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# Selection of Satisfied Association Rules via Aggregation of Linguistic Satisfied Degrees

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**ABSTRACT** Many association rule mining algorithms have been well-established, such as Apriori, Eclat, FP-Growth, or LCM algorithms. However, the challenge is that the huge size of association rules is extracted by using these algorithms, and it is difficult for users to select satisfied association rules from them. In this paper, a new method is proposed to select satisfied association rules, which is based on the aggregation of fuzzy linguistic satisfied degrees of extracted association rules. To this end, two problems must be solved, one is which interesting measures are utilized to obtain fuzzy linguistic satisfied degrees of association rules and the other is how to aggregate them. For the first problem, many objective and subjective interesting measures have been proposed, which are generally included in [0, 1] or others universes and easily calculated by support, confidence, or other measures, these interesting measures cannot be directly aggregated, because different interesting measures represent different satisfied degrees of association rules. In this paper, a new transformation function is proposed to transform these interesting measures into fuzzy linguistic satisfied degrees, such as *dissatisfied*, *fair*, *satisfied*, and so on. For the second problem, by considering different weights of objective and subjective interesting measures, linguistic aggregation operators are designed to aggregate these fuzzy linguistic satisfied degrees of association rules. Accordingly, satisfied association rules are selected by using order on the aggregation results of linguistic satisfied degrees. In cases' study, Apriori, Eclat, FP-Growth, and LCM algorithms are first utilized to extract association rules with higher support or confidence measures from Chess, Connect, Mushroom, and T40I10D100K databases, then the proposed method is applied to obtain fuzzy linguistic satisfied degrees of extracted association rules and aggregate and select satisfied association rules from those extracted association rules, and comparison and analysis show that the proposed method is a useful and alternative tool to select satisfied association rules from extracted association rules.

**INDEX TERMS** Association rule mining, objective interesting measures, subjective interesting measures, fuzzy linguistic satisfied degree, linguistic aggregation operator.

## I. INTRODUCTION

After Agrawal proposed association rule mining [1], it has become one of the most popular data mining techniques and contributed to many advances in the area of knowledge discovery, by which implicit, previously unknown and potentially useful knowledge can be discovered from large datasets. Generally, association rule mining consists of two phases, one is to mine itemsets (or called patterns) according to support measure, the other is to generate association rules from mined itemsets according to confidence measure, which are formally explained as follows: Let $(U, \mathcal{A})$ be a transaction database, where $U$ be a non-empty finite set of transactions, $\mathcal{A}$ a non-empty finite set of items and each transaction $u \in U$ such that $u \subseteq \mathcal{A}$. Subset $A' \subseteq \mathcal{A}$ is called as an itemset and $u$ contains $A'$ if $A' \subseteq u$. The count of $A'$ is the number of transactions in $U$ that contain $A'$, suppose that the total number of transactions is $|U| = n$, support measure of $A'$ is defined by $Sup(A') = \frac{|\{u \in U | A' \subseteq u\}|}{n}$. $A'$ is called as frequent itemset (FI) if $Sup(A') \geq r$ ($r$ is a given minimum support). Theoretically, all FIs are mined from subsets of $\mathcal{A}$ by scanning $U$ many times to obtain their counts, *i.e.*, $2^{|\mathcal{A}|} - 1$ nonempty

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subsets of $\mathcal{A}$ and their counts. In addition, for each FI $F_I \subseteq \mathcal{A}$ such that $A_1 \cup A_2 = F_I$ and $A_1 \cap A_2 = \emptyset$, $A_1 \rightarrow A_2$ is an association rule generated by $F_I$, $A_1$ is as left side of the rule and $A_2$ as right side of the rule. Suppose $F_I$ with $l$ items, then there are $2^l - 2$ nonempty subsets can be used to generate association rules, in which, confidence measure $Con(A_1 \rightarrow A_2) = \frac{Sup(F_I)}{Sup(A_1)}$ of $A_1 \rightarrow A_2$ is widely used to evaluate "usefulness" of these association rules. It is obvious that mining all FIs and association rules become NP-hard problem when $U$ or $\mathcal{A}$ are in large datasets.

Existed association rule mining algorithms show that the challenge is the huge size of extracted association rules and many of them are redundant or useless in practical applications. Theoretically, the problem can be solved by utilizing interesting measures [2]–[4], *i.e.*, only those association rules satisfied by interesting measures are generated. Roughly, interesting measures of association rules can be divided into objective and subjective interesting measures. Objective interesting measures involve analyzing the association rules' structure, predictive performance and statistical significance, such as support and confidence measures which are mostly used in association rule mining, in addition, interest factor, certainty factor and entropy measures and so on. In [5], a comparative study has been made for twenty-one objective interesting measures, it seems that objective interesting measures may provide conflicting information in many situations, one should examine their properties in order to select the right objective interesting measure in association rule mining. Subjective interesting measures take into account users' the knowledge and interests in association rule mining, in [6], unexpectedness and actionability interesting measures are provided, where unexpectedness means that association rules are interesting if they are unknown for users or contradict with users' existing knowledge (or expectations), actionability means that association rules are interesting if users can do something with them, in practical applications, actionability is partially handled through unexpectedness because actionable rules are either expected or unexpected.

Compared objective interesting measures with subjective interesting measures in association rule mining methods, objective interesting measures are paid more attention due to their explicit definitions and statistical significance [7]–[18]. In fact, by considering objective interesting measures as a fitness function, association rule mining is transformed into multi-objective optimization problem, and many efficient algorithms can be designed, such as the cuckoo optimization algorithm [19], weighted rule-mining technique [20], evolutionary algorithm [21], principal components analysis [22], structured association map [23], multi-tier granule structures [24] and many fuzzy optimization algorithm [25]–[34]. Because decision making methods are similar with multi-objective optimization, recently, they become useful and alternative tools to mine association rules, such as in [35], multiple-criteria decision making method is utilized to evaluate objective interesting measures and help users to choose interesting measures in the context of association rules. In [36], performance, memory space and response time of mining algorithms are considered as decision criteria, then multiple-criteria decision analysis is utilized to choose the best association rule mining algorithm, the selected algorithm can be applied to extract association rules from medical records. In [37], based on involved criteria and covered examples by association rules, a new measure is presented to evaluate the similarity between two rules and a new genetic algorithm is provided to obtain a reduce set of different positive and negative quantitative association rules. In [38], an adaptive relational association rule mining method is proposed to discover interesting relational association rules from the set of extracted association rules, which was established by mining the data before the feature set changed and preserving the completeness. As our best knowledge, association rule mining algorithms based on support and confidence measures are well-established and widely used in large datasets, such as Apriori [1], Eclat [39], FP-Growth [40] or LCM [41] algorithms, adding others interesting measures in these algorithms to mine satisfied association rules generally face memory space or response time problem. Hence decision making methods may be useful and alternative tools to select satisfied association rules from the set of extracted association rules.

In the paper, fuzzy linguistic satisfied degrees are proposed to explain several objective and subjective interesting measures, which are generally used to evaluate interestingness of extracted association rules [5] and mine satisfied association rules, then fuzzy linguistic satisfied degrees are aggregated by considering different weights of objective and subjective interesting measures, accordingly, satisfied association rules are selected by using order on linguistic aggregation results of extracted association rules. Major contributions of the paper are summarized as follows:

1) Transform selection of satisfied association rules into a decision making problem, where criteria are objective and subjective interesting measures, which are easily calculated by support measure, confidence measure or structure of extracted association rule, alternatives are the set of extracted association rules which are generated by existed association rule mining algorithms;
2) Present a new transformation function to transform objective and subjective interesting measures of extracted association rules into fuzzy linguistic satisfied degrees. In practical applications, numbers in $[0, 1]$ or others universes of objective and subjective interesting measures cannot be directly aggregated according to properties of interesting measures, and fuzzy linguistic satisfied degrees is natural or artificial language, which can been easily understood by users and processed by linguistic information processing methods;
3) Propose different linguistic aggregation operators to aggregate fuzzy linguistic satisfied degrees of each

extracted association rule, which are based on different weights of objective and subjective interesting measures, then order on linguistic aggregation results are utilized to select satisfied association rules.

The rest of this paper is structured as follows: In Section II, interesting measures of association rules are discussed and 2-tuple fuzzy linguistic representation model is briefly reviewed, then selection of satisfied association rules is formalized as a decision making problem. In Section III, a new transformation function is proposed to transform interesting measures into fuzzy linguistic satisfied degrees of extracted association rules. In Section IV, linguistic aggregation operators according to weights of objective and subjective interesting measures are proposed to aggregate fuzzy linguistic satisfied degrees of extracted association rules, then selection of satisfied association rules is carried out by linguistic aggregation results. In Section V, Chess, Connect, Mushroom and T40I10D100K databases are utilized to experiment and analyze the proposed method, extracted association rules are generated by Apriori, Eclat, FP-Growth or LCM algorithms. Section VI is conclusion of the paper.

## II. PRELIMINARIES

In the section, forms and properties of interesting measures of association rules and 2-tuple fuzzy linguistic representation model are briefly reviewed, then satisfied association rule mining is processed as a decision making problem.

### A. INTERESTING MEASURES OF ASSOCIATION RULES

Interesting measures play an essential role in association rule mining, which are utilized to extract and reduce association rules from databases. In [2], interestingness is explained as it is a broad concept that emphasizes conciseness, coverage, reliability, peculiarity, diversity, novelty, surprisingness, utility and actionability, if or not an association rule is interesting is determined by these characteristic. According to existed studies, interesting measures of association rules are divided into objective and subjective interesting measures. In [42], a semantic measure is considered as the semantics and explanations of association rule, which is a special type of subjective interesting measure due to semantic involving domain knowledge from users.

#### 1) OBJECTIVE INTERESTING MEASURES

Theoretically, many objective interesting measures can be obtained from the raw database by using probability, statistics or information theory, where support measure $Sup(A \to B)$ represents generality of association rule and confidence measure $Con(A \to B)$ represents reliability of association rule, which are the most basic interesting measures and many other objective interesting measures can be calculated by them, such as cosine measure $C(A \to B) = \frac{Sup(A\to B)}{\sqrt{Sup(A)Sup(B)}}$ [2], conviction of association rule $CA(A \to B) = \frac{1-Sup(B)}{1-Con(A\to B)}$ [43], and weighted relative accuracy $W(A \to B) = Sup(A)(Con(A \to B) - Sup(B))$ [44]. Meanwhile, the properties of objective interesting measures, such as symmetric, monotonic, dependent, invariant and consistent, have been analyzed in [2], [5], [8], [21], [22]. Table 1 shows some objective interesting measures, in which transaction database is $(U, \mathcal{A})$, for any $A \subseteq \mathcal{A}$, $\overline{A} = \mathcal{A} - A$. In Table 1, $Add(A \to B)$ and $Lif(A \to B)$ can also be used to represent reliability of $A \to B$. $Acc(A \to B)$ is used to represent accuracy or veracity of $A \to B$. $Cer(A \to B)$ is used to represent variation of the probability that $B$ is in an example considering only those where $A$ is present, $Cer(A \to B) > 0$ means that $B$ is satisfied more frequently when $A$ is satisfied than it is generically, $Cer(A \to B) = 0$ means that $B$ is satisfied with the same frequency when $A$ is satisfied as it is generically, $Cer(A \to B) < 0$ means that $B$ is satisfied less frequently when $A$ is satisfied than it is generically [10]. $Int(A \to B)$ is used to represent surprise of association rule for user, which discovers not only the rules with higher frequency but also the rules comparatively less frequency in the database [48].

#### 2) SUBJECTIVE INTERESTING MEASURES

Different with objective interesting measures, which are data-driven and only take into account the data cardinalities, subjective interesting measures are user-driven in the sense that take into account the user's a priori knowledge. In [6], unexpectedness and actionability of association rules have been discussed as the two main subjective interesting measures, and actionability is partially handled through unexpectedness because actionable rules are either expected or unexpected. From the granular computing point of view, the user's a priori knowledge on the set of transactions or items are generally represented by information granules, which can be induced by indiscernible, equivalent or similar relations on transactions or items [49]–[51]. For sake of simplicity, suppose that information granule $A_e \subset \mathcal{A}$ of a transaction database $(U, \mathcal{A})$ is the user's expected knowledge and information granule $A_{\overline{e}} \subset \mathcal{A}$ is the user's unexpected knowledge, where $A_e \cap A_{\overline{e}} = \emptyset$ and $A_e \cup A_{\overline{e}} \subseteq \mathcal{A}$. If $A_e \cup A_{\overline{e}} = \mathcal{A}$, then we call that user owns distinct knowledge about $(U, \mathcal{A})$, *i.e.*, expected and unexpected knowledge are distinct. Otherwise, there exists indeterminate knowledge in the user's a priori knowledge. Accordingly, the following subjective interesting measures of association rules are proposed, *i.e.*, for any extracted association rule $A \to B$,

- Complete expected degree of association rule: Complete expected degree of $A \to B$ can be defined by

$$\mu_{ce}(A \to B) = \frac{|(A \cup B) \cap A_e|}{|A \cup B|}, \tag{1}$$

where $|\cdot|$ is cardinality of a set. If $\mu_{ce}(A \to B) = 1$ (or $A \cup B \subseteq A_e$), then the association rule $A \to B$ is called as a complete expected association rule, *i.e.*, both left and right sides of $A \to B$ are conformed to the user's expected knowledge, which is also called as a conforming association rule [6];

**TABLE 1. Some objective interesting measures of association rules.**

| ID | Measure name | Definition | Domain |
|---|---|---|---|
| 1 | The relative support of $A \subseteq \mathcal{A}$ [1] | $Sup(A) = \frac{\lvert\{u \in U \mid A \subseteq u\}\rvert}{n}$ | [0, 1] |
| 2 | The relative support of association rule | $Sup(A \to B) = Sup(A \cup B)$ | [0, 1] |
| 3 | The confidence of association rule [1] | $Con(A \to B) = \frac{Sup(A \to B)}{Sup(A)}$ | [0, 1] |
| 4 | Accuracy of association rule [2] | $Acc(A \to B) = Sup(A \to B) + Sup(\overline{A} \to \overline{B})$ | [0, 1] |
| 5 | Added value or change of support [17] | $Add(A \to B) = Con(A \to B) - Sup(B)$ | [-0.5, 1] |
| 6 | Lift of association rule [45] | $Lif(A \to B) = \frac{Sup(A \to B)}{Sup(B)}$ | [0, 1] |
| 7 | Certainty Factor of association rule [46] | $Cer(A \to B) = \begin{cases} \frac{Con(A \to B) - Sup(B)}{1 - Sup(B)}, \text{if } Con(A \to B) > Sup(B), \\ \frac{Con(A \to B) - Sup(B)}{Sup(B)}, \text{if } Con(A \to B) \leq Sup(B). \end{cases}$ | [-1, 1] |
| 8 | Interestingness of association rule [47] | $Int(A \to B) = \frac{Sup(A \to B)}{Sup(A)} \times \frac{Sup(A \to B)}{Sup(B)} \times (1 - Sup(A \to B))$ | [0, 1] |

