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Sliding Mode Observer-Based Control for Uncertain Singular Biological Economic System With Invasion of Alien Species

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ABSTRACT In this paper, the controller based on sliding mode observer (SMO) is studied for a singular biological economic model with stage structure and uncertain parameters. First, a biological economic system is established by the singular system with uncertain parameters for the invasion of alien species. And SMO is designed by use of control inputs and measurement outputs. Then, the integral sliding surface is constructed for the error system and the SMO system. Second, by constructing an augmented system, it is ensured that the system is regular, free of impulse, and stable by use of the linear matrix inequality technique and Lyapunov stability theory. Third, a sliding mode control (SMC) law is designed to guarantee the considered system reaching the sliding mode surface. Finally, the effectiveness of the proposed method is illustrated by a simulation example.

INDEX TERMS Stage structure, singular biological economic system, sliding mode observer.

I. INTRODUCTION

With the increasing research of modern control theory and the penetration into other disciplines such as aviation, energy, network, environmental protection, etc., a more generalized dynamic system has been discovered. This is the generalized system. The concept of the generalized system was first proposed in the 1970s. Although it has only been studied for more than 30 years, it has achieved fruitful results and has developed into an independent branch of modern control theory. Since the twentieth century, with the research and efforts of many scholars, the biological sciences have achieved great results. Over the years, many disciplines related to biological sciences have also emerged. Biomathematics is a discipline between biology and mathematics. It studies and solves biological problems from the perspective of mathematics and mechanics, and organically combines dynamics with mathematical thinking. Biomathematics has a complete mathematical theoretical foundation, including set theory, probability theory, statistical mathematics, etc. The application of singular systems is very extensive, mainly applied in information technology, aerospace, physics, biology, economics [1]–[5]. Singular systems have many special properties, and their application in biodynamic systems [1]–[4] has attracted the interest of many scholars. The control problem of a bio-economic model with H_∞ performance has been considered in [1]. References [2] and [3] analyzed the bifurcation problem of bio-economic systems. Their ideas originate from the economic theory in [5]: Net Economic Profit = Total Revenue - Total Cost. Bioeconomic models with invasion of alien species have been developed in [6]. In fact, we all know that the population is divided into stages in the whole growth process and will be affected by the external environment. Thus, we have established a bio-economic model with stage structure and uncertain parameters. Since the T-S fuzzy model has the ability to approximate a smooth nonlinear function with arbitrary precision under any compact set, it is worthy of our theoretical research and its application research. In the past few years, [7]–[9] have reached some conclusions related to T-S fuzzy models. For example, stability and stabilization [7], [14], robust H_∞ control [13] observer design [11], [12], sliding mode control [10].

Sliding mode control can handle many complex systems. It has the advantages of fast response, robustness to uncertainties, external disturbances, etc., and is suitable for systems with uncertainties, nonlinearities and disturbances [15]–[20]. As we know, sliding mode control is widely used in information, aerospace, robotics, etc. [21]–[23]. At the same time,

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it also provides effective control methods for [24] systems with time-delay and uncertain [25]. In addition, the variable structure control has many results through the method of fuzzy sliding mode control [26], such as back stepping based on sliding mode control [27], adaptive sliding mode control [15]. Therefore, we will continue to study variable structure control.

We all know that in reality, the state of the system is usually not easy to obtain. To solve this problem, we construct observers [28]–[30] by considering control inputs and measurement outputs. The emergence of state observers solves many of its problems and provides a broader space for the development of sliding mode control systems, such as replicating disturbances to achieve complete compensation for disturbances.

In this paper, we study biological systems with uncertainties. For the first time, we apply the observer theory to the problem of alien species to obtain the population density of alien species. This is also the main contribution of this paper.

II. MODELING

The following models are introduced in [15]:

$$\begin{cases} \dot{x}_1(t) = rx_2(t) - a_1x_1(t) - bx_1(t) - c_1x_1^2(t) \\ \dot{x}_2(t) = bx_1(t) - a_2x_2(t) - c_2x_2^2(t). \end{cases} \tag{1}$$

where $x_1(t), x_2(t)$ are, respectively, the density of the juveniles, adults of the fish population at time t. The rate of transformation of the adults is proportional to the existing juveniles population with proportionality constant b. a_1, a_2 are the death rates of the juveniles and adults of the fish population respectively. c_1 and c_2 represent the population internal control factor at different stages respectively.

There is a time interval between early stage and full outbreak of alien species. We set it as $\tau(t)$.

We will add the economic benefits equation related to capture to the system (3): NER=TR-TC, with time t varying, harvested effort $E_1(t)$ switches, then the following equation can be obtained

$$E_1(t)(x(t)p - c) = m(t). \tag{2}$$

In summary, the following bioeconomic models have been established,

$$\begin{aligned} \dot{x}_1(t) &= rx_2(t) - a_1x_1(t) - bx_1(t) - c_1x_1^2(t) \\ \dot{x}_2(t) &= bx_1(t) - a_2x_2(t) - E_1(t)x_2(t) \\ &\quad - c_2x_2^2(t) - \eta x_2(t)x_3(t-\tau(t)) \\ \dot{x}_3(t) &= ax_3(t-\tau(t)) - hx_4(t) \\ \dot{x}_4(t) &= \beta x_3(t-\tau(t)) - \theta x_2(t) - wx_4(t) \\ 0 &= E_1(t)(x_2(t)p - c) - m(t). \end{aligned} \tag{3}$$

where $x_3(t)$ and $x_4(t)$ respectively represent the density of the alien species, the harvested capability to alien species at time t. a is the intrinsic growth rate of the invasive species. η is the restriction rate for invasive species on adult fish populations. $-hx_4(t)$ represents the quantity of purification to the invasive species. $\beta x_3(t)$ represents the targeted purification of alien species, and θ represents the effect of purification on adults of the fish population. According to the actual situation, we do not consider the impact of purification on juvenile populations. $\omega x_4(t)$ represents the cost of capture for alien species. $\tau(t)$ is a time-varying function, and it is assumed that $0 \le \tau_m \le \tau(t) \le \tau_M$ and $\dot{\tau}(t) \le \tau_d < 1$, where τ_m, τ_M are constants and τ_d is a positive constant.

Because the system is affected by the external environment, in order to make the system closer to reality, we get the following parameter uncertain model,

$$\begin{aligned} \dot{x}_1(t) &= (r+\Delta r(t))x_2(t) - a_1x_1(t) - (b+\Delta b(t))x_1(t) \\ &\quad - c_1x_1^2(t) \\ \dot{x}_2(t) &= (b+\Delta b(t))x_1(t) - a_2x_2(t) - E_1(t)x_2(t) \\ &\quad - c_2x_2^2(t) - \eta x_2(t)x_3(t-\tau(t)) \\ \dot{x}_3(t) &= (a+\Delta a(t))x_3(t-\tau(t)) - hx_4(t) + u(t) \\ \dot{x}_4(t) &= (\beta+\Delta\beta(t))x_3(t-\tau(t)) - \theta x_2(t) - wx_4(t) \\ 0 &= E_1(t)(x_2(t)p - c) - m(t). \end{aligned} \tag{4}$$

where $\|\Delta r(t)\| \le \delta_1, \|\Delta b(t)\| \le \delta_2, \|\Delta a(t)\| \le \delta_3, \|\Delta\beta(t)\| \le \delta_4$.

