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LPV Modeling and Identification of Unsteady Aerodynamics for Fast Maneuvering Aircrafts

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ABSTRACT Fast maneuvers at large angles of attack are necessary for modern fighter aircrafts. The aerodynamic coefficients in this situation are unsteady, with the characteristics of hysteresis and fast time-varying. We propose an identification method based on the linear parameter varying (LPV) model. The model takes into account the hysteresis with respect to the angle of attack, and fast time varying with respect to the motion of post-stall maneuvers. By introducing the angular velocity as an exogenous variable, the model can guarantee the tracking ability of fast time-varying variations. We use the instrumental variable least squares (IVLS) algorithm to deal with the estimation problem. Finally, the experimental data obtained from maneuvers at large angles of attack from wind tunnel tests are used to validate the applicability and effectiveness of the proposed method.

INDEX TERMS Instrumental variable least squares (IVLS) algorithm, large angle of attack, linear parameter varying (LPV) model, system identification, unsteady aerodynamics.

I. INTRODUCTION

Ehanced maneuverability and high agility is required by modern fighter aircrafts. It is necessary to increase the aerodynamic angles of attack for maneuvering. Large angle-of-attack maneuvers can help the aircrafts change the attitude and direction dramatically in a small space over a short time, and then the pilots can quickly turn the head of aircrafts and realize the rapid nose pointing in air combat. Identification of aerodynamics plays a crucial role in the overall design of control systems, flight tests, flight simulation systems and fault detection [1]–[4]. The coefficients of aerodynamic forces and moments are represented as functions of flight state parameters including angle of attack, sideslip angle and the Mach number [5], [6]. Identification theory is an efficient method to determine the aerodynamic coefficients for establishing the equations of motion of aircrafts [7]. The identification method for aerodynamic coefficients at the range of small angles of attack is mature, however, the method cannot handle the situations beyond the traditional range, because the flow topology at large angles of attack is more complex. Identification of aerodynamic coefficients at large angles of attack has attracted attentions from researchers because the aerodynamics with respect to the angle of attack is complicated, which is nonlinear and unsteady [8]–[10].

A large number of wind tunnel tests for research of large angle-of-attack aerodynamic characteristics have been developed, such as the tests at Central Aerohydrodynamic Institute (TsAGI), Russia and Defence and Evaluation Research Agency (DERA Bedford), UK. Based on the nonlinear and unsteady aerodynamic data obtained from the tests, mathematical models are established by means of identification methods. The identification methods in general fall into two categories. One combines physical mechanism in the establishment of mathematical models, and the other is divorced from physical mechanism. The former is adopted more in practice, because the latter contains more uncertainties which makes the performance analysis complex and brings troubles to the design of control system. A series of unsteady aerodynamic modeling methods combining the knowledge of physical mechanism exist, including reduced frequency model [11], step response model [12], [13], state space model [14], [15], differential equation model [16], [17], etc.

Hysteresis is usually observed at angles of attack close to the stall angle. The flow separation causes sharp drop

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in the lift and notable increase in the drag. Since the past history has an influence on the flow, it is also responsible for aerodynamic hysteresis [18], [19]. Aerodynamic coefficients have strong connections with the angle of attack, which are determined not only by the value of the angle of attack, but also by the variation of the angle of attack. Aerodynamic characteristics always present a phenomenon of hysteresis which is history dependent, and the connections between aerodynamic coefficients and the angle of attack are multi-valued mappings. The phenomenon is especially obvious at large angles of attack.

Physical and mathematical features of hysteresis of Soyuz spacecraft were analyzed by [20] according to the flight test data and the hysteresis phenomenon was found to be the result of damping. The effect of profile asymmetry, Reynolds number on the hysteresis loop shape was studied in [21] according to the wind tunnel test data of rectangular wings. Hysteresis of NACA 0012 airfoil with a simple asymmetric foil structure at the angles of attack close to stall angle was studied in [22]. By conducting an experimental study on a NASA low speed GA(W)-1 foil, the authors in [23] found that the hysteresis had a close relationship with the phenomenon of laminar boundary layer separation and transition over the airfoil. However, the phenomenon is still not well understood.

Rapid online identification is also required for the reason that flight state parameters may be fast time-varying with rapidly changing rate at large angles of attack. Therefore, the identification method should be able to track the fast time-varying parameters or even a sudden change which usually happens in the post-stall maneuvers. The existing methods mainly focus on the hysteresis problem which can get a good performance in the longitudinal maneuvers. The fast time-varying problem, which plays an important role in the aerodynamic characteristics under fast maneuvers, however, still remains to be solved.

This paper studies the identification of aerodynamic coefficients under fast maneuvers at large angles of attack, in which case, a clear hysteresis and fast time-varying features exist. In the paper, we mainly discuss the aerodynamics under lateral maneuvers.

