22}(\tilde{n}_h) \end{bmatrix} x + \begin{bmatrix} b_{11}(\tilde{n}_h) \\ b_{21}(\tilde{n}_h) \end{bmatrix} u \\ y &= [0 \quad 1] x \end{aligned} \quad (12)$$

where $x = [\Delta n_1 \quad \Delta n_h]^T$, $u = \Delta W_f$, $y = \Delta n_h$. By comparing the simulation results, this paper discovers that the 3rd-order polynomial fitting model satisfies the accuracy and efficiency of the fitting. The corresponding mathematical expression is

$$\begin{cases} a_{11}(\tilde{n}_h) = -16.8153\tilde{n}_h^3 + 17.7903\tilde{n}_h^2 - 3.0473\tilde{n}_h - 3.4468 \\ a_{12}(\tilde{n}_h) = -7.6978\tilde{n}_h^3 + 7.7052\tilde{n}_h^2 - 0.3495\tilde{n}_h + 1.2398 \\ a_{21}(\tilde{n}_h) = 4.2784\tilde{n}_h^3 + 0.0964\tilde{n}_h^2 - 3.7480\tilde{n}_h + 0.8222 \\ a_{22}(\tilde{n}_h) = -4.3705\tilde{n}_h^3 + 0.4025\tilde{n}_h^2 + 2.6493\tilde{n}_h - 2.5416 \\ b_{11}(\tilde{n}_h) = -1.2335\tilde{n}_h^3 + 4.6068\tilde{n}_h^2 - 3.4020\tilde{n}_h + 1.1687 \\ b_{21}(\tilde{n}_h) = 0.0239\tilde{n}_h^3 - 0.3288\tilde{n}_h^2 + 0.1294\tilde{n}_h + 0.4840 \end{cases} \quad (13)$$

The fitting curves are shown in Figure 2 and Figure 3. We find there is a certain error in the fitted lines. However, due to the strong robustness of the PI control, subsequent simulations show that it has little impact on the control effect.

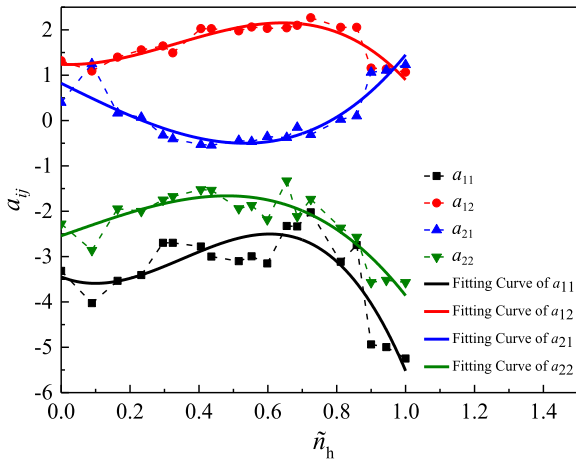


FIGURE 2. Fitting curves of a_{ij} .

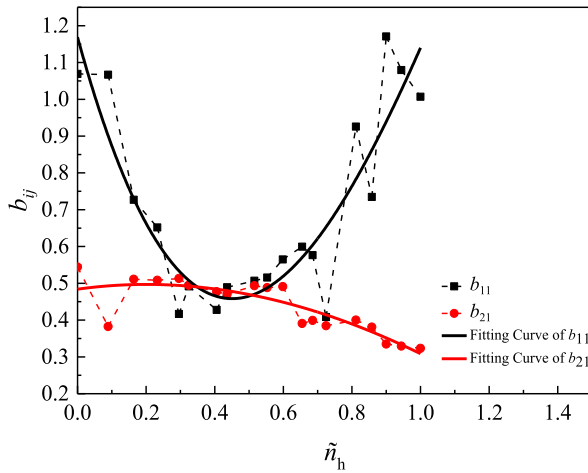


FIGURE 3. Fitting curves of b_{ij} .

This article selects two arbitrarily speed points ($n_h = 0.9114$ and $n_h = 0.9603$) to verify the accuracy of the LPV model. Figure 4 and Figure 5 show high pressure speed response curves of LPV model and nonlinear model under the same step input for $n_h = 0.9114$ and $n_h = 0.9603$ respectively. The curves show that LPV model has a good tracking effect. Although there is a certain steady-state error, the error is negligible, and can be eliminated by PI control described later.

III. GUARDIAN MAPS THEORY

This section introduces the guardian maps theory to provide a theoretical basis for subsequent control design. guardian maps are scalar valued maps defined on the set of $n \times n$ real matrices (or polynomials) that take non-zero values on the set of “stable” matrices (or polynomials) and vanish on its boundary. Matrix generalized stability set is defined by

$$S(\Omega) = \{M \in \mathbf{R}^{n \times n} : \Lambda(M) \subset \Omega\} \quad (14)$$

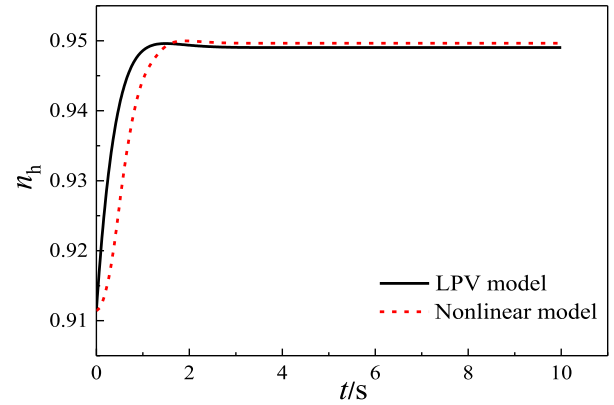


FIGURE 4. High pressure speed response curves when $n_h = 0.9114$.

