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Sum Rate Maximization for Multi-Carrier SWIPT Relay System With Non-Ideal Power Amplifier and Circuit Power Consumption

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ABSTRACT In this paper, we investigate the resource optimization algorithm design in a multicarrier relay system with simultaneous wireless information and power transfer (SWIPT). The relay is capable of harvesting energy from the source's signals by using the power splitting method. The non-ideal energy consumption including both the non-ideal power amplifier and non-ideal circuit power consumption is considered. First, we study the transmission rate maximization problem (TRMP) in an asymmetric decode-and-forward (DF) relay transmission, where the transmission power at the source, the transmission power at the relay, the power splitting ratio, and the transmission time are jointly optimized. The formulated problem is a non-convex problem, and it is generally quite difficult to solve it. By exploiting the structure of the problem, we propose two methods (logarithmic operation on constraints and logarithmic change of variables) to transform it into the corresponding difference of convex (DC) optimization problems. Then, we extend the TRMP to an amplify-and-forward (AF) relay transmission. Furthermore, we propose an effective algorithm to solve the DC optimization problem and prove that the algorithm can converge to a stationary point. Finally, extensive simulations are conducted to verify the performance of the proposed algorithm. The simulation results show that the asymmetric DF relay transmission achieves the highest sum rate and the AF relay transmission achieves a much lower sum rate than both the asymmetric and symmetric DF relay transmissions under different conditions.

INDEX TERMS Non-ideal power consumption, non-convex optimization, power splitting, relay transmission, simultaneous wireless information and power transfer.

I. INTRODUCTION

Wireless relay transmission has been widely used in wireless sensor networks, where the relay node helps transmit one node's information to another. Commonly, the relay node is supplied energy by a capacity constrained battery, which makes the available energy limited, and thus further restricts the network performance improvement. To solve the energy deficiency problem, replacing battery is one possible method, but it is usually with high cost due to the large number of sensor nodes, and it is not always feasible, e.g., in the unsafe environment. Recently, simultaneous wireless information and power transfer (SWIPT) has been proposed and drawn extensive attention [1]–[3]. Different from traditional relay

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transmission without SWIPT, with this technology, terminals can not only obtain information but also harvest energy from the radiated signals, thus the energy deficiency problem in relay transmission can be resolved to some extent.

To achieve SWIPT, time switching (TS) and power splitting (PS) are two well-known schemes. Based on these two schemes, many works have devoted to wireless relay transmission with SWIPT. For TS scheme, different network performance is studied in recent works. The authors in [4] derived the analytical expression of the throughput with variable time of energy harvesting, and showed the performance with variable time of energy harvesting is better than that with fixed time of energy harvesting. The authors in [5] analyzed the signal-to-noise ratio (SNR), outage and throughput, and gave the optimal energy harvesting time to achieve throughput maximization. The authors in [6] proposed a bisection search based algorithm for optimal transmission power and time allocation to achieve sum rate maximization in multicarrier relay networks. The authors in [7] proposed a new method to enhance the network performance of [6] by joint optimization of subcarrier pairing, power allocation, and time switching allocation, where the ordered subcarrier pairing is proven to be optimal, and both a global optimal algorithm and a fast asymptotic optimal algorithm are proposed. There are also some works investigating the energy harvesting relay transmission with PS scheme. The authors in [8] investigated the joint power splitting ratio and power allocation problem to achieve sum rate maximization in orthogonal frequency-division multiplexing (OFDM) relay networks, and the Lagrangian dual decomposition method is adopted to propose an efficient algorithm. Then the considered problem is extended to include transmission mode selection. The authors in [9] studied the sum rate maximization problem with the consideration of power splitting and power allocation, where one searching algorithm for finding the optimal power splitting ratio and an algorithm based on alternative convex search are proposed. The authors in [10] studied the joint transmission power allocation and power splitting ratio in multi-cell relay networks, where three different objectives, i.e., sum rate maximization, max-min throughput fairness, and sum-power minimization, are studied.

In wireless relay transmission, decode-and-forward (DF) and amplify-and-forward (AF) are two well known relay transmission strategies. In the existing works devoting to the performance analysis and resource optimization for wireless relay transmission with SWIPT, both DF [6]–[8] and AF [5], [9] relay transmissions are investigated, and the performance comparison of both the DF and AF relay transmissions is studied in [4], [10]. But most of those works study the symmetric relay transmission model for DF relay, by which the transmission from the source to the relay and that from the relay to the destination have the same time duration. There are few works studying the asymmetric relay transmission in energy harvesting relay networks [10]. The authors in [10] investigated the network performance for multicell relay networks with asymmetric DF relay transmission. In fact, the asymmetric DF relay transmission is able to provide much better performance than the symmetric DF relay transmission due to the flexible transmission time allocation [11], but the resource optimization for asymmetric DF relay transmission with SWIPT is not well studied.

Power consumption is an important consideration for network performance analysis in energy constrained SWIPT relay networks. But all the above mentioned works only consider the transmission power consumption, the circuit power consumption is totally ignored. This is consistent with traditional relay networks without SWIPT. However, in SWIPT networks the transmission distance is usually much shorter (from several meters to dozens of meters) than that of the pure information transmission in traditional relay networks, which makes the circuit power consumption comparable with the transmission power consumption [12], [13], thus the circuit power consumption cannot be omitted. Without this consideration, the practical network model cannot be well described, and the obtained results cannot fully reflect the true performance of the system. Recently, there are some works considering the circuit power consumption in SWIPT direct transmission networks without relaying [14]–[18]. The works [14]–[16] considered the on-off transmitter power consumption model, which shows that when the transmitter transmits, the consumed power is the sum of the transmit power and a constant circuit power, and when the transmitter turns off, the consumed power is zero; and the work [17], [18] studied the rate-dependent circuit power consumption model. However, the studied models in those works do not consider the effect of the non-ideal power amplifier (PA), thus they are not sufficient to model the complex non-ideal power consumption. As far as we know, the aggressive effect of the non-ideal power amplifier and the non-ideal circuit power consumption in multicarrier relay transmission with SWIPT has not been investigated so far. Motivated by these observations, we desire to explore the sum rate performance in SWIPT relay transmissions with a more realistic power consumption model, including both the non-ideal PA and non-ideal circuit power consumption. The non-ideal PA makes the PA power consumption non-linear to the transmission power. The non-ideal circuit power makes the circuit power consumption non-linear and non-convex, because the circuit power consumption is usually proportional to the data transmission rate. For the asymmetric DF relay transmission, the data transmission rate itself is a non-convex function with the variables of transmission power, transmission time, and PS factor, which are highly coupled. The non-convex circuit power constraints make the constraint set of the considered resource optimization problem non-convex. This dramatically increases the difficulty to solve the resource optimization problem, and no existing methods can be directly used to solve the problem here.

Inspired by all the mentioned considerations, in this paper we investigate the sum rate maximization problem in SWIPT relay networks with PS scheme by taking into account a more realistic energy consumption model, with the considerations of non-ideal power amplifier and non-ideal circuit power. To provide a comprehensive study of both the DF and AF relay transmissions in SWIPT networks with a more realistic power consumption model, and compare their performance under different conditions, both the asymmetric DF relay and AF relay transmissions are studied. The sum rate maximization problems for both asymmetric DF and AF are formulated with the considerations of the source power constraint, the relay's available power constraint, and the power splitting ratio constraints. One more constraint, i.e., time allocation constraint, is also included for the asymmetric DF relay. The integration of the asymmetric DF /the AF relay transmission with the non-ideal power consumption makes the resource optimization problem non-convex and thus very difficult to solve. Therefore, we propose two effective methods based on logarithmic operation on constraints and logarithmic change of variables respectively to equivalently transform the problems into difference of convex (DC) programming problems. Then an algorithm for solving the DC optimization problem is proposed and the convergence property of the algorithm is analyzed. Simulations are carried out to compare the achieved sum rate for the asymmetric DF relay transmission, AF relay transmission, and the benchmark symmetric DF relay transmission by employing the proposed algorithms and the baseline algorithm under different system settings.

