IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received June 22, 2019, accepted July 11, 2019, date of publication July 16, 2019, date of current version August 7, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2929302

Consensus Based Strong Tracking Adaptive Cubature Kalman Filtering for Nonlinear System Distributed Estimation

ZHENG ZHANG[1], QINGDONG LI[1], LIANG HAN[2], (Member, IEEE), XIWANG DONG[1], (Member, IEEE), YUAN LIANG[1], AND ZHANG REN[1]

[1]Science and Technology on Aircraft Control Laboratory, School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China
[2]School of Sino-French Engineer, Beihang University, Beijing 100191, China

Corresponding author: Liang Han (liang_han@buaa.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grants 61873011, Grant 61803014, Grant 61503009, and Grant 61333011, in part by the Beijing Natural Science Foundation under Grant 4182035, in part by the Young Elite Scientists Sponsorship Program by the CAST under Grant 2017QNRC001, in part by the Aeronautical Science Foundation of China under Grants 2016ZA51005 and Grant 20170151001, in part by the Special Research Project of Chinese Civil Aircraft, in part by the State Key Laboratory of Intelligent Control and Decision of Complex Systems, and in part by the Fundamental Research Funds for the Central Universities under Grant YWF-18-BJ-Y-73.

ABSTRACT The distributed cubature Kalman filter is widely used in the field of target tracking, however, the presence of model uncertainties will undermine its tracking stability and effectiveness for tracking maneuvering target. In order to eliminate this effect on maneuvering target tracking, this paper develops a distributed consensus based strong tracking cubature Kalman filter scheme. First, each node obtains its local estimation with the usage of local observations via strong tracking cubature Kalman filter, where the suboptimal fading factor and adaptive factor are introduced for adaptively modifying the filter gain. Then, the designed filter gain is used for updating the local state estimation. Second, after all, nodes have achieved its local estimation, each node exchanges its local estimation to its neighbors and updates its local estimation according to the consensus communication protocol. It can be further proved that the distributed interaction between neighbors will contribute to enhancing the tracking stability. The detailed proof for stochastic boundedness of the estimation error is analyzed by introducing a stochastic process. Simulation results demonstrate that the proposed algorithm can achieve higher tracking accuracy than the existing methods for tracking a maneuvering target.

INDEX TERMS Adaptive cubature kalman filtering (ACKF), consensus method, distributed estimation, stochastic boundedness.

I. INTRODUCTION

The distributed state estimation (DSE) problem for nonlinear system has received extensive attention in past few years [1]–[3]. Based on the fast development of large scale mobile sensor network, DSE has been widely used in target localization and tracking, surveillance [4]–[6]. Compared with traditional centralized estimation, DSE does not need a central node to process large amounts of communication data in real time [7]. The communication topology of the mobile sensor network need not to be connected completely and each sensor node in the network only communicates with its neighbor [8]. The distributed structure can reduce the communication burden of sensor network, which makes the filter algorithm based on consensus method more effective in dealing with practical problems. Olfati-saber et al. proposed the distributed Kalman filtering algorithm (DKF) for distributed estimation in discrete linear system [9]. Distributed estimation based on extend Kalman filter named distributed extend Kalman filter (DEKF) is proposed for nonlinear system [10]. However, DEKF has low precision when dealing with high-order nonlinear system because extend Kalman filter algorithm expands the nonlinear function into Taylor series and ignores the second-order and above terms.

The consensus algorithm is an effective method for solving DSE problem and it can almost achieve the same performance

The associate editor coordinating the review of this manuscript and approving it for publication was Rui-Jun Yan.

as centralized method. G. Battistelli et al. propose consensus-based linear and nonlinear filtering [11] and W. Y. Li et al. propose a weighted average consensus-based unscented kalman filtering [12]. The main idea of consensus based distributed estimation is to reach consensus of the state estimation and measurements by spreading the available information between sensor nodes [13]. The consensus-based algorithm for DSE problem include consensus on information (CI), consensus on measurements (CM), and consensus on estimation (CE) [11]. The CE algorithm, which is based on the idea of spreading the available information over sensor network by performing average consensus of the local estimation at each time instant, can be used to solve the DSE problem. The consensus based distributed information filtering algorithm is proposed in [14], which has the advantage of consensus estimation and information filtering fusion. Consensus based estimation for nonlinear system is more challenging and has been a focal point of state estimation research for many years [15], [16].

In the past few decades, the filtering algorithm for nonlinear system has been studied widely [16]–[18]. Ienkaran Arasaratnam et al. have proposed the cubature Kalman filters (CKF) [19]. Then CKF algorithm become a popular filter algorithm to solve the DSE problem because it has an excellent performance while dealing with the high-order nonlinear system. Ding et al. propose distributed algorithm-based CKF algorithm, which can achieve the high-precision filtering and strongly robust to node failures based on the information filter and weight-average consensus method [20]. Adaptive cubature Kalman filter (ACKF) is proposed by Tan et al. and it can adjust the weight matrix between node and its neighbor to improve the convergence rate of consensus iteration adaptively [21]. Both of the algorithm mentioned above can achieve target tracking in a certain degree, but neither of them can reduce the effect of the previous bad measurement on the current estimation, which is important for tracking a maneuvering target. Y. H. et al. propose adaptive tracking algorithm based on modified strong tracking filter (MSTF), which is suitable for tracking maneuvering target and non-maneuvering target [22]. However, the MSTF algorithm does not consider the distributed state estimation for maneuvering target.

It has been realized for many years that traditional filtering method has low accuracy for state estimation of maneuvering target and the result of state estimation is very susceptible to measurement noise [3], [23]. How to improve the accuracy of state estimation for maneuvering target has been a central topic in state estimation. It is, however, impossible to eliminate the effect of noise completely. Besides, centralized estimation can achieve high tracking accuracy at the expense of increasing the burden of network communication. It is a fact that available techniques cannot solve the problems mentioned above at the same time.

This paper proposes a consensus based strong tracking adaptive cubature Kalman filter (CSTA-CKF) algorithm for mobile sensor network to solve DSE problem of nonlinear system, which takes advantage of strong tracking adaptive cubature Kalman filter and consensus algorithm. The algorithm is consisted of two steps. The first step is local filter stage. In that stage, the suboptimal fading factor is introduced firstly to adjust the filter gain, which makes the residual sequences orthogonal to achieve the strong tracking for maneuvering target. Then, we introduce a bounded adaptive factor to balance the weight of state prediction and measurement when the measurement is affected largely by noise. The second step is consensus process. In this step, each node exchanges its local estimation with neighbor node and then updates its local estimation according to the consensus communication protocol to achieve global optimal estimation of target state. Besides, the communication topology need not to be connected completely and the burden of communication is reduced largely for each sensor node only communicates with its neighbor in the consensus process.

The rest of the paper is organized as follows. Section 2 gives some basic concepts of graph theory and nonlinear system. Section 3 introduces the standard CKF filtering algorithm and strong tracking adaptive cubature kalman filter. Section 4 introduces the consensus algorithm and gives the consensus based strong tracking adaptive cubature Kalman filter algorithm and compared it with some other filter algorithms for nonlinear system. The stochastic boundedness of the algorithm is analyzed and the detailed proof is given in Section 5. Section 6 gives a simulation example. Section 7 concludes the paper.

II. PROBLEM DESCRIPTION

In this section, some basic concepts and results on graph theory are introduced and the problem description is presented.

A. BASIC CONCEPTS ON GRAPH THEORY

Let $G = \{V, E\}$ be an undirected graph of order N, where $V = \{1, 2, \ldots, N\}$ is the node set and $E \subset V \times V$ is the edge set. Two nodes are said to be connected if they can communicate directly with each other. The adjacency matrix A with elements a_{ij} is defined by $a_{ij} > 0$ if and only if $(j, i) \in E$, and $a_{ij} = 0$ otherwise. For an undirected graph, if $(j, i) \in E$, then $(i, j) \in E$. The set of nodes connected with a certain node i is called the neighbor set of node i and the set is denoted by N_i. Let $\deg_{in}(i) = \sum_{j=1}^{N} a_{ij}$ denote the in-degree of node i. Denote $D = \mathrm{Diag}(\deg_{in}(i), i = 1, 2, \ldots N)$ the degree matrix of G. The Laplacian matrix of the graph is defined as $L = D - A$. If an undirected graph is connected, the Laplacian has a single zero eigenvalue, and the other eigenvalues can be listed in an increasing order $0 = \lambda_1(L) < \lambda_2(L) \leqslant \ldots \leqslant \lambda_N(L)$.

B. PROBLEM DESCRIPTION

Assume that a set of nodes are evenly distributed in space. All the nodes cannot obtain the real state of the target directly. However, the nodes can get the distance and angle of the target through certain sensors. Then the following equations (1) and (2) can describe the discrete-time nonlinear system:

$$x_k = f(x_{k-1}) + w_{k-1}, \tag{1}$$
$$z_k^i = h^i(x_k) + v_k^i, \quad i = 1, 2, \ldots, N, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector of the system, $z_k^i \in \mathbb{R}^m$ represents the observation vector of node i, k is the sample instant time, $f(\cdot)$ and $h^i(\cdot)$ are the nonlinear state transition function and measurement function respectively. w_k represents the process noise and v_k^i represents the observation noise, which covariance matrices are Q_k and R_k^i.