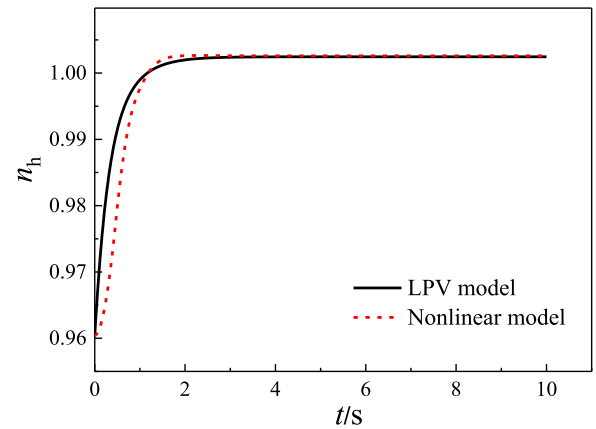


FIGURE 5. High pressure speed response curves when $n_h = 0.9603$.

where Ω represents an open subset of the complex plane of interest, $\Lambda(M)$ represents the set of the eigenvalues of M [18]. $S(\Omega)$ is the set of all matrices which are stable with respect to the matrix M in the region of Ω .

Definition 1: Let v map $\mathbf{R}^{n \times n}$ into complex domain \mathbf{C} , if sufficient and necessary condition of $M \in \partial S(\Omega)$ is that $v(M) = 0$, then mapping v is a guardian map of $S(\Omega)$, and it is a scalar mapping on real matrices [18].

Some guardian maps are given for typical regions (Figure 6). The guardian map of Figure 6(a) ($\text{Re}(z) < \alpha$) is guarded by

$$v_\alpha(M) = \det(M \odot I - \alpha I \odot I) \det(M - \alpha I) \quad (15)$$

where \odot represents the Bialternate product. The guardian map of Figure 6(b) (conic sector with inner angle 2θ) is given by

$$v_\xi(M) = \det[M^2 \odot I + (1 - 2\xi^2)M \odot M] \det(M) \quad (16)$$

where $\xi = \cos \theta$.

The guardian map of Figure 6(c) (radius ω) is given by

$$v_\omega(M) = \det(M \odot M - \omega^2 I \odot I) \det(M - \omega I) \det(M + \omega I) \quad (17)$$

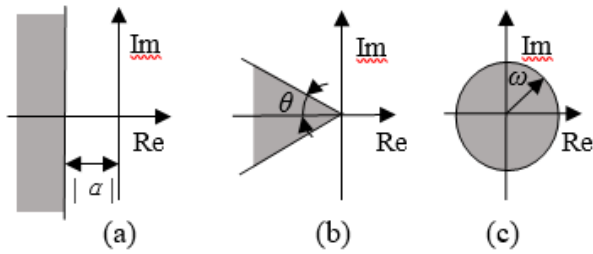


FIGURE 6. Typical regions for guardian maps.

The guardian maps for other regions can be obtained by the above typical guardian maps. For example, assume the guardian maps expressions of $S(\Omega_1), S(\Omega_2), \dots, S(\Omega_n)$ are $\nu_1, \nu_2, \dots, \nu_n$, thus the guardian maps expression of $S(\Omega_1 \cap \Omega_2 \cdots \cap \Omega_n)$ are $\nu = \nu_1 \nu_2 \cdots \nu_n$.

Lemma 1: Let $M(x) = M_0 + xM_1 + \dots + x^k M_k$ be a polynomial matrix, where x is an uncertain parameter, M_i is a given constant matrix for $i = 1, \dots, k$. Assume that $M(x_0)$ is stable with respect to Ω and let ν_Ω be a guardian map of $S(\Omega)$. Then $M(x)$ is stable with respect to Ω for $x \in (x^-, x^+)$ obtained by (18) and (19) [19].

$$x^- \approx \sup\{x < x_0 : \nu_\Omega[M(x)] = 0\} \quad (\text{or } -\infty \text{ if none exists}) \quad (18)$$

$$x^+ \approx \inf\{x > x_0 : \nu_\Omega[M(x)] = 0\} \quad (\text{or } +\infty \text{ if none exists}) \quad (19)$$

According to Lemma 1, suitable control parameters can be achieved to maintain the closed-loop system stable in respect to the target region Ω .

IV. PARAMETER TUNING ALGORITHM

We describe the gain preposition algorithms based on guardian maps. The algorithms are used to design global controller for single parameter LPV model.

A. DEFINITIONS

Let Ω be a set of stability region which illustrated by Figure 7(a). Define:

$$\Omega \triangleq \Omega(\alpha, \xi, \omega) = \{\lambda \in \mathbf{C} : \text{Re}(\lambda) \leq \alpha, \xi(\lambda) \geq \xi, |\lambda| \leq \omega\} \quad (20)$$

where $\xi(\lambda)$ represents the damping ratio of the complex number λ .

Similarly, we define Ψ as a set of unstable region which is illustrated by Figure 7(b):

$$\Psi \triangleq \Psi(\alpha, \omega) = \{\lambda \in \mathbf{C} : \text{Re}(\lambda) \leq \alpha, |\lambda| \leq \omega\} \quad (21)$$

Definition 2: Assume $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ is a set of all eigenvalues of Hurwitz stable matrix M . Define $\Omega_\Lambda \triangleq \Omega(\alpha_\Lambda, \xi_\Lambda, \omega_\Lambda)$ which is the smallest region containing all the eigenvalues of the matrix M , where $\alpha_\Lambda = \max\{\text{Re}(\lambda_i)\}$, $\xi_\Lambda = \min\{\xi(\lambda_i)\}$ and $\omega_\Lambda = \max\{|\lambda_i|\}$. Assume $\Omega_t = \Omega(\alpha_t, \xi_t, \omega_t)$ is the target region defined by (20) then define

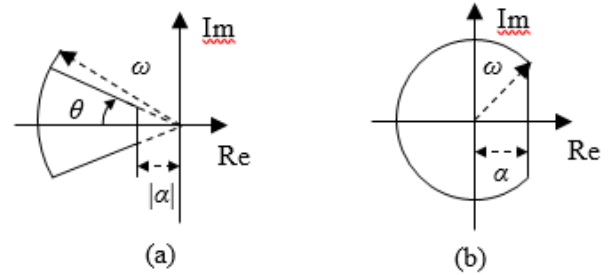


FIGURE 7. Stability and unstable region.

$\Omega_u = \Omega(\alpha_u, \xi_u, \omega_u)$ where $\alpha_u = \max\{\alpha_t, \alpha_\Lambda\}$, $\xi_u = \min\{\xi_t, \xi_\Lambda\}$ and $\omega_u = \max\{\omega_t, \omega_\Lambda\}$ [20].

