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A New Geometric Mean FMEA Method Based on Information Quality

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ABSTRACT Failure mode and effect analysis (FMEA) is used for risk assessment. The risk priority number (RPN) is the product of the three indicators of severity (S), probability of occurrence (O), and detection (D), which is an important measure to determine the risk priority. A new geometric mean FMEA method based on information quality is presented. First, a fuzzy evaluation distribution form is proposed, which constructs a more flexible and reasonable expression of experts' opinions in decision-making. Second, a geometric mean method to combine several probability distributions based on information quality is proposed to calculate the RPN. Finally, a numerical case study is illustrated to show the efficiency of the proposed method.

INDEX TERMS Failure mode and effect analysis (FMEA), risk priority number (RPN), generalized information quality, probability distribution, geometric mean.

I. INTRODUCTION

The world is full of uncertainty, and risk is the possibility of an event having unintended consequences. One of the most used risk management methods is the failure mode and effect analysis (FMEA) method [1]. The FMEA was first applied to the Apollo missions in the aerospace industry in 1960, and was recognized by the US military in the 1980s as a military specification [MIL-STD-1629A] [2]. It's widely used in environmental domain [3], medical system [4], [5], industry engineering [6], engineering design process [7], [8] and so on. The purpose of the FMEA is to improve the reliability of products and manufacturing. It is pointed out that the reliability of design can be improved in the design stage, thus improving product quality and reducing cost loss.

One of the most important parameters is Risk Priority Number (RPN), which is the product of severity (S), occurrence probability (O) and detection (D) [9]. The value of the three indicators is between 1-10 according to the degree, so the RPN score is between 1-1000. The higher the score, the higher the risk of a failure mode and the higher the priority of attention. This traditional risk assessment method is simple, but it also has the following weaknesses [10]: First, this is a traditional risk assessment method, but it ignores the relative importance among S, O and D. These three indicators are considered of equal importance, but this assumption may not be true in practical applications. Then the most controversial drawback is that the traditional FMEA is the same RPN value that may be generated by different values of S, O and D, while the meaning of risk may be completely different.

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A. PREVIOUS WORKS

Recently, some methods are combined with FMEA to improve the efficiency of FMEA. For example, ambiguity measure weighted risk priority number (AMWRPN) considers the relative weight of different risk factors by measuring the fuzziness of expert evaluation [11]. To handle the uncertainty in the complexity system and to model the domain experts' subjective opinion, it is necessary to present a more reasonable mathematical tool to deal with the uncertainty and fuzziness [12], [13]. Fuzzy sets is efficient to deal with linguistic variable [14], [15]. A larger number of methods based on linguistic terms have been proposed by many researchers [16]–[18]. For example, in Kutlu *et al.*'s work [19], a fuzzy approach allows experts to use linguistic variables for determining S, O and D. Some similar works include grey relational projection [20]–[22], Z numbers [23], TOPSIS (technique for order preference by similarity to an ideal solution) [2], cloud model [24], [25], TODIM (an acronym in Portuguese of interactive and multicriteria decision making) [26] and Soft Set Theory [27]. Due to the advantage to process nonspecificity, evidence theory is widely used in data fusion [28], [29], which is the key step in fuzzy evidential FMEA [30]–[35]. Based on belief entropy [36], [37], some other evidential FMEA model is presented [38]. In addition, D numbers [39], as the generalization of basic probability assignment, are combined with FMEA [40], [41].

B. OUR WORK

However, these previous methods do not take the impact of information quality into account. In order to solve this problem, this paper proposes a new FMEA method, which combines fuzzy probability distribution, generalized information quality and geometric mean to overcome the shortcomings of traditional RPN. Some advantages of the proposed method are briefly introduced as follows:

1) Fuzzy probability distribution provides a more flexible way for decision makers to evaluate S, O and D indicator.
2) Both the information quality of S, O and D indicator and the credibility among the three indicators are considered in generalized information quality.
3) Geometric mean is efficient to combine S, O and D indicator to obtain final RPN.