The main contributions are summarized as follows:

- A more realistic energy consumption model including both the non-ideal power amplifier and non-ideal circuit power consumption is incorporated into the sum rate maximization problem, which is much closer to the practical system. As far as we know, this is the first time to consider this energy consumption model in SWIPT relay networks.
- In all of the existing works studying the multicarrier relay transmission with power splitting, either AF relay transmission or DF relay transmission with equal transmission time are studied. To further explore the network performance, we study the asymmetric DF relay transmission with unequal transmission time, which increases one new dimensional freedom, and can greatly enhance the sum rate performance.
- To solve the non-convex sum rate maximization problems, we propose two methods to transform them into equivalent DC optimization problems by exploiting the structure of these problems, and an effective algorithm is then designed to solve the DC optimization problems. Moreover, we prove that the proposed algorithm can monotonously converge to a stationary point.
- To verify the performance of the proposed algorithms for different relay transmissions, besides the asymmetric DF relay and AF relay, the symmetric DF relay transmission is also simulated. In addition, a baseline algorithm is also simulated for performance comparison with the proposed algorithms. Numerical simulation results show that the proposed algorithm based on the designed two methods can always achieve almost the same sum rate performance for either asymmetric DF relay transmission or AF relay transmission. The convergence speed of the algorithm with logarithmic operation on constraints is often much faster than that with logarithmic change of variables for the three different relay transmissions. Under different simulation parameters, the asymmetric DF relay transmission always obtains the highest sum rate, the AF relay transmission admits the lowest sum rate, and the symmetric DF relay transmission has a sum rate in between. The proposed algorithm outperforms the corresponding baseline algorithm in terms of sum rate under different conditions.

The remainder of the paper is organized as follows. In section II, the system model is introduced, and the sum rate maximization problem for the asymmetric DF relay transmission is formulated. In section III, two effective methods are proposed to transform the formulated optimization problem to DC optimization problems. In section IV, the considered problem is extended to AF relay transmission and two effective methods are designed to change the formulated problem to DC optimization problems. In section V, an efficient algorithm is proposed to solve the DC optimization problems, and the convergence of the proposed algorithm is analyzed. Numerical simulation results are presented in section VI. Finally, section VII concludes the paper.

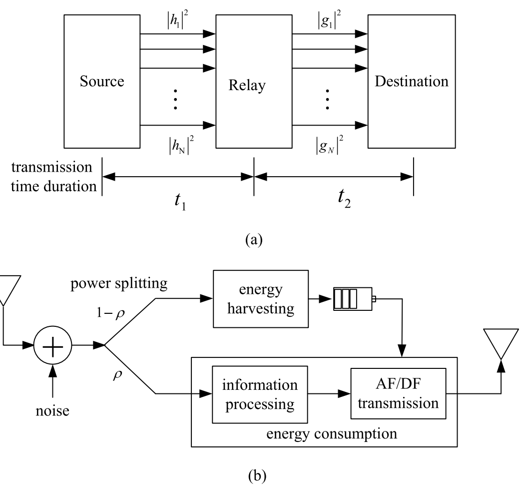

FIGURE 1. (a) Multi-carrier relay transmission. (b)Architecture for the relay with PS scheme.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a two-hop DF relay network where the source transmits signals to the destination with the help of an energy harvesting relay. The total bandwidth B is equally divided into N orthogonal subcarriers. Each subcarrier has a bandwidth of $\frac{B}{N}$ Hz. Let $\mathcal{N}$ be the set of all subcarriers, that is, $\mathcal{N}=\{1, 2, \cdots, N\}$. It is assumed that the relay is capable of harvesting energy from the received signals from the source before forwarding transmission. PS scheme is adopted by the relay for SWIPT. The PS based relay transmission protocol is shown in Fig. 1. The transmission time from the source to the destination is fixed as T. In the first-hop transmission, the source sends signals, the relay uses ρ part of the received signals from the source for information receiving and the remaining $1-\rho$ part of signals for energy harvesting, where ρ denotes the PS factor and it satisfies that $0<\rho<1$. In the second-hop transmission, the relay uses the energy harvested from the first-hop to transmit the source information to the destination. It is assumed that during the time duration T the relay can harvest enough energy to help the source to finish a data block transmission. Moreover, it is assumed that the direct transmission from the source to the destination is impossible due to the obstacles between them [4], [7]. The channel state information is assumed to be perfectly known, and the energy used for obtaining the channel state information at the relay is assumed from a dedicated energy supplier that does not consume the harvested energy [19].

A. INFORMATION TRANSMISSION

The transmission rate at the relay on subcarrier n can be expressed by [20]

$$R_{s,r,n}=t_1\log_2\left(1+\frac{\rho p_{s,n}|h_n|^2}{\frac{B}{N}\left(\rho N_{r,a,n}+N_{r,b,n}\right)}\right), \tag{1}$$

where t_1 is the time duration factor for the transmission from the source to the relay and it satisfies $0<t_1<1$, $p_{s,n}$ is the transmission power at the source on subcarrier n, h_n the channel gain from the source to the relay on subcarrier n, and $N_{r,a,n}$ the noise power spectral density of baseband additive white Gaussian noise (AWGN) due to receiving antenna at the relay, $N_{r,b,n}$ noise power spectral density of AWGN due to RF band to baseband signal conversion at the relay on subcarrier n [3], [21]. In practice, the antenna noise is often much less than the baseband signal conversion noise, thus the antenna noise has a negligible impact on both the energy harvesting and information transmission rate [22]. For simplicity, we will ignore the effect of antenna noise at the relay in the following derivation by setting $N_{r,a,n}=0$ [22], [23].

The relay uses the harvested energy and the initial stored energy to forward the received signal from the source to the destination in the remaining time duration t_2T, where t_2 satisfies $t_2=1-t_1$. The transmission rate from the relay to the destination on subcarrier n is given by

$$R_{r,d,n}=t_2\log_2\left(1+\frac{p_{r,n}|g_n|^2}{\frac{B}{N}\left(N_{d,a,n}+N_{d,b,n}\right)}\right), \tag{2}$$

where $p_{r,n}$ is the transmission power at the relay on subcarrier n, g_n the channel gain from the relay to the destination on subcarrier n, and $N_{d,a,n}$ and $N_{d,b,n}$ the noise power spectral density of antenna AWGN and conversion AWGN at the destination on subcarrier n, respectively. Without loss of generally and for ease of notation, it is assumed that $N_a=N_{r,a,n}=N_{d,a,n}$ and $N_b=N_{r,b,n}=N_{d,b,n}$, $\forall n\in\mathcal{N}$ [10], [23], [24].

For the asymmetric DF relay transmission, the transmission rate from the source to the destination is the minimum of transmission rates achieved in the two consecutive time durations, which can be expressed by [6], [8],

$$R=\min\left\{R_{s,r},R_{r,d}\right\}, \tag{3}$$

where $R_{s,r}$ and $R_{r,d}$ are the transmission rate achieved from the source to the relay transmission and that from the relay to the destination transmission, respectively, and they are given by

$$R_{s,r}=\sum\nolimits_{n=1}^{N}R_{s,r,n} \tag{4}$$

$$R_{r,d}=\sum\nolimits_{n=1}^{N}R_{r,d,n}. \tag{5}$$

B. ENERGY HARVESTING

In the first-hop transmission of time duration t_1T, the harvested energy at the relay on subcarrier n from the source signal is

$$E_n=t_1T\varsigma(1-\rho)p_{s,n}|h_n|^2, \tag{6}$$

where ς denotes the energy conversion efficiency and it satisfies $0<\varsigma\le 1$.

The relay gathers all the harvested energy E_n from all subcarriers $n\in\mathcal{N}$. The total harvested energy at the relay is denoted by $E_r=\sum_{n=1}^{N}E_n$. The relay uses the harvested energy in the first-hop and its initial stored energy for relay transmission in the second-hop. The total available power at the relay for the second-hop transmission in the duration t_2T is given by

$$\begin{aligned}P_{r,tot}&=\frac{E_r}{t_2T}+\frac{Q_{ini}}{t_2T}\\&=\frac{t_1\sum_{n=1}^{N}\varsigma(1-\rho)p_{s,n}|h_n|^2}{t_2}+\frac{Q_{ini}}{t_2T},\end{aligned} \tag{7}$$

where Q_{ini} is the initial stored energy at the relay. It should be noted that it is assumed the initial energy Q_{ini} is not sufficient enough in the paper [25], hence energy harvesting at the relay is still necessary to improve the transmission rate from the source to the destination.

C. NON-IDEAL POWER CONSUMPTION

The energy consumption of the transmitter includes the transmission energy consumption and the circuit power consumption for mixers, filters, and analog-to-digital converters. In our model, both the power amplifier (PA) used for signal transmission and the circuit power consumption are modeled as a realistic nonlinear model. The total power consumption of a transmitter in a transmission can be expressed by

$$P=P_{con,tr}+P_{cir}, \tag{8}$$

where $P_{con,tr}$ and P_{cir} are the energy consumptions for signal transmission and circuit, respectively.

