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Lubrication Oil Anti-Wear Property Degradation Modeling and Remaining Useful Life Estimation of the System Under Multiple Changes Operating Environment

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ABSTRACT Anti-wear property (AWP) is one of the critical characteristics that define the lubrication performance of the lubrication oil. The AWP deterioration is characterized by the wear rate variation and is often related to the operating environment. Since a low AWP can lead to system failure, proactive means to predict the remaining useful life (RUL) considering the environmental factors is an important practical relevance. This paper presents a stochastic model to determine the oil AWP deterioration in order to predict the RUL of the related system. The model assumes that the operating environment behaves as a continuous-time Markov chain (CTMC). A Bayesian methodology using three sources of information (online degradation information, observed degradation, and environmental data) is applied to update dynamically the RUL. In order to demonstrate the applicability of the proposed model, a case of study is presented. Furthermore, to show the accuracy and effectiveness of the proposed approach, a comparative study is conducted with a previously developed model, which does not consider the operating environment.

INDEX TERMS Anti-wear property, Bayesian updating, lubrication oil, varying operating environment.

I. INTRODUCTION

Modern industrial facilities are more vulnerable to wear and friction because of the high integration and complexity of rotating machines. Lubrication oil is a useful tool that aims to control wear, friction, corrosion and temperature [1]. In modern commercialized lubrication oils, the anti-wear and extreme pressure additives are used to improve the lubricant performance and durability. Extreme pressure and anti-wear properties of the lubrication oil is characterized by the prevention of metal to metal contact [2]. The lubricating performance of the lube-oil in boundary lubrication is determined by the anti-wear and extreme pressure properties; therefore, if a defection of one of this properties is observed, the lubricated system can exhibit severe wear that can lead to failure [3]. The anti-wear property (AWP) of the lubricant is one of the important parameters of interest that determines the health of the system and is often characterized by the wear rate variation [4]. A low AWP of the lubricant leads to an increase in the wear rate, which can damage the lubricated component in the long term. Yadav *et al.* [5] investigated the Anti-Wear Extreme Pressure (AWEP) of used and unused engine lubrication oil using a four-ball machine. The results show that the used lube-oil presented a lower AWEP compared to the unused oil and the wear rate increase with the operating time. Spiller *et al.* [6] conducted a study on the wear tribofilms protection capability of a formulated fresh and artificially aged engine oil. After reaching a steady wear rate, the engine lubricant was replaced by a free-additive oil to assess the resistance of the formed tribofilms. They found that the artificially aged oil provides better anti-wear protection compared to the fresh oil because of its lower wear rate. As noticed in the above literature, the wear rate can be used to evaluate the anti-wear property of lubrication

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oil and the related system actual health. Few research papers have been published on the lubrication oil anti-wear property degradation [7]–[10], and still none of them proposed a stochastic model to describe the lubrication oil anti-wear property depletion.

Lubrication oil reflects the system performance through its properties, while properties depletion of the lube-oil can represent a danger for the lubricated system health as well [5]. Here, the lubricated system refers to the oil itself and the machine where it is used. Therefore, prediction of the evolution of these properties are extremely vital for the safety and economy of machine operation. In many industrial applications, such as marine structure, power plant, aircraft engine, etc. Lubricating oil carries an important information source for early fault detection of rotational systems [11]. Some research works have shown that lubrication oil analysis using sensors is much more efficient to warn machine failure in comparison with vibration-based machine health monitoring techniques for rotational systems. It provides precise information concerning the effectiveness of current lubrication protection and system health [12]. The online oil monitoring system is a technology that uses sensors to analyze the wear particles generation rate, concentration and size of lubricant while the machine is operating [13]–[16]. The oil wear particles characteristics provide accurate information not only about the lubricant condition but also about the lubricated system health condition [17]. The online visual ferrograph (OLVF) is a digital on-line ferrograph sensor, integrated with a *CMOS* image sensor developed at Xian Jiaotong University by T. H. Wu *et al*. (2009) [18]. It is an innovative and efficient lubrication oil-monitoring tool with a measurement index IPCA (Index of Particle Covered Area). Wear particles information, such as debris concentration rate, size, and morphology can be obtained from an OLVF sample due to its high sampling frequency and automatic monitoring system [19], [20]. Its effectiveness has been proved in many applications [21]. Here in this paper, the OLVF information (IPCA signal) will be used to describe the lubrication oil anti-wear property depletion path. In an earlier study [22], it has been proved that the IPCA signal is able to assess the AWP of the lubrication oil. The idea was based on the assumption that the wear particles concentration in the oil was proportional to the wear rate, which is approximately equal to the IPCA. More recently, Fan *et al.* [13] investigated the relationship between wear rate and real-time IPCA signal. A new mathematical model that can map the wear rate to the wear debris concentration has been introduced. From the findings, one can say that the IPCA curve can explicitly describe a lubricated system wear rate variation. According to the fact that the wear rate variation can reflect the oil anti-wear property, in our model, the IPCA signal will be used to determine the lubrication oil actual anti-wear protection and observe the related degradation of the lubricated system.

Since the lubricated system, operating with a lubrication oil having a low anti-wear property can cause irreversible damage in the inner components and can lead to reduced performance, fatal and costly failure; it is of importance to know when the lubrication oil no longer fulfills its functions [17]. In this paper, it is assumed that the system exhibits a failure when the wear rate exceeds a certain threshold level [13], [23]. The system degradation path related to the AWP of the lubrication oil is considered as a continuously observed signal that draws the degradation process from a brand "New State" to a completely "Failure State." It is assumed that this process is mainly caused by wear protection depletion and physical properties deterioration. The wear rate usually manifests a stochastic behavior caused by the interaction with the operating environment, which involves factors such as the lubrication system load, rotational speed, temperature, humidity, pressure, to name only a few [24]. Hence, to achieve an efficient degradation modeling, all those possible details influencing the degradation process should be integrated into the degradation signal-modeling equation.

In industrial application, lubrication oil particle monitoring is essential for machine operation, since the particle accumulation in the oil is related to the machine wear rate. Despite the fact that several research works have been presented in the field of lubrication oil analysis and systems Remaining Useful Life (RUL) estimation [25], [26]; few have been directed to a statistical approach to model the lubrication oil properties degradation and the estimation of the related system RUL. Among those papers, Zhu *et al.* [27] presented a physical model to describe the lubrication oil deterioration and estimated the RUL using particle filtering. Du *et al.* [28] applied a hidden Markov chain (HMM) to model the oil degradation states using two monitoring indices with three states (healthy, unhealthy, and failure state). The degradation information was provided by an OLVF. Valis and Zak [29] developed two degradation models using respectively the Wiener process with drift and the Ornstein-Uhlenbeck based on the concentration of soot in the lube-oil using oil analysis sensors data. They derived that a high concentration of soot determines the deterioration state of the oil. More recently, Valis *et al.* [30] developed a Wiener degradation signal based on the anti-oxidant anti-wear particles (AOWP) to determine the oil degradation path and observe the related degradation of the lubricated system. They assumed that the AOWP depletion is related to the operating time and calendar time. The RUL was estimated according to a determined crucial level of AOWP in the lubricant. It can be noticed that most of the works mentioned above used a stochastic model to estimate the RUL using oil sensor data, and still, none of them considered the effect of the operating environment in the degradation path and the RUL prediction of the lube-oil.

