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A Novel Iterative Learning Control Approach Based on Steady-State Kalman Filtering

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ABSTRACT This paper presents a novel off-line iterative learning control algorithm for multiple-input–multiple-output time-varying discrete stochastic systems. Using the steady-state Kalman filtering method, we provide a novel framework for the selection of optimal/sub-optimal fixed learning gain matrices in real applications, which is convenient for engineers. Meanwhile, this framework considerably decreases the calculation about the operations of inverting matrix by introducing a matrix Riccati equation at every iteration. It is strictly proved that the input error covariance converges to its steady-state value asymptotically in the mean square sense, and accordingly, the tracking error covariance also converges. The numerical simulations verify the theoretical results.

INDEX TERMS Iterative learning control, steady-state Kalman filtering, sub-optimal fixed learning gain, matrix Riccati equation.

I. INTRODUCTION

Iterative learning control (ILC) originates from the pioneering work of Arimoto *et al.* [1] in terms of tracking in robotic systems. The term ''iterative learning'' stems from ''practice makes perfect''. By capitalizing on the errors in previous iterations, an ILC algorithm can continuously correct the control input to achieve perfect tracking performance. Thanks to its simple but effective control structure, ILC has been widely applied various systems such as robotic systems [2], [3], multi-agent systems [4]–[6], quantized control systems [7]–[9], event-triggered control systems [10], [11] and networked control systems [12], [13].

From recent surveys [14], [15], it is observed that an overwhelming majority of ILC algorithms have employed on-line real-time learning in fixing the problems of actual engineering. Theoretically, it is natural to study convergence and stability of robust ILC, adaptive ILC, and stochastic ILC. Nevertheless, in practical applications, we usually implement a fixed learning structure so as to save the computation burden and seek for fast convergence rate; however, few literature focuses on off-line optimality of ILC methods according to the selection of learning gain matrices. In [16], [17], off-line ILC schemes were applied to seek the optimal gain of PID. In [18], [19], ILC mechanism, available to produce initial input signals through an off-line optimal tuning method, was employed to achieve the desired final output.

Most ILC algorithms update their input signals $\boldsymbol{u}(t, i)$ through an iterative form $\boldsymbol{u}(t, i+1) = \boldsymbol{u}(t, i) + Kf(\boldsymbol{e}(\cdot, i))$ (t and i is the iteration number and time index) and the learning gain matrix K is usually required to meet certain necessary conditions such as $\|I - KL\| < 1$ (where the matrix L is an input-output-coupling matrix of the system) in a bid to guarantee the convergence of ILC. This necessary condition serves as the selection guideline of learning gain matrix. However, it is less discussed more specific design of the learning gain matrix K according to some given optimization index for practical implementations. For all the tangible advances in the online computation methods, the optimal/sub-optimal learning gain matrix K is generally dependent on the iteration number and time instant. Notably, it is not always convenient for engineers to adopt an iteration-time-varying gain matrix in practice. Thus, we are motivated to find a proper fixed learning gain matrix K according to some specified index. In other words, our main objective in this paper is to provide an optimal/sub-optimal off-line computed learning gain matrix.

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Especially in the era featured "big data", many industrial processes produce a huge amount of input/output (I/O) data. Those online ILC algorithms are required to solve inverse matrices with high dimensions continuously to generate the learning gain matrix, resulting in a considerably large calculation burden. To solve this problem, the steady-state Kalman filtering technique has been proved an effective approach [20]. In particular, this technique replaces the varying Kalman gain matrix with a constant matrix, consequently decreasing the online computation time while applied to big data processes. In this direction, it is important to establish a steady-state gain matrix according to some specified index. Meanwhile, similar results are derived for single-input-single-output systems [21], [22], undoubtedly motivating us to consider an off-line ILC for general multi-input-multi-output (MIMO) systems.

Motivated by the above observations, this paper considers to establish an off-line learning framework with a constant learning gain matrix, which is designed according to the steady-state Kalman filtering theory. As a result, this framework is proved effective in increasing convergence rate and achieving acceptable tracking performance. Comparing with the existing literature, our novelties are summarized as follows:

- A novel ILC algorithm is proposed on the basis of the steady-state Kalman filtering method.
- The calculation burden is remarkably decreased by solving a matrix Riccati equation off-line.
- The advantage of the proposed algorithm compared those with the arbitrarily selected constant gain matrix is demonstrated by simulations.

The rest of this paper is organized as follows. The problem formulation is given in Section II. In Section III, derivations of the ILC algorithm based on a 2D model is presented. A matrix Riccati equation and the generation of the steady-state learning gain matrix are given in Section IV. Convergence results of the proposed scheme is provided in Section V. Illustrative simulations are given in Section VI. Section VII concludes the paper.

II. PROBLEM FORMULATION

Consider the following discrete time-varying linear MIMO system:

$$\begin{aligned} \boldsymbol{x}(t+1,i) &= A(t)\boldsymbol{x}(t,i) + B(t)\boldsymbol{u}(t,i) + \boldsymbol{\eta}(t,i), \\ \boldsymbol{y}(t,i) &= C(t)\boldsymbol{x}(t,i) + \boldsymbol{\xi}(t,i), \end{aligned} \tag{1}$$

where i denotes the iteration number and $t \in \{0, 1, \ldots, T\}$ with T being the trial length. The input vector, state vector and output vector are denoted by $\boldsymbol{u}(t,i) \in \mathbb{R}^k$, $\boldsymbol{x}(t,i) \in \mathbb{R}^s$ and $\boldsymbol{y}(t,i) \in \mathbb{R}^o$, respectively, and $A(t) \in \mathbb{R}^{s\times s}$, $B(t) \in \mathbb{R}^{s\times k}$, and $C(t) \in \mathbb{R}^{o\times s}$ are system matrices with proper dimensions. $\boldsymbol{\eta}(t,i) \in \mathbb{R}^s$ and $\boldsymbol{\xi}(t,i) \in \mathbb{R}^o$ are stochastic system noises and measurement noises, respectively.

For further analysis, the following assumptions are imposed for system (1).

Assumption 1: For all t, the matrix $C(t+1)B(t)$ is of full-column rank.