- Left expected degree of association rule: Left expected degree of $A \to B$ can be defined by

$$\mu_{le}(A \to B) = \frac{|A \cap A_e|}{|A|}. \tag{2}$$

If $\mu_{le}(A \to B) = 1$ (or $A \subseteq A_e$), then the association rule $A \to B$ is called as a left expected association rule, *i.e.*, left side of $A \to B$ is conformed to the user's expected knowledge;

- Right expected degree of association rule: Right expected degree of $A \to B$ can be defined by

$$\mu_{re}(A \to B) = \frac{|B \cap A_e|}{|B|}. \tag{3}$$

If $\mu_{re}(A \to B) = 1$ (or $B \subseteq A_e$), then the association rule $A \to B$ is called as a right expected association rule, *i.e.*, right side of $A \to B$ is conformed to the user's expected knowledge;

- Complete unexpected degree of association rule: Complete unexpected degree of $A \to B$ can be defined by

$$\mu_{cu}(A \to B) = \frac{|(A \cup B) \cap A_{\overline{e}}|}{|A \cup B|}, \tag{4}$$

If $\mu_{cu}(A \to B) = 1$ (or $A \cup B \subseteq A_{\overline{e}}$), then the association rule $A \to B$ is called as a complete unexpected association rule, *i.e.*, both left and right sides of $A \to B$ are conformed to the user's unexpected knowledge;

- Left unexpected degree of association rule: Left unexpected degree of $A \to B$ can be defined by

$$\mu_{lu}(A \to B) = \frac{|A \cap A_{\overline{e}}|}{|A|}. \tag{5}$$

If $\mu_{lu}(A \to B) = 1$ (or $A \subseteq A_{\overline{e}}$), then the association rule $A \to B$ is called as a left unexpected association rule, *i.e.*, left side of $A \to B$ is conformed to the user's unexpected knowledge;

- Right unexpected degree of association rule: Right unexpected degree of $A \to B$ can be defined by

$$\mu_{ru}(A \to B) = \frac{|B \cap A_{\overline{e}}|}{|B|}. \tag{6}$$

If $\mu_{ru}(A \to B) = 1$ (or $B \subseteq A_{\overline{e}}$), then the association rule $A \to B$ is called as a right unexpected association rule, *i.e.*, right side of $A \to B$ is conformed to the user's unexpected knowledge.

It is obvious that subjective interesting measures $\mu_*(A \to B)$ is in [0, 1] according to Eqs. (1)-(6). In practical applications, the more $\mu_*(A \to B)$ is, the more expected or unexpected association rule is. Formally, the above mentioned objective and subjective interesting measures as criteria can be utilized to select *interesting*, *non-redundant*, *maximal-minimal*, *actionable*, *expected* or *unexpected association rules* and so on.

In this paper, $Acc(A \to B)$, $Cer(A \to B)$ and $Int(A \to B)$ are selected as objective evaluation criteria and six subjective interesting measures $\mu_*(A \to B)$ defined by Eqs. (1)-(6) as subjective evaluation criteria, the set of extracted association rules is generated by Apriori, Eclat, FP-Growth or LCM algorithms with higher $Sup(A \to B)$ and $Con(A \to B)$. Then selection of satisfied association rules is transformed into a decision making problem, *i.e.*, extracted association rules as alternatives are evaluated by criteria $Acc(A \to B)$, $Cer(A \to B)$, $Int(A \to B)$ and six subjective interesting measures $\mu_*(A \to B)$, decision making table can be constructed in Table 2. Accordingly, decision making method can be utilized to select satisfied association rules instead of multi-objective optimization models, which are widely used to mine satisfied association rules.

It can be noticed from Table 1 that universes of $Acc(A \to B)$ and $Cer(A \to B)$ are different, *i.e.*, [0, 1] and $[-1, 1]$, in addition, $Int(A \to B)$ is general less than $Acc(A \to B)$, this means that they correspond to different satisfied degrees despite $Int(A \to B)$ and $Acc(A \to B)$ have the same universe [0, 1]. Similarly, expected and unexpected degrees of association rules also correspond to different satisfied degrees on universes [0, 1], hence numbers of these interesting measures cannot be directly aggregated. In the paper, these numbers of interesting measures are transformed into fuzzy linguistic satisfied degrees of extracted association rules, or they are normalized as linguistic satisfied degrees of extracted association rules.

### B. 2-TUPLE LINGUISTIC REPRESENTATION MODEL

Linguistic variable is the foundation of computing with words or linguistic information processing [51], in which information is represented by linguistic values, which are natural or artificial language and consisted by names of linguistic values (or called as linguistic terms) and fuzzy sets, fuzzy

**TABLE 2.** Decision making table of selection of satisfied association rules.

| Rules | Objective and subjective interesting measures | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Object interesting measures | | | Subjective interesting measures | | | | | |
| | $Acc()$ | $Cer()$ | $Int()$ | $\mu_{ce}()$ | $\mu_{le}()$ | $\mu_{re}()$ | $\mu_{cu}()$ | $\mu_{lu}()$ | $\mu_{ru}()$ |
| $R_1 : A_1 \rightarrow B_1$ | $v_{11}$ | $v_{12}$ | $v_{13}$ | $v_{14}$ | $v_{15}$ | $v_{16}$ | $v_{17}$ | $v_{18}$ | $v_{19}$ |
| $R_2 : A_2 \rightarrow B_2$ | $v_{21}$ | $v_{22}$ | $v_{23}$ | $v_{24}$ | $v_{25}$ | $v_{26}$ | $v_{27}$ | $v_{28}$ | $v_{29}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $R_m : A_m \rightarrow B_m$ | $v_{m1}$ | $v_{m2}$ | $v_{m3}$ | $v_{m4}$ | $v_{m5}$ | $v_{m6}$ | $v_{m7}$ | $v_{m8}$ | $v_{m9}$ |

sets are utilized to explain linguistic terms and linguistic terms are utilized to describe fuzzy sets. Up to now, linguistic decision making is an important application of computing with words or linguistic information processing [52]–[57], because fuzzy linguistic values provide a more direct way to represent imprecise or uncertain information in decision making problems, the representation is closest to human being's cognitive processes that occurs in real life. In linguistic decision making, linguistic decision making methods based on 2-tuple linguistic representation model [58] have been paid more attention due to its' computational simplicity, no loss information, the accuracy and understandability [59]–[67]. 2-tuple linguistic representation model can be formally expressed as follows:

Let $L = \{s_0, s_1, \cdots, s_g\}$ be an initial linguistic term set, whose semantics are provided by fuzzy sets on a universe of discourse, a total order on $L$ is for any $s_i, s_j \in S$, $s_i \leq s_j$ if and only if $i \leq j$, operators on $S$ are $Neg(s_i) = s_j$ where $j = g - i$, $max\{s_i, s_j\} = s_j$ and $min\{s_i, s_j\} = s_i$ if $s_i \leq s_j$; $\alpha$ is a numerical value that represents the value of the symbolic translation, *i.e.*, $\alpha = [-0.5, 0.5)$ if $s_i \in \{s_1, \cdots, s_{g-1}\}$; $\alpha = [0, 0.5)$ if $s_i = s_0$; $\alpha = [-0.5, 0)$ if $s_i = s_g$. Theoretically, 2-tuple fuzzy linguistic representation model provides transformation from numerical values of $[0, g]$ to 2-tuple linguistic terms on linguistic term set $L$:

$$\Delta : [0, g] \longrightarrow L \times [-0.5, 0.5), \beta \mapsto \Delta(\beta) = (s_j, \alpha). \qquad (7)$$

where $j = round(\beta)$, $\alpha = \beta - j \in [-0.5, 0.5)$ and $round(\cdot)$ is the usual rounding operation, $s_j \in L$ is the linguistic term that is mostly close to $\beta$ and $\alpha$ represents the symbolic translation value. $\Delta$ is an one-to-one mapping, its inverse function transforms 2-tuple linguistic terms to its equivalent numerical values, *i.e.*,

$$\Delta^{-1} : L \times [-0.5, 0.5) \longrightarrow [0, g],$$
$$(s_j, \alpha) \longmapsto \Delta^{-1}(s_j, \alpha) = j + \alpha \in [0, g]. \qquad (8)$$

Formally, the 2-tuple linguistic term $(s_i, \alpha)$ can be used to represent continues linguistic information on the universe of discourse, such as in Fig.1, fuzzy set of each linguistic term $s_i \in L = \{s_0, \cdots, s_6\}$ is defined on the universe of discourse $[0, 1]$, fuzzy set of 2-tuple linguistic term $(s_3, 0.4)$ on $L$ can be induced by $\Delta$ or $\Delta^{-1}$, where $u \in [0, 1]$ is the universe of discourse of linguistic terms $L$, $\mu$ is membership degree of fuzzy set, for example membership degree of $u = 0.75$ corresponding to fuzzy set of linguistic term $s_5$ is 0.6. In the paper,

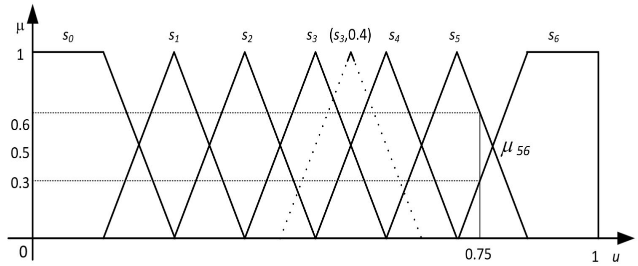


**FIGURE 1.** The fuzzy set of 2-tuple linguistic term $(s_3, 0.4)$ on $[0, 1]$.

denote all 2-tuple linguistic terms as $L_{[0,g]} = \{(s_i, \alpha)|s_i \in L, \alpha \in [-0.5, 0.5)\}$.

Combined with linguistic decision making method based on 2-tuple linguistic terms, selection of satisfied association rules is consisted by the following five steps: 1) Intelligence: Alternatives of the problem are the set of extracted association rules from a database by using Apriori, Eclat, FP-Growth or LCM algorithms, the objective is to evaluate extracted association rules and select satisfied association rules; 2) Modeling: The framework is shown in Fig.2, where each extracted association rule is evaluated by objective and subjective interesting measures, which represent different satisfaction of association rules; 3) Information gathering: Table 2 represents information gathering, each $v_{ij}(i = 1, \cdots, m, j = 1, \cdots, 9)$ can be calculated by the raw database and the user's domain or background knowledge; 4) Analysis: Evaluation values of Table 2 cannot be aggregated directly due to their different satisfied degrees, which are transformed into 2-tuple linguistic terms on initial linguistic satisfied degrees set. Then linguistic aggregation operators are provide to aggregate these linguistic satisfied degrees; 5) Selection: According to linguistic satisfied degree results, satisfied association rules can be selected. The five steps are also shown in Fig.(2).

## III. LINGUISTIC SATISFIED DEGREES OF EXTRACTED ASSOCIATION RULES

In this section, initial linguistic satisfied degrees are designed and a new transformation function is provide to transform evaluation values of Table 2 into 2-tuple linguistic satisfied degrees of extracted association rules.

### A. INITIAL LINGUISTIC SATISFIED DEGREE TERMS

In the paper, initial linguistic satisfied degrees are designed as $L = \{s_0$ (*very dissatisfied*), $s_1$ (*dissatisfied*), $s_2$ (*slightly dissatisfied*), $s_3$ (*fair*), $s_4$ (*slightly satisfied*), $s_5$ (*satisfied*),

