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H-Infinity Fault Detection for Delta Operator Systems With Random Two-Channels Packet Losses and Limited Communication

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ABSTRACT This paper is concerned with the H-infinity fault detection for time-delays delta operator systems with random two-channels packet losses and limited communication. The considered networked control systems (NCSs) are modeled as the time-delays delta operator systems with random packet losses and limited communication, where the packet losses exist in sensor-to-controller (S-C link) or controller-to-actuator (C-A link), and the limited communication occurs in the controller-to-actuator link. The random two-channels packet losses are described by the Markov chain process and limited communication is transformed into system state by feedback control in time-delays delta operator systems. The above delta operator systems are modeled as the Markovian jump systems, and the designed method of H-infinity fault detection filter is proposed under arbitrary transition probabilities matrix. The sufficient conditions for the asymptotical stability of the residual systems with H-infinity performance for the considered delta operator systems are presented by Lyapunov–Krasovskii functional in delta domain, and the gain matrices of the designed fault detection filter can be easily determined by linear matrix inequalities (LMIs). Finally, comparative examples are provided to illustrate the effectiveness of the proposed method.

INDEX TERMS Delta operator, fault detection, limited communication, networked control systems, packet losses, stability analysis, two-channels.

I. INTRODUCTION

Networked control systems (NCSs) are feedback control systems wherein the control loops are closed through a communication network. In recent years, there exist many researches that focused on NCSs [1]–[3]. The main advantages of NCSs include the following 2-points: (i) NCSs eliminate unnecessary wiring reducing the complexity and total budget for constructing systems' platforms; (ii) The applications of the NCSs are numerous and cover a wide range of industries such as: space and terrestrial exploration, internet of things, industrial process, tele-diagnostics, process monitoring, robots, aircraft, automobiles and tele-control. However, due to complexity of the control systems' environment and insertion of the communication network for feedback control systems, there exist time-delays [4]–[6], packet losses [7], [8] and limited communication [9] in communications network which receive more and more attention on some challenging issues for fault detection techniques in NCSs, such as stability analysis [10], robust control [11], nonlinear systems [10], [12] and so on.

During the past decades, the studies on the above research frontiers and key technologies for fault detection in NCSs have been investigated in many previous literatures based on traditional shift operator. Hu and Zhu [4] proposed the design method of long delays for two-channels in NCSs. To solve two-channel random delays in NCSs, the constructed closed loop systems are transformed into jump systems with two Markov chains in [5]. The design issue of time-varying delays in NCSs has been investigated in [6]. Lu *et al.* [7] proposed the systems' model with random two-channels packet losses by Bernoulli distribution, and another design method for random data missing by Markov chain has been addressed in [8]. The systems' composition of NCSs with limited communication and communication sequence has been considered in [9]. Wang *et al.* [10] investigated the stabilization issue of

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sampled-data controllers in nonlinear switched systems with arbitrary finite delays. The robust fault detection problem for the NCSs with packet losses and time-delays has been studied in [11]. Tong *et al.* [12] considered the design problem of output-feedback control for nonlinear systems.

On the other hand, delta operator systems [13], [14] based implementations have gained considerable interest due to its excellent performance than traditional shift operator. The three principal advantages are summarized as follows: (i) The delta operator model could avoid the ill conditioned issue caused by the traditional shift operator with the high sampling rate; (ii) The discrete-time model on delta operator will approach to the corresponding continuous-time model when the sampling rate is fast; (iii) Delta operator has the advantage of better numerical properties based on finite word length performance at high sampling rate. Figure 1 and example 1 show that the stability region and trajectories on delta operator model will approach to the corresponding Laplace domain of continuous systems when sampling time is very small, respectively. Therefore, based on the above reasons, many important researches about delta operator have been investigated in excellent literatures for the past several years. Li and Gevers [15] indicated that delta operator has the advantage of parameterizations compare with traditional shift operator in finite word length effects according to mathematical theory. Yang *et al.* [16] presented the systems' model with delta operator in NCSs and proposed the methods for solving the problems of fault detection filter design, robust control with uncertainty, constructing Lyapunov-Krasovskii functional, stability analysis and so on.

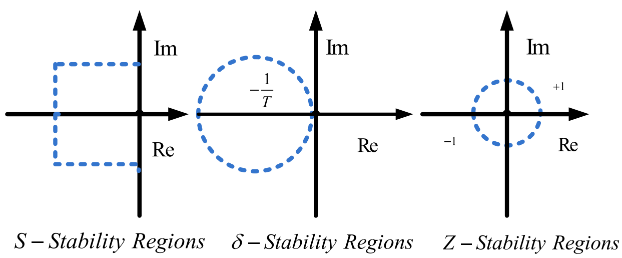

FIGURE 1. Stability regions for the continuous Laplace plane, discrete delta operator plane and traditional shift operator plane.

Given the importance of fault detection in NCSs and advantages of delta operator, there have been rich achievements focused on delta operator systems [17]–[19] with time-delays, packet losses or limited communication. Gao and Zhang [20] proposed the delta operator systems with random single-channel (S-C link) delays for fault detection by Bernoulli distribution. The systems' model with two-channels time-varying delays based on delta operator has been investigated in [21]. Li *et al.* [22] considered stabilization issue for the robust fault detection in time-varying delays delta operator systems with two-channels data missing. The more complicated model based on delta operator with two-channel random packet losses has been addressed in [23]. Zhao and Zhang [24] studied the issue of fault detection for the delta operator systems with random packet dropouts and limited communication in single-channel (S-C link) by Bernoulli distribution. Considered the time-delays, packet losses and limited communication in NCSs, the above existing results on delta operator only focus on 1 or 2 network faults based on single or two-channels. However, there must randomly exist all the above network faults in the real network environment with two-channels.

Motivated by the above analysis, this paper proposed the delta operator method of H-infinity fault detection from the real network environment perspective which covers the random time-delays, packet losses and limited communication on two-channels, where one main focus is to design the residual dynamic system based on above all network faults which random two-channels data missing are described by Markov chain process; The time-delays and limited communication are converted to control input of system feedback, and another main focus is to apply the Lyapunov-Krasovskii functional to obtain the sufficient conditions of the asymptotical stability for the proposed fault detection filter with H-infinity performance in delta domain. The example 1 and examples 3-4 show that trajectories of delta operator will approach to corresponding continuous domain and the design fault detection filter on delta operator can respond quickly to fault signal than the design method of traditional shift operator with the high sampling rate, respectively.

The main contributions of this paper could be summarized as follows:

- Designing the systems' model based on real time-delays network environment which includes the random packet losses and limited communication in two-channels, where the random two-channels data missing are formulated by two-states Markov chain; The time-delays and limited communication are changed into input of system feedback control.
- Constructing the Lyapunov-Krasovskii functional in delta domain to obtain the sufficient conditions of stability analysis, which ensure that the proposed fault detection filter is asymptotically stable and satisfies the H-infinity performance.
- The proposed H-infinity fault detection filter on delta operator could detect the fault signal more accurate and faster than the existing methods by traditional shift operator at the high sampling rate.