II. LINEAR TIME VARYING (LPV)

In many practical situations, a system is usually associated to some exogenous variables which may have physical meanings. Then the system can be modeled as a function of these fundamental parameters [24], [25]. One typical model is the linear parameter varying (LPV) model, first introduced by Shamma and Athans [26]. A system can be parameterized by the LPV model as

$$H(q^{-1},p)=A(q^{-1},p)^{-1}B(q^{-1},p), \tag{1}$$

where q^{-1} is the delay operator with the relationship as $q^{-1}y(k)=y(k-1)$, p is a time-varying exogenous parameter which can be taken as a function of discrete time k as $p=p(k)$, and

$$\begin{aligned} A(q^{-1},p) &= 1+a_1(p)q^{-1}+\cdots+a_{n_a}(p)q^{-n_a},\\ B(q^{-1},p) &= b_0(p)+b_1(p)q^{-1}+\cdots+b_{n_b}(p)q^{-n_b}, \end{aligned} \tag{2}$$

where n_a is the order of model output, n_b is the order of model input and $n_a \geq n_b$.

$a_i(p)(i=1,\ldots,n_a)$ and $b_j(p)(j=1,\ldots,n_b)$ are the non-linear mappings with respect to $p(k)$. It can be seen that the number of unknown functions to be identified is n_a+n_b+1. $p(k)$ is always chosen to be the parameter which can be measured or easily predicted.

Equation (1) can also be represented as

$$A(q^{-1},p(k))y(k)=B(q^{-1},p(k))u(k), \tag{3}$$

where $u(k)$ is the input to the system, and $y(k)$ is the output of the system.

$p(k)\in P\subset \mathbb{R}^{n_p}$. P is the parameter space for $p(k)$, and n_p is the number of variables chosen as the exogenous parameters. $p(k)$ is written as p_k, $u(k)$ as u_k, $y(k)$ as y_k in the discussion behind.

Assume that the unknown functions $\{a_i(\cdot)\}$, $\{b_j(\cdot)\}$ in (2) are linear combinations of a set of basis functions $\{f_1,\ldots,f_N\}$, which can be written as

$$a_i(p)=a_i^1f_1(p)+\cdots+a_i^Nf_N(p),\quad i=1,\ldots,n_a, \tag{4}$$

$$b_j(p)=b_j^1f_1(p)+\cdots+b_j^Nf_N(p),\quad j=1,\ldots,n_b, \tag{5}$$

where a_i^l and b_j^l $(l=1,\ldots,N)$ are constants, and N is the number of basis functions.

The identification problem is then transformed into the estimation of a_i^l and b_j^l. Basis functions can be chosen from a variety of different forms, not limited to polynomial basis functions $f_l(p)=p^{l-1}$ and Fourier basis functions $f_l(p)=\cos(2\pi p(l-1))$. Taking the polynomial function as the basis function, (4) and (5) can be rewritten as

$$a_i(p)=a_i^1+a_i^2p+\cdots+a_i^Np^{N-1},\quad i=1,\ldots,n_a. \tag{6}$$

$$b_j(p)=b_j^1+b_j^2p+\cdots+b_j^Np^{N-1},\quad j=1,\ldots,n_b. \tag{7}$$

Defining an $n\times N$ matrix Θ consisting of parameters to be identified as

$$\begin{bmatrix} a_1^1 & \cdots & a_1^N\\ \vdots & \ddots & \vdots\\ a_{n_a}^1 & \cdots & a_{n_a}^N\\ b_0^1 & \cdots & b_0^N\\ \vdots & \ddots & \vdots\\ b_{n_b}^1 & \cdots & b_{n_b}^N \end{bmatrix}. \tag{8}$$

Defining the regressor matrix Φ_k determined by values of input and output data, together with the values of basis functions with respect to p_k as

$$\Phi_k = \begin{bmatrix} -y_{k-1} \\ \vdots \\ -y_{k-n_a} \\ u_k \\ \vdots \\ u_{k-n_b} \end{bmatrix} \begin{bmatrix} 1 & p_k & \cdots & p_k^{N-1} \end{bmatrix}. \tag{9}$$

By defining the matrices, (3) is then described by the form of inner product

$$y_k = \langle \Theta, \Phi_k \rangle . \tag{10}$$

According to the representation above, the model finally obtained is linear with respect to the unknown parameters. Then, the commonly used estimation algorithms can be applied, such as the recursive least-square (RLS) algorithm and the least mean-square (LMS) algorithm [27].

On the selection of p, it is convenient to choose the variable which can be measured directly during the process. If p is uncertain, a controller should be designed based on the priori knowledge of p, such as the range of potential variations.

III. PROPOSED FRAMEWORK

A. LPV-BASED MODELING STRATEGY

The fast time-varying characteristics of aerodynamic parameters are caused by the rapid maneuvers of aircrafts. Vortex generating area and flow separating area interact with each other in the lateral maneuvers such as rolling at large angles of attack, which leads to complex flow topology.

Vortex breakdown has great influence on aerodynamic characteristics, resulting in loss of lateral stability. The asymmetric expansion of flow separation on the wing surface brings asymmetric aerodynamic loads. Then, the asymmetric flow topology leads to large rolling and yawing moments, and the values change rapidly at large angles of attack. The aerodynamic parameters appear to be rapidly time-varying. The linear superposition principle is no longer applicable in this situation. Then the traditional aerodynamic derivatives model and other derived models based on the linear superposition principle cannot be used.

