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CSE: Complex-Valued System With Evolutionary Algorithm

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ABSTRACT Complex-valued system identification has the ability to provide the basis for system analysis. So as to demonstrate the potential and internal mechanism of the complex-valued system, this paper proposes a novel complex-valued hybrid evolutionary (CSE) algorithm to optimize the complex-valued expression model (CEM). Complex-valued gene expression programming (CVGEP) is proposed to optimize the architectures of the CEM. Complex-valued water wave optimization (CVWWO) is first proposed to optimize complex-valued coefficients and constants of the CEM. The two artificial complex-valued function approximation problems and non-minimum phase equalization problem are utilized to test the performance of the proposed algorithm. The results demonstrate that this method could identify complex-valued systems more correctly than complex-valued neural networks, which could obtain above 90% smaller mean-squared error (MSE) performance than other complex-valued models. With 10% Gaussian white noise, the CSE could identify accurate complex-valued structure and parameters containing coefficients and constants. The CVWWO has better convergence performance than complex-valued particle swarm optimization and crow search algorithm.

INDEX TERMS Complex-valued expression model, gene expression programming, water wave optimization.

I. INTRODUCTION

In the fields of engineering technology, economic management, natural sciences and social sciences, there are complex systems and non-linear phenomena that change with time. When it comes to the complex systems, such issue has been widely and successfully utilized in several fields, including price fluctuation, weather change, population growth, etc [1]–[4]. The reasonable models have been established with the observation data to provide the basis for system analysis, design and future state prediction. The understanding of dynamic behavior of the unknown system has become an important research topic in this field [5]–[8].

It was pointed that complex-valued system identification has been widely utilized in several fields, including hydrodynamics, aerodynamics, elasticity theory, electrostatic field, circuit theory [9]–[11]. Particularly, black-box modeling technology based on input-output complex-valued data is one of the hotspots and has attracted wide attention. Savitha et al. proposed an improved complex-valued back propagation (CVBP) algorithm to resolve complex-valued XOR and synthetic function approximation problems [12]. Deng et al. proposed complex-valued minimal radial basis function neural networks to solve nonlinear channel equalization problems [13]. Cousseau et al. proposed a two-dimensional simplified canonical piecewise linear filter to identify complex-valued nonlinear Wiener model [14]. In order to identify complex-valued Wiener system more accurately, Hong et al. proposed a complex-valued B-spline neural network method [15]. The black-box modeling methods of complex-valued systems have some disadvantages, such as difficult structure design, unclear internal interpretation and so on.

In order to discover and demonstrate the internal mechanism of the system deeply and clearly, the essential and significant rules and theories have been abstracted from

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the intricate external phenomena. Moreover, these can be demonstrated with the mathematical forms, including equations or other similar methods. Computational models have been applied to identify systems in real-valued application fields [16]. Chen et al. proposed a hybrid evolutionary algorithm based on additive tree model and random search algorithm for linear/nonlinear polynomials identification [17]. Feijoo et al. proposed associated linear equations to identify the systems of an electrostrictive actuator and a Duffing oscillator [18]. Gennemark et al. proposed parameter optimization algorithm and model selection to evolve ordinary differential equation (ODE) to identify metabolic pathway and genetic network [19]. Kreiberg et al. proposed structural equation modeling to identify errors-in-variables single-input single-output (SISO) system [20].

In order to express the internal mechanism of complex-value system more clearly, a novel hybrid evolutionary algorithm is proposed to optimize complex-valued mathematical expression model (CEM) in order to identify nonlinear complex-valued systems. In this hybrid algorithm, complex-valued gene expression programming (CVGEP) is proposed to optimize the architectures of CEM. In order to search the optimal complex-valued coefficients and constants of CEM, complex-valued water wave optimization (CVWWO) is utilized. The two artificial complex-valued functions and one real system are utilized to test the performance of our proposed algorithm.

II. METHOD

A. COMPLEX-VALUED GENE EXPRESSION PROGRAMMING

Gene expression programming (GEP) is a structure-based evolutionary algorithm based on genetic algorithm (GA) and genetic programming (GP), which owns the advantages of GA and GP, and has stronger the ability of problem solving [21]. In GEP, the structure of chromosome is simple, linear and compact, so it has been applied for function mining problems [22]. Thus in this paper, as complex-valued version of GEP, complex-valued gene expression programming (CVGEP) is proposed to identify mathematical expression model of complex-valued system.

In order to create complex-valued chromosome, complex-valued function set $F = \{+, -, *, /, \sin, \cos, \ln, e^{x+yi}\}$ and complex-valued terminal set $T = \{z_1, z_2, \ldots, z_n, R\}$ are defined beforehand. In CVGEP, each chromosome contains one or more genes. Each gene is represented by a fixed-length string of symbols, which includes head part and tail part. The head part could include function symbols and terminal symbols, while the tail part only includes terminal symbols. For example, an example of complex-valued expression model $z = \frac{z_2 z_3}{\sin(z_3)+0.6+1.2\,i}$ is encoded, which is depicted in Fig. 1. In this gene, the length of head part is 5 and the length of tail part is set as 6. The length of gene is 11 and the region after the expression is non-coding.