For convenience of research, we have organized the system (4) into the following forms:

$$E\dot{x}(t) = (A+\Delta A)x(t) + (A_d+\Delta A_d)x(t-\tau(t)) + BU(t) \tag{5}$$

where $\|\Delta A(t)\| \le a$, $\|\Delta A_d(t)\| \le a_d$

$$A+\Delta A = \begin{bmatrix} \begin{matrix}-(a_1+b+\Delta b)\\ -c_1x_1(t)\end{matrix} & r+\Delta r & 0 & 0 & 0 \\ b+\Delta b & \begin{matrix}-a_2\\ -c_2x_2(t)\end{matrix} & 0 & 0 & -x_2(t) \\ 0 & 0 & 0 & -h & 0 \\ 0 & -\theta & 0 & -w & 0 \\ 0 & pE_1(t) & 0 & 0 & -c \end{bmatrix}$$

$$A_d+\Delta A_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\eta x_2 & 0 & 0 \\ 0 & 0 & a+\Delta a & 0 & 0 \\ 0 & 0 & \beta+\Delta\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -m(t) \end{bmatrix}$$

$$U(t) = \begin{bmatrix} 1 & 1 & u(t) & 1 & 1 \end{bmatrix}^T,$$

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \dot{x}_4 & \dot{E}_1 \end{bmatrix}^T,$$

$$x(t-\tau(t)) = \begin{bmatrix} 0 & 0 & 0 & x_3(t-\tau(t)) & 0 \end{bmatrix}^T$$

Let

$$z_1(t) = -(a_1 + b + \Delta b) - c_1 x_1(t),$$
$$z_2(t) = -a_2 - c_2 x_2(t), \quad z_3(t) = -x_2(t), \; z_4 = pE_1(t).$$

So

$$\begin{aligned}
&\max z_1(t) = -(a_1 + b + \Delta b) - c_1 {x_1}^{\min}(t),\\
&\min z_1(t) = -(a_1 + b + \Delta b) - c_1 {x_1}^{\max}(t),\\
&\max z_2(t) = -a_2 - c_2 {x_2}^{\min}(t),\\
&\min z_2(t) = -a_2 - c_2 {x_2}^{\max}(t),\\
&\max z_3(t) = -{x_2}^{\min}(t), \quad \min z_3(t) = -{x_2}^{\max}(t),\\
&\max z_4 = p{E_1}^{\max}(t), \quad \min z_4 = p{E_1}^{\min}(t).
\end{aligned}$$

Using the maximum and minimum values, $z_1(t)$, $z_2(t)$, $z_3(t)$ and $z_4(t)$ can be represented by

$$\begin{aligned}
z_1(t) &= M_{11}(z_1(t)) \max z_1(t) + M_{12}(z_1(t)) \min z_1(t),\\
z_2(t) &= M_{21}(z_2(t)) \max z_2(t) + M_{22}(z_2(t)) \min z_2(t),\\
z_3(t) &= M_{31}(z_3(t)) \max z_3(t) + M_{32}(z_3(t)) \min z_3(t),\\
z_4(t) &= M_{41}(z_4(t)) \max z_4(t) + M_{42}(z_4(t)) \min z_4(t).
\end{aligned}$$

where $M_{i1} + M_{i2} = 1, i = 1, 2, 3, 4$ and M_{ij} represents the membership function.

Then, by using the standard fuzzy blending method, the overall T-S descriptor model is inferred as follows:

$$\begin{aligned}
E\dot{x}(t) &= \sum_{i=1}^{16} h_i(\theta)\{(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - \tau(t))\\
&\quad + BU(t)\}. \qquad (6)
\end{aligned}$$

where $h_i(\theta)$ are the weight functions. We define by $h_i(\theta) = \frac{\omega_i(\theta)}{\sum_{i=1}^{l} \omega_i(\theta)}$, $\omega_i(\theta) = \prod_{j=1}^{g} \mu_{ij}(\theta_j)$. $\theta = [\theta_1, \theta_2 \ldots \theta_g]$ is assumed to be measurable. $\mu_{ij}(\theta_j)$ represents the membership degrees of θ_j in the fuzzy set μ_{ij}. Noting that $h_i(\theta)$ satisfies $h_i(\theta) \geq 0$, $\sum_{i=1}^{8} h_i(\theta) = 1$.

Assumption 1: We assume that rank $\begin{bmatrix} E \\ C \end{bmatrix} = n$, *so there exists a matrix* $\begin{bmatrix} T_1 & T_2 \end{bmatrix}$ *such that* $T_1 E + T_2 E = I_n$.

Below we introduce some definitions and lemmas, giving the following systems (7)

$$\begin{aligned}
E\dot{x}(t) &= A_i x(t) + A_d x(t - \tau(t))\\
x(t) &= \varphi(t), \quad t \in [-\tau_M, 0]. \qquad (7)
\end{aligned}$$

Lemma 1: For any positive constant ε *and positive definite matrix* P, *the inequalities below are applicable to any proper dimension matrix* D_1, D_2:

$$2D_1^T D_2 \leq \varepsilon D_1^T P D_1 + \varepsilon^{-1} D_2^T P^{-1} D_2.$$

III. MAIN RESULTS

In this part, we design SMO and two suitable sliding surface for the system (6) and present the conditions to ensure admissibility.

For the purposes of our research, we add the output vector to the system (6):

$$\begin{aligned}
E\dot{x}(t) &= \sum_{i=1}^{16} h_i(\theta)\,[(A_i + \Delta A_i)x(t)\\
&\quad + (A_{di} + \Delta A_{di})x(t - \tau(t)) + BU(t)]\\
y(t) &= Cx(t)\\
x(t) &= \varphi(t), \quad t \in [-\tau_M, 0]. \qquad (8)
\end{aligned}$$

where $\varphi(t)$ is an initial function.

A. SLIDING MODE OBSERVER DESIGN

This paper mainly studies the population density of alien species. Because of the rapid propagation of alien species, the actual population density is difficult to measure or has a large error. Thus, the observer is usually designed to reconstruct the state variables of the original system. The SMO is based on SMC strategy in this paper. Therefore, the following SMO is designed.

$$\begin{aligned}
E\hat{\dot{x}} &= \sum_{i=1}^{16} h_i(\theta)[A_i \hat{x}(t) + A_{di}\hat{x}(t - \tau(t)) + BU(t)\\
&\quad + L_i(y(t) - \hat{y}(t)) + Bv(t)]\\
\hat{y} &= Cx(t)\\
x(t) &= \phi(t), \quad t \in [-\tau_M, 0]. \qquad (9)
\end{aligned}$$

where $\hat{x}(t) \in R^n$ is the state estimation of $x(t)$, $\hat{y}(t)$ is the output vector of the observer, $L_i \in R^{n\times p}$ is the gain matrix to be designed in the sequel, $\phi(t)$ is an initial function , $v(t)$ is the nonlinear input vector.