Chen and Tsui [31] presented a degradation process of rotational bearing with a two-phase exponential degradation model. They assumed that after normal and stable degradation stages, the machine starts to experience a severe continuous degradation at a certain unknown change point. A Bayesian framework that combined historical data and real-time degradation signal is used to update the remaining life. Wang *et al.* [32] proposed an RUL estimation method for the aviation axial piston pump. A Wiener process-degradation model based on the return oil flow of the pump has been used. The RUL was also updated using only the historical degradation data. In literature [31], [32], one can observe that only the signal degradation history has been used to update the RUL. In this paper, we would like to develop how the IPCA curve can be used to represent the oil anti-wear degradation path considering the operating environment and estimate the residual life associated to the lubricated system using three sources of information.

To improve the degradation process modeling and at the same time the RUL estimation, this paper presents an online degradation signal based-oil anti-wear property that considers: (a) the wear protection degradation (b) the interaction with the environmental conditions and (c) the jump occurring at different transition period. In the modeling process, the environmental conditions we assumed that evolve randomly as a continuous time Markov chain (*CTMC*). The *CTMC* has been used in the past to model the environment process, as seen in the following literature [33],]34]. The online oil sensor OLVF provided the degradation data based-oil anti-wear property. To expect an accurate estimation result of the RUL, we applied a Bayesian updating methodology combining three sources of information: the real-time degradation signal, the environmental and historical degradation data. Fig. 1 illustrates an overview of the proposed model, from the degradation signal modeling to the RUL estimation.

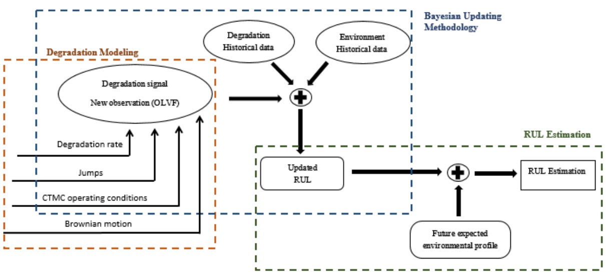

FIGURE 1. **Overall flowchart to illustrate the proposed methodology.**

Our research aims to estimate the remaining life of a lubricated system in real time via OLVF sensor according to the random operating environment. The novelty of the presented model is the stochastic description of the anti-wear property degradation of the lubrication oil considering the environmental factors and the application of three relevant sources of information to estimate the related RUL of the lubricated system. The foul-ball tribometer is known as an efficient and performant tool to assess AWP of lubricants in sliding contact operating under boundary lubrication. It has been used in many research papers for such purpose [2], [5], [28], [35]. In the case study, a four-ball tribometer is used for the experiments.

The remaining parts of this paper are organized as follows: Section 2 introduces the model used to build the degradation signal. In this section, we used a *CTMC* to model the dynamic environment profiles. Section 3 presents the RUL updating process using the Bayesian methodology. In Section 4, we presented the estimation methodology of the RUL. Section 5 shows a case study, which demonstrates the capability of our proposed prediction approach using real practical data. Finally, a conclusion is drawn in Section 6.

II. CTMC DEGRADATION MODEL

A. CTMC DYNAMIC ENVIRONMENT MODELING

A large number of mechanical processes occur in continuous time (e.g., weather, operating environment, mechanical component, and system failure times); hence, a discrete-time definition may not be appropriate. A *CTMC* is a stochastic process that experiences transitions from one state to another taking value in a finite or countable state space, according to certain probabilistic rules [36]. In other words, knowledge of the previous state is all that is necessary to determine the probability distribution of the current state. In the presented degradation model, the environment is expected to evolve dynamically as a continuous time discrete state Markov chain and, the system is assumed to operate up to time t. At each t, the environment can occupy solely one unique state in the finite state space $S = \{1, 2, \ldots, s\}$ *with* $S \subseteq \mathbb{N}, s < \infty$. To avoid the complexity of state number estimation, it is assumed that the distinct state number s is a known value.

Assume that $\{\Omega(t) : t \geq 0\}$ is a *CTMC* process on the finite state space S, where $\Omega(t)$ is the random state of the environment. Let $\mathcal{M} = [m_{i,j}]$ be the transition rate matrix of the process $\{\Omega(t) : t \geq 0\}$ with $m_{i,j} : ij \in S$ the rate of environment transitions from i to $j, j \neq i$. The negative elements of the infinitesimal generator matrix diagonal $-m_{i,i}$ represent the total rate of leaving the state i. The rate of leaving must be the sum of the rates to go at any state j. The rows of the generator must sum to zero, and the total rate of leaving state i is given by:

$$m_i = -m_{i,i} = -\sum_{j \neq i} m_{i,j}$$

According to Markov property [37], when the operating environment shift to state $i \in S$, independent of the past, it remains for a continuous and very positive random length of time called the holding time. In the current model, the holding time $\mathcal{S}_i$ is an exponentially distributed random variable described by:

$$\mathcal{S}_i \sim \exp(m_i)$$

with mean

$$\mathcal{S}_{M_i} = \frac{1}{m_i}$$

A detailed description of the degradation signal affected by the CTMC environmental condition is presented in the next subsection.

B. DEGRADATION MODEL DESCRIPTION

The oil analysis sensors information are often perturbed by many factors that cannot be explained by the environmental condition such as external interference, incomplete monitoring information, and so on [13]. The related degradation signal is frequently infected with noise, which perturbs the characteristics and the recognition of the lube-oil healthy state [19], [21]. Thus, to reduce or neutralize these perturbations, a standard Brownian motion process is used for extracting efficient and effective degradation information. Over time, the lubricating oil AWP degrades due to normal usage and the influence of its operating environment. Therefore, it is of capital importance to include the effect of the environment on its degradation signal modeling. The operating environment is supposed to evolve as a *CTMC*. Without loss of generality, the initial AWP degradation of the lube-oil is assumed to be zero. The degradation signal is denoted by $Z(t)$. As for lubrication oil used in rotational machines, it is considered that the degradation signal may experience certain jump magnitudes caused by the operating environment changes. Therefore, a function $\mathcal{H}(\Omega(t))$ is defined as the function of the jump, which can occur at any transition period. A rate function $\mathcal{D}$ of the degradation associated with each environment state is constructed, so that $\mathcal{D}(\Omega(t))$ denotes the rate of the lube-oil AWP degradation at time t. Whenever $\Omega(t)$ stand at any $j \in S$, the lubricant AWP degrades at a unique and invariable rate $\mathcal{D}(j)$ with $\mathcal{D}(j) > 0$. By an increasing ordination of the degradation rate, the distinct states of S are ordered as follow: if $i < j, \mathcal{D}(i) < \mathcal{D}(j)$. To determine the current number of transition, Λ_j is defined as the *jth* environment transition, and $\mathcal{C}(t)$ is the cumulative number of environment transitions up to time t. The *CTMC* is supposed to be regular, meaning that the number of transition at any finite length of time is finite with probability one ($\mathcal{P}(\mathcal{C}(t) < \infty) = 1$). Consequently, the degradation signal of the lubrication oil is given:

$$Z(t) = \sum_{j=1}^{C(t)} \mathcal{H}\left(\Omega\left(\Lambda_j\right)\right) + \int_0^t \mathcal{D}\left(\Omega\left(r\right)\right) dr + \gamma B\left(t\right) \tag{1}$$

where $\gamma B(t)$ is a Brownian Motion (**BM**) following a normal distribution $N(0, \gamma^2 t)$, and represents random errors in the stochastic process that cannot be ascribed to the environmental behaviors. $\gamma, (\gamma > 0)$ is the diffusion parameter. The present degradation signal based wear rate represents the AWP deterioration of the lubrication oil that will be used to estimate the associated RUL of the lubricated system.