1) NON-IDEAL POWER AMPLIFIER

In multi-carrier communication systems, such as OFDM systems, the modulated signals exhibit high peak-to-average power ratio, thus suffering from severe nonlinear power amplifier (PA) effects [26]. To model the nonlinear PA, the total input power required for mean output transmission power can be approximated as [27],

$$P_{con,tr} = \frac{\sqrt{P_{max}}}{\eta_{max}}\sqrt{P_{tr}}, \tag{9}$$

where $P_{con,tr}$ is the total consumed power for information transmission, P_{tr} is the transmission power, P_{max} is the maximum output power of the PA and η_{max} is the maximum PA efficiency.

2) NON-IDEAL CIRCUIT POWER CONSUMPTION

The circuit power is modeled by a dynamic linear rate-dependent part and a static part used for driving hardware [28], i.e.,

$$P_{cir} = \kappa R_t + P_c, \tag{10}$$

where κ denotes the dynamic power consumption per unit data rate, R_t is the transmission rate, and P_c refers to the constant power consumption for driving hardware.

D. PROBLEM FORMULATION

The objective is to maximize the total rate over all subcarriers constrained by the total source power, the available power based on energy harvesting at the relay, the PS factor, and the dynamic time allocation. The problem under consideration is formulated as follows,

$$\max_{\{\rho,t_1,t_2,p_{s,n},p_{r,n}\}} R = \min\left\{\sum_{n=1}^{N} t_1 \log_2\left(1+\frac{\rho p_{s,n}\left|h_n\right|^2}{\frac{B}{N}N_b}\right),\right. \left.\sum_{n=1}^{N} t_2 \log_2\left(1+\frac{p_{r,n}\left|g_n\right|^2}{\frac{B}{N}\left(N_a+N_b\right)}\right)\right\}$$

$$\text{subject to: } \frac{\sqrt{P_{s,max}}}{\eta_{s,max}}\sqrt{\sum\nolimits_{n=1}^{N} p_{s,n}+P_{s,c}} + \kappa R_{s,r} \leqslant P_{s,tot}, \tag{11a}$$

$$\frac{\sqrt{P_{r,max}}}{\eta_{r,max}}\sqrt{\sum\nolimits_{n=1}^{N} p_{r,n}+P_{r,c}+\kappa R_{r,d}} \leqslant \frac{t_1\sum_{n=1}^{N}\varsigma(1-\rho)p_{s,n}\left|h_n\right|^2}{t_2}+\frac{Q_{ini}}{t_2 T}, \tag{11b}$$

$$0 < \rho < 1, \tag{11c}$$

$$t_1 + t_2 = 1, \tag{11d}$$

$$0 \le p_{s,n}, 0 \le p_{r,n}, \forall n \in \mathcal{N}, \tag{11e}$$

$$0 < t_1, 0 < t_2, \tag{11f}$$

where $P_{s,tot}$ is the power threshold at the source, $P_{s,max}$ and $P_{r,max}$ are the maximum output power of the PA at the source and the relay, respectively, $\eta_{s,max}$ and $\eta_{r,max}$ are the maximum PA efficiency of the source and the relay, respectively, and $P_{s,c}$ and $P_{r,c}$ are the constant circuit power consumption for driving hardware at the source and the relay, respectively. (11a) is the total power constraint at the source, (11b) is the total power constraint at the relay, (11c) is the constraint of the power splitting ratio ρ, (11d) indicates the transmission time allocation constraint, (11e) reveals that the power allocation variables $p_{s,n}, p_{r,n}$ are non-negative, and (11f) shows the time allocation factor t_1 and t_2 are positive.

Problem (11) is a non-convex optimization problem, since the objective function is not a concave function, the left-hand side of constraint (11a) is not a convex function, and both the left-hand side and right-hand side of (11b) are neither concave or convex functions. And the variables are highly coupled as shown in the right-hand side of (11b). It is generally quite difficult to solve this kind of non-convex optimization problem. As far as we know, there exist no practical methods that can guarantee to converge to the global optimal solution. Therefore, in the following we will propose two efficient methods to transform the problem into DC problems by exploiting the problem's special structure.

III. PROBLEM TRANSFORMATION

For ease of problem transformation, let us introduce a new variable z, and let $z = \min\{R_{s,r}, R_{r,d}\}$. Then problem (11) can be equivalently written as

$$\max_{\{\rho,t_1,t_2,p_{s,n},p_{r,n},z\}} z$$

$$\text{subject to : } z \le t_1 \sum\nolimits_{n=1}^{N} \log_2\left(1+\frac{\rho p_{s,n}\left|h_n\right|^2}{\frac{B}{N}N_b}\right), \tag{12a}$$

$$z \le t_2 \sum_{n=1}^{N} \log_2\left(1+\frac{p_{r,n}\left|g_n\right|^2}{\frac{B}{N}\left(N_a+N_b\right)}\right), \tag{12b}$$

$$\text{(11a)-(11f).}$$

In the following, two methods will be proposed to transform problem (12) into equivalent DC problems.

A. METHOD 1 WITH LOGARITHMIC OPERATION ON CONSTRAINTS

To facilitate the following analysis, define $q_{s,n} = \rho p_{s,n}$. We also employ new variables y, x, and ω to indicate the terms in constraints (11a) and (11b), then problem (12) can be written as,

$$\max_{\{\rho,t_1,t_2,q_{s,n},p_{r,n},x,y,z,\omega\}} z$$

$$\text{subject to: } z \le t_1 \sum\nolimits_{n=1}^{N} \log_2\left(1+\frac{q_{s,n}\left|h_n\right|^2}{\frac{B}{N}N_b}\right), \tag{13a}$$

$$z \leq t_2 \sum_{n=1}^{N} \log_2 \left(1 + \frac{p_{r,n} |g_n|^2}{\frac{B}{N}(N_a + N_b)}\right), \tag{13b}$$

$$y + P_{s,c} + \kappa R_{s,r} \leq P_{s,tot}, \tag{13c}$$

$$\frac{\sqrt{P_{s,max}}}{\eta_{s,max}} \sqrt{\frac{\sum_{n=1}^{N} q_{s,n}}{\rho}} \leq y, \tag{13d}$$

$$\frac{\sqrt{P_{r,max}}}{\eta_{r,max}} \sqrt{\sum_{n=1}^{N} p_{r,n} + P_{r,c}} + \kappa\omega \leqslant x + \frac{Q_{ini}}{t_2 T}, \tag{13e}$$

$$R_{r,d} \leq \omega, \tag{13f}$$

$$x \leqslant \frac{t_1(1-\rho)}{t_2\rho} \sum_{n=1}^{N} \varsigma q_{s,n} |h_n|^2 \tag{13g}$$

$$0 < \rho < 1, \tag{13h}$$

$$t_1 + t_2 = 1, \tag{13i}$$

$$0 \leq q_{s,n}, 0 \leq p_{r,n}, \forall n \in \mathcal{N}, \tag{13j}$$

$$0 \leq z, 0 \leq x, \tag{13k}$$

$$0 < t_1, 0 < t_2. \tag{13l}$$

Since the right-hand side of the constraints of (13a), (13b) and (13g) are not concave functions, and the left-hand sides of the constraints (13c) and (13d) are not convex functions, the constraint set is a non-convex set, problem (13) is still a non-convex problem, which is still difficult to solve. To make the problem solvable, we will transform the problem to a DC optimization problem by exploiting its structure. The definitions of the DC optimization problem and DC function are shown in Appendix.