Define $e(t) = x(t) - \hat{x}(t)$ is the state estimation error and $e_y(t) = y(t) - \hat{y}(t) = Ce(t)$ is the output estimation.

Multiplying E on both sides of the equation $e(t) = x(t) - \hat{x}(t)$, we can get the following expression

$$E\dot{e}(t) = E\dot{x}(t) - E\hat{\dot{x}}(t). \qquad (10)$$

Then, the error system can be obtained as follows:

$$\begin{aligned}
E\dot{e}(t) &= \sum_{i=1}^{16} h_i(\theta)\{(A_i - L_i C)e(t) + A_{di}e(t - \tau(t))\\
&\quad + \Delta A_i x(t) + \Delta A_{di} x(t - \tau(t)) - Bv(t)\}. \qquad (11)
\end{aligned}$$

B. CONSTRUCTION OF SLIDING SURFACES

The aim of our note is to develop a SMO and observer-based SMC such that the closed-loop system is admissible. Therefore, we should design two sliding surfaces and two controllers. In this subsection, the suitable sliding surfaces are designed for the error system (9) and the SMO system (11) respectively.

Firstly, we propose an integral sliding surface for the error system as follows:

$$s(t) = GEe(t) + G\int_0^t \sum_{i=1}^{16} h_i(\theta)(L_iCe(z))dz. \tag{12}$$

where $G \in R^{m\times n}$ is known matrix, satisfying $\det(GB) \neq 0$ and $rank\begin{bmatrix} GE \\ C \end{bmatrix} = rank(C)$. L_i is the gain matrix.

Considering the solution of (11), we get

$$\begin{aligned} Ee(t) = Ee(0) + \int_0^t \sum_{i=1}^{16} h_i(\theta)[A_i - L_iC)e(z) \\ + A_{di}e(z-\tau(z)) + \Delta A_i x(z) + \Delta A_{di}x(z-\tau(z)) \\ - Bv(z)]dz. \end{aligned} \tag{13}$$

From (12) and (13), it follows that

$$\begin{aligned} s(t) = GEe(0) + G\int_0^t \sum_{i=1}^{16} h_i(\theta)\{A_ie(z) + A_{di}e(z-\tau(z)) \\ + \Delta A_i x(z) + \Delta A_{di}x(z-\tau(z)) - Bv(z)\}dz. \end{aligned} \tag{14}$$

Then we can obtain that

$$\begin{aligned} \dot{s}(t) = G\sum_{i=1}^{16} h_i(\theta)\{A_ie(t) + A_{di}e(t-\tau(t)) + \Delta A_i x(t) \\ + \Delta A_{di}x(t-\tau(t)) - Bv(t)\}. \end{aligned} \tag{15}$$

From $\dot{s}(t) = 0$, we obtain the following equivalent control laws

$$\begin{aligned} v_{eq}(t) = (GB)^{-1}G\sum_{i=1}^{16} h_i(\theta)\{A_ie(t) + A_{di}e(t-\tau(t)) \\ + \Delta A_i x(t) + \Delta A_{di}x(t-\tau(t))\}. \end{aligned} \tag{16}$$

By substituting equation (16) into the error system (11), the sliding equation (17) can be obtained.

$$\begin{aligned} E\dot{e}(t) = \sum_{i=1}^{16} h_i(\theta)\{(\bar{R}A_i - L_iC)e(t) + \bar{R}A_{di}e(t-\tau(t)) \\ + \bar{R}\Delta A_i x(t) + \bar{R}\Delta A_{di}x(t-\tau(t))\}. \end{aligned} \tag{17}$$

where $\bar{R} = I - R$ and $R = B(GB)^{-1}G$.

Secondly, we propose an integral sliding surface for the SMO system as follows:

$$\begin{aligned} s_1(t) = G_1E\hat{x}(t) - G_1\int_0^t \sum_{i=1}^{16} h_i(\theta)\{(A_i + BQ_i)\hat{x}(z) \\ + L_iCe(z)\}dz. \end{aligned} \tag{18}$$

where $G_1 \in R^{m\times n}$ is known matrix satisfying $\det(G_1B) \neq 0$ and L_i is the gain matrix. $Q_i \in R^{m\times n}$ is chosen so that A_i+BQ_i is Hurwitz.

By using (9), the solution of $E\hat{x}(t)$ can be expressed as follows:

$$\begin{aligned} E\hat{x}(t) = E\hat{x}(0) + (t)\int_0^t \sum_{i=1}^{16} h_i(\theta)\{A_i\hat{x}(z) + A_{di}\hat{x}(z-\tau(z)) \\ + BU(z) + L_i(y(z) - \hat{y}(z) + Bv(z)\}dz. \end{aligned} \tag{19}$$

According to (18), (19) and taking the derivation of $s_1(t)$, we can get

$$\begin{aligned} \dot{s}_1(t) = G_1\sum_{i=1}^{16} h_i(\theta)\{(-BQ_i)\hat{x}(t) + A_{di}\hat{x}(t-\tau(t)) \\ + BU(t) + Bv(t)\}. \end{aligned} \tag{20}$$

Following the same approach, the equivalent control law yields as

$$\begin{aligned} U_{eq}(t) = -(G_1B)^{-1}G_1\sum_{i=1}^{16} h_i(\theta)\{(-BQ_i)\hat{x}(t) \\ + A_{di}\hat{x}(t-\tau(t))\} - v(t). \end{aligned} \tag{21}$$

By substituting equation (20) into the observer system (9), the sliding equation (22) can be obtained.

$$\begin{aligned} E\hat{x}(t) = \sum_{i=1}^{16} h_i(\theta)\{(A_i + BQ_i)\hat{x}(t) + \bar{R}_1A_{di}\hat{x}(t-\tau(t)) \\ + L_i(y(t) - \hat{y}(t))\}. \end{aligned} \tag{22}$$

where $\bar{R}_1 = I - R_1$ and $R_1 = B(G_1B)^{-1}G$.

C. ADMISSIBILITY ANALYSIS OF THE SLIDING MOTIONS

In this section, we will simultaneously analyze the acceptability of the error system (17) and the observer system (22).