The flight states change rapidly at large angles of attack under complex flow structures, meanwhile the aerodynamic parameters change dramatically. Based on the physical process, we design our modeling framework by considering that the unsteady aerodynamic coefficients are generated by the static aerodynamic coefficients through a rapidly time-varying dynamic process, which can be discretized into a form of difference equations,

$$\begin{aligned} &C_i(k) + a_1 C_i(k-1) + \cdots + a_{n_a} C_i(k-n_a) \\ &\quad = b_0 C_{i0}(k) + b_1 C_{i0}(k-1) + \cdots + b_{n_b} C_{i0}(k-n_b), \end{aligned} \tag{11}$$

where $C_i(k)$ is the aerodynamic coefficient at time k, and $C_{i0}(k)$ is the static aerodynamic coefficient corresponding to

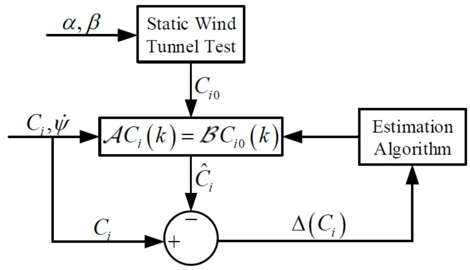


FIGURE 1. The identification framework based on the LPV model. The rolling moment and yawing moment are the major concerns in the large amplitude yawing motion. C_i is C_l or C_n.

the flight states at time k including the angle of attack and sideslip angle which can be obtained by the static wind tunnel test ($p, q, r = 0$).

According to the analysis above, the dynamic process is characterized by the fast time-varying flight states, thus the fast time-varying features in the flight states determine the values of $a_1, \cdots, a_{n_a}, b_0, \cdots, b_{n_b}$ in (11). The angular velocity is the key feature in the dynamic process. We choose the angular velocity in the main direction as the exogenous variable p in the LPV model (1). Taking the large amplitude yawing motion as an example, the yawing rate $\dot{\psi}$ is chosen to be the exogenous variable p. Then (11) can be rewritten to be the form of (3) by defining $C_{i0}(k)$ as the input and $C_i(k)$ as the output. The identification framework based on the LPV model is shown in Fig. 1. The model allows tracking of flight states which change rapidly by introducing the angular velocity as an exogenous variable.

B. PARAMETER ESTIMATION

The problem of estimating the LPV model $y(k) = \boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{\varphi}(k) + v(k)$ is considered, with the definitions,

$$\begin{aligned} y(k) &= C_i(k); \\ \boldsymbol{\varphi}(k) &= [-C_i(k-1), -C_i(k-1) f_1(p_{k-1}), \cdots, -C_i(k \\ &\quad -1) f_N(p_{k-1}), \cdots, -C_i(k-n_a) f_N(p_{k-n_a}), \\ &\quad C_{i0}(k), \cdots, C_{i0}(k) f_N(p_k), \cdots, C_{i0}(k-n_b), \\ &\quad C_{i0}(k-n_b) f_1(p_{k-n_b}), \cdots, C_{i0}(k-n_b) f_N(p_{k-n_b}))]^{\mathrm{T}}; \\ \boldsymbol{\theta} &= [a_1^1, \cdots, a_1^N, \cdots, a_{n_a}^0, \cdots, a_{n_a}^N, b_0^1, \cdots, b_0^N, \cdots, \\ &\quad b_{n_b}^1, \cdots, b_{n_b}^N]^{\mathrm{T}}. \end{aligned} \tag{12}$$

Then the least squares (LS) estimation gives the following result

$$\hat{\boldsymbol{\theta}}_{\mathrm{LS}} = \left[\sum_{k=1}^{M} \boldsymbol{\varphi}(k) \boldsymbol{\varphi}(k)^{\mathrm{T}} \right]^{-1} \sum_{k=1}^{M} \boldsymbol{\varphi}(k) y(k). \tag{13}$$

The LS algorithm is based on the assumption that $v(k)$ is white noise. When the coloured noise is contained in the data, the estimation result will be biased, given by

$$\hat{\boldsymbol{\theta}}_{\mathrm{LS}} = \boldsymbol{\theta}_0 + \left[\sum_{k=1}^{M} \boldsymbol{\varphi}(k) \boldsymbol{\varphi}(k)^{\mathrm{T}} \right]^{-1} \sum_{k=1}^{M} \boldsymbol{\varphi}(k) v(k). \tag{14}$$

where $\boldsymbol{\theta}_0$ is the true value of $\boldsymbol{\theta}$.