In the next section, we will provide a methodology to update continuously the remaining service life through the Bayesian approach.

III. RUL UPDATING USING BAYESIAN APPROACH

To proceed with the Bayesian updating process, one first needs to provide a prior distribution from the prior information, which results from a data gathering process. The posterior distribution or reviewed prior distribution is provided based on historical observations and new signal information [38], [39]. In this section, the RUL of the lubrication oil is updated using the Bayesian methodology according to the updated degradation signal and environmental activity. In the current model, the degradation signal is observed discretely at a vector of time $t = (t_0, t_1 t_k)$ such that $0 = t_0 \leq t_1 \leq \ldots \leq t_k$. The discrete observation of the signal is defined by $z(t_i), i = 0, 1, \ldots, k$ where t_k represents the last observation time. Let $z_k \in R_{k+1}$ represents the set of signal observations where $z_k = (z(0), z(t_1), \ldots, z(t_k))$. In addition, a sampling process of jump magnitude during environment transitions is carried out to integrate its effects on the AWP degradation of the lube-oil. Therefore, as well as the set of degradation signal observations z_k at time t_k, the set of environmental profiles experienced during the lifetime is denoted by $\mathcal{E}_{t_k}$, where $\mathcal{E}_{t_k} = \{(\mathcal{A}_j, \Lambda_j) : j = 1, 2, \ldots, \mathcal{C}(t_k)\}$. According to this definition, it is assumed that during the transition interval $\left[\Lambda_{j-1}, \Lambda_j\right), j = 1, 2, \ldots, \mathcal{C}(t_k)$ with $\Lambda_0 = 0$, he environment remains at state $\Omega(\Lambda_{j-1})$. In what follows we described gradually the Bayesian framework applied to update the RUL.

A. OPERATING ENVIRONMENT UPDATING

In most mechanical systems, the operating environment has an impact on the machines aging or the length of the RUL [12]. The operating environment can be deterministic or unknown. Recall that we consider the operating environment evolving randomly as a CTMC. Due to its dynamism, the environment updating is an important and crucial phase in the lubrication oil RUL updating process. Next, the environment process $\{\Omega(t) : t \geq 0\}$ updating methodology is presented. It assumed that the matrix $\mathcal{M}$ elements are random. Let $\boldsymbol{\lambda} = \left\{m_{i,j} : i, j \in S, j \neq i\right\}$ be the set of non-diagonal elements of $\mathcal{M}$, its prior distribution is defined by $\vartheta_{\mathcal{M}}(\boldsymbol{\lambda})$. As described above, for $i, j \in S, j \neq i$, let $C_{i,j}(t_k)$ and $\mathcal{S}_i^T(t_k)$ represent respectively the number of environment transitions in the interval $[0, t_k]$ and the total value of the holding time in state i at t_k. From [18], the likelihood function of $\boldsymbol{\lambda}$ for a continuous time discrete state Markov chain can be expressed by:

$$L(\boldsymbol{\lambda}) = \prod_{i=1}^{s} \prod_{j \neq i} m_{i,j}^{C_{i,j}(t_k)} exp\left(-m_{i,j}\mathcal{S}_i^T(t_k)\right) \tag{2}$$

We assumed that the infinitesimal generator matrix $(i, j)th$ elements have a Gamma prior distribution. Among distributions, we make use of Gamma distribution for the following reasons.

i. Gamma distribution flexibility to cover a significant number of distribution.
ii. Real positive number of $\boldsymbol{\lambda}$ elements.
iii. The great skewness ability of Gamma shape and scale parameters.

The Probability Density Function (PDF) of the transition probability $m_{i,j}$ is given by:

$$u_{i,j}(x) = \begin{cases} \dfrac{x^{k_{i,j}-1}}{\Gamma\left(k_{i,j}\right)\theta_{i,j}^{k_{i,j}}} \exp\left(-\dfrac{x}{\theta_{i,j}}\right), & x > 0 \\ 0, & x \leq 0 \end{cases} \tag{3}$$

where $k_{i,j}$ and $\theta_{i,j}$ are respectively the shape and scale parameters. By the Bayes' formula, the posterior distribution $\Theta_{\mathcal{M}}$, is obtained:

$$\begin{aligned}
\Theta_{\mathcal{M}}(\boldsymbol{\lambda}|\mathcal{E}_{t_k}) &\propto \vartheta_Q(\boldsymbol{\lambda}) \times L(\boldsymbol{\lambda}).\\
&\propto \prod_{i=1}^{\mathcal{S}}\prod_{j\neq i}\left(m_{i,j}^{k_{i,j}(t_k)}\exp\left(-\frac{m_{i,j}}{\theta_{i,j}}\right)\times\frac{1}{\Gamma\left(k_{i,j}\right)\theta_{i,j}^{k_{i,j}}}\right)\\
&\quad\times\prod_{i=1}^{\mathcal{S}}\prod_{j\neq i}m^{C_{i,j}(t_k)}\exp\left(-m_{i,j}\mathcal{S}_i^T\left(t_k\right)\right)\\
&\propto\left(\prod_{i=1}^{\mathcal{S}}\prod_{j\neq i}m_{i,j}^{k_{i,j}-1}\times m_{i,j}^{C_{i,j}(t_k)}\right.\\
&\quad\left.\times\exp\left(-m_{i,j}\left(\frac{1}{\theta_{i,j}}+S_i^T\left(t_k\right)\right)\right)\right)\\
&\quad\times\prod_{i=1}^{\mathcal{S}}\prod_{j\neq i}\frac{1}{\Gamma\left(k_{i,j}\right)\theta_{i,j}^{k_{i,j}}}
\end{aligned}$$

We normalized $\Theta_{\mathcal{M}}(\boldsymbol{\lambda}|\mathcal{E}_{t_k}))$ to make our computation more accessible and feasible

$$\begin{aligned}
&\int_{\boldsymbol{\lambda}}^{\Theta_{\mathcal{M}}(\boldsymbol{\lambda}\,|\,\mathcal{E}_{t_k})=1}\\
&\Longrightarrow\Theta_{\mathcal{M}}\left(\boldsymbol{\lambda}\,|\,\mathcal{E}_{t_k}\right)=\prod_{i=1}^{\mathcal{S}}\prod_{j\neq i}\left(m_{i,j}^{\left(\tilde{k}_{i,j}\right)-1}\exp\left(-\frac{m_{i,j}}{\tilde{\theta}_{i,j}}\right)\right.\\
&\qquad\left.\times\frac{1}{\Gamma\left(\tilde{k}_{i,j}\right)\tilde{\theta}_{i,j}^{k_{i,j}}}\right)
\end{aligned}\tag{4}$$

where

$$\tilde{k}_{i,j}=k_{i,j}+\mathcal{C}_{i,j}\text{ and }\tilde{\theta}_{i,j}=\left[\theta_{i,j}^{-1}+\mathcal{S}_i^T(t_k)\right]^{-1}$$