The remaining of this paper is organized as follows. Section 2 introduces some preliminaries. In Section 3, a new geometric mean of RPN for FMEA is proposed. A numerical case study about the preference of cause failures of steel production process is illustrated to show the advantage of the new method in Section 4. Finally, the conclusion is given in Section 5.

II. PRELIMINARIES

In this section, some basic preliminaries on FMEA [42], information quality [43] and Generalized information quality [44] are introduced.

A. FMEA

In FMEA, for each failure mode, the team has to determine the amount of RPN. The RPN is obtained by multiplying the three numerical value (*Severity*, *Occurrence*, *Detection*) ratings:

$$RPN = S \times O \times D \tag{1}$$

There is 1 to 10 score for each of likelihood of occurrence, detection, and severity [10].

- Severity: 1 = not severe, 10 = very severe
- Occurrence: 1 = not likely, 10 = very likely
- Detection: 1 = easy to detect, 10 = difficult to detect

Evaluate the results and use RPNs to plan improvement efforts (develop action plan). Then, determine appropriate activities to address potential failures with high risk priority number. Identify the failure modes and their causes with the top 10 highest RPNs. The minimum amount of score can be 1 and the maximum 1,000. Determination of high-risk failure modes is the most important part of the risk reduction process. The low-risk failure modes do not affect the overall process very much, and they should therefore be at the bottom of the list of priorities. Finally, the flowchart of FMEA is depicted in FIGURE 1.

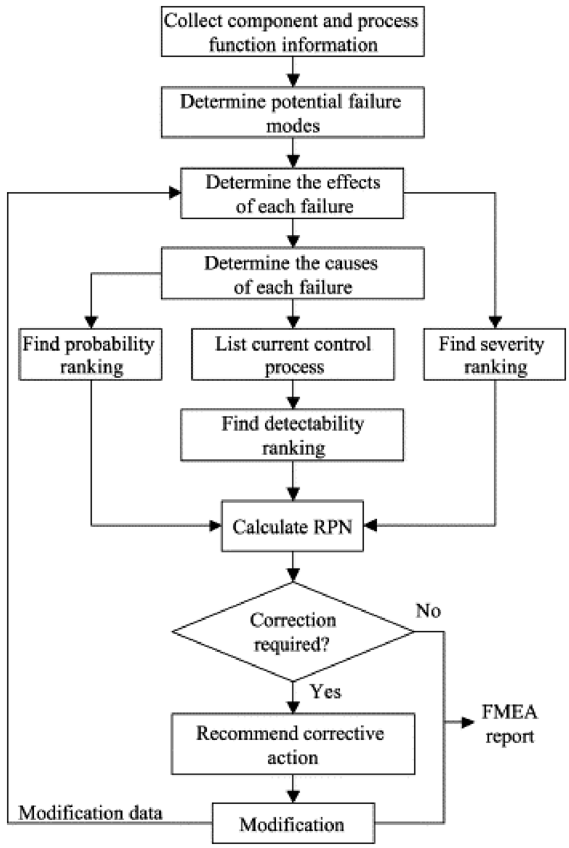

FIGURE 1. FMEA process [45].

B. INFORMATION QUALITY

In many cases, for ease of calculation, it is advisable to use vector to represent probability distribution [46].

Information quality is widely used in decision making [47] and fault diagnosis [48], [49]. There are many ways of defining information quality today, and we chose to use the information quality defined by Yager to conduct RPN research [43], [50].

Definition 1: Let p_i is the vector form of probability distribution, the information quality is defined as follows [43],

$$||p_i|| = \sqrt{p_i \times p_i} = (\sum_{t=1}^{m} (p_{it})^2)^{\frac{1}{2}} \tag{2}$$

$$||p_i||^2 = \sum_{t=1}^{m} (p_{it})^2 \tag{3}$$

where m is the number of distributions of p_i; p_{it} is the value of the t^{th} probability distribution in p_i.