Augmented System: The sliding motions are given as

$$\begin{aligned} E\dot{e}(t) &= \sum_{i=1}^{16} h_i(\theta)\{(\bar{R}A_i - L_iC)e(t) + \bar{R}A_{di}e(t-\tau(t)) \\ &\quad + \bar{R}\Delta A_i x(t) + \bar{R}\Delta A_{di}x(t-\tau(t))\} \\ E\hat{x}(t) &= \sum_{i=1}^{16} h_i(\theta)\{(A_i + BQ_i)\hat{x}(t) + \bar{R}_1A_{di}\hat{x}(t-\tau(t)) \\ &\quad + L_i(y(t) - \hat{y}(t))\}. \end{aligned} \tag{23}$$

In order to analyze the admissibility of (17) and (22), simultaneously, we consider the following augmented system

$$\begin{aligned} \bar{E}\dot{x}_e(t) &= \sum_{i=1}^{16} h_i(\theta)\{(\bar{A}_i + \Delta\bar{A}_i)x_e(t) \\ &\quad + (\bar{A}_{di} + \Delta\bar{A}_{di})x_e(t-\tau(t))\} \\ e_y(t) &= \bar{C}x_e(t). \end{aligned} \tag{24}$$

where

$$x_e(t) = \begin{bmatrix} e(t) \\ \hat{x}(t) \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix},$$

$$\bar{A}_i = \begin{bmatrix} \bar{R}A_i - L_iC & 0 \\ L_iC & A_i + BQ_i \end{bmatrix},$$

$$\bar{A}_{di} = \begin{bmatrix} \bar{R}A_{di} & 0 \\ 0 & \bar{R}_1A_{di} \end{bmatrix},$$

$$\Delta \bar{A}_i = \begin{bmatrix} \bar{R}\,\Delta A_i & \bar{R}\,\Delta A_i \\ 0 & 0 \end{bmatrix},$$
$$\Delta \bar{A}_{di} = \begin{bmatrix} \bar{R}\,\Delta A_{di} & \bar{R}\,\Delta A_{di} \\ 0 & 0 \end{bmatrix}.$$

Firstly, we consider the nominal case (that is, $\Delta \bar{A}_i = 0, \Delta \bar{A}_{di} = 0$) of the augmented system (24).

$$\bar{E}\,\dot{x}_e(t) = \sum_{i=1}^{16} h_i(\theta)\{\bar{A}_i\,x_e(t) + \bar{A}_{di}\,x_e(t-\tau(t))\}$$
$$e_y(t) = \bar{C}\,x_e(t). \tag{25}$$

Theorem 1: Given $\gamma > 0$, positive definite matrix $P \in R^{n\times m}$, and normal number ε_2, the inequalities below are set up

$$\bar{E}^T P = P^T \bar{E}$$
$$\begin{bmatrix} W & P^{-1}\bar{A}_{di}{}^T \\ \bar{A}_{di}P^{-1} & -\varepsilon_2 I \end{bmatrix} < 0 \tag{26}$$

where $W = \bar{A}_i P^{-1} + P^{-1}\bar{A}_i{}^T + \varepsilon_2 I$.

The nominal case of the system (24) is admissible

Proof: The evidence is divided into two parts. Firstly, we prove the regularity and the no-impulse of the systems. denoting

$$\bar{E} = \begin{bmatrix} I_g & 0 \\ 0 & 0 \end{bmatrix}$$

where of $rank(\bar{E}) = rank(I_g) = g \le n$, there is a nonsingular matrix U, V such that

$$U\bar{E}\,V = \begin{bmatrix} I_g & 0 \\ 0 & 0 \end{bmatrix}, \quad U\bar{A}_i\,V = \begin{bmatrix} \bar{A}_{i11} & \bar{A}_{i12} \\ \bar{A}_{i21} & \bar{A}_{i22} \end{bmatrix},$$
$$U^{-T}\bar{P}\,U^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}. \tag{27}$$

By (26) and (27), we get $P_{12} = 0$ and P_{22} is nonsingular. By (26), we know $W < 0$. Because P is a positive definite matrix, we have

$$P^T\bar{A}_i + \bar{A}_i{}^T P < 0. \tag{28}$$

Moreover, pre-multiplying and post-multiplying Eq. (28) by U^T and U

$$\begin{bmatrix} * & * \\ * & sym(\bar{P}_{22}{}^T\bar{A}_{22}) \end{bmatrix} < 0.$$

Therefore

$$\det(s\bar{E} - \sum_{i=1}^{16} h_i\bar{A}_i) = \det(U^{-1})\det(sI_g - \sum_{i=1}^{16} h_i\bar{A}_{i11} + \sum_{i=1}^{16} h_i\bar{A}_{i12}(\sum_{i=1}^{16} h_i\bar{A}_{i22})^{-1}\sum_{i=1}^{16} h_i\bar{A}_{i21}) \times \det(-\sum_{i=1}^{16} h_i\bar{A}_{i22})\det(V^{-1}) \neq 0.$$

And

$$\deg(\det(s\bar{E} - \sum_{i=1}^{16} h_i\bar{A}_i)) = g.$$

By Definition 1, we get the system (24) in the nominal case is regular, impulse-free.

Next, we will prove that the formula (11) is asymptotically stable. To this end, we choose a Lyapunov-Krasovskii functional

$$V(x(t)) = x_e{}^T(t)P\bar{E}\,x_e(t) + \int_{t-\tau}^{t} x_e{}^T(s)Qx_e(s)ds. \tag{29}$$

where Q is the definite positive definite matrix. Find, (29) the derivative along the system (24) is as follows

$$\dot{V}(x) = 2x_e{}^T(t)P\bar{E}\,\dot{x}_e(t) + x_e{}^T(t)Qx_e(t) - x_e{}^T(t-\tau)Qx_e(t-\tau(t)).$$

Into equation (24)

$$\dot{V}(x) = x_e{}^T(t)[2P\sum_{i=1}^{16} h_i\bar{A}_i + Q]x_e(t) + \varepsilon_2 x_e{}^T(t)2P\sum_{i=1}^{16} h_i\bar{A}_{di}x_e(t-\tau(t)) - x_e{}^T(t-\tau(t))Qx_e(t-\tau(t)). \tag{30}$$

From Lemma 1, we know(24)

$$2x^T P\sum_{i=1}^{16} h_i\bar{A}_{di}\,x_e(t-\tau(t)) \le \varepsilon_2 x_e{}^T(t)PPx_e(t) + \varepsilon_2^{-1}x_e{}^T(t-\tau(t))\sum_{i=1}^{16} h_i\bar{A}_{di}{}^T\bar{A}_{di}x_e(t-\tau(t)). \tag{31}$$

By (30), (31), denoting

$$Q = \varepsilon_2^{-1}\sum_{i=1}^{16} h_i\bar{A}_{di}{}^T\bar{A}_{di}.$$

Therefore

$$\dot{V}(x) \le x_e{}^T(t)\sum_{i=1}^{16} h_i[2P\bar{A}_i + \varepsilon_2^{-1}\bar{A}_{di}{}^T\bar{A}_{di} + \varepsilon_2 PP]x_e(t) = x_e{}^T(t)\sum_{i=1}^{16} h_i\Gamma x_e(t).$$

If $\Gamma < 0$, then it can be shown that the system (11) is stable. By using Schur lemma, $\Gamma < 0$ is equivalent to

$$\begin{bmatrix} 2P\bar{A}_i + \varepsilon_2 PP & \bar{A}_{di}^T \\ \bar{A}_{di} & -\varepsilon_2 I \end{bmatrix} < 0. \tag{32}$$

for elementary transformation, there is a nonsingular matrix $diag(P^{-1}, I)$ such that

$$\begin{bmatrix} W & P^{-1}\bar{A}_{di}^T \\ \bar{A}_{di}P^{-1} & -\varepsilon_2 I \end{bmatrix} < 0. \tag{33}$$

where

$$W = \bar{A}_i P^{-1} + P^{-1}\bar{A}_i^T + \varepsilon_2 I.$$

From (33), we have $\dot{V} < 0$. In combination with the definition of 1, the sliding motion (24) is admissible.