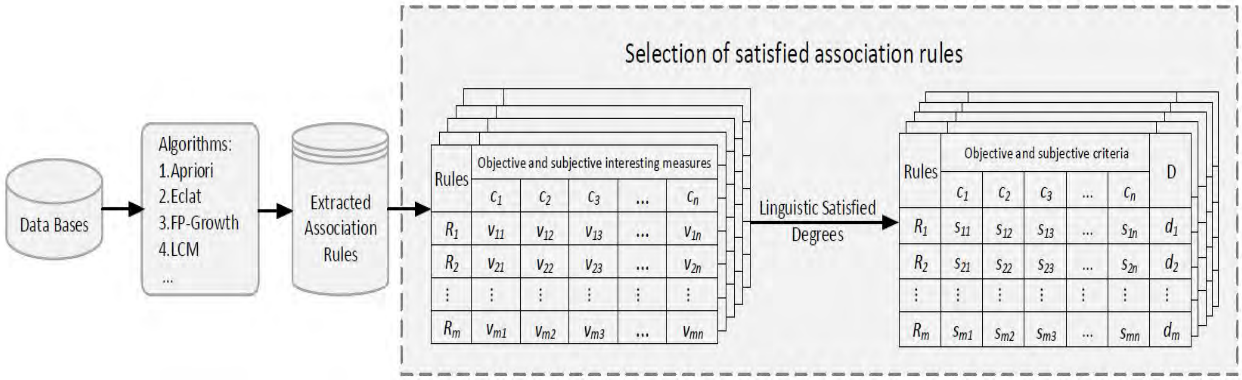


**FIGURE 2.** Selection of satisfied association rules via aggregation of fuzzy linguistic satisfied degrees.

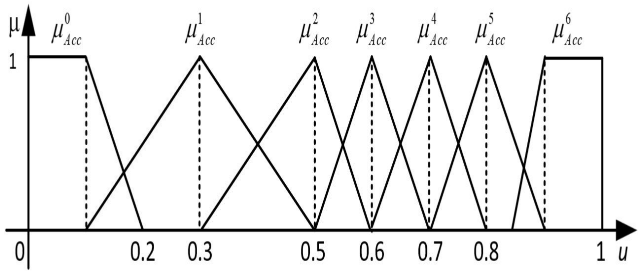


**FIGURE 3.** Fuzzy sets of linguistic satisfied degrees for accuracy.

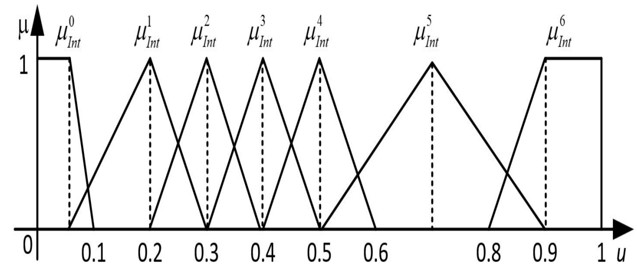


**FIGURE 5.** Fuzzy sets of linguistic satisfied degrees for interestingness.

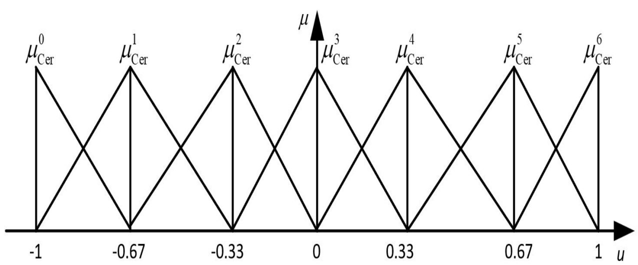


**FIGURE 4.** Fuzzy sets of linguistic satisfied degrees for certainty factor.

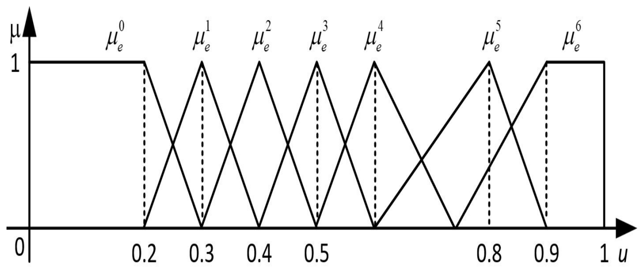


**FIGURE 6.** Fuzzy sets of linguistic satisfied degrees for expected degree.

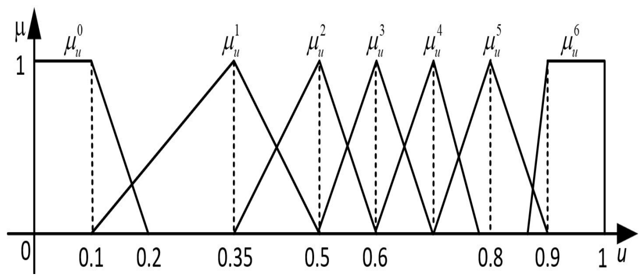


**FIGURE 7.** Fuzzy sets of linguistic satisfied degrees for unexpected degree.

$s_6$ (*very satisfied*)}, which are utilized to evaluate satisfied degrees of extracted association rules according to interesting measures of $Acc()$, $Cer()$, $Int()$, $\mu_{ce}()$, $\mu_{le}()$, $\mu_{re}()$, $\mu_{cu}()$, $\mu_{lu}()$ and $\mu_{ru}()$ in Table 2. Based on analysis of objective and subjective interesting measures, different interesting measures of Table 2 correspond to different linguistic satisfied degrees, hence fuzzy sets of linguistic satisfied degree term for different interesting measures are different, in the paper, triangular or trapezoidal fuzzy sets of linguistic satisfied degree term for different interesting measures are designed, such as fuzzy sets of linguistic satisfied degrees for accuracy of association rule are $\{\mu^0_{Acc}, \cdots, \mu^6_{Acc}\}$ shown in Fig.(3), where $u \in [0, 1]$ is domain of accuracy of association rule. Others triangular or trapezoidal fuzzy sets of linguistic satisfied degree terms are shown in Figs.(4)-(7), respectively, where $u \in [-1, 1]$ of Fig.(4) is domain of certainty factor of association rule, $u \in [0, 1]$ of Fig.(5) is domain of interestingness of association rule, $u \in [0, 1]$ of Fig.(6) is domain of expected degree of association rule and $u \in [0, 1]$ of Fig.(7) is domain of unexpected degree of association rule, $\mu$ of Figs.(3)-(7) is membership degree of fuzzy set. Theoretically, these fuzzy sets can be utilized to transform each interesting measure of association rule into linguistic satisfied degree $s_i \in L$ of the association rule.

### B. A NEW TRANSFORMATION FUNCTION

To obtain linguistic satisfied degree of an association rule for each interesting measure $v_{ij}$ in Table 2, $v_{ij}$ needs to be transform into 2-tuple linguistic term on initial linguistic satisfied degrees $L$ according to their fuzzy sets shown in Figs.(3)-(7) on domains of interesting measures, respectively. Existed transformation methods are summarized as follows:

- In fuzzy set theory [51], $v_{ij}$ can be transformed into linguistic value $s_k(k = 0, \cdots, 6)$ according to the maximum membership degree principle, *i.e.*, $\mu_k(v_{ij}) = max\{\mu_0(v_{ij}), \cdots, \mu_6(v_{ij})\}$, such as in Fig.2, $v_{ij}$ can be described by linguistic term $s_8$ with membership degree $\mu_8(v_{ij})$.
- In 2-tuple linguistic representation model [58], $v_{ij}$ can be transformed into 2-tuple linguistic term $(s_k, \alpha)$ $(k = 0, \cdots, 6)$ on $L = \{s_0, \cdots, s_6\}$ according to membership degrees of $v_{ij}$ in all fuzzy sets of linguistic satisfied degrees, *i.e.*, $(s_k, \alpha) = \Delta(\sum_{k=0}^{6} \frac{\mu_k(v_{ij})\times k}{\sum_{k=0}^{6}\mu_k(v_{ij})})$.

In this paper, a new transformation function is proposed to obtain 2-tuple linguistic term $(s_k, \alpha_k)$ of interesting measure $v_{ij}$ in Table 2, which is formalized as follows: Let initial linguistic terms $L = \{s_0, \cdots, s_g\}$ are defined on universe $[a_1, a_2]$, triangular or trapezoidal fuzzy sets of initial linguistic terms are $\{\mu_0, \cdots, \mu_g\}$ on $[a_1, a_2]$ and their centers are $\{v_0, \cdots, v_g\}$, *i.e.*, for any $k \in \{0, \cdots, g\}$, center $v_k$ of $\mu_k$ is the center of the set $\{v|\mu_k(v) = 1\}$. Denote cross point of triangular fuzzy membership functions $\mu_{k-1}$ and $\mu_k$ is $\mu_{(k-1)k}$, such as in Fig.3, the cross point of triangular fuzzy sets $\mu_2$ and $\mu_3$ of linguistic satisfied degree $s_2$ and $s_3$ is $\mu_{23}$. Then for any $v \in [a_1, a_2]$, 2-tuple linguistic term $(s_k, \alpha_k)$ of $v$ is decided by

$$\begin{aligned} \Delta : [a_1, a_2] &\longrightarrow L_{[0,g]} \\ v &\longmapsto (s_k, \alpha_k), \end{aligned} \tag{9}$$

$s_k$ satisfies $\mu_k(v) = max\{\mu_0(v), \cdots, \mu_g(v)\}$ and $\alpha_k$ is

$$\alpha_k = \frac{S(v-v_k)(\mu_k(v)-1)}{(1-S(v-v_k))\mu_{(k-1)k} + (1+S(v-v_k))\mu_{k(k+1)} - 2}$$

where $S(v - v_k)$ is sign function, *i.e.*, $S(v - v_k) = 1$ if $v - v_k \geq 0$ and $S(v - v_k) = -1$ if $v - v_k < 0$.

*Property 1:* In Eq.(9), $\alpha_k$ satisfies the following propositions:

1) If $v - v_k < 0$, then $\alpha_k \in [-0.5, 0)$;
2) If $v - v_k = 0$, then $\alpha_k = 0$;
3) If $v - v_k > 0$, then $\alpha_k \in (0, 0.5]$.

*Proof:* 1) If $v-v_k < 0$, then $S(v-v_k) = -1$ and $v$ is placed at the left of the center $v_k$. in such case,

$$\begin{aligned} \alpha_k &= \frac{-(\mu_k(v)-1)}{(1-(-1))\mu_{(k-1)k} + (1+(-1))\mu_{k(k+1)} - 2} \\ &= \frac{1-\mu_k(v)}{2\mu_{(k-1)k} - 2} = -\frac{1}{2} \times \frac{1-\mu_k(v)}{1-\mu_{(k-1)k}}. \end{aligned}$$

Because $\mu_k(v) = max\{\mu_0(v), \cdots, \mu_g(v)\}$ and $\mu_{(k-1)k}$ is the cross point of $\mu_{k-1}$ and $\mu_k$, we have $\mu_{(k-1)k} \leq \mu_k(v)$, *i.e.*, $0 < \frac{1-\mu_k(v)}{1-\mu_{(k-1)k}} \leq 1$ and $\alpha_k = -\frac{1}{2} \times \frac{1-\mu_k(v)}{1-\mu_{(k-1)k}} \in [-0.5, 0)$, especially, if $\mu_k(v) = \mu_{(k-1)k}$, then $\alpha_k = -0.5$.

2) If $v - v_k = 0$, then $v = v_k$, *i.e.*, $\mu_k(v) = \mu_k(v_k) = 1$ and $S(v - v_k)(\mu_k(v) - 1) = 0$, hence $\alpha_k = 0$.

3) If $v - v_k > 0$, then $S(v - v_k) = 1$ and $v$ is placed at the right of the center $v_k$. in such case,

$$\begin{aligned} \alpha_k &= \frac{-(\mu_k(v)-1)}{(1-(-1))\mu_{(k-1)k} + (1+(-1))\mu_{k(k+1)} - 2} \\ &= \frac{\mu_k(v)-1}{2\mu_{k(k+1)} - 2} = \frac{1}{2} \times \frac{1-\mu_k(v)}{1-\mu_{k(k+1)}}. \end{aligned}$$

Because $\mu_k(v) = max\{\mu_0(v), \cdots, \mu_g(v)\}$ and $\mu_{(k-1)k}$ is the cross point of $\mu_k$ and $\mu_{k+1}$, we have $\mu_{k(k+1)} \leq \mu_k(v)$, *i.e.*, $0 < \frac{1-\mu_k(v)}{1-\mu_{k(k+1)}} \leq 1$ and $\alpha_k = \frac{1}{2} \times \frac{1-\mu_k(v)}{1-\mu_{k(k+1)}} \in (0, 0.5]$, especially, if $\mu_k(v) = \mu_{k(k+1)}$, then $\alpha_k = 0.5$.

Intuitively, Property 1 shows that the transformation function $\Delta$ based on Eq.(9) can be utilized to transform any number $v$ in $[a_1, a_2]$ into 2-tuple linguistic term $(s_k, \alpha_k)$ on initial linguistic terms $L$ with fuzzy sets defined on $[a_1, a_2]$, where 1) the number $v$ with the membership degree in $[\mu_{(k-1)k}, 1)$ is described by the 2-tuple linguistic term $(s_k, \alpha_k)(\alpha_k \in [-0.5, 0))$, and $v$ with the membership degree $\mu_{(k-1)k}$ is described by $(s_k, -0.5)$; 2) $v = v_k$ with the membership degree 1 is exactly described by $s_k$; 3) the number $v$ with the membership degree in $[\mu_{k(k+1)}, 1)$ is described by the 2-tuple linguistic term $(s_k, \alpha_k)(\alpha_k \in (0, 0.5])$, and $v$ with the membership degree $\mu_{k(k+1)}$ is described by $(s_k, 0.5) = (s_{k+1}, -0.5)$, it is coincide with our sense.

*Example 1:* In Fig.1, for $v = 0.75$, according to the maximum membership degree principle, $v = 0.75$ is transformed into $s_5$ because membership degree 0.6 of $v$ is maximum.

Based on 2-tuple linguistic representation model, $v = 0.75$ is transformed into 2-tuple linguistic term $(s_5, 0.33)$ due to $\Delta(\frac{0.6\times5+0.3\times6}{0.6+0.3}) \doteq \Delta(5.33) = (s_5, 0.33)$.

Based on the new transformation function $\Delta$ defined by Eq.(9), $v = 0.75$ is transformed into 2-tuple linguistic term $(s_5, 0.4)$ due to $v - v_5 > 0$ and $\alpha = \frac{0.6-1}{2\times0.5-2} = 0.4$.

Based on the transformation function $\Delta$ defined by Eq.(9), decision making table of selection of satisfied association rules shown in Table 2 can be transformed into linguistic satisfied degree decision making table of association rules, which is shown in Table 3.

## IV. AGGREGATION LINGUISTIC SATISFIED DEGREES AND SELECTION OF SATISFIED ASSOCIATION RULES

In this section, based on Table 3 and weights of objective and subjective interesting measures, we propose linguistic aggregation operators to aggregate linguistic satisfied degrees of association rules, then satisfied association rules can be selected from extracted association rules according to order on linguistic evaluation results of association rules.

**TABLE 3.** Linguistic satisfied degree decision making table of association rules.

| Rules | Linguistic satisfied degrees | | | | | | | | | $D$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Objective Criteria | | | Subjective Criteria | | | | | | |
| | $Acc()$ | $Cer()$ | $Int()$ | $\mu_{ce}()$ | $\mu_{le}()$ | $\mu_{re}()$ | $\mu_{cu}()$ | $\mu_{lu}()$ | $\mu_{ru}()$ | |
| $R_1 : A_1 \to B_1$ | $s_{11}$ | $s_{12}$ | $s_{13}$ | $s_{14}$ | $s_{15}$ | $s_{16}$ | $s_{17}$ | $s_{18}$ | $s_{19}$ | $d_1$ |
| $R_2 : A_2 \to B_2$ | $s_{21}$ | $s_{22}$ | $s_{23}$ | $s_{24}$ | $s_{25}$ | $s_{26}$ | $s_{27}$ | $s_{28}$ | $s_{29}$ | $d_2$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $R_m : A_m \to B_m$ | $s_{m1}$ | $s_{m2}$ | $s_{m3}$ | $s_{m4}$ | $s_{m5}$ | $s_{m6}$ | $s_{m7}$ | $s_{m8}$ | $s_{m9}$ | $d_m$ |

### A. 2-TUPLE LINGUISTIC AGGREGATION OPERATOR

Existed 2-tuple linguistic aggregation operators can be roughly divided into two categories: 1) non-considering weight information; 2) considering weight information. Here, three 2-tuple linguistic aggregation operators [59] are reviewed. Formally, let $L = \{s_0, \cdots, s_g\}$ be an initial linguistic term set, 2-tuple linguistic terms $X = \{(s_1, \alpha_1), \cdots, (s_n, \alpha_n)\} \subseteq L_{[0,g]}$ are aggregated.

(1) 2-tuple linguistic arithmetic mean $\overline{x} : L^n_{[0,g]} \to L_{[0,g]}$, which is the non-considering weight information linguistic aggregation operator and formalized as

$$\begin{aligned}\overline{x}((s_1,\alpha_1),\cdots,(s_n,\alpha_n)) &= \Delta(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}(s_j,\alpha_j))\\ &= \Delta(\frac{1}{n}\sum_{j=1}^{n}\beta_j), \qquad (10)\end{aligned}$$

The 2-tuple linguistic arithmetic mean allows us to compute the mean of 2-tuple linguistic terms in a linguistic and precise way without any approximation process.

(2) 2-tuple linguistic weighted averaging operator $\overline{x}_w : L^n_{[0,g]} \to L_{[0,g]}$, which is the considering weight information linguistic aggregation operator and formalized as

$$\begin{aligned}\overline{x}_w((s_1,\alpha_1),\cdots,(s_n,\alpha_n)) &= \Delta(\sum_{j=1}^{n}w_j\Delta^{-1}(s_j,\alpha_j))\\ &= \Delta(\sum_{j=1}^{n}w_j\beta_j). \qquad (11)\end{aligned}$$

in which, $\{w_1, \cdots, w_n\}$ such that each $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$ are weights of 2-tuple linguistic terms in $X$.