The remainder of this paper is organized as follows: The systems' model of time-delays NCSs with random two-channels packet losses and limited communication in delta domain is provided in Section II. The residual dynamic systems based on delta operator are proposed in Section III and the sufficient conditions which make the considered system to be asymptotically stable and meet H-infinity performance are investigated in Section IV. And then, the results of comparative examples 1-4 are presented in Section V. Finally, the paper is concluded in Section VI.

Notation. Throughout this paper, T denotes a sampling period; The superscript $(\cdot)^h$ denotes matrix transposition;

The symbol'$-$' defines the measured value and symbol '$\wedge$' represents the estimated value.

II. PROBLEM FORMULATION

The linear time invariant NCSs are given as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Df(t) + Ed(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the vector of system state; $u(t) \in R^m$ is the vector of control input; $y(t) \in R^r$ is the vector for output signal; $d(t) \in R^d$ is the external disturbance which belongs to $L_2[0, +\infty)$; $f(t) \in R^s$ is the fault signal; A, B, C, D, E are the constant matrices with the appropriate dimensions.

The structure for the considered NCSs via delta operator is shown in Figure 2.

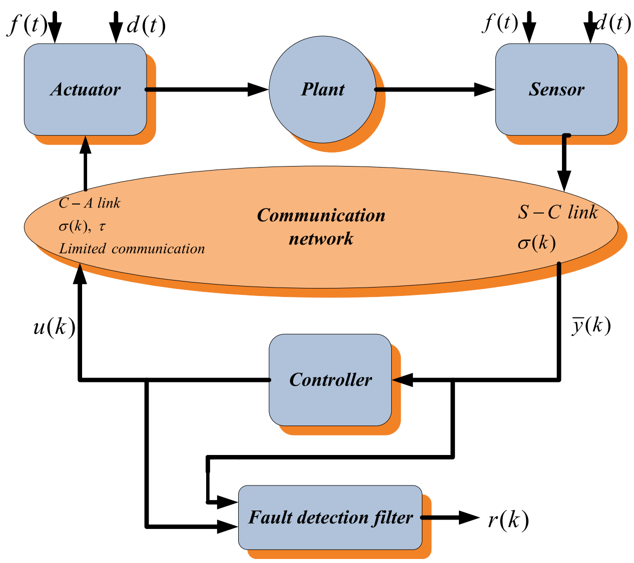

FIGURE 2. Structure for the considered NCSs.

Then the discretization method for delta operator is given as [13]:

$$\delta x(t) = \begin{cases} \dfrac{dx(t)}{dt}, & T = 0, \\ \dfrac{x(t+T) - x(t)}{T}, & T \neq 0, \end{cases} \tag{2}$$

where $T > 0$ is a sampling period.

Some reasonable assumptions for NCSs of this paper are given as follows:

Assumption 1: The sensor is clock-driven, but the controller and actuator are event-driven. And the NCSs based on delta operator are the time-delays systems;

The τ is the total time-delays, and meet $0 < \tau < T$. Due to the time-delays, the control input signal cannot be updated in time. Then the $u(k)$ which reaches the actuator at the kth sampling period can be updated to:

$$u(k) = \begin{cases} u(k-1), & kT \leq t < kT + \tau. \\ u(k), & kT + \tau \leq t < (k+1)T. \end{cases} \tag{3}$$

From (1)-(3), the delta operator model for NCSs (1) with the time-delays τ is written as:

$$\begin{cases} \delta x(k) = A_\delta x(k) + B_{\delta 2}\bar{u}(k) + B_{\delta 1}\bar{u}(k-1) \\ \qquad + D_\delta f(k) + E_\delta d(k), \\ y(k) = Cx(k). \end{cases} \tag{4}$$

where,

$$A_\delta = \frac{e^{AT} - I}{T}, \quad B_{\delta 1} = \frac{\int_{T-\tau}^{T} e^{AT} B dt}{T},$$
$$B_{\delta 2} = \frac{\int_0^{T-\tau} e^{AT} B dt}{T},$$
$$D_\delta = \frac{\int_0^T e^{AT} D dt}{T}, \quad E_\delta = \frac{\int_0^T e^{AT} E dt}{T}.$$

Assumption 2: The packet losses randomly exist in the S-C link or C-A link, and the limited communication occurs in the C-A link.

There exist registers for each sensor, controller and actuator, and the purpose is to store the information of the last available measurement.

Due to random two-channels packet losses, the output signal is expressed as following [8]:

$$\bar{y}(k) = \begin{cases} \text{the last available measurement}, & \sigma(k) = 1 \\ y(k), & \sigma(k) = 2 \end{cases} \tag{5}$$

Similarly, the control input signal can be updated as follows in the C-A link:

$$u(k) = \begin{cases} u(k-1), & \sigma(k) = 1. \\ u(k), & \sigma(k) = 2. \end{cases} \tag{6}$$

where $\sigma(k) = 1$ means that there exist packet losses; $\sigma(k) = 2$ indicates that the information transmission is successful in S-C link or C-A link.

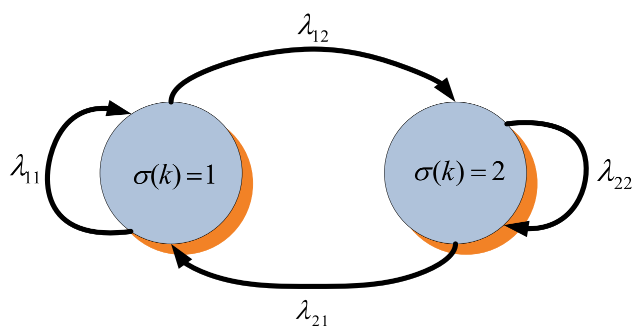

FIGURE 3. Markov model in the NCSs.

Definition of $\sigma(k) \in \{1, 2\}$ is used to describe random two-channels packet losses which satisfy the Markov chain process. The structure of the Markov model considered in this paper is shown in Figure 3, and the above Markov chain process is defined as follows:

$$\begin{cases} \lambda_{ij} = P_T\{\sigma(k+1) = j / \sigma(k) = i\}, \\ \sum_{j=1}^{2} \lambda_{ij} = 1, \quad \forall i, j \in \{1, 2\}, \ \lambda_{ij} > 0. \end{cases} \tag{7}$$

where the transition probabilities matrix of Markov jump systems is given as follows:

$$\lambda_k = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

Based on (4)-(6), the discretization model via delta operator for the time-delays system (4) with random two-channels packet losses is written as:

$$\begin{cases} \delta x(k) = A_\delta x(k) + \vec{B}_{\sigma(k)} \vec{U}(k) + D_\delta f(k) \\ \qquad +E_\delta d(k), \\ y(k) = Cx(k). \end{cases} \tag{8}$$

where,

$$\vec{U}(k) = [\bar{u}^h(k), \bar{u}^h(k-1)]^h,\ \vec{B}_1 = [0, B_{\delta 1}],\ \vec{B}_2 = [B_{\delta 2}, 0].$$

According to *Assumption 2*, there is a total of m transmission paths on the communication network in the C-A link for the NCSs [24], which means that there exists m controllers and m actuators. With each sampling k-th, the input signals from i_{th} controller which send to the i_{th} actuator can be expressed as $\bar{u}_i(k) = \theta_{ik} u_i(k)$. where $\theta_{ik} = 1$ means that signals for control input $u_i(k)$ can be successfully transmitted to the i_{th} actuator at the k_{th} time; while $\theta_{ik} = 0$, it represents the opposite status with the above. The updated control input signals are given as:

$$\bar{u}(k) = Mu(k)$$

$$M = \begin{bmatrix} \theta_{1k} & 0 & \cdots & 0 \\ 0 & \theta_{2k} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \theta_{mk} \end{bmatrix} \tag{9}$$

where $\theta_{1k} \in \{0, 1\}$, $\sum_{i=1}^{m} \theta_{ik} \leq m$.