Definition 3: Assume $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ is a set of all eigenvalues of Hurwitz stable matrix M . We define $\Psi_\Lambda \triangleq \Psi(\alpha_\Lambda, \omega_\Lambda)$ which is the smallest region containing all the eigenvalues of the matrix M . Assume $\Omega_t = \Omega(\alpha_t, \xi_t, \omega_t)$ is the target region defined by (12), then define $\Psi_u = \Psi(\alpha_u, \omega_u)$ where $\alpha_u = \alpha_\Lambda$ and $\omega_u = \max\{\omega_t, \omega_\Lambda\}$ [20].

The dynamic performance of the system mainly depends on the position of the system poles, which not only determines the stability of the system, but also determines other dynamic performance of the system. When the pole placement of the system is as Figure 7(a) (resp. Figure 7(b)), the system is stable (resp. unstable). According to the requirements of aeroengine control target, the detailed design indexes of target region are determined to ensure that the poles are allocated in stability region. We design gain preset algorithm to guarantee the closed-loop poles on desired place by tuning the gain vector in target region. Thus the desired performance is guaranteed by controller design based on the algorithm.

B. ALGORITHM

The flow chart of gain preset algorithm of target region based on guardian maps theory is presented in Figure 8 [21]. The controller gain vector K can be calculated according to the arbitrarily initial controller gain, and the closed-loop poles of the control system are located in the target region $\Omega_t = \Omega(\alpha_t, \xi_t, \omega_t)$ to achieve the desired performance by the algorithm.

The algorithm consists of the following steps:

- 1) Initialization. Set a fixed control architecture with gain vector $K = [K_j]$ for $j = 1, \dots, m$ with a target region $\Omega_t = \Omega(\alpha_t, \xi_t, \omega_t)$. Let $K^0 = [K_j^0]$ be an initial gain vector, and set the counter $q = r = 0$, thus $K^q = K^r = K^0$.
- 2) Compute the set of eigenvalues of state matrix $A_{cl}(K^q)$ of closed-loop system i.e. $\Lambda^q = \{\lambda_1, \lambda_2, \dots\}$. If $\Lambda^q \subset \Omega_t$, then jump the loop iterations.
- 3) Building new stable region according to the stability of $A_{cl}(K^q)$. If $A_{cl}(K^q)$ is stable, then construct the new region $\Omega_q = \Psi_u = \Psi(\alpha_q, \xi_q, \omega_q)$ by **Definition 2**; else $\Omega_q = \Psi_u = \Psi(\alpha_q, \omega_q)$ calculated by **Definition 3**.
- 4) The following cyclic iteration process is performed based on new region Ω_q and K^q :

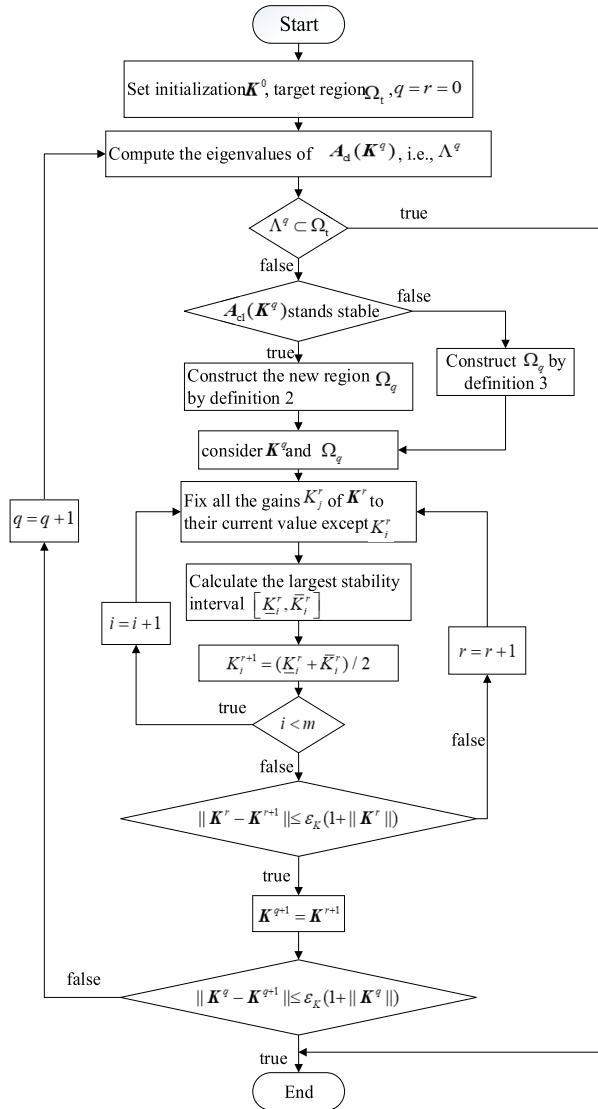


FIGURE 8. Flow chart of gain preset algorithm.

- 5) Let K_i^r be a unique variable parameter of K^r , then $\nu_{\Omega_{ij}}(K^r)$ is merely related to K_i^r .
- 6) The largest stability interval $[K_i^r, \bar{K}_i^r]$, which contains K_i^r , can be obtained by **Lemma 1**.
- 7) Let $K_i^{r+1} = (K_i^r + \bar{K}_i^r)/2$, $i = 1, 2, \dots, m$. If $\|K^r - K^{r+1}\| \leq \epsilon_K(1 + \|K^r\|)$ (ϵ_K is a smaller positive number), stop iteration, and let $K^{q+1} = K^{r+1}$. Else set $r = r + 1$ and continue this iteration.
- 8) If $\|K^q - K^{q+1}\| \leq \epsilon_K(1 + \|K^q\|)$, stop the loop. Else, set $q = q + 1$, and go back to step 2.

C. CONTROL PROCESS OF LPV MODEL

It is critical to design a global controller with respect to scheduling parameters $\rho \in [\rho_{\min}, \rho_{\max}]$ which satisfies the desired performance requirements based on LPV model. The flow chart of global controller design process for single parameter LPV model based on guardian maps theory is shown in Figure 9 [22].