Scanning the characters in the gene one by one from left to right, an expression tree (ET) is constructed by hierarchy traversal order, which is shown in Fig. 2. The mathematical expression is calculated by traversing ET with in-order traversal.

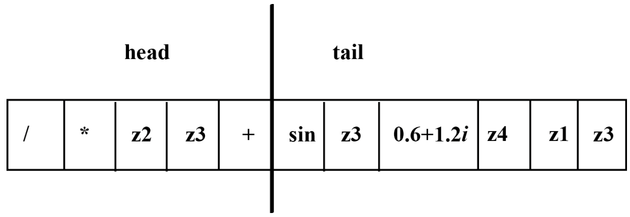


FIGURE 1. A gene example of the chromosome in CVGEP. F is set as $\{+, -, \times, /, \sin\}$ and T is set as $\{z_1, z_2, z_3, z_4, 0.6 + 1.2\,i\}$.

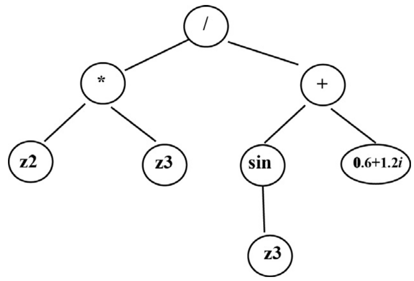


FIGURE 2. The decoding expression tree of gene in chromosome.

In CVGEP, many chromosomes constitute the population. In order to gain the optimal solution of problem, genetic operators are implemented to make population evolve generation by generation, which contain selection, mutation and recombination. The flowchart of CVGEP algorithm is depicted in Fig. 3.

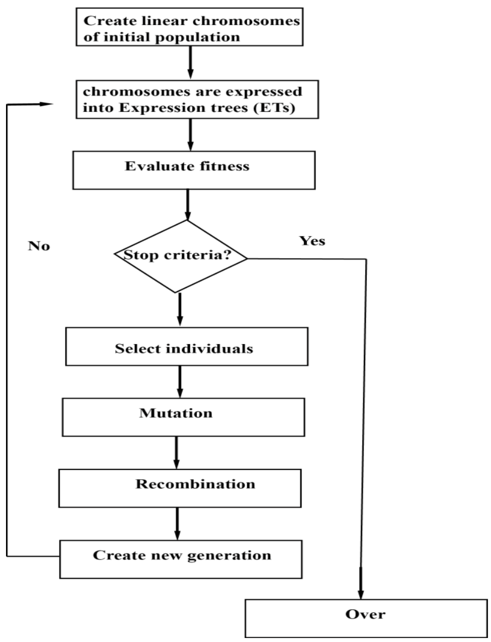


FIGURE 3. The outlines of CVGEP.

B. COMPLEX-VALUED PARAMETERS OPTIMIZATION

1) WATER WAVE OPTIMIZATION

Water wave optimization (WWO) algorithm is a novel evolutionary algorithm based on shallow water wave theory, which was proposed by Zheng in 2015 [23]. By simulating the propagation, refraction, breaking waves and other water wave motions, the optimal solution is searched in high-dimensional space [24]. WWO has some advantages, such as simple structure, less control parameters and less computational overhead, so it performs better than some new evolutionary algorithms, such as invasive weed optimization (IWO), biogeography-based optimization (BBO), crow search algorithm (CSA), etc [25].

In WWO, one water wave corresponds to a solution and has two attributes: wave height h and wave length λ. When the population are initialized, the wavelength and height of each wave are initialized to the constants (0.5 and $h_{\max}$). The fitness value of water wave is inversely proportional to the distance from sea level. The closer the distance to sea level is, the higher the fitness of water waves is; otherwise the lower the fitness is. Propagation, refraction and breaking waves are utilized to solve the optimization problem.

(1) Propagation

Suppose that the dimension of the problem is D and the current position of the water wave is X. The new position of the water wave X is updated as follows.

$$X' = X + r \times \lambda L. \tag{1}$$

where, r is a random number ranges from -1 to 1, λ is the wavelength of the water wave X and L is the length of the water wave in the search space. If the length of the new water wave exceeds the search limit, it needs to be reassigned a random position in the search space.