Therefore, in order to ensure the estimation result to be unbiased, according to the form of (14), the following equation need to be satisfied,

$$\lim_{M\to\infty}\sum_{k=1}^{M}\boldsymbol{\varphi}(k)v(k)=\mathbf{0}. \tag{15}$$

We adopt the instrumental variable least squares (IVLS) algorithm to ensure the consistency. By introducing an instrumental variable $\boldsymbol{z}(k)$, we get

$$y(k)\boldsymbol{z}(k)^{\mathrm{T}}=\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(k)\boldsymbol{z}(k)^{\mathrm{T}}+v(k)\boldsymbol{z}(k)^{\mathrm{T}}. \tag{16}$$

The estimation result of the IVLS algorithm is

$$\hat{\boldsymbol{\theta}}_{\mathrm{IV}}=\boldsymbol{\theta}_0+\left[\sum_{k=1}^{M}\boldsymbol{z}(k)\boldsymbol{\varphi}(k)^{\mathrm{T}}\right]^{-1}\sum_{k=1}^{M}\boldsymbol{z}(k)v(k). \tag{17}$$

Two conditions need to be satisfied to ensure the result to be unbiased,

$$\left|\lim_{M\to\infty}\sum_{k=1}^{M}\boldsymbol{z}(k)\boldsymbol{\varphi}(k)^{\mathrm{T}}\right|\neq 0. \tag{18}$$

$$\lim_{M\to\infty}\sum_{k=1}^{M}\boldsymbol{z}(k)v(k)=\mathbf{0}. \tag{19}$$

The instrumental variable $\boldsymbol{z}(k)$ must be correlated with $\boldsymbol{\varphi}(k)$, meanwhile non-correlated with $\boldsymbol{v}(k)$. Through analysis of the elements in the vector $\boldsymbol{\varphi}(k)$, $C_{i0}(k)$ and p_k are correlated with $\boldsymbol{v}(k)$, and $C_i(k)$ is correlated with $\boldsymbol{v}(k)$. Hence, the instrumental variable is constructed,

$$\begin{aligned}\boldsymbol{z}(k)=[&-x(k-1),-x(k-1)f_1(p_{k-1}),\cdots,\\&-x(k-1)f_N(p_{k-1}),\cdots,-x(k-n_a)f_N(p_{k-n_a}),\\&C_{i0}(k),\cdots,C_{i0}(k)f_N(p_k),\cdots,C_{i0}(k-n_b),\\&C_{i0}(k-n_b)f_1(p_{k-n_b}),\cdots,C_{i0}(k-n_b)f_N(p_{k-n_b}))]^{\mathrm{T}},\end{aligned} \tag{20}$$

where $x(k)$ is $\hat{C}_i(k)$, the estimate of $C_i(k)$, satisfying

$$A(q^{-1},p(k))x(k)=B(q^{-1},p(k))C_{i0}(k), \tag{21}$$

where A, B are constructed by the LS result $\hat{\boldsymbol{\theta}}_{\mathrm{LS}}$.

The recursive algorithm is then,

$$\hat{\boldsymbol{\theta}}(k)=\hat{\boldsymbol{\theta}}(k-1)+\boldsymbol{P}(k)\boldsymbol{z}(k)\left[y(k)-\boldsymbol{\theta}^{\mathrm{T}}(k-1)\boldsymbol{\varphi}(k)\right], \tag{22}$$

$$\boldsymbol{P}^{-1}(k)=\boldsymbol{P}^{-1}(k-1)+\boldsymbol{z}(k)\boldsymbol{\varphi}^{\mathrm{T}}(k), \tag{23}$$

$$\begin{aligned}\boldsymbol{z}(k)=[&-\hat{C}_i(k-1),-\hat{C}_i(k-1)f_1(p_{k-1}),\cdots,\\&-\hat{C}_i(k-1)f_N(p_{k-1}),\cdots,-\hat{C}_i(k-n_a)f_N(p_{k-n_a}),\\&C_{i0}(k),\cdots,C_{i0}(k)f_N(p_k),\cdots,C_{i0}(k-n_b),\\&C_{i0}(k-n_b)f_1(p_{k-n_b}),\cdots,C_{i0}(k-n_b)f_N(p_{k-n_b}))]^{\mathrm{T}}.\end{aligned} \tag{24}$$

In terms of conventional maneuvers, the estimation algorithm is required to be less sensitive to measurement noise, while in term of fast maneuvers such as post-stall maneuvers, the estimation algorithm is required to converge in a short time [28]. Therefore, we define an index to indicate whether a fast change happens, as in

$$\frac{\epsilon(k)-\epsilon(k-1)}{\epsilon(k-1)}>\eta, \tag{25}$$

where $\epsilon(k)$ is the prediction error at time k, and η is a predefined threshold.

When the inequality holds, which means a fast time-varying motion is detected, then the proportion of history information influence on the estimation result should be reduced, and meanwhile rapid convergence should be mainly encouraged.

C. PERFORMANCE ANALYSIS

Weighted sum of a set of basis functions is usually used to approximate the nonlinear function [29]. There are many choices of basis functions types, such as Gaussian function, polynomial, S function, wavelet function, etc.

The number of parameters to be estimated in the proposed model is $(n_a+n_b+1)N$, which increases dramatically with the increase of n_a, n_b and N. Excessive number will lead to high time complexity and space complexity. Additionally, the model will suffer from overfitting. Hence selection of model terms is required. We used the selection method discussed in [30].

Let $\hat{a}_i(p_k)(i=1,\ldots,n_a)$, $\hat{b}_j(p_k)(j=0,1,\ldots,n_b)$ denote the approximation functions. Let Δa_i and Δb_j denote the residual functions which are calculated by $a_i-\hat{a}_i$ and $b_j-\hat{b}_j$ respectively.