Assumption 2: The desired output trajectory $\boldsymbol{y}_d(t)$ can be realized in the sense that there exists a unique $\boldsymbol{u}_d(t)$ and $\boldsymbol{x}_d(t)$ satisfying the following equation:

$$\begin{aligned} \boldsymbol{x}_d(t+1) &= A(t)\boldsymbol{x}_d(t) + B(t)\boldsymbol{u}_d(t), \\ \boldsymbol{y}_d(t) &= C(t)\boldsymbol{x}_d(t). \end{aligned} \tag{2}$$

Assumption 3: Both stochastic system noises $\{\boldsymbol{\eta}(t,i)\}$ and measurement noises $\{\boldsymbol{\xi}(t,i)\}$ are independent and identically distributed random variables with zero-mean normal distribution such that $\mathbb{E}(\boldsymbol{\eta}(t,i)\boldsymbol{\eta}^T(t,i)) = Q_t$ is a positive-semidefinite matrix and $\mathbb{E}(\boldsymbol{\xi}(t,i)\boldsymbol{\xi}^T(t,i)) = R_t$ is a positive-definite matrix, $\forall i$. Moreover, $\{\boldsymbol{\eta}(t,i)\}$ is independent of $\{\boldsymbol{\xi}(t,i)\}$, $\forall t, i$.

Remark 1: From Assumption 1, the system relative degree is one. The dimension of the system output is not smaller than that of the system input, i.e., $o \geq k$. Hence, according to the desired reference, a unique input can be defined as follows:

$$\boldsymbol{u}_d(t) = [(C(t+1)B(t))^T C(t+1)B(t)]^{-1}(C(t+1)B(t))^T \times [\boldsymbol{y}_d(t+1) - C(t+1)A(t)\boldsymbol{x}_d(t)].$$

Assumption 3 implies $\mathbb{E}(\boldsymbol{\eta}(t,i)\boldsymbol{\xi}^T(t+1,i)) = 0$.

Assumption 4: The initial state error $\boldsymbol{x}_d(0) - \boldsymbol{x}(0,i)$ and the initial input error $\boldsymbol{u}_d(t) - \boldsymbol{u}(t,0)$ are random variables, subject to zero-mean normal distribution such that $\mathbb{E}[(\boldsymbol{x}_d(0) - \boldsymbol{x}(0,i))(\boldsymbol{x}_d(0) - \boldsymbol{x}(0,i))^T]$ is a positive-semidefinite matrix and $\mathbb{E}[(\boldsymbol{u}_d(t) - \boldsymbol{u}(t,0))(\boldsymbol{u}_d(t) - \boldsymbol{u}(t,0))^T]$ is a symmetrical positive-definite matrix. In addition, $\boldsymbol{x}_d(0) - \boldsymbol{x}(0,i)$ have no correlation with $\boldsymbol{u}_d(t) - \boldsymbol{u}(t,0)$, $\boldsymbol{\eta}(0,i)$ and $\boldsymbol{\xi}(0,i)$.

The control objective of this paper is to derive a suitable constant learning gain matrix such that an ILC algorithm is established to generate the input signal enabling that the output approximate to the desired trajectory as iteration number increases. The details are given in the next section.

III. CONVENTIONAL KALMAN FILTERING-BASED FRAMEWORK

In this section, we revisit the derivations in [24], [25] for the iteration-time-varying Kalman filtering-based ILC algorithm. These derivations establish the basis of our framework in the next section.

The tracking error $\boldsymbol{e}(t,i)$ is defined as follows:

$$\boldsymbol{e}(t,i) = \boldsymbol{y}_d(t) - \boldsymbol{y}(t,i), \quad 0 < t \leqslant N.$$

The learning update is given by

$$\boldsymbol{u}(t,i+1) = \boldsymbol{u}(t,i) + K(t,i)\boldsymbol{e}(t+1,i) \tag{3}$$

where $K(t,i) \in \mathbb{R}^{k\times o}$ is the iteration- and time-dependent learning gain matrix.

The following 2D model was given in [24], [25]:

$$\begin{bmatrix} \delta\boldsymbol{u}(t,i+1) \\ \delta\boldsymbol{x}(t+1,i) \end{bmatrix} = \begin{bmatrix} I - K(t,i)C^+B(t) & -K(t,i)C^+A(t) \\ B(t) & A(t) \end{bmatrix} \begin{bmatrix} \delta\boldsymbol{u}(t,i) \\ \delta\boldsymbol{x}(t,i) \end{bmatrix}$$

$$+\begin{bmatrix} K(t,i)C^{+} & K(t,i) \\ -I & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{\eta}(t,i) \\ \boldsymbol{\xi}(t+1,i) \end{bmatrix}. \quad (4)$$

where $C^{+} \triangleq C(t+1)$, $\delta\boldsymbol{u}(t,i) = \boldsymbol{u}_d(t) - \boldsymbol{u}(t,i)$, and $\delta\boldsymbol{x}(t,i) = \boldsymbol{x}_d(t) - \boldsymbol{x}(t,i)$.

For the sake of simplicity, let

$$X^{+} = \begin{bmatrix} \delta\boldsymbol{u}(t,i+1) \\ \delta\boldsymbol{x}(t+1,i) \end{bmatrix}, \quad X = \begin{bmatrix} \delta\boldsymbol{u}(t,i) \\ \delta\boldsymbol{x}(t,i) \end{bmatrix}, \; Z = \begin{bmatrix} \boldsymbol{\eta}(t,i) \\ \boldsymbol{\xi}(t+1,i) \end{bmatrix}$$

and

$$\Phi = \begin{bmatrix} I - K(t,i)C^{+}B(t) & -K(t,i)C^{+}A(t) \\ B(t) & A(t) \end{bmatrix},$$
$$\Psi = \begin{bmatrix} K(t,i)C^{+} & K(t,i) \\ -I & 0 \end{bmatrix}.$$

Then, (4) is reduced as

$$X^{+} = \Phi X + \Psi Z. \quad (5)$$

Applying the conventional Kalman filtering technique, we attempt to search an optimal learning gain matrix $K(t,i)$, which minimizes the trace of the error covariance matrix $P^{+} = \mathbb{E}[X^{+}X^{+T}]$. From (5), we obtain

$$\begin{aligned} P^{+} &= \mathbb{E}[(\Phi X + \Psi Z)(\Phi X + \Psi Z)^T] \\ &= \Phi\mathbb{E}[XX^T]\Phi^T + \Phi\mathbb{E}[XZ^T]\Psi^T \\ &\quad + \Psi\mathbb{E}[ZX^T]\Phi^T + \Psi\mathbb{E}[ZZ^T]\Psi^T. \end{aligned} \quad (6)$$

Let

$$P = \mathbb{E}[XX^T] = \begin{bmatrix} P_{11,i} & P_{12,i} \\ P_{12,i}^T & P_{22,i} \end{bmatrix}$$
$$Q = \mathbb{E}[ZZ^T] = \begin{bmatrix} Q_t & 0 \\ 0 & R_{t+1} \end{bmatrix},$$

where $P_{11,i} = \mathbb{E}[\delta\boldsymbol{u}(t,i)\delta\boldsymbol{u}^T(t,i)]$, $P_{12,i} = \mathbb{E}[\delta\boldsymbol{u}(t,i)\delta\boldsymbol{x}^T(t,i)]$, $P_{22,i} = \mathbb{E}[\delta\boldsymbol{x}(t,i)\delta\boldsymbol{x}^T(t,i)]$, $Q_t = \mathbb{E}[\boldsymbol{\eta}(t,i)\boldsymbol{\eta}^T(t,i)]$, $R_{t+1} = \mathbb{E}[\boldsymbol{\xi}(t+1,i)\boldsymbol{\xi}^T(t+1,i)]$.

