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Multi-Scale Quantum Harmonic Oscillator Algorithm With Centroid Motion for Global Numerical Optimization Problems

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ABSTRACT This paper proposes a new approach for function optimization using a new variant of multi-scale quantum harmonic optimization algorithm (MQHOA). The new approach introduces a centroid motion to improve the convergence efficiency, which is called MQHOA with centroid motion (CM-MQHOA). Instead of replacing the worst particle by the current best individual in the quantum harmonic oscillator process in MQHOA, the weakest player is replaced by a current centroid position in the proposed algorithm. Simple mechanisms are added to maintain the diversity of the population and help achieve the global optima in difficult unimodal and multimodal search spaces. The benefits of the proposed algorithm are improved performance in terms of effectiveness, reliability, accuracy, and efficiency. The approach appears to be able to efficiently deal with several unimodal and multimodal benchmark functions. A variety of standard benchmark functions are used to illustrate the proposed approach. The experimental results are compared with several state-of-the-art optimization algorithms. The comparative results indicate the competitiveness of the proposed algorithm and suggest a viable and attractive addition to the portfolio of computational intelligence techniques.

INDEX TERMS Multi-scale quantum harmonic oscillator algorithm, heuristic algorithm, population-based algorithm, global optimization, centroid motion.

I. INTRODUCTION

Global optimization problems universally exist in the real-world scenarios, such as maximization of benefit or minimization of cost in science, engineering and business. These issues are often complicated or even NP hard. In the past decades, global optimization problems have broadly aroused interest of researchers to develop a large number of optimization algorithms. Typically, these approaches can be categorized as deterministic (direct search) algorithms [1], [2] and stochastic algorithms [3], [4]. Deterministic algorithms obtain the same output when given the same input and locate to the global optimum with a level of assurance. Most determined methods like coordinate search algorithm [2] and branch and bound [5] are local search techniques which are time-consuming and cannot prevent from falling into local optima.

Although stochastic algorithms output different solutions with the same input, they obtain results with an acceptable calculation error within polynomial time [6]. Among stochastic algorithms, nature inspired heuristic algorithms are compelling for their remarkable robust, fast convergence, parallelism and their global search capability [7]–[9]. Typical heuristics include single solution heuristics such as Simulated Annealing (SA) [10], [11], and population-based heuristics like Genetic Algorithm (GA) [12], Ant Colony Optimization (ACO) [13], Artificial Bee Colony (ABC) [14], [15], Bat Algorithm (BA) [16], Harmony Search (HS) [17], Fireworks Algorithm (FA) [18] and Particle Swarm Optimization (PSO) [19], [20].

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For researchers, pursuing better performance of an algorithm never stops. Generally, improvements of an algorithm may be summarized as hybridising of some algorithms, parameter regulation and introducing new mechanisms into the original algorithms. For instance, quantum behavior is integrated with classic heuristic algorithms to perform efficient parallel computation [21]. These algorithms include Quantum-inspired Differential Evolution (QDE) [22], Quantum-inspired Genetic Algorithm (QGA) [23], Quantum-inspired Ant Colony Optimization (QACO) [24], Quantum-inspired Artificial Fish Swarm Algorithm (QAFSA) [25], Quantum-behaved Particle Swarm Optimization (QPSO) [26], [27], and Multi-scale Quantum Harmonic Oscillator Algorithm (MQHOA) [28], [29] and etc.

MQHOA is a population-based heuristic algorithm proposed recently and proved effective in dealing with unimodal and multimodal problems [29], [30]. However, there are defects in the original MQHOA. For one thing, it adopts the mechanism to substitute the worst solution with the current optimum in each iteration cycle, which will weaken the diversity of particles. For another, particles in MQHOA repeatedly explore promising areas one after another which is time-consuming and inefficient. In this paper, a centroid motion mechanism is introduced into MQHOA to improve the optimization performance. The proposed approach is evaluated from different aspects and compared with several state-of-the-art algorithms. Experiments are carried out by evaluating the algorithms on several well-defined benchmark functions from IEEE Congress on Evolutionary Computation [31], [32]. The computational results indicate the competitiveness of the proposed algorithm. The contribution of the proposal can mainly be summarized as follows:

First, a new Multi-scale Quantum Harmonic Oscillator Algorithm with centroid motion (CM-MQHOA) is proposed. Second, the efficiency of convergence process is validated by comparing the proposed algorithm with the original MQHOA, the trajectory of their convergence path indicates the significant improvement of CM-MQHOA. Third, a fallback mechanism is proposed and added to the proposed algorithm to jump out local optima. Forth, the convergence process is theoretically analysed and proved. Last but not least, the proposed algorithm is sufficiently evaluated on several unimodal and multimodal benchmark functions, and the experiments are compared with several state-of-the-art optimization algorithms. The comparative results indicate the competitiveness of the proposed algorithm.

The remainder of this paper is organized as follows: Section II briefly states the mathematical and physical background of MQHOA, followed by the introduction of the proposed algorithm in Section III. Then, Section IV extensively evaluates the effectiveness and efficiency of CM-MQHOA by testing it on several well defined benchmark functions and comparing it with several state-of-the-art algorithms. Finally, Section V outlines the conclusion and our research work in the near future.

II. OVERVIEW OF MQHOA

A. THEORETICAL BACKGROUND

An optimization problem $f(x)$ in this paper is designated as follows.

$$Minimize \quad f(x) \quad subject \quad to \quad x_i \in [x_l, x_u]^D \tag{1}$$

where $f(x)$ is an objective function, x_i is a decision variable, x_l and x_u are the lower and upper bounds for each decision variable, D is the dimension of the decision variable. In quantum system, the motion of particles can be defined by Schrödinger equation as follows [33], [34].

$$E\psi(x) = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi(x) \tag{2}$$

(2) is an eigenvalue equation, where E is the system energy of stationary state $\psi(x)$, $\psi(x)$ is probability amplitude and $|\psi|^2$ designates the probability distribution of the particles in the quantum space. $\hbar = h/2\pi$ (h is the Planck constant), $V(x)$ is the potential energy and a bound in the quantum space.

Inspired by quantum theory [34], [35] and quantum annealing method [36], the main idea of MQHOA is that the process of solving an optimization problem $f(x)$ can be regarded as particles in quantum system transferring from high energy levels to the ground state under a potential well $V(x)$ [29].

Correspondingly, the probability of finding the global optimum in the evaluation of an optimization problem is deemed as the probability of locating to the ground state in quantum system. As wavefunction in quantum physics reflects the probability density of particles appear in the ground state [33], the probability of particles appears in the quantum space can be demonstrated as follows [29].

$$|\psi_n(x)|^2 = \frac{1}{2^n n!}(\frac{m\omega}{\pi\hbar})^{\frac{1}{2}} \cdot exp(-\frac{m\omega x^2}{\hbar}) \cdot |H_n(\sqrt{\frac{m\omega}{\hbar}}x)|^2 \tag{3}$$

When $n \to 0$, (3) is equal to:

$$|\psi_0|^2 = (\frac{m\omega}{\pi\hbar})^{\frac{1}{2}} \cdot exp(-\frac{m\omega x^2}{\hbar}) \tag{4}$$

where (4) is a form of Gaussian equation:

$$\psi(x) = \frac{1}{\sqrt{2\pi}\sigma}exp(-\frac{(x-\mu)^2}{2\sigma^2}) \tag{5}$$

Accordingly, for n particles, (5) can be rewritten as follows:

$$\psi_n(x) = \sum_{i=1}^{n}\psi(i) = \sum_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}exp(-\frac{(x_i-\mu_i)^2}{2\sigma^2}) \tag{6}$$

where μ is the mean value of the optimal solutions, σ is the standard deviation of the current optimal solutions. The smaller the σ is, the narrower the search space will be.

It can be seen in (3) and (4), from high energy levels to the ground state, the wavefunction of quantum harmonic oscillator changes from n scattered and intertwined Gaussian functions in (3) to an overlapped Gaussian function in (4).

Wavefunction plays an important role in quantum-behaved algorithms. In QPSO, the ground state wavefunction δ potential well works as a sampling probability density function [27]. While in MQHOA wavefunction of harmonic oscillator potential is employed as a sampling probability density function. Wavefunction reflects the convergence process of the proposed algorithm. For simple object functions, few iterations are needed for particles to jump from high energy levels to the ground state. While for sophisticated functions, a large number of running generations will be required.