The NCSs are memoryless state feedback systems and which could be expressed as follows:

$$u(k) = Kx(k) \tag{10}$$

where $K \in R^{m\times n}$ is the state feedback controller gain to be determined such that the delta operator systems are asymptotically stable for any $0 \leq \tau \leq T$.

From (9) to (10), the control input signal is expressed as:

$$\bar{u}(k) = MKx(k) \tag{11}$$

Consider systems (8) based on the control law (11), then the resulting time-delays delta operator systems with random two-channels packet losses and limited communication can be rewritten as follows:

$$\begin{cases} \delta x(k) = A_\delta x(k) + \vec{B}_{\sigma(k)} \begin{bmatrix} MKx(k) \\ MKx(k-1) \end{bmatrix} \\ \qquad +D_\delta f(k) + E_\delta d(k), \\ y(k) = Cx(k). \end{cases} \tag{12}$$

In summary, the NCSs are transformed into Markovian jump systems which combine the random two-channels packet losses model with limited communication model in time-delays delta operator systems.

III. RESIDUAL DYNAMIC SYSTEMS

This paper will focus attention to design an H_∞ fault detection filter with Markovian jump systems based on the observer in delta domain:

$$\begin{cases} \delta \hat{x}(k) = A_\delta \hat{x}(k) + \vec{B}_{\sigma(k)} \begin{bmatrix} MK\hat{x}(k) \\ MK\hat{x}(k-1) \end{bmatrix} \\ \qquad +L\{\bar{y}(k) - \hat{y}(k)\}, \\ \hat{y}(k) = C\hat{x}(k), \\ r(k) = \bar{y}(k) - \hat{y}(k), \end{cases} \tag{13}$$

where $\hat{x}(k)$ is the state estimation for the fault detection filter; $\bar{y}(k)$ is the measurement output; $\hat{y}(k)$ is the estimation of the measurement output; $r(k)$ is the residual signal; L is the gain matrix to be determined for the H_∞ fault detection filter for delta operator systems.

According to the state estimation error $e(k) = x(k) - \hat{x}(k)$, the following residual dynamic systems with $\sigma(k) = 1$ can be expressed:

$$\begin{cases} \delta e(k) = (A_\delta - LC)\, e(k) + (B_{\delta 1}MK)e(k-1) \\ \qquad +D_\delta f(k) + E_\delta d(k), \\ r(k) = Ce(k). \end{cases} \tag{14}$$

when $\sigma(k) = 2$, the residual dynamic systems are given as:

$$\begin{cases} \delta e(k) = (A_\delta - LC + B_{\delta 2}MK)\, e(k) + D_\delta f(k) \\ \qquad +E_\delta d(k), \\ r(k) = Ce(k). \end{cases} \tag{15}$$

Let $w(k) = [f^h(k), d^h(k)]^h$, then the residual dynamic systems can be updated to

$$\begin{cases} \delta e(k) = A^* e(k) + B^* e(k-1) + C^* w(k), \\ r(k) = Ce(k). \end{cases} \tag{16}$$

where

$$A^* = A_\delta - LC + \vec{B}_{\sigma(k)}\bar{M}, \quad B^* = \vec{B}_{\sigma(k)}\tilde{M}$$

$$C^* = [D_\delta, E_\delta] \quad A_\delta = \begin{bmatrix} A_{\delta 1} & A_{\delta 2} \\ A_{\delta 2}^h & A_{\delta 3} \end{bmatrix}, \quad B_{\delta 1} = \begin{bmatrix} B_{\delta 11} \\ B_{\delta 12} \end{bmatrix},$$

$$B_{\delta 2} = \begin{bmatrix} B_{\delta 21} \\ B_{\delta 22} \end{bmatrix}, \quad K = \begin{bmatrix} K_1 & K_2 \end{bmatrix},$$

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \bar{M} = \begin{bmatrix} MK \\ 0 \end{bmatrix},$$

$$\tilde{M} = \begin{bmatrix} 0 \\ MK \end{bmatrix}, \quad D_\delta = \begin{bmatrix} D_{\delta 1} \\ D_{\delta 2} \end{bmatrix},$$

$$E_\delta = \begin{bmatrix} E_{\delta 1} \\ E_{\delta 2} \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}.$$

For the H_∞ fault detection of delta operator systems, the evaluation functional J_γ and the threshold J_{th} are expressed as follows:

$$J_\gamma(k) = \|r(k)\|_{2.Win} = \left[\sum_0^N r^h(k) r(k)\right]^{1/2}$$

$$J_{th} = \sup_{f(k)=o, w(k)\in l_2} \|r_e\|_2 = \left[\sum_0^N r^h(k) r(k)\right]^{1/2}$$

where *Win* is an evaluation window.

Then, the conditions for the fault detection are given as follows:

$$J_r > J_{th} \Rightarrow Alarm$$
$$J_r \leq J_{th} \Rightarrow No\ Fault.$$

IV. STABILITY ANALYSIS

In this section, an LMIs approach is proposed to solve the H_∞ fault detection for the time-delays delta operator systems with random two-channels packet losses and limited communication.

Lemma 1 [16]: The delta operator systems (16) satisfy asymptotical stability if the following conditions hold:

(i) $V[x(k)] \geq 0$, with equality if and only if $x(k) = 0$;
(ii) $\delta V[x(k)] = \frac{E\{V[x(k+T)]-V[x(k)]\}}{T} < 0$

where $V[x(k)]$ is the Lyapunov functional in delta domain.

Lemma 2 [25]: The fault detection filter with asymptotically stable based on the delta operator systems will meet the H_∞ performance if the following requirements are satisfied:

(i) The residual dynamic systems (16) are asymptotically stable;

(ii) With any nonzero $w(k) \in [0, \infty)$, the following condition is satisfied:

$$E\left\{\sum_{k=0}^{\infty} r^h(k)r(k)\right\} < \gamma^2 E\left\{\sum_{k=0}^{\infty} w^h(k)w(k)\right\}. \qquad (17)$$

where the index $\gamma > 0$ and $E\{\cdot\}$ is the expectation.

Lemma 3 [26], [27]: For some symmetric matrices S_{11}, S_{12} and S_{22}, the following conditions are equivalent:

(i) $\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} < 0$
(ii) $S_{11} < 0, S_{22} - S_{12}^h S_{11}^{-1} S_{12} < 0$
(iii) $S_{22} < 0, S_{11} - S_{12}^h S_{22}^{-1} S_{12} < 0$

Theorem 1: Considering the delta operator systems with the given scalar $\gamma > 0$, if there exist the matrices $P_i > 0$, $Q > 0$, such that the following LMI holds:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & P_iB^* & P_iC^* \\ * & \Pi_{22} & P_iB^* & P_iC^* \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \qquad (18)$$

where

$$\Pi_{11} = T^2 \sum_{j=1}^{n} \lambda_{ij}P_j + TP_i - 2P_i,$$
$$\Pi_{12} = T \sum_{j=1}^{n} \lambda_{ij}P_j + P_iA^*,$$
$$\Pi_{22} = \sum_{j=1}^{n} \lambda_{ij}P_j + A^{*h}P_i + P_iA^* + Q + NTQ + C^hC., \quad n \in \{1, 2\}.$$

Then the residual dynamic systems (16) are asymptotically stable and satisfy the H_∞ performance.