After the propagation process, the fitness function f is utilized to calculate the fitness value of the new water wave (X'). If $f(X') < f(X)$, X' is utilized to replace X, and the wave height of X' is set as $h_{\max}$; otherwise, the original wave X is retained and the wave height is reduced by 1. After each iteration, the wavelength of each water wave is updated as follows:

$$\lambda = \lambda \times a^{-\frac{f(X)-f_{\min}+\varepsilon}{f_{\max}-f_{\min}+\varepsilon}}. \tag{2}$$

where, a denotes the wavelength attenuation coefficient, $f_{\max}$ and $f_{\min}$ are the maximum and minimum fitness values of the current population, respectively. ε is the minimum positive number.

(2) Refraction

If the wave could not be improved after multiple propagation operations, the height of the water wave decreases to 0, and the wave will stop searching. For such water waves, refraction can be utilized. For water wave X, refraction operation is defined as follows.

$$X' = Gaussian(\frac{X_{best} + X}{2}, \frac{|X_{best} - X|}{2}). \tag{3}$$

where, X_{best} is the best water wave in the current population.

After refraction, the height of the new water wave X' is reset to $h_{\max}$ and the wavelength is updated as follows.

$$\lambda' = \lambda \frac{f(X)}{f(X')}. \tag{4}$$

(3) Wave breaking operation

According to the theory of water wave, if the energies of water waves continue to increase, their wave summits become steeper until they break up into a series of solitary waves in order to improve the diversity of the population. In WWO, only the optimal solution (X_{best}) could be broken. Select the $i-th$ dimension randomly and its wave breaking operation is defined as follows.

$$X'_{best,i} = X_{best,i} + Guassian(0, 1) \times \beta L_i. \tag{5}$$

where, β is the breaking wave coefficient. k wave breaking operations are implemented repeatedly and k sub-waves could be obtained. If the fitness values of k sub-waves are better than that of the optimal solution X_{best}, the optimal solution in the population is replaced by the optimal solution in sub-waves.

2) COMPLEX-VALUED WWO

As a new evolutionary algorithm, the convergence speed of WWO is related to the diversity of the initial population. It is unstable and easy to fall into the local optimum. Fig. 1 shows that chromosome in CVGEP may contain complex-valued constants. And the optimized expression model could include complex-valued coefficients. In order to optimize complex-valued coefficients and constants of CEM, complex-valued water wave optimization (CVWWO) algorithm is proposed. In CVWWO, each water wave is encoded by complex numbers containing two parts: real part and imaginary part. Real and imaginary parts could be optimized in parallel. The pseudo code of CVWWO is described in **Algorithm 1**.

C. SUMMARY OF LEARNING COMPLEX-VALUED EXPRESSION MODEL

(1) Create an initial CEM population randomly, which contain complex-valued chromosomes in CVGEP and their corresponding complex-valued coefficients and constants.
(2) Structure optimization by selection, recombination and mutation in CVGEP.
(3) At some iterations, complex-valued coefficients and constants are optimized by complex-valued water wave optimization. In this process, the structure of CEM is fixed.
(4) If the maximum iteration is reached, learning process is stopped; otherwise go to step (2).

III. EXPERIMENT

The two artificial complex-valued function approximation problems and non-minimum phase equalization problem are utilized to test the performance of our proposed algorithm. The parameters in CVGEP and CVWWO are listed in Table 1. Mean squared error (MSE) is utilized to evaluate the performance of methods.

$$MSE = \frac{1}{N}\sum_{i=1}^{N} E_i^2 \tag{6}$$

Algorithm 1 Pseudo Code of CVWWO

```
Initialize N complex-valued water waves [X_1, X_2, ..., X_N](X_k = x_k^{1,R} + x_k^{1,I} i, x_k^{2,R} + x_k^{2,I} i, ... x_k^{n,R} + x_k^{n,I} i)) with the n dimension and parameters (α, β, t_max and h_max)
while t < t_max do
  for k = 1; k < N-k ++ do
    Calculate the fitness
  end for
  Search the optimal wave X_best;
  for i = 1: i ≤ N; i++ do
    X_R^i(t+1) ← X_R^i(t) + r * λ_i * L_R^I;
    X_I^i(t+1) ← X_I^i(t) + r * λ_i * L_I^I;
    if f(X_i(t+1)) < f_i(X(t)) then
      h_i ← h_max
      if f(X_i(t+1)) < f(X_best) then
        for j = 1; j ≤ k; j++ do
          s ← select a dimension;
          O_{j,s}^R ← X_{best,s}^R + Guassian(0,1) * β * L_i^R
          O_{j,s}^I ← X_{best,s}^I + Guassian(0,1) * β * L_i^I
        end for
        if f(O_1) < f(X_best) and (O_2) < f(X_best) and .... and (O_k) < f(X_best) then
          X_best ← the optimal solution of O_1, O_2, ..., O_k;
        end if
      end if
    else
      X_i(t+1) ← X_i(t)
      h_i ← h_i − 1;
      if hi == 0 then
        X_i^R(t+1) ← Guassian((X_best^R + X_i^R(t))/2, |(X_best^R − X_i^R(t))/2|)
        X_i^I(t+1) ← Guassian((X_best^I + X_i^I(t))/2, |(X_best^I − X_i^I(t))/2|)
        λ_i = λ_i * f(X_i(t+1)) / f(X_best)
      end if
    end if
    λ_i^R ← λ_i^R * α^(−(f(X_i(t+1)) − f_min + ε)/(f_max − f_min + ε))
    λ_i^I ← λ_i^I * α^(−(f(X_i(t+1)) − f_min + ε)/(f_max − f_min + ε))
  end for
end while
Store the best solution obtained;
```

where E_i is the error between the $i-th$ actual and predicted data.