Theorem 2: Normal number $\varepsilon_1, \varepsilon_2, \varepsilon_3$, system (24) is admissible under all parameter uncertainties, if satisfying

$$\begin{bmatrix} W & P^{-1}\bar{A}_{di}^T & \bar{G} & aP^{-1} & a_d P^{-1} \\ \bar{A}_{di}P^{-1} & -\varepsilon_2 I & 0 & 0 & 0 \\ G^{-T} & 0 & -(\varepsilon_1+\varepsilon_2)I & 0 & 0 \\ aP^{-1} & 0 & 0 & -\varepsilon_1 I & 0 \\ a_d P^{-1} & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{34}$$

where

$$W = \bar{A}_i P^{-1} + P^{-1}\bar{A}_i^T + \varepsilon_2 I.$$

Proof: solving (29) the derivative along the system (24), into equation (24)

$$\begin{aligned}\dot{V}(x) &= x_e^T(t)(2P\sum_{i=1}^{16} h_i \bar{G}\Delta A_i + Q)x_e(t) \\ &\quad + x_e^T(t)(2P\sum_{i=1}^{16} h_i \bar{A}_{di} + 2P\sum_{i=1}^{16} h_i \bar{G}\Delta \bar{A}_{di})x_e(t-\tau(t)) \\ &\quad - x_e^T(t-\tau(t))Qx_e(t-\tau(t)). \end{aligned} \tag{35}$$

There is a Lemma 1, we know(36)

$$\begin{aligned} 2P\bar{G}\sum_{i=1}^{16} h_i\Delta A_i(t) &= 2(\bar{G}^T P)^T \sum_{I=1}^{16} h_i \Delta A_i(t) \\ &\le \varepsilon_1(\bar{G}^T P)^T \bar{G}^T P + \varepsilon_1^{-1}\sum_{I=1}^{16} h_i\Delta A_i^T(t) \\ &\le \varepsilon_1 P\bar{G}\bar{G}^T P + a^2\varepsilon_1^{-1} I. \\ 2x_e^T(t)P\bar{G}\sum_{i=1}^{16} h_i \Delta \bar{A}_{di} x_e(t-\tau(t)) \\ &\le \varepsilon_3 x_e^T(t)P\bar{G}\bar{G}^T Px_e(t) \\ &\quad + \varepsilon_3^{-1} x_e^T(t-\tau(t))\sum_{i=1}^{16} h_i\Delta \bar{A}_{di}^T x_e(t-\tau(t)) \\ &\le \varepsilon_3 x_e^T(t)P\bar{G}\bar{G}^T Px_e(t) \\ &\quad + a_d^2\varepsilon_3^{-1} x_e^T(t-\tau(t))x_e(t-\tau(t)). \end{aligned} \tag{36}$$

By (35), (31), (36), we make

$$Q = \varepsilon_2^{-1}\sum_{i=1}^{r} h_i \bar{A}_{di}^T \bar{A}_{di} + a_d^2\varepsilon_3^{-1} I > 0.$$

Therefore

$$\begin{aligned}\dot{V}(x) &\le x_e^T(t)\sum_{i=1}^{16} h_i[2P\bar{A}_i + \varepsilon_2^{-1}\bar{A}_{di}^T\bar{A}_{di} + a_d^2\varepsilon_3^{-1} I \\ &\quad + \varepsilon_1 P\bar{G}\bar{G}^T P + a^2\varepsilon_1^{-1} I + \varepsilon_2 PP + \varepsilon_3 P\bar{G}\bar{G}^T P]x_e(t) \\ &= x_e^T(t)\Gamma x_e(t). \end{aligned}$$

According to the proving process of Theorem 1, (34) is obtained LMI.

IV. SMC LAWS DESIGN

In this section, we will design two new SMC rules to ensure the establishment of reachability conditions.

Theorem 3: For appropriate matrices G, T_1 and T_2 as shown previously, the reachability condition can be guaranteed by the SMC law

$$\begin{aligned} v(t) &= (GB)^{-1}\sum_{i=1}^{16} h_i(\theta)\{GA_iT_2e_y(t) \\ &\quad + GA_{di}T_2e_y(t-\tau(t)) + \rho_i\frac{s(t)}{\|s(t)\|}\}. \end{aligned} \tag{37}$$

where

$$\begin{aligned}\rho_i &= (\|GA_iT_1E\| + a\|G\| + \|GA_{di}T_1E\| + a_d\|G\|)\lambda \\ &\quad + a\|G\|\left\|\hat{x}(t)\right\| + a_d\|G\|\left\|\hat{x}(t-\tau(t)\right\| + \mu_0. \end{aligned} \tag{38}$$

with μ_0 is a positive constant.

Proof: If the LMI in Theorem 1 is solvable, then it implies that $e(t)$ is bounded, and for some small $\lambda > 0$, we have $\sup_{0\le t<\infty}\|e(t)\| \le \lambda$. Choosing the Lyapunov function $V_s(t) = \frac{1}{2}s^T(t)s(t)$ and taking the derivation of $V_s(t)$, then

$$\begin{aligned}\dot{V}(s) &= s^T(t)\dot{s}(t) \\ &= s^T(t)\sum_{i=1}^{16} h_i(\theta)\{GA_ie(t) + GA_{di}e(t-\tau(t)) \\ &\quad + G\Delta A_i x(t) + G\Delta A_{di}x(t-\tau(t)) - GBv(t)\}. \end{aligned} \tag{39}$$

By substituting $x(t) = \hat{x}(t) + e(t)$ into (39) and using $T_1E + T_2C = I_n$, we can obtain the equation as follows:

$$\begin{aligned}\dot{V}_s(t) &= s^T(t)\sum_{i=1}^{16} h_i(\theta)\{GA_i(T_1E+T_2C)e(t) + G\Delta A_ie(t) \\ &\quad + GA_{di}(T_1E+T_2C)e(t-\tau(t)) + G\Delta A_{di}e(t-\tau(t)) \\ &\quad + G\Delta A_i\hat{x}(t) + G\Delta A_{di}\hat{x}(t-\tau(t)) - GBv(t)\}. \end{aligned} \tag{40}$$

Then from (37), we get

$$\dot{V}_s(t) = s^T(t)\sum_{i=1}^{16} h_i(\theta)\{GA_iT_1Ee(t)+G\Delta A_ie(t) + GA_{di}T_1Ee(t-\tau(t))+G\Delta A_{di}e(t-\tau(t))+G\Delta A_i\hat{x}(t) + G\Delta A_{di}\hat{x}(t-\tau(t))\} - s^T(t)\sum_{i=1}^{16} h_i(\theta)\rho_i\frac{s(t)}{\|s(t)\|}. \tag{41}$$

Thus, we have

$$\dot{V}_s(t) \le \|s(t)\|\sum_{i=1}^{16} h_i(\theta)\{(\|GA_iT_1E\| + a\|G\| + \|GA_{di}T_1E\| + a_d\|G\|\lambda)+a\|G\|\left\|\hat{x}(t)\right\|+a_d\|G\|\left\|\hat{x}(t-\tau(t))\right\|\} - \|s(t)\|\sum_{i=1}^{16} h_i(\theta)\rho_i. \tag{42}$$

Substituting (38) into (42), we get

$$\dot{V}(t) \le -\mu_0\|s(t)\|, \quad \forall\|s(t)\| \ne 0. \tag{43}$$

which implies that the reachability condition can be guaranteed, thus ends the proof.

