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# Stabilization for Markovian Jump Distributed Parameter Systems With Time Delay

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**ABSTRACT** The problems of stochastic stability and stabilization for a class of Markovian jump distributed parameter systems with time delay are researched in this paper. First, taking advantage of a combination of Poincare inequality and Green formula, a stochastic stability criterion is presented by a linear matrix inequality (LMI) approach. Then, a state feedback controller is designed. Based on the proposed results, the sufficient conditions of the close-loop systems' stochastic stability are given in terms of a set of LMIs by constructing the appropriate Lyapunov functionals, calculating the weak infinitesimal generator, and using the Schur complement lemma. The sufficient conditions could be solved directly and applied to engineering practice conveniently. The obtained results generalize and enrich the theory of distributed parameter systems with time delay. The model of Markovian jump distributed parameter systems is more fitting the actual systems' requirements and has wider application scope. Finally, numerical examples are used to demonstrate the validity of the method.

**INDEX TERMS** Markovian jump, distributed parameter systems, stochastic stability, linear matrix inequality.

## I. INTRODUCTION

The research of control system has a long history, and a lot of results have been achieved, including system stability [1], [2], optimization [3]–[7], dynamic behavior analysis [8], [9] and so on. In reality, the state space of most systems is infinite dimensional [10], [11], and the phenomenon of time delay is everywhere. In mathematical theory, distributed parameter systems are infinite dimensional dynamical systems described by partial differential equations (PDE) [12]–[17], functional differential equations or differential equations in abstract space [18]. In the real world, for example, elastic aircraft with mass distribution, chemical reaction process, population dynamics and many other fields [19]–[22], the mathematical model is a differential dynamic system described by distributed parameter system. Distributed parameter systems with time delays have become an important research direction of modern control theory.

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Many researchers have paid more attention to the time-delay distributed parameter systems [23]–[27]. The main research methods of time-delay distributed parameter systems are semi-group theory [28], matrix norm theory [29], [30] and linear matrix inequality theory (LMI) [31]–[34]. Semi-group method is to transform a concrete practical system into an abstract equation of development, use the properties of semi-groups to get conclusions, and then convert the results back to the original system. It is difficult to ensure the system has good dynamic quality and performance index through the deviation formed by two transformations. The results derived from using matrix norm theory are generally not easy to test, and the application in practical problems is not good. With the rapid development of computer application technology, the conclusions obtained by linear matrix inequality method are simple to be applied in engineering practice.

Based on linear matrix inequality method, the problems of exponential stability [31], exponential stabilization [32] and robust control [33] of distributed parameter systems with time-delay are discussed by constructing appropriate

Lyapunov functional. Using linear matrix inequality approach, the feedback control of distributed parameter systems with continuous distributed delays is investigated [34]. Some scholars have paid attention to the distributed parameters switched systems [35], [36]. The problems of exponential stability, feedback stabilization and robust fault-tolerant control of distributed parameters switched systems with time-delay are studied, and the results are described by a group of linear matrix inequalities (LMIs).

At the same time, there are a large number of dynamic systems in reality. The jump of the system is caused by random mutation phenomena, such as abrupt change of working environment conditions, parameter changes and so on. Through a lot of research work, it is found that these random changes usually follow the law of Markovian jump process. Up to present, many results of Markovian jump systems with time-delay have emerged [37]–[53], such as finite-time stability and stabilization [37], Finite-time H$\infty$ filtering [38], Stability and stabilization [39], Exponential stability [40], and stochastic stability [41]. Due to singular Markovian jump time-delay systems [42] and neutral Markovian jump systems with time-varying delays [43], some scholars have discussed the Exponential stability. In order to deal with getting less conservatism, delay-dependent H$\infty$ control [44], [45] of Markovian jump systems with time-delay has been proposed. Under delay-dependent circumstance, robust stability [46], [47] and Exponential stability [48] of time-delay Markovian jump systems have been studied. As we know, it is not easy to obtain all the accurate transition rate in practical application due to huge cost and lack of technology. In [49]–[53], Markovian jump systems with general unknown transition probability matrices have been reported.

In summary, the results of time-delay distributed parameter systems and Markovian jump systems have been carried out from different aspects, and the research contents and methods are also various. For the study of distributed parameter systems with time delays, although some literatures involve stochastic factors, such as Itô type, they basically do not involve Markovian jump. The stochastic stability analysis and control of distributed parameter systems with Markovian jump have a lot of work to be solved and improved.