A. PREDICTION RESULTS

1) SYNTHETIC EXAMPLE 1

The first complex-valued system is a synthetic problem, which is descried as follows [12].

$$Z = Z_1^2 + Z_2^2 \tag{7}$$

TABLE 1. Parameters in CVGEP and CVWWO.

Parameters	Values
Population size in CVGEP	30
Maximum iteration in CVGEP	100
Head length in CVGEP	8
Tail length in CVGEP	9
Population size in CVWWO	30
Maximum iteration t_{max} in CVWWO	100
λ in CVWWO	0.5
h_{max} in CVWWO	12
α in CVWWO	1.0026
β in CVWWO	[0.001, 0.25]

In order to compare the performance fairly, the setting of experiment are as the same as the Ref [26]. Complex-valued variables Z_1 and Z_2 are selected randomly from the interval $[-2.5 - 2.5_{i,2.5+2.5}i]$. 4000 samples are created randomly, in which 3000 samples and 1000 samples are utilized as training and testing datasets, respectively. Through 30 runs, the optimal complex-valued system is obtained as Eq.(8). Compared Eq.(7) and Eq.(8), it could be clearly seen that these two systems have the same structures and complex-valued coefficients are very close, which reveal that our proposed hybrid evolutionary algorithm could identify complex-valued system accurately from complex-valued dataset.

$$Z = (0.999997 + 0\ i)Z_1^2 + (0.999999 - 0.000004\ i)Z_2^2 \tag{8}$$

The predicted error distributions of real and imaginary parts are depicted in Fig. 4. From the results, we can see that the predicted errors are extremely small and concentrate mainly around zero. The predicted performance results with seven methods are listed in Table 2. From Table 2, it could be clearly shown that our proposed method has the smallest MSE performance among seven complex-valued methods. And our method could improve at least 99% predicted accuracy.

In order to test the ability of noise tolerance of our method, we add 1%, 5%, 10%, 15% and 20% Gaussian white noise into complex-valued resampling data. The optimal CEMs obtained are listed in Table 3. Compared Table 3 with Eq.(7), it could be clearly seen that when the noise rate reaches 20%, our method could identify the same structure as the standard complex-valued system by ignoring the items with the smaller complex-valued coefficients. When the noise rate is between 1% and 10%, the coefficients of CEMs obtained are close to those of standard model. But when the noise rate

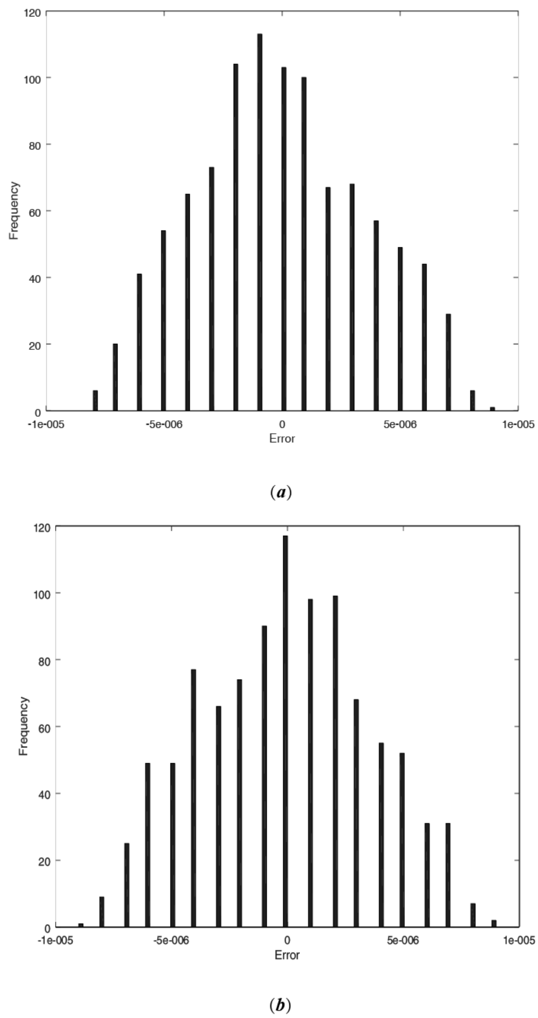


FIGURE 4. Prediction error distributions of real parts (a) and imaginary parts (b) for the first complex-valued system.