Theorem 4: For appropriate matrix G, T_1 and T_2 as shown previously, the reachability condition can be guaranteed by the SMC law:

$$U(t) = \sum_{i=1}^{16} h_i(\theta)\{Q_i\hat{x}(t) - (G_1B)^{-1}G_1A_{di}\hat{x}(t-\tau(t)) - (GB)^{-1}GA_iT_2e_y(t) - (GB)^{-1}GA_{di}T_2e_y(t-\tau(t)) - (GB)^{-1}\alpha_i\frac{s(t)}{\|s(t)\|}\}. \tag{44}$$

where

$$\alpha_i = \|G_1B\|\left\|(GB)^{-1}\right\|\rho_i + \mu. \tag{45}$$

with μ_1 is a positive constant.

Proof: Choosing the Lyapunov function $V_{s_1}(t) = \frac{1}{2}s_1^T(t)s_1(t)$ and taking the derivation of $V_{s_1}(t)$, we can get

$$\dot{V}_{s_1}(t) = s_1^T(t)\dot{s}_1(t) = s_1^T(t)\sum_{i=1}^{16} h_i(\theta)\{-G_1BQ_i\hat{x}(t) + G_1A_{di}\hat{x}(t-\tau(t)) + G_1BU(t) + G_1Bv(t)\}. \tag{46}$$

Substituting (37) and (44) into (46), we have

$$\dot{V}_{s_1}(t) = s_1^T(t)\sum_{i=1}^{16} h_i(\theta)\{G_1B)(GB)^{-1}\rho_i\frac{s(t)}{\|s(t)\|}\} - s_1^T(t)\sum_{i=1}^{16} h_i(\theta)\alpha_i\frac{s_1(t)}{\|s_1(t)\|} \le \|s_1(t)\|\sum_{i=1}^{16} h_i(\theta)\{\|G_1B\|\left\|(GB)^{-1}\right\|\rho_i - \alpha_i. \tag{47}$$

FIGURE 1. **Water hyacinth infestation.**

Then from (45), we get

$$\dot{V}_{s_1}(t) \le -\mu_1\|s_1(t)\|, \quad \forall\|s_1(t)\| \ne 0. \tag{48}$$

which implies that the reachability condition can be guaranteed, thus ends the proof.

V. SIMULATION EXAMPLE

Biological invasion is a hot topic of concern to the international community. As a major trading country, China has become one of the most biologically invasive countries in the world. It is reported that there are more than 620 alien invasive species in China. The international union for conservation of nature has named the world's 100 most threatening alien species. 51 species have been found in China, causing more than 200 billion yuan in economic losses.

Case: Water hyacinth

Water hyacinth, originated in Brazil, is now widely distributed in the Yangtze River, Yellow River Basin and South China provinces. Although water hyacinth itself has a strong ability to purify sewage, a large number of water hyacinths cover the river surface, which is easy to cause water quality deterioration and affects the growth of underwater organisms. Water hyacinth reproduction speed is very fast, growth will consume a lot of dissolved oxygen, almost become a synonym for "pollution."

In order to facilitate the calculation, the selected data is in units of 1000.

First, we selected the following parameters in some news reports:

$$r = 0.05, a_1 = 0.01, b = 0.02, c_1 = 0.01, a_2 = 0.01, c_2 = 0.01,$$
$$\eta = 0.02, a = 0.06, \beta = 0.04,$$
$$c = 1h = 0.001, \theta = 0.01, w = 0.02, p = 4$$

Then we can get the system:

$$\dot{x}_1(t) = (0.05 + \Delta r(t))x_2(t) - 0.01x_1(t) - (0.02 + \Delta b(t))x_1(t) - 0.01x_1^2(t) \tag{49}$$
$$\dot{x}_2(t) = (0.02 + \Delta b(t))x_1(t) - 0.02x_2(t) - E_1(t)x_2(t) - 0.01x_2^2(t)\,0.02x_2(t)x_3(t-\tau(t))$$
$$\dot{x}_3(t) = (0.06 + \Delta a(t))x_3(t-\tau(t)) - 0.001x_4(t) + u(t)$$

$$\dot{x}_4(t) = (0.04 + \Delta\beta(t))\,x_3(t - \tau(t)) - 0.01x_2(t) - 0.02x_4(t)$$
$$0 = E_1(t)(x_2(t)4 - 1) - m(t). \tag{50}$$

Based on the actual situation, we have established the following inequalities

$$0 \le x_1 \le 25, \quad 0 \le x_2 \le 25,\ 0 \le x_3 \le 25,\ 0 \le x_4 \le 25,$$
$$0 \le E_1 \le 25, \quad \Delta r(t) = \Delta b(t) = \Delta a(t) = \Delta\beta(t) = \sin(t).$$

Calculating the maximum and minimum values, $z_1(t)$, $z_2(t)$, $z_3(t)$ and $z_4(t)$ are obtained as follows:

$$\max z_1(t) = -0.03,\ \min z_1(t) = -0.28,\ \max z_2(t) = -0.01,$$
$$\min z_2(t) = -0.26,\ \max z_3(t) = 0,\ \min z_3(t) = -25,$$
$$\max z_4 = 100,\ \min z_4 = 0.$$

Finally, we establish the following fuzzy models:

$$E\dot{x}(t) = \sum_{i=1}^{16} h_i(\theta)\{(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - \tau(t)) + BU(t)\}. \tag{51}$$

$$M_{11} = \frac{-0.03 - z_1}{0.25},\quad M_{12} = \frac{z_1 + 0.28}{0.25},\quad M_{21} = \frac{-0.01 - z_2}{0.25},$$
$$M_{22} = \frac{z_2 + 0.26}{0.25},\quad M_{31} = \frac{-z_3}{25},\quad M_{32} = \frac{z_3 + 25}{25},$$
$$M_{41} = \frac{100 - z_4}{100},\quad M_{42} = \frac{z_4}{100}$$