Entropy function plays an important role in uncertainty measure [51]–[54]. Yager's information quality is based on Gini entropy [55].

C. GENERALIZED INFORMATION QUALITY

In order to reflect the credibility relations among the collected probability distribution, lots of different types of credibility functions have been presented [43], [56]. It is reasonable to calculate the degree of credibility with the use of the similarity of the probability distribution, determined by the degree of support. The generalized information quality is shown as follows [44].

$$Qu(p_i) = e^{crd(p_i)} \times ||p_i||^2 \tag{4}$$

The calculation process is detailed in Algorithm 1, as follows.

Algorithm 1 The Generalized Information Quality

Some probability distributions, $p_1, p_2, \ldots, p_n$

Step 1 $d(p_i, p_j) = \sqrt{(\overrightarrow{p_i} - \overrightarrow{p_j})(\overrightarrow{p_i} - \overrightarrow{p_j})^{\mathrm{T}}}$

Step 2 $sim(p_i, p_j) = 1 - d(p_i, p_j)$

Step 3 $\sup(p_i) = \sum_{j=1, j\neq i}^{n} sim(p_i, p_j)$

Step 4 $crd(p_i) = \frac{\sup(p_i)}{\sum_{k=1}^{n} \sup(p_k)}$

Step 5 $||p_i||^2 = \sum_{t=1}^{m} (p_{it})^2$

Step 6 $Qu(p_i) = e^{crd(p_i)} \times ||p_i||^2$

It needs the calculation of distance function. For more detailed information, refer [44].

III. A NEW RPN OF FMEA

Suppose a fuzzy decision-making problem with M failure modes (A_i) to the three indicators (S, O, D). We assume that all three indicators are equally important. Moreover, the judgments are represented by fuzzy probability distributions. The proposed method is composed of the following steps:

Step 1 List all failure modes (FMs) and cause of failure modes (CFs) throughout the system by historical data, past experiences, and expert opinions.

Step 2 Construct the fuzzy assessment matrix. The occurrence, probability, and severity of the associated effects and detection to each failure mode are considered as risk factors in the assessment matrix. The judgment for each A_i versus each indicator is modeled as fuzzy belief structure.

Definition 2: The fuzzy judgments are represented by probability distributions as fuzzy probability distributions matrix:

$$M = \begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_M \end{array} \begin{bmatrix} p_{S1} & p_{O1} & p_{D1} \\ \vdots & \vdots & \vdots \\ p_{Si} & p_{Oi} & p_{Di} \\ \vdots & \vdots & \vdots \\ p_{SM} & p_{OM} & p_{DM} \end{bmatrix} \tag{5}$$

Each judgment is expressed such as fuzzy probability distribution with q evaluation grades:

$$p_i = (p_{i1}, p_{i2}, \ldots, p_{iq}) = (\text{“}L_1\text{”}, \text{“}L_2\text{”}, \ldots, \text{“}L_q\text{”}) \tag{6}$$

where p_{it} is the probability distribution of “L_t”, $t \in [1, q]$. “L_t” is the decision maker’s rating of indicators (S, O, D).

Example 1: p_{S1} = $(0.8, 0.1, 0.1)$ = (“good”, “average”, “poor”), which means the decision maker is 80 % sure that the assigned amount of CF_1 is good, 10 % is average, and 10 % is poor with respect to the first indicator-S.

Step 3 Use Algorithm 1 to calculate the generalized information quality of each element in fuzzy probability distribution matrix, shown in Def. 2.