TABLE 2. The testing MSE performances of seven methods for the first complex-valued system.

Methods	Testing MSE
CNN [27]	0.9344
CRBF [28]	0.623
CMRAN [29]	0.0614
CELM [30]	0.704
FC-RBF [26]	0.003
FC-RBF with KMC [26]	0.00137
CEM	1.6×10^{-11}

TABLE 3. CEMs of our method with different noise rates for the first complex-valued system.

Noise	CEMs
1%	$Z=(0.998889+0.000441i)Z_2^2+(0.999904-0.000231\,i)Z_1^2+(-0.000241-0.000858i)\dfrac{Z_1}{Z_2}$
5%	$Z=(0.979668-0.002584\,i)Z_1^2+(0.978159-0.00046\,i)Z_2^2+(-0.005632-0.00339i)Z_1Z_2$
10%	$Z=(0.921687+0.007217\,i)Z_1^2+(0.916679-0.001458\,i)Z_2^2+0.008515+0.003479i$
15%	$Z=(0.844933-0.001981\,i)Z_1^2+(0.837456+0.013381\,i)Z_2^2+(-0.000897+0.0114)\cos(Z_2)$
20%	$Z=(0.751892+0.000879\,i)Z_1^2+(0.73995-0.002471\,i)Z_2^2$

TABLE 4. Training and testing performance of our method and CNN with different noise rates for the first complex-valued system.

Noise	Our method		CNN	
	Training MSE	Testing MSE	Training MSE	Testing MSE
1%	0.00084285	0.00084163	0.94161	0.96850
5%	0.02143969	0.02244063	0.969772	1.02011
10%	0.08273734	0.08263037	0.973837	1.02923
15%	0.17801648	0.1751732	0.987635	1.02352
20%	0.28960111	0.30050021	1.005389	1.09774

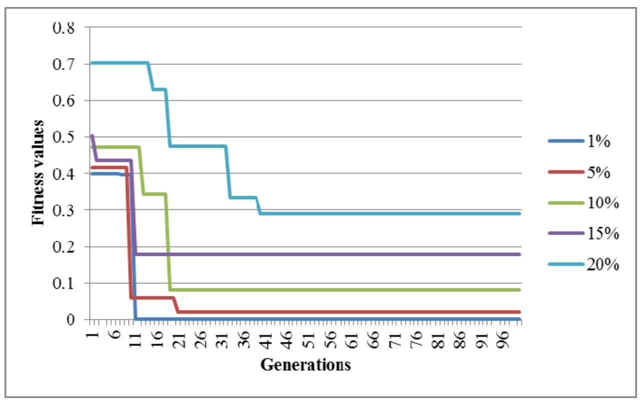
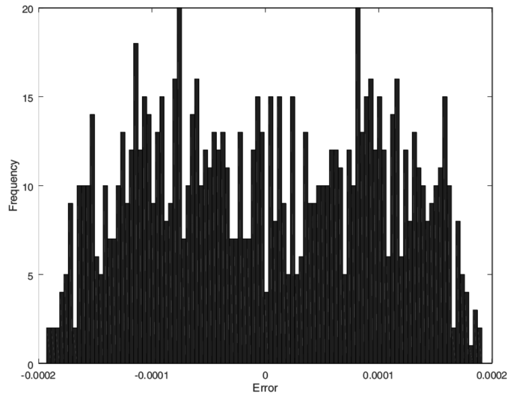


FIGURE 5. Evolutionary curves of our method with different noise rates for the first complex-valued system.

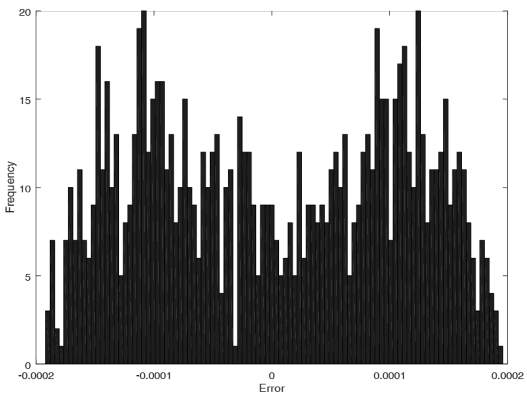
exceeds 15%, the errors between the coefficients of models optimized and standard model are very large.

Training and testing performance of our method and CNN with different noise rates are listed in Table 4, which reveal that our method has better training and testing performances than CNN with noise data on the whole. With different noise rates, CNN has similar MSE values. However our method has worse performances along with the increment of noise rate.