(3) 2-tuple linguistic ordered weighted averaging operator $\overline{x}_o : L^n_{[0,g]} \to L_{[0,g]}$, which is the considering partial known weight information linguistic aggregation operator and formalized as

$$\overline{x}_o((s_1,\alpha_1),\cdots,(s_n,\alpha_n)) = \Delta(\sum_{j=1}^{n}w_j\beta_j), \qquad (12)$$

in which, $\beta_j$ is the $j$th largest value in $\{\beta_{j'} = \Delta^{-1}(s_{j'}, \alpha_{j'}) | j' = 1, 2, \cdots, n\}$ and weight $w_j$ is decided by fuzzy linguistic quantifier [68], *i.e.*,

$$w_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n}), \quad j = 1, 2, \cdots, n. \qquad (13)$$

Function $Q : [0, 1] \longrightarrow [0, 1]$ such that $\forall x, y \in [0, 1]$, $Q(x) \leq Q(y)$ if $x \leq y$ and

$$Q(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq a,\\ \dfrac{x-a}{b-a}, & \text{if } a < x \leq b,\\ 1, & \text{if } b < x \leq 1,\end{cases} \qquad (14)$$

parameters $a, b \in [0, 1]$ and $a < b$, different $a$ and $b$ mean different fuzzy linguistic quantifier, such as "*Most*" with $a = 0.3$ and $b = 0.8$, "*At least half*" with $a = 0$ and $b = 0.5$ and " *As many as possible* " with $a = 0.5$ and $b = 1$.

### B. AGGREGATING LINGUISTIC SATISFIED DEGREES

Theoretically, linguistic satisfied degrees of extracted association rules in Table 3 can be aggregated by any linguistic aggregation operator. However, in this paper, the user's expected and unexpected knowledge satisfies $A_e \cap A_{\overline{e}} = \emptyset$, *i.e.*, linguistic satisfied degrees of expected and unexpected degrees are exclusive, hence for each extracted association rule $R_i$, denote $S_{i2} = \{s_{i4}, s_{i5}, s_{i6}, s_{i7}, s_{i8}, s_{i9}\}$ and

$$s_{max}(R_i) = maxS_{i2} = max\{s_{i4}, s_{i5}, s_{i6}, s_{i7}, s_{i8}, s_{i9}\}. \qquad (15)$$

In practical applications, the more the value $s_{max}(R_i)$ is, the more the rule $R_i$ is expected exclusive-or unexpected. According to Table 3 and Eqs.(10)-(12), the following linguistic aggregation operators are provided to aggregate linguistic satisfied degrees of extracted association rules.

#### 1) NON-CONSIDERING WEIGHTS OF EVALUATION CRITERIA

Because evaluation criteria of extracted association rules are divided into object criteria and subjective criteria, the following two situations are considered in 2-tuple linguistic arithmetic mean of linguistic satisfied degrees.

*(1) Non-considering weights of object criteria and subjective criteria, i.e., object criteria and subjective criteria have the same important degree, then linguistic arithmetic mean of linguistic satisfied degrees of extracted association rules is*

$$\begin{aligned}d_i &= \overline{x}_1(\{s_{i1}, s_{i2}, s_{i3}\}, s_{max}(R_i))\\ &= \Delta(\frac{1}{2}\overline{x}(s_{i1}, s_{i2}, s_{i3}) + \frac{1}{2}\beta^i_{max})\\ &= \Delta(\frac{1}{6}\beta_{i1} + \frac{1}{6}\beta_{i2} + \frac{1}{6}\beta_{i3} + \frac{1}{2}\beta^i_{max}), \qquad (16)\end{aligned}$$

in which, $\beta_{i1} = \Delta^{-1}(s_{i1})$, $\beta_{i2} = \Delta^{-1}(s_{i2})$, $\beta_{i3} = \Delta^{-1}(s_{i3})$ and $\beta^i_{max} = \Delta^{-1}(s_{max}(R_i))$. If considering each element of

$S_{i2}$ has the the same important degree, then Eq.(16) becomes

$$d_i = \overline{x}_1(\{s_{i1}, s_{i2}, s_{i3}\}, S_{i2}) = \Delta(\frac{1}{2}\overline{x}(s_{i1}, s_{i2}, s_{i3}) + \frac{1}{2}S_{i2}) = \Delta(\frac{1}{6}\beta_{i1} + \frac{1}{6}\beta_{i2} + \frac{1}{6}\beta_{i3} + \frac{1}{12}\beta_{i4} + \cdots + \frac{1}{12}\beta_{i9}).$$

*(2) Non-considering weights of Acc(), Cer(), Int() and $s_{max}$(), then linguistic arithmetic mean of linguistic satisfied degrees of extracted association rules is*

$$d_i = \overline{x}_2(s_{i1}, s_{i2}, s_{i3}, s_{max}(R_i)) = \Delta(\frac{1}{4}\beta_{i1} + \frac{1}{4}\beta_{i2} + \frac{1}{4}\beta_{i3} + \frac{1}{4}\beta^i_{max}). \quad (17)$$

If non-considering weights of $Acc()$, $Cer()$, $Int()$ and $S_{i2}$, then Eq.(17) becomes

$$d_i = \overline{x}_2(s_{i1}, s_{i2}, s_{i3}, S_{i2}) = \Delta(\frac{1}{9}\beta_{i1} + \frac{1}{9}\beta_{i2} + \cdots + \frac{1}{9}\beta_{i9}).$$

### 2) CONSIDERING WEIGHTS OF EVALUATION CRITERIA

Because weights are stemmed from different sources, hence known weights and partial known weights of evaluation criteria are considered in the following linguistic weighted aggregation operators.

*(1) 2-tuple linguistic weighted averaging operators of linguistic satisfied degrees are as follows:*

- Considering known weights of object criteria and subjective criteria, *i.e.*,

$$d_i = \overline{x}^1_w(\{s_{i1}, s_{i2}, s_{i3}\}, s_{max}(R_i)) = \Delta(w_1\overline{x}_w(s_{i1}, s_{i2}, s_{i3}) + w_2\beta^i_{max}) = \Delta(w_1(\omega_1\beta_{i1}+\omega_2\beta_{i2}+\omega_3\beta_{i3})+w_2\beta^i_{max}), \quad (18)$$

in which, $w_1$ and $w_2$ are weights of object criteria and subjective criteria, non-negative numbers $\omega_1, \omega_2$ and $\omega_3$ are weights of $Acc()$, $Cer()$ and $Int()$ such that $\omega_1 + \omega_2 + \omega_3 = 1$. Similarly, if weights of $S_{i2} = \{s_{i4}, \cdots, s_{i9}\}$ such that $\omega_4 + \cdots + \omega_9 = 1$ are known, then

$$d_i = \overline{x}^2_w(\{s_{i1}, s_{i2}, s_{i3}\}, S_{i2}) = \Delta(w_1\overline{x}_w(s_{i1}, s_{i2}, s_{i3}) + w_2\overline{x}_w(S_{i2})) = \Delta(w_1(\omega_1\beta_{i1} + \omega_2\beta_{i2} + \omega_3\beta_{i3}) + w_2(\omega_4\beta_{i4} + \omega_5\beta_{i5} + \omega_6\beta_{i6} + \omega_7\beta_{i7} + \omega_8\beta_{i8} + \omega_9\beta_{i9})), \quad (19)$$

in which, $\beta_{ik} = \Delta^{-1}(s_{ik})(k = 4, 5, 6, 7, 8, 9)$.

- Considering known weights of $Acc()$, $Cer()$, $Int()$ and $s_{max}()$, *i.e.*,

$$d_i = \overline{x}^3_w(s_{i1}, s_{i2}, s_{i3}, s_{max}(R_i)) = \Delta(w_1\beta_{i1} + w_2\beta_{i2} + w_3\beta_{i3} + w_4\beta^i_{max}), \quad (20)$$

in which, non-negative weights are such that $w_1 + w_2 + w_3 + w_4 = 1$. If weights of $Acc()$, $Cer()$ , $Int()$ and elements of $S_{i2}$ are known, *i.e.*, $w_1 + w_2 + \cdots + w_9 = 1$, then

$$d_i = \overline{x}^4_w(s_{i1}, s_{i2}, \cdots, s_{i9}) = \Delta(w_1\beta_{i1} + w_2\beta_{i2} + \cdots + w_9\beta_{i9}). \quad (21)$$

**TABLE 4.** A transaction database ($U$, $\mathcal{A}$).

| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 | 1 |
| 3 | 1 | 1 | 0 | 1 | 1 |
| 4 | 1 | 1 | 1 | 0 | 1 |
| 5 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 1 | 1 | 1 | 0 |

*(2) 2-tuple linguistic ordered weighted averaging operators of linguistic satisfied degrees are as follows:*

- Considering partial known weights of object criteria and subjective criteria, *i.e.*,

$$d_i = \overline{x}^1_o(\{s_{i1}, s_{i2}, s_{i3}\}, s_{max}(R_i)) = \overline{x}_o(\overline{x}_o(s_{i1}, s_{i2}, s_{i3}), s_{max}(R_i)) = \Delta(w_1\beta^1_i + w_2\beta^2_i), \quad (22)$$

in which, $\overline{x}_o(s_{i1}, s_{i2}, s_{i3}) = \Delta(\omega_1\beta'_{i1} + \omega_2\beta'_{i2} + \omega_3\beta'_{i3})$, $\beta'_{i1}, \beta'_{i2}, \beta'_{i3} \in \{\beta_{i1}, \beta_{i2}, \beta_{i3}\}$ and $\beta'_{i1} \geq \beta'_{i2} \geq \beta'_{i3}$. $\beta^1_i, \beta^2_i \in \{\omega_1\beta'_{i1} + \omega_2\beta'_{i2} + \omega_3\beta'_{i3}, \beta^i_{max}\}$ and $\beta^1_i \geq \beta^2_i$, $\omega_1$, $\omega_2$ and $\omega_3$ or $w_1$ and $w_2$ are decided are decided by Eq.(14) with fixed $a$ and $b$. Similarly, if weights of $S_{i2} = \{s_{i4}, \cdots, s_{i9}\}$ are partial known, then

$$d_i = \overline{x}^2_o(\overline{x}_o(s_{i1}, s_{i2}, s_{i3}), \overline{x}_o(s_{i4}, \cdots, s_{i9})) = \Delta(w_1\beta^1_i + w_2\beta^2_i), \quad (23)$$

where, $\overline{x}_o(s_{i4}, \cdots, s_{i9}) = \Delta(\omega_4\beta'_{i4} + \cdots + \omega_9\beta'_{i9})$, $\beta'_{ik} \in \{\beta_{i4}, \cdots, \beta_{i9}\}$ and $\beta'_{i4} \geq \cdots \geq \beta'_{i9}$. $\beta^1_i, \beta^2_i \in \{\omega_1\beta'_{i1} + \omega_2\beta'_{i2} + \omega_3\beta'_{i3}, \omega_4\beta'_{i4} + \cdots + \omega_9\beta'_{i9}\}$ and $\beta^1_i \geq \beta^2_i$.

- Considering partial known weights of $Acc()$, $Cer()$ , $Int()$ and $s_{max}()$, *i.e.*,

$$d_i = \overline{x}^3_o(s_{i1}, s_{i2}, s_{i3}, s_{max}(R_i)) = \Delta(w_1\beta^1_i + w_2\beta^2_i + w_3\beta^3_i + w_4\beta^4_i), \quad (24)$$

in which, $\beta^1_i, \beta^2_i, \beta^3_i, \beta^4_i \in \{\beta_{i1}, \beta_{i2}, \beta_{i3}, \beta^i_{max}\}$ and $\beta^1_i \geq \beta^2_i \geq \beta^3_i \geq \beta^4_i$. If weights of $Acc()$, $Cer()$ , $Int()$ and elements of $S_{i2}$ are partial known, then

$$d_i = \overline{x}^4_o(s_{i1}, \cdots, s_{i9}) = \Delta(w_1\beta^1_i + \cdots + w_9\beta^9_i), \quad (25)$$

where, $\beta^k_i \in \{\beta_{i1}, \cdots, \beta_{i9}\}$ and $\beta^1_i \geq \cdots \geq \beta^9_i$.

According to linguistic aggregation operators Eqs.(15)-(25), linguistic satisfied degrees of each association rule $R_i$ can be aggregated, *i.e.*, 2-tuple linguistic satisfied degree $d_i$ of each association rule $R_i$ in Table 3 can be obtained.

### C. SELECTION OF SATISFIED ASSOCIATION RULES

Based on Table 3, extracted association rules can be ranked by 2-tuple linguistic satisfied degrees $D = \{d_1, \cdots, d_m\}$, *i.e.*, for any two association rules $R_i$ and $R_{i'}$,