Fcctness, define $A = A(t)$, $B = B(t)$, $C^{+} = C(t+1)$, $K_i = K(t,i)$, $\Phi_1 = I - K_iC^{+}B$, $\Phi_2 = K_iC^{+}A$, $\Psi_1 = -K_iC^{+}B$, and $\Psi_2 = -K_iC^{+}A$. Expanding the terms on the right-hand side of (6), we obtain the equation displayed at the top of the next page. As a consequence, the trace of P^{+} in (6) can be calculated as follows:

$$\begin{aligned} &\text{trace}(P^{+}) \\ &= \text{trace}\{(\Phi_1P_{11,i} + \Phi_2P_{12,i}^T)\Phi_1^T + (\Phi_1P_{12,i} + \Phi_2P_{22,i})\Phi_2^T \\ &\quad + (BP_{11,i} + AP_{12,i}^T)B^T + Q_t + (BP_{12,i} + AP_{22,i})A^T \\ &\quad + K_iC^{+}Q_t(K_iC^{+})^T + K_iR_{t+1}K_i^T\}. \end{aligned}$$

Let $F_1 = \left(C^{+}B, \; C^{+}A\right)$, $F_2 = \left(B, \; A\right)$, and $F_3 = \left(I, \; 0\right)$. Then, the above equation becomes

$$\begin{aligned} \text{trace}(P^{+}) = \text{trace}\{&K_iF_1PF_1^TK_i^T + F_2PF_2^T + F_3PF_3^T \\ &- K_iF_1PF_3^T - F_3PF_1^TK_i^T \\ &+ K_i[(C^{+}Q_tC^{+T} + R_{t+1})]K_i^T + Q_t\}. \end{aligned} \quad (7)$$

Therefore, calculating the derivative of trace(P^{+}) with respect to K_i leads to

$$\frac{\partial\text{trace}(P^{+})}{\partial K_i} = 2K_iF_1PF_1^T - 2F_3PF_1^T + 2K_i[(C^{+}Q_tC^{+T} + R_{t+1})].$$

Letting $\frac{\partial\text{trace}(P^{+})}{\partial K_i} = 0$ yields that

$$2K_iF_1PF_1^T - 2F_3PF_1^T + 2K_i[(C^{+}Q_tC^{+T} + R_{t+1})] \equiv 0.$$

Then, the optimal learning gain matrix K_i, which is time-varying and iteration-varying, is given by:

$$K_i = F_3PF_1^T\Gamma^{-1}, \quad (8)$$

where $\Gamma = (F_1PF_1^T + C^{+}Q_tC^{+T} + R_{t+1})$.

From (8), it is worth pointing out that K_i is an optimal learning gain matrix by applying the conventional Kalman filtering technique, whereas in real applications K_i is likely replaced with a fixed gain to save control efforts. Therefore, we are interested in establishing an ILC algorithm with a fixed gain, which is optimal/sub-optimal according to certain index. This is the major contribution in this study.

IV. STEADY-STATE KALMAN FILTERING-BASED FRAMEWORK

In this section, we apply the steady-state (or limiting) Kalman filtering technique to calculate the limiting learning gain matrix $\overrightarrow{K}$, which is employed to replace K_i in the ILC algorithm. This replacement avoids online calculation of the iteration-dependent gain matrix as in the previous section and thus saves computation resources. Furthermore, we show that this limiting gain matrix is an optimal/sub-optimal selection among the set of constant learning gain matrices.

Using the derivations in [24], [25], we obtain

$$\begin{aligned} \delta\boldsymbol{x}(t,i) &= \left[\prod_{m=0}^{t-1} A^T(m)\right]^T \delta\boldsymbol{x}(0,i) \\ &\quad + \sum_{l=0}^{t-1}\left[\prod_{n=l}^{t-2} A^T(n+1)\right]^T [B(l)\delta\boldsymbol{u}(l,i) - \boldsymbol{\eta}(l,i)], \\ \delta\boldsymbol{u}(t,i) &= \left[\prod_{m=0}^{i-1} \Upsilon_{1,m}^T\right]^T \delta\boldsymbol{u}(t,0) \\ &\quad + \sum_{m=0}^{i-1}\left[\prod_{n=l}^{i-2} \Upsilon_{1,n+1}^T\right]^T [\Upsilon_{2,l}\delta\boldsymbol{x}(t,l) - h_{t,l}]. \end{aligned}$$

Define

$$\begin{aligned} h_{t,l} &\triangleq K_iC^{+}\boldsymbol{\eta}(t,i) + K_i\boldsymbol{\xi}(t+1,i), \\ \Upsilon_{1,i} &\triangleq I - K_iC^{+}B(t), \\ \Upsilon_{2,i} &\triangleq -K_iC^{+}A(t). \end{aligned}$$

If $m > i-2$, we denote $\prod_{k=m}^{i-2}[\text{item}] = I$. By Assumptions 3 and 4, there is irrelevance among $\delta\boldsymbol{x}(0.i)$, $\delta\boldsymbol{u}(0,i)$, $\boldsymbol{\eta}(0 \leqslant m \leqslant t-1, i)$, $\boldsymbol{\eta}(t, 0 \leqslant m \leqslant i-1)$ and $\boldsymbol{\xi}(t+1, 0 \leqslant m \leqslant i-1)$.