Definition 3: The generalized information quality matrix is defined as,

$$Qu(M) = \begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_M \end{array} \begin{bmatrix} Qu(p_{S1}) & Qu(p_{O1}) & Qu(p_{D1}) \\ \vdots & \vdots & \vdots \\ Qu(p_{Si}) & Qu(p_{Oi}) & Qu(p_{Di}) \\ \vdots & \vdots & \vdots \\ Qu(p_{SM}) & Qu(p_{OM}) & Qu(p_{DM}) \end{bmatrix} \tag{7}$$

Step 4 Calculate the geometric mean weight of the new RPN. First, find the maximum value in the generalized information quality matrix, shown in Def.3, and divide each element in the matrix by the maximum value to obtain the geometric mean weight value of the new RPN.

Definition 4: The geometric mean weight value is defined as,

$$w = \begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_M \end{array} \begin{bmatrix} w_{S1} & w_{O1} & w_{D1} \\ \vdots & \vdots & \vdots \\ w_{Si} & w_{Oi} & w_{Di} \\ \vdots & \vdots & \vdots \\ w_{SM} & w_{OM} & w_{DM} \end{bmatrix} = \frac{Qu(M)}{\max} \tag{8}$$

where “max” is the maximum value in the generalized information quality matrix $Qu(M)$. Each element in the w matrix is between 0 and 1.

Step 5 Calculate the new RPN by using the matrix w as the weight of the geometric mean of RPN.

Definition 5: The geometric mean RPN of A_{iq} is defined as,

$$RPN_{iq} = \left(S_{iq}{}^{w_{S_i}} \times O_{iq}{}^{w_{O_i}} \times D_{iq}{}^{w_{D_i}}\right)^{\frac{1}{w_{S_i}+w_{O_i}+w_{D_i}}} \tag{9}$$

Step 6 In order to facilitate decision making, RPN under probability distribution is converted into a numerical value by weighted sum. Choose the A_i as a suitable option according to the measure RPN_i. Note that RPN_i is a negative indicator; therefore, set the A_i with largest RPN_i as the riskiest failure mode.

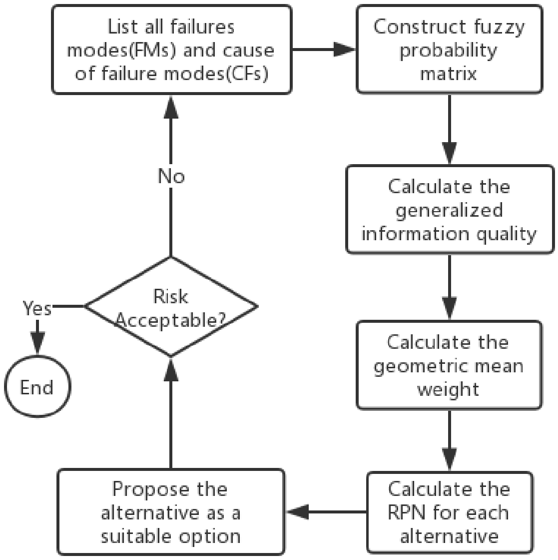

FIGURE 2. The flowchart of proposed FMEA process.

Step 7 Using the result of ranking, analyze the results and provide suggestions to plan improvement efforts. Reassess the severity, probability, and detection and review the revised RPNs after provided action.

Finally, the flowchart of the proposed new FMEA method based on the geometric mean of generalized information quality for group decision-making problems is depicted in FIGURE 2.

IV. CASE STUDY

In this section, an application of the proposed method in FMEA is used to illustrate the efficiency. The results and comparisons are briefly discussed.

A. AN APPLICATION OF THE PROPOSED METHOD

The steps of above method are described in the following case study. In this case, ten options of sheet steel production process in a steel factory (steel factory of guilan) are evaluated by the proposed method with respect to the three indicators. The failure modes of this case study are previously evaluated by Deshpande and Modak [57]. The judgment in assessment matrix is taken by experts. The indicators are related to their occurrence probability, severity of the associated effects, and detection to each failure mode as shown in FIGURE 3. The aim is to find high-risk options among the ten failure modes. The indicators are evaluated by a set of standard with three fuzzy evaluation grades. We utilize Generalized Information Quality [58] to rank our case study failure modes.