$$A_1 = \begin{bmatrix} -0.03 - \sin(t) & 0.05 + \sin(t) & 0 & 0 & 0 \\ 0.02 + \sin(t) & -0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.001 & 0 \\ 0 & -0.01 & 0 & -0.02 & 0 \\ 0 & 100 & 0 & 0 & -1, \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.28 - \sin(t) & 0.05 + \sin(t) & 0 & 0 & 0 \\ 0.02 + \sin(t) & -0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.001 & 0 \\ 0 & -0.01 & 0 & -0.02 & 0 \\ 0 & 100 & 0 & 0 & -1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.28 - \sin(t) & 0.05 + \sin(t) & 0 & 0 & 0 \\ 0.02 + \sin(t) & -0.26 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.001 & 0 \\ 0 & -0.01 & 0 & -0.02 & 0 \\ 0 & 100 & 0 & 0 & -1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -0.28 - \sin(t) & 0.05 + \sin(t) & 0 & 0 & 0 \\ 0.02 + \sin(t) & -0.26 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.001 & -25 \\ 0 & -0.01 & 0 & -0.02 & 0 \\ 0 & 100 & 0 & 0 & -1 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} -0.03 - \sin(t) & 0.05 + \sin(t) & 0 & 0 & 0 \\ 0.02 + \sin(t) & -0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.001 & -25 \\ 0 & -0.01 & 0 & -0.02 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.06 + sin(t) & 0 & 0 \\ 0 & 0 & 0.04 + sin(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0.06 + sin(t) & 0 & 0 \\ 0 & 0 & 0.04 + \sin(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T,$$

$$C = \begin{bmatrix} -0.5 & 0 & 1 & -2 & 1 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

The time-varying delay $\tau(t) = 0.3 + 0.2\sin(t)$. Thus, $0.1 \le \tau(t) \le 0.5$, $\tau_m = 0.1$, $\tau_M = 0.5$ and $\dot{\tau}(t) \le \tau_d = 0.2 < 1$. Then given the initial condition $\varphi(t) = \phi(t) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1.5 \end{bmatrix}^T$, we get the state variable of the system (51), as shown in Figure 2–6.

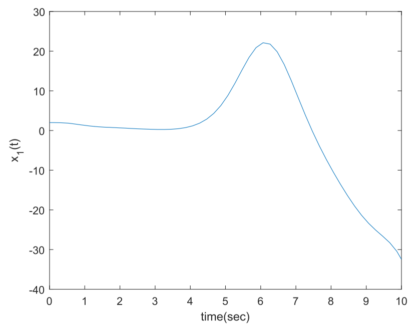

FIGURE 2. Response of state $x_1(t)$.

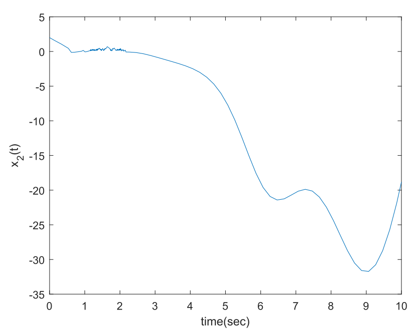

FIGURE 3. Response of state $x_2(t)$.

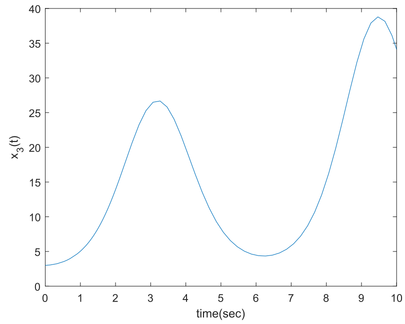

FIGURE 4. **Response of state x_3 (t).**

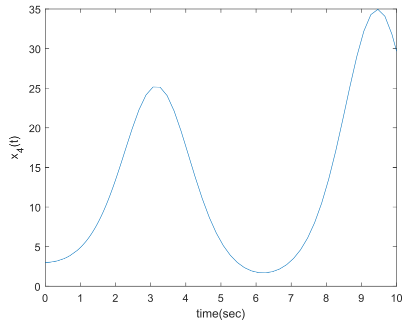

FIGURE 5. **Response of state x_4 (t).**

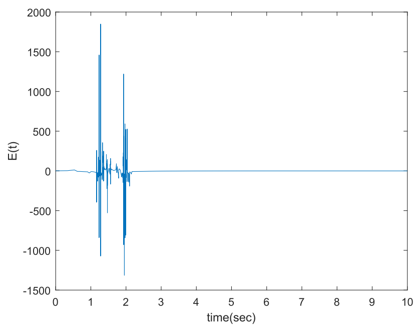

FIGURE 6. **Response of state E_1 (t).**

From Fig. 2-6, we can find that the state variables of the unforced system (51) are not stable.

We set

$$G=\begin{bmatrix}1 & 0 & 1 & 2 & 0\end{bmatrix},\quad G_1=\begin{bmatrix}1 & 0 & 1 & 0 & 0\end{bmatrix}.$$

then $GB = 1$ and $G_1B = 1$ are nonsingular. Furthermore, we set

$$Q_1=\begin{bmatrix}-1 & 2 & -1 & 0 & -1\end{bmatrix},$$
$$Q_2=\begin{bmatrix}1 & 0 & -1 & 0 & 2\end{bmatrix},$$
$$Q_3=\begin{bmatrix}1 & -2 & -1 & -1 & -1\end{bmatrix}.$$

By solving the inequality (34), a feasible solution can be obtained

$$L_1=\begin{bmatrix}-0.8729 & 0.8680\\ 5.7762 & 0.7128\\ 1.7410 & -1.7211\\ -2.2043 & 2.4112\\ -6.5270 & 16.8446\end{bmatrix},$$
$$L_2=\begin{bmatrix}-0.8337 & 0.8517\\ 5,8042 & 0.6615\\ 1.7440 & -1.7300\\ -2.2122 & 2.4086\\ -6.5079 & 16.8325\end{bmatrix},$$
$$L_3=\begin{bmatrix}-0.8251 & 0.8355\\ 5.7723 & 0.7797\\ 1.7268 & -1.6972\\ -2.2153 & 2.3951\\ -6.3845 & 16.6911\end{bmatrix}$$

As we said above, our purpose was to design SMC laws $v(t)$ and $U(t)$. Therefore, taking the matrices

$$T_1=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 0.5 & 0 & 0 & 0\\ -1 & 0 & -0.5 & 0 & 0\\ 0 & 1 & 0 & 1 & 0\\ 0 & 0.5 & 1 & 0 & 0\end{bmatrix},\quad T_2=\begin{bmatrix}0 & 0\\ 0 & 0.25\\ -2 & 4\\ 0 & 0.5\\ 0 & -0.25\end{bmatrix}$$

we get the state variable of the error system, as shown in Figure 7–10.