Proof: For the residual dynamic systems, the Lyapunov functionals are proposed in delta domain as follows:

$$V[e(k)] = V_1[e(k)] + V_2[e(k)] + V_3[e(k)] \qquad (19)$$

where

$$V_1[e(k)] = e^h(k) P_{\sigma(k)} e(k),$$
$$V_2[e(k)] = T \sum_{i=1}^{n} e^h(k-i) Qe(k-i),$$
$$V_3[e(k)] = T^2 \sum_{i=1}^{n} \sum_{j=1}^{i} e^h(k-j) Qe(k-j), \quad n \in \{1, 2\}.$$

where $V[x(k)]$ is the Lyapunov functional in delta domain; the time-delays and the random two-channels packet losses exist in $V_2[e(k)]$; there exist the time-delays and the limited communication in $V_3[e(k)]$.

According to the definition of delta operator (2), the delta operator of the Markov chain process $e(k)$, acting on $V(k)$ at the sampling point k-th which could be updated as follows:

$$\delta V[e(k)] = \frac{E[V(e(k+1)) - V(e(k))]}{T} \qquad (20)$$

Case: For the Markovian jump systems: Model state is $\sigma(k) = i$ when sampling point is tk; The delta operator systems may jump any model $\sigma(k+1) = j$ at the sampling point $tk + T$.

Where $\forall i, j \in \{1, 2\}$; $k \in [0, N)$, $N = [0, 1, 2, \cdots)$.

By constructing the $E\{P_j\} = T\sum_{j=1}^{n} \lambda_{ij}P_j + P_i$, and which could be updated as follows:

$$E\{P_{k+1}\} = T\sum_{j=1}^{n} \lambda_{ij}P_{k+1} + P_k \qquad (21)$$

From (19) to (21), there exists:

$$\begin{aligned}\delta V_1[e(k)] &= \frac{E\{V_1[e(k+1)] - V_1 e(k)\}}{T} \\ &= \sum_{j=1}^{n} \lambda_{ij} e^h(k+1)P_j e(k+1) \\ &\quad + \frac{1}{T}[e^h(k+1)P_i e(k+1) \\ &\quad - e^h(k)P_i e(k)].\end{aligned} \qquad (22)$$

For the definition of the delta operator, we can obtain:

$$\delta e(k) = \frac{e(k+1) - e(k)}{T}, \quad T \neq 0 \qquad (23)$$

which is equivalent to

$$e(k+1) = T\delta e(k) + e(k). \qquad (24)$$

From (22) to (24), it can be obtained that

$$\begin{aligned}\delta V_1(e(k)) &= T^2 \sum_{j=1}^{n} \lambda_{ij}\delta^h e(k)P_j\delta e(k) \\ &+ T\sum_{j=1}^{n} \lambda_{ij}\delta^h e(k)P_j e(k) \\ &+ T\sum_{j=1}^{n} \lambda_{ij} e^h(k)P_j\delta e(k) \\ &+ \sum_{j=1}^{n} \lambda_{ij} e^h(k)P_j e(k) + T\delta^h e(k)P_i\delta e(k) \\ &+ \delta^h e(k)P_i e(k) + e^h(k)P_i\delta e(k).\end{aligned} \qquad (25)$$

where

$$\delta^h e(k)P_i e(k) = [A^* e(k) + B^* e(k-1) + C^* w(k)]^h P_i e(k)$$
$$= [e^h(k)A^{*^h} + e^T(k-1)B^{*h} + w^h(k)C^{*h}]P_i e(k)$$
$$e^h(k)P_i\delta e(k) = e^h(k)P_i[A^* e(k) + B^* e(k-1) + C^* w(k)]$$

Taking the time-delays delta operator systems with random two-channels packet losses of $V_2[e(k)]$, we can obtain as follows:

$$\begin{aligned}\delta V_2[e(k)] &= \frac{1}{T}E\{V_2[e(k+1)] - V_2[e(k)]\}\\ &= \frac{1}{T}\{T\sum\nolimits_{i=1}^{n} e^h(k-i+1)Qe(k-i+1)\\ &\quad - T\sum\nolimits_{i=1}^{n} e^h(k-i)Qe(k-i)\}\\ &= e^h(k)Qe(k)\\ &\quad - e^h(k-n)Qe(k-n)\\ &\quad + T\sum\nolimits_{i=1}^{n} e^h(k-i)Qe(k-i). \end{aligned} \tag{26}$$

By using the above methods of Lyapunov functionals, $\delta V_3[e(k)]$ which includes the time-delays and limited communication can be easily expressed as:

$$\begin{aligned}\delta V_3[e(k)] &= \frac{1}{T}E\{V_3[e(k+1)] - V_3[e(k)]\}\\ &= T\sum\nolimits_{i=1}^{n}\{T\sum\nolimits_{j=1}^{i} e^h(k-j+1)Qe(k-j+1)\\ &\quad - T\sum\nolimits_{j=1}^{i} e^h(k-j)Qe(k-j)\}\\ &= T\sum\nolimits_{i=1}^{n}\{e^h(k)Qe(k)\\ &\quad - e^h(k-i)Qe(k-i)\}\\ &\le nTe^h(k)Qe(k)\\ &\quad - T\sum\nolimits_{i=1}^{n} e^h(k-n)Qe(k-n). \end{aligned} \tag{27}$$

where $n = NT$, $i = nT$, $i \in [1, 2]$.

Considering the Markovian jump systems with two states, the system state $x(k)$ of $k - i_{th}$ is equivalent to state value of $k - n_{th}$.

According to the definition $e(k) = x(k) - \hat{x}(k)$ and the above discuss, then the state estimation error of proposed fault detection filter is also the same at above moments.

For any positive definite real matrix P_i, the following equation is satisfied:

$$\begin{aligned} 0 &= \delta^h e(k)P_i[-\delta e(k) + A^* e(k)\\ &\quad + B^* e(k-1) + C^* w(k)] + [-\delta e(k)\\ &\quad + A^* e(k) + B^* e(k-1)\\ &\quad + C^* w(k)]^h P\delta e(k)_i. \end{aligned} \tag{28}$$

According to the *Lemma 1* and (19) to (28), the following inequality is proposed by Lyapunov functional with $w(k) = 0$.

$$\delta V[e(x)] \le \begin{bmatrix} \delta e(k) \\ e(k) \\ e(k-n) \end{bmatrix}^h \Omega \begin{bmatrix} \delta e(k) \\ e(k) \\ e(k-n) \end{bmatrix}$$

where Ω, as shown at the bottom of this page.

Due to the *Lemma 3*, it means that the $\Omega < 0$. Then it is equivalent to $\delta V[e(x)] < 0$. Then the delta operator systems of this paper are asymptotically stable by *Lemma 1*.