$$\Phi\mathbb{E}[XX^T]\Phi^T = \begin{bmatrix}\Phi_1 & \Phi_2\\ B & A\end{bmatrix}\begin{bmatrix}P_{11,i} & P_{12,i}\\ P_{12,i}^T & P_{22,i}\end{bmatrix}\begin{bmatrix}\Phi_1^T & B^T\\ \Phi_2^T & A^T\end{bmatrix}$$
$$= \begin{bmatrix}\begin{pmatrix}\Phi_1P_{11,i}\Phi_1^T + \Phi_2P_{12,i}^T\Phi_1^T\\ +\Phi_1P_{12,i}\Phi_2^T + \Phi_2P_{22,i}\Phi_2^T\end{pmatrix} & \begin{pmatrix}\Phi_1P_{11,i}B^T + \Phi_2P_{12,i}^TB^T\\ +\Phi_1P_{12,i}A^T + \Phi_2P_{22,i}A^T\end{pmatrix}\\ \begin{pmatrix}BP_{11,i}\Phi_1^T + AP_{12,i}^T\Phi_1^T\\ +BP_{12,i}\Phi_2^T + AP_{22,i}\Phi_2^T\end{pmatrix} & \begin{pmatrix}BP_{11,i}B^T + AP_{12,i}^TB^T\\ +BP_{12,i}A^T + AP_{22,i}A^T\end{pmatrix}\end{bmatrix}$$
$$\Psi\mathbb{E}[ZZ^T]\Psi^T = \begin{bmatrix}K_iC^+ & K_i\\ -I & 0\end{bmatrix}\begin{bmatrix}Q_t & 0\\ 0 & R_{t+1}\end{bmatrix}\begin{bmatrix}(K_iC^+)^T & -I\\ (K_i)^T & 0\end{bmatrix}$$
$$= \begin{bmatrix}\begin{pmatrix}K_iC^+Q_t(K_iC^+)^T\\ +K_iR_{t+1}(K_i)^T\end{pmatrix} & -K_iC^+Q_t\\ -Q_t(K_iC^+)^T & Q_t\end{bmatrix}.$$

Furthermore, the term $\delta\boldsymbol{u}(0 \leqslant m \leqslant t-1, i)$ is irrelevant to $\delta\boldsymbol{x}(t, 0 \leqslant m \leqslant i-1)$. This is because there is no functional relationship between these terms and anyone cannot represent the other. Hence, we have $P_{12,i} = 0$.

Rewriting (8) as

$$K_i = P_{11,i}L^T[LP_{11,i}L^T + S_{1,i}]^{-1}. \tag{9}$$

The input error covariance matrix becomes

$$P_{11,i+1} = (I - K_iL)P_{11,i}(I - K_iL)^T + K_iS_{1,i}^{-1}K_i^T, \tag{10}$$

where $S_{1,i} = [(C^+A)P_{22,i}(C^+A)^T + C^+Q_tC^{+T} + R_{t+1}]$ and $L = C^+B$.

Moreover, from (4), we can get

$$\begin{aligned}P_{11,i+1} &= \mathbb{E}[\delta\boldsymbol{u}(t,i+1)\delta\boldsymbol{u}^T(t,i+1)]\\ &= F_3PF_3^T + K_iF_1PF_1^TK_i^T - K_iF_1PF_3^T\\ &\quad - F_3PF_1^TK_i^T + K_i(C^+Q_tC^{+T} + R_{t+1})K_i^T\\ &= P_{11,i} - K_iLP_{11,i} - P_{11,i}L^TK_i^T\\ &\quad + K_i[LP_{11,i}L^T + S_{1,i}]K_i^T\\ &= (I - K_iL)P_{11,i}.\end{aligned} \tag{11}$$

Obviously, even if the system model (1) is simple, it is imperative to invert a matrix at each iteration to get the optimal gain K_i in the equation (9). In real applications, it is incredibly imperative to replace K_i in equation (9) by a constant gain matrix so as to reduce calculation, thereby saving computation time.

To this end, we borrow the idea of steady-state Kalman filtering theory. In other words, a steady-state ILC framework is provided by replacing K_i with its corresponding limit $\overrightarrow{K}$. Here, $\overrightarrow{K}$ is called the steady-state gain matrix.

We rewrite (3) as follows:

$$\boldsymbol{u}(t, i+1) = \boldsymbol{u}(t, i) + \overrightarrow{K}\boldsymbol{e}(t+1, i). \tag{12}$$

In [25], it is observed that the sequence K_i does converge as the iteration number increases. In fact, $tr\|K_i - \overrightarrow{K}\|^2$ tends to zero exponentially. This observation, to certain extent, tells us that the replacement of K_i with $\overrightarrow{K}$ may not affect the final tracking performance much. However, due to the existence of random noise, the input error will no longer retain a stable convergence.

According to the definition of K_i in (9), in a bid to study the convergence of K_i, it is necessary that we will study the convergence of $P_{11,i+1} := P_{11,i}$ as $i \to \infty$.

From (9) and (10), we have

$$\begin{aligned}P_{11,i+1} &= (I - K_iL)P_{11,i}(I - K_iL)^T + K_i(C^+Q_tC^{+T}\\ &\quad + R_{t+1})K_i^T\\ &= (I - K_iL)(I - K_{i-1}L)P_{11,i-1}(I - K_iL)^T\\ &\quad + K_i(C^+Q_tC^{+T} + R_{t+1})K_i^T\\ &= (I - K_iL)[I - P_{11,i-1}L^T(LP_{11,i-1}L^T\\ &\quad + C^+Q_tC^{+T} + R_{t+1})^{-1}L]P_{11,i-1}(I - K_iL)^T\\ &\quad + K_i(C^+Q_tC^{+T} + R_{t+1})K_i^T.\end{aligned}$$

Because $P_{11,i+1} := P_{11,i}$ as $i \to \infty$, the above equation can be rewritten as follows:

$$\begin{aligned}P_{11,i} &= (I - K_iL)(P_{11,i-1} - P_{11,i-1}L^T(LP_{11,i-1}L^T\\ &\quad + C^+Q_tC^{+T} + R_{t+1})^{-1}LP_{11,i-1})(I - K_iL)^T\\ &\quad + K_i(C^+Q_tC^{+T} + R_{t+1})K_i^T.\end{aligned} \tag{13}$$

Remark 2: To further simplify the calculation, the term $S_{1,i}$ in (9) and (10) can be reduced to $C^+Q_tC^{+T} + R_{t+1}$, where the term $(C^+A)P_{22,i}(C^+A)^T$ is omitted. Such calculation results in a sub-optimal selection, as has been proved in [24]. Details are omitted here for saving space.

By setting

$$\Omega(T) = H(T - TL^T(LTL^T + Q')^{-1}LT)H + K_iQ'K_i^T, \tag{14}$$

where $H = I - K_iL$ and $Q' = C^+Q_tC^{+T} + R_{t+1}$, it is apparent that $P_{11,i}$ satisfies the following recursion,

$$P_{11,i} = \Omega(P_{11,i-1}). \tag{15}$$

This relation is known as a matrix Riccati equation. While $P_{11,i} \to \overrightarrow{P}$ as $i \to \infty$, $\overrightarrow{P}$ meets the above relation, i.e.,

$$\overrightarrow{P} = \Omega(\overrightarrow{P}). \tag{16}$$

Consequently, we can solve the above equation for $\overrightarrow{P}$ iteratively and define

$$\overrightarrow{K} = \overrightarrow{P} L^T (L \overrightarrow{P} L^T + C^+ Q_t C^{+T} + R_{t+1})^{-1}. \quad (17)$$

Accordingly, $K_i \rightarrow \overrightarrow{K}$ as $i \rightarrow \infty$. Because $P_{11,i}$ is a symmetric matrix, so is $\Omega(P_{11,i})$.