B. DEGRADATION SIGNAL UPDATING

The AWP degradation signal Z(t) directly related to the operating environment is updated at each sampling period t_k. The updating process of the degradation signal requires the updating of the parameters associated with the function of jump $\mathcal{H}$, the degradation rate function $\mathcal{D}$, and BM diffusion parameter γ. the signal model is updated based on the degradation signal and the environmental condition historical data. By employing the Bayesian rule, one can express the posterior distribution of $(\mathcal{D}, \mathcal{H}, \gamma)$, conditional on the observed $z_k\mathcal{E}_{t_k}$ as follows:

$$\Theta_Z\left(\mathcal{H},\mathcal{D},\gamma\,|\,z_k,\mathcal{E}_{t_k}\right)\propto\vartheta_Z(\mathcal{H},\mathcal{D},\gamma)L_z(z_k|\mathcal{E}_{t_k},\mathcal{H},\mathcal{D},\gamma)\tag{5}$$

where $\Theta_Z\left(\mathcal{H},\mathcal{D},\gamma\,|\,z_k,\mathcal{E}_{t_k}\right)$ represents the posterior distribution, $\vartheta_Z\left(\mathcal{H},\mathcal{D},\gamma\right)$ is the joint prior distribution and $L_z(z_k|\mathcal{E}_{t_k},\mathcal{H},\mathcal{D},\gamma)$ is the likelihood function.

C. RUL UPDATING

The RUL also named remaining service life, or remnant life is the usage time from the current observation time to the end of the useful life. The RUL prediction of operating systems has been the subject of attention of researchers over the last decades due to the increasing demand of industrial companies for lower expenses, higher efficiency, and a longer production time [17]. In this subsection, the RUL of the lubricated system is going to update dynamically according to the historical data of the degradation signal and the environment activity. More specially, the updated RUL is expressed using the posterior distribution of the degradation signal and the operating environment. It is assumed that the lubrication oil no longer fulfills its function when the degradation signal reaches or crosses the fixed failure threshold level D^T, which can be obtained from expert knowledge or industrial standards. The random lifetime of the lubricated system is defined as:

$$L=inf\{t:Z\left(t\right)\geq D^T|Z\left(0\right)<D^T\}$$

Assume that the degradation signal crosses the threshold level D^T for the first time at $F_k=L-t_k$; the remaining life of the lubrication oil at time t_k can be expressed under the concept of first passage time as:

$$F_k=inf\left\{u>0:Z\left(t_k+u\right)\geq D^T\right\}$$

Using the conditional on observation histories, the updated conditional Cumulative Distribution Function (CDF) of RUL is given by:

$$\begin{aligned}
F_k^{cd}&=\int_{\mathcal{D},\mathcal{H},\gamma}\int_{\boldsymbol{\lambda}}\mathcal{P}\left(F_k\leq t-t_k|z_k,\mathcal{E}_{t_k},\mathcal{H},\mathcal{D},\gamma,\boldsymbol{\lambda}\right)\\
&\quad\times\Theta_Z\left(\mathcal{H},\mathcal{D},\gamma|z_k,\mathcal{E}_{t_k}\right)\times\Theta_{\mathcal{M}}\left(\boldsymbol{\lambda}|\mathcal{E}_{t_k}\right)\\
&=\int_{\mathcal{D},\mathcal{H},\gamma}\int_{\boldsymbol{\lambda}}\mathcal{P}\left(F_k\leq t-t_k|z_k,\mathcal{E}_{t_k},\mathcal{H},\mathcal{D},\gamma,\boldsymbol{\lambda}\right)\\
&\quad\times\prod_{i=1}^{\mathcal{S}}\prod_{j\neq i}m_{i,j}^{\tilde{k}_{i,j}-1}\exp\left(-\frac{m_{i,j}}{\tilde{\theta}_{i,j}}\right)\\
&\quad\times\left(\Gamma\left(\tilde{k}_{i,j}\right)\tilde{\theta}_{i,j}^{k_{i,j}}\right)^{-1}\\
&\quad\times\vartheta_Z(\mathcal{H},\mathcal{D},\gamma)L_z\left(z_k|\mathcal{E}_{t_k},\mathcal{H},\mathcal{D},\gamma\right)
\end{aligned}\tag{6}$$

IV. RUL ESTIMATION

When the operating environment is a CTMC process, it is difficult to figure out which state the process will jump to at the next transition. To overcome this difficulty, the future environment state can be predicted using a Markov renewal process. The predicted environment state will allow us to estimate the RUL of the lubricated system. To proceed, one needs to estimate the RUL in a future determined environment, and the result will be used as a basis to estimate the RUL in a dynamic environment.

A. RUL UNDER DETERMINED FUTURE ENVIRONMENT

In this subsection, the first passage probability of the **BM** through a piecewise linear boundary is used to estimate the RUL. The boundary crossing probability for **BM** is an important tool in stochastic modeling, and its efficiency has been proved in numerous research fields, such as epidemiology, environmental science, physics, as well as in mechanical engineering, [40], [41]. In what follows, the future environment is totally determined.

Considering the linear boundary case, it is supposed that $c(r) = a + br$ is linear in r on a certain interval $[0, \mathcal{T}]$..Wang and Pötzelberger [42] established a specific formula for the boundary crossing probability for piecewise linear boundaries when the function $c(r)$ is piecewise linear without accounting for the random jumps occurrence on the interval $[0, \mathcal{T}]$. Evolving on the basis that upward and downward jump can occur, the number of jumps occurrences up to time $\mathcal{T}$ is defined by $n(\mathcal{T})$, with $n(\mathcal{T}) = n$ and partitioned $[0, \mathcal{T}]$ such that $[0, \mathrm{T}] = \bigcup_{d}^{n} [r_{d-1} r_d)$, where r_d is the time of the d^{th} jump occurrence. It is assumed that $c(r)$ is linear on any single interval $[r_{d-1} r_d))$, $d = 1, 2, \ldots, n$ and fulfills $c(0) > 0$. Then for each $r \in [0, \mathcal{T}]$, the one side linear boundary of the form $c(r) = a + br$ is considered. For easy understanding, let $m_d \equiv min\{c_d, c_d^-\}$, where $c_d = c(r_d), c_d^- = c(r_d^-)$, and the notation r_d^- indicates the left-hand limit of r_d; that is the time just before the d^{th} environmental transition, with $d = 0, 1, \ldots, n$, and $let \mathbf{c} = (c_0 c_0^-, c_1, c_1^-, \ldots, c_n c_n^-)$. Given that for any $r \in [0, \mathrm{T}], c(r) = a + br$ is one side linear boundary on the interval $[0, T]$, then applying the well-known formula of Siegmund for the standard **BM** boundary crossing probability in [27] we obtain:

$$\mathcal{P}(\boldsymbol{B}(r) < a + br); r < \mathcal{T} | \boldsymbol{B}(\mathcal{T}) = x) = 1 - \exp\left[-\frac{2a(a + b\mathcal{T} - x)}{\mathcal{T}}\right]$$

for our model,

$$\begin{aligned}\mathcal{P}(\gamma \boldsymbol{B}(r) < c(r), r \leq \mathcal{T}) &= \mathcal{P}\left(\boldsymbol{B}(r) < \frac{c(r)}{\gamma}, r \leq \mathcal{T}\right) \\ &= \int_{-\infty}^{m_1/\gamma} \mathcal{P}\left(\boldsymbol{B}(r) < \frac{c(r)}{\gamma}, r < r_1 | \boldsymbol{B}(r_1) = x_1\right) \\ &\quad . \quad \times \mathcal{P}\left(\boldsymbol{B}(r) < \frac{c(r)}{\gamma}, r < r_1 \leq \mathcal{T} | \boldsymbol{B}(r_1) \right. \\ &\quad \left. = x_1\right) \mathrm{d}\mathbb{P}_{r_1}(x_1) \end{aligned} \tag{7}$$

with

$$\frac{\mathrm{d}\mathcal{P}_r(x)}{\mathrm{d}x} = \frac{1}{\sqrt{2\pi r}} \exp\left(\frac{-x^2}{2r}\right);$$