$$R_i \succeq R_{i'} \quad \text{if and only if } d_i \geq d_{i'}. \quad (26)$$

**TABLE 5.** Objective and subjective interesting measures of twenty two association rules.

| Rules | Evaluation Criteria | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Object Criteria | | | Subjective Criteria | | | | | |
| $(minSup = 0.5, minCon = 0.8)$ | $Acc()$ | $Cer()$ | $Int()$ | $\mu_{ce}()$ | $\mu_{le}()$ | $\mu_{re}()$ | $\mu_{cu}()$ | $\mu_{lu}()$ | $\mu_{ru}()$ |
| $R_1 : a_1 \to a_2\ (\frac{2}{3}, 1)$ | 0.83 | 0 | 0.22 | 0 | 0 | 0 | 1 | 1 | 1 |
| $R_2 : a_1 \wedge a_4 \to a_2\ (\frac{1}{2}, 1)$ | 0.67 | 0 | 0.25 | 0.33 | 0.5 | 0 | 0.67 | 0.5 | 1 |
| $R_3 : a_1 \wedge a_2 \wedge a_4 \to a_5\ (\frac{1}{2}, 1)$ | 0.67 | 1 | 0.3 | 0.25 | 0.33 | 0 | 0.5 | 0.67 | 0 |
| $R_4 : a_1 \wedge a_4 \wedge a_5 \to a_2\ (\frac{1}{2}, 1)$ | 0.67 | 0 | 0.25 | 0.25 | 0.33 | 0 | 0.5 | 0.33 | 1 |
| $R_5 : a_1 \wedge a_4 \to a_2 \wedge a_5\ (\frac{1}{2}, 1)$ | 0.67 | 1 | 0.3 | 0.25 | 0.5 | 0 | 0.5 | 0.5 | 0.5 |
| $R_6 : a_2 \wedge a_4 \wedge a_5 \to a_1\ (\frac{1}{2}, 1)$ | 0.67 | 1 | 0.375 | 0.25 | 0.33 | 0 | 0.5 | 0.33 | 1 |
| $R_7 : a_4 \wedge a_5 \to a_1 \wedge a_2\ (\frac{1}{2}, 1)$ | 0.67 | 0 | 0.375 | 0.25 | 0.5 | 0 | 0.5 | 0 | 1 |
| $R_8 : a_1 \wedge a_2 \to a_5\ (\frac{2}{3}, 1)$ | 0.83 | 1 | 0.27 | 0 | 0 | 0 | 0.67 | 1 | 0 |
| $R_9 : a_1 \wedge a_5 \to a_2\ (\frac{2}{3}, 1)$ | 0.83 | 0 | 0.22 | 0 | 0 | 0 | 0.67 | 0.5 | 1 |
| $R_{10} : a_1 \to a_2 \wedge a_5\ (\frac{2}{3}, 1)$ | 0.83 | 1 | 0.27 | 0 | 0 | 0 | 0.67 | 1 | 0.5 |
| $R_{11} : a_2 \wedge a_5 \to a_1\ (\frac{2}{3}, 0.8)$ | 0.83 | 0.4 | 0.27 | 0 | 0 | 0 | 0.67 | 0.5 | 1 |
| $R_{12} : a_5 \to a_1 \wedge a_2\ (\frac{2}{3}, 0.8)$ | 0.83 | 0.4 | 0.27 | 0 | 0 | 0 | 0.67 | 0 | 1 |
| $R_{13} : a_1 \wedge a_4 \to a_5\ (\frac{1}{2}, 1)$ | 0.67 | 1 | 0.3 | 0.33 | 0.5 | 0 | 0.33 | 0.5 | 0 |
| $R_{14} : a_4 \wedge a_5 \to a_1\ (\frac{1}{2}, 1)$ | 0.67 | 1 | 0.375 | 0.33 | 0.5 | 0 | 0.33 | 0 | 1 |
| $R_{15} : a_1 \to a_5\ (\frac{2}{3}, 1)$ | 0.83 | 1 | 0.27 | 0 | 0 | 0 | 0.5 | 1 | 0 |
| $R_{16} : a_5 \to a_1\ (\frac{2}{3}, 0.8)$ | 0.83 | 0.4 | 0.27 | 0 | 0 | 0 | 0.5 | 0 | 1 |
| $R_{17} : a_3 \to a_2\ (\frac{2}{3}, 1)$ | 0.83 | 0 | 0.22 | 0.5 | 1 | 0 | 0.5 | 0 | 1 |
| $R_{18} : a_3 \wedge a_5 \to a_2\ (\frac{1}{2}, 1)$ | 0.67 | 0 | 0.25 | 0.33 | 0.5 | 0 | 0.33 | 0 | 1 |
| $R_{19} : a_4 \to a_2\ (\frac{2}{3}, 1)$ | 0.83 | 0 | 0.22 | 0.5 | 1 | 0 | 0.5 | 0 | 1 |
| $R_{20} : a_4 \wedge a_5 \to a_2\ (\frac{1}{2}, 1)$ | 0.67 | 0 | 0.25 | 0.33 | 0.5 | 0 | 0.33 | 0 | 1 |
| $R_{21} : a_2 \to a_5\ (\frac{5}{6}, 0.83)$ | 1 | 0 | 0.14 | 0 | 0 | 0 | 0.5 | 1 | 0 |
| $R_{22} : a_5 \to a_2\ (\frac{5}{6}, 1)$ | 1 | 0 | 0.14 | 0 | 0 | 0 | 0.5 | 0 | 1 |

**TABLE 6.** 2-tuple linguistic satisfied degrees of twenty two association rules.

| Rules | Evaluation Criteria | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Object Criteria | | | Subjective Criteria | | | | | |
| | $Acc()$ | $Cer()$ | $Int()$ | $\mu_{ce}()$ | $\mu_{le}()$ | $\mu_{re}()$ | $\mu_{cu}()$ | $\mu_{lu}()$ | $\mu_{ru}()$ |
| $R_1$ | $(s_5, 0.36)$ | $s_3$ | $(s_1, 0.24)$ | $s_0$ | $s_0$ | $s_0$ | $s_6$ | $s_6$ | $s_6$ |
| $R_2$ | $(s_4, -0.43)$ | $s_3$ | $(s_2, -0.5)$ | $(s_1, 0.3)$ | $s_3$ | $s_0$ | $(s_4, -0.36)$ | $s_2$ | $s_6$ |
| $R_3$ | $(s_4, -0.43)$ | $s_6$ | $(s_2, 0)$ | $(s_1, -0.5)$ | $(s_1, 0.3)$ | $s_0$ | $s_2$ | $(s_4, -0.36)$ | $s_0$ |
| $R_4$ | $(s_4, -0.43)$ | $s_3$ | $(s_2, -0.5)$ | $(s_1, -0.5)$ | $(s_1, 0.3)$ | $s_0$ | $s_2$ | $(s_1, 0.18)$ | $s_6$ |
| $R_5$ | $(s_4, -0.43)$ | $s_6$ | $(s_2, 0)$ | $(s_1, -0.5)$ | $s_3$ | $s_0$ | $s_2$ | $s_2$ | $s_2$ |
| $R_6$ | $(s_4, -0.43)$ | $s_6$ | $(s_3, -0.21)$ | $(s_1, -0.5)$ | $(s_1, 0.3)$ | $s_0$ | $s_2$ | $(s_1, 0.18)$ | $s_6$ |
| $R_7$ | $(s_4, -0.43)$ | $s_6$ | $(s_3, -0.21)$ | $(s_1, -0.5)$ | $s_3$ | $s_0$ | $s_2$ | $s_0$ | $s_6$ |
| $R_8$ | $(s_5, 0.36)$ | $s_6$ | $(s_2, -0.16)$ | $s_0$ | $s_0$ | $s_0$ | $(s_4, -0.36)$ | $s_6$ | $s_0$ |
| $R_9$ | $(s_5, 0.36)$ | $s_3$ | $(s_1, 0.24)$ | $s_0$ | $s_0$ | $s_0$ | $(s_4, -0.36)$ | $s_2$ | $s_6$ |
| $R_{10}$ | $(s_5, 0.36)$ | $s_6$ | $(s_2, -0.16)$ | $s_0$ | $s_0$ | $s_0$ | $(s_4, -0.36)$ | $s_6$ | $s_4$ |
| $R_{11}$ | $(s_5, 0.36)$ | $(s_4, 0.21)$ | $(s_2, -0.16)$ | $s_0$ | $s_0$ | $s_0$ | $(s_4, -0.36)$ | $s_4$ | $s_6$ |
| $R_{12}$ | $(s_5, 0.36)$ | $(s_4, 0.21)$ | $(s_2, -0.16)$ | $s_0$ | $s_0$ | $s_0$ | $(s_4, -0.36)$ | $s_0$ | $s_6$ |
| $R_{13}$ | $(s_4, -0.43)$ | $s_6$ | $(s_2, 0.4)$ | $(s_1, 0.3)$ | $s_3$ | $s_0$ | $(s_1, 0.18)$ | $s_2$ | $s_0$ |
| $R_{14}$ | $(s_4, -0.43)$ | $s_6$ | $(s_3, -0.21)$ | $(s_1, 0.3)$ | $s_3$ | $s_0$ | $(s_1, 0.18)$ | $s_0$ | $s_6$ |
| $R_{15}$ | $(s_5, 0.36)$ | $s_6$ | $(s_2, -0.16)$ | $s_0$ | $s_0$ | $s_0$ | $s_2$ | $s_6$ | $s_0$ |
| $R_{16}$ | $(s_5, 0.36)$ | $(s_4, 0.21)$ | $(s_2, -0.16)$ | $s_0$ | $s_0$ | $s_0$ | $s_2$ | $s_0$ | $s_6$ |
| $R_{17}$ | $(s_5, 0.36)$ | $s_3$ | $(s_1, 0.24)$ | $s_3$ | $s_6$ | $s_0$ | $s_2$ | $s_0$ | $s_6$ |
| $R_{18}$ | $(s_4, -0.43)$ | $s_3$ | $(s_2, -0.5)$ | $(s_1, 0.3)$ | $s_3$ | $s_0$ | $(s_1, 0.18)$ | $s_0$ | $s_6$ |
| $R_{19}$ | $(s_5, 0.36)$ | $s_3$ | $(s_1, 0.24)$ | $s_3$ | $s_6$ | $s_0$ | $s_2$ | $s_0$ | $s_6$ |
| $R_{20}$ | $(s_4, -0.43)$ | $s_3$ | $(s_2, -0.5)$ | $(s_1, 0.3)$ | $s_3$ | $s_0$ | $(s_1, 0.18)$ | $s_0$ | $s_6$ |
| $R_{21}$ | $s_6$ | $s_3$ | $(s_1, -0.38)$ | $s_0$ | $s_0$ | $s_0$ | $s_2$ | $s_6$ | $s_0$ |
| $R_{22}$ | $s_6$ | $s_3$ | $(s_1, -0.38)$ | $s_0$ | $s_0$ | $s_0$ | $s_2$ | $s_0$ | $s_6$ |

Then satisfied association rules can be selected from ranking association rules, *i.e.*,

$$\mathcal{R}_{max} = \{R_i | d_i = maxD = max\{d_1, d_2, \cdots, d_m\}\}. \tag{27}$$

or $s$-level satisfied association rules in practical applications,

$$\mathcal{R}_s = \{R_i | d_i \geq s\}, \tag{28}$$

in which $s$ is a fixed linguistic satisfied degree.

*Example 2:* Let a transaction database be $(U, \mathcal{A})$ shown in Table 4, where $U = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{A} = \{a_1, a_2, a_3, a_4, a_5\}$. Let $A_e = \{a_3, a_4\}$, $A_{\overline{e}} = \{a_1, a_2\}$, $minSup() = 0.5$ and $minCon() = 0.8$. By using Apriori algorithm, twenty two association rules are generated, according to Table 1 or Eqs.(1)-(6), objective and subjective interesting measures of twenty two association rules can be obtained, which are shown in Table 5. Based on Figs.(3)-(7), objective and subjective interesting measures of twenty two association rules are transformed in 2-tuple linguistic satisfied degrees on $L = \{s_0$ (*very dissatisfied*), $s_1$ (*dissatisfied*), $s_2$ (*slightly dissatisfied*), $s_3$ (*fair*), $s_4$ (*slightly satisfied*), $s_5$ (*satisfied*), $s_6$ (*very satisfied*)$\}$, which are shown in Table 6.

(1) Non-considering weights of evaluation criteria. Linguistic aggregation operators $\overline{x}_1$ and $\overline{x}_2$ defined by Eqs.(16) and (17) are utilized to aggregate 2-tuple linguistic satisfied degrees of twenty two association rules.

**TABLE 7. All aggregation results $d_i$ of twenty two association rules.**

| Rules | $D$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\overline{x}_1$ | $\overline{x}_2$ | $\overline{x}_w^1$ | $\overline{x}_w^3$ | $\overline{x}_w^4$ | $\overline{x}_o^1$ | $\overline{x}_o^3$ | $\overline{x}_o^4$ |
| $R_1$ | $(s_5, -0.4)$ | $(s_4, -0.1)$ | $(s_5, -0.104)$ | $(s_5, -0.44)$ | $(s_3, 0.208)$ | $(s_4, 0.013)$ | $(s_4, -0.23)$ | $(s_3, -0.47)$ |
| $R_2$ | $(s_4, 0.345)$ | $(s_4, -0.48)$ | $(s_5, -0.26)$ | $(s_4, 0.30)$ | $(s_3, 0.07)$ | $(s_4, 0.00)$ | $(s_3, 0.39)$ | $(s_3, -0.15)$ |
| $R_3$ | $(s_4, -0.25)$ | $(s_4, -0.2)$ | $(s_4, -0.1)$ | $(s_4, -0.14)$ | $(s_2, 0.46)$ | $(s_4, -0.26)$ | $(s_4, -0.05)$ | $(s_2, 0.16)$ |
| $R_4$ | $(s_4, 0.345)$ | $(s_4, -0.48)$ | $(s_5, -0.26)$ | $(s_4, 0.30)$ | $(s_3, -0.36)$ | $(s_4, 0.00)$ | $(s_3, 0.39)$ | $(s_2, 0.20)$ |
| $R_5$ | $(s_3, 0.43)$ | $(s_4, -0.36)$ | $(s_3, 0.17)$ | $(s_3, 0.44)$ | $(s_3, -0.28)$ | $(s_3, 0.23)$ | $(s_3, 0.42)$ | $(s_3, -0.26)$ |
| $R_6$ | $(s_5, 0.06)$ | $(s_5, -0.41)$ | $(s_5, 0.08)$ | $(s_5, -0.23)$ | $(s_3, -0.12)$ | $(s_5, -0.39)$ | $(s_5, -0.18)$ | $(s_2, 0.45)$ |
| $R_7$ | $(s_5, 0.06)$ | $(s_5, -0.41)$ | $(s_5, 0.08)$ | $(s_5, -0.23)$ | $(s_3, -0.22)$ | $(s_5, -0.39)$ | $(s_5, -0.18)$ | $(s_3, -0.49)$ |
| $R_8$ | $(s_5, 0.2)$ | $(s_5, -0.35)$ | $(s_5, 0.22)$ | $(s_5, -0.08)$ | $(s_3, -0.48)$ | $(s_5, -0.11)$ | $(s_5, 0.26)$ | $(s_2, -0.38)$ |
| $R_9$ | $(s_5, -0.4)$ | $(s_4, -0.1)$ | $(s_5, -0.104)$ | $(s_5, -0.44)$ | $(s_3, -0.39)$ | $(s_4, 0.013)$ | $(s_4, -0.23)$ | $(s_2, -0.05)$ |
| $R_{10}$ | $(s_5, 0.2)$ | $(s_5, -0.35)$ | $(s_5, 0.22)$ | $(s_5, -0.08)$ | $(s_3, -0.23)$ | $(s_5, -0.11)$ | $(s_5, 0.26)$ | $(s_2, 0.31)$ |
| $R_{11}$ | $(s_5, -0.1)$ | $(s_4, 0.23)$ | $(s_5, 0.03)$ | $(s_5, -0.26)$ | $(s_3, -0.3)$ | $(s_5, -0.46)$ | $(s_4, 0.43)$ | $(s_2, 0.25)$ |
| $R_{12}$ | $(s_5, -0.1)$ | $(s_4, 0.23)$ | $(s_5, 0.03)$ | $(s_5, -0.26)$ | $(s_2, 0.44)$ | $(s_5, -0.46)$ | $(s_4, 0.43)$ | $(s_2, -0.43)$ |
| $R_{13}$ | $(s_3, 0.495)$ | $(s_4, -0.36)$ | $(s_3, 0.17)$ | $(s_3, 0.44)$ | $(s_2, 0.37)$ | $(s_3, 0.23)$ | $(s_3, 0.42)$ | $(s_2, -0.31)$ |
| $R_{14}$ | $(s_5, 0.06)$ | $(s_5, -0.41)$ | $(s_5, 0.08)$ | $(s_5, -0.23)$ | $(s_3, -0.31)$ | $(s_5, -0.39)$ | $(s_5, -0.18)$ | $(s_2, 0.38)$ |
| $R_{15}$ | $(s_5, 0.2)$ | $(s_5, -0.35)$ | $(s_5, 0.22)$ | $(s_5, -0.08)$ | $(s_2, 0.35)$ | $(s_5, -0.11)$ | $(s_5, 0.26)$ | $(s_1, 0.39)$ |
| $R_{16}$ | $(s_5, -0.1)$ | $(s_4, 0.23)$ | $(s_5, 0.03)$ | $(s_5, -0.26)$ | $(s_2, 0.27)$ | $(s_5, -0.46)$ | $(s_4, 0.43)$ | $(s_1, 0.34)$ |
| $R_{17}$ | $(s_5, -0.4)$ | $(s_4, -0.1)$ | $(s_5, -0.104)$ | $(s_5, -0.44)$ | $(s_3, 0.208)$ | $(s_4, 0.013)$ | $(s_4, -0.23)$ | $(s_3, -0.31)$ |
| $R_{18}$ | $(s_4, 0.345)$ | $(s_4, -0.48)$ | $(s_5, -0.26)$ | $(s_4, 0.30)$ | $(s_2, 0.46)$ | $(s_4, 0.00)$ | $(s_3, 0.39)$ | $(s_2, 0.13)$ |
| $R_{19}$ | $(s_5, -0.4)$ | $(s_4, -0.1)$ | $(s_5, -0.104)$ | $(s_5, -0.44)$ | $(s_3, 0.208)$ | $(s_4, 0.013)$ | $(s_4, -0.23)$ | $(s_3, -0.31)$ |
| $R_{20}$ | $(s_4, 0.345)$ | $(s_4, -0.48)$ | $(s_5, -0.26)$ | $(s_4, 0.30)$ | $(s_2, 0.46)$ | $(s_4, 0.00)$ | $(s_3, 0.39)$ | $(s_2, 0.13)$ |
| $R_{21}$ | $(s_5, -0.397)$ | $(s_4, -0.2)$ | $(s_5, -0.06)$ | $(s_5, -0.23)$ | $(s_2, 0.30)$ | $(s_4, -0.02)$ | $(s_4, 0.14)$ | $(s_1, 0.08)$ |
| $R_{22}$ | $(s_5, -0.397)$ | $(s_4, -0.2)$ | $(s_5, -0.06)$ | $(s_5, -0.23)$ | $(s_2, 0.30)$ | $(s_4, -0.02)$ | $(s_4, 0.14)$ | $(s_1, 0.08)$ |