Since the function $\log_2(a + \gamma)$ with variable satisfying $\gamma \geq 0$ and a given scalar a satisfying $a > 0$ is a concave function, and the nonnegative weighted sum of concave functions is also a concave function [29], we get that $\sum_{n=1}^{N} \log_2 \left(1 + \frac{q_{s,n}|h_n|^2}{\frac{B}{N}N_b}\right)$ and $\sum_{n=1}^{N} \log_2 \left(1 + \frac{p_{r,n}|g_n|^2}{\frac{B}{N}(N_a+N_b)}\right)$ are concave functions. So constraints (13a)-(13c) and (13f) contain the term with the form of a variable multiplying a concave function. And constraints (13d) and (13g) have the term with the form of multiple variables' multiplication and (or) division. With the help of the logarithmic operations on these constraints, we can transform these constraints into DC constraints. After logarithmic operations on these constraints, problem (13) becomes,

$$\max_{\{\rho,t_1,t_2,q_{s,n},p_{r,n},x,y,z,\omega\}} z$$

$$\text{subject to: } \log(z) \leq \log(t_1) + \log\left(\sum_{n=1}^{N} \log_2 \left(1 + \frac{q_{s,n}|h_n|^2}{\frac{B}{N}N_b}\right)\right), \tag{14a}$$

$$\log(z) \leq \log(t_2) + \log\left(\sum_{n=1}^{N} \log_2 \left(1 + \frac{p_{r,n}|g_n|^2}{\frac{B}{N}(N_a+N_b)}\right)\right), \tag{14b}$$

$$\log(t_1) + \log\left(\sum_{n=1}^{N} \log_2 \left(1 + \frac{q_{s,n}|h_n|^2}{\frac{B}{N}N_b}\right)\right) \leq \log\left(\frac{P_{s,tot} - P_{s,c} - y}{\kappa}\right), \tag{14c}$$

$$\log\left(P_{s,max}\sum_{n=1}^{N} q_{s,n}\right) \leq 2\log\left(\eta_{s,max}y\right) + \log(\rho), \tag{14d}$$

$$\frac{\sqrt{P_{r,max}}}{\eta_{r,max}} \sqrt{\sum_{n=1}^{N} p_{r,n} + P_{r,c}} + \kappa\omega \leqslant x + \frac{Q_{ini}}{t_2 T}, \tag{14e}$$

$$\log(t_2) + \log\left(\sum_{n=1}^{N} \log_2 \left(1 + \frac{p_{r,n}|g_n|^2}{\frac{B}{N}(N_a+N_b)}\right)\right) \leq \log(\omega), \tag{14f}$$

$$\log(t_2) + \log(\rho) + \log(x) \leqslant \log(t_1) + \log(1-\rho) + \log\left(\sum_{n=1}^{N} \varsigma q_{s,n}|h_n|^2\right), \tag{14g}$$

$$(13h) - (13l). \tag{14h}$$

Since $g_1 = \sum_{n=1}^{N} \log_2 \left(1 + \frac{q_{s,n}|h_n|^2}{\frac{B}{N}N_b}\right)$ and $g_2 = \sum_{n=1}^{N} \log_2 \left(1 + \frac{p_{r,n}|g_n|^2}{\frac{B}{N}(N_a+N_b)}\right)$ are concave functions, by applying the composition rule that for $f(\gamma) = h(g(\gamma))$, $\mathbf{dom} f = \{\gamma \in \mathbf{dom} g | g(\gamma) \in \mathbf{dom} h\}$, f is concave if h is concave and nondecreasing, and g is concave [29], it is readily obtained that $\log(g_1)$, $\log(g_2)$ are also concave functions. It should be pointed out that $\log(\cdot)$ indicates the natural logarithm throughout the paper. Besides, $\sqrt{\sum_{n=1}^{N} p_{r,n}}$ with $p_{r,n} \geq 0$ is a concave function and $1/t_2$ with $t_2 > 0$ is a convex function too. Hence, all the constraints in problem (14) can be written as either linear functions (14h) or DC functions ((14a)-(14g)). Therefore, problem (14) becomes a DC optimization problem.

B. METHOD 2 WITH LOGARITHMIC CHANGE OF VARIABLES

Besides method 1, we will propose another method to transform problem (12) to a DC optimization problem as well. To facilitate the following transformation, new variables u, v, and ω are introduced to problem (12). Thus problem (12) can be rewritten as,

$$\max_{\{\rho,t_1,t_2,p_{s,n},p_{r,n},z,u,v,\omega\}} z$$

$$\text{subject to: } z \leq t_1 \sum_{n=1}^{N} \log_2 \left(1 + \frac{\rho p_{s,n} |h_n|^2}{\frac{B}{N}N_b}\right), \tag{15a}$$

$$z \leq t_2 \sum_{n=1}^{N} \log_2 \left(1 + \frac{p_{r,n} |g_n|^2}{\frac{B}{N}(N_a + N_b)}\right), \tag{15b}$$

$$\frac{\sqrt{P_{s,max}}}{\eta_{s,max}}\sqrt{\sum_{n=1}^{N} p_{s,n}}+P_{s,c}+\kappa u \leqslant P_{s,tot}, \tag{15c}$$

$$R_{s,r} \leq u, \tag{15d}$$

$$v+P_{r,c}+\kappa\omega \leqslant \frac{t_1}{t_2}\sum_{n=1}^{N}\varsigma(1-\rho)p_{s,n}\left|h_n\right|^2+\frac{Q_{ini}}{t_2 T}, \tag{15e}$$

$$R_{r,d} \leq \omega, \tag{15f}$$

$$\frac{\sqrt{P_{r,max}}}{\eta_{r,max}}\sqrt{\sum_{n=1}^{N} p_{r,n}} \leq v, \tag{15g}$$

$$0 \leq z, \tag{15h}$$

$$(11c)-(11f). \tag{15i}$$

Define new variables as $\tilde{z}=\ln z$, $\tilde{v}=\ln v$, $\tilde{u}=\ln u$, $\tilde{p}_{s,n}=\ln p_{s,n}$, $\tilde{p}_{r,n}=\ln p_{r,n}$, $\tilde{\rho}=\ln\rho$, $\tilde{t}_1=\ln t_1$, $\tilde{t}_2=\ln t_2$, and $\tilde{\omega}=\ln\omega$, problem (15) becomes,

$$\max_{\{\tilde{\rho},\tilde{t}_1,\tilde{t}_2,\tilde{p}_{s,n},\tilde{p}_{r,n},\tilde{z},\tilde{u},\tilde{v},\tilde{\omega}\}} e^{\tilde{z}}$$

subject to:

$$e^{\tilde{z}-\tilde{t}_1}-\sum_{n=1}^{N}\log_2\left(1+\frac{e^{\tilde{\rho}+\tilde{p}_{s,n}}\left|h_n\right|^2}{\frac{B}{N}N_b}\right) \leq 0, \tag{16a}$$

$$e^{\tilde{z}-\tilde{t}_2}-\sum_{n=1}^{N}\log_2\left(1+\frac{e^{\tilde{p}_{r,n}}\left|g_n\right|^2}{\frac{B}{N}\left(N_a+N_b\right)}\right) \leq 0, \tag{16b}$$

$$\frac{\sqrt{P_{s,max}}}{\eta_{s,max}}\sqrt{\sum_{n=1}^{N} e^{\tilde{p}_{s,n}}}+P_{s,c}+\kappa e^{\tilde{u}} \leqslant P_{s,tot}, \tag{16c}$$

$$\sum_{n=1}^{N}\log_2\left(1+\frac{e^{\tilde{\rho}+\tilde{p}_{s,n}}\left|h_n\right|^2}{\frac{B}{N}N_b}\right)-e^{\tilde{u}-\tilde{t}_1} \leq 0, \tag{16d}$$

$$e^{\tilde{v}+\tilde{t}_2}+P_{r,c}e^{\tilde{t}_2}+\kappa e^{\tilde{t}_2+\tilde{\omega}} +\varsigma\sum_{n=1}^{N} e^{\tilde{\rho}+\tilde{t}_1+\tilde{p}_{s,n}}\left|h_n\right|^2 -\frac{Q_{ini}}{T}-\varsigma\sum_{n=1}^{N} e^{\tilde{t}_1+\tilde{p}_{s,n}}\left|h_n\right|^2 \leqslant 0 \tag{16e}$$

$$\sum_{n=1}^{N}\log_2\left(1+\frac{e^{\tilde{p}_{r,n}}\left|g_n\right|^2}{\frac{B}{N}\left(N_a+N_b\right)}\right)-e^{\tilde{\omega}-\tilde{t}_2} \leq 0, \tag{16f}$$

$$\sum_{n=1}^{N} e^{\tilde{p}_{r,n}} \leq \frac{\eta_{r,max}^2 e^{2\tilde{v}}}{P_{r,max}}, \tag{16g}$$

$$e^{\tilde{\rho}}<1, \tag{16h}$$

$$e^{\tilde{t}_1}+e^{\tilde{t}_2} \leqslant 1, \tag{16i}$$

$$1-e^{\tilde{t}_1}-e^{\tilde{t}_2} \leq 0, \tag{16j}$$

Before showing the property of problem (16), we will first analyze the convexity of the function $\sqrt{\sum_{n=1}^{N} e^{\tilde{p}_{s,n}}}$ in (16c).