Notice that $\mathrm{d}\mathcal{P}_r(x)/dx$ is the PDF of $\boldsymbol{B}(r)$. The product of (7) holds since $\{\boldsymbol{B}(t) : t \geq 0\}$ respect the strong Markov property. According to Siegmund's results in [43], the first term in the integral of (7) is given by:

$$\mathcal{P}\left(\boldsymbol{B}(r) < \frac{c(r)}{\gamma}, r < r_1 | \boldsymbol{B}(r_1) = x_1\right) = 1 - \exp\left[\frac{2(c_0/\gamma)\left(c_1^-/\gamma\right) - x_1}{r_1}\right]$$

Given that $B(r_1) = x_1$, it is obvious that $\boldsymbol{B}(r + r_1) - x_1$ is again a **BM** process starting from the zero. The non-crossing boundary probability of our **BM** process , $\boldsymbol{B}(r)$ with diffusion parameter γ is given by:

$$\begin{aligned}\boldsymbol{B}(r) < \frac{c(r)}{\gamma} | z0, \mathcal{E}_{t_0}, \mathcal{H}, \mathcal{D}, \gamma) \\ = \mathbb{E}g[(\boldsymbol{B}(r_1), \boldsymbol{B}(r_2), \ldots, \boldsymbol{B}(r_n); \boldsymbol{c})]\end{aligned} \tag{8}$$

where

$$\begin{aligned} g(x_1, x_2, \ldots, x_n; \boldsymbol{c}) \\ = \prod_{j=1}^{n} \mathbf{I}(x_d < m_d/\gamma) \\ \times \left(1 - \exp\left[-\frac{2(c_{d-1}/\gamma - x_{d-1})\left(c_d^-/\gamma - x_d\right)}{(r_d - r_{d-1})}\right]\right) \end{aligned}$$

and $\mathbf{I}(\_)$ is the indicator function equal to 1 when the conditions inside are respected and 0 otherwise.

Assume that the deterministic Markov process $\{\Omega(t) : t_k < t \leq \mathcal{T}\}$ represents the future environmental profile, and the degradation signal sampled at time $0, t_1 t_k$. Then for any $t \in [t_k, \mathcal{T}]$, the determined part of the degradation signal in (1) can be written as:

$$\xi^k(t) = z(t_k) + \sum_{d \in V_k(t)} \mathcal{H}(\Omega(r_d)) + \int_{t_k}^{t} \mathcal{D}(\Omega(r))\, dr \tag{9}$$

where $V_k(t) \equiv \{d : r_d \in (t_k t]\}$ is the set of jump occurrence times. Hence, from (9) the lubrication oil degradation signal $Z(t)$ given $(z_k \mathcal{E}_{t_k}, \mathcal{H}, \mathcal{D}, \gamma)$ can be rewritten as follows:

$$Z(t) \,|\, \left(z_k, \mathcal{E}_{t_k}, \mathcal{H}, \mathcal{D}, \gamma\right) = \xi^k(t) + y\, \boldsymbol{B}(t - t_k)$$

The CDF of the remaining life according to the degradation signal first passage time probability to the boundary $c_k(t)$ with $c_k(t) \equiv D^{\mathcal{T}} - \xi^k(t)$ for $t_k < t \leq \mathcal{T}$, can be obtained using (8) as (10), as shown at the bottom of this page, with $m_d^k = min\left\{c_k(r_d), c_k\left(r_d^-\right)\right\}$ for $d \in V_k(T)$.

The probability function $\mathcal{P}(F_k \leq \mathcal{T} - t_k \,|\, z_k, \mathcal{E}_{t_k}, \mathcal{H}, \mathcal{D}, \gamma, \boldsymbol{\lambda})$ is difficult to calculate due to the multi-dimensional

$$\begin{aligned} \mathcal{P}&\left(F_k \leq \mathcal{T} - t_k \,|\, z_k, E_{t_k}, H, D, \gamma, \boldsymbol{\lambda}\right) \\ &= \mathbf{1} - E\left[\prod_{d \in V_k(t)} \boldsymbol{B}\left(r_d - t_k < \frac{m_d^k}{y}\right)\right. \\ &\quad \left. \times \left(1 - \exp\left(\frac{2\left[c_k(r_{d-1})/y - \boldsymbol{B}(r_{d-1} - t_k)\right]\left[c_k\left(r_d^-\right)/y - \boldsymbol{B}(r_d - t_k)\right]}{(r_d - r_{d-1})}\right)\right)\right] \end{aligned} \tag{10}$$

integration; therefore, to make it easy, we used the Monte Carlo simulation [44] to make the estimation.

B. RUL UNDER RANDOM ENVIRONMENT

To obtain the RUL under random environment, an estimation of the future environment state is conducted applying the renewal process. The renewal process is a stochastic process during which a certain event appears many times, and the time between appearances are independent and identically distributed [45], [46]. Assume that $0 = t_0 \leq t_1 \leq \ldots \leq t_k \leq T$ are the finite operating environment state occurrence times, the number of occurrence in the interval $[0, T]$ is:

$$\mathcal{C}(t) = \sum_{k=1}^{\infty} 1\left(t_k \leq \mathcal{T}\right) \quad \mathcal{T} > 0;$$

Let the Markov renewal process denoted by $\mathcal{C}_j(t), j = 1, 2 \ldots m$ defines the number of recurrences or the number of visits to a certain state j. It is assumed that Markov chains of interest are regular, meaning that the number of transitions in any finite length of time is finite with probability 1. Assume that $Q_{i,j}$ is the presumed number of visits to state j and given the environment current state $\psi(t_k) = i$ The Markov renewal function [26] is given:

$$Q_{i,j}(t) \equiv \mathbb{E}\left(\mathcal{C}_j(t) \mid U_0 = i\right), \quad i, j \in S, \quad t \geq 0,$$

Let $\boldsymbol{Q}(t)$ be the matrix renewal function with elements $Q_{i,j}(t)$, its Laplace–Stieltjes Transform is given by:

$$\hat{\boldsymbol{Q}}(s) = \int_0^{\infty} e^{-st} \mathrm{d}\boldsymbol{Q}(t) \tag{11}$$

From literature [47], The Laplace–Stieltjes Transform $\hat{\boldsymbol{Q}}(s)$ can be rewritten as:

$$\hat{\boldsymbol{Q}}(s) = \hat{\boldsymbol{U}}(s)\left[I^{\mathbf{D}} - \hat{\boldsymbol{U}}(S)\right]^{-1},$$

where $I^{\mathbf{D}}$ is the identity matrix and $\hat{\boldsymbol{U}}(s) = \left[\hat{\boldsymbol{U}}_{i,j}(s)\right]$ with

$$\hat{U}_{i,j}(s) = \begin{cases} \dfrac{m_{ij}}{m_i + s}, j \neq i, & m_i \neq 0 \\ 0, & otherwise \end{cases}$$