According the *Lemma 2*, considering following condition with the H_∞ performance index γ.

$$J = E\sum\nolimits_{k=0}^{\infty}\left[r^h(k)r(k) - \gamma^2 w^h(k)w(k)\right]$$

where $k \in [o, N)$, $N = [0, 1, 2, \cdots)$.

When $w(k) \neq 0$, $k = 0$, $V[e(0)] = 0$ and $k \to \infty$, $V[e(k)] > 0$. The above condition could be rewritten as follows:

$$J \le E\sum\nolimits_{k=0}^{\infty}\left[r^h(k)r(k) - \gamma^2 w^h(k)w(k) + \delta V[e(k)]\right] \tag{29}$$

Then, it can be updated to

$$J \le \begin{bmatrix} \delta e(k) \\ e(k) \\ e(k-n) \\ w(k) \end{bmatrix}^h \prod \begin{bmatrix} \delta e(k) \\ e(k) \\ e(k-n) \\ w(k) \end{bmatrix}$$

where

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & P_i B^* & P_i C^* \\ * & \Pi_{22} & P_i B^* & P_i C^* \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

From the *Lemma 3*, we can easily obtain that the $\Pi < 0$, which is equivalent to $J < 0$. According to the *Lemma 1* and *Lemma 2*, it means that the residual dynamic systems are asymptotically stable and satisfy H_∞ performance under the condition $w(k) \neq 0$. The proof is completed.

$$\Omega = \begin{bmatrix} T^2\sum\limits_{j=1}^{n}\lambda_{ij}P_j + TP_i - 2P_i & T\sum\limits_{j=1}^{n}\lambda_{ij}P_j + P_iA^* & P_iB^* \\ * & \begin{matrix}\sum\limits_{j=1}^{n}\lambda_{ij}P_j + A^{*h}P_i + P_iA^* \\ +Q + NTQ + C^hC\end{matrix} & P_iB^* \\ * & * & -Q \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} T^2\sum_{j=1}^{n}\lambda_{ij}Z_{1j}+(T-2)Z_{1i} & 0 & \Sigma_{13} & \Sigma_{14} & Z_{1i}[\vec{B}_{11}\tilde{M}_{11}+\vec{B}_{12}\tilde{M}_{21}] & Z_{1i}[\vec{B}_{11}\tilde{M}_{12}+\vec{B}_{12}\tilde{M}_{22}] & Z_{1i}D_{\delta 1} & Z_{1i}E_{\delta 1} & 0 \\ * & T^2\sum_{j=1}^{n}\lambda_{ij}Z_{3j}+(T-2)Z_{3i} & \Sigma_{23} & \Sigma_{24} & Z_{3i}[\vec{B}_{21}\tilde{M}_{11}+\vec{B}_{22}\tilde{M}_{21}] & Z_{3i}[\vec{B}_{21}\tilde{M}_{12}+\vec{B}_{22}\tilde{M}_{22}] & Z_{3i}D_{\delta 2} & Z_{3i}E_{\delta 2} & 0 \\ * & * & \Sigma_{33} & \Sigma_{34} & Z_{1i}[\vec{B}_{11}\tilde{M}_{11}+\vec{B}_{12}\tilde{M}_{21}] & Z_{1i}[\vec{B}_{11}\tilde{M}_{12}+\vec{B}_{12}\tilde{M}_{22}] & Z_{1i}D_{\delta 1} & Z_{1i}E_{\delta 1} & C_1^h \\ * & * & \Sigma_{43} & \Sigma_{44} & Z_{3i}[\vec{B}_{21}\tilde{M}_{11}+\vec{B}_{22}\tilde{M}_{21}] & Z_{3i}[\vec{B}_{21}\tilde{M}_{12}+\vec{B}_{22}\tilde{M}_{22}] & Z_{3i}D_{\delta 2} & Z_{3i}E_{\delta 2} & C_2^h \\ * & * & * & * & -R_1 & -R_2 & 0 & 0 & 0 \\ * & * & * & * & -R_{2h} & -R_3 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & * & \gamma I & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{30}$$

$$\Pi = \begin{bmatrix} T^2\sum_{j=1}^{n}\lambda_{ij}P_j+TP_i-2P_i & T\sum_{j=1}^{n}\lambda_{ij}P_j+P_iA^* & P_iB^* & P_iC^* \\ * & \sum_{j=1}^{n}\lambda_{ij}P_j+A^{*h}P_i+P_iA^*+Q+NTQ & P_iB^* & P_iC^* \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} 0 \\ C^h \\ 0 \\ 0 \end{bmatrix} I \begin{bmatrix} 0 & C & 0 & 0 \end{bmatrix}$$

Next, the main research will focus on obtaining the gain matrix L of the H_∞ fault detection filter for NCSs in delta domain.

Theorem 2: Consider the residual dynamic systems (16) in delta domain, then the gain matrix L could be solved if there exist index $\gamma > 0$, symmetric positive definite matrices Z_{1i}, Z_{3i}, Z_{1j}, Z_{3j}, R_1, R_3 and the invertible matrix R_2 which could make the LMI (30), as shown at the top of this page, hold and the residual dynamic systems (16) are asymptotically stable and meet H_∞ performance.

The filter gain L and state feedback gain K with meeting H_∞ performance are updated to:

$$L = SC^{-1}, \quad K = \vec{B}_{\sigma(k)}^{-1}\bar{M}^{-1}G$$

where

$$\Sigma_{13} = T\sum_{j=1}^{n}\lambda_{ij}Z_{1j} + Z_{1i}[A_{\delta 1} - L_1C_1 + \vec{B}_{11}\bar{M}_{11} + \vec{B}_{12}\bar{M}_{21}]$$
$$\Sigma_{14} = Z_{1i}[A_{\delta 2} - L_1C_2 + \vec{B}_{11}\bar{M}_{12} + \vec{B}_{12}\bar{M}_{22}]$$
$$\Sigma_{23} = Z_{3i}[A_{\delta 2}^h - L_2C_1 + \vec{B}_{21}\bar{M}_{11} + \vec{B}_{22}\bar{M}_{21}]$$
$$\Sigma_{24} = T\sum_{j=1}^{n}\lambda_{ij}Z_{1j} + Z_{3i}[A_{\delta 3} - L_2C_2 + \vec{B}_{21}\bar{M}_{12} + \vec{B}_{22}\bar{M}_{22}]$$
$$\begin{aligned}\Sigma_{33} &= \sum_{j=1}^{n}\lambda_{ij}Z_{1j} + [A_{\delta 1} - L_1C_1 + \vec{B}_{11}\bar{M}_{11} + \vec{B}_{12}\bar{M}_{21}]Z_{1i} \\ &\quad + Z_{1i}[A_{\delta 1} - L_1C_1 + \vec{B}_{11}\bar{M}_{11} + \vec{B}_{12}\bar{M}_{21}] \\ &\quad + (NT+1)R_1\end{aligned}$$
$$\begin{aligned}\Sigma_{34} &= [A_{\delta 2}^h - L_2C_1 + \vec{B}_{21}\bar{M}_{11} + \vec{B}_{22}\bar{M}_{21}]Z_{3i} + Z_{1i}[A_{\delta 2}^h \\ &\quad - L_2C_1 + \vec{B}_{21}\bar{M}_{11} + \vec{B}_{22}\bar{M}_{21}] + (NT+1)R_2\end{aligned}$$
$$\begin{aligned}\Sigma_{43} &= [A_{\delta 2} - L_1C_1 + \vec{B}_{11}\bar{M}_{12} + \vec{B}_{12}\bar{M}_{22}]Z_{1i} + Z_{3i}[A_{\delta 2}^h \\ &\quad - L_2C_1 + \vec{B}_{21}\bar{M}_{11} + \vec{B}_{22}\bar{M}_{21}] + (NT+1)R_2^h\end{aligned}$$
$$\begin{aligned}\Sigma_{44} &= \sum_{j=1}^{n}\lambda_{ij}Z_{3j} + [A_{\delta 3} - L_2C_2 + \vec{B}_{21}\bar{M}_{12} + \vec{B}_{22}\bar{M}_{22}]Z_{3i} \\ &\quad + Z_{3i}[A_{\delta 3} - L_2C_2 + \vec{B}_{21}\bar{M}_{12} + \vec{B}_{22}\bar{M}_{22}] \\ &\quad + (NT+1)R_3\end{aligned}$$