Remark 3: From (14)*, we notice the noise term $K_i Q_t K_i^T$ in the matrix Riccati equation. It is difficult for us to solve this equation because K_i is coupled in this equation. For practical applications, we solve the equation iteratively. In particular, we first set the term $K_i Q_t K_i^T$ as zero, because it has been proved $K_i \rightarrow 0$ uniformly in $[0, n]$ as $i \rightarrow \infty$ [25]. Then, we solve the Riccati equation and generate a corresponding gain matrix K^*. Next, we replace K_i by K^* to solve the Riccati equation again. Repeating the above process leads to $\overrightarrow{P}$ and $\overrightarrow{K}$.*

V. CONVERGENCE ANALYSIS

In this section, convergence of the proposed framework is analyzed in detail. We give the following claim first.

Claim 1: Suppose that both matrices $\overrightarrow{P}$ and $S_{1,i}$ are symmetric and positive definite, then for all iterations i and $t \in T$, we can easily get,

$$I - \overrightarrow{K} L = [I + \overrightarrow{P} L^T S_{1,i}^{-1} L]^{-1}.$$

The proof is similar to that of Claim 1 in [25] and thus is omitted.

Lemma 1: If the matrix L is of full-column rank, adopting the ILC framework defined by (12), (14), *and* (17)*, the eigenvalues of $(I - \overrightarrow{K} L)$ are positive and less than one.*

Proof: Notice that the matrix $\overrightarrow{P}$ ia a symmetric positive-definite matrix. Denote $D = I - \overrightarrow{K} L$. Using Claim 1, we have $D = [I + \overrightarrow{P} L^T S_{1,i}^{-1} L]^{-1}$. Because L is of full-column rank and $S_{1,i}$ is a symmetric positive-definite matrix, we have that $L^T S_{1,i}^{-1} L$ is symmetric and positive definite. Hence, $\overrightarrow{P} L^T S_{1,i}^{-1} L$ is symmetric and all eigenvalues of $\overrightarrow{P} L^T S_{1,i}^{-1} L$ are positive. Then, the eigenvalues of $I + \overrightarrow{P} L^T S_{1,i}^{-1} M$ are greater than one. Therefore, these eigenvalues of $I - \overrightarrow{K} L$ are less than one. □

Remark 4: Since all the eigenvalues of $(I - \overrightarrow{K} L)$ are positive and strictly less than one over the entire time interval $t \in [0, n]$, there exists a consistent norm $\| \cdot \|$ such that, $\forall t \in [0, n]$

$$\|I - \overrightarrow{K} L\| < 1. \quad (18)$$

Theorem 1: If there exists a solution P for the following discrete algebraic Riccati equation (DARE):

$$P = A^T PA - A^T PB(B^T PB + R)^{-1} B^T PA + Q$$

with $P = P_{11,i} = \overrightarrow{P}$, $A = H$, $B = L, R = Q'$ and $Q = K^* Q' K^{*T}$, then $\lim_{i \to \infty} P_{11,i} = \overrightarrow{P}$.

Proof: According to (11) and Claim 1, we can easily obtain $\overrightarrow{P} = P_{11,i+1} \leq P_{11,i} \leq \cdots \leq P_{11,0}$ [25]. In order to prove the theorem, we will introduce the formula

$$\frac{d}{ds} A^{-1} = -A^{-1} [\frac{d}{ds} A(s)] A^{-1}(s).$$

Setting $T(s) = P_{11,i-1} + s(P_{11,i-1} - P_{11,i})$, we can obtain $\Omega(P_{11,i-1}) \geq \Omega(P_{11,i})$, shown at the bottom of the next page. Notice that $P_{11,1} = \mathbb{E}[\delta \boldsymbol{u}(t, 1) \delta \boldsymbol{u}^T (t, 1)] \geq 0$. Moreover, both $P_{11,i}$ and $P_{11,0}$ are symmetric. According to $\Omega(P_{11,i-1}) \geq \Omega(P_{11,i})$, we can obtain

$$\begin{aligned} P_{11,2} &= \Omega(P_{11,1}) \leq P_{11,1} \\ &\cdots \\ \overrightarrow{P} &= P_{11,i} \leq \Omega(P_{11,i-1}) \leq P_{11,i-1} \end{aligned}$$

Hence,$\{P_{11,i}\}$ is monotonic decreasing and bounded by $\overrightarrow{P}$. □

Remark 5: The value of $\overrightarrow{P}$ is a small non-negative constant matrix, $\overrightarrow{P} \geq 0$. For different systems, $\overrightarrow{P}$ also varies. It is feasible for practical applications while algorithms with $\overrightarrow{P}$ offer sufficient tracking precision.

Lemma 2 [27]: For the following DARE:

$$A^T PA - P - A^T PB(B^T PB + R)^{-1} B^T PA + Q = 0$$

with $Q = C^T C$, if (A, B) is stabilizable and (A, C) is observable, then there exists a unique stabilizing solution $P = P^T > 0$ for the above equation.

Theorem 2: In the proposed steady-state Kalman filtering-based ILC framework, if L is of full rank, then there always exists a unique positive definite solution P of DARE given in Theorem 1.

Proof: Let us define matrices

$$\begin{aligned} U_{ii} &= \sqrt{Q_{ii}}, \quad U_{ij} = 0, \quad \text{when } i \neq j \\ \overline{A} &:= \overline{H}; \quad \overline{B} = C^+ B = L; \quad \overline{C} := U. \end{aligned}$$

Because $\overline{B}$ is a full-rank matrix and is invertible, there exists a matrix $\overline{K}$ such that $\lambda(\overline{A} - \overline{BK}) < 0$. Since $\overline{A}$ and $\overline{C}$ are non-zero diagonal matrices, the pair $(\overline{A}, \overline{C})$ is detectable. Hence, by Lemma 2, the conclusion is proved. □

Remark 6: From [28], in addition to the conditions of Lemma 2, if the pair (A, B) is controllable, then we always have $\lambda(\overline{A} - \overline{BK}) < 0$. Since the pair $(\overline{A}, \overline{B})$ is controllable, we can easily obtain $\lambda(\overline{H} - \overline{LK}) < 0$.