$\boldsymbol{Q}(t)$ is calculated, by numerically inverting $\hat{\boldsymbol{Q}}(s)$ in (11) using the stable inversion algorithms in [48]. Knowing that the current environment state $\Omega(t_k) = i$, the presumed number of environment transition in the interval is $(t_k, T]$ given by:

$$\tau_i^k \equiv \mathbb{E}\left(\mathcal{C}_j(\mathcal{T} - t_k) \mid \Omega(t_k) = i\right) = \sum_{j=1}^{s} Q_{i,j}(\mathcal{T} - t_k), \quad 0 < t_k \leq \mathcal{T} \tag{12}$$

After determining the presumed number of visits to each state; by inverting (11), one builts the resulting different state sequences and removed all sequences with one-step transitions from state j to j. Let $u = \{1, 2, \ldots, \vartheta\}$ be the set of the residual state sequences and E_n be the u^{th} state sequence such that $E_n = \left\{e_0^u, e_1^u, \ldots, e_{\tau_i^k}^u\right\}$ where $e_0^u = i$, for any sequence u. According to the Markov property, the probability $P(E_n)$ that sequence E_n occur is:

$$P(E_n) = \prod_{j=1}^{\tau_i^k} P\left(e_j^u \mid e_{j-1}^u\right), \quad u = 1, , 2, \ldots, \vartheta \tag{13}$$

According to (13), the estimated RUL is given by:

$$P\left(F_k \leq \mathcal{T} - t_k \mid z_k, \mathcal{E}_{t_k}, \mathcal{H}, \mathcal{D}, \gamma, \boldsymbol{\lambda}\right) \approx \sum_{u=1}^{\vartheta} P_n\left(F_k \leq T - t_k \mid z_k, \mathcal{E}_{t_k}, \mathcal{H}, \mathcal{D}, \gamma, \boldsymbol{\lambda}\right) P(E_n) \tag{14}$$

where $P_n\left(F_k \leq T - t_k \mid z_k, \mathcal{E}_{t_k}, \mathcal{H}, \mathcal{D}, \gamma, \boldsymbol{\lambda}\right)$ is the remaining life at t_k given the nth state sequence and is computed by (10).

C. PARAMETER ESTIMATION

The parameters are updated at each time t_k when degradation data are available. Next, we estimated the CTMC dynamic environment infinitesimal generator matrix. Estimating a transition rate matrix is a relatively straightforward process, given that the state sequence of each sampled signal and the respective state holding time has been observed, one can compute the transition rate from any state i to any state j. Knowing that the sum of rows of the transition matrix is equal to zero, we can compute the transition rate of a specific jump from state i at time t_{k-1} to state j at t_k using the next formula:

$$\hat{m}_{i,j} = \begin{cases} \dfrac{C_{i,j}}{\mathcal{S}_i^T} & i \neq j \\ -\sum_j \hat{m}_{i,j} & i = j \end{cases} \tag{15}$$

On the basis that the infinitesimal generator matrix elements follow a Gamma distribution, from (3), the likelihood function is obtained by:

$$\frac{\hat{m}_{i,j}^{k_{i,j}-1}}{\Gamma\left(k_{i,j}\right) \theta_{i,j}^{k_{i,j}}} \exp\left(-\frac{\hat{m}_{i,j}}{\theta_{i,j}}\right) \tag{16}$$

By (4), the posterior distribution of $\boldsymbol{\lambda}$ according to the set of historical data E_{t_k} up to time t_k is given:

$$\Theta_{\mathcal{M}}(\boldsymbol{\lambda} \mid \mathcal{E}_{t_k}) = \prod_{i=1}^{s} \prod_{j \neq i} m_{i,j}^{\tilde{k}_{i,j}-1} \exp\left(-m_{i,j}/\tilde{\theta}_{i,j}\right) \times \frac{1}{\Gamma\left(\tilde{k}_{i,j}\right) \tilde{\theta}_{i,j}^{k_{i,j}}},$$

Further, to determine the parameters of the degradation model, the functions $\mathcal{D}$ and $\mathcal{H}$ is assumed as follow:

$$\begin{aligned} \mathcal{H}(\Omega(r_d)) &= \rho\left[\Omega\left(r_d^+\right) - \Omega(r_d)\right] \\ \mathcal{D}(\Omega(r)) &= X\Omega(r) + \Psi \; and \end{aligned}$$

where $X\Psi$ and ρ are the respective random parameters and are mutually independent. The prior parameters distributions are described as $\rho \sim N\left(\mu_\rho, \sigma_\rho^2\right), X \sim N\left(\mu_X, \sigma_X^2\right),$

$\Psi \sim N\left(\mu_{\Psi}, \sigma_{\Psi}^{2}\right)$, and $\gamma \sim N(\mu_{\gamma}, \sigma_{\gamma}^{2})$, where μ_i and σ_i are the respective mean and variance. The join prior distribution is defined by $\vartheta_z(\rho, X, \Psi, \gamma)$, and the likelihood function of the degradation model is derived to obtain the estimation of the posterior distribution of (ρ, X, Ψ, γ) Recall that the degradation signal is discredited as z_k vectors. The likelihood function of z_k is:

$$L_z\left(z_k | \mathcal{E}_{t_k}, \rho, X, \Psi, \gamma\right) = \prod_{i=1}^{k} f_i\left(z\left(t_i\right) - z\left(t_{i-1}\right)\right) \tag{17}$$

where f_i is the PDF of the random variable with mean and variance.

$$\left(\int_{t_{i-1}}^{t_i} \rho\left[\Omega\left(r_d^{+}\right) - \Omega\left(r_d\right)\right] + \left(X\Omega\left(r\right) + \Psi\right) dr, \quad \gamma^2\left(t_i - t_{i-1}\right)\right)$$

The posterior distribution is obtained as:

$$\Theta_Z\left(\rho, X, \Psi, \gamma \mid z_k, \mathcal{E}_{t_k}\right) = \frac{\vartheta_z\left(\rho, X, \Psi, \gamma\right) \times \prod_{i=1}^{k} f_i\left(z\left(t_i\right) - z\left(t_{i-1}\right)\right)}{\int_{\alpha,\beta,\eta,\gamma}^{\vartheta_z(\rho,X,\Psi,\gamma)\times} \prod_{i=1}^{k} f_i\left(z\left(t_i\right) - z\left(t_{i-1}\right)\right)}$$

where

$$\vartheta_z\left(\rho, X, \Psi, \gamma\right) = e_\rho\left(\rho\right) e_X\left(X\right) e_\Psi\left(\Psi\right) e_\gamma\left(\gamma\right),$$

and

$$e_i = \frac{1}{\sqrt{2\pi\sigma_i^2}} exp\left[-\frac{x_i - \mu_i}{2\sigma_i^2}\right], \quad i = \rho X \Psi, \gamma$$

Therefore using the degradation signal samples, the updated RUL is

$$F_k^{cd} = \int_{X,\Psi,\rho,\gamma} \int_{\boldsymbol{\lambda}} \mathcal{P}\left(F_k \leq t - t_k | z_k, \mathcal{E}_{t_k}, \rho, X, \Psi, \gamma, \boldsymbol{\lambda}\right) \times \theta_Z\left(\rho, X, \Psi, \gamma | z_k, \mathcal{E}_{t_k}\right) \times \theta_{\mathcal{M}}\left(\boldsymbol{\lambda} | \mathcal{E}_{t_k}\right)$$

In this formula, the first term is estimated by (10), the second term by Markov chain Monte Carlo approach [44], and the last term by (4).