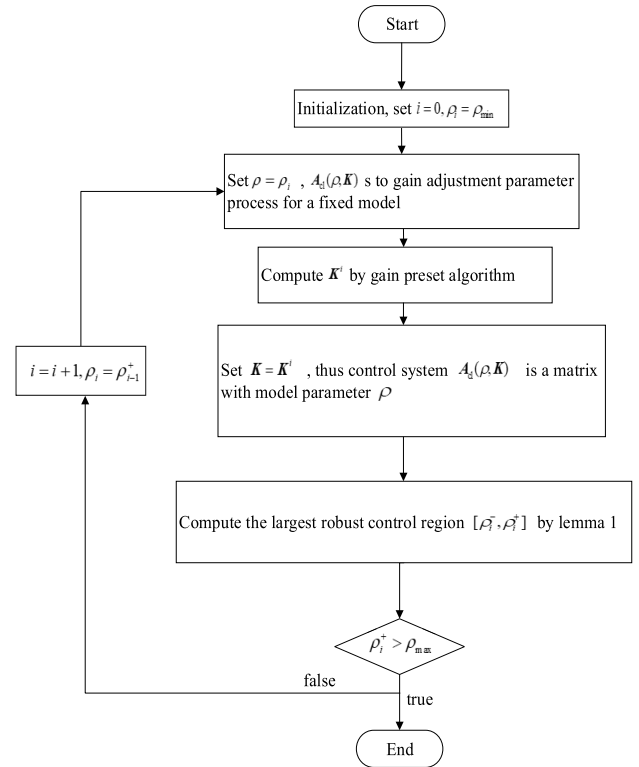


FIGURE 9. Flow chart of LPV/PI control.

The algorithm consists of the following steps:

- 1) Let $A_{cl}(\rho, K)$ be the state matrix of closed-loop system and $K = [K_j](j = 1, 2, \dots, m)$ is gain vector of the system. Set $\rho = \rho_0 = \rho_{\min}$, thus the controlled object turns to a fixed parameter model.
- 2) Compute gain vector K^i which meets the performance requirements by gain preset algorithm.
- 3) Set $K = K^i$, and compute the largest stability interval $[\rho_i^-, \rho_i^+]$ by **Lemma 1**.
- 4) If $\rho_i^+ > \rho_{\max}$, stop the loop iterations. Else set $i = i + 1$, $\rho_i = \rho_{i-1}^+$, and continue the loop.

According to the algorithm, the controller's gain vector $\{K^0, K^2, \dots, K^{n-1}, K^n\}$ corresponding to $\{[\rho_{\min}, \rho_0^+], \dots, [\rho_i^-, \rho_i^+], \dots, [\rho_n^-, \rho_{\max}]\}$ can be obtained respectively. Finally, the parameter values of the controllers are computed in the entire range of the scheduling parameters.

V. DESIGN AND SIMULATION

A. DESIGN PROCESS OF LPV/PI CONTROLLER BASED ON GUARDIAN MAPS THEORY

In engineering practice, PID control is one of the most widely used control methods, it is still widely used in aeroengine control system due to its various advantages. Therefore, PI control is utilized in this paper because it enables the system performance to meet the requirement of none steady-state error, meanwhile, it also has the advantage in dynamic response. The structure of the closed-loop system is shown in Figure 10.

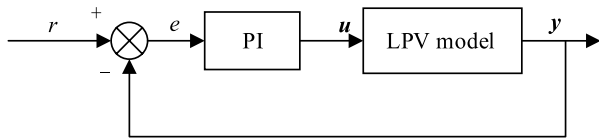


FIGURE 10. Controller structure.

The transfer function of the PI controller is

$$K(s) = K_p + K_i/s \tag{22}$$

This article utilizes the algorithm based on the guardian map theory to design the controller $K(s)$, so that the LPV system can meet the control requirements in the entire range of the scheduling parameters.

The most significant issue in the design process of aeroengine control system is that the designed control system should meet the performance requirements. The design index of the controller takes into account the attenuation coefficient α , damping ratio ξ and natural frequency ω , which can be utilized to characterize the steady-state and dynamic performance of the closed-loop system. To ensure the good performance of the control system, we set the desired performance requirements as $\alpha \leq -1.7$, $\omega \leq 8$ and $\xi \geq 0.85$ respectively according to the target requirements of aeroengine control, and the target area is constructed by (20). The corresponding guardian map of the target region is

$$\nu_{\Omega_1}(A_{cl}) = \nu_{\alpha}(A_{cl})\nu_{\xi}(A_{cl})\nu_{\omega}(A_{cl}) \tag{23}$$

The controller is designed based on LPV aero-engine model at point $H = 3.5\text{km}$, $Ma = 0.5$. Firstly, let $\tilde{n}_h = 0$, set initial value of PI Controller $K_p = K_i=1$, and we get $K_p = 17.81137$, $K_i = 30.84448$ by the algorithm in section B, which guarantee the poles of the closed loop system can be restricted to the target region to achieve the desired performance. Secondly, parameters of the controller can be obtained by Lemma 1. The maximum interval of \tilde{n}_h , which keeps the system stable with respect to Ω_1 is $[-0.0154, 0.1308]$. The poles of the closed loop system are all located in the target area within the variable section of the scheduling parameters. Then select upper range value $\tilde{n}_h = 0.0445$, compute new PI controller parameter values and the corresponding stability intervals by the approach in section IV. Keep operating the cycle calculation until there are controllers corresponding to all the \tilde{n}_h in the $\tilde{n}_h \in [0, 1]$. The iterative results of the controller design are shown in Table 1.

From Table 1 we can see that there is a certain overlap between the calculated stable intervals of each step. In order to guarantee the stability of the system with respect to the stability region and the efficiency of the controller in the whole range of scheduling parameters, we choose to switch the controller at the midpoint of the overlap interval. The switching process is shown in Figure 11.