Each node can obtain the global optimal state estimation based on local measurements and neighboring information using the consensus Kalman filter. In the consensus process, the state estimation and intermediate states are exchanged among the nodes to reach the estimation consensus. Consider that each observation node should exchange their local state estimation at every sampling time instant, which cause the sensor network have to process a great deal of data. As a result, it will inevitably aggravate the burden of sensor network. Although exchanging information at every sampling instant contributes to the accuracy of the state estimation, it is a good choice to decrease the times of communication on the premise of satisfying the accuracy requirements in order to reduce the burden of the network.

III. STRONG TRACKING ADAPTIVE CUBATURE KALMAN FILTER

Cubature Kalman filtering (CKF) algorithm has been one of the most popular filter algorithms for nonlinear system [19]. However, the CKF algorithm has low precision when estimating the state of maneuvering target. The strong tracking adaptive cubature Kalman filtering (STA-CKF) algorithm can estimate the state of maneuvering target with high precision for the suboptimal fading factor and the adaptive factor are introduced to weaken the effect of the previous measurement on the current estimation and balance the weight of prediction and measurement of state respectively. In this section, the CKF algorithm is given first, then the STA-CKF is introduced.

A. CUBATURE KALMAN FILTER

In this section, the standard CKF filtering algorithm is introduced first.

1) PREDICT

For each sensor node i $(i \in N)$, which is included in the sensor network, one initialize the estimation of state $\hat{x}_0^i$ and error variance matrix P_0^i according to the following equations

$$\hat{x}_0^i = E(x_0), \tag{3}$$
$$P_0^i = E[(x_0 - \hat{x}_0^i)(x_0 - \hat{x}_0^i)^T, \tag{4}$$

where $i = 1, 2, \cdots, N$.

By using the Cholesky decomposition approach, one gets the equation

$$P_{k-1}^i = \sqrt{P_{k-1}^i}\sqrt{P_{k-1}^i}^T. \tag{5}$$

Then $2n$ cubature points can be obtained according to the following equation

$$x_{k-1}^{i,r} = \sqrt{P_{k-1}^i}\zeta_r + \hat{x}_{k-1}^i, \tag{6}$$

where $r = 1, 2, \cdots 2n$, ζ_r is the rth element of the following equation

$$\sqrt{n}\left\{\underbrace{\begin{pmatrix}1\\0\\ \cdot\\ \cdot\\0\end{pmatrix}\begin{pmatrix}0\\1\\ \cdot\\ \cdot\\0\end{pmatrix}\cdots\begin{pmatrix}0\\0\\ \cdot\\ \cdot\\1\end{pmatrix}\begin{pmatrix}-1\\0\\ \cdot\\ \cdot\\0\end{pmatrix}\begin{pmatrix}0\\-1\\ \cdot\\ \cdot\\0\end{pmatrix}\cdots\begin{pmatrix}0\\0\\ \cdot\\ \cdot\\-1\end{pmatrix}}_{2n}\right\}$$

Then the propagated cubature points can be obtained by the following transform function

$$x_{k|k-1}^{i,r} = f(x_{k-1}^{i,r}). \tag{7}$$

Next the one step prediction can be completed according to

$$\hat{x}_{k|k-1}^i = \frac{1}{2n}\sum_{r=1}^{2n} x_{k|k-1}^{i,r}, \tag{8}$$

$$P_{k|k-1}^i = \frac{1}{2n}\sum_{r=1}^{2n} x_{k|k-1}^{i,r}(x_{k|k-1}^{i,r})^T - \hat{x}_{k|k-1}^i(\hat{x}_{k|k-1}^i)^T + Q_{k-1}. \tag{9}$$

2) UPDATE

The obtained cubature points can be transformed into the forms as below based on the measurement function

$$Z_k^{i,r} = h^i(x_{k|k-1}^{i,r}), \tag{10}$$

where $r = 1, 2, \cdots 2n$, $Z_k^{i,r}$ is the transformed points. Then the prediction of measurement and error covariance can be obtained according to the equations below.

$$\hat{z}_k^i = \frac{1}{2n}\sum_{r=1}^{2n} Z_k^{i,r}, \tag{11}$$

$$P_{z_k,z_k}^i = \frac{1}{2n}\sum_{r=1}^{2n} Z_k^{i,r}(Z_k^{i,r})^T - \hat{z}_k^i(\hat{z}_k^i)^T + R_k^i, \tag{12}$$

$$P_{x_k,z_k}^i = \frac{1}{2n}\sum_{r=1}^{2n} x_{k|k-1}^{i,r}(Z_k^{i,r})^T - \hat{x}_{k|k-1}^i(\hat{z}_k^i)^T. \tag{13}$$

The filter gain can be calculated by

$$K_k^i = P_{x_k,z_k}^i(P_{z_k,z_k}^i)^{-1}. \tag{14}$$

In the final step, the state estimation and error variance of node i at time k can be updated according to the following equations

$$\hat{x}_k^i = \hat{x}_{k|k-1}^i + K_k^i(z_k^i - \hat{z}_k^i), \tag{15}$$
$$P_k^i = P_{k|k-1}^i - K_k^i P_{z_k,z_k}^i (K_k^i)^T. \tag{16}$$

Remark 1: CKF and UKF algorithm are based on Bayesian filter for nonlinear system. CKF gets $2n$ cubature points based on three-order spherical radial criterion and then calculates the Gaussian weighted integral. UKF calculates the Gaussian weights through a set of sigma points. The sigma points have one more center points than cubature points and those points occupy a higher weights. The center points is negative while approximating the high-dimensional nonlinear system. For example, the covariance is proved non-positive and indefinite due to those negative center sigma points with high weight. Meanwhile, EKF is another filter algorithm for nonlinear system and it has low precision when dealing with second-order nonlinear system due to it expands the nonlinear function into Taylor series and ignores the second-order and above terms. In that case, CKF is better than EKF.

B. STRONG TRACKING ADAPTIVE CUBATURE KALMAN FILTER ALGORITHM

The strong tracking adaptive cubature kalman filter algorithm has been one of the most popular algorithm for solving target tracking problem. H. W. Zhang et al. propose strong tracking SCKF based on adaptive CS model for maneuvering aircraft tracking [24] and R. Wang et al. propose a fusion algorithm based on adaptive cubature strong tracking filter for target tracking [25]. The general form of the strong tracking adaptive cubature Kalman filter is summarized in this section.

According to the CKF filtering algorithm, the one step prediction of state and variance can be obtained as follow equations

$$\hat{x}^i_{k|k-1} = \frac{1}{2n}\sum_{r=1}^{2n} x^{i,r}_{k|k-1}, \tag{17}$$

$$P^i_{k|k-1} = \frac{1}{2n}\sum_{r=1}^{2n} x^{i,r}_{k|k-1}(x^{i,r}_{k|k-1})^T - \hat{x}^i_{k|k-1}(\hat{x}^i_{k|k-1})^T + Q_k. \tag{18}$$

In order to track maneuvering targets accurately, we introduce a suboptimal fading factor λ_k to adjust the variance $P^i_{k+1|k}$ in real-time. Then adjust the filter gain to force the residual sequences orthogonal to achieve the strong tracking of the target. The suboptimal fading factor λ_k can be calculated by follow formulas.

$$\lambda_k = \begin{cases} \lambda_0, \ \lambda_0 > 1 \\ 1, \ \lambda_0 < 1 \end{cases} \quad \lambda_0 = \frac{tr[N_k]}{tr[M_k]}, \tag{19}$$

$$N_k = V_k - H_k Q_k H_k^T - \beta R, \tag{20}$$

$$M_k = P^i_{z_k,z_k} - V_k + N_k + (\beta - 1)R, \tag{21}$$

$$H_k = [P^i_{x_k,z_k}]^T [P^i_{z_k,z_k}]^{-1}. \tag{22}$$

where $tr[\cdot]$ is matrix trace. $\beta(\beta \geqslant 1)$ is the weaken factor, which can smooth the estimation. H_k is the Jacques matrix and V_k is the measurement residual covariance, which can be calculated by the equation

$$V_k = \begin{cases} \gamma_1\gamma_1^T, & k = 0 \\ \dfrac{\rho V_{k-1} + \gamma_k\gamma_k^T}{1+\rho}, & k \geqslant 1 \end{cases}. \tag{23}$$

where ρ is the forgotten factor and let $\rho = 0.95$ generally. γ_1 is the measurement covariance.