(2) Considering known weights of evaluation criteria, *i.e.*,

- linguistic aggregation operator $\overline{x}_w^1$ defined by Eq.(18) is utilized to aggregate 2-tuple linguistic satisfied degrees of twenty two association rules, in which $w_1 = 0.4$ and $w_2 = 0.6$, $\omega_1 = 0.4$, $\omega_2 = 0.2$ and $\omega_3 = 0.4$;
- linguistic aggregation operator $\overline{x}_w^3$ defined by Eq.(20) is utilized to aggregate 2-tuple linguistic satisfied degrees of twenty two association rules, in which $w_1 = 0.3$, $w_2 = 0.1$, $w_3 = 0.2$ and $w_4 = 0.4$;
- linguistic aggregation operator $\overline{x}_w^4$ defined by Eq.(21) is utilized to aggregate 2-tuple linguistic satisfied degrees of twenty two association rules, in which $w_1 = 0.2$, $w_2 = 0.05$, $w_3 = 0.15$, $w_4 = 0.2$, $w_5 = 0.05$, $w_6 = 0.05$, $w_7 = 0.2$, $w_8 = 0.05$ and $w_9 = 0.05$.

(3) Considering partial known weights of evaluation criteria, where fuzzy linguistic quantifier *Most* with $a = 0.3$ and $b = 0.8$ is selected to obtain weights, *i.e.*,

- linguistic aggregation operator $\overline{x}_o^1$ defined by Eq.(22)) is utilized to aggregate 2-tuple linguistic satisfied degrees of twenty two association rules, in which $w_1 = 0.4$ and $w_2 = 0.6$, $\omega_1 = \frac{1}{15}$, $\omega_2 = \frac{2}{3}$ and $\omega_3 = \frac{4}{15}$;
- linguistic aggregation operator $\overline{x}_o^3$ defined by Eq.(24) is utilized to aggregate 2-tuple linguistic satisfied degrees of twenty two association rules, in which $w_1 = 0$, $w_2 = \frac{2}{5}$, $w_3 = \frac{1}{2}$ and $w_4 = \frac{1}{10}$;
- linguistic aggregation operator $\overline{x}_o^4$ defined by Eq.(25) is utilized to aggregate 2-tuple linguistic satisfied degrees, in which $w_1 = 0$, $w_2 = 0$, $w_3 = \frac{1}{15}$, $w_4 = w_5 = w_6 = w_7 = \frac{2}{9}$, $w_8 = \frac{2}{45}$ and $w_9 = 0$.

All aggregation results $d_i$ are shown in Table 7. Based on Eqs.(27) and (28), the following satisfied association rules can be selected from twenty two association rules.

- Using $\overline{x}_1$: $\mathcal{R}_{max} = \{R_8, R_{10}, R_{15}\}$, $\mathcal{R}_{(s_5,0)} = \{R_6, R_7, R_8, R_{10}, R_{14}, R_{15}\}$;
- Using $\overline{x}_2$: $\mathcal{R}_{max} = \{R_8, R_{10}, R_{15}\}$, $\mathcal{R}_{(s_4,0)} = \{R_6, R_7, R_8, R_{10}, R_{11}, R_{12}, R_{14}, R_{15}, R_{16}\}$;
- Using $\overline{x}_w^1$: $\mathcal{R}_{max} = \{R_8, R_{10}, R_{15}\}$, $\mathcal{R}_{(s_5,0)} = \{R_6, R_7, R_8, R_{10}, R_{11}, R_{12}, R_{14}, R_{15}, R_{16}\}$;
- Using $\overline{x}_w^3$: $\mathcal{R}_{max} = \{R_8, R_{10}, R_{15}\}$, $\mathcal{R}_{(s_5,-0.3)} = \{R_6, R_7, R_8, R_{10}, R_{11}, R_{12}, R_{14}, R_{15}, R_{16}, R_{21}, R_{22}\}$;
- Using $\overline{x}_w^4$: $\mathcal{R}_{max} = \{R_1, R_{17}, R_{19}\}$, $\mathcal{R}_{(s_3,0)} = \{R_1, R_2, R_{17}, R_{19}\}$;
- Using $\overline{x}_o^1$: $\mathcal{R}_{max} = \{R_8, R_{10}, R_{15}\}$, $\mathcal{R}_{(s_5,-0.3)} = \{R_8, R_{10}, R_{15}\}$;
- Using $\overline{x}_o^3$: $\mathcal{R}_{max} = \{R_8, R_{10}, R_{15}\}$, $\mathcal{R}_{(s_5,0)} = \{R_8, R_{10}, R_{15}\}$;
- Using $\overline{x}_o^4$: $\mathcal{R}_{max} = \{R_2\}$, $\mathcal{R}_{(s_3,-0.5)} = \{R_2, R_5, R_7, R_{17}, R_{19}\}$;

In Example 2, it can be noticed that

- Some extracted association rules with higher *Sup*() and *Con*() are not included in satisfied association rules $\mathcal{R}_{max}$ or *s*-level satisfied association rules $\mathcal{R}_s$, such as $R_{13}$ with $Sup(R_{13}) = \frac{1}{2}$ and $Con(R_{13}) = 1$, $R_{21}$ with $Sup(R_{22}) = \frac{5}{6}$ and $Con(R_{22}) = 0.83$ and $R_{22}$ with $Sup(R_{22}) = \frac{5}{6}$ and $Con(R_{22}) = 1$ are not in $\mathcal{R}_{max}$ and $\mathcal{R}_s$. This means that satisfied association rules are not decided by interesting measures *Sup*() and *Con*();
- Different linguistic aggregation operator can be utilized to obtain different satisfied association rules from extracted association rules, such as $\mathcal{R}_{max}$ of $\overline{x}_w^4$, $\mathcal{R}_{max}$ of $\overline{x}_o^4$ and $\mathcal{R}_{max}$ of $\overline{x}_1$ or $\overline{x}_2$ are different each other. This means that linguistic aggregation operator and weights of objective and subjective interesting measures can be utilized to select different satisfied association rules.

It seems that linguistic aggregation operators provide an alternative and useful tool to select satisfied association rules from the huge size of extracted association rules, in which, different linguistic aggregation operator or weights of objective

**TABLE 8.** The characteristics and parameters of chess, connect, mushroom and T40I10D100K databases.

| Dataset | Items | Transactions | $(Sup(), Con())$ | Number | Expected knowledge $A_e$ | Unexpected knowledge $A_{\overline{e}}$ |
|---|---|---|---|---|---|---|
| Chess | 75 | 3196 | (0.8, 0.8) | 558825 | $\{1, \cdots, 15\}$ | $\{50, \cdots, 75\}$ |
| Connect | 129 | 67557 | (0.97, 0.8) | 8092 | $\{21, \cdots, 30\} \cup \{61, \cdots, 70\}$ | $\{51, \cdots, 60\} \cup \{81, \cdots, 100\}$ |
| Mushroom | 119 | 8124 | (0.3, 0.8) | 43558 | $\{31, \cdots, 50\}$ | $\{61, \cdots, 100\}$ |
| T40I10D100K | 1000 | 100000 | (0.013, 0.8) | 146451 | $\{1, \cdots, 100\} \cup \{450, \cdots, 600\}$ | $\{200, \cdots, 300\} \cup \{850, \cdots, 900\}$ |

**Algorithm 1** Selecting Satisfied Association Rules Based on Linguistic Aggregation Operators

**Input**: A transaction database $(U, \mathcal{A})$.
**Output**: Satisfied association rules.
Method:
1) Using Apriori, Eclat, FP-Growth and LCM algorithms to extract the set $\mathcal{R}$ of association rules with higher support or confidence measures.
2) **for** $i := 1 : |\mathcal{R}|$ **do**
3) **for** $j := 1 : 9$ **do**
4) $v_{ij} = Acc(R_i), Cer(R_i), Int(R_i), \mu_{ce}(R_i), \mu_{le}(R_i), \mu_{re}(R_i), \mu_{cu}(R_i), \mu_{lu}(R_i)$ and $\mu_{ru}(R_i)$
5) $\triangle(v_{ij}) = (s_k, \alpha_k)$ based on Eq.(9) and Figs.(3)-(7)
6) **end**
7) linguistic satisfied degree $d_i = Agg(\triangle(v_{i1}), \cdots, \triangle(v_{i9}))$ by linguistic aggregation operators Eqs.(16)-(25)
8) insert $d_i$ to $D$
9) **end for**
10) **sort** $D$ with the descending order of $d_i$
11) return satisfied association rules $\mathcal{R}_{max}$ or $\mathcal{R}_s$

and subjective interesting measures can help us to select satisfied association rules. Accordingly, Algorithm 1 based on linguistic aggregation operators can be designed to select satisfied association rules from the huge size of extracted association rules.

In the next section, four real databases are utilized to show selection of satisfied association rules via Algorithm 1.

## V. CASES STUDY

In the section, Chess, Connect, Mushroom and T40I10D100K databases are utilized to experiment selection of satisfied association rules via aggregation of linguistic satisfied degrees, where Chess, Connect and Mushroom are obtained from http://archive.ics.uci.edu/ml/index.php, T40I10D100K is high-dimensional and sparse (HiDS) matrix, which is generated using the generator from the IBM Almaden Quest research group. In real applications, HiDS matrices are commonly encountered in many big-data-related and industrial applications like recommender systems, recently, acquiring useful patterns or generating highly accurate predictions from them have become an important issue [69]–[73].

The characteristics of the four databases are shown in Table 8, our experiments were performed on a ThinkPad laptop with 2.3 GHz Intel i5-6200U CPU, 20 GB of memory, running 64-bit Windows 10. By using Apriori, Eclat, FP-Growth or LCM algorithms, extracted association rules are generated from Chess, Connect and Mushroom databases, their $(Sup(), Con())$ and the number of extracted association rules are shown in Table 8. Objective and subjective interesting measures of extracted association rules from three databases can be obtained according to Table 1 and Eqs. (1)-(6), linguistic satisfied degrees, weights of objective and subjective interesting measures and linguistic aggregation operators are similar with Example 2.

In Chess database, 558825 association rules are extracted according to $(Sup(), Con()) = (0.8, 0.8)$, in which, 34 extracted association rules with the maximum linguistic satisfied degree $(s_5, 0.05)$ are selected by linguistic aggregation operator $\overline{x}_1$, 34 extracted association rules with the maximum linguistic satisfied degree $(s_5, -0.41)$ are selected by $\overline{x}_2$, 34 extracted association rules with the maximum linguistic satisfied degree $(s_6, -0.48)$ are selected by $\overline{x}_w^1$, 1 extracted association rule with the maximum linguistic satisfied degree $(s_5, -0.11)$ is selected by $\overline{x}_w^3$, 5 extracted association rules with the maximum linguistic satisfied degree $(s_3, 0.33)$ are selected by $\overline{x}_w^4$, 1 extracted association rule with the maximum linguistic satisfied degree $(s_5, -0.02)$ is selected by $\overline{x}_o^1$, 1 extracted association rule with the maximum linguistic satisfied degree $(s_6, -0.48)$ is selected by $\overline{x}_o^3$ and 5 extracted association rules with the maximum linguistic satisfied degree $(s_3, 0.15)$ are selected by $\overline{x}_o^4$, the first five satisfied association rules of them are shown in Table 9. Fig.(8) shows analysis of time and spatial complexities by using the proposed method in Chess database.

In Connect database, 8092 association rules are extracted according to $(Sup(), Con()) = (0.97, 0.8)$, in which, 8 extracted association rules with the maximum linguistic satisfied degree $(s_5, 0.06)$ are selected by linguistic aggregation operator $\overline{x}_1$, 8 extracted association rules with the maximum linguistic satisfied degree $(s_5, -0.40)$ are selected by $\overline{x}_2$, 8 association rules with the maximum linguistic satisfied degree $(s_5, 0.10)$ are selected by $\overline{x}_w^1$, 8 association rules with the maximum linguistic satisfied degree $(s_5, -0.09)$ are selected by $\overline{x}_w^3$, 1 association rule with the maximum linguistic satisfied degree $(s_3, 0.34)$ is selected by $\overline{x}_w^4$, 5 association rules with the maximum linguistic satisfied degree $(s_5, 0.03)$ are selected by $\overline{x}_o^1$, 5 association rules with the maximum linguistic satisfied degree $(s_6, -0.40)$ are selected by $\overline{x}_o^3$ and 2 association rules with the maximum linguistic satisfied degree $(s_3, 0.15)$ are selected by $\overline{x}_o^4$, the first five satisfied association rules of them are shown in Table 10. Fig.(9) shows analysis of time and spatial complexities by using the proposed method in Connect database.