Proof: According to the Schur complement, the Π can be rewritten as shown at the top of this page.

From the *Lemma 3*, the Π can be updated to (31), as shown at the bottom of the next page.

To obtain the gain matrix L and index γ of the H_∞ fault detection filter, the following matrices need be proposed:

$$P_i = \begin{bmatrix} Z_{1i} & 0 \\ * & Z_{3i} \end{bmatrix}, \quad P_j = \begin{bmatrix} Z_{1j} & 0 \\ * & Z_{3j} \end{bmatrix},$$

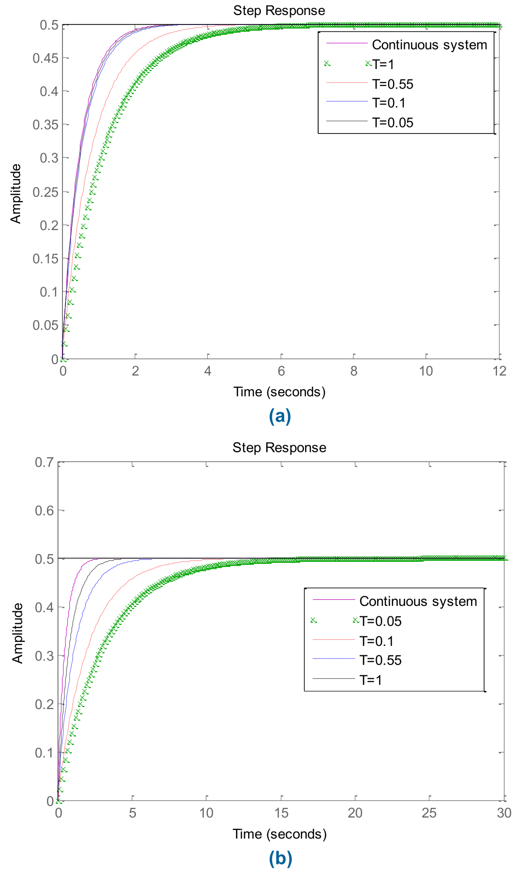

FIGURE 4. (a) Trajectories of the output on delta operator. (b) Trajectories of the output on traditional shift operator.

$$Q = \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix}, \quad \vec{B}_{\sigma(k)} = \begin{bmatrix} \vec{B}_{11} & \vec{B}_{12} \\ \vec{B}_{21} & \vec{B}_{22} \end{bmatrix},$$

$$\bar{M} = \begin{bmatrix} \bar{M}_{11} & \bar{M}_{12} \\ \bar{M}_{21} & \bar{M}_{22} \end{bmatrix}, \quad S = LC \quad \text{and } G = \bar{M}\vec{B}_{\sigma(k)}K.$$

where $Z_{1i}, Z_{3i}, Z_{1j}, Z_{3j}, R_1, R_3$ are symmetric positive definite matrices; R_2 is invertible matrix.

In summary, LMI (30) could be obtained according to the above equations. The gain matrix L and index γ are achieved by LMI (30) which make the residual dynamic systems asymptotically stable and meet H_∞ performance. The proof is completed.

V. NUMERICAL EXAMPLE

In this section, four examples are provided to illustrate the effectiveness and advantages of the H_∞ fault detection filter

TABLE 1. Evaluation function for sampling point in delta domain.

Sampling Point k	Evaluation function $J_r(k)/\sigma(k)=1$	Evaluation function $J_r(k)/\sigma(k)=2$
$k=55$	$J_r(k)=0.012413$	$J_r(k)=0.007818$
$k=56$	$J_r(k)=0.012434$	$J_r(k)=0.007819$
$k=57$	$J_r(k)=0.012460$	$J_r(k)=0.007822$
$k=58$	$J_r(k)=0.012483$	$J_r(k)=0.008318$
$k=59$	$J_r(k)=0.012589$	$J_r(k)=0.008366$
$k=60$	$J_r(k)=1.000487$	$J_r(k)=1.000557$
$k=61$	$J_r(k)=1.407677$	$J_r(k)=1.405821$
$k=62$	$J_r(k)=1.715076$	$J_r(k)=1.713184$
$k=63$	$J_r(k)=1.972598$	$J_r(k)=1.981813$
$k=64$	$J_r(k)=2.199001$	$J_r(k)=2.220255$
$k=65$	$J_r(k)=2.402964$	$J_r(k)=2.424577$

in delta domain. The example 1 is the approaching experiment by two output curve graphs on delta operator and traditional shift operator, respectively; The example 2 is residual analysis which compares the delta operator with traditional shift operator based on the same network environment assumed in this paper; The example 3 and example 4 are comparative experiments of delta operator systems with existing results [8], [11].

Example 1:

Consider the continuous system as following:

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

By using delta operator and traditional shift operator in sampling the continuous system, two output curve graphs of trajectories with different sampling rates are obtained in Figure 4(a) and 4(b).

According to the above results with different sampling periods, it is clear to see that advantage of delta operator systems which is closer to the continuous system with the high sampling rate.

Example 2:

Consider the networked control systems (1) with the following data:

$$A = \begin{bmatrix} -55 & 12 \\ 8 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ 4 \end{bmatrix}, \quad C = \begin{bmatrix} 4.5 & 0.78 \end{bmatrix},$$

$$D = \begin{bmatrix} 3.9 \\ 3.7 \end{bmatrix}, \quad E = \begin{bmatrix} 3.7 \\ 5.8 \end{bmatrix}$$

$$\Xi = \begin{bmatrix} T^2\sum_{j=1}^{n}\lambda_{ij}P_j + TP_i - 2P_i & T\sum_{j=1}^{n}\lambda_{ij}P_j + P_iA^* & P_iB^* & P_iC^* & 0 \\ * & \sum_{j=1}^{n}\lambda_{ij}P_j + A^{*h}P_i + P_iA^* + Q + NTQ & P_iB^* & P_iC^* & C^h \\ * & * & -Q & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & C & * & * & -I \end{bmatrix} \tag{31}$$