After introducing the suboptimal fading factor λ_k, the prediction step covariance matrix can be calculated by the follow equation according to equation (6), (10), (11) - (13)

$$P^i_{k|k-1} = \lambda_k \left(\frac{1}{2n}\sum_{r=1}^{2n} x^{i,r}_{k|k-1}(x^{i,r}_{k|k-1})^T - \hat{x}^i_{k|k-1}(\hat{x}^i_{k|k-1})^T \right) + Q_{k-1}. \tag{24}$$

Remark 2: The suboptimal fading factor λ_k is introduced to reduce the influence of past data on the current data and make the filter gain can be adjusted online and achieve targets stable tracking. Moreover, λ_k has no affect on the convergence speed of covariance due to it can be regarded as a constant, which can be calculate online.

Under normal measurement circumstance, the true value of measurement covariance equals the measured value. The formula below is satisfied.

$$\gamma_k\gamma_k^T = P_{z_k z_k} = P_{z_k} + R. \tag{25}$$

When bad measurement occurs, equation (25) cannot be satisfied. By introducing an adaptive factor to form adaptive cubature Kalman filter, the equation below is satisfied.

$$\gamma_k\gamma_k^T = P_{z_k z_k} = P_{z_k} + \mu_k R. \tag{26}$$

Assumed that the adaptive factor μ_k is bounded and it can be calculated by

$$\mu_k = \frac{tr[\gamma_k\gamma_k^T] - tr[P_z]}{tr[R]}. \tag{27}$$

The adaptive factor can balance the weight of prediction and measurement of state, and fade the effect of target maneuvering on filter. Thus improving the filter accuracy.

The ACKF algorithm is applied only when the target has maneuvering and the normal CKF algorithm is used under normal circumstance. The algorithm is chosen according to the measurement value. Assumed that the measurement value obey the χ^2 distribution of n: $Y \sim \chi^2(n)$. Select the confidence interval α: $P\{\chi^2 < \chi^2_{\alpha,M}\} = 1 - \alpha$, $0 < \alpha < 1$. The threshold of the measured value detection can be determined according to the confidence level and the criterion for adaptive selection switching is: CKF: $Y_k \leqslant \chi^2_{\alpha,M}$;ACKF: $Y_k > \chi^2_{\alpha,M}$.

Next update the measurement to achieve STA-CKF algorithm, one can get

$$P^i_{z_k,z_k} = \frac{1}{2n}\sum_{r=1}^{2n} Z^{i,r}_k(Z^{i,r}_k)^T - \hat{z}^i_k(\hat{z}^i_k)^T + \mu_k R^i_k, \tag{28}$$

$$P^i_{x_k,z_k} = \frac{1}{2n}\sum_{r=1}^{2n} x^{i,r}_{k|k-1}(Z^{i,r}_k)^T - \hat{x}^i_{k|k-1}(\hat{z}^i_k)^T, \tag{29}$$

$$K^i_k = P^i_{x_k,z_k}(P^i_{z_k,z_k})^{-1}, \tag{30}$$

$$\hat{x}^i_k = \hat{x}^i_{k|k-1} + K^i_k(z^i_k - \hat{z}^i_k), \tag{31}$$

$$P^i_k = P^i_{k|k-1} - K^i_k P^i_{z_k,z_k}(K^i_k)^T. \tag{32}$$

IV. CONSENSUS ALGORITHM AND CONSENSUS BASED STRONG TRACKING ADAPTIVE CUBATURE KALMAN FILTER

A. CONSENSUS ALGORITHM

The consensus method has received extensive attention due to its wide applications [26]–[28]. The distributed state estimation problem for sensor network can be treated as a consensus problem [15]. In the local filtering stage, each node gets an estimation of the target state by applying the filtering algorithm based on the local prediction and measurement information. In the consensus iteration process, each sensor node interacts information with its neighbor. In the final of the consensus process, each node reaches consensus. The state estimation $\hat{x}^i_k$ and covariance matrix P^i_k are selected as information pairs, which can reach a same level in the final of the consensus process. According to [12], the consensus method can be summarized as follow equation.

$$(\hat{x}^*_k, P^*_k) = \lim_{l\to\infty}(\hat{x}^i_{k,l}, P^i_{k,l}), \tag{33}$$

where l represents the iteration times of consensus. $\hat{x}^i_k$ and P^i_k are the state estimation and covariance matrix of node i at time k. $\hat{x}^i_{k,l}$ and $P^i_{k,l}$ are the variables of lth step in the consensus process, which satisfies

$$\hat{x}^i_{k,l+1} = \pi^{i,i}\hat{x}^i_{k,l} + \sum_{j\in N_i}\pi^{i,j}\hat{x}^j_{k,l}, \tag{34}$$

$$P^i_{k,l+1} = \pi^{i,i}P^i_{k,l} + \sum_{j\in N_i}\pi^{i,j}P^j_{k,l}, \tag{35}$$

where N_i is the neighbor of node i. $\pi^{i,j}(j \in N_i)$ is the consensus weight, which satisfy

$$\pi^{i,i} + \sum\nolimits_{j\in N_i}\pi^{i,j} = 1, \pi^{i,j} \geqslant 0. \tag{36}$$

Based on the nonlinear filtering algorithm, each node obtains the local state estimation. In the consensus step, each node interacts estimation information with its neighbor. Finally, each node gets estimation of the target state according to the local measurements and information from neighbor to reach the estimation consensus. Noticed that the distributed sensor nodes only exchange information with its neighbor during the consensus process, which can decrease the burden of communication network. In the centralized model, the center node cannot deal with the huge real-time dataflow when the communication network is large enough.

B. CONSENSUS BASED STRONG TRACKING ADAPTIVE CUBATURE KALMAN FILTER ALGORITHM AND SOME OTHER CONSENSUS BASED ALGORITHMS

According to equation (28)-(32), we can complete the CSTA-CKF algorithm by following two steps. First, one initialize $\hat{x}^i_{k,0} = \hat{x}^i_k, P^i_{k,0} = P^i_k$. For $l = 0, 1, \cdots, L-1$, then broadcast the information pairs $(\hat{x}^i_{k,l}, P^i_{k,l})$ with its neighbor and fuse the state estimation and error covariance according to the consensus algorithm below

$$\hat{x}^i_{k,l+1} = \pi^{i,i}\hat{x}^i_{k,l} + \sum_{j\in N_i}\pi^{i,j}\hat{x}^j_{k,l}, \tag{37}$$

$$P^i_{k,l+1} = \pi^{i,i}P^i_{k,l} + \sum_{j\in N_i}\pi^{i,j}P^j_{k,l}. \tag{38}$$

Finally, the state estimation of target based on the CSTA-CKF algorithm can be obtained by

$$\hat{x}^i_k = \hat{x}^i_{k,L}, \tag{39}$$

$$P^i_k = P^i_{k,L}. \tag{40}$$

Remark 3: The Algorithm 1 is summarized under the circumstance of fixed topology. The Algorithm is also suitable for switching topologies with the conditions that the isolated nodes do not exist in the switched topologies and the topology weights satisfying $W_k 1 = 1, 1^T W_k = 1^T$.

The CSTA-CKF proposed in this paper is summarized in Algorithm 1. For comparison, the consensus based extended Kalman filtering (CEKF) algorithm is summarized in Algorithm 2 and the consensus based unscented Kalman filtering (CUKF) algorithm is summarized in Algorithm 3.

V. STOCHASTIC BOUNDEDNESS OF THE ESTIMATION ERROR

The most commonly used criterion for the filter performance is the algorithm's boundedness in the sense of mean square error (MSE). The estimation error of the standard CKF algorithm has been proven to be bounded. In this section, some basic preliminaries on stochastic boundedness are presented and the boundedness proof of the CSTA-CKF algorithm under mean square error is proposed. Consider the system described by equation (1) and (2). Motivated by the technique in [2], [4], [28], we derive a pseudo system matrix and observation matrix in this sensor network in order to analyze the boundedness of CSTA-CKF algorithm. S^i_{k-1} and O^i_k can be defined by

$$S^i_{k-1} \triangleq (P^i_{x_{k-1}x_{k|k-1}})^T(P^i_{k-1})^{-1}, \tag{41}$$

$$O^i_k \triangleq (P^i_{x_k z_k})^T(P^i_{k|k-1})^{-1}. \tag{42}$$

Meanwhile, one introduce the compensation diagonal matrices $\alpha^i_k = diag(\alpha^i_{k,1}, \alpha^i_{k,2}, \ldots, \alpha^i_{k,n})$, $\beta^i_k = diag(\beta^i_{k,1}, \beta^i_{k,2}, \ldots, \beta^i_{k,n})$ to compensensate the error caused by the approximation. Then the system can be rewritten as