Evolutionary curves of our method with different noise rates are depicted in Fig. 5. The maximum generation is set as 100. From Fig. 5, it could be seen that our method could gain the optimal solutions with different noise rates when the



(a)



(b)

FIGURE 6. Prediction error distributions of real parts (a) and imaginary parts (b) for the second complex-valued system.

evolutionary generation achieves 50. It could be proved that our method could converge with few generations.

2) SYNTHETIC EXAMPLE 2

The second complex-valued system is synthetized by our team, which is more complex than the first complex-valued system, and contains complex-valued division and sine functions.

$$Z = (1.2 + 0.8i)\frac{Z_1}{Z_2} + (1 + i)\sin(Z_3) \tag{9}$$

Complex-valued variables Z_1, Z_2 and Z_3 are created randomly. 3000 samples and 1000 samples are utilized as training and testing datasets, respectively. Through 30 runs, the optimal complex-valued system is searched (Eq.(10)). Compared Eq.(9) and Eq.(10), two systems are very close and the coefficients are nearly the same.

$$\begin{aligned} Z = (1.200074 + 0.799879i)\frac{Z_1}{Z_2} &+ (1.000014 \\ + 1.000007i)\sin(Z_3) &+ (0.000033 - 0.00002i)Z_1 \end{aligned} \tag{10}$$

The predicted error distributions of real and imaginary parts are depicted in Fig. 6. The predicted errors manly concentrate in the interval [−0.0002, 0.0002], which reveal that our method could predict accurately complex-valued data. The predicted MSE performances of CNN, CRBF and our method are listed in Table 5. Compared the performances of three methods, our method has better performance than CNN and CRBF models.

TABLE 5. The testing MSE performances of CNN, CRBF and our method for the second complex-valued system.

Methods	Testing MSE
CNN [27]	1.063
CRBF [28]	0.539
CEM	1.08×10^{-8}

We also add 1%, 5%, 10%, 15% and 20% Gaussian white noise into the data from the second complex-valued system. The optimal CEMs gained are listed in Table 6. Compared with standard model (Eq.(9)), our method could identify the accurate structures when the noise rate reaches 20%. And our method could identify accurate complex-valued coefficients when the noise rate reaches 10%.

TABLE 6. CEMs of our method with different noise rates for the second complex-valued system.

Noise	CEMs
1%	$Z = (1.199357 + 0.799765i)\frac{Z_1}{Z_2} + (0.999989 + 0.999623i)\sin(Z_3) + (-0.000615 + 0.001325i)\cos(Z_3)$
5%	$Z = (1.190090 + 0.792259i)\frac{Z_1}{Z_2} + (0.990946 + 0.990372i)\sin(Z_3)$
10%	$Z = (1.149758 + 0.757281i)\frac{Z_1}{Z_2} + (0.971488 + 0.969822i)\sin(Z_3) + (-0.001638 - 0.006557i)\cos(Z_1)$
15%	$Z = (0.622831 + 0.008079i)\frac{Z_1}{Z_2} + (0.353598 + 0.982796i)\sin(Z_3) + (-0.018188 - 0.009214i)\sin(Z_2)$
20%	$Z = (0.04196 - 0.00568i)\frac{Z_1}{Z_2} + (0.901242 + 0.92086i)\sin(Z_3)$

Training and testing performances of our method and CNN with different noise rates are listed in Table 7. As the noise rates increase, CNN has similar MSE values and our method has worse performances. In all, our method has smaller MSE values than CNN when noise rates vary from 1% to 20%. Evolutionary curves of our method with different noise rates are depicted in Fig. 7, which reveals that our method could

TABLE 7. Training and testing performance of our method and CNN with different noise rates for the second complex-valued system.

Noise	Our method		CNN	
	Training MSE	Testing MSE	Training MSE	Testing MSE
1%	0.0007751	0.0007726	0.91778	0.90709
5%	0.0188049	0.0190787	0.91929	0.90831
10%	0.0742747	0.0749253	0.92117	0.92117
15%	0.7755990	0.7793782	0.92967	0.92777
20%	1.1284557	1.1270922	0.94193	0.94067

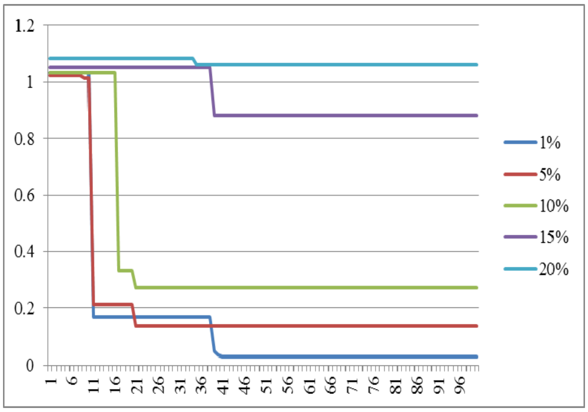


FIGURE 7. Evolutionary curves of our method with different noise rates for the second complex-valued system.

gain the optimal solutions with few generations for noise complex-valued datasets.