Proposition 1: $\sqrt{\sum_{n=1}^{N} e^{\tilde{p}_{s,n}}}$ *is a convex function.*

Proof: Let $f=\sqrt{\sum_{n=1}^{N} e^{\tilde{p}_{s,n}}}$, then the first-order derivative of f with $\tilde{p}_{s,n}$ is

$$\frac{\partial f}{\partial \tilde{p}_{s,n}}=\frac{1}{2}\left(\sum_{n=1}^{N} e^{\tilde{p}_{s,n}}\right)^{-\frac{1}{2}} e^{\tilde{p}_{s,n}}. \tag{17}$$

And the second-order derivatives of f are given by

$$\frac{\partial^2 f}{\partial \tilde{p}_{s,n}^2}=-\frac{1}{4}\left(\sum_{n=1}^{N} e^{\tilde{p}_{s,n}}\right)^{-\frac{3}{2}}\left(e^{\tilde{p}_{s,n}}\right)^2 +\frac{1}{2}\left(\sum_{n=1}^{N} e^{\tilde{p}_{s,n}}\right)^{-\frac{1}{2}} e^{\tilde{p}_{s,n}}, \tag{18}$$

$$\frac{\partial^2 f}{\partial \tilde{p}_{s,n}\partial \tilde{p}_{s,m}}=-\frac{1}{4}\left(\sum_{n=1}^{N} e^{\tilde{p}_{s,n}}\right)^{-\frac{3}{2}} e^{\tilde{p}_{s,n}}e^{\tilde{p}_{s,m}}. \tag{19}$$

Based on the expressions of (18)-(19), the Hessian matrix of f can be written as

$$\nabla^2 f(\tilde{p}_{s,n})=\frac{1}{4\left(\mathbf{1}^T\tau\right)^{\frac{3}{2}}}\left[2\left(\mathbf{1}^T\tau\right)\mathbf{diag}\left(\tau\right)-\tau\tau^T\right], \tag{20}$$

where $\tau=[e^{\tilde{p}_{s,1}}, e^{\tilde{p}_{s,2}}, \cdots, e^{\tilde{p}_{s,N}}]^T$. To verify the convexity of f, we must show that for all $\mathbf{c}$, $\mathbf{c}^T\nabla^2 f(\tilde{p}_{s,n})\mathbf{c} \geq 0$, that is,

$$\begin{aligned}
&\mathbf{c}^T\nabla^2 f(\tilde{p}_{s,n})\mathbf{c}\\
&=\frac{\mathbf{c}^T}{4\left(\mathbf{1}^T\tau\right)^{\frac{3}{2}}}\left[\left(\mathbf{1}^T\tau\right)\mathbf{diag}\left(\tau\right)+\left(\mathbf{1}^T\tau\right)\mathbf{diag}\left(\tau\right)-\tau\tau^T\right]\mathbf{c}\\
&=\frac{1}{4\left(\mathbf{1}^T\tau\right)^{\frac{3}{2}}}\left[\left(\sum_{n=1}^{N}\tau_n\right)\left(\sum_{n=1}^{N}\mathbf{c}_n^2\tau_n\right)\right.\\
&\quad\left.+\left(\sum_{n=1}^{N}\tau_n\right)\left(\sum_{n=1}^{N}\mathbf{c}_n^2\tau_n\right)-\left(\sum_{n=1}^{N}\mathbf{c}_n\tau_n\right)^2\right]\\
&\geq 0,
\end{aligned} \tag{21}$$

where τ_n and $\mathbf{c}_n$ denote the nth element in vectors τ and $\mathbf{c}$, respectively. The inequality in (21) is obtained because 1) $\tau>0$, so $\left(\mathbf{1}^T\tau\right)^{\frac{3}{2}}>0$ and $\left(\sum_{n=1}^{N}\tau_n\right)\left(\sum_{n=1}^{N}\mathbf{c}_n^2\tau_n\right) \geq 0$, 2) $\left(\sum_{n=1}^{N}\tau_n\right)\left(\sum_{n=1}^{N}\mathbf{c}_n^2\tau_n\right)-\left(\sum_{n=1}^{N}\mathbf{c}_n\tau_n\right)^2 \geq 0$, which is resulted from the Cauchy-Schwarz inequality $\left(\mathbf{a}^T\mathbf{a}\right)\left(\mathbf{b}^T\mathbf{b}\right) \geq \left(\mathbf{a}^T\mathbf{b}\right)^2$ applied to the two vectors with component $\mathbf{a}_n=\sqrt{\tau_n}$ and $\mathbf{b}_n=\mathbf{c}_n\sqrt{\tau_n}$. □

Since e^{γ} is a convex function with variable γ, we get that the objective function $e^{\tilde{z}}$ of problem (16) is a convex function, and $e^{\tilde{u}}$, $e^{\tilde{p}_{r,n}}$, $e^{\tilde{\rho}}$, $e^{\tilde{t}_1}$, and $e^{\tilde{t}_2}$ in the constraints are convex functions. Since the composition rule with affine mapping does not change the function's convexity [29], we obtain that $e^{\tilde{z}-\tilde{t}_1}$, $e^{\tilde{z}-\tilde{t}_2}$, $e^{\tilde{u}-\tilde{t}_1}$, $e^{\tilde{v}+\tilde{t}_2}$, $e^{\tilde{t}_2+\tilde{\omega}}$, $e^{\rho+\tilde{t}_1+\tilde{p}_{s,n}}$, $e^{\tilde{t}_1+\tilde{p}_{s,n}}$, $e^{\tilde{\omega}-\tilde{t}_2}$, $e^{2\tilde{v}}$ in the constraints are convex functions as well. $\sum_{n=1}^{N} e^{\tilde{p}_{r,n}}$ is also a convex function since the sum of convex functions is also a convex function. $\log_2(1+e^{\gamma})$ is a convex function with respect to variable γ, which can be verified by showing that its second-order derivative is positive. By applying the composition rule with affine mapping, one can obtain that $\log_2\left(1+\frac{e^{\tilde{\rho}+\tilde{p}_{s,n}}\left|h_n\right|^2}{\frac{B}{N}N_b}\right)$

is a convex function. Since the nonnegative weighted sums of convex functions preserves convexity, we have $\sum_{n=1}^{N} \log_2\left(1+\frac{e^{\tilde{\rho}+\tilde{p}_{s,n}}|h_n|^2}{\frac{B}{N}N_b}\right)$, $\sum_{n=1}^{N} \log_2\left(1+\frac{e^{\tilde{p}_{r,n}}|g_n|^2}{\frac{B}{N}(N_a+N_b)}\right)$, and $\sum_{n=1}^{N} \log_2\left(1+\frac{e^{\tilde{\rho}+\tilde{p}_{s,n}}}{\frac{B}{N}N_b}\ |h_n|^2\right)$ are also convex functions. Hence, the objective function of problem (16) is a DC function, all the constraints are DC functions, thus problem (16) is a DC optimization problem.

Now the non-convex resource allocation problem for the asymmetric DF relay has been equivalently transformed to DC optimization problems. Before designing an efficient algorithm to solve the DC problems, we will first analyze the resource allocation problem for multi-carrier AF relay with PS scheme in the next section.

IV. AF RELAY

A. AF RELAY TRANSMISSION

In AF relay transmission, the total transmission time T from the source to the destination is equally divided into two parts, i.e., $\frac{T}{2}$ of time duration is used for the signal transmission from the source to the relay, and the other $\frac{T}{2}$ of time duration is adopted for the signal transmission from the relay to the destination. It should be noted that in the AF relay transmission, the relay amplifies the received information signals from the source and directly forwards them to the destination, thus the symmetric transmission (equal transmission time) during the two hops is necessary [10], [30]. The SNR from the source to the destination by ignoring the antenna noise at the relay on subcarrier n can be approximated by [21]

$$\gamma_n^{AF} = \frac{\rho p_{s,n}\frac{|h_n|^2}{\frac{B}{N}(\rho N_a+N_b)}p_{r,n}\frac{|g_n|^2}{\frac{B}{N}(N_a+N_b)}}{1+\rho p_{s,n}\frac{|h_n|^2}{\frac{B}{N}(\rho N_a+N_b)}+p_{r,n}\frac{|g_n|^2}{\frac{B}{N}(N_a+N_b)}} \approx \frac{\rho p_{s,n}\frac{|h_n|^2}{\frac{B}{N}N_b}p_{r,n}\frac{|g_n|^2}{\frac{B}{N}(N_a+N_b)}}{1+\rho p_{s,n}\frac{|h_n|^2}{\frac{B}{N}N_b}+p_{r,n}\frac{|g_n|^2}{\frac{B}{N}(N_a+N_b)}}. \tag{22}$$