B. FRAMEWORK OF MQHOA

The structure of MQHOA is concise, including quantum harmonic oscillator process (QHO process) and multi-scale process (M process). In QHO process, particles explore new neighbor fields of current optimal positions to exploit better optimal solutions. While in M process, the search domain is narrowed by half. The framework of MQHOA is demonstrated in Fig. 1 [29].

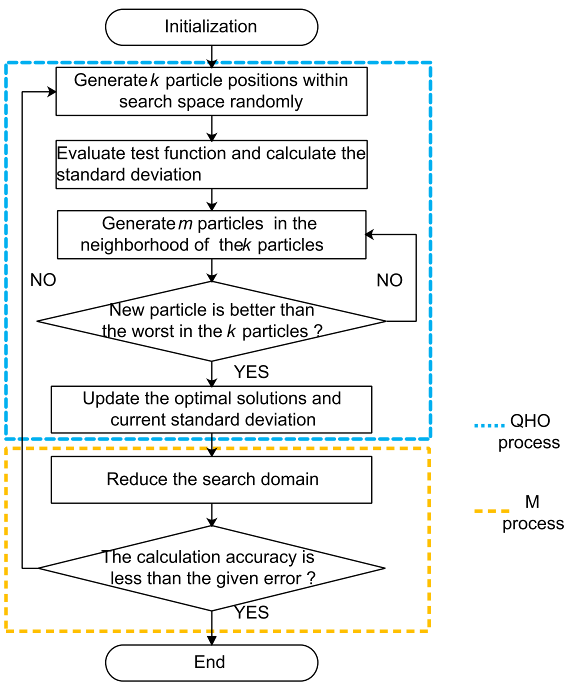


FIGURE 1. Framework of Multi-scale Quantum Harmonic Oscillator Algorithm.

In Fig.1, for each generation of convergence within the current search domain, elitist strategy is employed to obtain the present optimal solutions. When all of the particles finish exploration and exploitation within an iteration cycle, the worst performer (particle with the largest fitness value) will be replaced by the particle with the fittest value. Then the standard deviation of the current optimal solutions is calculated and compared with the current search length (distance between the current upper bound and the lower bound). If the current standard deviation is less than the current search domain, the search domain will be reduced by the declining coefficient λ. Otherwise, the algorithm will return to generate new neighbour solutions for each particle. The whole optimization process ceases when the given stopping criteria are satisfied.

III. MQHOA WITH CENTROID MOTION

In MQHOA, the worst position in each iteration is replaced by the current best solution. This will weaken the diversity of the particles, because the current best solution may not be changed within several iterations. In this case, the worst position in each iteration process will be substituted by the same solution. In this section, we introduce an improved MQHOA with centroid motion (CM-MQHOA) to enhance the diversification of the particles and reduce the total iteration time.

A. MAIN IDEA

Centroid is the equilibrium point of a geometric figure or mass centre of a physical object. In mathematics, centroid is the geometric center of a plane figure [37]. In physics, centroid is the center of mass which is the arithmetic mean of all points weighted by the local density [37]. As a matter of fact, centroid motion is universally applied in global optimization [38]–[40]. The main idea of CM-MQHOA in this paper is to utilize the centroid of the propulation to guide an efficient exploration process toward the global optimum.

For a symmetric object, given k points in a search region $M \subseteq \mathbb{R}^N$ and a density function ρ, defined in M, the mass centroid y^* of M is defined as follows [38].

$$y^* = \frac{\int_M x\rho(x)dx}{\int_M \rho(x)dx} \tag{7}$$

where x is an independent variable. (7) indicates that the centroid of a continuous region is a combined action of all the particles. Meanwhile, for a finite k points in the continuous region M, the centroid can be defined as follows.

$$y^* = \frac{\sum_{i=1}^{k} m_i y_i}{\sum_{i=1}^{k} m_i} \tag{8}$$

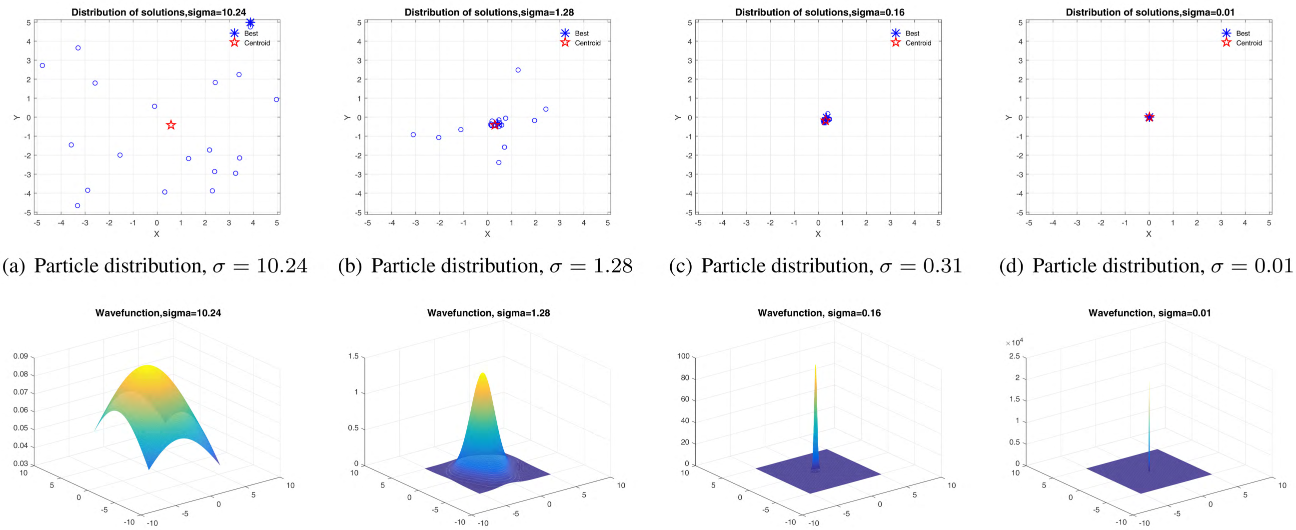
In CM-MQHOA, we apply the centroid to balance the particles in the search region.

B. MQHOA WITH CENTROID MOTION

In MQHOA, the trajectory of convergence is conducted by replacing the worst solutions with the current best ones. This process can be described as follows.

$$X|f_{worst} \leftarrow X|f_{best} \tag{9}$$

where $X = x_1, \ldots, x_d$, d is the dimension of the variable. As the $X|f_{best}$ may not be changed in several iteration cycles, there will be X_i, $(1 \leq i \leq k)$ changed by the same optimal solution. That will weaken the diversity of the particles. Meanwhile, current optimal solution in each iteration cycle obtains very little information from other particles, which will result in inefficiency of exploration and exploitation. In order



(a) Particle distribution, $\sigma = 10.24$ (b) Particle distribution, $\sigma = 1.28$ (c) Particle distribution, $\sigma = 0.31$ (d) Particle distribution, $\sigma = 0.01$

(e) Wavefunction, $\sigma = 10.24$ (f) Wavefunction, $\sigma = 1.28$ (g) Wavefunction, $\sigma = 0.31$ (h) Wavefunction, $\sigma = 0.01$

FIGURE 2. Wavefunction in the course of function evaluation.

to overcome these shortcomings, we apply the centroid of the particles to guide the convergence process toward the global optimal solution.

In CM-MQHOA, given k points $y_i, i = 1, \ldots, k$, we can define their associated exploration regions $\hat{M}_i, i = 1, \ldots, k$. Meanwhile, given the search regions $\hat{M}_i, i = 1, \ldots, k$, we can define their mass centroid $y_i^*, i = 1, \ldots, k$. As the particles $y_i, i = 1, \ldots, k$ serves as the generators, the mass centroids of the particle are themselves in the search region $\hat{M}_i$. In this case, we get

$$y_i^* = y_i \tag{10}$$

In CM-MQHOA, the mass of particle is ignored, the potential energy is considered. Analogy to (8), we define the centroid of CM-MQHOA in an iteration cycle as follows.

$$y^* = \frac{\sum_{i=1}^{k} x_i e^{-f_i(x)}}{\sum_{i=1}^{k} e^{-f_i(x)}} \tag{11}$$

where x_i is the variable in the search region, $f_i(x)$ is the fitness value of the ith particle, $e^{-f_i(x)}$ is defined as the *Potential Energy*, which will be decreasing as the fitness value $f_i(x)$ increases. Since the $e^{-f_i(x)}$ is not less than zero, the denominator of (11) is assured not to be zero.