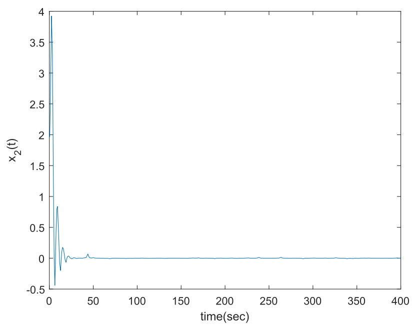

FIGURE 7. **Response of state $\hat{x}_1$.**

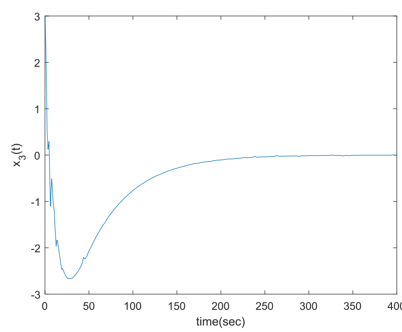

FIGURE 8. **Response of state $\hat{x}_2$.**

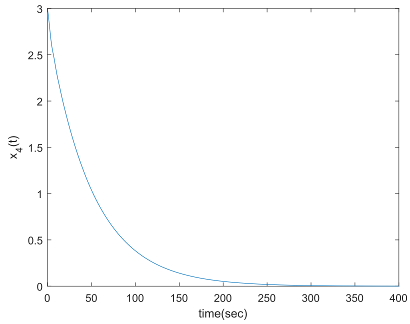

FIGURE 9. Response of state $\hat{x}_3$.

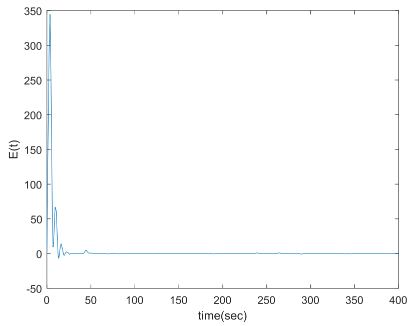

FIGURE 10. Response of state $E(t)$.

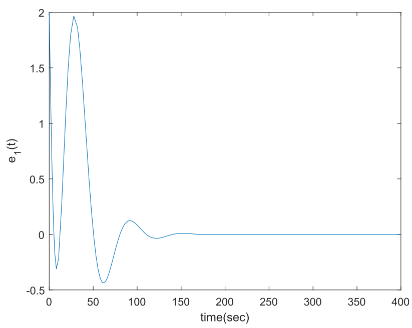

FIGURE 11. Estimation error variable $e_1(t)$.

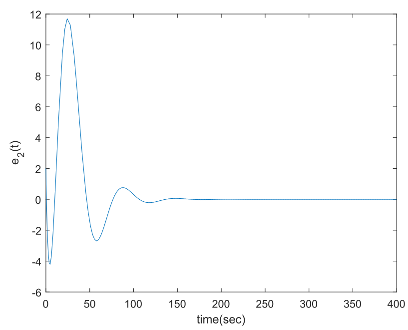

FIGURE 12. Estimation error variable $e_2(t)$.

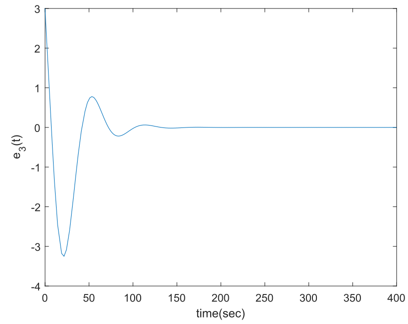

FIGURE 13. Estimation error variable $e_3(t)$.

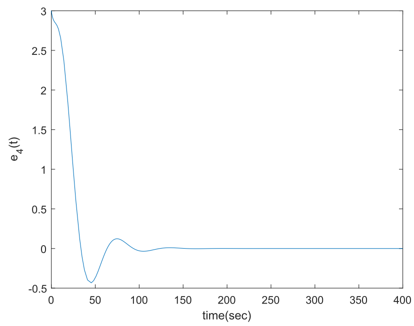

FIGURE 14. Estimation error variable $e_4(t)$.

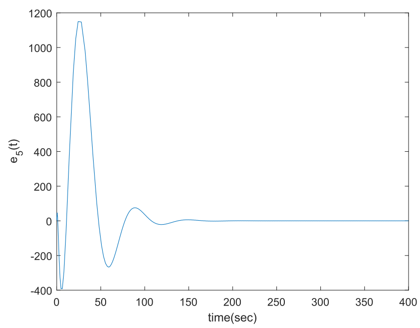

FIGURE 15. Estimation error variable $e_5(t)$.

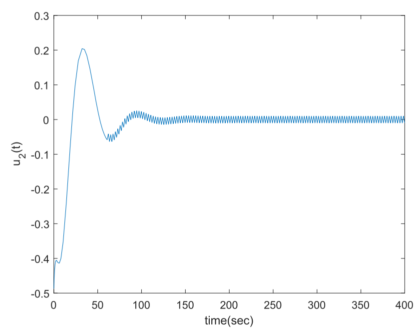

FIGURE 16. Sliding mode controller $v(t)$.

From Fig. 7-10, we can find that the state variables of error system converge to zero quickly. In other words, the observer is asymptotically stable.

From Fig. 11-15, we can find that the state variables of the error system converge to zero quickly. Therefore the proposed robust SMO based SMC scheme effectively guaranteed the

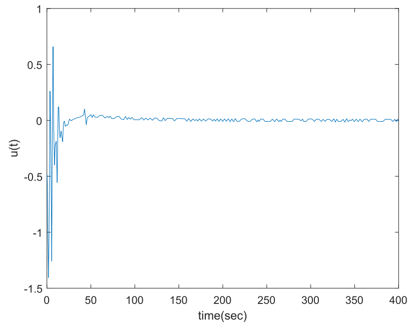

FIGURE 17. Sliding mode controller $U(t)$.

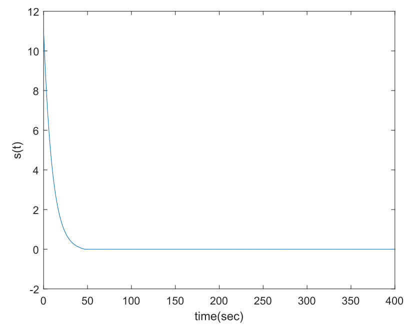

FIGURE 18. Sliding surface $s(t)$.

stability of the error system. That is to say, the observer is feasible.

Fig. 16-18 respectively plot the response of SMC laws $U(t)$, $v(t)$ and $s(t)$ sliding surfaces.

VI. CONCLUSION

In this paper, the design of sliding mode observer based control for singular biological economic model with stage structure and uncertain parameters is studied. Two novel integral sliding surfaces are constructed for the error system and the SMO system. Then some sufficient conditions have been obtained, which guarantees the admissibility of the sliding mode dynamics. Moreover, two SMC laws are presented such that the reachability conditions can be guaranteed respectively. Finally, the effectiveness of the results is illustrated by simulation examples.

From a biological point of view, our proposed SMO-based SMC method can effectively control the population of invasive alien species and protect local species to a certain extent. Therefore, this paper has made certain contributions to the introduction of alien species.

So far, we still need to invest a lot of human and financial resources to control the invasion of alien species. In the future, we will study how to turn waste into treasure so that invasive alien species can be used locally and bring benefits to the local community.

COMPETING INTERESTS

The authors declare that they have no competing interests.

AUTHOR'S CONTRIBUTIONS

The authors have achieved equal contributions. Both authors read and approved the manuscript.

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