The proposed method is applied to evaluate ten options of steel production process as follows:

step 1 List the CFs throughout the system versus three indexes as shown in TABLE 1.

step 2 Construct the group assessment matrix based on the expert opinion. The occurrence probability, severity of the associated effects, and detection to each failure mode are considered as indicators in the assessment matrix. Suppose there are ten failure modes A_1, $A_2, \ldots A_{10}$, three indicators (S, O, D). Each judgment is expressed such as fuzzy probability distribution with three evaluation grades H_1, H_2, H_3 = "good", "average", "poor". The greater the probability distribution corresponding to "poor" means that the score of the original RPN algorithm is closer to 10, indicating greater risk. The fuzzy

TABLE 1. The FMEA of the sheet steel production process in Guilan steel factory.

No.	Failure mode (FM)	Cause of failure (CF)
A_1	Non-acceptable formation	Non-conductive scrap
A_2	Nipple thread pitted	Proper coverage not obtained
A_3	Arc formation loss	Leakage of water, proper gripping loss
A_4	Burn-out electrode	Cooler not working properly
A_5	Breaking of house of pipe	Wearing of pipe due to use
A_6	Problem in movement of arm	Sever leakage
A_7	Refractory damage	Due to slag
A_8	Formation of steam	Roof leak
A_9	Refractory line damage	By hot gas
A_{10}	Movement of roof stop	Jam of plunger in un loader valve

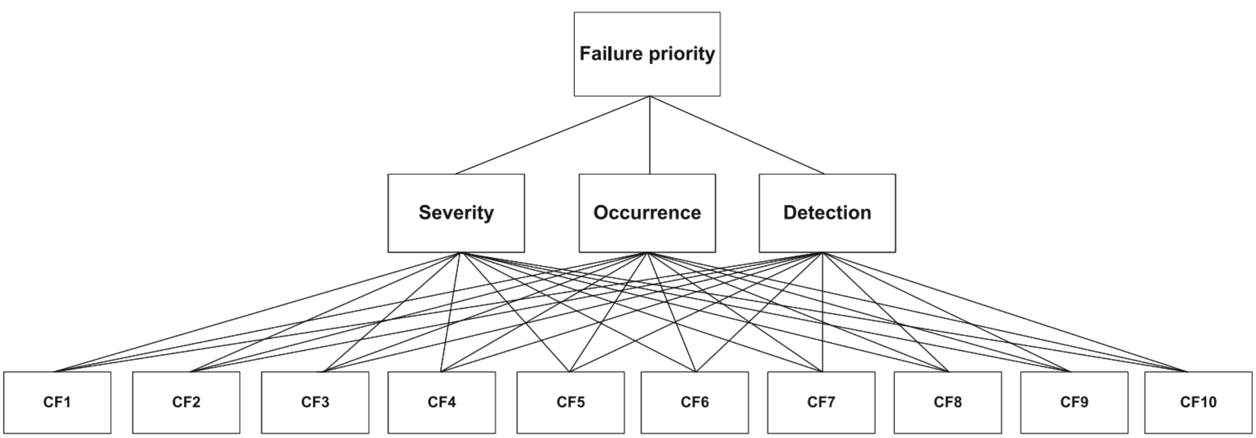

FIGURE 3. The preference of the CFs to find high-risk failure mode.

TABLE 2. Fuzzy probability distribution matrix [2].