C. WAVEFUNCTION

According to (6), the wavefunction of the population reflects the probability of appearance of particles in the search domain. As centroid represents the collective wisdom to some extent, the centroid motion will theoretically help the population move towards the global optimal landscape. As an example, Fig. 2 illustrates the variation of wavefunctions in the course of evaluating a benchmark function (Ackley, 100-dimensional).

As seen in Fig. 2, the figures in the first row depict the distribution of particles in the course of function evaluation. In 2-D coordinate system, the centroid of the population is closer to the global optimal solution (0,0) compared with the fittest particle (with the smallest fitness value). As the search domain narrows, the particles constantly aggregate to the global optimum. Correspondingly, the second row reveal the wavefunction of the population moving towards the global optimal area. As sigma decreases, the figure of wavefunction shrinks from a flat bell-shaped figure Fig.2(e) to a needle-shaped figure Fig.2(h).

D. CONVERGENCE OF CM-MQHOA

According to Chebyshev's inequality [41], for a random variable with finite mathematical expectation $E(X) = \mu$ and variance $D(X) = \sigma^2$, the probability of the variable can be written as:

$$P(|X - \mu| < \varepsilon) \geqslant 1 - \frac{\sigma^2}{\varepsilon^2} \tag{12}$$

where ε is a small arbitrary and positive number. In CM-MQHOA, μ is the best position $\vec{x}$ at current search domain, σ is the standard deviation of the best sampling points, ε in (12) is the computation accuracy ($\varepsilon = 1e - 6$). Then, we can rewrite (12) as follows:

$$\lim_{t \to \infty} P(|X - \vec{x}| < \varepsilon) \geqslant 1 - \frac{\sigma_s^2}{\varepsilon^2} \tag{13}$$

where t is the iteration time or iteration cycles in function evaluation. As iteration goes on, the search range σ_s will be narrowed by a reduction coefficient λ, and thus σ_s is reduced by σ_s/λ. As ε is a small positive constant, σ_s will be continuingly reduced until the ceasing condition is satisfied. In CM-MQHOA, the stopping criteria for function evaluation is met when the current solution domain σ_s is less than the computational accuracy ε. In this case, we get

$$\lim_{t\to\infty} \frac{\sigma_s^2}{\varepsilon^2} \Rightarrow 0 \tag{14}$$

That is

$$\lim_{t\to\infty} P(|X - \vec{x}| < \varepsilon) \geqslant 1 - \frac{\sigma_s^2}{\varepsilon^2} \Rightarrow 1 \tag{15}$$

(15) indicates that when the converging time is enough or the generation number is large enough, CM-MQHOA will converge to a local optimum.

E. PSEUDO CODE OF CM-MQHOA

The pseudo code of CM-MQHOA is shown in Algorithm 1.

Algorithm 1 : CM-MQHOA Pseudocode

Input: k, $X_i \in [d_l, d_u]^D$ (i=1,2,...,k), ε, λ, c
Output: the global optimum f_{best}, the optimal position X_{best}
Initialization, calculate fitness value $f_i = f(X_i)$ and the current minimum f_{best}=minimize $f(X)$
while *(* $\sigma_s > \varepsilon$*)* **do**
  while *(*$\sigma_k > \sigma_s$*)* **do**
    $\forall X_i \in X$, generate $X_i' \sim N(X_i, \sigma_s^2)$
    $\forall X_i$ and X_i', if $f(X_i') < f(X_i)$ then $X_i = X_i'$
    update X by $X_{worst} \leftarrow X_{centroid}$
    update σ_k;
    if $\sigma_k < \sigma_s$ **then**
      finish the iteration cycle
    else
      sNO=*sNO*+1
    end
    if *sNO* > *100* **then**
      initialize the X_{worst} and sigma=c*sigma
      sNO=0
    else
      continue
    end
  end
  $\sigma_s = \sigma_s/\lambda$
end

In Algorithm 1, k is the number of optimal solutions X_i $(i = 1, ..., k)$, X_i=$x_i^1, x_i^2, ..., x_i^D$, D is the dimension, $f(X_i)$ is the fitness value, λ is the factor of scale reduction, σ_s is the current standard deviation of the population fitness, d_l and d_u are the lower and upper bounds of the search domain, ε is the computational accuracy, $\varepsilon = \sigma_{min}$, $N(x_i, \sigma_s^2)$ is the Gaussian distribution. c is the expansion coefficient (in this paper c=2.0). If new generated particle is out of boundaries, it is shifted onto the boundaries.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

In this section, the effectiveness and efficiency of the proposed algorithm are evaluated. Several well-defined benchmark functions are utilized to compare the performances of CM-MQHOA with the original MQHOA. Meanwhile, several state-of-the-art methods such as Stud Genetic Algorithm (StudGA) [42], Particle Swarm Optimization version 2011 (SPSO2011) [43], Comprehensive Learning Particle Swarm Optimization (CLPSO) [44], Quantum-behaved Particle Swarm Optimization (QPSO) [45] and Artificial Bee Colony (ABC) [15] are applied to compete with CM-MQHOA and the original MQHOA.

A. TEST FUNCTIONS

In order to sufficiently validate the characteristics of the proposed algorithm, several unimodal and multimodal benchmarks are applied in the simulation. The test functions are from the benchmark suit from IEEE Congress on Evolutionary Computation [31], [32], [46]. Function f_1-f_7 are unimodal functions. Function f_8-f_{12} are multimodal functions with many local optima. The test benchmark functions are listed in Table 1.

B. PARAMETER SETTING

Parameters used in every algorithm are set as same as possible. The number of particles (population size) is defined $N_p = 20$. The maximum iteration cycle is set according to the rule used in CEC2017 [47], the maximal iteration generation is defined as $maxFE = 10000 * dimension$, e.g., if the function dimension is 100, the maximal iteration will be 1000000. The search space $[d_l, d_u]^D$ for each benchmark function is set according to Table 1. The calculation accuracy (error) is set to $\varepsilon = 0.000001$. Meanwhile, for StudGA [42], the crossover probability is defined 1.0, the number of points in each crossover is 1, the mutation rate is set 0.01, maintaining 2 best individuals from one generation to the next. For SPSO2011 [43], the inertia weight $\omega = 1/2log(2)$, the learning factor $c1 = c2 = 0.5 + log(2)$. The parameters used in CLPSO [44] are the inertia weight linearly declined from 0.9 to 0.2, the accelerate constant $c1 = c2 = 1.49445$. For QPSO [26], [27], the contraction-expansion coefficient α increases linearly from 0 to 0.5. For ABC [14], [15], the size of the food sources is set half of the colony. The probability to choose a food source is defined as follows:

$$Prob = 0.9 * Fitness/max(Fitness) + 0.1 \tag{16}$$

where *Fitness* is a vector holding fitness (quality) values associated with food sources.

Meanwhile, the stopping criteria for all of the algorithms are uniformly defined as: the computational accuracy is less than 1e-6 or the maximal running generation is larger than 10000*D. All of the algorithms are coded in Matlab R2016a and executed on the same personal computer with an Intel core(TM) i5-4200U 64 bit, 2.3 GHz and windows 7 operation system.

C. EFFECTIVENESS EVALUATION

In order to evaluate the effectiveness of the proposed algorithm, several well-defined benchmark functions are utilized as test functions. To minimize the stochastic nature of the

TABLE 1. Benchmark functions.