The poles of the closed-loop system at different rotational speeds are calculated according to the designed controller, and the distribution graph is shown in Figure 12. It shows

TABLE 1. Controller design results.

i	K_p	K_i	$[\tilde{n}_h^-, \tilde{n}_h^+]$
1	17.81137	30.84448	$[-0.0735, 0.0445]$
2	11.18862	20.51906	$[0.0290, 0.1225]$
3	10.72503	20.22287	$[0.0843, 0.3553]$
4	14.9306	44.33968	$[0.2445, 0.9111]$
5	9.475348	20.45374	$[0.8367, 0.9820]$

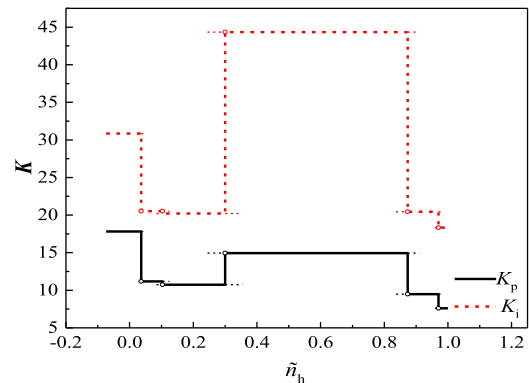


FIGURE 11. Variable regulation of parameter K with respect to \tilde{n}_h .

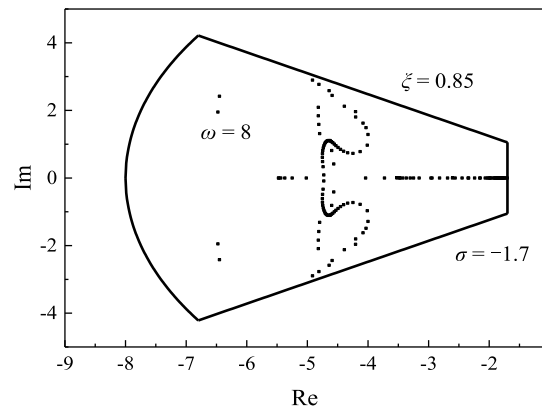


FIGURE 12. Poles of closed-loop system.

that the poles of the closed-loop system are located within the target region so the desired performance requirements are achieved.

B. DESIGN OF TRADITIONAL VARIABLE GAIN PI CONTROLLER BASED ON GENETIC ALGORITHM

Genetic Algorithms (GA) is an optimization algorithm based on the law of biological reproduction. This method draws the principle of biological evolution “survival of the fittest” into the optimization process [23]. In the process of GA, according to the selected fitness function, individuals are selected by genetic operation (replication, crossover and variation). Individuals with high adaptation values will be retained and

new optimization objects will be formed. The new group not only contains the characteristics of the previous generation, but also will be better than the previous generation. Through such a process, the individual's adaptability will continue increasing, and the process stops when a certain condition is reached [24]. GA is simple in calculation. It can be processed in parallel, and can get the global optimal solution with powerful functions. Since Professor Holland proposed Genetic Algorithm in 1962, it has been well developed in depth and breadth. In recent years, the improvement and application of GA are very active in various engineering fields. In this section, GA is used to tune the parameters of PI control. The specific steps of parameter tuning are described below.

1) SELECTION AND REPRESENTATION OF PARAMETERS

Firstly, the range of parameters should be defined, and then the parameters should be coded according to the accuracy requirement. Each parameter is represented by a binary string, and the relationship between the parameters and the binary string needs to be established. The operation object of genetic algorithm is a long binary string formed by connecting each binary string.

2) INITIAL POPULATION SELECTION

Since genetic algorithm has the characteristics of population, it is necessary to construct an initial population for subsequent operations. The initial population is generated randomly by computer. For binary coding, firstly, a series of uniformly distributed numbers is generated randomly between 0-1, then 0 is used to represent the random numbers in the range of 0-0.5, and 1 is used to represent the random numbers in the range of 0.5-1. Population size can be determined by the complexity of calculation.

3) FITNESS FUNCTION

The purpose of the optimization method is to obtain a series of satisfied parameters under the target conditions, and then find the optimal one from all the satisfied individuals. GA does not rely on any external information when it is in progress. It can make sure whether an individual is good or not only by the fitness function. Stability, accuracy and rapidity are usually considered as the measurement of the control system. The performance of the system in terms of rapidity can be reflected by the rising time of dynamic response. The shorter the rising time is, the better the quality of the system is. However, if only the dynamic characteristics of the system are guaranteed, the control signal may grow larger and larger, which makes the system unstable due to the saturation characteristics of the system. Therefore, adding the control value to the objective function can reduce the possibility of excessive control value. The ultimate optimal index is

$$J = \int_0^{\infty} (w_1 |e(t)| + w_2 u^2(t)) dt + w_3 \cdot t_u \quad (24)$$

where $e(t)$, $u(t)$, t_u represent system error, control input and rise time respectively, w_1 , w_2 and w_3 represent the

corresponding weights. If there is overshoot in the system, the overshoot can be added to the optimization index, i.e.

$$J = \int_0^{\infty} (w_1 |e(t)| + w_2 u^2(t) + w_4 |e(t)|) dt + w_3 \cdot t_u \quad (25)$$

where w_4 represents the corresponding weight, and $w_4 \gg w_1$. Due to the connection between the fitness function and the objective function, the objective function can be used as the fitness function for subsequent calculation.

4) SPECIFIC OPERATION PROCESS OF GENETIC ALGORITHM

The first genetic operation is reproduction, and the reproduction process can be described by the magnitude of the reproduction probability. The value of individual reproduction probability is closely linked with whether it can be retained or not. The larger the probability, the more offspring it will have. Otherwise, it will be eliminated. The goal of replication is to select excellent individuals from the existing group so that they may become paternal generations and have their own descendants. Then a single point crossover is carried out. The principle of crossover is that two matched chromosomes exchange some genes in some form. Crossover operation is the unique step of genetic algorithm and the first choice when generating new individuals. It is of great significance in the process of genetic algorithm. P_c represents the crossover probability. The crossover pool is formed by selecting individuals with the probability of P_c from the individuals obtained in the previous step, then the individuals in the crossover pool are matched randomly. The whole crossover process is random. Finally, the principle of mutation is to use different alleles in the same locus to replace the genes on the chromosome locus, thus forming a new individual. The mutation probability is P_m . The mutation bit number that need to be changed can be determined by P_m . Then, according to the mutation bit number, corresponding changes can be made, that is, 1 is changed to 0, or 0 is changed to 1. Variant sites are selected by random digits obtained from random functions. A new population can be obtained by replication, crossover and mutation of the original population and will be put into fitness function after decoding. If the termination criteria are met, the optimization will be stopped, otherwise, the above process will be recycled. The specific termination criteria are depended on the specific circumstances.