$$x_k = \alpha^i_{k-1}F^i_{k-1}x_{k-1} + \omega_{k-1}, \tag{43}$$

$$z^i_k = \beta^i_k H^i_k x_k + v^i_k, \quad i = 1, 2, \ldots, N. \tag{44}$$

Algorithm 1 Consensus Based STACKF Algorithm

1) Perform the local CKF filter and prediction step
$\hat{x}^i_{k|k-1} = \frac{1}{2n}\sum_{r=1}^{2n} x^{i,r}_{k|k-1}$,
$P^i_{k|k-1} = \frac{1}{2n}\sum_{r=1}^{2n} x^{i,r}_{k|k-1}(x^{i,r}_{k|k-1})^T - \hat{x}^i_{k|k-1}(\hat{x}^i_{k|k-1})^T + Q_{k-1}$.
2) Introduce a suboptimal fading factor λ_k and calculate the covariance
$P^i_{k|k-1} = \lambda_k\left(\frac{1}{2n}\sum_{r=1}^{2n} x^{i,r}_{k|k-1}(x^{i,r}_{k|k-1})^T - \hat{x}^i_{k|k-1}(\hat{x}^i_{k|k-1})^T\right) + Q_{k-1}$.
3) Introduce an adaptive factor μ_k, which satisfies
$\mu_k = \frac{tr[\gamma_k\gamma_k^T] - tr[P_z]}{tr[R]}$.
4) Update the measurement to achieve STACKF algorithm.
$P^i_{z_k,z_k} = \frac{1}{2n}\sum_{r=1}^{2n} Z^{i,r}_k(Z^{i,r}_k)^T - \hat{z}^i_k(\hat{z}^i_k)^T + \mu_k R^i_k$,
$P^i_{x_k,z_k} = \frac{1}{2n}\sum_{r=1}^{2n} x^{i,r}_{k|k-1}(Z^{i,r}_k)^T - \hat{x}^i_{k|k-1}(\hat{z}^i_k)^T$,
$K^i_k = P^i_{x_k,z_k}(P^i_{z_k,z_k})^{-1}$,
$\hat{x}^i_k = \hat{x}^i_{k|k-1} + K^i_k(z^i_k - \hat{z}^i_k)$,
$P^i_k = P^i_{k|k-1} - K^i_k P^i_{z_k,z_k}(K^i_k)^T$.
5) Apply consensus algorithm for $l = 0, 1, \cdots, L-1$.
   a) First initialize the $\hat{x}^i_{k,0} = \hat{x}^i_k$, $P^i_{k,0} = P^i_k$.
   b) Each node exchange the $(x^i_{k,l}, P^i_{k,l})$ information with its neighbors.
   c) Combine the state estimation and error covariance according to the consensus algorithm:
   $\hat{x}^i_{k,l+1} = \pi^{i,i}\hat{x}^i_{k,l} + \sum_{j\in N_i}\pi^{i,j}\hat{x}^j_{k,l}$,
   $P^i_{k,l+1} = \pi^{i,i}P^i_{k,l} + \sum_{j\in N_i}\pi^{i,j}P^j_{k,l}$.
6) Then the state estimation can be obtained by
$\hat{x}^i_k = \hat{x}^i_{k,L}$, $P^i_k = P^i_{k,l}$.

Assumption 1: Provided that $\underline{\alpha}, \underline{f}, \underline{\beta}, \underline{h}, \bar{\alpha}, \bar{f}, \bar{\beta}, \bar{h} \neq 0$, and then we can get the inequalities (45), (46), (47), (48) when each $k \geqslant 0$:

$$\underline{\alpha}^2 I \leqslant \alpha^i_k(\alpha^i_k)^T \leqslant \bar{\alpha}^2 I, \tag{45}$$
$$\underline{f}^2 I \leqslant F^i_k(F^i_k)^T \leqslant \bar{f}^2 I, \tag{46}$$
$$\underline{\beta}^2 I \leqslant \beta^i_k(\beta^i_k)^T \leqslant \bar{\beta}^2 I, \tag{47}$$
$$\underline{h}^2 I \leqslant H^i_k(H^i_k)^T \leqslant \bar{h}^2 I. \tag{48}$$

Assumption 2: There exist positive real numbers $p_{\min}$, $p_{\max} > 0, \underline{p}, \bar{p} > 0, \underline{r}, \bar{r} > 0$, so that the following inequalities are fulfilled:

$$p_{\min} \leq p^i \leq p_{\max}, \tag{49}$$
$$\underline{p}I \leqslant P^i_k \leqslant \bar{p}I, \tag{50}$$
$$\underline{q}I \leqslant Q_k \leqslant \bar{q}I, \tag{51}$$
$$\underline{r}I \leqslant \mu^i_k R^i_k \leqslant \bar{r}I. \tag{52}$$

Algorithm 2 Consensus Based EKF Algorithm

1) Calculate the one step prediction based on the EKF
$x_k = f(x_{k-1})$,
$P^i_{k|k-1} = FP^i_{k-1}F' + Q$.
2) Update the state and covariance matrix
$K^i_k = P^i_{k|k-1}H_i'(H_i P^i_{k|k-1}H_i' + R_i)^{-1}$,
$\hat{x}^i_k = \hat{x}^i_{k|k-1} + K^i_k(Z_{k,i} - h_i(\hat{x}^i_{k|k-1}, 0))$,
$P^i_k = (I - K^i_k H_i)P^i_{k|k-1}$.
3) Initialize the information pairs with the follow equations
$\hat{x}^i_{k,0} = x^i_k$, $P^i_{k,0} = P^i_k$.
4) Apply the consensus algorithm.
   a) Each node spread the local information of $\hat{x}^i_{k,l}$ and $P^i_{k,l}$ to neighbors
   b) Each node obtain the $(x^i_{k,l}, P^i_{k,l})$ information from neighbors.
   c) For $l = 0, 1, \ldots, L-1$, fuse the information from local and neighbors according to following equations:
   $\hat{x}^i_{k,l+1} = \pi^{i,i}\hat{x}^i_{k,l} + \sum_{j\in N_i}\pi^{i,j}\hat{x}^j_{k,l}$,
   $P^i_{k,l+1} = \pi^{i,i}P^i_{k,l} + \sum_{j\in N_i}\pi^{i,j}P^j_{k,l}$.
5) Set the state estimation as
$\hat{x}^i_k = \hat{x}^i_{k,L}$, $P^i_k = P^i_{k,l}$.

Lemma 1: [29] Provided that ς_k is a stochastic variable and $V(\varsigma_k)$ is a stochastic process, then there exist real numbers $v_{\min} > 0$, $v_{\max} > 0$ and $\mu > 0, 0 < \lambda \leqslant 1$, such that

$$v_{\min}\|\varsigma_k\|^2 \leqslant V(\varsigma_k) \leqslant v_{\max}\|\varsigma_k\|^2, \tag{53}$$
$$E\{V(\varsigma_k)|\varsigma_{k-1}\} - V(\varsigma_{k-1}) \leqslant \mu - \lambda V(\varsigma_{k-1}), \tag{54}$$

are satisfied. Moreover, the stochastic process can be proved to be bounded in the sense of mean square error, which means

$$E\left\{\|\varsigma_k\|^2\right\} \leqslant \frac{v_{\max}}{v_{\min}} E\left\{\|\varsigma_0\|^2\right\}(1-\lambda)^k + \frac{\mu}{v_{\min}}\sum_{i=1}^{k-1}(1-\lambda)^i. \tag{55}$$

Lemma 2: [30] Provided that $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$, and $C \in \mathbb{R}^{n\times n}$, if $A > 0$ and $C > 0$, one can get

$$A^{-1} > (A + C)^{-1}. \tag{56}$$
$$A^{-1} > B(B^T AB + C)^{-1}B^T. \tag{57}$$

Lemma 3: [13] If Assumption 1 and Assumption 2 hold, then there exist definite positive matrices $\underline{Y}$, $\bar{Y}$, $\underline{Y}^+$ and $\bar{Y}^+$, which satisfied that $0 < \underline{Y} \leqslant Y^i_k \leqslant \bar{Y}$ and $0 < \underline{Y}^+ \leqslant Y^i_{k|k-1} \leqslant \bar{Y}^+$ for any $k \geqslant 1$ and $i \in N$.