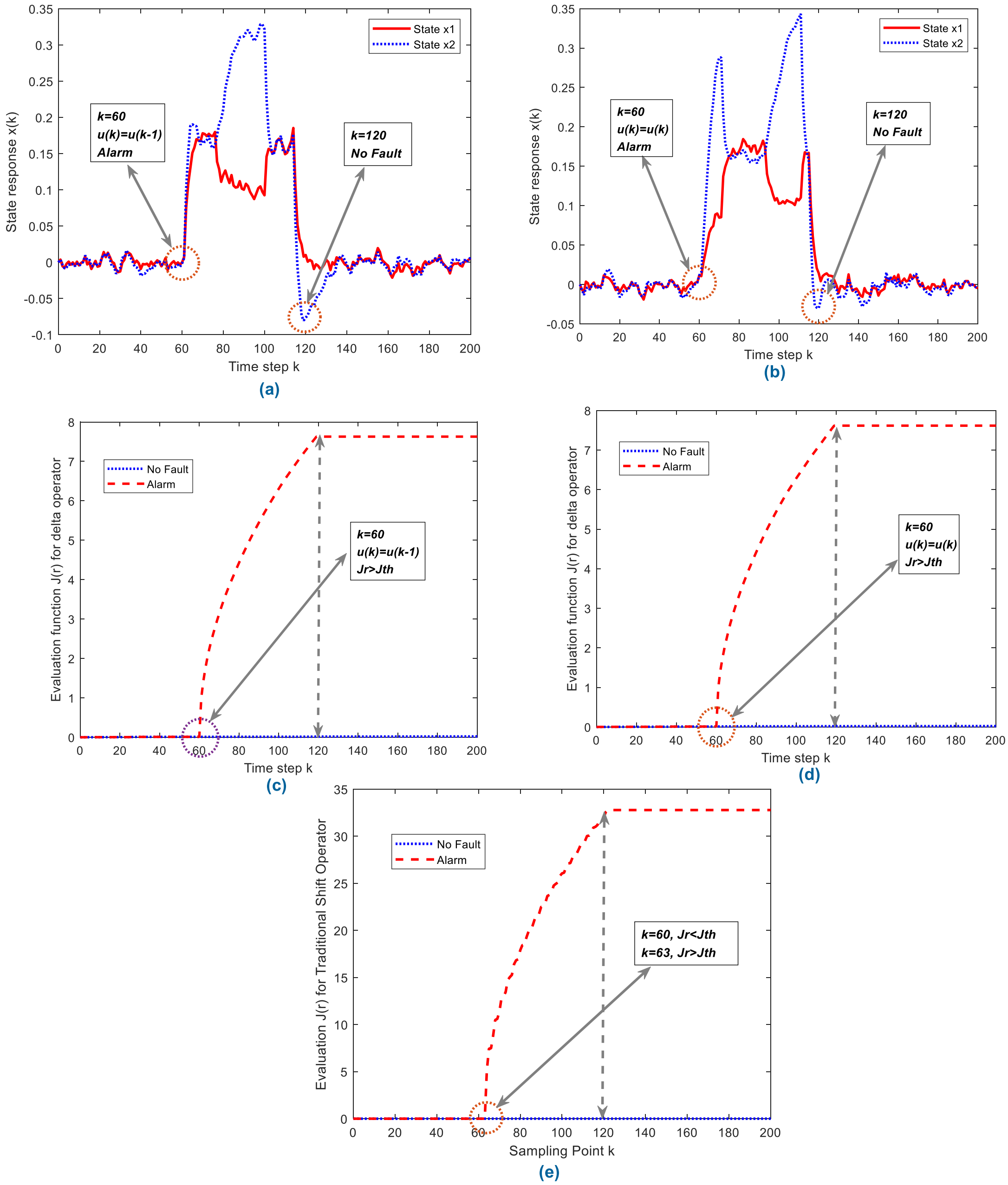

FIGURE 5. **(a) System state response in delta domain on u(k) = u(k-1). (b) System state response in delta domain on u(k) = u(k). (c) Residual evaluation function in delta domain on u(k) = u(k-1). (d) Residual evaluation function in delta domain on u(k) = u(k). (e) Residual evaluation function for traditional shift operator.**

The fault signal $f(k)$ is given as follows:

$$f(k) = \begin{cases} 1, & k = 60, \ldots, 120 \\ 0 & others \end{cases}$$

The transition probabilities matrix based on Markov chain process is given as:

$$\lambda_k = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.9 \end{bmatrix}$$

It can be obtained that fault detection filter gain L, state feedback controller gain K and index γ from the LMIs (30):

when $\sigma(k) = 1$, the parameters of filter are:

$$L = \begin{bmatrix} -2.3949 \\ 0.0156 \end{bmatrix}, \quad K = \begin{bmatrix} 2.1476 & -0.4439 \end{bmatrix}$$

and $\gamma = 1.0600$.

when $\sigma(k) = 2$, the parameters of filter are:

$$L = \begin{bmatrix} -0.3084 \\ -2.2861 \end{bmatrix}, \quad K = \begin{bmatrix} -0.3864 & 1.0028 \end{bmatrix},$$

and $\gamma = 1.0033$

The threshold J_{th} is given with $f(k) = 0$ as follows:

$$J_{th/\sigma(k)=1} = \sup_{f(k)=o,w(k)\in l_2} \|r_e\|_2 = \left[\sum_0^N r^h(k)r(k)\right]^{1/2} = 0.0230,$$

$$J_{th/\sigma(k)=2} = \sup_{f(k)=o,w(k)\in l_2} \|r_e\|_2 = \left[\sum_0^N r^h(k)r(k)\right]^{1/2} = 0.0189.$$

According to the Table 1 and Figure 5(a) to 5(d), the residual dynamic systems have a great change when the fault occurred at sampling point $k = 60$.

In order to prove the advantage of the delta operator when the sampling interval is very small. In the Figure 5(e), a fault detection filter based on observer is proposed by traditional shift operator. The fault signal is $f(k) = 1$ with the sampling interval at $60 \leq k \leq 120$.

For the traditional shift operator with the same data in the NCSs, a threshold $J_{th}(TSO)$ can be obtained as:

$$J_{th}(TSO) = \sup_{f(k)=o,w(k)\in l_2} \|r_e\|_2 = \left[\sum_0^N r^h(k)r(k)\right]^{1/2} = 0.0019.$$

TABLE 2. Evaluation function with traditional shift operator.

Sampling Point k	*Evaluation function $J_rTHO(k)$*
$k=55$	$J_r = 1.8712*10^{-3}$
$k=56$	$J_r = 1.8713*10^{-3}$
$k=57$	$J_r = 1.8719*10^{-3}$
$k=58$	$J_r = 1.8729*10^{-3}$
$k=59$	$J_r = 1.8743*10^{-3}$
$k=60$	$J_r = 1.8758*10^{-3}$
$k=61$	$J_r = 1.8775*10^{-3}$
$k=62$	$J_r = 1.8825*10^{-3}$
$k=63$	$J_r = 4.9610$
$k=64$	$J_r = 4.9635$
$k=65$	$J_r = 7.1404$

Compared Figure 5(c-d) with 5(e) and Table 1 with Table 2, respectively. It can be obtained that:

(i) In delta operator systems. In the case of $\sigma(k) = 1$, the evaluation function $J_r(k = 60) = 1.000487 > J_{th} = 0.0230$; In the case of $\sigma(k) = 2$, the evaluation function $J_r(k = 60) = 1.000557 > J_{th} = 0.0189$. Then the proposed method with delta operator in different modes is effective.