In this paper, we are considering the problems of stochastic stability and stabilization for a class of Markovian jumping time-delay distributed parameter systems. By constructing appropriate stochastic Lyapunov functional, based on linear matrix inequality method, using Green's formula and Poincare inequality, sufficient conditions for the stochastic stability of Markovian jumping distributed parameter systems are given in group of linear matrix inequalities.

## II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of Markovian jump distributed parameter systems with time delay of the following form

$$\begin{aligned}\frac{\partial}{\partial t}W(x,t) &= D(r_t)\Delta W(x,t) + A(r_t)W(x,t)\\ &\quad + A_1(r_t)W(x,t-\tau) + B(r_t)u(x,t) \qquad (1)\end{aligned}$$

where

$$(x,t) \in \Omega \times \mathrm{R}_+, \Omega = \{x, \|x| < l < \infty\} \in \mathrm{R}^m$$

is the bounded domain with smooth boundary $\partial\Omega$ and $mes\Omega > 0$.

$$\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_m}\right)$$

is the gradient operator and

$$\Delta = \sum_{k=1}^{m} \frac{\partial^2}{\partial x_k^2}$$

is the Laplace operator on $\Omega$. State function

$$W(x,t) = col(w_1(x,t), w_2(x,t), \ldots, w_n(x,t)) \in \mathrm{R}^n$$
$$\nabla W(x,t) = col(\nabla w_1(x,t), \nabla w_2(x,t), \ldots, \nabla w_n(x,t)).$$

The initial value and boundary value conditions satisfy

$$W(x,t) = 0, (x,t) \in \partial\Omega \times [-\tau, +\infty) \qquad (2)$$
$$W(x,t) = \varphi(x,t), (x,t) \in \Omega \times [-\tau, 0] \qquad (3)$$
$$\frac{\partial W(x,t)}{\partial n} = 0, (x,t) \in \partial\Omega \times [-\tau, +\infty) \qquad (4)$$

where $n$ is the unit outward normal vector of $\partial\Omega$ and $\varphi(x,t)$ is the suitable smooth function. $D(r_t) > 0$ and $\tau > 0$ are constants, $A(r_t)$ and $A_1(r_t)$ are constant matrices.

Let $\{r_t, t \geq 0\}$ be a right-continuous Markov process and take values in $S = \{1, 2, \ldots, N\}$ with the transition probabilities matrix $\pi = (\pi_{ij})_{N\times N}$ given by

$$P_r(r_{t+\delta} = j | r_t = i) = \begin{cases} \pi_{ij}\delta + o(\delta), & i \neq j \\ 1 + \pi_{ii}\delta + o(\delta), & i = j \end{cases}$$

where $\delta > 0$ and

$$\lim_{\delta\to 0} \frac{o(\delta)}{\delta} = 0, \quad \pi_{ij} \geq 0, \ i \neq j$$

is the transition rate from mode $i$ to mode $j$ in the time interval $\delta$ with

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}, \quad i = j$$

For each $r_t = i \in S$,
Let

$$A(r_t) = A_i, \quad A_1(r_t) = A_{1i}, \quad D(r_t) = D_i$$

and

$$B(r_t) = B_i.$$

Then, the system (1) can be rewritten as

$$\begin{aligned}\frac{\partial}{\partial t}W(x,t) &= D_i\Delta W(x,t) + A_iW(x,t)\\ &\quad + A_{1i}W(x,t-\tau) + B_iu(x,t) \qquad (5)\end{aligned}$$

We design state feedback controller for system (5) and it is described as

$$u(x,t) = K_iW(x,t) \qquad (6)$$

For convinces, we study the Markovian jump distributed parameter systems without control firstly.

$$\frac{\partial}{\partial t}W(x,t)=D_i\Delta W(x,t)+A_iW(x,t)+A_{1i}W(x,t-\tau) \tag{7}$$

In order to obtain the main results, the following lemmas are introduced.

*Lemma 1 [54] (Green Formula):* Let $\Omega \subset \mathrm{R}^n$ be the bounded domain with smooth boundary $\partial\Omega$, $n$ is the unit outward normal vector of $\partial\Omega$, $G \subset \Omega$ is the smooth subdomain. If $u, v \in C^2(\overline{G})$, then

$$\int_G u\Delta v\mathrm{d}x=\int_{\partial\Omega}u\frac{\partial v}{\partial n}\mathrm{d}s\text{-}\int_\Omega \nabla u\nabla v\mathrm{d}x$$

where $\nabla$ is Hamilton operator, ds is the area elements over the boundary region.