3) REAL-WORLD PROBLEM: NON-MINIMUM PHASE EQUALIZATION PROBLEM

The complex-valued non-minimum phase channel problem is utilized to test our method, which is three-order with nonlinear distortion for 4-QAM signaling [31]. The model is described as follows [26].

$$z_n = o_n + 0.1o_n^2 + 0.05o_n^3 + v_n, \quad v_n \aleph(0, 0.01), \quad (11)$$

$$o_n = (0.34 - 0.27i)s_n + (0.87 + 0.43i)s_{n-1} + (0.34 - 0.21)s_{n-2}. \quad (12)$$

where $\aleph(0, 0.01)$ is white Gaussian noise with mean 0 and variance 0.01. Complex-valued input variable s_n is selected randomly from the interval $[-7.5\text{-}7.5i, 7.5\text{+}7.5i]$. 3000 samples and 1000 samples are utilized as training and testing datasets, respectively. Through 30 runs, we gain the optimal complex-valued model as follows.

$$z_n = (0.018569 + 0.077476\,i)\,s_{n-1} + (0.331945 - 0.204832\,i) \times (s_n + s_{n-2}) + (0.832976 + 0.468700\,i)s_{n-1} \quad (13)$$

The predicted error distributions of real and imaginary parts for real-word problem are shown in Fig. 8, which reveal that most of predicted errors concentrate around zero and our method could predict accurately complex-valued data from real-word problem. The predicted MSE performances of seven complex-valued methods are listed in Table 8. From the results, we can see that in terms of MSE, our method is 10.5% smaller than CNN, 98.56% smaller than CRBF, 99.34% smaller than CMRAN, 98.5% smaller than CELM, 97.9% smaller than FC-RBF and 97.52% smaller than FC-RBF with KMC. Our method greatly improves the prediction accuracy of complex-valued data.

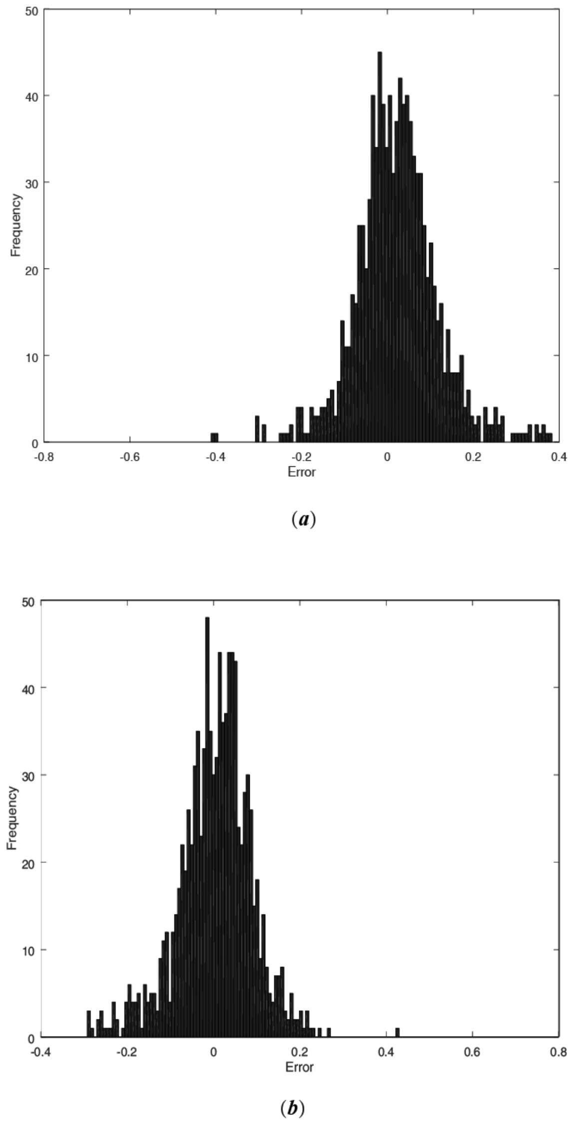


FIGURE 8. Prediction error distributions of real parts (a) and imaginary parts (b) for the real-word problem.

B. OPTIMIZATION ABILITY INVESTIGATION OF CVWWO

In order to investigate the optimization ability of CVWWO, we make the comparison experiments with the complex-valued versions (CVPSO [32] and CVCSA [33]) of classical swarm intelligent algorithm (particle swarm optimization) and new evolutionary algorithm (crow search algorithm). The maximum iteration is set as 100 and population size is

TABLE 8. The testing MSE performances of seven methods for the real-word problem.