Function Name	Benchmark Function	D	Range	Optimum
Sphere	$f_1 = \sum_{i=1}^{n} x_i^2$	n	[-5.12,5.12]	$f(0,\ldots,0) = 0$
Sum Squares	$f_2 = \sum_{i=0}^{n-1} ix_i^2$	n	[-10,10]	$f(0,\ldots,0) = 0$
Rotated Hyper-Ellipsoid	$f_3 = \sum_{i=1}^{n}(\sum_{j=1}^{i} x_i)^2$	n	[-65.54,65.54]	$f(0,\ldots,0) = 0$
Ellipsoidal	$f_4 = \sum_{i=1}^{n}(x_i - i)^2$	n	[-100,100]	$f(1,2,\ldots n) = 0$
Sum of Different Power	$f_5 = \sum_{i=1}^{n}(\|x_i\|^{i+1}$	n	[-100,100]	$f(0,\ldots,0) = 0$
Zakharov	$f_6 = \sum_{i=1}^{n} x_i^2 + (\sum_{i=1}^{n} 0.5ix_i)^2 + (\sum_{i=1}^{n} 0.5ix_i)^4$	n	[-5,10]	$f(0,\ldots,0) = 0$
High Conditioned Elliptic	$f_7 = \sum_{i=1}^{n}(10^6)^{\frac{i-1}{n-1}} x_i^2$	n	[-10,10]	$f(0,\ldots,0) = 0$
Ackley	$f_8 = -20exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - exp(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)) + 20 + e$	n	$[-32.77, 32.77]$	$f(0,\ldots,0) = 0$
Griewank	$f_9 = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	n	[-100,100]	$f(0,\ldots,0) = 0$
Levy	$f_{10} = sin^2(\pi\omega_1) + \sum_{i=1}^{n-1}(\omega_i - 1)^2[1 + 10sin^2(\pi\omega_i + 1)] + (\omega_n - 1)^2[1 + sin^2(2\pi\omega_n)]$,where $\omega_i = 1 + \frac{x_i - 1}{4}$,for all $i = 1,\ldots,n$	n	[-10,10]	$f(1,\ldots,1) = 0$
Rastrigin	$f_{11} = 10n + \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i)]$	n	[-5.12,5.12]	$f(0,\ldots,0) = 0$
Modified Schwefel	$f_{12} = 418.9829 \times D - \sum_{i=1}^{n} g(z_i) \quad z_i = x_i + 420.9687462275036, where$	n	[-5.12,5.12]	$f(0,\ldots,0)$
	$g(z_i) = \begin{cases} z_i sin(\|z_i\|^{1/2}) & if \quad \|z_i\| \le 500 \\ (500 - mod(z_i,500))sin(\sqrt{(}\|500 - mod(z_i,500)\|) - \frac{(z_i-500)^2}{10000n} & if \quad z_i > 500 \\ (mod(\|z_i\|,500) - 500)sin\sqrt{(}\|mod(\|z_i\|,500) - 500\|) - \frac{(z_i+500)^2}{10000n} & if \quad z_i < -500 \end{cases}$			=0.000012727*D

algorithms, 51 independent trials on each function are conducted and the reported results for each function are the average values. In this subsection, the experiments includes two parts: the trajectory evaluation of the algorithms and the successful rate (SR) evaluation over 51 independent trails.

1) TRAJECTORY OF CONVERGENCE

To reveal the different convergence performances between the CM-MQHOA and the original MQHOA, we traced the trajectory of their optimal solutions in multi-dimensional plane figures in the experiments. The evaluations were conducted on well-defined benchmark functions. Without losing of generality, the dimension of the benchmarks was set 10. The trajectory pace and fitness-iteration figures are shown in Fig.3 and Fig.4.

In Fig.3 and Fig.4, the first row of the figures records the motion path of the best particles of MQHOA. The second row shows the motion trajectory of the best players of CM-MQHOA. The third row illustrates the convergence tendency of the two algorithms, and so on.

As illustrated in Fig. 3(a), in evaluation of function f_1, the motion path of particles in MQHOA is more roundabout than the trajectory of particles in CM-MQHOA in Fig. 3(e), which indicates that particles in CM-MQHOA move faster than MQHOA toward the optimal landscape. In Fig. 3(a), the fitness-iteration chart of the MQHOA is above CM-MQHOA, and the fitness-iteration graph of CM-MQHOA is much closer to the horizontal coordinate axis which indicate more efficient of CM-MQHOA in finding the global optimal solution. Similar situations happen to the rest functions.

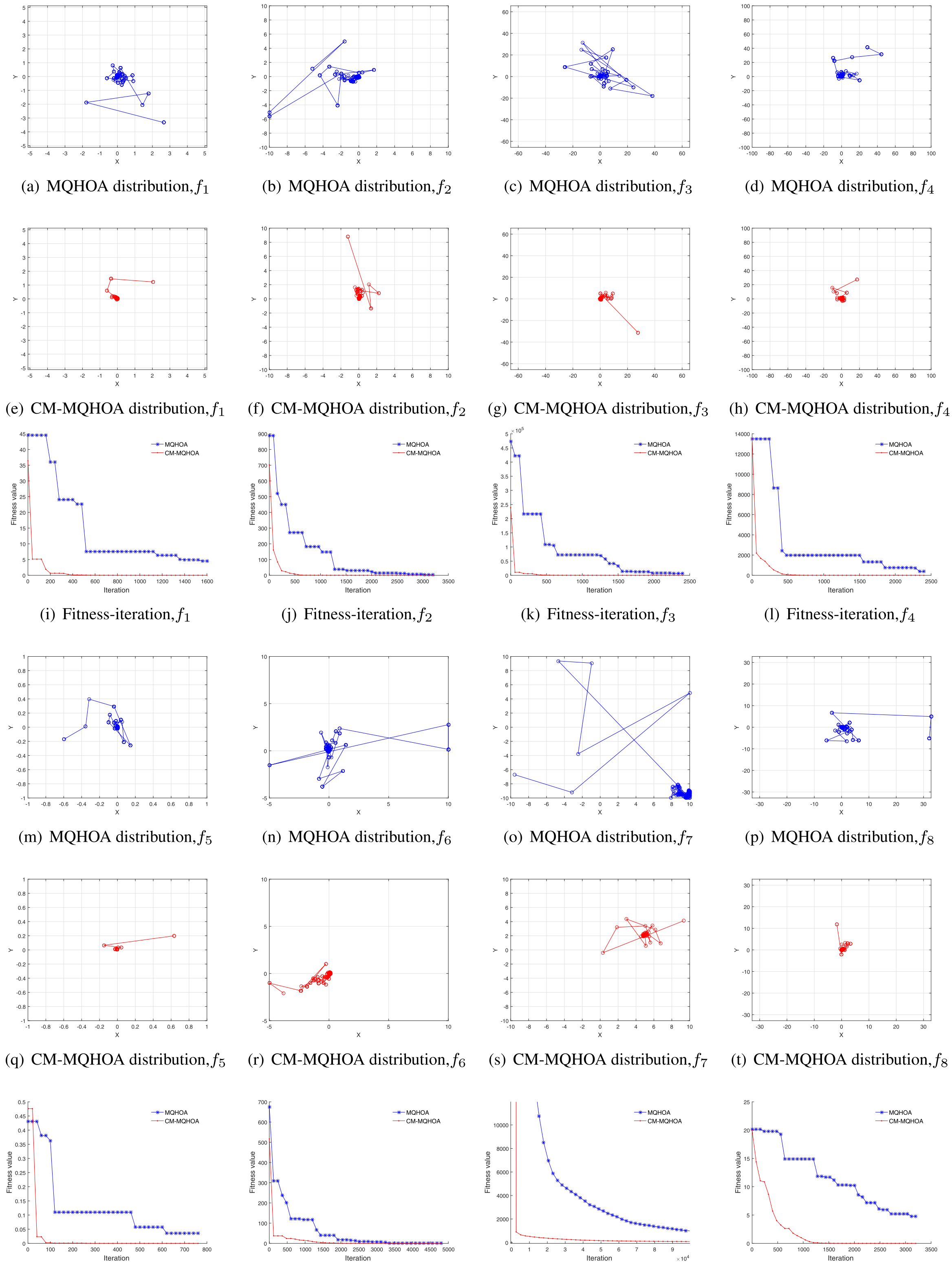
Meanwhile, in the evaluation of unimodal function f_1-f_7, though both MQHOA and CM-MQHOA are able to locate to the global optimal landscape, CM-MQHOA performs much better than MQHOA. The fitness-iteration curves of MQHOA is much higher than that of CM-MQHOA. In the evaluation of multimodal functions f_8-f_{12} the difference between MQHOA and CM-MQHOA is more significant. As shown in Fig. 3(p) and (t), in evaluation of function f_8, the trajectory of best individuals in MQHOA is more dispersive than that of CM-MQHOA. Meanwhile, Fig. 3(x) demonstrated that CM-MQHOA converges closely to horizontal axis within about 1200 iterations (function evaluations), while MQHOA is not able to converge within 3500 generations. Similarly, in the evaluation of function f_9,f_{10} and f_{12}, the CM-MQHOA converges much faster than MQHOA. In evaluation of function f_{11}, though both MQHOA and CM-MQHOA are not capable of finding the global optimum within the maximal function evaluation *maxFE*, CM-MQHOA performs much better than MQHOA, because the fitness-iteration of CM-MQHOA is much lower than that of MQHOA in Fig. 4(k).