*Lemma 2 [54][56] (Poincare's Inequality):* Let $w \in C_0^1(\Omega)$ and $\Omega$ be included in closed region $\Omega_1$ : $0 \le x_i \le l\ (i=1,2,\ldots,n)$, then

$$\int_\Omega w^2(x)\,\mathrm{d}x \le \int_\Omega \sum_{i=1}^n\left(\frac{\partial w}{\partial x}\right)^2\mathrm{d}x=c\int_\Omega|\nabla w|^2\mathrm{d}x$$

where $c=l^2/n$.

*Remark 1:* This is a very useful lemma, it is also called Friedrichs inequality.

*Lemma 3 [55]:* Let $V_1, V_2, V_3$ be real matrices, and $V_3=V_3^{\mathrm{T}}>0$, then for an arbitrary given scalar $\alpha>0$, the following inequality holds.

$$V_2^{\mathrm{T}}V_1+V_1^{\mathrm{T}}V_2 \le \alpha^{-1}V_1^{\mathrm{T}}V_3^{-1}V_1+\alpha V_2^{\mathrm{T}}V_3V_2.$$

## III. MAIN RESULTS

*Theorem 1:* For an arbitrary Markovian jump mode $i \in S$, if exist positive symmetric matrices $Q$, $P_i$, such that the following linear matrix inequalities (LMIs) hold, then Markovian jump distributed parameter systems (7) is stochastic stable.

$$\begin{pmatrix} A_i^{\mathrm{T}}P_i+P_iA_i+Q+\sum_{j=1}^N\pi_{ij}P_j & A_{1i}\\ A_{1i}^{\mathrm{T}} & -Q\end{pmatrix}<0 \tag{8}$$

*Proof:* Choose the following stochastic Lyapunov functionals for system (7)

$$V_i(t,W(x,t))=\int_\Omega W^{\mathrm{T}}(x,\theta)P_iW(x,\theta)\,\mathrm{d}x+\int_\Omega\int_{t-\tau}^t W^{\mathrm{T}}(x,\theta)QW(x,\theta)\,\mathrm{d}\theta\mathrm{d}x \tag{9}$$

Let $L$ be the weak infinitesimal generator, then we calculate

$$LV(x,t,i)=V_t+\sum_{j=1}^N\pi_{ij}V(x,t,j)$$

So, it is obtained that

$$\begin{aligned}
&LV_i(t,W(x,t))\\
&=\int_\Omega \dot{W}^{\mathrm{T}}(x,t)P_iW(x,t)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t)P_i\dot{W}(x,t)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t)QW(x,t)\,\mathrm{d}x\\
&\quad-\int_\Omega W^{\mathrm{T}}(x,t-\tau)QW(x,t-\tau)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t)\left(\sum_{j=1}^N\pi_{ij}P_j\right)W(x,t)\,\mathrm{d}x\\
&=2D_i\int_\Omega W^{\mathrm{T}}(x,t)P_i\Delta W(x,t)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t)\left(A_i^{\mathrm{T}}P_i+P_iA_i\right)W(x,t)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t)P_iA_{1i}W(x,t-\tau)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t-\tau)A_{1i}^{\mathrm{T}}P_iW(x,t)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t)QW(x,t)\,\mathrm{d}x\\
&\quad-\int_\Omega W^{\mathrm{T}}(x,t-\tau)QW(x,t-\tau)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t)\left(\sum_{j=1}^N\pi_{ij}P_j\right)W(x,t)\,\mathrm{d}x
\end{aligned} \tag{10}$$

Applying Lemma 1 and Lemma 2,

$$\begin{aligned}
&\int_\Omega W^{\mathrm{T}}(x,t)P_l\Delta W(x,t)\mathrm{d}x\\
&=\sum_{i=1}^n\sum_{j=1}^n P_{l_{ij}}\left[\int_{\partial\Omega}w_i(x,t)\frac{\partial w_j(x,t)}{\partial n}\mathrm{d}s\right.\\
&\quad\left.-\int_\Omega \nabla w_i(x,t)\nabla w_j(x,t)\,\mathrm{d}x\right]\\
&=-\sum_{i=1}^n\sum_{j=1}^n P_{l_{ij}}\int_\Omega \nabla w_i(x,t)\nabla w_j(x,t)\,\mathrm{d}x\\
&=-\int_\Omega \nabla W^{\mathrm{T}}(x,t)P_l\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x
\end{aligned} \tag{11}$$