No.	Severity (Good, Average, Poor)	Occurrence (Good, Average, Poor)	Detectability (Good, Average, Poor)
A_1	(0.8193, 0.0771, 0.1033)	(0.0545, 0.3105, 0.6346)	(0.2191, 0.4894, 0.2914)
A_2	(0.7224, 0.1373, 0.1399)	(0.0545, 0.3104, 0.6346)	(0.8250, 0.0776, 0.1040)
A_3	(0.8709, 0.1039, 0.0253)	(0.0233, 0.0722, 0.2042)	(0.2191, 0.4892, 0.2914)
A_4	(0.3669, 0.4475, 0.1854)	(0.0233, 0.0722, 0.9043)	(0.0545, 0.3104, 0.6346)
A_5	(0.3669, 0.4475, 0.1854)	(0.1504, 0.4446, 0.4045)	(0.7546, 0.1373, 0.1075)
A_6	(0.3669, 0.4475, 0.1854)	(0.1504, 0.4446, 0.4045)	(0.7546, 0.1373, 0.1075)
A_7	(0.7227, 0.1374, 0.1400)	(0.1504, 0.4446, 0.4045)	(0.0545, 0.3104, 0.6346)
A_8	(0.8709, 0.1039, 0.0253)	(0.0233, 0.0722, 0.9042)	(0.2191, 0.4894, 0.2914)
A_9	(0.3669, 0.4475, 0.1854)	(0.1504, 0.4446, 0.4045)	(0.7546, 0.1373, 0.1075)
A_{10}	(0.7966, 0.1070, 0.1135)	(0.3092, 0.1103, 0.5671)	(0.3365, 0.4158, 0.2475)

TABLE 3. Generalized information quality matrix.

No.	Severity	Occurrence	Detectability
A_1	0.7083	0.5181	0.3874
A_2	0.5799	0.5181	0.7180
A_3	0.7897	0.0488	0.3871
A_4	0.3844	0.8357	0.5180
A_5	0.3844	0.3990	0.6201
A_6	0.3844	0.3990	0.6201
A_7	0.5804	0.3990	0.5180
A_8	0.7897	0.8355	0.3874
A_9	0.3844	0.3990	0.6201
A_{10}	0.6798	0.4442	0.3620

TABLE 4. *w* matrix.

No.	w_S	w_O	w_D
A_1	0.8476	0.6199	0.4635
A_2	0.6939	0.6199	0.8592
A_3	0.9449	0.0584	0.4633
A_4	0.4599	1.0000	0.6199
A_5	0.4599	0.4775	0.7420
A_6	0.4599	0.4775	0.7420
A_7	0.6945	0.4775	0.6199
A_8	0.9449	0.9998	0.4635
A_9	0.4599	0.4775	0.7420
A_{10}	0.8134	0.5315	0.4332

probability distribution matrix after the fusion of multiple expert opinions is presented in TABLE 2, [2]. For the case study, the decision maker is 81.93% sure that the assigned amount of alternative CF_1 is good, 7.71% is average, and 10.33% is poor with respect to the first criterion(S).

step 3 Calculate the generalized information quality of each element in the fuzzy probability matrix by Algorithm 1. The generalized information quality matrix is presented in TABLE 3.

step 4 The maximum value in the Generalized information quality matrix (shown in TABLE 3) is 0.8357. Then we can get the geometric mean weight value of the new RPN. The *w* matrix is presented in TABLE 4.

TABLE 5. The geometric mean RPNs of 10 failure modes.

No.	RPN_{-Good}	$RPN_{-Average}$	RPN_{-Poor}	RPN	Rank
A_1	0.2501	0.1879	0.2373	0.2238	7
A_2	0.3642	0.1383	0.1915	0.1928	8
A_3	0.4875	0.1671	0.0595	0.1346	10
A_4	0.0552	0.1669	0.5732	0.3995	1
A_5	0.3916	0.2650	0.1819	0.2278	3
A_6	0.3916	0.2650	0.1819	0.2278	4
A_7	0.1945	0.2491	0.3133	0.2822	2
A_8	0.1485	0.1204	0.1787	0.1582	9
A_9	0.3916	0.2650	0.1819	0.2278	5
A_{10}	0.4866	0.1503	0.2220	0.2269	6

TABLE 6. The results of the classical method.