Theorem 3: Assume that Assumptions 1-4 hold for system (1) *and apply the novel ILC algorithm defined by* (12), (14), *and* (17)*, then $P_{11,i} \rightarrow \overrightarrow{P}$ and*

$$\begin{aligned} P_{22,i} \rightarrow & \left[\prod_{m=0}^{t-1} A^T(m) \right]^T \mathbb{E}[\delta \boldsymbol{x}(0, i) \delta \boldsymbol{x}^T (0, i)] \left[\prod_{m=0}^{t-1} A^T(m) \right] \\ & + \sum_{l=0}^{t-1} \left[\prod_{n=l}^{t-2} A^T(n+1) \right]^T (Q_t + \overrightarrow{P}) \sum_{l=0}^{t-1} \left[\prod_{n=l}^{t-2} A^T(n+1) \right] \end{aligned}$$

as $i \rightarrow \infty$.

Proof: From Theorem 1, we obtain $\lim_{i \to \infty} P_{11,i} = \lim_{i \to \infty} \mathbb{E}[\delta \boldsymbol{u}(t, i) \delta \boldsymbol{u}^T (t, i)] = \overrightarrow{P}$.

Furthermore,

$$\delta x(t,i) = \left[\prod_{m=0}^{t-1} A^T(m)\right]^T \delta x(0,i) + \sum_{l=0}^{t-1}\left[\prod_{n=l}^{t-2} A^T(n+1)\right]^T \cdot [B(l)\delta u(l,i) - \eta(l,i)]$$

Notice that $x_d(0)-x(0,i)$ is uncorrelated with $u_d(t)-u(t,0)$, $\eta(0,i)$, and $\xi(0,i)$, thus we obtain the equation displayed at the bottom of this page. Using this property of $\lim_{i\to\infty} P_{11,i} = \lim_{i\to\infty} \mathbb{E}[\delta u(l,i)\delta u^T(l,i)] = \overrightarrow{P}$ leads to

$$\mathbb{E}[\delta x(t,i)\delta x^T(t,i)] = \left[\prod_{m=0}^{t-1} A^T(m)\right]^T \mathbb{E}[\delta x(0,i)\delta x^T(0,i)] \cdot \left[\prod_{m=0}^{t-1} A^T(m)\right] + \sum_{l=0}^{t-1}\left[\prod_{n=l}^{t-2} A^T(n+1)\right]^T \cdot (Q_t + \overrightarrow{P}) \sum_{l=0}^{t-1}\left[\prod_{n=l}^{t-2} A^T(n+1)\right].$$

The proof is completed. □

Remark 7: By Theorem 3, along with the increase of the iteration number, $P_{22,i}$ converges to a fixed matrix. If the system is free of noise and varying initial state, then $Q_t \to 0$ and $P_{22,i} \to 0$ as $i \to \infty$. Furthermore, from (4), we obtain

$$P_{22,i}(t+1) = AP_{22,i}(t)A^T + BP_{11,i}(t)B^T + Q_t. \tag{19}$$

The specific algorithm is given as follows:

Algorithm 1 ILC Based on Steady-State Kalman Filtering

(1) The initial input error covariance $P_{11,0}(t)$ is designed as zI with $z > 0$, for all t and the initial state error covariance $P_{22,i}(0)$ is designed as zero diagonal matrix, for all i;
(2) using Equation (14), compute $\overrightarrow{P_{11}}$;
(3) using Equation (19), compute $P_{22,i}(t+1)$;
(4) using Equation (17), compute learning gain $\overrightarrow{K}$;
(5) using Equation (12), update the control $u(t, i+1)$;
(6) $i = i+1$, repeat whole process until the stop rule is satisfied.

VI. ILLUSTRATIVE SIMULATION

A. EXAMPLE 1

To illustrate the effectiveness of the novel ILC scheme, we consider a time-varying linear MIMO system, where the system matrices $(A(t), B(t), C(t))$ are given as follows:

$$A(t) = \begin{bmatrix} \frac{3}{50}\sin(\frac{3t}{10}) & -\frac{1}{10} & \frac{3t}{100} \\ \frac{1}{10} & -\frac{t}{50} & -\frac{1}{20}\cos(\frac{3t}{10}) \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10}+\frac{2}{25}\cos(\frac{3t}{10}) \end{bmatrix},$$

$$B(t) = \begin{bmatrix} \frac{3}{2}-\frac{2}{5}\cos^2(\frac{2\pi t}{5}) & 0 \\ \frac{t}{50} & \frac{t}{50} \\ 0 & \frac{1}{5}\sin(\frac{2\pi t}{5}) \end{bmatrix},$$

$$C(t) = \begin{bmatrix} \frac{2}{5}+\frac{1}{5}\sin^2(\frac{3\pi t}{10}) & \frac{1}{10} & -\frac{2}{5} \\ 0 & \frac{2}{5} & \frac{2}{5}-\frac{1}{5}\sin(\frac{3\pi t}{10}) \end{bmatrix}.$$

$$\begin{aligned}
\Omega(P_{11,i-1}) - \Omega(P_{11,i}) &= \int_0^1 \frac{d}{ds}\Omega(P_{11,i-1} + s(P_{11,i-1} - P_{11,i})) \\
&= H\Big\{\int_0^1 \frac{d}{ds}\{(P_{11,i-1} + s(P_{11,i-1} - P_{11,i})) - (P_{11,i-1} + s(P_{11,i-1} - P_{11,i})L^T \\
&\quad \cdot [L((P_{11,i-1} + s(P_{11,i-1} - P_{11,i}))L^T + Q']^{-1} L(P_{11,i-1} + s(P_{11,i-1} - P_{11,i}))\}ds\Big\}H^T \\
&= H\Big\{\int_0^1 [P_{11,i-1} - P_{11,i} - (P_{11,i-1} - P_{11,i})L^T(LT(s)L^T + Q')^{-1}LT(s) \\
&\quad - T(s)L^T(LT(s)L^T + Q')^{-1}L(P_{11,i-1} - P_{11,i}) + T(s)L^T(LT(s)L^T + Q')^{-1} \\
&\quad \cdot L(P_{11,i-1} - P_{11,i})L^T(LT(s)L^T + Q')^{-1}LT(s)]ds\Big\}H^T \\
&= H\Big\{\int_0^1 [T(s)L^T(LT(s)L^T + Q')^{-1}L](P_{11,i-1} - P_{11,i}) \\
&\quad \cdot [T(s)L^T(LT(s)L^T + Q')^{-1}L]^T ds\Big\}H \\
&\geq 0.
\end{aligned}$$