Theorem 1: Consider a nonlinear system described by (1) and (2). In the case that Assumption 1 and Assumption 2 are established, the topology remains unchanged during the consensus iteration and the consensus weight matrix $\prod$ is row stochastic and primitive at time instant k, the error of state

Algorithm 3 Consensus Based UKF Algorithm

1) Collect a set of points and weights ω^i using sampling rules of UKF.
$X^i_{k-1}=[\hat{X}_{k-1}\ \hat{X}_{k-1}+\sqrt{(n+\lambda)P_{k-1}}\ \hat{X}_{k-1}-\sqrt{(n+\lambda)P_{k-1}}]$.
2) Calculate one-step prediction of points set.
$\hat{X}_{k|k-1}=\sum_{i=1}^{2n}\omega^i X^i_{k|k-1}$,
$P_{k|k-1}=\sum_{i=1}^{2n}\omega^i[\hat{X}_{k|k-1}-X^i_{k|k-1}][\hat{X}_{k|k-1}-X^i_{k|k-1}]^T+Q$.
3) Calculate the prediction and covariance of the state.
$\hat{X}_{k|k-1}=\sum_{i=1}^{2n}\omega^i X^i_{k|k-1}$,
$P_{k|k-1}=\sum_{i=1}^{2n}\omega^i[\hat{X}_{k|k-1}-X^i_{k|k-1}][\hat{X}_{k|k-1}-X^i_{k|k-1}]^T+Q$.
4) Generate a new set of points according to the one-step prediction based on UT transform.
$X^i_{k|k-1}=[\hat{X}_{k|k-1}\ \hat{X}_{k|k-1}+\sqrt{(n+\lambda)P_{k|k-1}}\ \hat{X}_{k|k-1}-\sqrt{(n+\lambda)P_{k|k-1}}]$.
5) For $i=1,2,\cdots,2n+1$, apply the points set obtained from (4) into the measurement equation and get the measurement value.
$Z^i_{k|k-1}=h(X^i_{k|k-1})$.
6) Then calculate the covariance and mean prediction of system by following equations
$\bar{Z}_{k|k-1}=\sum_{i=0}^{2n}\omega^i Z^i_{k|k-1}$,
$P_{z_k,z_k}=\sum_{i=0}^{2n}\omega^i[Z^i_{k|k-1}-\bar{Z}_{k|k-1}][Z^i_{k|k-1}-\bar{Z}_{k|k-1}]^T+R$,
$P_{x_k,z_k}=\sum_{i=0}^{2n}\omega^i[X^i_{k|k-1}-\bar{Z}_{k|k-1}][Z^i_{k|k-1}-\bar{Z}_{k|k-1}]^T$.
7) The Kalman gain can be obtained by
$K_k=P_{x_k,z_k}P^{-1}_{z_k,z_k}$.
8) Next update the prediction and covariance of state by
$\hat{X}_k=\hat{X}_{k|k-1}+K_k[Z_k-\hat{Z}_{k|k-1}]$,
$P_k=P_{k|k-1}-K_kP_{z_k,z_k}K_k^T$.
9) Apply the consensus algorithm.
   a) Each node spread the local information of $\hat{x}^i_{k,l}$ and $P^i_{k,l}$ to neighbors.
   b) Each node obtain the $(x^i_{k,l},P^i_{k,l})$ information from neighbors.
   c) For $l=0,1,\ldots,L-1$, fuse the information from local and neighbors according to following equations: $\hat{x}^i_{k,l+1}=\pi^{i,i}\hat{x}^i_{k,l}+\sum_{j\in N_i}\pi^{i,j}\hat{x}^j_{k,l}$,
   $P^i_{k,l+1}=\pi^{i,i}P^i_{k,l}+\sum_{j\in N_i}\pi^{i,j}P^j_{k,l}$.
10) Set the state estimation as
$\hat{x}^i_k=\hat{x}^i_{k,L}$, $P^i_k=P^i_{k,l}$.

estimation satisfies

$$E\left\{\left\|\tilde{x}^i_k\right\|^2\right\}\leqslant e,$$

where $\tilde{x}^i_k\triangleq x_k-\hat{x}^i_k$, e represents a constant value.

Proof: At node i, first define the prediction error as $\tilde{x}^i_{k+1|k}=x_{k+1}-\hat{x}^i_{k+1|k}$, then define the estimation error as $\tilde{x}^i_k=x_k-\hat{x}^i_k$. Besides, one define $\tilde{x}_{k+1|k}=col(\tilde{x}^i_{k+1|k},\ i\in N)$, $\tilde{x}_k=col(x^i_k,\ i\in N)$.

Let $P=(p^1,\cdots,p^i,\cdots p^N)^T$ be the Perron-Frobenius left eigenvector of the matrix W^L. Where p^i represents a positive element and satisfies $P^TW^L=P^T$, then one can get

$$\sum_{j\in N}p^j\omega_L^{i,j}=p^i. \tag{58}$$

A stochastic process is selected as below:

$$V(\tilde{x}_{k+1|k})=\sum p^i(\tilde{x}^i_{k+1|k})^T(P^i_{k+1|k})^{-1}\tilde{x}^i_{k+1|k}. \tag{59}$$

According to the Assumption 1 and Lemma 2, one can get

$$(\bar{p}\bar{\alpha}\bar{f}^2+\bar{q})^{-1}I\leqslant(P^i_{k+1|k})^{-1}\leqslant(\underline{p}\underline{\alpha}^2\underline{f}^2+\underline{q})^{-1}I. \tag{60}$$

From Assumption 2, we get

$$\frac{p_{\min}}{\bar{p}\bar{\alpha}\bar{f}^2+\bar{q}}\left\|\tilde{x}_{k+1|k}\right\|^2\leqslant V(\tilde{x}_{k+1|k})\leqslant\frac{p_{\max}}{\underline{p}\underline{\alpha}^2\underline{f}^2+\underline{q}}\left\|\tilde{x}_{k+1|k}\right\|^2. \tag{61}$$

Meanwhile, one can get

$$\begin{aligned}
\tilde{x}^i_{k+1|k}
&=x_{k+1}-x^i_{k+1|k},\\
&=\alpha^i_kS^i_k(x_k-\hat{x}^i_k)+w_k,\\
&=\alpha^i_kS^i_k(\sum_{j\in N}w_L^{i,j}x_k-\sum_{j\in N}w_L^{i,j}\hat{x}^j_{k,0})+w_k,\\
&=\alpha^i_kS^i_k(\sum_{j\in N}w_L^{i,j}(x_k-\hat{x}^j_{k,0}))+w_k,\\
&=\alpha^i_kS^i_k[\sum_{j\in N}w_L^{i,j}(x_k-\hat{x}^j_{k|k-1}-K^j_k(z^j_k-\hat{z}^j_k))]+w_k,\\
&=\alpha^i_kS^i_k[\sum_{j\in N}w_L^{i,j}(I-K^j_k\beta^j_kO^j_k)(x_k-x^j_{k|k-1})\\
&\quad-\sum_{j\in N}w_L^{i,j}K^j_kv^j_k]+w_k,\\
&=\sum_{j\in N}\Gamma^{i,j}_k\tilde{x}^j_{k|k-1}+\sum_{j\in N}\Xi^{i,j}_kv^j_k+w_k.
\end{aligned} \tag{62}$$

where $\Gamma^{i,j}_k=w_L^{i,j}\alpha^i_kS^i_k(I-K^j_k\beta^j_kO^j_k)$, $\Xi^{i,j}_k=-w_L^{i,j}\alpha^i_kS^i_kK^j_k$.

Then, substituting (62) to (59), one can get

$$\begin{aligned}
&E\{(V(\tilde{x}_{k+1|k})|\tilde{x}_{k|k-1})\}\\
&\quad=E\{\sum_{i\in N}p^i(\tilde{x}^i_{k+1|k})^T(P^i_{k+1|k})^{-1}\tilde{x}^i_{k+1|k}|\tilde{x}_{k|k-1}\},\\
&\quad=\Phi^x_{k+1}+\Phi^v_{k+1}+\Phi^w_{k+1}.
\end{aligned} \tag{63}$$

where

$$\begin{aligned}
\Phi^x_{k+1}&=E\{\sum_{i\in N}p^i(\sum_{j\in N}\Gamma^{i,j}_k\tilde{x}^j_{k|k-1})^T(P^i_{k+1|k})^{-1}\\
&\quad\times(\sum_{j\in N}\Gamma^{i,j}_k\tilde{x}^j_{k|k-1})|\tilde{x}_{k|k-1}\}.
\end{aligned} \tag{64}$$

$$\Phi_{k+1}^{v} = E\{\sum_{i\in N} p^i(\sum_{j\in N} \Xi_{i,j}^{k} v_k^j)^T (P_{k+1|k}^i)^{-1} \times(\sum_{j\in N} \Xi_k^{i,j} v_k^j)|\tilde{x}_{k|k-1}\}. \quad (65)$$

$$\Phi_{k+1}^{w} = E\{\sum_{i\in N} p^i w_k^T (P_{k+1|k}^i)^{-1} w_k|\tilde{x}_{k|k-1}\}. \quad (66)$$

Under the circumstance of noise-free. One can get

$$(P_{k+1|k}^i)^{-1} = [\alpha_k^i S_k^i P_k^i(\alpha_k^i S_k^i)^T + Q_k]^{-1} \leqslant \tilde{\tau}(\alpha_k^i S_k^i)^{-T}(P_k^i)^{-1}(\alpha_k^i S_k^i)^{-1}, \quad (67)$$

where $\tilde{\tau} < 1$.

$$\begin{aligned}\Phi_{k+1}^{x} &= E\{\sum_{i\in N} p^i(\sum_{j\in N}\Gamma_k^{i,j}\tilde{x}_{k|k-1}^j)^T (P_{k+1|k}^i)^{-1} \times(\sum_{j\in N}\Gamma_k^{i,j}\tilde{x}_{k|k-1}^j)|\tilde{x}_{k|k-1}\},\\ &\leqslant \tilde{\tau}E\{\sum_{i\in N} p^i(\sum_{j\in N} w_L^{i,j}(P_{k|k-1}^j)^{-1}x_{k|k-1}^j)^T (P_k^i)^{-1} \times\sum_{j\in N} w_L^{i,j}(P_{k|k-1}^j)^{-1}x_{k|k-1}^i\}. \quad (68)\end{aligned}$$