No.	S	O	D	RPN	Rank
A_1	2.1345	7.3184	5.2887	82.6154	3
A_2	2.6680	7.3184	2.1490	41.9603	9
A_3	1.6181	2.2221	5.2877	19.0123	10
A_4	4.2730	8.5230	7.3179	266.5090	1
A_5	4.2730	6.0139	2.4086	61.8947	6
A_6	4.2730	6.0139	2.4086	61.8947	7
A_7	2.6697	6.0139	7.3179	117.4911	2
A_8	1.6181	8.5221	5.2887	72.9291	4
A_9	4.2730	6.0139	2.4086	61.8947	8
A_{10}	2.3531	5.9646	4.6430	65.1659	5

step 5 Calculate the new RPN by using the matrix *w* as the weight of geometric mean of RPN by Def.4. The new RPN is shown in TABLE 5. For the case study, the RPN of A_1 "good" is 0.2501, the RPN of A_1 "average" is 0.1879, the RPN of A_1 "poor" is 0.2373. The RPN of A_1 "good" is calculated as follow:

$$w_S + w_O + w_D = 0.8476 + 0.6199 + 0.4635 = 1.931$$
$$RPN_{1-good} = (S^{w_s} \times O^{w_o} \times D^{w_D})^{\frac{1}{w_S+w_O+w_D}}$$
$$= \left(0.8193^{0.8476} \times 0.0545^{0.6199} \times 0.2191^{0.4635}\right)^{\frac{1}{1.931}}$$
$$= 0.2501$$

step 6 In order to facilitate decision making, RPN under probability distribution is converted into a numerical value by weighted sum. The weighted sum for this case is $RPN_i = 0.1 \times RPN_{i-good} + 0.3 \times RPN_{i-average} + 0.6 \times RPN_{i-poor}$. The RPNs are shown

TABLE 7. The results of Li and Chen's method.

No.	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Rp_i	0.2662	0.2611	0.3754	0.6000	0.2004	0.2004	0.1614	0.3754	0.2004	0.1219
Rank	4	5	2	1	6	7	9	3	8	10