Since at the high SNR, γ_n^{AF} can be further approximated by ignoring the term '1' in the denominator of (22), which makes (22) become

$$\gamma_n^{AF} \approx \frac{\rho p_{s,n}\frac{|h_n|^2}{\frac{B}{N}N_b}p_{r,n}\frac{|g_n|^2}{\frac{B}{N}(N_a+N_b)}}{\rho p_{s,n}\frac{|h_n|^2}{\frac{B}{N}N_b}+p_{r,n}\frac{|g_n|^2}{\frac{B}{N}(N_a+N_b)}} = \frac{\rho p_{s,n}|h_n|^2 p_{r,n}|g_n|^2}{\rho p_{s,n}|h_n|^2\frac{B}{N}(N_a+N_b)+p_{r,n}|g_n|^2\frac{B}{N}N_b}. \tag{23}$$

The transmission rate of the AF relay from the source to the destination can be expressed by [21]

$$R^{AF} = \frac{1}{2}\sum_{n=1}^{N}\log_2(1+\gamma_n^{AF}), \tag{24}$$

where $1/2$ is because the relay is half-duplex and the transmission time from the source to the relay and that from the relay to the destination are $T/2$, respectively.

B. PROBLEM FORMULATION

The joint optimization of power allocation at both the source and the relay, and the power splitting ratio to achieve transmission rate maximization can be formulated as

$$\max_{\{\rho,p_{s,n},p_{r,n}\}} \quad R^{AF}$$

$$\text{subject to: } \frac{\sqrt{P_{s,max}}}{\eta_{s,max}}\sqrt{\sum_{n=1}^{N}p_{s,n}}+P_{s,c}+\kappa R_{s,r}^{AF} \leqslant P_{s,tot}, \tag{25a}$$

$$\frac{\sqrt{P_{r,max}}}{\eta_{r,max}}\sqrt{\sum_{n=1}^{N}p_{r,n}}+P_{r,c}+\kappa R^{AF} \leqslant \sum_{n=1}^{N}\varsigma(1-\rho)p_{s,n}|h_n|^2+\frac{Q_{ini}}{\frac{T}{2}}, \tag{25b}$$

$$0<\rho<1, \tag{25c}$$

$$0\le p_{s,n},\ 0\le p_{r,n},\ \forall n\in\mathcal{N}, \tag{25d}$$

where $R_{s,r}^{AF}$ is the transmission rate from the source to the relay, and it is given by

$$R_{s,r}^{AF} = \frac{1}{2}\sum_{n=1}^{N}\log_2(1+\frac{\rho p_{s,n}|h_n|^2}{\frac{B}{N}N_b}). \tag{26}$$

Inequality (25a) is the source transmission power constraint. (25b) is the power constraint at the relay, which indicates that the consumed power cannot be higher than the maximum available power coming from energy harvesting and initial energy storage. (25c) describes the lower and upper bounds of the power splitting ratio, and (25d) shows the transmission power at both the source and the relay should be nonnegative.

The objective function of (25) is not a concave function with respect to the variables $\rho, p_{s,n}$, and $p_{r,n}$ since it contains the term $\rho p_{s,n}$. The left-hand sides of constraints (25a) and (25b) contain concave functions in the form of $\sqrt{\gamma}$ with $\gamma > 0$, and $R_{s,r}^{AF}$ and R^{AF} in constraints (25a) and (25b) are either concave or convex functions (the result can be verified by showing that their Hessian matrixes are indefinite), thus the constraint set is non-convex. Considering both the non-concave objective function and non-convex set, we get that problem (25) is a non-convex optimization problem. Generally, it is hard to find its optimal solution directly. Fortunately, by exploiting its special structure, we can transform it to DC problems, which can be efficiently solved. The proposed two methods that can transform it to DC optimization problems are described in the following.

C. Problem Transformation

1) METHOD 1 WITH LOGARITHMIC OPERATION ON CONSTRAINTS

Define $q_{s,n} = \rho p_{s,n}$, γ_n^{AF} can be rewritten as

$$\gamma_n^{AF} = \frac{q_{s,n}|h_n|^2 p_{r,n}|g_n|^2}{q_{s,n}|h_n|^2\frac{B}{N}(N_a+N_b)+p_{r,n}|g_n|^2\frac{B}{N}N_b}. \tag{27}$$

By introducing new variables s, y, l_n, problem (25) can be equivalently written as,

$$\max_{\{\rho,q_{s,n},p_{r,n},s,y,l_n\}} \frac{1}{2}\sum_{n=1}^{N}\log_2(1+l_n)$$

$$\text{subject to: } l_n \leqslant \gamma_n^{AF}, \forall n \in \mathcal{N}, \quad (28a)$$

$$y + P_{s,c} + \kappa R_{s,r}^{AF} \leqslant P_{s,tot}, \quad (28b)$$

$$\frac{\sqrt{P_{s,max}}}{\eta_{s,max}}\sqrt{\frac{\sum_{n=1}^{N} q_{s,n}}{\rho}} \leq y \quad (28c)$$

$$\frac{\sqrt{P_{r,max}}}{\eta_{r,max}}\sqrt{\sum\nolimits_{n=1}^{N} p_{r,n} + P_{r,c}} + \kappa\frac{1}{2}\sum_{n=1}^{N}\log_2(1+l_n) \leqslant s + \frac{2Q_{ini}}{T}, \quad (28d)$$

$$s \leqslant \frac{\sum_{n=1}^{N}\varsigma(1-\rho)q_{s,n}|h_n|^2}{\rho} \quad (28e)$$

$$0 < \rho < 1, \quad (28f)$$

$$0 \leq q_{s,n}, 0 \leq p_{r,n}, 0 \leq l_n, \forall n \in \mathcal{N}, \quad 0 \leq s. \quad (28g)$$

Constraint (28a) can be equivalently changed to $\log(l_n) + \log(A_n) \leqslant \log(B_n)$, where A_n and B_n are the denominator and numerator of γ_n^{AF} shown in (27), respectively. Constraint (28c) can be equivalently written as $\log(P_{s,max}\sum_{n=1}^{N} q_{s,n}) \leq 2\log(\eta_{s,max}y) + \log(\rho)$. And constraint (28e) is equivalent to $\log(\rho)+\log(s) \leqslant \log((1-\rho))+\log(\sum_{n=1}^{N}\varsigma q_{s,n}|h_n|^2)$. After the equivalent transformation, problem (28) becomes

$$\max_{\{\rho,q_{s,n},p_{r,n},s,y,l_n\}} \frac{1}{2}\sum_{n=1}^{N}\log_2(1+l_n)$$

$$\text{subject to: } \log(l_n) + \log(A_n) \leqslant \log(B_n), \forall n \in \mathcal{N}, \quad (29a)$$

$$y + P_{s,c} + \kappa R_{s,r}^{AF} \leqslant P_{s,tot}, \quad (29b)$$

$$\log\left(P_{s,max}\sum_{n=1}^{N} q_{s,n}\right) \leq 2\log\left(\eta_{s,max}y\right) + \log\left(\rho\right), \quad (29c)$$

$$\frac{\sqrt{P_{r,max}}}{\eta_{r,max}}\sqrt{\sum\nolimits_{n=1}^{N} p_{r,n} + P_{r,c}} + \kappa\frac{1}{2}\sum_{n=1}^{N}\log_2(1+l_n) \leqslant s + \frac{2Q_{ini}}{T}, \quad (29d)$$

$$\log\left(\rho\right) + \log\left(s\right) \leqslant \log\left(1-\rho\right) + \log\left(\sum\nolimits_{n=1}^{N}\varsigma q_{s,n}|h_n|^2\right), \quad (29e)$$

$$(28f) - (28g), \quad (29f)$$

Since $\log_2(1+\gamma)$ with variable $\gamma \geq 0$ is a concave function, and the sum of concave functions is also a concave function [29], hence we have that the objective function of problem (29) is concave. $R_{s,r}^{AF}$ in (29b) is also concave, which can be obtained by replacing $\rho p_{s,n}$ with $q_{s,n}$ and from the fact that the sum of concave functions is also concave. And $\sqrt{\sum_{n=1}^{N} p_{r,n}}$ with $0 \leq p_{r,n}$ in constraint (29d) is a concave function as well. In addition, $\log(\gamma)$ with variable $\gamma > 0$ is also a concave function, thus all the constraints of problem (29) can be written as either DC functions (29a)-(29e) or linear functions (29f) with the variables. Hence problem (29) becomes a DC optimization problem.