V. CASE STUDY

A. CASE DESCRIPTION

Wear Protection deterioration is considered as one of the main causes of failures and performance lost in lubrication oil. The depletion of AWP increases the wear rate, which can lead to machine failure. In earlier studies [28], [35], the four-ball machine has been used to determine and evaluate the AWP of the lubrication oil. The present case study describes a four-ball test rig operating under a dynamic operating environment in order to predict the online *RUL* afterwards, therefore, for an effective real-time monitoring, an online oil analyzer, the *OLVF* sensor has been used to continuously monitor the lubrication oil particles concentration. During the operational life, the monitoring data was collected at intervals under conditions determined by a specific methodology: through the means of a micro-flow pump, the lubrication oil was conducted from the four-ball machine to the *OLVF* sensor, which is a digital on-line ferrograph sensor, integrated with an *CMOS* image sensor. Inside the *OLVF* sensor, the ferric particles were attracted and fixed to a lens by an electromagnet having a considerable magnetic pitch. A picture of the deposited particles was captured and transmitted to a computer for analysis. Then, the electromagnet was demagnetized so that the fixed particles can rejoin the lubrication system. The index of particle-covered area (*IPCA*) is used to identify the wear rate variation coming from the direct wear of the four-ball machine and to determine the AWP degradation of the lubrication oil. The IPCA was expressed as:

$$IPCA = \frac{\sum c_i}{fw}$$

where $\sum c_i$ denotes the object pixel number of wear particle in the image and f, w represent the height and the width of the image respectively.

TABLE 1. OLFV operating parameters.

Magnetomotive force (Ampere turn)	Flow rate (ml/min)	Sampling volume (ml)	Sampling period (min)
1200	5	15	4

The operating parameters of the OLVF are summarized in Table 1. The current experimentation considers the rotational speed and the load as the primary operating environment factors that affect the AWP of the lubrication oil. Four operating states have been applied. The operating states are ordered regarding the effect on the oil AWP degradation rate such that state1< state2 < state3< state4. Here, the lube-oil is supposed to manifest a failure when the degradation path reaches a fixed threshold level [28]. The critical amount of wear particle in the solvent is fixed to 0.15 IPCA. A total number of 21 degradation data is obtained from the four-ball machine experimentation. Among them, three oil data (Oil1, Oil2, Oil3) are presented to demonstrate the accuracy and effectiveness of the presented model. Oil1 and Oil2 are run to failure, and oil3 is stopped before reaching the critical level. Table 2 summarize the different operating conditions and the respective operating time of Oil1, Oil2, and Oil3 in the different operating states.

B. RESULTS AND COMPARISON

We obtained 21 data histories from the experimentation (14 failure data and 7 suspension data). The failure data corresponds to the experiment that ends when the failure threshold is reached, and the suspension data corresponds to the experiment that is stopped without reaching the failure threshold. During each experiment, different operating condition profiles (different state variations) is applied.

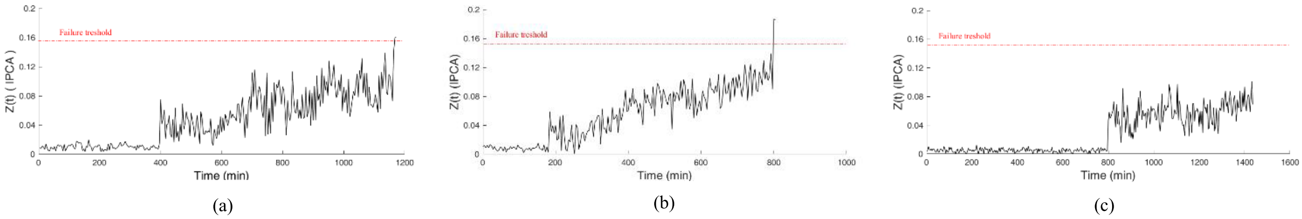

FIGURE 2. **Degradation signal. (a) Oil1. (b) Oil2. (c) Oil3.**

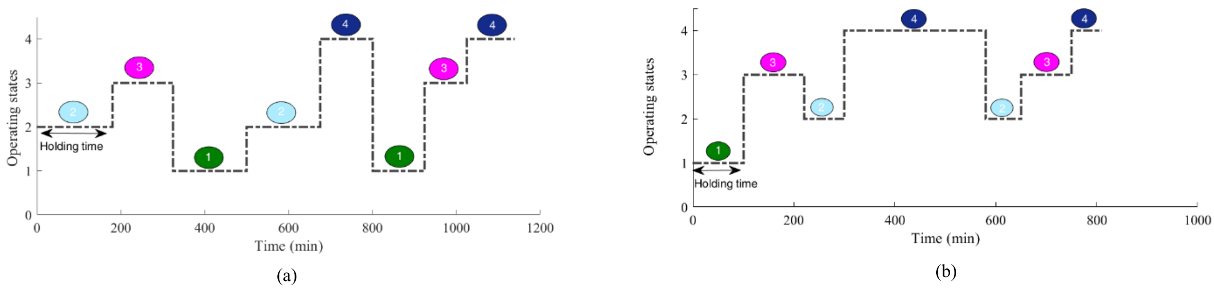

FIGURE 3. **Operating conditions. (a) Oil1. (b) Oil2.**

TABLE 2. **Operating conditions of the experimentation.**

Operating Conditions / State No.	Loads / N	Rotational Speeds / rpm	Operating Time /mins		
			Oil1	Oil2	Oil3
1	1500	1000	300	100	-
2	1500	2000	355	150	1440
3	2000	1000	245	220	-
4	2000	2000	240	340	-

Three degradation signal samples are presented below. Fig. 2 shows the degradation signal of Oil1, Oil2, Oil3, and the threshold level. The corresponding operating environments of Oil1 and Oil2 are presented in Fig. 3. Associating Fig. 2 (a) and Fig. 3 (a), one can observe that the fluctuation of wear rate increases when the operating condition becomes harsh, which reflect a decrease of the AWP of oil. In Fig. 2 (b) the same behavior is observed with Oil2, the wear rate increases drastically, a while after the operating environment enters in state 4 (harsh). Oil2 being subject to severe environmental condition reached first the critical threshold and failed before Oil1. Fig. 2 (c) shows the degradation signal of Oil3 run under an unchanged operating environment (state 2). It is observed that the degradation path does not reach the failure threshold after 14 40 operating time. This higher lifetime can be explained by the fact that Oil3 is run under softer conditions compared to Oil1 and Oil2. In Fig. 2, it can be observed that Oil 2 fails before Oil1, which can be explained by the fact that, Oil2 enters the operating state 4 earlier and has a longer operating time therein. From the above observations, one can say that the operating environment has a considerable impact on the AWP depletion of the oil.

Next, the procedure of prior parameters and RUL estimation are presented. First, the transition rate matrix of the operating environment is determined using the environment data history. Then, the prior parameters of the signal are estimated using the degradation and the environment data histories. Applying (15), one computed the maximum likelihood function according to the environment data history. Moreover using these samples, the gamma distribution shape and scale parameters are determined using (16). The Matrix M, as shown at the bottom of this page, provides the prior mean of the environment transition rates. Equation (17) is utilized to estimate the degradation signal prior parameters. Table 3 presents the estimated parameters.