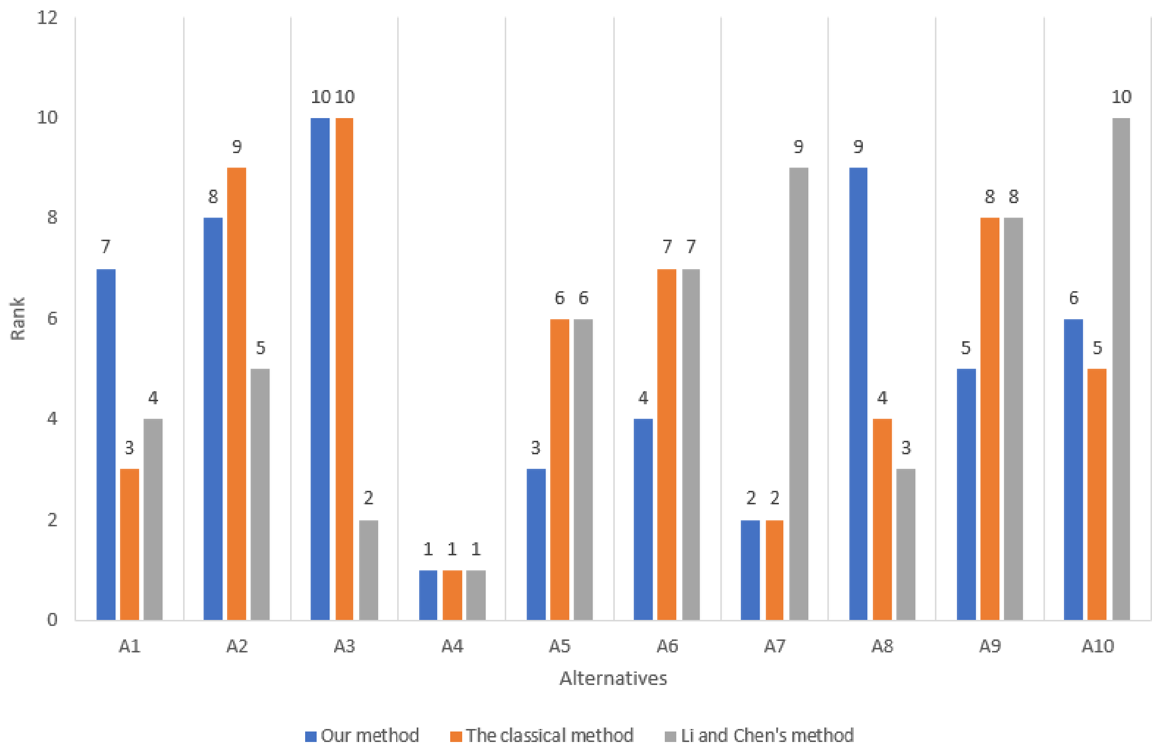

FIGURE 4. Contrast among three methods.

in TABLE 5. Because risk is a negative concept, set the A_i with largest RPN_i as the riskiest failure mode. For this case, RPN_4 is largest, so A_4 (Cooler not working properly) is the riskiest failure mode and ranked first; A_7 is ranked second; A_5, A_6, and A_9 are ranked third; A_2 is ranked fourth; A_1 is ranked fifth; A_7 is ranked sixth; and A_8 is ranked seventh; and A_4 is ranked eighth.

step 7 Using the results from TABLE 5, analyze the results and provide suggestions to plan improvement efforts. Reassess the severity, probability, and detection and review the revised RPNs after provided action.

B. THE RESULTS AND COMPARISONS

To illustrate the validation of this novel method, the results generated for the FMEA using the proposed approach is collated with the results obtained from the classical method and from Li and Chen's method [20].

1) THE CLASSICAL METHOD

We first convert the probability distribution shown in TABLE 2 into the score values between 1 and 10. We define that the scores of the probability distribution corresponding to "poor", "average" and "good" are 9, 5, 1, respectively. For example, the calculation of S in A_1 is: $S = 0.8193 \times 1 + 0.0771 \times 5 + 0.1033 \times 9 = 2.1345$. The scores of the three indicators after transformation are shown in TABLE 6. Then we can get RPN through the product of the three indicators. For example, the calculation of RPN in A_1 is: $RPN = 2.1345 \times 7.3184 \times 5.2887 = 82.6154$. The RPNs of 10 failure modes are shown in TABLE 6. Finally, the ranking of risk can be obtained by descending the scores of 10 RPNs, shown in TABLE 6.

2) LI AND CHEN'S METHOD

Li and Chen use an evidential FMEA integrating fuzzy belief structure and grey relational projection method (GRPM) to calculate RPN. Their results are shown in TABLE 7.

3) ANALYSIS OF RESULTS

The comparison result is displayed in FIGURE 4. Note that, all three methods consider A_4 as the riskiest, and the other failure modes have similar risk levels, which can show the rationality of the proposed method. The new method makes use of information quality under probability distribution, and this RPN algorithm is more reasonable. Its main advantages are: first, experts can express their opinions in a more flexible way; Second, generalized information quality is taken into consideration in the geometric mean of the RPN; Third, the weighted sum of RPN under the obtained probability distribution is conducted to generate the final ranking in a more comprehensive way, which overcomes the limitations of traditional RPN.

V. CONCLUSION

This paper presents a geometric mean FMEA method based on information quality. Its main contributions are: experts can express their opinions in a more flexible way, also generalized information quality is taken into consideration in the geometric mean of the RPN, what's more, the weighted sum of RPN under the obtained probability distribution is conducted to generate the final ranking in a more comprehensive way, which overcomes the limitations of traditional RPN. This new method is relatively simple to calculate and can effectively evaluate risks. One of the ongoing works is to explore the other efficient data fusion model to determine RPN in FMEA.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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