Then

$$\begin{aligned}
&LV_i(t,W(x,t))\\
&\le\int_\Omega W^{\mathrm{T}}(x,t)\left(A_i^{\mathrm{T}}P_i+P_iA_i+Q+\sum_{j=1}^N\pi_{ij}P_j\right)\\
&\quad\times W(x,t)\,\mathrm{d}x-\int_\Omega W^{\mathrm{T}}(x,t-\tau)QW(x,t-\tau)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t)P_iA_{1i}W(x,t-\tau)\,\mathrm{d}x\\
&\quad+\int_\Omega W^{\mathrm{T}}(x,t-\tau)A_{1i}^{\mathrm{T}}P_iW(x,t)\,\mathrm{d}x\\
&\le\int_\Omega \xi^{\mathrm{T}}(x,t)\Lambda\xi(x,t)\,\mathrm{d}x<0,
\end{aligned} \tag{12}$$

where

$$\xi^{\mathrm{T}}(x,t)=\left[W^{\mathrm{T}}(x,t),W^{\mathrm{T}}(x,t-\tau)\right],$$

$$\Lambda=\begin{bmatrix} A_i^{\mathrm{T}}P_i+P_iA_i+Q+\sum_{j=1}^{N}\pi_{ij}P_j & P_iA_{1i} \\ A_{1i}^{\mathrm{T}}P_i & -Q \end{bmatrix}<0.$$

The proof is end.

Using the feedback controller (6), the systems (5) is converted into

$$\frac{\partial}{\partial t}W(x,t)=D_i\Delta W(x,t)+(A_i+B_iK_i)W(x,t)+A_{1i}W(x,t-\tau). \quad (13)$$

Taking advantage of the conclusion of theorem 1, we obtain the theorem 2 immediately.

*Theorem 2:* For any Markovian jump mode$i \in S$ and given matrices $A_i, A_{1i}, B_i$, The controller systems of Markovian jump distributed systems with time-delay Delay (5) is stochastic stability. If there exist a matrix $K_i$ and positive symmetric matrices $Q, P_i$, such that the following LMIs hold.

$$\begin{pmatrix} (A_i+B_iK_i)^{\mathrm{T}}P_i+P_i(A_i+B_iK_i)+Q+\sum_{j=1}^{N}\pi_{ij}P_j & P_iA_{1i} \\ A_{1i}^{\mathrm{T}}P_i & -Q \end{pmatrix}<0 \quad (14)$$

When the system is single mode, that is, there is no mode switching. Taking $P_i = I$, a sufficient condition for asymptotic stability of time-delay distributed parameter system is obtained by Theorem 2.

$$\begin{pmatrix} A+A^{\mathrm{T}}+BK+B^{\mathrm{T}}K^{\mathrm{T}}+Q & A_1 \\ A_1^{\mathrm{T}} & -Q \end{pmatrix}<0$$

*Remark 2:* The above results is consistent with the conclusion of Theorem 1 in reference [33], when we take $\beta = 1$. So Theorem 2 of this paper is more representative.The expansion and extension of [33] is more suitable for the needs of practical systems.

*Theorem 3:* Given a scalar $\beta > 0$ and matrices $A_i$, $A_{1i}$, and $B_i$, the system (5) is stochastic stability under the controller (6). If there exist a matrix$K_i$and positive symmetric matrices $Q, P_i$, such that

$$\begin{pmatrix} A_i^{\mathrm{T}}P_i+P_iA_i+\beta Q+\sum_{j=1}^{N}\pi_{ij}P_j+P_i & K_i^{\mathrm{T}}B_i^{\mathrm{T}} & P_iA_{1i} \\ B_iK_i & -P_i^{-1} & 0 \\ A_{1i}^{\mathrm{T}}P_i & 0 & -\beta Q \end{pmatrix}<0 \quad (15)$$

*Proof:* Choose the following stochastic Lyapunov functionals

$$V_i(t,W(x,t))=\int_{\Omega}W^{\mathrm{T}}(x,\theta)P_iW(x,\theta)\,\mathrm{d}x+\beta\int_{\Omega}\int_{t-\tau}^{t}W^{\mathrm{T}}(x,\theta)QW(x,\theta)\,\mathrm{d}\theta\mathrm{d}x \quad (16)$$