Then define $\lambda = 1 - \tilde{\tau}$,according to $(P_k^i)^{-1} \geqslant \sum_{j\in\{N_i,i\}} w_L^{i,j}(P_{k|k-1}^j)^{-1}$, we can write that

$$\begin{aligned}\Phi_{k+1}^{x} &\leqslant \tilde{\tau}E\{\sum_{i\in N} p^i \sum_{j\in N} w_L^{i,j}(\tilde{x}_{k|k-1}^j)^T (P_{k|k-1}^j)^{-1}x_{k|k-1}^j\},\\ &\leqslant \tilde{\tau}E\{\sum_{i\in N} p^i(\tilde{x}_{k|k-1}^j)^T(P_{k|k-1}^j)^{-1}x_{k|k-1}^j\}\\ &= (1-\lambda)E\{V(\tilde{x}_{k|k-1})\}. \quad (69)\end{aligned}$$

Consider the process noise and measurement noise, one can get

$$\begin{aligned}&\Phi_{k+1}^{v} + \Phi_{k+1}^{w}\\ &= E\{\sum_{i\in N} p^i(\sum_{j\in N}\Xi_k^{i,j}V_k^j)^T (P_{k+1|k}^i)^{-1}(\sum_{j\in N}\Xi_k^{i,j}V_k^j)\\ &\quad + \sum_{i\in N} p^i w_k^T (P_{k+1|k}^i)^{-1} w_k|\tilde{x}_{k|k-1}\}\\ &\leqslant \|\bar{Y}^+\|\{\left\|\underline{Y}\right\|^{-1}\bar{\alpha}^2\bar{f}^2\bar{\beta}^2\bar{h}^2N^2\bar{r}/\underline{r}^2[\sum_{i,j\in N}p^i(\omega_L^{i,j})^2]m\\ &\quad + \bar{q}(\sum_{i\in N}p^i)n\} \triangleq \mu \quad (70)\end{aligned}$$

where n and m represent the dimensions of x_k and z_k^i, N is the number of the nodes.

According to equation (70) and (54),

$$E\{V_{k+1}(\tilde{x}_{k+1|k})|\tilde{x}_{k|k-1}\} - V_k(\tilde{x}_{k|k-1}) \leqslant \mu - \lambda V_k(\tilde{x}_{k|k-1}). \quad (71)$$

According to Lemma 3, it can be concluded that the stochastic process $\tilde{x}_{k+1|k}$ is bounded in the sense of mean square error. So the $\tilde{x}_{k+1|k}^i$ is bounded in the sense of mean square error. Besides, according to the equation

$$\tilde{x}_{k+1|k}^i = \alpha_k^i F_k^i(x_k - \hat{x}_k^i) + w_k,$$

and finally one gets

$$E\left\{\left\|\tilde{x}_k^i\right\|^2\right\} \leqslant \underline{\alpha}^{-2}\underline{f}^{-2}(E\left\{\left\|\tilde{x}_{k+1|k}^i\right\|^2\right\} - E\left\{\|w_k\|^2\right\}). \quad (72)$$

Taking the same method, it shows that w_k is also bounded in the sense of mean square error. Finally, We can prove that the estimation error is bounded in the sense of mean square error. The proof is completed. ■

VI. ILLUSTRATIVE EXAMPLE

In this section, a simulation example is given to compare the availability of the STACKF, CEKF, CUKF, CCKF and CSTACKF. The mobile sensor network on the ground is designed to track the target in the sky. The network consist one moving node and three static nodes. The initial position of the static nodes is (2500,1000) km, (3500,1000) km, (4500,1000) km and the position of moving node is (1500,0) km with a 10 km/s speed in the X direction and 20 km/s speed in the Y direction. The height of the sensor node is 0.2 km. The initial position of the target is (2000,10000) km with a height of 0.5 km. Each sensor node could obtain the directions and distance of the target.

The target's state can be described as $x_k = [\,\xi_k\ \dot{\xi}_k\ \eta_k\ \dot{\eta}_k\,]$. ξ and η are the position in the X direction and Y direction, $\dot{\xi}$ and $\dot{\eta}$ is the velocity in the X direction and Y direction. The turning speed can be represented as Ω and w_{k-1} is the process noise. The angular speed of turning is $0.5°/s$ and covariance of the process noise is $10^{-5}I_2$. The time interval for each estimation is 0.5s.

The measurement function can be express as the equation

$$z_k^i = \begin{bmatrix} r_k^i \\ \psi_k^i \\ \theta_k^i \end{bmatrix} = \begin{bmatrix} \sqrt{(\xi_k - x_k^i)^2 + (\eta_k - y_k^i)^2} \\ \arctan((\eta_k - y_k^i)/(\xi_k - x_k^i)) \\ \arctan(\Delta h/d) \end{bmatrix} + v_k^i, \quad (73)$$

where $d = \sqrt{(\xi_k - x_k^i)^2 + (\eta_k - y_k^i)^2 + \Delta h^2}$, Δh is the height difference between target and sensor node. v_k^i is the measurement noise at time k. ψ_k^i and θ_k^i is the azimuth angle and pitch angle, respectively. r_k^i represents the distance between target and sensor node.

The topology of the sensor network can be illustrated by Fig. 1 and each node can only communicates with its neighbor in the consensus process. Besides, $\pi^{i,j}$ represents the consensus weights in the consensus weight matrix Π. According to the Metropolis weights rule, we get

$$\pi^{i,j} = \begin{cases} \dfrac{i}{1+\max\{d_i, d_j\}}, & if\ \{i,j\} \in E\ , \\ 1 - \sum\limits_{i,j\in E}\pi^{i,j}, & if\ i = j, \\ 0, & \text{otherwise}. \end{cases} \quad (74)$$

where d_i is the degree of node i.

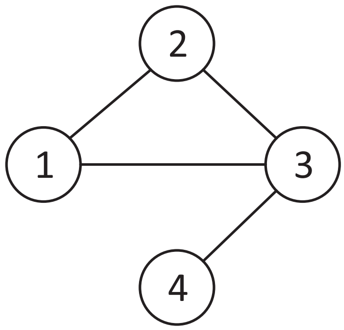


FIGURE 1. Topology of sensor network.

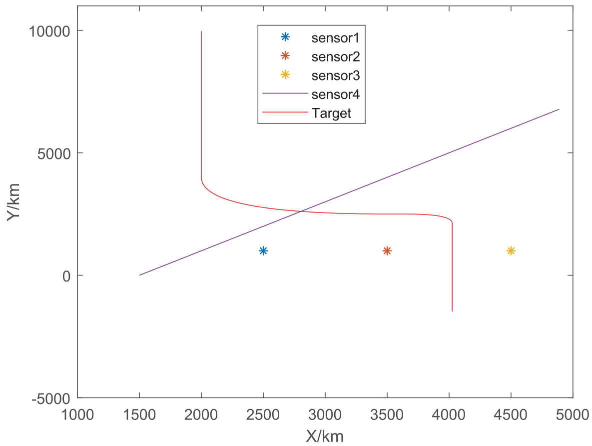


FIGURE 2. The trajectory of sensor and target in 2-D.

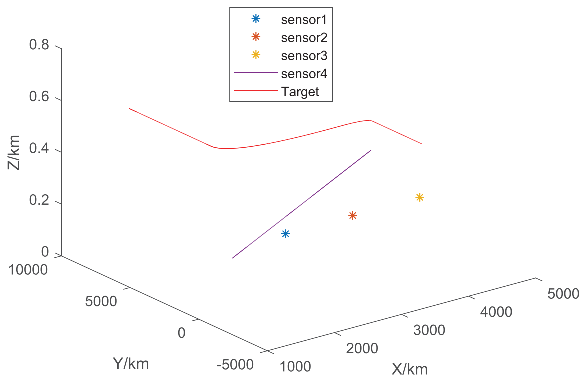


FIGURE 3. The trajectory of sensor and target in 3-D.

The consensus weight matrix can be selected as

$$\Pi = \begin{bmatrix} \frac{2}{15} & \frac{1}{3} & \frac{1}{4} & 0 \\ \frac{1}{3} & \frac{5}{12} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix} \tag{75}$$

Figs. 2 and 3 illustrate the sensor 1's position, the sensor 2's position, the sensor 3's position, the sensor 4's trajectory and target's trajectory in 2-dimensional and 3-dimensional respectively. Note that the sensor node and the target are not at the same height, so it can be guaranteed that the trajectory of the moving node does not intersect with the trajectory of the target. Besides, it can be seen from Fig. 2 that the target makes maneuvering at the position (2000,4000) km and position (4000,2500) km.