2) SUCCESSFUL PROPORTION

In order to reveal the effectiveness and reliability of the CM-MQHOA, experiments are carried out by evaluating the successful proportions in locating to the global optimum within 51 independent trials. Meanwhile, to reveal the different performance of the proposed algorithm, experiments are carried out on 10-dimensional, 30-dimensional, 50-dimensional, 80-dimensional and 100-dimensional benchmark functions. The experimental results are compared with several state-of-the-art algorithms, such as StudGA [42], SPSO2011 [43], CLPSO [44], QPSO [45] and ABC [15]. The statistical results are displayed in Table 2.

As illustrated in Table 2, from an overall perspective, ABC algorithm performs the best among the compared techniques, obtaining 52 100%s in total, followed by CM-MQHOA, StudGA, MQHOA, SPSO2011, QPSO and CLPSO, getting 37, 33, 33, 29, 18 and 7 times of 100% respectively.

For unimodal functions f_1-f_6 and multimodal function f_{12}, both CM-MQHOA and MQHOA perform excellent,



(a) MQHOA distribution, f_1 (b) MQHOA distribution, f_2 (c) MQHOA distribution, f_3 (d) MQHOA distribution, f_4

(e) CM-MQHOA distribution, f_1 (f) CM-MQHOA distribution, f_2 (g) CM-MQHOA distribution, f_3 (h) CM-MQHOA distribution, f_4

(i) Fitness-iteration, f_1 (j) Fitness-iteration, f_2 (k) Fitness-iteration, f_3 (l) Fitness-iteration, f_4

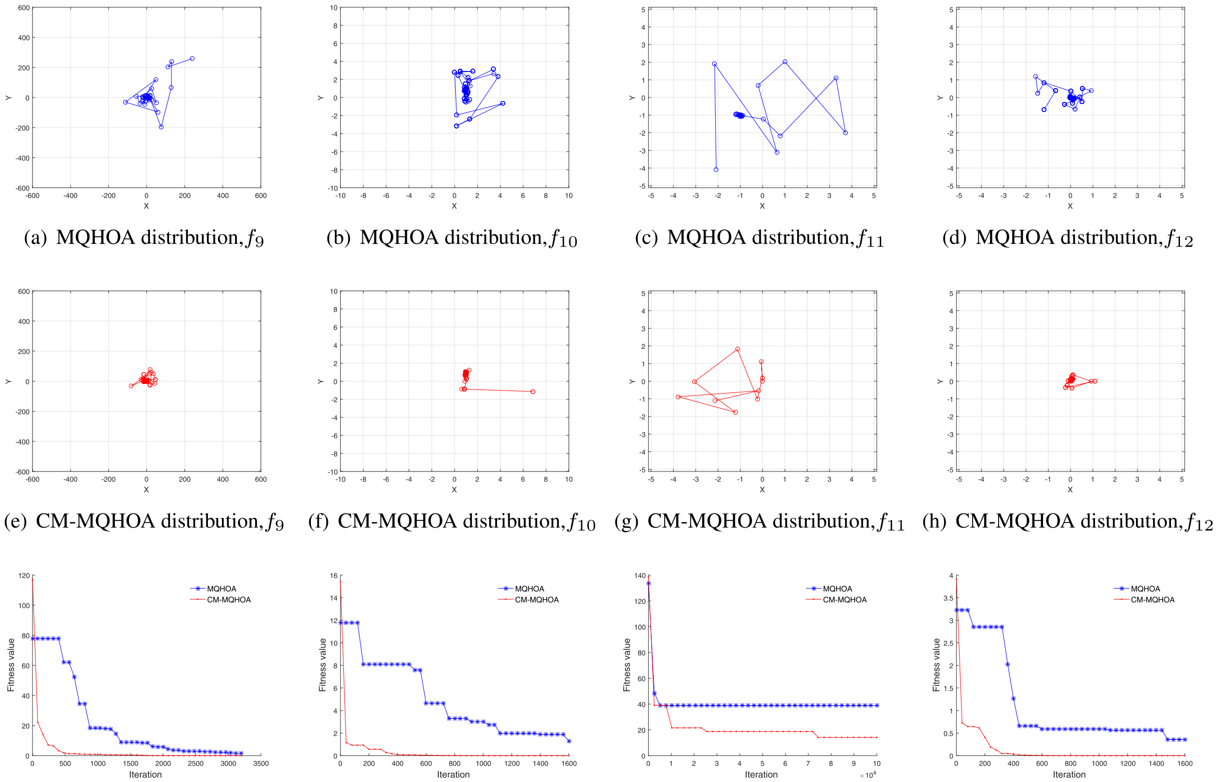
(m) MQHOA distribution, f_5 (n) MQHOA distribution, f_6 (o) MQHOA distribution, f_7 (p) MQHOA distribution, f_8

(q) CM-MQHOA distribution, f_5 (r) CM-MQHOA distribution, f_6 (s) CM-MQHOA distribution, f_7 (t) CM-MQHOA distribution, f_8

(u) Fitness-iteration, f_5 (v) Fitness-iteration, f_6 (w) Fitness-iteration, f_7 (x) Fitness-iteration, f_8

FIGURE 3. Trajectory of the convergence process for MQHOA and CM-MQHOA (part 1).

successfully locating to the global optima in every experiment, except that MQHOA obtains 98.04% and 90.20% successful proportions in the evaluation of 80-dimensional and 100-dimensional function f_2 and gets 50% SR in the evaluation of 100-dimensional function f_6. It should be noticed that in the evaluation of function f_6, CM-MQHOA and MQHOA



(a) MQHOA distribution, f_9 (b) MQHOA distribution, f_{10} (c) MQHOA distribution, f_{11} (d) MQHOA distribution, f_{12}

(e) CM-MQHOA distribution, f_9 (f) CM-MQHOA distribution, f_{10} (g) CM-MQHOA distribution, f_{11} (h) CM-MQHOA distribution, f_{12}

(i) Fitness-iteration, f_9 (j) Fitness-iteration, f_{10} (k) Fitness-iteration, f_{11} (l) Fitness-iteration, f_{12}

FIGURE 4. Trajectory of the convergence process for MQHOA and CM-MQHOA (part 2).

are the only two algorithms which are capable of finding the global optima. And CM-MQHOA is the only approach which is able to find the global optimum in the evaluation of 100-dimensional function f_6.

In the evaluation of function f_7-f_{11}, CM-MQHOA and MQHOA are not doing as well as in function f_1-f_6 and f_{12}. They are not able to find the global optima with large successful proportions in these function evaluations. In the evaluation of function f_7, StudGA and ABC algorithms are the only two algorithms which are able to find the global optima every time in the 51 independent trials, while other algorithms nearly fail to locate to the global minima. Similar situation happens in the evaluation of function f_{11}.

In the evaluation of function f_8, ABC is the only technique which is capable of fully finding the global optima in the 10, 30, 50, 80 and 100 dimensional experiments, while other algorithms are nearly unable to find the global optima especially in the 80 and 100 dimensional experiments. Similarly, in evaluation of function f_9, ABC obtains the highest SR in the multi-dimensional function evaluations, and it is the only one which is able to find the global minima with 100% SR in the 30, 50, 80 and 100 dimensional trials (80.39% SR in 10-dimensional function evaluation).

In the evaluation of function f_{11}, though all of the algorithms are able to completely find the global optimum with 100% SR in the 10-dimensional experiments, the successful proportions of CLPSO, QPSO, MQHOA and CM-MQHOA are decreasing when the function dimension is larger than 30. When the dimension increases to 100, all of the algorithms are unable to find the global optima except for StudGA and ABC algorithm. StudGA and ABC are the only two approaches which are able to successfully find the global optima with 100% SR in the 51 independent trials.

D. EFFICIENCY EVALUATION

Section IV-C qualitatively reveals the effectiveness and improvements of CM-MQHOA. However, it is difficult to figure out the better performer when the competitors obtain the same successful proportion in Table 2. In this section, the efficiency of CM-MQHOA is evaluated by detailed records and fitness-iteration relations.

1) COMPUTATIONAL PRECISION

To estimate the efficiency of the proposed algorithm, several items are considered such as the best fitness, mean fitness (the average of the fitness value obtained by the population), standard deviation of the fitness value, iteration time (CPU run time, time from start to finish the function evaluation) and function evaluation number (FE). Without losing of generality, the experiments were carried out on 100-dimensional function evaluations. Meanwhile, to reveal the differences among the CM-MQHOA and some congeneric

TABLE 2. Successful proportions for StudGA, SPSO2011, CLPSO, QPSO, ABC, MQHOA and CM-MQHOA in 10, 30, 50, 80 and 100 dimensional function evaluations. The results are the average of the 51 independent trials.