The tuning process of GA parameters above can be described by Figure 13.

In this section, the traditional variable gain PI controller design method is used to design the controller at the design point $H = 3.5\text{km}$, $Ma = 0.5$. Firstly, the rotating speed points are selected with equal spacing in the range of dispatching parameters $n_h \in [0.85, 1.05]$ as design points based on the non-linear model of a turbofan engine, and the system is linearized at each point to obtain its state variable model as in section II-A. Then, PI control method is adopted and

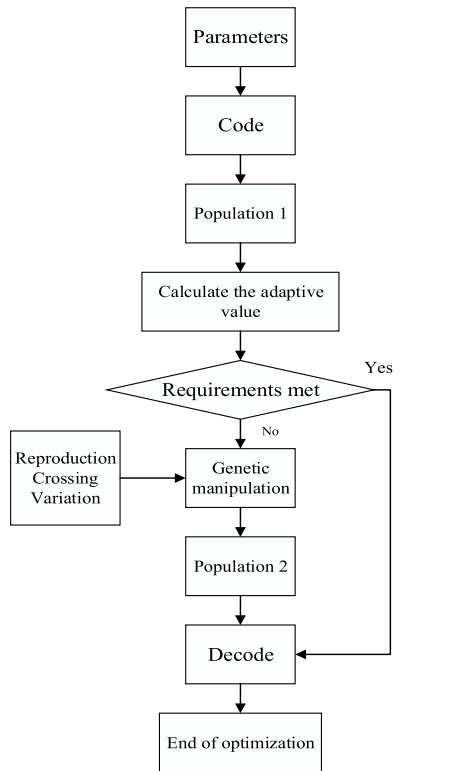


FIGURE 13. Process of genetic algorithm.

PI parameters are tuned by genetic algorithm to obtain local controllers at each design point. Finally, all local controllers are connected by interpolation.

Within the scope of dispatching parameter $n_h \in [0.85, 1.05]$, operating points are selected at 0.04 intervals as design points, and the state variable model is used to establish the state variable model at each point. The controller is designed with the controller structure shown in Figure 14. In the process of tuning PI parameters by genetic algorithm, the number of samples is 30, the crossover probability P_c is 0.7, the mutation probability P_m is 0.03, the weights w_1, w_2, w_3 and w_4 are 0.99, 0.001, 2 and 900 respectively, and the iteration steps are $g = 100$. The variation curve of the optimum index J at the design point is shown in Figure 15.

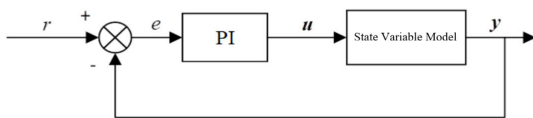


FIGURE 14. Controller structure.

PI parameters with $H = 3.5 \text{ km}$, $Ma = 0.5$ obtained by genetic algorithm are shown in Table 2.

In order to verify the control effect of PI controller designed by traditional variable gain control method in closed-loop linear system, the designed rotor speed point and non-designed rotor speed point are selected for simulation verification. Firstly, the obtained controller is brought into the

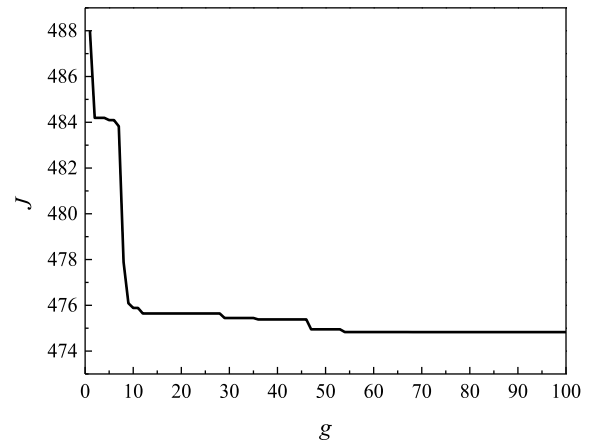


FIGURE 15. Optimum index with $H = 3.5 \text{ km}$, $Ma = 0.5$, $n_h = 0.93$ PI parameters with $H = 3.5 \text{ km}$, $Ma = 0.5$ obtained by genetic algorithm are shown in Table 2.

TABLE 2. Traditional PI parameters with $H = 3.5 \text{ km}$, $Ma = 0.5$.

Rotor speed	K_p	K_i
0.85	17.4487	28.3241
0.89	10.0293	19.5652
0.93	23.5093	43.6773
0.97	7.8006	21.5579
1.01	13.0792	32.6544
1.05	19.138	37.8731

closed-loop control system shown in Figure 14. Then, a unit step signal is given as the input signal. Finally, the control effect of the controller is analyzed according to the output response of the system. The closed-loop linear simulation results can provide reference for the follow-up nonlinear simulation. The closed-loop linear simulation results are shown in Figure 16-17.

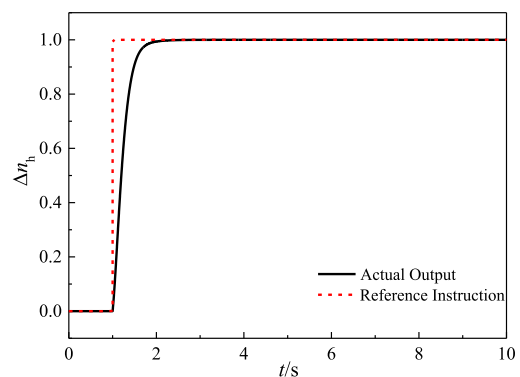


FIGURE 16. Simulation with $H = 3.5 \text{ km}$, $Ma = 0.5$, $n_h = 0.93$.