**TABLE 9.** The first five satisfied association rules with the maximum linguistic satisfied degrees in Chess database.

| Op | E-Rules | L-degree | Op | E-Rules | L-degree | Op | E-Rules | L-degree |
|---|---|---|---|---|---|---|---|---|
| $\overline{x}_1$ | $7 \wedge 29 \wedge 58 \wedge 66 \rightarrow 60$ | $(s_5, 0.05)$ | $\overline{x}_2$ | $7 \wedge 29 \wedge 58 \wedge 66 \rightarrow 60$ | $(s_5, -0.41)$ | $\overline{x}_w^1$ | $7 \wedge 29 \wedge 58 \wedge 66 \rightarrow 60$ | $(s_6, -0.48)$ |
| | $7 \wedge 29 \wedge 62 \rightarrow 58$ | $(s_5, 0.05)$ | | $7 \wedge 29 \wedge 62 \rightarrow 58$ | $(s_5, -0.41)$ | | $7 \wedge 29 \wedge 62 \rightarrow 58$ | $(s_6, -0.48)$ |
| | $7 \wedge 40 \wedge 62 \rightarrow 58$ | $(s_5, 0.05)$ | | $7 \wedge 40 \wedge 62 \rightarrow 58$ | $(s_5, -0.41)$ | | $7 \wedge 40 \wedge 62 \rightarrow 58$ | $(s_6, -0.48)$ |
| | $7 \wedge 52 \wedge 58 \wedge 66 \rightarrow 60$ | $(s_5, 0.05)$ | | $7 \wedge 52 \wedge 58 \wedge 66 \rightarrow 60$ | $(s_5, -0.41)$ | | $7 \wedge 52 \wedge 58 \wedge 66 \rightarrow 60$ | $(s_6, -0.48)$ |
| | $7 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_5, 0.05)$ | | $7 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_5, -0.41)$ | | $7 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_6, -0.48)$ |
| $\overline{x}_w^3$ | $62 \rightarrow 25$ | $(s_5, -0.11)$ | $\overline{x}_w^4$ | $52 \wedge 60 \wedge 62 \rightarrow 58$ | $(s_3, 0.33)$ | $\overline{x}_o^1$ | $62 \rightarrow 58$ | $(s_5, -0.02)$ |
| | $29 \wedge 40 \wedge 62 \rightarrow 58$ | $(s_5, -0.12)$ | | $52 \wedge 58 \wedge 66 \rightarrow 60$ | $(s_3, 0.33)$ | | $29 \wedge 40 \wedge 62 \rightarrow 58$ | $(s_5, -0.03)$ |
| | $29 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_5, -0.12)$ | | $52 \wedge 66 \rightarrow 60$ | $(s_3, 0.33)$ | | $29 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_5, -0.03)$ |
| | $29 \wedge 62 \rightarrow 58$ | $(s_5, -0.12)$ | | $62 \rightarrow 58$ | $(s_3, 0.33)$ | | $29 \wedge 62 \rightarrow 58$ | $(s_5, -0.03)$ |
| | $40 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_5, -0.12)$ | | $66 \rightarrow 60$ | $(s_3, 0.33)$ | | $40 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_5, -0.03)$ |
| $\overline{x}_o^3$ | $62 \rightarrow 58$ | $(s_6, -0.48)$ | $\overline{x}_o^4$ | $52 \wedge 60 \wedge 62 \rightarrow 58$ | $(s_3, 0.15)$ | | | |
| | $29 \wedge 40 \wedge 62 \rightarrow 58$ | $(s_5, 0.49)$ | | $52 \wedge 58 \wedge 66 \rightarrow 60$ | $(s_3, 0.15)$ | | | |
| | $29 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_5, 0.49)$ | | $52 \wedge 66 \rightarrow 60$ | $(s_3, 0.15)$ | | | |
| | $29 \wedge 62 \rightarrow 58$ | $(s_5, 0.49)$ | | $62 \rightarrow 58$ | $(s_3, 0.15)$ | | | |
| | $40 \wedge 52 \wedge 62 \rightarrow 58$ | $(s_5, 0.49)$ | | $66 \rightarrow 60$ | $(s_3, 0.15)$ | | | |

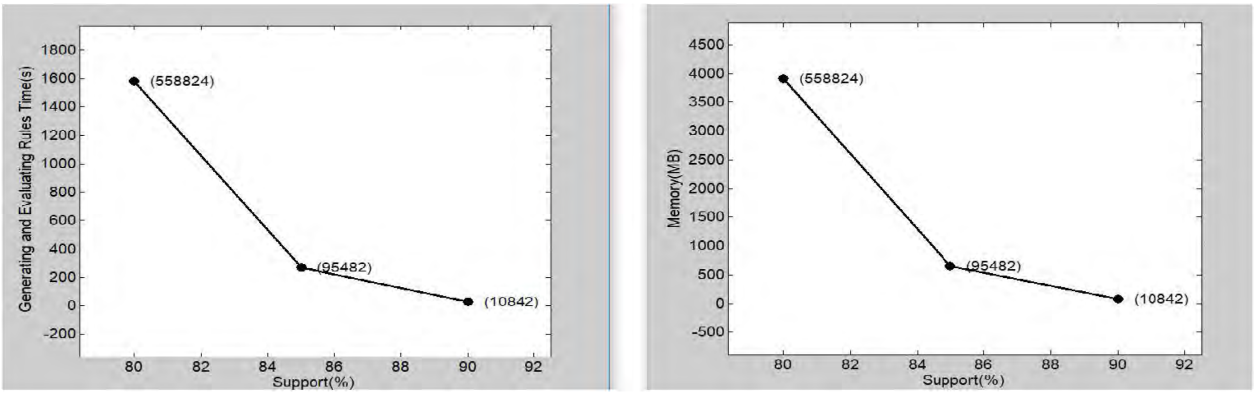


**FIGURE 8.** Time and spatial complexities of satisfied association rules via aggregation of fuzzy linguistic satisfied degrees from Chess database.

**TABLE 10.** The first five satisfied association rules with the maximum linguistic satisfied degrees in Connect database.

| Op | E-Rules | L-degree | Op | E-Rules | L-degree | Op | E-Rules | L-degree |
|---|---|---|---|---|---|---|---|---|
| $\overline{x}_1$ | $19 \wedge 37 \wedge 88 \rightarrow 91$ | $(s_5, 0.06)$ | $\overline{x}_2$ | $19 \wedge 37 \wedge 88 \rightarrow 91$ | $(s_5, -0.40)$ | $\overline{x}_w^1$ | $19 \wedge 37 \wedge 88 \rightarrow 91$ | $(s_5, 0.10)$ |
| | $19 \wedge 55 \wedge 88 \rightarrow 91$ | $(s_5, 0.06)$ | | $19 \wedge 55 \wedge 88 \rightarrow 91$ | $(s_5, -0.40)$ | | $19 \wedge 55 \wedge 88 \rightarrow 91$ | $(s_5, 0.10)$ |
| | $75 \wedge 88 \wedge 106 \wedge 109 \rightarrow 91$ | $(s_5, 0.06)$ | | $75 \wedge 88 \wedge 106 \rightarrow 91$ | $(s_5, -0.40)$ | | $75 \wedge 88 \wedge 106 \wedge 109 \rightarrow 91$ | $(s_5, 0.10)$ |
| | $37 \wedge 55 \wedge 88 \rightarrow 91$ | $(s_5, 0.06)$ | | $37 \wedge 55 \wedge 88 \rightarrow 91$ | $(s_5, -0.40)$ | | $37 \wedge 55 \wedge 88 \rightarrow 91$ | $(s_5, 0.10)$ |
| | $75 \wedge 88 \wedge 106 \rightarrow 91$ | $(s_5, 0.06)$ | | $75 \wedge 88 \wedge 106 \wedge 109 \rightarrow 91$ | $(s_5, -0.40)$ | | $75 \wedge 88 \wedge 106 \rightarrow 91$ | $(s_5, 0.10)$ |
| $\overline{x}_w^3$ | $19 \wedge 37 \wedge 88 \rightarrow 91$ | $(s_5, -0.09)$ | $\overline{x}_w^4$ | $88 \rightarrow 91$ | $(s_3, 0.34)$ | $\overline{x}_o^1$ | $75 \wedge 88 \rightarrow 91$ | $(s_5, 0.03)$ |
| | $19 \wedge 55 \wedge 88 \rightarrow 91$ | $(s_5, -0.09)$ | | $55 \wedge 88 \rightarrow 91$ | $(s_3, 0.33)$ | | $88 \rightarrow 91$ | $(s_5, 0.03)$ |
| | $37 \wedge 55 \wedge 88 \rightarrow 91$ | $(s_5, -0.09)$ | | $55 \rightarrow 91$ | $(s_3, 0.33)$ | | $88 \wedge 109 \rightarrow 91$ | $(s_5, 0.03)$ |
| | $75 \wedge 88 \wedge 106 \rightarrow 91$ | $(s_5, -0.09)$ | | $91 \rightarrow 55$ | $(s_3, 0.29)$ | | $88 \wedge 109 \wedge 127 \rightarrow 91$ | $(s_5, 0.03)$ |
| | $75 \wedge 88 \wedge 106 \wedge 109 \rightarrow 91$ | $(s_5, -0.09)$ | | $88 \rightarrow 55$ | $(s_3, 0.28)$ | | $88 \wedge 127 \rightarrow 91$ | $(s_5, 0.03)$ |
| $\overline{x}_o^3$ | $75 \wedge 88 \rightarrow 91$ | $(s_6, -0.40)$ | $\overline{x}_o^4$ | $55 \wedge 88 \rightarrow 91$ | $(s_3, 0.15)$ | | | |
| | $88 \rightarrow 91$ | $(s_6, -0.40)$ | | $88 \rightarrow 91$ | $(s_3, 0.15)$ | | | |
| | $88 \wedge 109 \rightarrow 91$ | $(s_6, -0.40)$ | | $55 \rightarrow 91$ | $(s_3, 0.15)$ | | | |
| | $88 \wedge 109 \wedge 127 \rightarrow 91$ | $(s_6, -0.40)$ | | $88 \rightarrow 55$ | $(s_3, 0.15)$ | | | |
| | $88 \wedge 127 \rightarrow 91$ | $(s_6, -0.40)$ | | $88 \wedge 91 \rightarrow 55$ | $(s_3, 0.15)$ | | | |

In Mushroom database, 43558 association rules are extracted according to $(Sup(), Con()) = (0.3, 0.8)$, in which, 3 association rules with the maximum linguistic satisfied degree $(s_5, 0.04)$ are selected by linguistic aggregation operator $\overline{x}_1$, 3 association rules with the maximum linguistic satisfied degree $(s_5, -0.43)$ are selected by $\overline{x}_2$, 6 association rules with the maximum linguistic satisfied degree $(s_5, 0.09)$ are selected by $\overline{x}_w^1$, 3 association rules with the maximum linguistic satisfied degree $(s_5, -0.12)$ are selected by $\overline{x}_w^3$, 3 association rules with the maximum linguistic satisfied degree $(s_3, 0.33)$ are selected by $\overline{x}_w^4$, 3 association rules with the maximum linguistic satisfied degree $(s_5, 0.0)$ are selected by $\overline{x}_o^1$, 3 association rule with the maximum linguistic satisfied degree $(s_5, 0.49)$ are selected by $\overline{x}_o^3$ and 4 association rules with the maximum linguistic satisfied degree $(s_3, 0.14)$ are selected by $\overline{x}_o^4$, the first five satisfied association rules of them are shown in Table 11. Fig.(10) shows analysis of time and spatial complexities by using the proposed method in Mushroom database.

In T40I10D100K database, 146451 association rules are extracted according to $(Sup(), Con()) = (0.013, 0.8)$, in which, 3 association rules with the maximum linguistic sat-

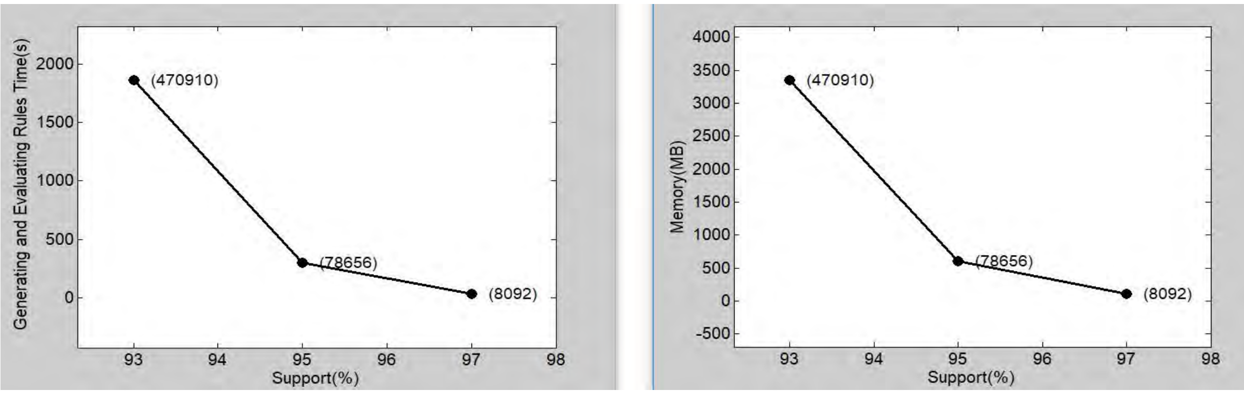


**FIGURE 9.** Time and spatial complexities of satisfied association rules via aggregation of fuzzy linguistic satisfied degrees from connect database.