Once the new degradation signal, as well as the current operating environment state became available, the new information up to date is used to compute the posterior distribution of the stochastic parameters using (4) and (18). To estimate the actual remaining life, one first estimated the future environmental feature. By inverting (11), the estimated number of visits to each state $\{1, 2, 3, 4\}$ is calculated. Then applying (12), the total transition number of the operating

$$M = \begin{bmatrix} -5.55 \times 10^{-2} & 2.64 \times 10^{-3} & 4.12 \times 10^{-2} & 1.17 \times 10^{-2} \\ 1.13 \times 10^{-2} & -4.61 \times 10^{-2} & 1.41 \times 10^{-2} & 2.07 \times 10^{-2} \\ 2.72 \times 10^{-2} & 4.63 \times 10^{-2} & -8.22 \times 10^{-2} & 8.73 \times 10^{-3} \\ 3.29 \times 10^{-2} & 1.61 \times 10^{-2} & 3.12 \times 10^{-2} & -8.02 \times 10^{-2} \end{bmatrix}$$

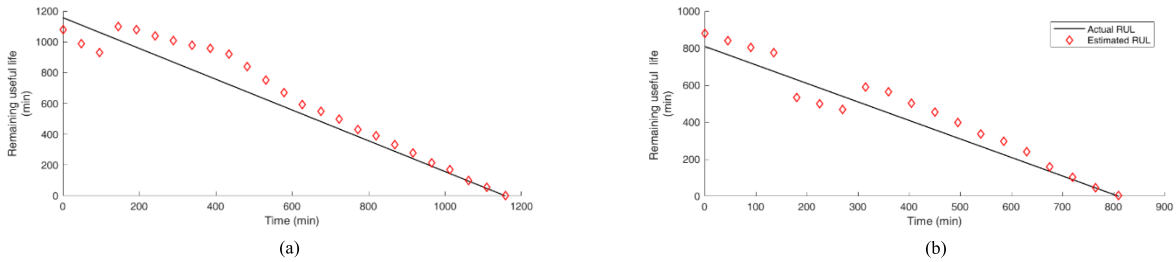

FIGURE 4. The actual and estimated RUL. (a) Oil1. (b) Oil2.

TABLE 3. Estimated prior parameters.

Parameters	Prior Means	Prior Variances
ρ	$\mu_\rho = 2.01 \times 10^{-2}$	$\sigma_\rho = 3.01 \times 10^{-3}$
$\mathbf{X}$	$\mu_X = 1.21 \times 10^{-4}$	$\sigma_X = 2.82 \times 10^{-3}$
Ψ	$\mu_\Psi = -7.89 \times 10^{-4}$	$\sigma_\Psi = 8.92 \times 10^{-4}$
γ	$\mu_\gamma = 1.03 \times 10^{-3}$	$\sigma_\gamma = 5.16 \times 10^{-3}$

environment is determined. Based on the state sequence occurrence probability, the future operating environment feature is determined using (13). Finally, the updated RUL of the lubricated system is computed using (14). The prediction period of the RUL is fixed at every 45 minutes of the operating time. Fig. 4 illustrates the performance of the presented RUL predictive model against the full life cycle data of Oil1 and Oil2.

It is shown in Fig. 4 that the prediction precision increase with the operating time. The estimated value of RUL is brought closer to its real value as more online information is provided. That is, the longer the test duration the higher the RUL estimation precision. For instance, as shown in Fig. 4 (a) at t = 900 min of the lifetime, the relative error between the estimated and the actual RUL is 0.1012. That reveals that the estimated value gets closer to the actual RUL at only 78.9% of the lifetime. That prediction accuracy is explained by the use of three sources of information for the RUL updating and estimation process. With this close value and fast estimation of the RUL, proactive means can be undertaken to avoid a potentially hard and costly failure. To access the proposed model effectiveness, a comparative study is conducted with a Wiener process model presented by Wang *et al.* [32]. The main reason we chose this model among others is that the model does not take into account the operating environment, and the RUL is updated using the Bayesian methodology. However, unlike the presented model, only the degradation data history is considered. Therefore, any performance improvement accomplished from the proposed model can be assigned to the consideration of the environment effect in the degradation model and the RUL estimation.

To quantify the estimation performance of the method developed in the current paper with Wang's model, two comparison metric, namely the relative error and convergence are computed. The relative error is a metric that indicates the uncertainty in the prediction value. It is calculated as L_p percentile of the operating life with $L_p = (15, 35, 55, 75, 95)$. The formula is defined as: $\varepsilon_k = \frac{|L - t_k - \widehat{F}_k|}{L}$, where L represents the actual failure time, t_k is the current sampling period and $\widehat{F}_k$ is the estimated median of the remaining life at time t_k. The convergence is a metric that characterizes the rate of accuracy improvement with time; the lower the value, the faster the convergence [49]. In the current case, it is calculated at the time indexes from the middle to end of the operating life.

The comparative results are drawn in Table 4. The results in Table 4 indicates that the score of the relative error deviation in Wang's model is quite significant, which is not appropriate for an accurate RUL estimation of the lubricated system. However, the proposed model shows a better relative error deviation, which means that our model presents a more accurate estimation of the RUL. The convergence value of the proposed model is also lower, compared to Wang's model, meaning that our model converges faster towards the real value of the RUL. Regarding literature [49], the current model has better confidence for a larger prediction horizon. From the above analysis, one can infer that Wang *et al.* [32] having not considered the effects of the working environment in Wang's model shows a prediction of RUL with a higher relative error and convergence value, consequently less accurate and none appropriate for the RUL estimation of the lubricated system.

The current model incorporates the novelty of considering the operating environment in the degradation model and the advantages of using three sources of information in the residual life updating and estimation process. Therefore, Thanks to these new properties, our model performs a better estimation result for the RUL of the lubricated system using oil analysis data.

In conclusion, the current case study shows that our developed model is suitable and effective for the RUL prediction of lubrication oil under a dynamic environment using oil sensor data. In other words, we proved conclusively that the RUL prediction is more accurate when the effects of the operating environment are considered. The current study can be used for two essential purposes in industrial plants, namely maintenance optimization of rotating machines and service time maximization.

TABLE 4. RUL prediction error comparison.

		Prediction error ε_k					
Lifetime Percentile		15%	35%	55%	75%	95%	Convergence
Wang's model	Oil A	0.5632	0.4401	0.4112	0.3817	0.3749	198.73
Our model	Oil A	0.2073	0.1546	0.1274	0.1058	0.0871	121.09
Wang's model	Oil B	0.5143	0.4209	0.3988	0.3942	0.3772	361.68
Our model	Oil B	0.1829	0.1523	0.1508	0.1211	0.0939	204.49

VI. CONCLUSION

This paper presents an approach based online AWP degradation of lubrication oil in order to estimate the RUL of the lubricated system operating in a changing operating environment. In the degradation model, the operating environment, it is considered as a CTMC stochastic process with a finite state space. By applying the Bayesian updating methodology, the system RUL is updated using the real-time degradation signal and two historical data sources (the historical operating environment data and the historical degradation signal data). The results show that the proposed model succeeded in minimizing the relative error in the RUL estimation. The comparative study conducted with an earlier developed model that does not consider the operating environment proved conclusively that our model is more effective and accurate in the system RUL estimation using oil analysis data. The currently developed model can be used in industrial systems to optimize the maintenance and prolong the machine useful life.

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