Through calculating the weak infinitesimal generator, it follows that

$$\begin{aligned} &LV_i(t,W(x,t)) \\ &=2D_i\int_{\Omega}W^{\mathrm{T}}(x,t)P_i\Delta W(x,t)\,\mathrm{d}x \\ &+\int_{\Omega}W^{\mathrm{T}}(x,t)\left(A_i^{\mathrm{T}}P_i+P_iA_i\right)W(x,t)\,\mathrm{d}x \\ &+\int_{\Omega}W^{\mathrm{T}}(x,t)P_iA_{1i}W(x,t-\tau)\,\mathrm{d}x \\ &+\int_{\Omega}W^{\mathrm{T}}(x,t-\tau)A_{1i}^{\mathrm{T}}P_iW(x,t)\,\mathrm{d}x \\ &+\beta\int_{\Omega}W^{\mathrm{T}}(x,t)QW(x,t)\,\mathrm{d}x \\ &-\beta\int_{\Omega}W^{\mathrm{T}}(x,t-\tau)QW(x,t-\tau)\,\mathrm{d}x \\ &+\int_{\Omega}\left[W^{\mathrm{T}}(x,t)P_iB_iu(x,t)+u^{\mathrm{T}}(x,t)B_i^{\mathrm{T}}P_iW(x,t)\right]\mathrm{d}x \\ &+\int_{\Omega}W^{\mathrm{T}}(x,t)\left(\sum_{j=1}^{N}\pi_{ij}P_j\right)W(x,t)\,\mathrm{d}x \end{aligned} \quad (17)$$

With the help of Lemma 3, we get

$$\begin{aligned} &\int_{\Omega}W^{\mathrm{T}}(x,t)P_iA_{1i}W(x,t-\tau)\,\mathrm{d}x \\ &\quad+\int_{\Omega}W^{\mathrm{T}}(x,t-\tau)A_{1i}^{\mathrm{T}}P_iW(x,t)\,\mathrm{d}x \\ &\le\beta^{-1}\int_{\Omega}W^{\mathrm{T}}(x,t)P_iA_{1i}Q^{-1}A_{1i}^{\mathrm{T}}P_iW(x,t)\,\mathrm{d}x \\ &\quad+\beta\int_{\Omega}W^{\mathrm{T}}(x,t-\tau)QW(x,t-\tau)\,\mathrm{d}x. \end{aligned} \quad (18)$$

And

$$\begin{aligned} &\int_{\Omega}\left[W^{\mathrm{T}}(x,t)P_iB_iu(x,t)+u^{\mathrm{T}}(x,t)B_i^{\mathrm{T}}P_iW(x,t)\right]\mathrm{d}x \\ &=\int_{\Omega}-\left[P_i^{\frac{1}{2}}B_iu(x,t)-P_i^{\frac{1}{2}}W(x,t)\right]^{\mathrm{T}} \\ &\quad\times\left[P_i^{\frac{1}{2}}B_iu(x,t)-P_i^{\frac{1}{2}}W(x,t)\right]\mathrm{d}x \\ &\quad+\int_{\Omega}u^{\mathrm{T}}(x,t)B_i^{\mathrm{T}}P_iP_iB_iu(x,t)\,\mathrm{d}x \\ &\quad+\int_{\Omega}W^{\mathrm{T}}(x,t)P_iW(x,t)\mathrm{d}x \\ &\le\int_{\Omega}u^{\mathrm{T}}(x,t)B_i^{\mathrm{T}}P_iP_iB_iu(x,t)\,\mathrm{d}x \\ &\quad+\int_{\Omega}W^{\mathrm{T}}(x,t)P_iW(x,t)\mathrm{d}x \\ &\le\int_{\Omega}W^{\mathrm{T}}(x,t)K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_iP_iB_iK_iW(x,t)\,\mathrm{d}x \\ &\quad+\int_{\Omega}W^{\mathrm{T}}(x,t)P_iW(x,t)\mathrm{d}x \end{aligned} \quad (19)$$

Based on (17), (18) and (19), by the Schur complement lemma, we have

$$\begin{aligned} &LV_i(t, W(x,t)) \\ &\leq \int_\Omega W^{\mathrm{T}}(x,t)\Bigg(A_i^{\mathrm{T}}P_i + P_iA_i + \beta Q + P_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_iB_iK_i \\ &\quad + \beta^{-1}P_iA_{1i}Q^{-1}A_{1i}^{\mathrm{T}}P_i + \sum_{j=1}^{N}\pi_{ij}P_j\Bigg)W(x,t)\,\mathrm{d}x \\ &\leq \int_\Omega W^{\mathrm{T}}(x,t)\Lambda W(x,t)\,\mathrm{d}x \\ &< 0 \end{aligned}$$

where

$$\Lambda = \begin{pmatrix} A_i^{\mathrm{T}}P_i + P_iA_i + \beta Q + \sum_{j=1}^{N}\pi_{ij}P_j + P_i & K_i^{\mathrm{T}}B_i^{\mathrm{T}} & P_iA_{1i} \\ B_iK_i & -P_i^{-1} & 0 \\ A_{1i}^{\mathrm{T}}P_i & 0 & -\beta Q \end{pmatrix}.$$