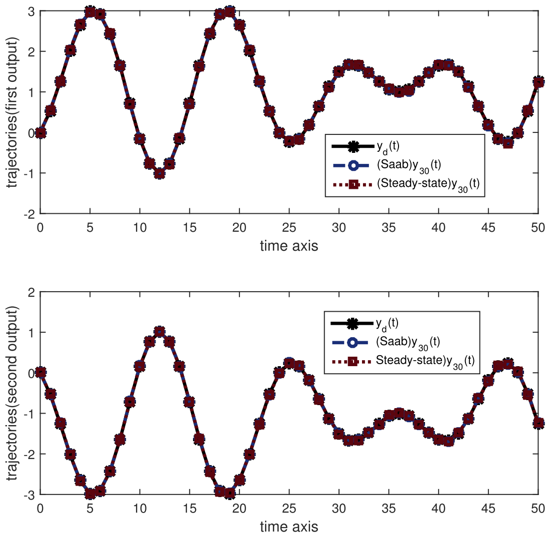

FIGURE 1. The desired trajectory and the output at the 30th iteration, where the upper and lower plots correspond to the first and second dimensions of the output.

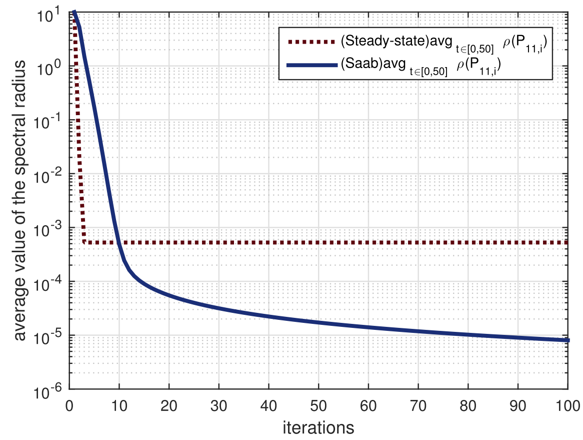

FIGURE 2. Comparison of $\mathrm{avg}_{t\in[0,50]}\rho(P_{11,i}(t))$ for the proposed algorithm and Saab's algorithm.

The desired trajectory is presented in the following:

$$\boldsymbol{y}_d(t) = \begin{bmatrix} \sin(\frac{\pi t}{8}) + \frac{11}{10} - \frac{11}{10}\cos(\frac{\pi t}{6}) \\ -\sin(\frac{\pi t}{8}) - \frac{11}{10} - \frac{11}{10}\cos(\frac{\pi t}{6}) \end{bmatrix}, \quad t \in \mathbb{Z}_{50}.$$

Both the stochastic system noise $\eta(t, i)$ and measurement noise $\xi(t, i)$ are subject to normal distribution $N(0, 0.1^2)$.

The conventional online stochastic ILC based on Kalman filtering method in [25] is also simulated for comparison. These algorithms run for 100 iterations.

From Fig. 1, we can clearly see that the proposed ILC algorithm performs acceptable tracking performance compared with the conventional stochastic ILC [25]. Fig. 2 illustrates that the proposed steady-state ILC has a faster convergence rate than that of Saab's algorithm; however, the convergence precision is not as good as Saab's algorithm. These observations coincide with our theoretical analysis. Furthermore, to demonstrate the optimality of the proposed ILC, we select two different fixed gain matrices:

$$\text{gain } 1 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix} \quad \text{and} \quad \text{gain } 2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

The maximum absolute tracking error profiles along the iteration axis are provided in Figs. 3 and 4 for the first and second dimension of the output, respectively, compared with those generated by the proposed steady-state ILC algorithm. It is observed that the tracking performance of the proposed algorithm is significantly better than the other two.

$$\begin{aligned}
\mathbb{E}[\delta\boldsymbol{x}(t,i)\delta\boldsymbol{x}^T(t,i)] &= \left[\prod_{m=0}^{t-1} A^T(m)\right]^T \mathbb{E}[\delta\boldsymbol{x}(0,i)\delta\boldsymbol{x}^T(0,i)] \left[\prod_{m=0}^{t-1} A^T(m)\right] \\
&\quad + \sum_{l=0}^{t-1}\left[\prod_{n=l}^{t-2} A^T(n+1)\right]^T \mathbb{E}\left\{[B(l)\delta\boldsymbol{u}(l,i) - \boldsymbol{\eta}(l,i)]\right. \\
&\quad \left.\cdot [B(l)\delta\boldsymbol{u}(l,i) - \boldsymbol{\eta}(l,i)]^T\right\} \sum_{l=0}^{t-1}\left[\prod_{n=l}^{t-2} A^T(n+1)\right] \\
&= \left[\prod_{m=0}^{t-1} A^T(m)\right]^T \mathbb{E}[\delta\boldsymbol{x}(0,i)\delta\boldsymbol{x}^T(0,i)] \left[\prod_{m=0}^{t-1} A^T(m)\right] \\
&\quad + \sum_{l=0}^{t-1}\left[\prod_{n=l}^{t-2} A^T(n+1)\right]^T \left\{B(l)\mathbb{E}[\delta\boldsymbol{u}(l,i)\delta\boldsymbol{u}^T(l,i)]B(l)^T\right. \\
&\quad \left. + \mathbb{E}[\boldsymbol{\eta}(l,i)\boldsymbol{\eta}^T(l,i)]\right\} \sum_{l=0}^{t-1}\left[\prod_{n=l}^{t-2} A^T(n+1)\right].
\end{aligned}$$

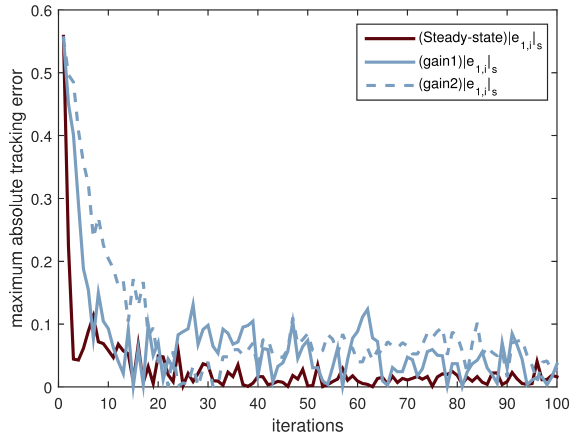

FIGURE 3. Comparison of maximum absolute tracking error with two fixed constant gain matrices: The first dimension of the output.