The measurement noise $v(k)$ is assumed to be bounded, which is a standard assumption in identification problems,

$$\|v(k)\|_h\leq\mu,\ h\in[1,\infty). \tag{26}$$

The error bound is an important criterion for evaluation of model prediction performance. We discuss the error bound below. First the following two assumptions are assumed:

A-1 For any Δa_i and Δb_j, Lipschitz condition is satisfied in the space P with the Lipschitz constants l_i,

$$\begin{aligned}\Delta a_i\in\mathscr{L}(l_i)=\{f:|f(p1)-f(p2)|\leq l_i\|p1-p2\|_2,\\\forall p1,p2\in P\},\ i=1,\ldots,n_a.\end{aligned} \tag{27}$$

$$\begin{aligned}\Delta b_j\in\mathscr{L}(l_{n_a+j})=\{f:|f(p1)-f(p2)|\leq l_{n_a+j}\|p1-p2\|_2,\\\forall p1,p2\in P\},\ j=1,\ldots,n_b.\end{aligned} \tag{28}$$

A-2 The residual functions are bounded on the observation data,

$$|\Delta a_i(p_k)|\leq\epsilon_i,\quad i=1,\ldots,n_a. \tag{29}$$

$$|\Delta b_j(p_k)|\leq\epsilon_{n_a+j},\quad j=1,\ldots,n_b. \tag{30}$$

The two assumption conditions hold in case of interpolation of basis functions [31]. Then the error bound is given by the theorem.

FIGURE 2. FL-8 wind tunnel test section [32].

Theorem 1: For all $q \in P$, $a_i(q), i = 1, \ldots, n_a$ and $b_j(q), j = 1, \ldots, n_b$ can be estimated by the representations of (4) and (5). Let M denote the number of training data. Then the error bounds are given by $\min\limits_{k=1,\ldots,M} (\epsilon_i + l_i\|q - p_k\|_2)$, $i = 1, \ldots, n_a$ and $\min\limits_{k=1,\ldots,M} \left(\epsilon_{n_a+j} + l_{n_a+j}\|q - p_k\|_2\right)$, $j = 1, \ldots, n_b$ respectively.

Proof: Under the condition (A-1), $\forall q \in P, k = 1, \ldots, M$,

$$\Delta a_i(q) - \Delta a_i(p_k) \leq l_i\|q - p_k\|_2. \tag{31}$$

Then, with the training data it satisfies that

$$\Delta a_i(q) \leq \Delta a_i(p_k) + l_i\|q - p_k\|_2. \tag{32}$$

According to the condition (A-2),

$$\Delta a_i(q) \leq \epsilon_i + l_i\|q - p_k\|_2. \tag{33}$$

Hence,

$$\Delta a_i(q) \leq \min_{k=1,\ldots,M} (\epsilon_i + l_i\|q - p_k\|_2). \tag{34}$$

The proof also holds for (30). ■

IV. EXPERIMENTAL RESULTS

FL-8 wind tunnel is used for large amplitude yawing oscillation tests and static tests of the corresponding flight states (Fig. 2). The experiments are done on the model with the fourth-generation fighter configuration (Fig. 3). The physical parameters are shown in Table 1. The experimental speed is 30m/s. The deflection angle of the leading edge flap is 30°, and the angles of other flight control surfaces are zero. The oscillation frequency is 1Hz. The sampling period is 0.02s.

First we obtain the interpolation table of the static aerodynamic coefficients C_{i0} with respect to the angles of attack and yawing angles (α, ψ). Then we use ten groups of large amplitude oscillation data with different testing conditions. In the large amplitude yawing oscillation test, the notable variations of aerodynamic coefficients are mainly reflected on the rolling moment C_l and the yawing moment C_n.

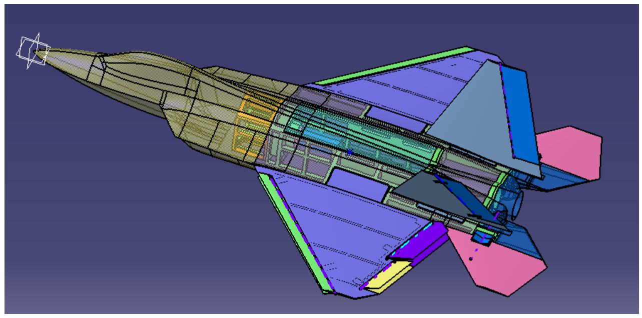


FIGURE 3. 3D configuration of the experimental prototype [32].

TABLE 1. Physical parameters of the experimental prototype.

Physical Parameter	Value
Weight	$\leq$ 8kg
Length	1.1825m
Wing Span	0.8475
Wing Area	0.3047m^2
Mean Aerodynamic Chord	0.4269m
Position of Aerodynamic Center	0.6926m

Thus, we validate the effectiveness of the proposed approach according to the aerodynamic data of C_l and C_n.

The order n_a and n_b are determined by comparison of fitting degree of different models with observation data, which is evaluated by the sum of squared errors,

$$J = \sum_{k=1}^{M} \left(y(k) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\varphi}(k)\right)^{\mathrm{T}} \left(y(k) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\varphi}(k)\right) \tag{35}$$

where $\hat{\boldsymbol{\theta}}$ is the estimate of the model with given n_a and n_b.