As can be seen from Figure 16-17, the PI controller designed by the traditional variable gain controller design method has a good control effect at the designed rotor

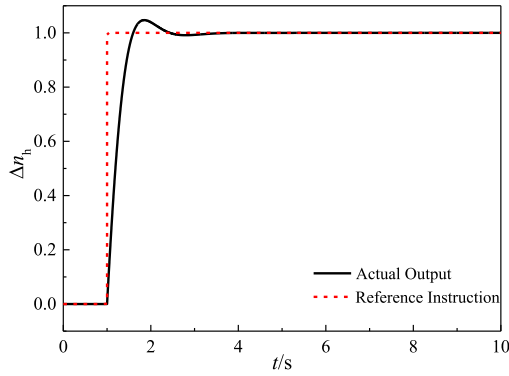


FIGURE 17. Simulation with $H = 3.5\text{km}$, $Ma = 0.5$, $n_h = 0.8738$.

speed point. The output response can track the reference instructions well. The adjustment time is less than 1s, the steady-state error is zero, the PI controller has a good dynamic response. However, there is a large overshoot in the speed response at the non-designed rotor speed point, and the overshoot is more than 5%. Therefore, the controller designed by the traditional variable gain controller design method has a good control effect on the closed-loop linear system at the designed rotor speed point, but the control effect at the non-designed point is not ideal, which is also one of the shortcomings of the traditional variable gain controller.

C. ANALYSIS AND COMPARISON OF SIMULATION

Due to limited space of this paper and in order to guarantee the control effect, at the operating mode $H = 3.5\text{km}$, $Ma = 0.5$, Monte Carlo simulation is utilized to verify the control effect, and the noise signal is added to verify its stability. Simultaneously, we also consider the situation of fuel flow saturation which often occurs in practice.

Monte Carlo method is called randomization method, sometimes it is also called random sampling technique or statistical testing method, which is based on probability and statistics theory [24]. The parameter settings in operating mode are shown in Table 3.

TABLE 3. Parameter settings.

Parameters	Range	Distribution type
$H(\text{km})$	3.0-4.0	Gaussian distribution
Ma	0.4-0.6	Gaussian distribution
$W_f(\text{kg/s})$	0.4-0.46	even distribution
$A(\text{m}^2)$	0.28-0.3	even distribution

Where W_f stands for the fuel flow and A stands for the area of nozzle. Then the iteration is set to be 1000 and the simulation is shown in Figure 18.

From Figure 18, it can be seen that PI control effect based on guardian maps theory has good robustness and stability.

The actual working environment of aeroengine is very bad. There are various kinds of disturbances such as

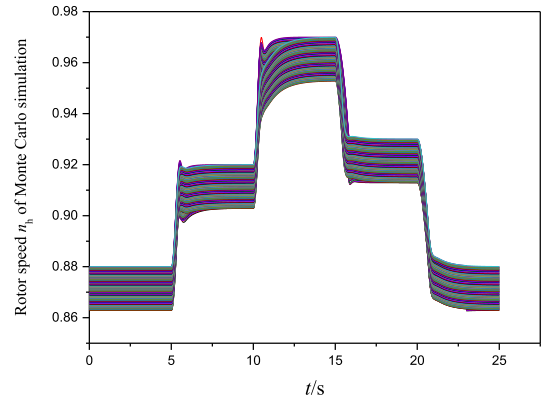


FIGURE 18. The Monte Carlo simulation of high pressure rotor speed.

vibration, electromagnetic, etc. in the working condition, so the interference in the actual working condition is indispensable. So there will be interference signals in actual engine operating conditions, the two simulations with increasing noise signal are also presented in Figure 19 to compare the control effect.

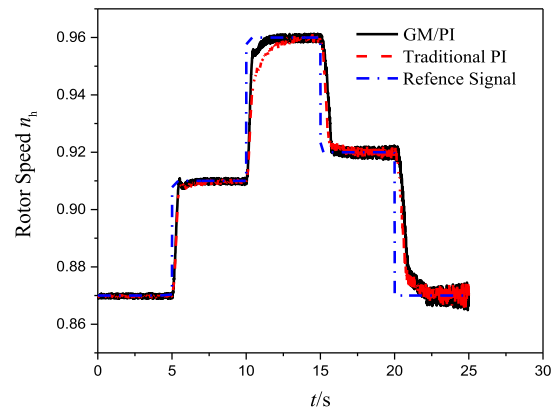


FIGURE 19. High pressure fuel flow with $H = 3.5\text{km}$, $Ma = 0.5$.

From Figure 19 it can be seen that the two control methods both maintain good stability in the presence of interference signals, but the response time of PI control under guardian maps theory is less than 2s, while the response time of traditional PI control is less than 3s.

In actual situation, there are restrictions on the cross-sectional area for fuel nozzle of an aeroengine, that is, fuel saturation. This kind of situation will also affect the performance and control effect of the whole aeroengine, so in the simulation, we also need to consider the saturation of fuel input. At the actual working condition of aeroengine, the fuel flow will often be restricted, especially when a large range variation occurs in the rotor speed. The maximum fuel flow is set to 1kg/s, shown in Figure 20, the simulation is shown in Figure 21.

From Figure 21, it can be seen that fuel input saturation occurs when the rotor speed step is large, which leads to a rise in the response time. However, the response time of the

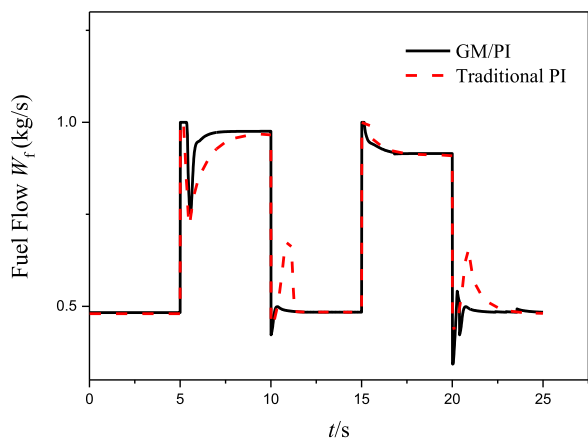


FIGURE 20. Large range variation of high pressure fuel flow with saturation.

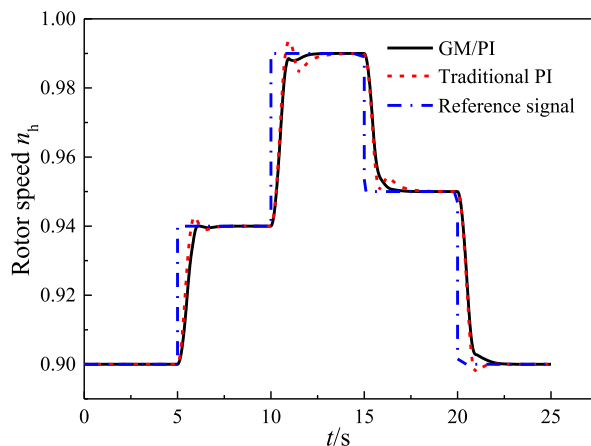


FIGURE 22. High pressure rotor speed with $H = 6\text{km}$, $Ma = 0.6$.