Func	DIM	StudGA	SPSO2011	CLPSO	QPSO	ABC	MQHOA	CM-MQHOA
f_1	10	100.00%	100.00%	98.04%	100.00%	100.00%	100.00%	100.00%
	30	100.00%	100.00%	0.00%	100.00%	100.00%	100.00%	100.00%
	50	100.00%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	80	100.00%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	100	100.00%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
f_2	10	100.00%	100.00%	45.45%	100.00%	100.00%	100.00%	100.00%
	30	100.00%	45.45%	0.00%	100.00%	100.00%	100.00%	100.00%
	50	100.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	80	100.00%	0.00%	0.00%	0.00%	100.00%	98.04%	100.00%
	100	98.04%	0.00%	0.00%	0.00%	100.00%	90.20%	100.00%
f_3	10	80.39%	100.00%	0.00%	100.00%	100.00%	100.00%	100.00%
	30	19.61%	100.00%	0.00%	100.00%	100.00%	100.00%	100.00%
	50	5.88%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	80	0.00%	23.53%	0.00%	0.00%	100.00%	100.00%	100.00%
	100	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%
f_4	10	19.61%	100.00%	0.00%	100.00%	100.00%	100.00%	100.00%
	30	0.00%	100.00%	0.00%	100.00%	100.00%	100.00%	100.00%
	50	0.00%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	80	0.00%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	100	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%
f_5	10	92.16%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	30	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	50	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	80	100.00%	100.00%	100.00%	0.00%	100.00%	100.00%	100.00%
	100	100.00%	100.00%	100.00%	0.00%	100.00%	100.00%	100.00%
f_6	10	0.00%	100.00%	0.00%	100.00%	0.00%	100.00%	100.00%
	30	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%
	50	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%
	80	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%
	100	0.00%	0.00%	0.00%	0.00%	0.00%	78.43%	100.00%
f_7	10	100.00%	0.00%	0.00%	1.96%	100.00%	0.00%	1.96%
	30	100.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	50	100.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	80	100.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	100	100.00%	0.00%	0.00%	0.00%	94.12%	0.00%	0.00%
f_8	10	0.00%	100.00%	0.00%	100.00%	100.00%	96.08%	98.04%
	30	0.00%	88.24%	0.00%	100.00%	100.00%	33.33%	66.67%
	50	0.00%	31.37%	0.00%	0.00%	100.00%	3.92%	11.76%
	80	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	100	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
f_9	10	0.00%	35.29%	0.00%	0.00%	80.39%	0.00%	17.65%
	30	0.00%	74.51%	0.00%	0.00%	100.00%	25.49%	90.20%
	50	0.00%	0.00%	0.00%	0.00%	100.00%	80.39%	70.59%
	80	0.00%	0.00%	0.00%	0.00%	100.00%	62.75%	62.75%
	100	0.00%	0.00%	0.00%	0.00%	100.00%	62.75%	74.51%
f_{10}	10	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	30	100.00%	100.00%	0.00%	100.00%	100.00%	98.04%	100.00%
	50	100.00%	100.00%	0.00%	0.00%	100.00%	92.16%	98.04%
	80	100.00%	100.00%	0.00%	0.00%	100.00%	86.27%	88.24%
	100	100.00%	98.04%	0.00%	0.00%	100.00%	86.27%	90.20%
f_{11}	10	100.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	30	100.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	50	100.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	80	100.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	100	100.00%	0.00%	0.00%	0.00%	94.12%	0.00%	0.00%
f_{12}	10	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	30	100.00%	100.00%	0.00%	100.00%	100.00%	100.00%	100.00%
	50	100.00%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	80	100.00%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	100	100.00%	100.00%	0.00%	0.00%	100.00%	100.00%	100.00%
100%s	-	33	29	7	18	52	33	37

algorithms, the experimental results are compared with MQHOA and several state-of-the-art optimization algorithms such as StudGA [42], SPSO2011 [43], CLPSO [44], QPSO [45] and ABC [15]. In the experiments, for all of the competitors, the stopping criterion were defined as the maximal function evaluation (*maxFE*) was larger than 10000*dimension or the computational accuracy is less than 1e-6. The experimental results are the average over the 51 independent trials and listed in Table 3.

In Table 3, the items of the table are defined as follows: *Best* denotes the averaged best fitness in the 51 independent trials, *Mean* is the average of the mean fitness values, *Std* represents the average of standard deviation obtained in every independent trial, *IterNO.* denotes the averaged function evaluation times and *Time* represents the average of CPU run time in the 51 independent trial. As illustrated in Table 3, from an overall perspective, StudGA, ABC, MQHOA and CM-MQHOA perform better than SPSO2011, CLPSO and QPSO in the 100-dimensional function evaluations. StudGA obtains the largest number of the best fitness values, such as in the evaluation of function $f_3, f_4, f_8, f_9, f_{10}$ and f_{12}. ABC performs excellently in most of the function evaluations, especially on function f_7 and f_8 that ABC is the only one which is able to find the global optima. Although CM-MQHOA does not find the smallest fitness values, it spends the least time and takes the fewest iteration (function evaluation) in the evaluation of function $f_1, f_3, f_4, f_5, f_9, f_{10}$ and f_{12}.

2) FITNESS-ITERATION EVALUATION

Although Table2 and Table3 reflect the performances of CM-MQHOA to some extent, it is not enough to figure out the efficiency of the algorithms. In this subsection, fitness-iteration evaluation is introduced to estimate the efficiency of the algorithms. The fitness-iteration curve records the fitness value as the run times (function evaluation times) increases. The fitness-iteration figures are shown in Fig.5.

In Fig.5, the horizontal axis represents the function evaluation times, the vertical coordinate denotes the fitness value (semilogy). As illustrated in Fig.5 (a)-(l), though MQHOA and CM-MQHOA perform similarly in Table 2 and Table 3, CM-MQHOA converges much faster than MQHOA in the evaluation of function f_1-f_{12}.

In Fig. 5(a), in the evaluation of function f_1, CM-MQHOA converges much faster than other algorithms, followed by ABC, SPSO2011 and StudGA. Although ABC converges slower than SPSO2011 in the first 10000 generations, it outperforms SPSO2011 after that. The performance of MQHOA and StudGA is approximate, both converge faster than QPSO and CLPSO. Similar situation happens in the evaluation of function f_3, f_4, f_9 and f_{10}.

As demonstrated in Fig.5(b) that in the evaluation of function f_2, ABC is the only algorithm which is able to converge to the global optimum within 50000 generations. Though CM-MQHOA cannot find the global optimum within 50000 iterations, it converges more efficient than other algorithms except for ABC. For function f_5, Fig.5(e) reveals the superiority of CM-MQHOA which is the most efficient algorithm converging to the global minimum within 600 iterations. In the evaluation of function f_6, CM-MQHOA and MQHOA perform much better than other algorithms, both of them converge to the global optimum within 720000 generations.

In the evaluation of function f_8, CM-MQHOA converges much faster than other algorithms, positioning the global optimal landscape within 15000 generations. While algorithms except CM-MQHOA converge slowly within the first 15000 function evaluation. As shown in Fig.5(k), the ABC algorithm performs the best in the evaluation of function f_{11}, which is the only algorithm that is able to converge to the global minimum within 200000 generations. Although CM-MQHOA converges much faster than other algorithms in the first 10000 generations, it converges very slowly after that and its fitness-iteration curve remains a horizontal line. In the evaluation of function f_{12} (Fig.5), CM-MQHOA outperforms other algorithms, which converges quickly to the global optimal landscape within 7500 generations.

E. BRIEF DISCUSSION

In above sections, the characteristics of the proposed algorithm are evaluated from several aspects. The trajectory plots demonstrated in Fig. 3 reveal that the centroid motion applied in CM-MQHOA benefits the algorithm from shortening the converging path and enhancing the converging efficiency. To evaluate the performances of the proposed algorithm, the effectiveness and efficient evaluations are carried out on several multi-dimensional benchmark functions. The experimental results are compared with the original MQHOA. The comparative results indicate that generally CM-MQHOA and MQHOA are matched in the successful rate of locating to the global optima in the evaluation of unimodal functions. But CM-MQHOA converges much more efficient than the original MQHOA. Meanwhile, CM-MQHOA obtains much higher successful rate in evaluation of multimodal functions and converges much faster than MQHOA.