**TABLE 11.** The first five satisfied association rules with the maximum linguistic satisfied degrees in Mushroom database.

| Op | E-Rules | L-degree | Op | E-Rules | L-degree | Op | E-Rules | L-degree |
|---|---|---|---|---|---|---|---|---|
| $\overline{x}_1$ | $85 \wedge 86 \wedge 90 \rightarrow 34$ | $(s_5, 0.04)$ | $\overline{x}_2$ | $85 \wedge 86 \wedge 90 \rightarrow 34$ | $(s_5, -0.43)$ | $\overline{x}_w^1$ | $34 \wedge 85 \rightarrow 86$ | $(s_5, 0.09)$ |
| | $86 \wedge 90 \rightarrow 34 \wedge 85$ | $(s_5, 0.04)$ | | $86 \wedge 90 \rightarrow 34 \wedge 85$ | $(s_5, -0.43)$ | | $34 \rightarrow 85 \wedge 86$ | $(s_5, 0.09)$ |
| | $86 \wedge 90 \rightarrow 34$ | $(s_5, 0.04)$ | | $86 \wedge 90 \rightarrow 34$ | $(s_5, -0.43)$ | | $85 \wedge 86 \wedge 90 \rightarrow 34$ | $(s_5, 0.09)$ |
| | $24 \wedge 53 \rightarrow 90 \wedge 94$ | $(s_5, 0.03)$ | | $24 \wedge 53 \rightarrow 90 \wedge 94$ | $(s_5, -0.45)$ | | $86 \wedge 90 \rightarrow 34 \wedge 85$ | $(s_5, 0.09)$ |
| | $34 \rightarrow 85 \wedge 86$ | $(s_5, 0.03)$ | | $34 \rightarrow 85 \wedge 86$ | $(s_5, -0.45)$ | | $34 \rightarrow 86$ | $(s_5, 0.09)$ |
| $\overline{x}_w^3$ | $34 \wedge 85 \rightarrow 86$ | $(s_5, -0.12)$ | $\overline{x}_w^4$ | $34 \wedge 85 \rightarrow 86$ | $(s_3, 0.33)$ | $\overline{x}_o^1$ | $34 \wedge 85 \rightarrow 86$ | $(s_5, 0.0)$ |
| | $34 \rightarrow 85 \wedge 86$ | $(s_5, -0.12)$ | | $34 \rightarrow 85 \wedge 86$ | $(s_3, 0.33)$ | | $34 \rightarrow 85 \wedge 86$ | $(s_5, 0.0)$ |
| | $34 \rightarrow 86$ | $(s_5, -0.12)$ | | $34 \rightarrow 86$ | $(s_3, 0.33)$ | | $34 \rightarrow 86$ | $(s_5, 0.0)$ |
| | $85 \wedge 86 \rightarrow 34$ | $(s_5, -0.14)$ | | $85 \wedge 86 \rightarrow 34$ | $(s_3, 0.32)$ | | $85 \wedge 86 \rightarrow 34$ | $(s_5, -0.05)$ |
| | $86 \rightarrow 34 \wedge 85$ | $(s_5, -0.14)$ | | $86 \rightarrow 34 \wedge 85$ | $(s_3, 0.32)$ | | $86 \rightarrow 34 \wedge 85$ | $(s_5, -0.05)$ |
| $\overline{x}_o^3$ | $34 \wedge 85 \rightarrow 86$ | $(s_5, 0.49)$ | $\overline{x}_o^4$ | $34 \wedge 85 \rightarrow 86$ | $(s_3, 0.14)$ | | | |
| | $34 \rightarrow 85 \wedge 86$ | $(s_5, 0.49)$ | | $34 \rightarrow 85 \wedge 86$ | $(s_3, 0.14)$ | | | |
| | $34 \rightarrow 86$ | $(s_5, 0.49)$ | | $34 \rightarrow 86$ | $(s_3, 0.14)$ | | | |
| | $85 \wedge 86 \rightarrow 34$ | $(s_5, 0.42)$ | | $86 \rightarrow 34 \wedge 85$ | $(s_3, 0.14)$ | | | |
| | $86 \rightarrow 34 \wedge 85$ | $(s_5, 0.42)$ | | $34 \wedge 85 \wedge 90 \rightarrow 86$ | $(s_3, 0.13)$ | | | |

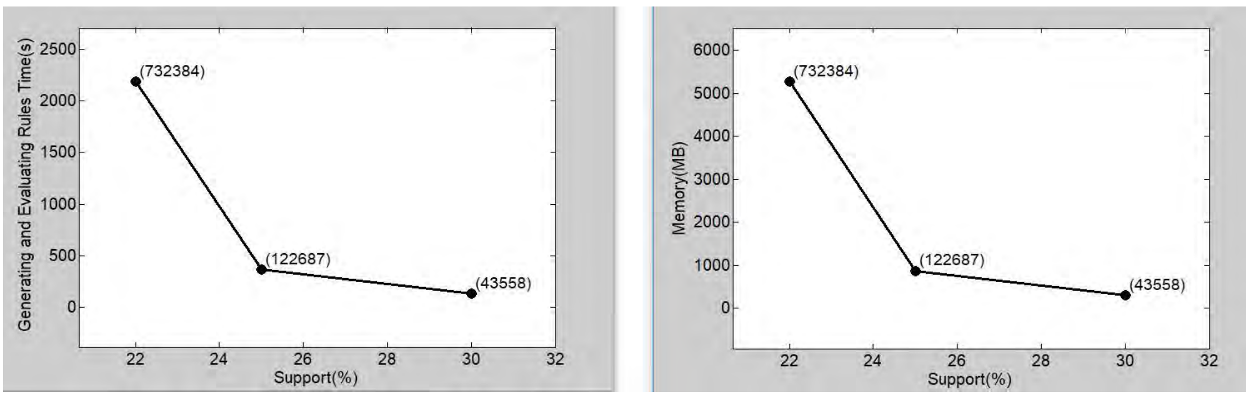


**FIGURE 10.** Time and spatial complexities of satisfied association rules via aggregation of fuzzy linguistic satisfied degrees from mushroom database.

isfied degree $(s_6, 0.47)$ are selected by linguistic aggregation operator $\overline{x}_1$, 1 association rules with the maximum linguistic satisfied degree $(s_6, -0.29)$ are selected by $\overline{x}_2$, 3 association rules with the maximum linguistic satisfied degree $(s_7, -0.45)$ are selected by $\overline{x}_w^1$, 10 association rules with the maximum linguistic satisfied degree $(s_5, 0.4)$ are selected by $\overline{x}_w^3$, 1 association rules with the maximum linguistic satisfied degree $(s_5, -0.37)$ are selected by $\overline{x}_w^4$, 10 association rules with the maximum linguistic satisfied degree $(s_6, 0.29)$ are selected by $\overline{x}_o^1$, 1 association rule with the maximum linguistic satisfied degree $(s_7, -0.42)$ are selected by $\overline{x}_o^3$ and 50 association rules with the maximum linguistic satisfied degree $(s_4, 0.03)$ are selected by $\overline{x}_o^4$, the first five satisfied association rules of them are shown in Table 12. Fig.(11) shows analysis of time and spatial complexities by using the proposed method in T40I10D100K database.

It can be noticed from Tables 9-12 that linguistic aggregation operators $\overline{x}_1$ and $\overline{x}_2$ seem own the similar effect in

**TABLE 12.** The first five satisfied association rules with the maximum linguistic satisfied degrees in T40I10D100K database.

| Op | E-Rules | L-degree | Op | E-Rules | L-degree |
|---|---|---|---|---|---|
| $\overline{x}_1$ | $509 \wedge 544 \wedge 598 \rightarrow 680 \wedge 876$ | $(s_6, 0.47)$ | $\overline{x}_2$ | $509 \wedge 544 \wedge 598 \rightarrow 680 \wedge 876$ | $(s_6, -0.29)$ |
| | $680 \wedge 876 \wedge 900 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_6, 0.47)$ | | $680 \wedge 876 \wedge 900 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_6, -0.30)$ |
| | $509 \wedge 544 \wedge 598 \rightarrow 876 \wedge 900 \wedge 989$ | $(s_6, 0.47)$ | | $509 \wedge 544 \wedge 598 \rightarrow 876 \wedge 900 \wedge 989$ | $(s_6, -0.30)$ |
| | $509 \wedge 544 \wedge 598 \rightarrow 618 \wedge 876 \wedge 989$ | $(s_6, 0.46)$ | | $509 \wedge 544 \wedge 598 \rightarrow 618 \wedge 876 \wedge 989$ | $(s_6, -0.31)$ |
| | $618 \wedge 876 \wedge 989 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_6, 0.46)$ | | $618 \wedge 876 \wedge 989 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_6, -0.31)$ |
| $\overline{x}_w^1$ | $509 \wedge 544 \wedge 598 \rightarrow 680 \wedge 876$ | $(s_7, -0.45)$ | $\overline{x}_w^3$ | $509 \wedge 544 \wedge 598 \rightarrow 618 \wedge 876 \wedge 989$ | $(s_5, 0.40)$ |
| | $680 \wedge 876 \wedge 900 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_7, -0.45)$ | | $618 \wedge 876 \wedge 989 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_5, 0.40)$ |
| | $509 \wedge 544 \wedge 598 \rightarrow 876 \wedge 900 \wedge 989$ | $(s_7, -0.45)$ | | $509 \wedge 544 \wedge 598 \rightarrow 680 \wedge 876$ | $(s_5, 0.40)$ |
| | $509 \wedge 544 \wedge 598 \rightarrow 618 \wedge 876 \wedge 989$ | $(s_7, -0.46)$ | | $680 \wedge 876 \wedge 900 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_5, 0.40)$ |
| | $618 \wedge 876 \wedge 989 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_7, -0.46)$ | | $509 \wedge 544 \wedge 598 \rightarrow 680 \wedge 876 \wedge 966$ | $(s_5, 0.40)$ |
| $\overline{x}_w^4$ | $205 \wedge 339 \wedge 544 \rightarrow 509 \wedge 598 \wedge 900$ | $(s_5, -0.37)$ | $\overline{x}_o^1$ | $509 \wedge 544 \wedge 598 \rightarrow 618 \wedge 876 \wedge 989$ | $(s_6, 0.29)$ |
| | $509 \wedge 598 \wedge 900 \rightarrow 205 \wedge 339 \wedge 544$ | $(s_5, -0.38)$ | | $618 \wedge 876 \wedge 989 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_6, 0.29)$ |
| | $205 \wedge 544 \wedge 598 \rightarrow 339 \wedge 509 \wedge 900$ | $(s_5, -0.40)$ | | $509 \wedge 544 \wedge 598 \rightarrow 680 \wedge 876$ | $(s_6, 0.29)$ |
| | $205 \wedge 544 \wedge 900 \rightarrow 339 \wedge 509 \wedge 598$ | $(s_5, -0.40)$ | | $680 \wedge 876 \wedge 900 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_6, 0.29)$ |
| | $205 \wedge 598 \wedge 900 \rightarrow 339 \wedge 509 \wedge 544$ | $s_5, -0.40)$ | | $509 \wedge 544 \wedge 598 \rightarrow 680 \wedge 876 \wedge 966$ | $(s_6, 0.29)$ |
| $\overline{x}_o^3$ | $509 \wedge 544 \wedge 598 \rightarrow 680 \wedge 876$ | $(s_7, -0.42)$ | $\overline{x}_o^4$ | $205 \wedge 509 \wedge 598 \rightarrow 339 \wedge 544$ | $(s_4, 0.03)$ |
| | $680 \wedge 876 \wedge 900 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_7, -0.44)$ | | $205 \wedge 544 \wedge 598 \rightarrow 339 \wedge 509$ | $(s_4, 0.03)$ |
| | $509 \wedge 544 \wedge 598 \rightarrow 876 \wedge 900 \wedge 989$ | $(s_7, -0.44)$ | | $205 \wedge 544 \rightarrow 339 \wedge 509 \wedge 598$ | $(s_4, 0.03)$ |
| | $509 \wedge 544 \wedge 598 \rightarrow 618 \wedge 876 \wedge 989$ | $(s_7, -0.46)$ | | $339 \wedge 509 \wedge 598 \rightarrow 205 \wedge 544$ | $(s_4, 0.03)$ |
| | $618 \wedge 876 \wedge 989 \rightarrow 509 \wedge 544 \wedge 598$ | $(s_7, -0.46)$ | | $339 \wedge 544 \rightarrow 205 \wedge 509 \wedge 598$ | $(s_4, 0.03)$ |

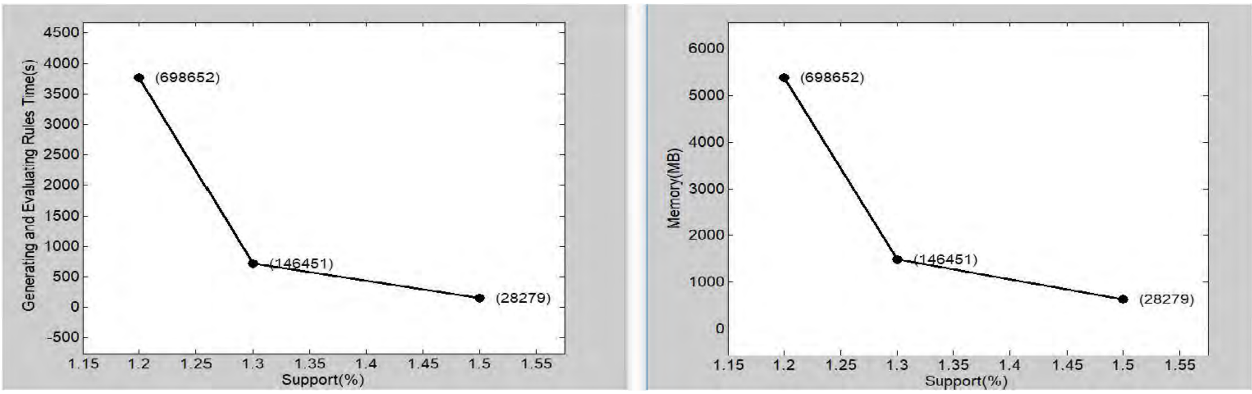


**FIGURE 11.** Time and spatial complexities of satisfied association rules via aggregation of fuzzy linguistic satisfied degrees from T40I10D100K database.

selecting satisfied association rules from extracted association rules, because satisfied association rules selected from Chess, Connect, Mushroom or T40I10D100K databases by using $\overline{x}_1$ and $\overline{x}_2$ are almost the same. 2-tuple linguistic weighted averaging operators and 2-tuple linguistic ordered weighted averaging operators seem own different effect in selecting satisfied association rules from extracted association rules, because satisfied association rules selected from Chess, Connect, Mushroom or T40I10D100K databases by using them are very different. Satisfied association rules selected from Chess, Connect, Mushroom or T40I10D100K databases via aggregation of linguistic satisfied degrees further show that linguistic aggregation operator and weights of objective and subjective interesting measures can be utilized to select different satisfied association rules from the huge size of extracted association rules.

## VI. CONCLUSION

Apriori, Eclat, FP-Growth or LCM algorithms have been well-established, by which the huge size of association rules with higher support or confidence measures are extracted from databases. In the paper, a new method based on aggregating linguistic satisfied degrees of extracted association rules has been proposed, which can be utilized to select satisfied association rules from extracted association rules. In the new method, objective and subjective interesting measures of extracted association rules are firstly transformed into linguistic satisfied degrees, then linguistic aggregation operators are designed according to weights of objective and subjective interesting measures, which are utilized to aggregate linguistic satisfied degrees of each extracted association rule, finally, order on linguistic aggregation results of extracted association rules is used to select satisfied association rules. Chess, Connect, Mushroom and T40I10D100K databases are utilized to experiment the new method, in which association rules are extracted from them by Apriori, Eclat, FP-Growth or LCM algorithms, results show that different linguistic aggregation operator and weights of objective and subjective interesting measures can be utilized to select different satisfied association rules from the huge size of extracted association rules.

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