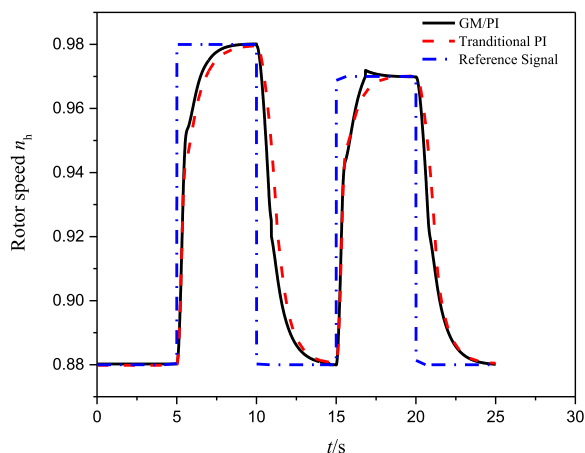


FIGURE 21. Large range variation of high pressure rotor speed with saturation.

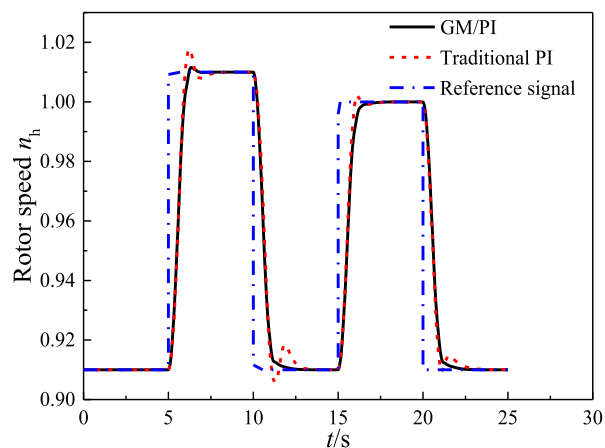


FIGURE 23. Large range variation of high pressure rotor speed with $H = 6\text{km}$, $Ma = 0.6$.

control under guardian maps theory is less than 2.7 seconds, and that of the traditional PI control is less than 3.2 seconds. The guardian maps theory still has some advantages.

From Figure 18-21, it can be seen that Monte Carlo simulation proves the control effect of guardian maps, under the interference of noise signal and in the case of fuel flow saturation guardian maps method still has some advantages. Several kinds of simulations with operating mode $H = 3.5\text{km}$, $Ma = 0.5$ further prove the reliability of PI control based on guardian maps theory. Simulations in the same way with several other operating modes are shown below from Figure 22-25.

From Figure 22-25, it can be seen that under the control of PI controller based on guardian maps theory, the setting time of response curve of the system is less than 2.2 seconds, the overshoot is less than 1.2% and the steady-state error is 0. However, under the control of the PI controller designed by the traditional variable gain control method, the setting time of time response curve is less than 3s, overshoot is up to 7.5%. Besides, the overshoot and tracking effect of fuel flow for guardian maps theory are more ideal. The simulation results

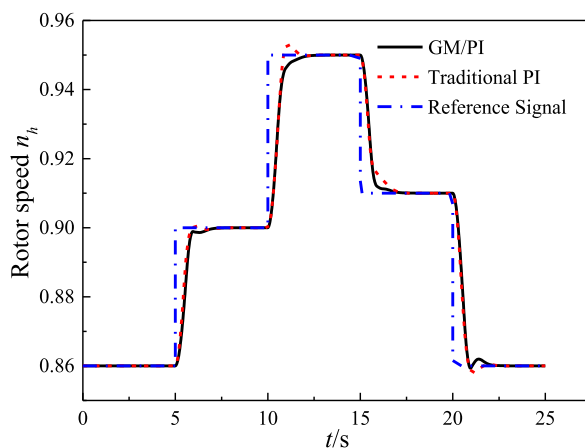


FIGURE 24. High pressure rotor speed with $H = 12\text{km}$, $Ma = 1.3$.

show that the control effect of the PI controller based on the guardian maps theory has better dynamic response compared with the traditional method, and the performance of overshoot and setting time is obviously improved.

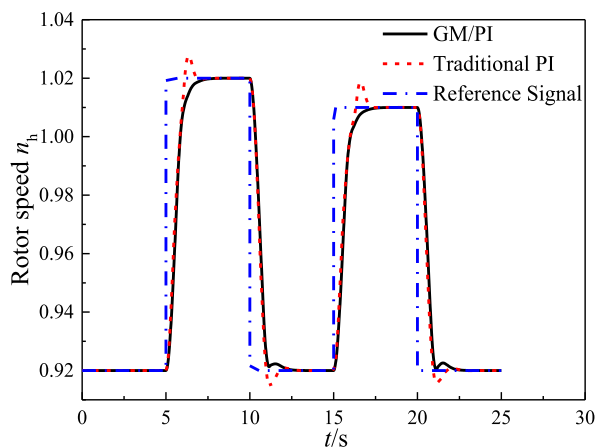


FIGURE 25. Large range variation of high pressure rotor speed with $H = 12\text{km}$, $Ma = 1.3$.

VI. CONCLUSION

This paper applies LPV/PI control method based on guardian maps theory to aeroengine control system. The theory of guardian maps and the related algorithms for aeroengine control system are elaborated in detail. Since the PI control is used for implementation, this method has a good engineering practicality. This approach can obtain the controller parameters with only one given initial controller parameter at the boundary point of the scheduling parameter. The controller avoided designing in a large number of balanced design points, and the global control performance is guaranteed. Finally, simulations based on nonlinear aero-engine model are performed and the results show that this approach is better than the traditional PI control method.

In the following research, the controller design based on guardian maps theory can be used in Multi-variable multi-output aeroengine control systems to verify the effectiveness of this method deeply. Meanwhile, some methods such as weighting algorithm may be utilized to control the process of controller parameters switching in the overlap region and analyze the performance of the switch system.

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