2) METHOD 2 WITH LOGARITHMIC CHANGE OF VARIABLES

To facilitate the following analysis, problem (25) can be equivalently rewritten as the following problem (30) by employing two kinds of new variables v and l_n, where γ_n^{AF} is with the expression shown in (23),

$$\max_{\{\rho,p_{s,n},p_{r,n},v,l_n\}} \frac{1}{2}\sum_{n=1}^{N}\log_2(1+l_n)$$

$$\text{subject to: } l_n \leqslant \gamma_n^{AF}, \forall n \in \mathcal{N}, \quad (30a)$$

$$\frac{\sqrt{P_{s,max}}}{\eta_{s,max}}\sqrt{\sum\nolimits_{n=1}^{N} p_{s,n} + P_{s,c} + \kappa R_{s,r}^{AF}} \leqslant P_{s,tot}, \quad (30b)$$

$$\frac{\sqrt{P_{r,max}}}{\eta_{r,max}}\sqrt{\sum\nolimits_{n=1}^{N} p_{r,n}} \leqslant v, \quad (30c)$$

$$v + P_{r,c} + \kappa R^{AF} \leqslant \sum\nolimits_{n=1}^{N}\varsigma(1-\rho)p_{s,n}|h_n|^2 + \frac{Q_{ini}}{\frac{T}{2}}, \quad (30d)$$

$$0 < \rho < 1, \quad (30e)$$

$$0 \leq p_{s,n}, 0 \leq p_{r,n}, 0 \leq l_n, \forall n \in \mathcal{N}, \quad (30f)$$

Define new variables as $\tilde{p}_{s,n} = \ln p_{s,n}, \tilde{p}_{r,n} = \ln p_{r,n}$, $\tilde{\rho} = \ln\rho$, $\tilde{v} = \ln v$ and $\tilde{l}_n = \ln l_n$. Then problem (30) can be expressed by

$$\max_{\left\{\tilde{\rho},\tilde{p}_{s,n},\tilde{p}_{r,n},\tilde{v},\tilde{l}_n\right\}} \frac{1}{2}\sum_{n=1}^{N}\log_2(1+e^{\tilde{l}_n})$$

$$\text{subject to: } e^{\tilde{l}_n}\tilde{A}_n \leq \tilde{B}_n, \quad (31a)$$

$$\frac{\sqrt{P_{s,max}}}{\eta_{s,max}}\sqrt{\sum\nolimits_{n=1}^{N} e^{\tilde{p}_{s,n}} + P_{s,c}} + \frac{\kappa}{2}\sum_{n=1}^{N}\log_2\left(1 + \frac{e^{\rho+p_{s,n}}|h_n|^2}{\frac{B}{N}N_b}\right) \leqslant P_{s,tot}, \quad (31b)$$

$$\sum\nolimits_{n=1}^{N} e^{\tilde{p}_{r,n}} - \frac{1}{P_{r,max}}(e^{\tilde{v}}\eta_{r,max})^2 \leq 0, \quad (31c)$$

$$e^{\tilde{v}} + P_{r,c} - \frac{2Q_{ini}}{T} + \frac{\kappa}{2}\sum_{n=1}^{N}\log_2\left(1+e^{\tilde{l}_n}\right) + \varsigma\sum\nolimits_{n=1}^{N} e^{\tilde{\rho}+\tilde{p}_{s,n}}|h_n|^2 - \varsigma\sum\nolimits_{n=1}^{N} e^{\tilde{p}_{s,n}}|h_n|^2 \leqslant 0, \quad (31d)$$

$$e^{\tilde{\rho}} < 1, \quad (31e)$$

where

$$\tilde{B}_n = e^{\tilde{p}_{s,n}+\tilde{p}_{r,n}+\tilde{\rho}} |h_n|^2 |g_n|^2,$$
$$\tilde{A}_n = e^{\tilde{\rho}+\tilde{p}_{s,n}} |h_n|^2 \frac{B}{N}(N_a+N_b) + e^{\tilde{p}_{r,n}} |g_n|^2 \frac{B}{N}N_b.$$

By doing similar analysis as that for problem (16) in Section III.B, we get that problem (31) is a DC optimization problem.

Now all the problems are transformed to DC optimization problems. In the following, we will propose an efficient algorithm to solve this kind of problems.

V. ALGORITHM DESIGN FOR THE DC OPTIMIZATION PROBLEM

It is observed from the formulation of the DC optimization problem given in Appendix that it becomes a convex problem unless $g_i, i = 0, \cdots, m$ is affine. Since it is generally difficult to solve DC optimization problems, to make the problems solvable, g_i will be approximated by an affine function. By using the first-order Taylor series expansion, we get $g_i(\mathbf{x})$ can be approximated at any feasible point $\mathbf{x}(k)$ by

$$g_i(\mathbf{x}) \approx g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T (\mathbf{x}-\mathbf{x}(k)), \tag{32}$$

where $\nabla g_i(\mathbf{x}(k))^T$ is the gradient of g_i at point $\mathbf{x}(k)$.

In the following, we will present the approximation problems of the considered DC optimizations for both asymmetric DF and AF relay transmission, respectively.

A. APPROXIMATION FOR ASYMMETRIC DF RELAY TRANSMISSION

1) METHOD 1

For the inequalities (14a) and (14b), $\log(z)$ can be approximated by

$$\log(z) \approx \log(z(k)) + \frac{1}{z(k)}(z - z(k)). \tag{33}$$

For inequality (14c), the left-hand side can be approximated by $\log(t_1) \approx \log(t_1(k)) + \frac{1}{t_1(k)}(t_1 - t_1(k))$ and

$$\begin{aligned}
&\log\left(\sum_{n=1}^N \log_2\left(1+\frac{q_{s,n}|h_n|^2}{\frac{B}{N}N_b}\right)\right) \\
&\approx \log\left(\sum_{n=1}^N \log_2\left(1+\frac{q_{s,n}(k)|h_n|^2}{\frac{B}{N}N_b}\right)\right) \\
&\quad + \frac{1}{\sum_{n=1}^N \log\left(1+\frac{q_{s,n}(k)|h_n|^2}{\frac{B}{N}N_b}\right)} \\
&\quad \times \sum_{n=1}^N \frac{1}{1+\frac{q_{s,n}(k)|h_n|^2}{\frac{B}{N}N_b}} \frac{|h_n|^2}{\frac{B}{N}N_b}\left(q_{s,n}-q_{s,n}(k)\right).
\end{aligned} \tag{34}$$

For inequality (14d), the concave function on the left-hand side can be approximated by

$$\begin{aligned}
&\log\left(P_{s,max}\sum_{n=1}^N q_{s,n}\right) \\
&\approx \log\left(P_{s,max}\sum_{n=1}^N q_{s,n}(k)\right) + \frac{\sum_{n=1}^N \left(q_{s,n}-q_{s,n}(k)\right)}{\sum_{n=1}^N q_{s,n}(k)}.
\end{aligned} \tag{35}$$

For inequality (14e), the concave function on the left-hand side of the inequality is approximated by

$$\sqrt{\sum_{n=1}^N p_{r,n}} \approx \sqrt{\sum_{n=1}^N p_{r,n}(k)} + \frac{\sum_{n=1}^N \left(p_{r,n}-p_{r,n}(k)\right)}{2\sqrt{\sum_{n=1}^N p_{r,n}(k)}}. \tag{36}$$

The convex function on the right hand side is approximated by

$$\frac{1}{t_1} \approx \frac{1}{t_1(k)} - \frac{1}{t_1^2(k)}(t_1 - t_1(k)). \tag{37}$$

For inequality (14f), the two terms in the left-hand side of inequality (14f) have the same structure as those of (14c), and thus they can be approximated by using the same method.

For inequality (14g), the concave functions $\log(t_2)$, $\log(\rho)$, and $\log(x)$ can be linearly approximated by the same method as (33).

2) METHOD 2

For problem (16), the approximations are presented in the following.