□

*Remark 3:* When the system is single mode, that is, there is no mode switching. The following conclusion is consistent with the results of Theorem 3 in [33], when we choose $Q = P$.

$$\begin{pmatrix} A^{\mathrm{T}}P + PA + \beta Q + P & K^{\mathrm{T}}B^{\mathrm{T}} & PA_1 \\ BK & -P^{-1} & 0 \\ A_1^{\mathrm{T}}P & 0 & -\beta Q \end{pmatrix} < 0.$$

*Remark 4:* There exists a term of $P_i^{-1}$ in theorem 3, which can not be solved directly by LMI toolbox in Matlab. We need to set $Q_i = P_i^{-1} > 0$, and then it can be transformed into

$$\begin{pmatrix} Q_i & I \\ I & P_i \end{pmatrix} > 0.$$

## IV. EXAMPLES

In 1994, Michael and Ryszard studied the cellular replication. The process of cellular replication just satisfied the time-delay distributed parameter systems [57] as follow:

$$\frac{\partial M}{\partial t} + \frac{\partial (v(m)M)}{\partial m} = c(t)\left[-M(t,m) + k'(m)M(t-\tau, k(m))\right]$$

Hidden Markov models are often used to biology application [58], the combination of the two models is described as Markovian jump distributed parameter systems in mathematical theory. So we give the following numerical examples to illustrate the effectiveness of the method. For simple, a distributed parameter system with two Markovian jump modes is considered.

*Example 1:* For Theorem 1, the transition rate matrix is given by

$$\Pi = \begin{pmatrix} -0.2 & 0.2 \\ 0.3 & -0.3 \end{pmatrix},$$

when $r_t = i = 1$, we consider system (7) with the following parameters:

$$A_1 = \begin{pmatrix} -25 & 2 \\ 1 & -20 \end{pmatrix}, \quad A_{11} = \begin{pmatrix} -11 & 0 \\ -9 & 11 \end{pmatrix}.$$

when $r_t = i = 2$, we choose

$$A_2 = \begin{pmatrix} -10 & 20 \\ -5 & -9 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 4 & 3 \\ -9 & 0 \end{pmatrix}.$$

By Theorem 1, using LMI toolbox to solve linear matrix inequalities (8), the following parameters are computed to be

$$P_1 = \begin{pmatrix} 17.3237 & 1.3197 \\ 1.3197 & 21.9271 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 35.0493 & 15.3697 \\ 15.3697 & 80.9490 \end{pmatrix},$$
$$Q = \begin{pmatrix} 430.0000 & 0.0000 \\ 0.0000 & 430.0000 \end{pmatrix}.$$

It is easy to verify that $P_1, P_2, Q$ is positive matrix.

*Example 2:* For theorem 3, we select $K = -I$, $B = I$, $\beta = 1$. $A_1, A_2, A_{11}, A_{12}$ and $\Pi$ are chosen as same as Example 1. Based on LMs (15), we obtain the following parameters

$$P_1 = \begin{pmatrix} 20.5164 & -1.4879 \\ -1.4879 & 15.9545 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 37.0899 & 4.2179 \\ 4.2179 & 70.8944 \end{pmatrix}$$
$$Q = \begin{pmatrix} 566.7060 & -95.2762 \\ -95.2762 & 233.9144 \end{pmatrix}$$
$$Q_1 = \begin{pmatrix} 295.3539 & -5.2792 \\ -5.2792 & 306.3389 \end{pmatrix}$$
$$Q_2 = \begin{pmatrix} 302.8284 & 16.7124 \\ 16.7124 & 267.1047 \end{pmatrix}$$

The results is easy to verify that conform to the requirement.

## V. CONCLUSION

In this paper, we give some criteria for stochastic stability of Markovian jumping distributed parameter system in the form of a set of linear matrix inequalities. by constructing Lyapunov stochastic functional, using boundary conditions, Schur complement lemma and matrix inequality knowledge, The results generalize and extend the conclusions in [33], and have a wider scope of application. The other more complicated mathematical concerns about neutral distributed parameter system with Markovian jump will discussed in the further study.

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