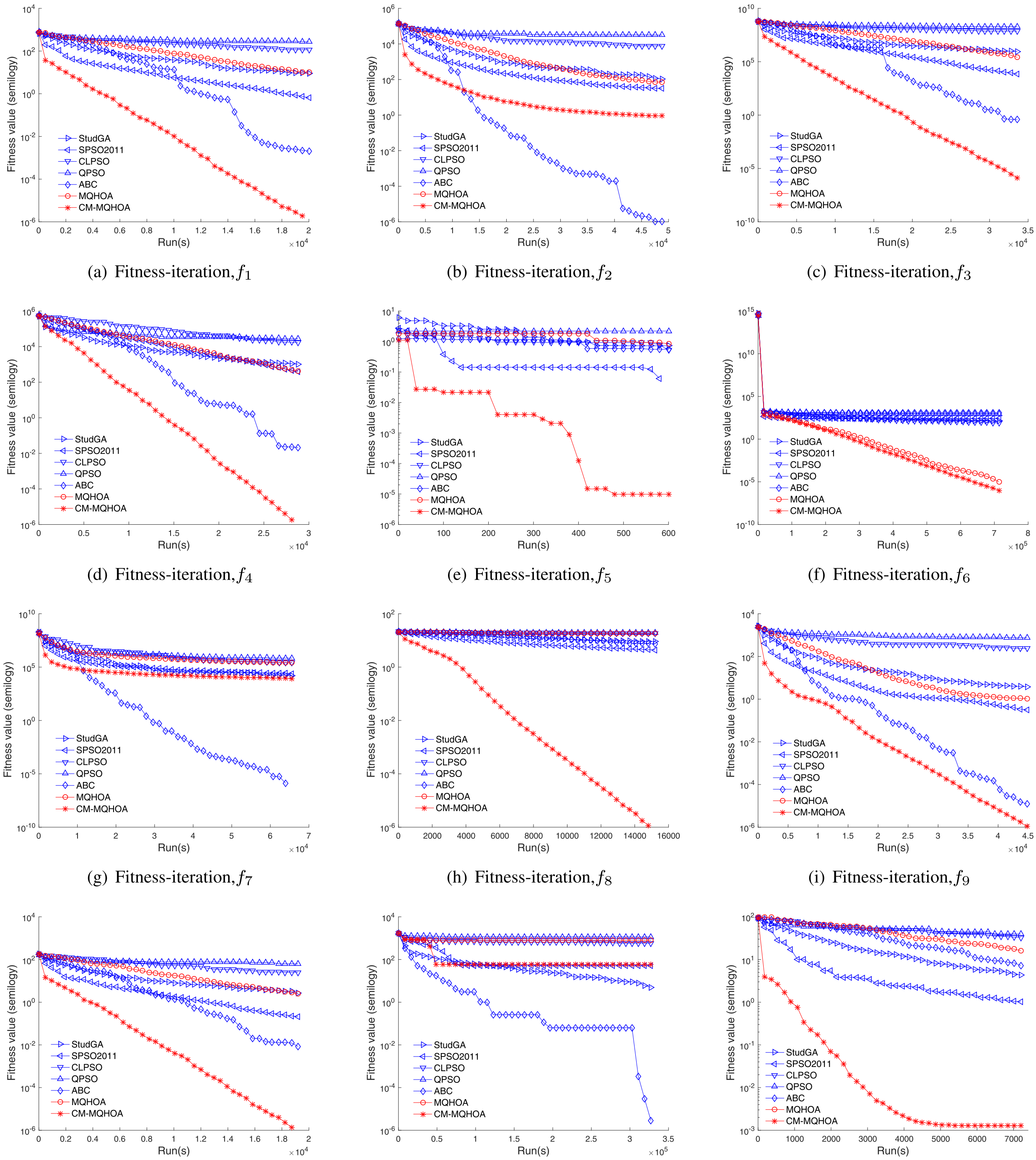
The detailed computational results in Table 3 reveal that CM-MQHOA spends less CPU run time and fewer generations in several function evaluations, which indicates the efficiency of CM-MQHOA in the course of converging to the global optima. These conclusions are verified in the fitness-iteration evaluations. In Fig.5, CM-MQHOA outperforms other techniques, converging much faster than other algorithms in all of the function evaluations except for function f_2, f_7 and f_{11}.

The reason for CM-MQHOA achieving competitive results can mainly be summarized as follows. First, the centroid in CM-MQHOA is the center of the potential system, the centroid motion helps the population move toward the global optimal landscape. Second, the centroid motion itself in essence is a mutation which helps to diversify the particles. Third, the mechanism to enlarge the search domain when particles fall into local searching for a long time helps to jump out stagnation. Last but not least, the centroid contains

TABLE 3. Detailed computational results comparison among StudGA, SPSO2011, CLPSO, QPSO, ABC, MQHOA and CM-MQHOA on 100-dimensional benchmark functions. The unit of *time* is second (s).