(ii) For the traditional shift operator, the evaluation function $J_rTSO(k = 60) = 1.8758 * 10^{-3} < J_{th} = 0.0019$. It means that there is no fault at sampling point $k = 60$. When the sampling point is $k = 63$, the residual signal has a change by $J_rTSO(k = 63) = 4.9610 > J_{th} = 0.0019$.

It is obvious that the proposed method for the H_∞ fault detection with the delta operator could detect the fault faster than the existing method of traditional shift operator with high sampling rate on the same network environment assumed in this paper.

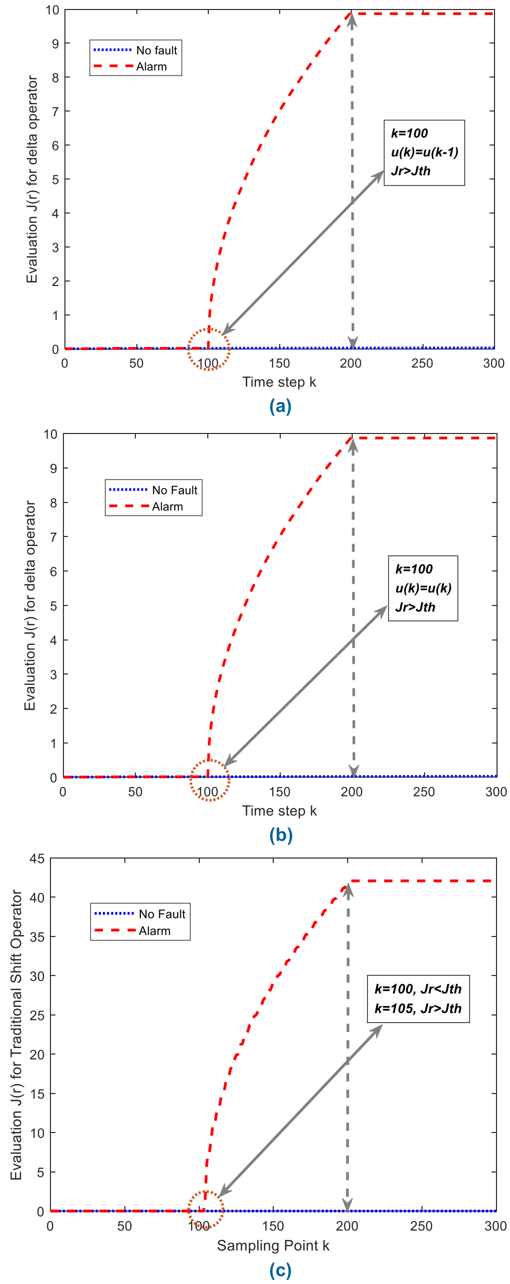

FIGURE 6. **(a) The proposed residual evaluation function on u(k) = u(k-1). (b) The proposed residual evaluation function on u(k) = u(k). (c) Residual evaluation function in [11].**

Example 3:

Consider the same systems' model with example 2, and the fault signal is given as:

$$f(k) = \begin{cases} 0.5, & k = 100, \ldots, 200 \\ 0 & others \end{cases}$$

With the same network environment [11] which covers the two-channels random packet losses and time-delays, residual evaluation functions with different methods of discretization are given in Figure 6(a-b) and 6(c), respectively.

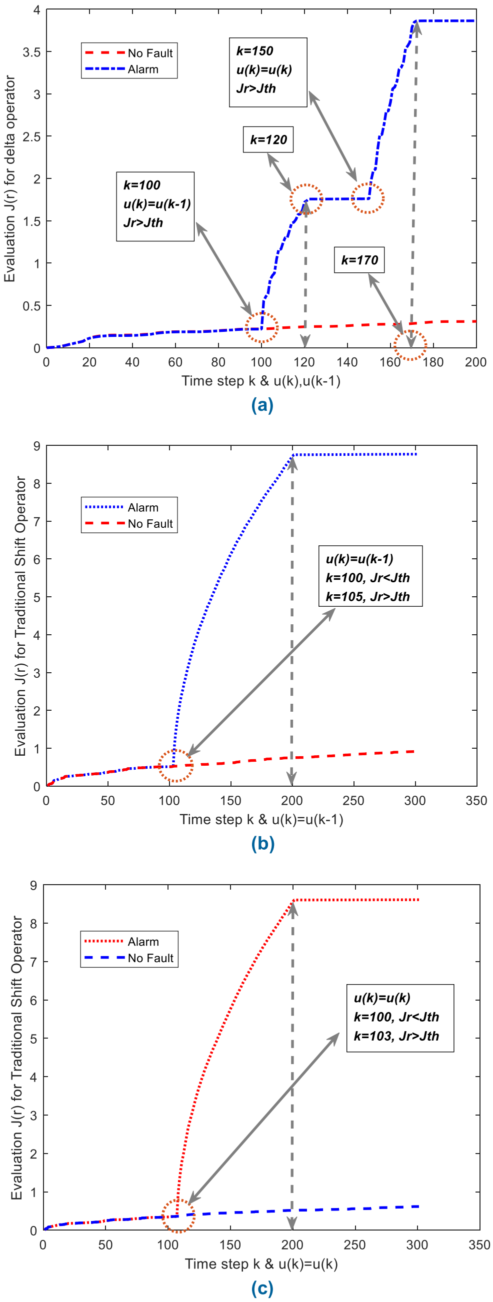

FIGURE 7. (a) The proposed residual evaluation function. (b) Residual evaluation function on u(k) = u(k-1) in [8]. (c) Residual evaluation function on u(k) = u(k) in [8].

From the above results Figure 6(a) to 6(c), it is easy to see that the proposed method on delta operator could detect the fault more accurate and faster than the existing method [11] by traditional shift operator at high sampling rate.

Example 4:

Given the same system model with example 2, and the new fault signal is:

$$f(k) = \begin{cases} 1 + 0.5\cos(4\pi k), & k = 100, \ldots, 200 \\ 0 & others \end{cases}$$

Consider the same network faults [8] which focus on random two-channels data missing and time-delays, and by using delta operator and traditional shift operator in sampling the continuous system, respectively. The residual evaluation functions with if there exist packet losses are obtained as Figure 7(a), 7(b) and 7(c).

The result of fault detection on delta operator is shown in Figure 7(a). Compared with Figure 7(b) and 7(c), it is obvious that the presented approach could detect 3 or 5 steps faster than the existing method [8] on traditional shift operator after the fault signal occurrence which means the proposed method has better property with high sampling rate.

VI. CONCLUSIONS

In this paper, we have addressed the H_∞ fault detection problem of time-delays delta operator systems with random two-channels packet losses and limited communication. It is the first to construct the residual dynamic model in delta domain, and then the sufficient conditions for the asymptotical stability of the residual systems with H_∞ performance based on delta operator systems are provided by using the Lyapunov functional technique. The results of four examples are presented to demonstrate the effectiveness and advantages of the proposed fault detection filter in delta domain than the existing method with traditional shift operator at high sampling rate. Furthermore, the proposed results can be extended to the fault detection with the uncertain delta operator systems.

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