The sum of squared errors J decreases with the increase of n_a and n_b. When the values of n_a and n_b are large enough, which may be larger than the real values, the notable decrease of J stops. Thus, the order of the model is determined based on the variation tendency of J. The determination is realized by gradually increasing n_a and n_b in sequence, and calculating the corresponding $\hat{\boldsymbol{\theta}}$ and J until no obvious decrease of J can be observed.

We first set $f_1, \ldots, f_N$ to be polynomial basis functions and $N = 4$. According to judgement discussed above, $n_a, n_b = 3$, and the number of parameters to be estimated is 28. We choose 8 groups of data as training data to identify the model with the method of the LS algorithm and the IVLS algorithm respectively. The testing conditions of the data for training the model are given in Table 2. Then we test the models obtained with the training data on 2 groups. The testing conditions of the data for testing the model are given in Table 3. The experiments have been repeated in the AVIC Aerodynamics Research Institute FL-8 wind tunnel [32]. The comparison results are shown in Fig. 4 and Fig. 5. From the result, the LPV model based on the polynomial basis functions can effectively track the fast time-varying variations of aerodynamics. Compared with the LS method, the IVLS method can estimate the model with higher accuracy.

TABLE 2. Testing conditions of the data for training the model.

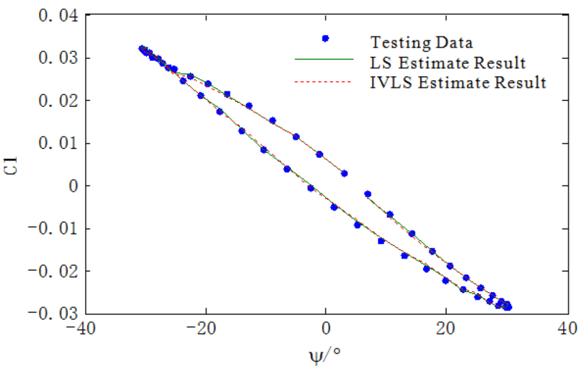
Group Number	$\psi_m/°$	$\alpha/°$
1	20	30
2	20	60
3	30	30
4	30	60
5	30	80
6	40	20
7	40	50
8	40	60

ψ_m is the amplitude of the oscillation.

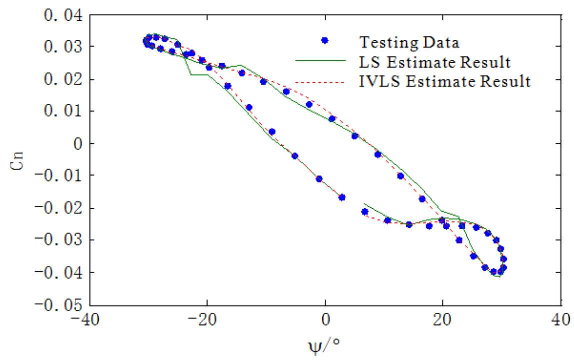
TABLE 3. Testing conditions of the data for testing the model.

Group Number	$\psi_m/°$	$\alpha/°$
1	30	50
2	40	40

ψ_m is the amplitude of the oscillation.



(a) Rolling moment coefficients



(b) Yawing moment coefficients

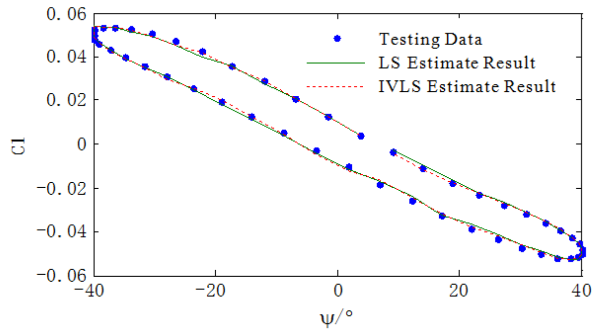
FIGURE 4. Comparison result on testing data of Group 1 in Table 3.

We then set the basis functions to be the form of Gaussian function,

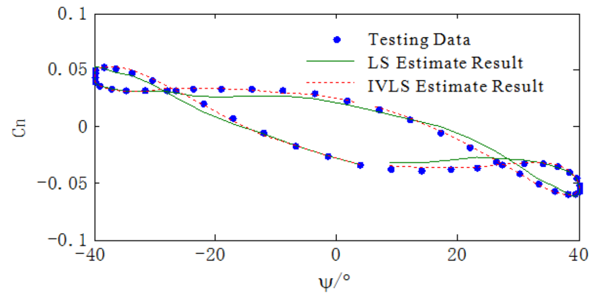
$$f_i(p_k) = \exp\left(\|p_k - p^i\|^2\right) \tag{36}$$

where $p^i \in \{-1, 0, 1\}$.