Method	Testing MSE
CNN [27]	0.0096
CRBF [28]	0.5972
CMRAN [29]	1.3114
CELM [30]	0.5772
FC-RBF[26]	0.4142
FC-RBF with KMC [26]	0.3476
CEM	0.00859

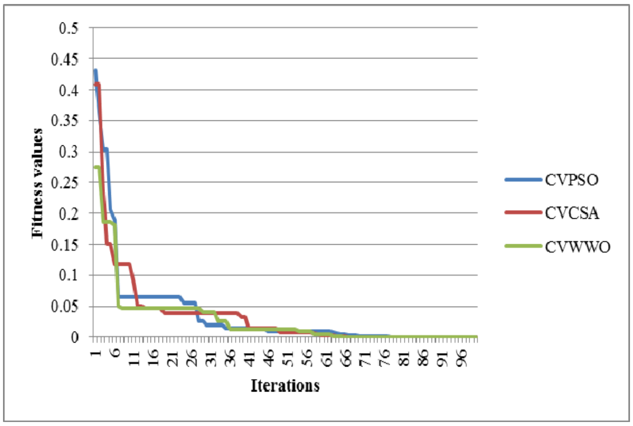


FIGURE 9. The training curves of CVPSO, CVCSA and CVWWO for the first complex-valued system.

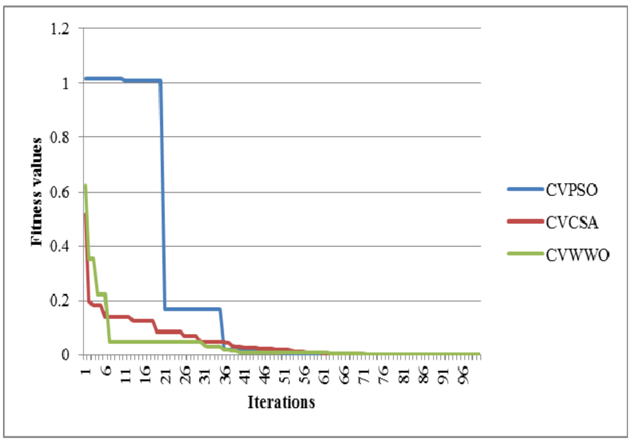


FIGURE 10. The training curves of CVPSO, CVCSA and CVWWO for the second complex-valued system.

set as 20. The fitness curves against generations with three complex-valued problems are plotted in Fig. 9, Fig. 10 and Fig. 11, respectively. Fig. 9 reveals that three optimization methods have similar performance and CVWWO has a slightly better convergence performance, which is because complex-valued coefficients are very simple in the first complex-valued system identification problem. As shown in Fig. 10 and Fig. 11, it could be obviously seen that

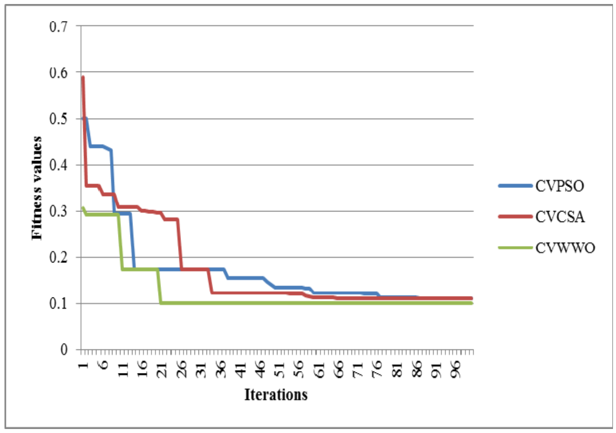


FIGURE 11. The training curves of CVPSO, CVCSA and CVWWO for the real-word problem.

CVWWO has better convergence performance than CVPSO and CVCSA.

IV. CONCLUSIONS

This paper proposes a novel complex-valued system identification method based on mathematical expression. In this approach, complex-valued gene expression programming is utilized to search the optimal architectures of expressions and complex-valued water wave optimization is proposed to optimize complex-valued coefficients and constants. The two artificial complex-valued function approximation problems and non-minimum phase equalization problem are utilized to test the performance of our proposed algorithm. The experiment results reveal that our method could identify complex-valued systems more correctly than complex-valued neural networks (CNN and CRBF) and complex-valued methods (CMRAN and CELM).

In order to test the ability of noise tolerance of our methods, our method is utilized to identify complex-valued systems with noise data. Our method could identify the accurate structures when the noise rate reaches 20%. And our method could identify accurate complex-valued coefficients when noise rate reaches 10%. When the noise rate is between 1% and 10%, the coefficients of models obtained are very close to those of standard models. The experiment results with noise data show that our method is noise-robust. In order to investigate the optimization ability of CVWWO, we make the comparison experiments with CVPSO and CVCSA. The experiments show that CVWWO has better convergence performance than CVPSO and CVCSA.

In the future, parallel methods will be applied for improving the identification speed of our proposed method. Our method will be utilized to identify more complex systems.

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