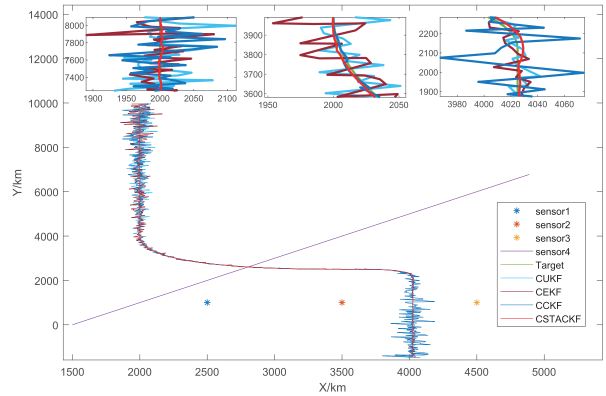


FIGURE 4. Filtered trajectory in 2-D.

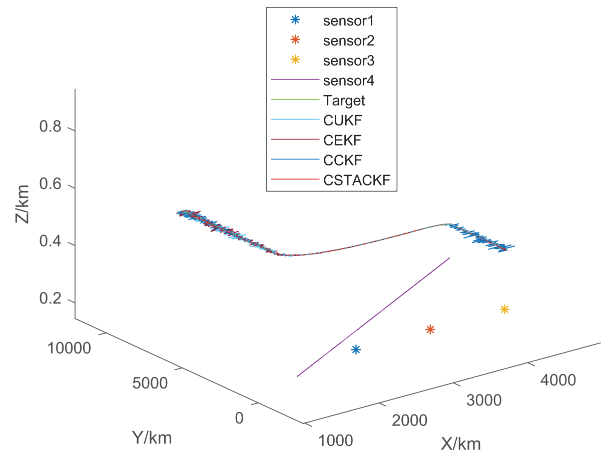


FIGURE 5. Filtered trajectory in 3-D.

The filtered trajectory in 2-dimensional and 3-dimensional based on the CEKF, CUKF, CCKF and CSTA-CKF are shown in Figs. 4 and 5, respectively. At the beginning, there exists a large tracking error between target trajectory and filtered trajectory. The tracking error of CSTA-CKF are approximately equivalent to those of the other filters. After a few steps, however, it can be seen from the simulation results that the CSTA-CKF algorithm has the best tracking performance among these algorithms when tracking the maneuvering target.

Fig. 6 shows the azimuth angle measured by four sensor nodes based on the CSTA-CKF algorithm. The azimuth angle measured by four sensors are different for the position of the four sensor nodes are different. The azimuth angle between sensor 4 and target is 180 degree when t = 200s, which can be seen from Fig. 6. Besides, sensor 1, sensor 2 and target form a horizontal line, which result the azimuth become 0 when t = 370s. The pitch angle is close to 0 for the height difference between target and sensor nodes is relatively small, so the picture of pitch angle is not given.

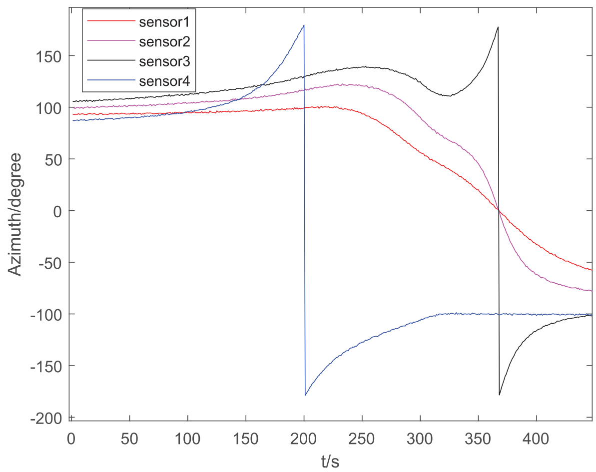


FIGURE 6. The azimuth measured by sensors.

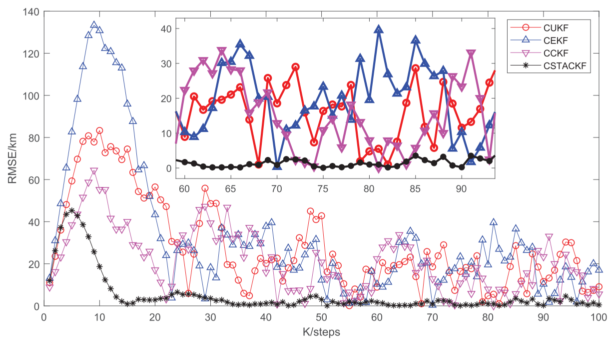


FIGURE 7. Comparison of RMSE.

In order to illustrate the performance of the proposed filtering algorithm, Monte Carlo simulation has been performed to validate the efficiency of the algorithm. The root-mean-square-error (RMSE) for node i at time instant k is defined as

$$RMSE_k = \left\{ \frac{1}{M} \sum_{n=1}^{M} \left[(\xi_{k|n} - \hat{\xi}_{k|n})^2 + (\eta_{k|n} - \hat{\eta}_{k|n})^2 \right] \right\}^{1/2},$$

where $[\,\xi_{k|n}\ \eta_{k|n}\,]^T$ and $[\,\hat{\xi}_{k|n}\ \hat{\eta}_{k|n}\,]^T$ are the true and estimated states at the n-th Monte Carlo algorithm performing respectively, M is the times of Monte Carlo algorithm performs.

Fig. 7 shows the RMSE of CUKF, CEKF, CCKF and CSTA-CKF. The error plots show the RMSE values averaged over 100 Monte Carlo algorithm runs. At the beginning, the RMSE of those algorithms are all relatively large. After a few steps, the RMSE of the CSTA-CKF become lower than the other algorithms by 60%. The convergence rate of CCKF and CSTA-CKF algorithm is shown in Fig. 8. Compared to the CCKF algorithm, the convergence rate of the CSTA-CKF is reduced by 50 %.

As shown in Fig. 9, another 300 times Monte Carlo simulation is conducted to compare the RMSE of centralized algorithm with CSTA-CKF algorithm. At the beginning, the RMSE of CSTA-CKF is larger than the RMSE of

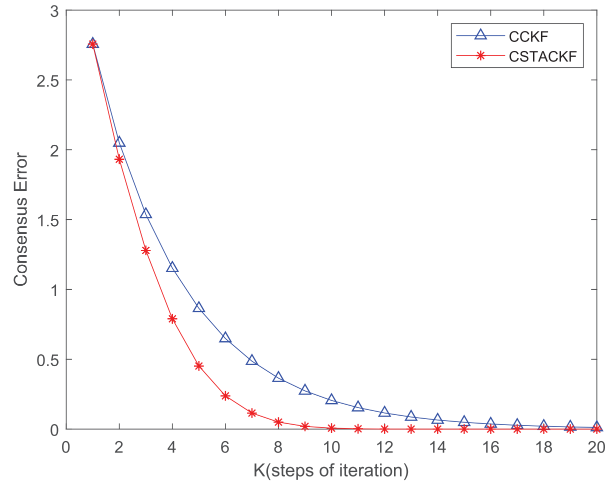


FIGURE 8. Convergence rate of consensus algorithm between CCKF and CSTA-CKF.

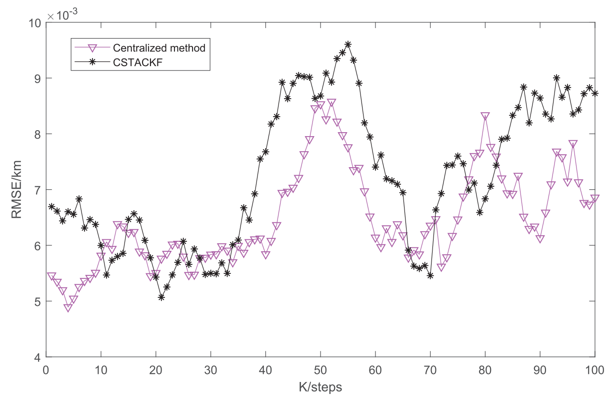


FIGURE 9. RMSE comparison of centralized method and CSTA-CKF.

centralized method. After some steps, the RMSE difference of both algorithms is very small and it can be kept within a reasonable range. In that sense, CSTA-CKF algorithm can achieve the same effect as centralized method.

VII. CONCLUSION

In this paper, CSTA-CKF algorithm was proposed to deal with the distributed state estimation problem for nonlinear system. The algorithm can track the maneuvering target accurately based on mobile sensor network by combining the advantage of consensus method and STA-CKF. The stochastic boundedness of the algorithm was analysed and the proof was given in detail. Then, we compared the results of tracking a maneuvering target based on CEKF, CUKF, CCKF and CSTA-CKF respectively, and the simulation results showed that the CSTA-CKF had the best performance in estimating the state of the target than those of the other algorithms. In this algorithm, communication topology of sensor network was undirected and fixed. Directed switching communication topology and communication delay might be studied in the future.

REFERENCES

[1] R. Carli, A. Chiuso, L. Schenate, and S. Zampieri, "Distributed Kalman filtering based on consensus strategies," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 4, pp. 622–633, May 2008.

[2] C. Zhang and Y. Jia, "Distributed estimation for stochastic hybrid systems with event-triggered sensor schedule," *IET Control Theory Appl.*, vol. 11, no. 2, pp. 173–181, Jan. 2017.