The model is identified according to the same training data corresponding to Table 2 by using the IVLS method, and then validated on the data of Group 2 in Table 2. The prediction errors of rolling moment coefficients and yawing moment coefficients between the model result and the real data are

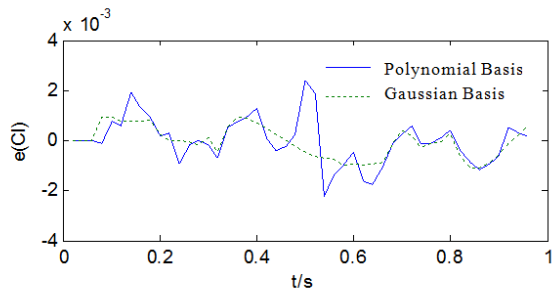


(a) Rolling moment coefficients

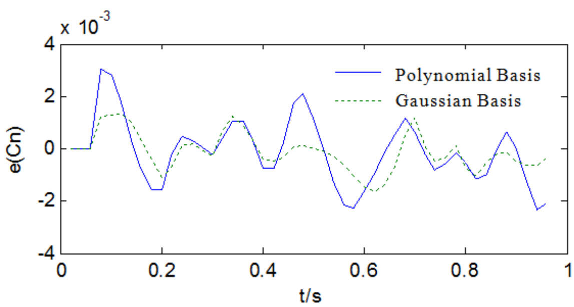


(b) Yawing moment coefficients

FIGURE 5. Comparison result on testing data of Group 2 in Table 3.



(a) Rolling moment coefficients



(b) Yawing moment coefficients

FIGURE 6. Contrast curve on testing data of Group 2 in Table 3.

compared. The contrast curve of prediction error $e(\cdot)$ is seen in Fig. 6. From the comparison result, using the same number of basis functions to construct the LPV model, Gaussian basis functions perform better.

V. CONCLUSION

By analyzing the physical mechanisms of aerodynamics under the lateral maneuver at large angles of attack, we find fast time-varying characteristic is the key problem in the identification of aerodynamic coefficients in addition to the phenomenon of hysteresis. Aiming at solving the tracking problem, we propose an LPV-based framework. Traditional approaches, such as the state space model and the step response method, handle the pitch movement well, however, not well in terms of the lateral movement. By introducing exogenous variable, the algorithm can track both fast and slow parameter changes, and handle the lateral movements well. The experimental results have demonstrated that the proposed algorithm dramatically improves the identification performance of aircrafts when the flight state parameters are fast time-varying at large angles of attack.

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REFERENCES

[1] X. Qiang and H. Duan, "Aerodynamic parameter identification of hypersonic vehicle via pigeon-inspired optimization," *Aircr. Eng. Aerosp. Technol.*, vol. 89, no. 3, pp. 425–433, 2017.

[2] J. Kou, W. Zhang, and M. Yin, "Novel Wiener models with a time-delayed nonlinear block and their identification," *Nonlinear Dyn.*, vol. 85, no. 4, pp. 2389–2404, 2016.

[3] X. Tian, J. Yan, and Q. Zhou, "A novel method of parameter identification based on nonlinear empirical model for vortex-induced vibration," *J. Eng. Res.*, vol. 5, no. 4, pp. 44–58, 2018.

[4] S. Li, H. Lei, L. Shao, and C. Xiao, "Multiple model tracking for hypersonic gliding vehicles with aerodynamic modeling and analysis," *IEEE Access*, vol. 7, pp. 28011–28018, 2019.

[5] P. Nian, B. Song, J. Xuan, W. Yang, and Y. Dong, "A wind tunnel experimental study on the flexible flapping wing with an attached airfoil to the root," *IEEE Access*, vol. 7, pp. 47891–47903, 2019.

[6] B. Zhang, Z. Feng, B. Xu, and T. Yang, "Free form deformation method applied to modeling and design of hypersonic glide vehicles," *IEEE Access*, vol. 7, pp. 61400–61413, 2019.

[7] Z. Wang, X. Wen, and M. Wu, "Identification of key nodes in aircraft state network based on complex network theory," *IEEE Access*, vol. 7, pp. 60957–60967, 2019.

[8] Q. Wang, K.-F. He, W.-Q. Qian, T.-J. Zhang, Y.-Q. Cheng, and K.-Y. Wu, "Unsteady aerodynamics modeling for flight dynamics application," *Acta Mechanica Sinica*, vol. 28, no. 1, pp. 14–23, 2012.

[9] H. Sun, H. Zhang, and Z. Liu, "Comparative evaluation of unsteady aerodynamics modeling approaches at high angle of attack," *Acta Aerodynamica Sinica*, vol. 29, no. 6, pp. 733–737, 2011.

[10] B. Zhao, G. Luo, and J. Zhu, "Improved boosting model for unsteady nonlinear aerodynamics based on computational intelligence," *Int. J. Cogn. Inform. Natural Intell.*, vol. 11, no. 1, pp. 46–59, 2017.

[11] G.-F. Lin, C. Lan, J. Brandon, G.-F. Lin, C. Lan, and J. Brandon, "A generalized dynamic aerodynamic coefficient model for flight dynamics applications," in *Proc. 22nd Atmos. Flight Mech. Conf.*, New Orleans, LA, USA, Aug. 1997, p. 3643.

[12] B. Zhao, G. Luo, and J. Zhu, "A weighted hybrid model for unsteady nonlinear aerodynamics," in *Proc. IEEE 15th Int. Conf. Cogn. Inform. Cogn. Comput.*, Palo Alto, CA, USA, Aug. 2016, pp. 219–225.