funcNO	Item	StudGA	SPSO2011	CLPSO	QPSO	ABC	MQHOA	CM-MQHOA
f_1	Best	**0.000E+00**	9.736E-07	9.082E-01	9.669E+01	7.617E-07	9.839E-07	9.900E-07
	Mean	1.322E+01	1.187E-06	2.956E+01	1.352E+02	2.021E-01	1.144E-06	**1.101E-06**
	Std	1.264E+01	1.001E-07	3.239E+01	6.399E+01	4.831E-01	8.769E-08	**4.759E-08**
	IterNO	2.608E+05	7.290E+04	1.000E+06	1.000E+06	3.996E+04	8.814E+04	**1.881E+04**
	Time	2.592E+02	2.391E+01	6.653E+01	5.099E+01	2.014E+00	3.397E+00	**1.009E+00**
f_2	Best	**0.000E+00**	2.845E+00	7.082E+01	1.034E+04	8.289E-07	9.985E-07	9.995E-07
	Mean	2.681E+03	2.914E+00	3.441E+03	1.662E+04	3.728E+00	**1.012E-06**	**1.002E-06**
	Std	2.766E+03	2.672E-02	4.080E+03	1.023E+04	7.388E+00	6.630E-09	**9.490E-10**
	IterNO	4.113E+05	1.000E+06	1.000E+06	1.000E+06	**4.926E+04**	8.183E+05	4.315E+05
	Time	3.712E+02	1.577E+03	6.050E+01	4.447E+01	**2.065E+00**	2.800E+01	1.930E+01
f_3	Best	5.102E+02	1.471E-03	6.784E+05	7.036E+07	**7.467E-07**	8.772E-07	8.858E-07
	Mean	9.274E+06	1.854E-03	2.504E+07	1.034E+08	2.562E+04	1.025E-06	**1.002E-06**
	Std	9.967E+06	1.697E-04	2.780E+07	4.127E+07	7.684E+04	6.621E-08	**4.323E-08**
	IterNO	1.000E+06	1.000E+06	1.000E+06	1.000E+06	6.981E+04	1.353E+05	**3.004E+04**
	Time	9.369E+02	1.677E+03	1.656E+02	1.524E+02	1.081E+01	1.951E+01	**4.664E+00**
f_4	Best	8.427E-01	1.130E-05	5.793E-03	4.639E+03	**8.194E-07**	9.728E-07	9.786E-07
	Mean	8.560E+03	1.378E-05	6.115E-01	1.025E+04	1.407E+01	**1.053E-06**	1.091E-06
	Std	9.457E+03	1.230E-06	1.404E+00	1.222E+04	3.192E+01	**3.490E-08**	4.536E-08
	IterNO	1.000E+06	1.000E+06	1.000E+06	1.000E+06	5.010E+04	1.160E+05	**2.820E+04**
	Time	8.260E+02	3.331E+02	6.826E+01	4.139E+01	2.006E+00	3.725E+00	**1.215E+00**
f_5	Best	**6.797E-08**	7.116E-07	7.850E-07	1.537E+00	5.345E-07	9.867E-07	8.217E-07
	Mean	1.492E-01	3.070E-02	1.397E-01	1.537E+00	4.545E-01	**1.042E-06**	1.158E-05
	Std	3.244E-01	1.074E-01	2.474E-01	3.936E+00	7.392E-01	**2.365E-08**	8.256E-06
	IterNO	6.230E+03	1.200E+04	3.138E+04	1.000E+06	2.468E+04	2.626E+04	**2.088E+03**
	Time	5.595E+00	3.908E+00	3.384E+00	6.658E+01	1.738E+00	1.590E+00	**1.612E-01**
f_6	Best	1.042E+02	2.409E+02	2.362E+04	7.458E+02	1.187E+03	1.033E-06	**9.974E-07**
	Mean	4.100E+09	8.525E+08	1.610E+10	1.143E+10	1.953E+14	1.046E-06	**1.008E-06**
	Std	1.138E+10	3.730E+09	5.593E+10	1.378E+12	3.655E+14	5.466E-09	**2.129E-09**
	IterNO	1.000E+06	1.000E+06	1.000E+06	1.000E+06	1.000E+06	8.440E+05	**7.278E+05**
	Time	9.993E+02	1.860E+03	9.377E+01	5.569E+01	5.354E+01	**3.419E+01**	3.985E+01
f_7	Best	**0.000E+00**	1.871E+04	1.250E+01	4.500E+04	7.406E-07	1.683E+04	8.047E+02
	Mean	3.747E+06	1.902E+04	3.157E+03	3.812E+05	**3.368E+00**	1.684E+04	8.048E+02
	Std	6.937E+06	1.311E+02	5.707E+03	2.245E+06	7.002E+00	6.103E+00	**2.091E-02**
	IterNO	3.678E+05	1.000E+06	1.000E+06	1.000E+06	**6.285E+04**	1.000E+06	1.000E+06
	Time	3.172E+02	1.543E+03	8.891E+01	6.847E+01	**4.187E+00**	5.777E+01	6.820E+01
f_8	Best	3.306E-01	2.537E+00	4.028E+00	1.567E+01	**7.461E-07**	1.783E+01	2.905E+00
	Mean	5.919E+00	2.537E+00	9.953E+00	1.696E+01	**5.503E-05**	1.783E+01	2.905E+00
	Std	3.438E+00	4.002E-09	4.321E+00	5.087E-01	9.581E-05	8.407E-10	**1.068E-15**
	IterNO	1.000E+06	1.000E+06	1.000E+06	1.000E+06	**8.835E+04**	1.000E+06	1.000E+06
	Time	8.329E+02	1.279E+03	6.385E+01	4.935E+01	**3.837E+00**	4.008E+01	4.850E+01
f_9	Best	1.380E-01	2.333E-03	3.365E+00	2.894E+02	**6.582E-07**	9.784E-07	4.227E-03
	Mean	3.941E+01	2.461E-03	1.046E+02	4.504E+02	1.254E-01	**1.111E-06**	4.227E-03
	Std	3.846E+01	5.851E-05	1.167E+02	2.127E+02	1.274E-01	7.058E-08	**8.749E-09**
	IterNO	1.000E+06	1.000E+06	1.000E+06	1.000E+06	**7.491E+04**	1.374E+05	5.922E+05
	Time	1.361E+03	2.605E+03	1.097E+02	8.750E+01	**6.926E+00**	9.188E+00	5.080E+01
f_{10}	Best	**1.500E-32**	9.779E-07	2.166E-01	2.207E+01	8.394E-07	8.231E-02	9.891E-07
	Mean	3.338E+00	1.194E-06	6.999E+00	3.057E+01	4.576E-02	8.231E-02	**1.086E-06**
	Std	3.157E+00	1.054E-07	7.502E+00	1.378E+01	1.116E-01	**3.302E-08**	4.220E-08
	IterNO	5.450E+05	7.258E+04	1.000E+06	1.000E+06	3.692E+04	1.680E+05	**1.908E+04**
	Time	5.902E+02	2.789E+01	1.257E+02	1.077E+02	4.099E+00	1.619E+01	**2.123E+00**
f_{11}	Best	**0.000E+00**	6.219E+01	4.263E+02	1.136E+03	7.407E-07	7.307E+02	3.732E+02
	Mean	2.962E+01	1.175E+02	8.266E+02	1.174E+03	**1.370E+01**	7.307E+02	3.966E+02
	Std	2.078E+01	1.719E+02	2.895E+02	8.801E+01	1.652E+01	**1.137E-13**	1.746E+01
	IterNO	**3.682E+05**	1.000E+06	1.000E+06	1.000E+06	4.255E+05	1.000E+06	1.000E+06
	Time	3.103E+02	1.530E+03	7.094E+01	4.684E+01	**1.699E+01**	3.850E+01	4.729E+01
f_{12}	Best	**1.273E-03**	**1.273E-03**	9.812E-02	1.140E+01	**1.273E-03**	**1.273E-03**	**1.273E-03**
	Mean	1.411E+00	**1.273E-03**	3.890E+00	1.613E+01	2.064E-03	**1.273E-03**	**1.273E-03**
	Std	1.393E+00	2.645E-08	4.284E+00	6.708E+00	2.196E-03	2.385E-08	**9.058E-09**
	IterNO	2.832E+05	7.083E+04	1.000E+06	1.000E+06	4.168E+04	8.594E+04	**1.809E+04**
	Time	2.621E+02	2.713E+01	1.512E+02	1.384E+02	5.674E+00	1.069E+01	**2.509E+00**

The bold mark indicates that they are the best results among the algorithms.



(a) Fitness-iteration, f_1 (b) Fitness-iteration, f_2 (c) Fitness-iteration, f_3

(d) Fitness-iteration, f_4 (e) Fitness-iteration, f_5 (f) Fitness-iteration, f_6

(g) Fitness-iteration, f_7 (h) Fitness-iteration, f_8 (i) Fitness-iteration, f_9

(j) Fitness-iteration, f_{10} (k) Fitness-iteration, f_{11} (l) Fitness-iteration, f_{12}

FIGURE 5. Fitness-iteration comparison among StudGA, SPSO2011, CLPSO, QPSO, ABC, MQHOA and CM-MQHOA for 100-dimensional function evaluations.

information from all of the individuals which helps to take advantage of the explorative power from the population.

The experimental results also indicate that the proposed algorithm does not perform as well as ABC in the evaluation of some multimodal benchmark functions, such as f_{11}. As the 3-D figure of function f_{11} in Fig. 6 shows, there are many local optimal solutions in the search domain. It is easy for a continuous optimization technique to fall into the local traps. But for ABC and StudGA, they do not get stumped by the problem. The reason for the good performance on function f_{11} may due to the reasons as follows.

In ABC algorithm [15], the employed bees randomly search for new food sources generated by a differential mechanism which helps to diversify the food sources. The onlooker

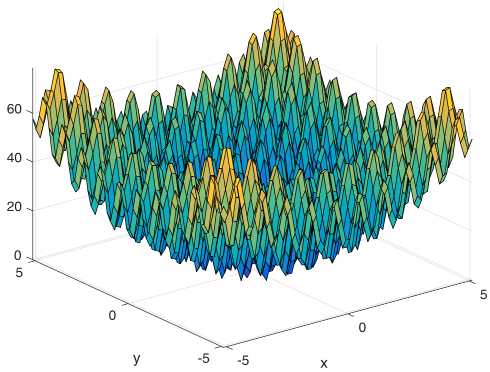


FIGURE 6. 3D figure of Rastrign function (f_{11}).

bees search for the best food source with a probability defined by using (16) which is a mutation mechanism to diversity the food source. For scout bees, new food sources are generated when the trial counter exceeds the limit value, which helps the algorithm to jump out local optima. In StudGA [42], different parents are randomly selected to mate with the stud, new children are created by the crossover mechanism. Non-elite population members are continuously replaced by new children. In the mutation process, the elites are protected not to be mutated. These mechanisms help StudGA perform excellent in the evaluation of function f_{11}, and they are deserved for reference in other optimization techniques.

V. CONCLUSION

This paper proposes a Multi-scale Quantum Harmonic Oscillator Algorithm with Centroid Motion (CM-MQHOA) to improve the convergence performance and enhance the exploration and exploitation ability. Effectiveness and efficiency of the proposed algorithm are estimated by evaluating several well-defined CEC benchmark problems from different aspects. The tracing of convergence path reveals the significant improvement of convergence performance in CM-MQHOA compared with the original MQHOA. The effectiveness evaluation indicates the reliability and the consistent repeatability of optimization performance for CM-MQHOA. The detailed computational results indicate the efficiency and accuracy of the proposed approach. The fitness-iteration evaluation reveals the efficiency of CM-MQHOA to converge to the global optima. The experimental results are compared with several state-of-the-art optimization algorithms such as StudGA, SPSO2011, CLPSO, QPSO and ABC. The comparative results indicate the competitiveness of the proposed technique and suggest a viable and attractive addition to the portfolio of computational intelligence techniques.

In the near future, many more state-of-the-art algorithms will be considered to further compare with the performance of the proposed technique, hybridizing of the proposed algorithm and several excellent algorithms such as ABC and SPSO2011 will be considered to improve the convergence performance. Meanwhile, application of the proposed algorithm to deal with multi-objective problems is in our research schedule.

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