[3] C. Liu, J. H. Xie, S. Bin, and X. N. Liu, "Distributed state estimation for networked spacecraft thermal experiments over sensor networks with randomly varying transmission delays," *IEEE Access*, vol. 6, pp. 56658–56665, 2018.

[4] U. A. Khan and A. Jadbabaie, "Networked estimation under information constraints," *IEEE Trans. Autom. Control*, vol. 109, no. 1, pp. 2346–2348, Nov. 2012.

[5] D.-J. Lee, "Nonlinear estimation and multiple sensor fusion using unscented information filtering," *IEEE Signal Process. Lett.*, vol. 15, pp. 861–864, Dec. 2008.

[6] X. Dong, Q. Tan, Q. Li, and Z. Ren, "Necessary and sufficient conditions for average formation tracking of second-order multi-agent systems with multiple leaders," *J. Franklin Inst.*, vol. 354, no. 2, pp. 611–626, Jan. 2016.

[7] Q. B. Ge, T. Shao, and Q. M. Yang, "Multisensor nonlinear fusion methods based on adaptive ensemble fifth-degree iterated cubature information filter for biomechatronics," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 46, no. 7, pp. 1–14, Jul. 2016..

[8] D. P. Spanos, R. Olfati-saber, and R. M. Murray, "Approximate distributed kalman filtering in sensor networks with quantifiable performance," presented at the 4th Int. Symp. Inf. Process. Sensor Netw., Boise, ID, USA, Apr. 2005.

[9] R. Olfati-saber, "Distributed kalman filtering for sensor networks," presented at the 46th IEEE Conf. Decis. Control, New Orleans, LA, USA, Dec. 2007.

[10] Z. Zhang and X. W. Dong, "Distributed extended Kalman-consensus filtering algorithm based on sensor network for nonlinear system," presented at the 37th Chin. Conrtol Conf., Wuhan, China, Jul. 2018.

[11] G. Battistelli, L. Chisci, G. Mugnai, A. Farina, and A. Graziano, "Consensus-based linear and nonlinear filtering," *IEEE Trans. Autom. Control*, vol. 60, no. 5, pp. 1410–1415, May 2015.

[12] W. Li, G. Wei, F. Han, and Y. Liu, "Weighted average consensus-based unscented Kalman filtering," *IEEE Trans. Cybern.*, vol. 46, no. 2, pp. 558–567, Feb. 2016.

[13] Q. Tan, X. Dong, Q. Li, and Z. Ren, "Distributed event-triggered cubature information filtering based on weighted average consensus," *IET Control Theory Appl.*, vol. 12, no. 1, pp. 78–86, Jan. 2018.

[14] D. J. Lee, "Unscented information filtering for distributed estimation and multiple sensor fusion," presented at the AIAA Guid., Navigat. Control Conf. Exhib., Honolulu, HI, USA, Aug. 2008.

[15] W. L. Li and Y. M. Jia, "Distributed consensus filtering for discrete-time nonlinear systems with non-gaussian noise," *Signal Process.*, vol. 92, no. 10, pp. 2464–2470, Oct. 2012.

[16] K. Xiong, H. Y. Zhang, and C. W. Chan, "Author's reply to "Comments on 'Performance evaluation of UKF-based nonlinear filtering,''" *Automatica*, vol. 42, no. 2, pp. 261–270, Mar. 2007.

[17] W. Li and Y. Jia, "Consensus-based distributed multiple model UKF for jump Markov nonlinear systems," *IEEE Trans. Autom. Control*, vol. 57, no. 1, pp. 227–233, Jan. 2012.

[18] R. Lefebvre, H. Bruyninckx, and J. D. Schuller, "Comment on "A new method for the nonlinear transformation of means and covariances in filters and estimators,''" *IEEE Trans. Autom. Control*, vol. 47, no. 8, pp. 1406–1409, Aug. 2002.

[19] I. Arasaratnam and S. Haykin, "Cubature Kalman filters," *IEEE Trans. Autom. Control*, vol. 54, no. 6, pp. 1254–1269, Jun. 2009.

[20] J. L. Ding, J. Xiao, and Y. Zhang, "Distributed algorithm-based CKF and its applications to target tracking," *Control Decis.*, vol. 30, no. 2, pp. 296–302, Feb. 2015.

[21] T. Qingke, D. Xiwang, L. Qingdong, and R. Zhang, "Adaptive distributed estimation for mobile sensor networks based on cubature Kalman filtering," presented at the 29th Chin. Control Decision Conf. (CCDC), Chongqing, China, May 2017.

[22] H. You, S. Qiang, D. Yun-long, and Y. Jian, "Adaptive tracking algorithm based on modified strong tracking filter," presented at the CIE Int. Conf. Radar, Shanghai, China, Oct. 2006.

[23] G.-Q. Duan and S.-L. Sun, "Self-Tuning distributed fusion filter for multi-sensor systems subject to unknown model parameters and missing measurement rates," *IEEE Access*, vol. 6, pp. 61519–61528, 2018.

[24] H. Zhang, J. Xie, J. Ge, W. Lu, and B. Liu, "Strong tracking SCKF based on adaptive CS model for manoeuvring aircraft tracking," *IET Radar, Sonar Navigat.*, vol. 12, no. 7, pp. 742–749, Jul. 2018.

[25] R. Wang and Q. B. Ge, "Information fusion algorithm based on adaptive cubature strong tracking filter," presented at the 36th Chin. Control Conf., Dalian, China, Jul. 2017.

[26] S.-Y. Tu and A. H. Sayed, "Diffusion strategies outperform consensus strategies for distributed estimation over adaptive networks," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6217–6234, Dec. 2012.

[27] W. C. David and B. Randy, "Distributed information filtering using consensus filters," presented at the Amer. Control Conf., St. Louis, MO, USA, Jun. 2009.

[28] Q. Chen, W. Wang, C. Yin, X. Jin, and J. Zhou, "Distributed cubature information filtering based on weighted average consensus," *Neurocomputing*, vol. 243, pp. 115–124, Jun. 2017.

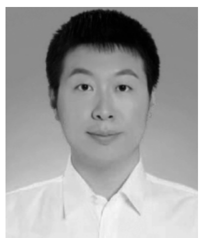
[29] K. Reif, S. Günther, E. Yaz, and R. Unbehauen, "Stochastic stability of the discrete-time extended Kalman filter," *IEEE Trans. Autom. Control*, vol. 44, no. 4, pp. 714–728, Apr. 1999.

[30] K. Xiong, H. Y. Zhang, and C. Chan, "Performance evaluation of UKF-based nonlinear filtering," *Automatica*, vol. 42, no. 2, pp. 261–270, 2006.

ZHENG ZHANG received the B.S. degree in automation science from the Shanxi University of Science and technology, Xi'an, Shanxi, in 2017. He is currently pursuing the M.S. degree with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China. His research interests include target tracking and distributed state estimation.

QINGDONG LI received the B.E. degree in automation and the M.E. and Ph.D. degrees in marine engineering from Northwestern Polytechnical University, Xi'an, China, in 2001, 2004, and 2008, respectively.

Since 2009, he has been a Lecturer with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China. His current research interests include aircraft guidance, navigation and control, fault detection, isolation and recovery, and cooperative control of multiagent systems.



LIANG HAN received the B.E. degree in automation from the Nanjing University of Science and Technology, Nanjing, China, in 2011, and the Ph.D. degree in control science and engineering from Beihang University, Beijing, China, in 2017, where he is currently a Lecturer with the School of Sino-French Engineer. He was a Research Scholar with the Aerospace Engineering Department, University of Michigan, Ann Arbor, from 2013 to 2014. His research interests include target tracking and cooperative control of multi-agent systems.

XIWANG DONG received the B.E. degree in automation from Chongqing University, Chongqing, China, in 2009, and the Ph.D. degree in control science and engineering from Tsinghua University, Beijing, China, in 2014.

He was a Research Fellow with the School of Electrical and Electrical Engineering, Nanyang Technological University, Singapore, from 2014 to 2015. He is currently a Lecturer with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China. His current research interests include consensus control, formation control, and containment control of multiagent systems.

YUAN LIANG received the B.S. and M.S. degrees in automation from the Beijing Institute of Technology, Beijing, China, in 2013 and 2016, respectively. He is currently pursuing the doctor's degree with the School of Automation Science and Electrical Engineering, Beihang University, Beijing. His research interests include integrated navigation systems and cooperative target tracking.

ZHANG REN received the B.E., M.E., and Ph.D. degrees in aircraft guidance, navigation, and simulation from Northwestern Polytechnical University, Xi'an, China, in 1982, 1985, and 1994, respectively, where he was a Professor with the School of Marine Engineering, from 1995 to 1998. From 1999 to 2000, he held visiting professor positions with the University of California, Riverside, and Louisiana State University, Baton Rouge, respectively. He is currently a Professor with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China. His research interests include aircraft guidance, navigation and control, fault detection, isolation and recovery, and cooperative control of multiagent systems. He was a recipient of the Chang Jiang Professorship awarded by the Education Ministry of China.