[13] S.-L. Yang, L.-P. Liang, H.-D. Liu, and K.-J. Xu, "Error analysis and new dual-cosine window for estimating the sensor frequency response function from the step response data," *Rev. Sci. Instrum.*, vol. 89, no. 3, 2018, Art. no. 035002. doi: 10.1063/1.5001689.

[14] H. Liu, R. Huang, Y. Zhao, and H. Hu, "Reduced-order modeling of unsteady aerodynamics for an elastic wing with control surfaces," *J. Aerosp. Eng.*, vol. 30, no. 3, 2017, Art. no. 04016083. doi: 10.1061/(ASCE)AS.1943-5525.0000682.

[15] M. Goman and A. Khrabrov, "State-space representation of aerodynamic characteristics of an aircraft at high angles of attack," *J. Aircr.*, vol. 31, no. 5, pp. 1109–1115, 1994.

[16] C. Li, T. Zhang, and J. Yang, "Attitude control of aircraft using only synthetic jet actuators when stall occurs," *IEEE Access*, vol. 6, pp. 37910–37917, 2018.

[17] C. Wang, "Modeling and adaptive control for multirotor subject to thruster dynamics," *IEEE Access*, vol. 7, pp. 44503–44513, 2019.

[18] T. Yu, L. Ma, and N. Qin, "Adaptive cooperative tracking control of multi-agent systems with unknown actuators hysteresis," *IEEE Access*, vol. 6, pp. 33015–33028, 2018.

[19] R. Xu, X. Zhang, H. Guo, and M. Zhou, "Sliding mode tracking control with perturbation estimation for hysteresis nonlinearity of piezo-actuated stages," *IEEE Access*, vol. 6, pp. 30617–30629, 2018.

[20] O. N. Khatuntseva, "Analysis of the reasons for an aerodynamic hysteresis in flight tests of the Soyuz reentry capsule at the hypersonic segment of its descent," *J. Appl. Mech. Tech. Phys.*, vol. 52, no. 4, pp. 52–62, Sep. 2011.

[21] I. V. Kolin, V. G. Markov, T. I. Trifonova, and D. V. Shukhovtsov, "Hysteresis in the static aerodynamic characteristics of a curved-profile wing," *Tech. Phys.*, vol. 49, no. 2, pp. 263–266, 2004.

[22] S. Mittal and P. Saxena, "Prediction of hysteresis associated with the static stall of an airfoil," *AIAA J.*, vol. 38, no. 5, pp. 933–935, 2000.

[23] Z. Yang, H. Igarashi, M. Martin, and H. Hu, "An experimental investigation on aerodynamic hysteresis of a low-Reynolds number airfoil," in *Proc. 46th AIAA Aerosp. Sci. Meeting Exhibit*, Reno, NV, USA, Jan. 2008, p. 315.

[24] H. Zhang, G. Zhang, and J. Wang, "H_∞ observer design for LPV systems with uncertain measurements on scheduling variables: Application to an electric ground vehicle," *IEEE/ASME Trans. Mechatronics*, vol. 21, no. 3, pp. 1659–1670, Jun. 2016.

[25] M. Li, Y. Jia, and J. Du, "LPV control with decoupling performance of 4WS vehicles under velocity-varying motion," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 5, pp. 1708–1724, Sep. 2014.

[26] J. S. Shamma and M. Athans, "Guaranteed properties of gain scheduled control for linear parameter-varying plants," *Automatica*, vol. 27, no. 3, pp. 559–564, May 1991.

[27] X. Li and H. Fan, "Linear prediction methods for blind fractionally spaced equalization," *IEEE Trans. Signal Process.*, vol. 48, no. 6, pp. 1667–1675, Jun. 2000.

[28] H. Zhang, J. Xie, J. Ge, W. Lu, and B. Zong, "Adaptive strong tracking square-root cubature Kalman filter for maneuvering aircraft tracking," *IEEE Access*, vol. 6, pp. 10052–10061, 2018.

[29] Y. Li, M. Y. Lei, Y. Z. Guo, Z. Y. Hu, and H. L. Wei, "Time-varying nonlinear causality detection using regularized orthogonal least squares and multi-wavelets with applications to EEG," *IEEE Access*, vol. 6, pp. 17826–17840, 2018.

[30] W. Zhang, J. Zhu, and D. Gu, "Identification of robotic systems with hysteresis using nonlinear autoregressive exogenous input models," *Int. J. Adv. Robot. Syst.*, vol. 14, no. 3, pp. 1–10, 2017. doi: 10.1177/1729881417705845.

[31] M. Milanese, J. Norton, H. Piet-Lahanier, and E. Walter, *Bounding Approaches to System Identification*. New York, NY, USA: Plenum Press, 1996.

[32] C. M. Liu, Z. J. Zhao, C. Bu, J. F. Wang, and W. Q. Mu, "Double degree-of-freedom large amplitude oscillation test technology in low speed wind tunnel," *Acta Aeronautica Astronautica Sinica*, vol. 37, no. 8, pp. 2417–2425, 2016.

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