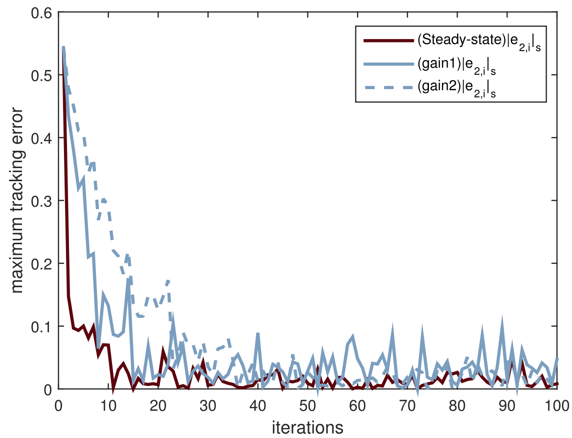

FIGURE 4. Comparison of maximum absolute tracking error with two fixed constant gain matrices: The second dimension of the output.

B. EXAMPLE 2

To further demonstrate the effectiveness of the proposed steady-state ILC algorithm for practical applications, the following model of permanent magnet linear motor (PMLM) is used [29]:

$$\begin{cases} x(t+1) = x(t) + v(t)\Delta \\ v(t+1) = v(t) - \Delta\dfrac{k_1 k_2 \psi_f^2}{Rm}v(t) + \Delta\dfrac{k_2\psi_f}{Rm}u(t) \\ y(t) = v(t) \end{cases}$$

where x denotes the motor position and v is rotor velocity. $\Delta = 0.01s$, $R = 8.6$, $m = 1.635kg$, $\psi_f = 0.35Wb$ are the discrete time interval, the resistance of stator, the rotor mass and the flux linkage, respectively. $k_1 = \pi/\tau$ and $k_2 = 1.5\pi/\tau$ with $\tau = 0.031m$ represent the pole pitches. It is effortless to find these coefficient matrices, $A = \begin{bmatrix} 1 & \Delta \\ 0 & 1 - \Delta\frac{k_1 k_2 \psi_f^2}{Rm} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \Delta\frac{k_2\psi_f}{Rm} \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$. It is clear that the output/input coupling matrix CB is full-column rank. Besides, the desired trajectory is presented by $y_d(t) = -\sin(\frac{t\pi}{20}) + \frac{1}{2} - \frac{1}{2}\cos(\frac{t}{50})$, $0 \le t \le \frac{1}{2}$.

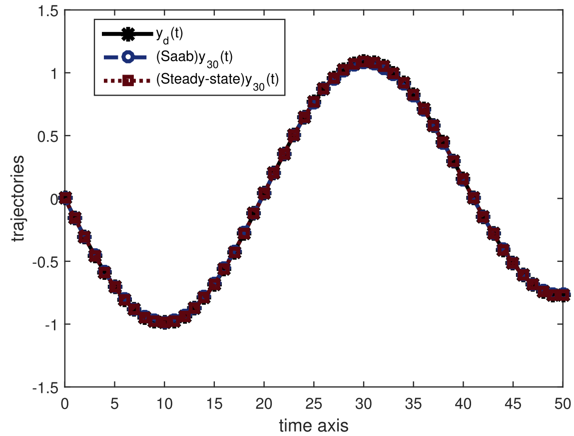

FIGURE 5. The desired trajectory and the output at the 30th iteration for the proposed algorithm and Saab's algorithm.

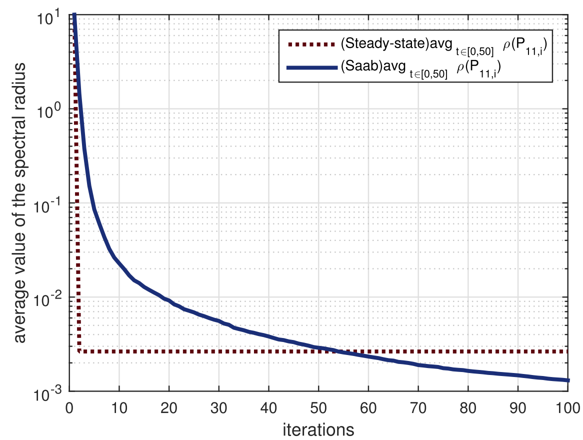

FIGURE 6. Comparison of $\text{avg}_{t\in[0,50]}\rho(P_{11,i}(t))$ for the proposed algorithm and Saab's algorithm.

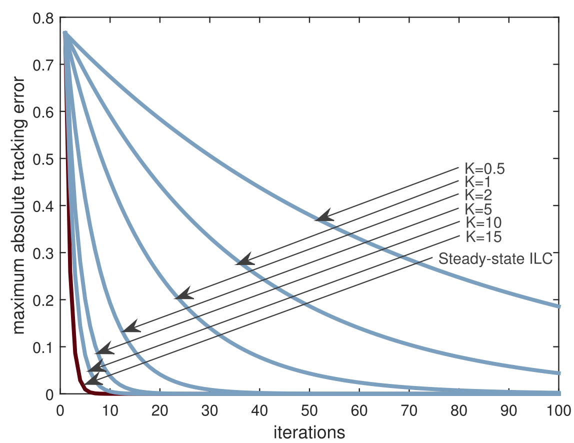

FIGURE 7. Comparison of maximum absolute tracking error with six fixed constant gains.

Fig. 5 illustrates the tracking performance of the proposed algorithm and Saab's algorithm at the 30th iteration, where we can observe that the output of the proposed algorithm affords a high tracking precision. The convergence rate and convergence precision of these two algorithms are shown in Fig. 6, where the advantages and disadvantages of the proposed algorithm can be observed. Lastly, Fig. 7 demonstrate the comparison of the proposed algorithm with other ILC algorithms using arbitrary fixed gain. In particular, we simulate the case of gain being 0.5, 1, 2, 5, 10, and 15. The results demonstrates the optimality of the proposed algorithm to certain extent.

VII. CONCLUSION

In this paper, we proposed a novel ILC algorithm based on the steady-state Kalman filtering theory for linear stochastic MIMO systems. The learning gain matrix is derived off-line and thus saves online computation resource and increases the convergence rate. To this end, we revisited the conventional Kalman filtering-based ILC and then compute the desired stead-state learning gain matrix by solving a DARE. The computed matrix is optimal/sub-optimal in the set of fixed gain matrices according to certain index. In other words, the proposed approach can help engineers select a good gain matrix according to real applications. Convergence of the proposed algorithm is strictly analyzed. For further research, it is of significance to extend the approach to nonlinear systems.

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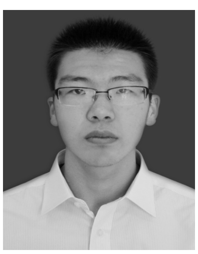

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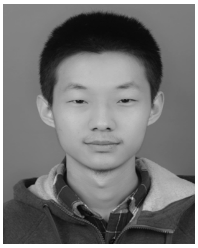

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