In constraint (16a), the approximation is given by

$$\begin{aligned}
&\sum_{n=1}^N \log_2\left(1+\frac{e^{\tilde{\rho}+\tilde{p}_{s,n}}|h_n|^2}{\frac{B}{N}N_b}\right) \\
&\approx \sum_{n=1}^N \log_2\left(1+\frac{e^{\tilde{\rho}(k)+\tilde{p}_{s,n}(k)}|h_n|^2}{\frac{B}{N}N_b}\right) \\
&\quad + \frac{1}{\log(2)}\left(\sum_{n=1}^N \frac{\frac{e^{\tilde{\rho}(k)+\tilde{p}_{s,n}(k)}|h_n|^2}{\frac{B}{N}N_b}}{1+\frac{e^{\tilde{\rho}(k)+\tilde{p}_{s,n}(k)}|h_n|^2}{\frac{B}{N}N_b}}\right)(\tilde{\rho}-\tilde{\rho}(k)) \\
&\quad + \frac{1}{\log(2)}\left(\sum_{n=1}^N \frac{\frac{e^{\tilde{\rho}(k)+\tilde{p}_{s,n}(k)}|h_n|^2}{\frac{B}{N}N_b}}{1+\frac{e^{\tilde{\rho}(k)+\tilde{p}_{s,n}(k)}|h_n|^2}{\frac{B}{N}N_b}}\left(\tilde{p}_{s,n}-\tilde{p}_{s,n}(k)\right)\right).
\end{aligned} \tag{38}$$

In constraint (16b), the approximation is presented by

$$\begin{aligned}
&\sum_{n=1}^N \log_2\left(1+\frac{e^{\tilde{p}_{r,n}}|g_n|^2}{\frac{B}{N}(N_a+N_b)}\right) \\
&\approx \sum_{n=1}^N \log_2\left(1+\frac{e^{\tilde{p}_{r,n}(k)}|g_n|^2}{\frac{B}{N}(N_a+N_b)}\right) \\
&\quad + \frac{1}{\log(2)}\sum_{n=1}^N\left(\frac{\frac{e^{\tilde{p}_{r,n}(k)}|g_n|^2}{\frac{B}{N}(N_a+N_b)}}{1+\frac{e^{\tilde{p}_{r,n}(k)}|g_n|^2}{\frac{B}{N}(N_a+N_b)}}\left(\tilde{p}_{r,n}-\tilde{p}_{r,n}(k)\right)\right).
\end{aligned} \tag{39}$$

In the objective function and constraints (16d), (16e), (16f), (16g), (16j), the terms with the form of e^{γ} are approximated by

$$e^{\gamma} \approx e^{\gamma(k)} + e^{\gamma(k)}(\gamma - \gamma(k)). \tag{40}$$

B. APPROXIMATION FOR AF RELAY TRANSMISSION

1) METHOD 1

In the constraint (29b), $R_{s,r}^{AF}$ can be approximated by

$$R_{s,r}^{AF} \approx \frac{1}{2}\sum_{n=1}^{N}\log_2(1+\frac{q_{s,n}(k)\,|h_n|^2}{\frac{B}{N}N_b}) + \frac{1}{2\log(2)}\sum_{n=1}^{N}\frac{\frac{|h_n|^2}{\frac{B}{N}N_b}\left(q_{s,n}-q_{s,n}(k)\right)}{1+\frac{q_{s,n}(k)|h_n|^2}{\frac{B}{N}N_b}}. \tag{41}$$

In the constraint (29c), $\log\left(P_{s,max}\sum_{n=1}^{N}q_{s,n}\right)$ can be approximated as (35). $\sqrt{\sum_{n=1}^{N}p_{r,n}}$ in constraint (29d) is approximated as (36), and in constraint (29d)

$$\frac{\kappa}{2}\sum_{n=1}^{N}\log_2(1+l_n) \approx \frac{\kappa}{2}\sum_{n=1}^{N}\log_2(1+l_n(k)) + \frac{\kappa}{2\log(2)}\sum_{n=1}^{N}\frac{l_n-l_n(k)}{1+l_n(k)} \tag{42}$$

For inequality (29e), the concave function $\log(\rho)$ and $\log(s)$ can be linearly approximated by the same method as (33).

2) METHOD 2

The objective function of problem (31) is a convex function, which can be approximated by

$$\frac{1}{2}\sum_{n=1}^{N}\log_2(1+e^{\tilde{l}_n}) \approx \frac{1}{2}\sum_{n=1}^{N}\log_2(1+e^{\tilde{l}_n(k)}) + \sum_{n=1}^{N}\frac{e^{\tilde{l}_n(k)}\left(\tilde{l}_n-\tilde{l}_n(k)\right)}{2\log(2)\left(1+e^{\tilde{l}_n(k)}\right)}. \tag{43}$$

$\tilde{B}_n$ in (31a) is approximated by

$$\tilde{B}_n \approx e^{\tilde{p}_{s,n}(k)+\tilde{p}_{r,n}(k)+\tilde{\rho}(k)}\,|h_n|^2\,|g_n|^2 + e^{\tilde{p}_{s,n}(k)+\tilde{p}_{r,n}(k)+\tilde{\rho}(k)}\,|h_n|^2\,|g_n|^2\left(\left(\tilde{p}_{s,n}-\tilde{p}_{s,n}(k)\right) + \left(\tilde{p}_{r,n}-\tilde{p}_{r,n}(k)\right) + \left(\tilde{\rho}-\tilde{\rho}(k)\right)\right). \tag{44}$$

And $\left(e^{\tilde{v}}\eta_{r,max}\right)^2/P_{r,max}$ in constraint (31c) and $\varsigma\sum_{n=1}^{N}e^{\tilde{p}_{s,n}}\,|h_n|^2$ in (31d) can be approximated similarly as that shown in (40).

C. ALGORITHM FOR DC OPTIMIZATION PROBLEM

Based on the approximation for all g_i, the DC problem becomes a convex problem at a given point $\mathbf{x}(k)$. If a feasible initial point is given, then at each iteration, a new solution will be obtained. The process continues until a stop criterion is satisfied. In each step of iterations, a convex problem needs to be solved, whose optimal solution can be efficiently obtained by standard methods, such as the interior point method. The whole algorithm for solving the DC optimization problem is described in Algorithm 1.

Algorithm 1 Algorithm for Solving the DC Optimization Problem

Give an initial feasible point $\mathbf{x}(0)$ for the DC problem, set the precision parameter ε to be a very small value, and let the iteration number $k = 0$.
repeat
  substitute $\mathbf{x}(k)$ to the approximately convex optimization problem,
  solve the approximately convex optimization problem and get $\mathbf{x}$,
  set $\mathbf{x}(k+1) = \mathbf{x}$,
  set $k = k+1$,
  compute the objective functions $J(\mathbf{x}(k))$ and $J(\mathbf{x}(k-1))$ of the approximated convex problem,
until $|J(\mathbf{x}(k)) - J(\mathbf{x}(k-1))| < \varepsilon$
$\mathbf{x}$ is the final solution.

D. CONVERGENCE ANALYSIS OF THE PROPOSED ALGORITHM

The convergence property of the proposed algorithm is analyzed in the following proposition, which indicates that the algorithm will finally get at least a stationary point when it is convergent.

Proposition 2: The proposed algorithm generates an improved sequence of feasible solutions and it will finally converge to a stationary point.

Proof: The DC constraint is in the form of $f_i(\mathbf{x}) - g_i(\mathbf{x}) \leq 0$, where $f_i(\mathbf{x})$ and $g_i(\mathbf{x})$ are convex functions, as shown in Appendix. Due to the convexity property, $g_i(\mathbf{x})$ satisfies that [29]

$$g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}-\mathbf{x}(k)) \leq g_i(\mathbf{x}), \quad \mathbf{x}, \mathbf{x}(k) \in \mathbf{dom} g_i, \tag{45}$$

Then we get that

$$f_i(\mathbf{x}) - g_i(\mathbf{x}) \leq f_i(\mathbf{x}) - \left[g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}-\mathbf{x}(k))\right], \quad \mathbf{x} \in \mathbf{dom} g_i \cap \mathbf{dom} f_i, \mathbf{x}(k) \in \mathbf{dom} g_i. \tag{46}$$

In the proposed algorithm, $f_i(\mathbf{x}) - g_i(\mathbf{x}) \leq 0$ is approximated by $f_i(\mathbf{x}) - \left[g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}-\mathbf{x}(k))\right] \leq 0$.

From (46) we know that the obtained solution $\mathbf{x}(k+1) = \mathbf{x}$ to the approximately convex optimization problem at each step k also makes $f_i(\mathbf{x}) - g_i(\mathbf{x}) \leq 0$ hold. Therefore, the obtained solution $\mathbf{x}(k+1) = \mathbf{x}$ at each iteration k is feasible for the DC problems (14), (16), (29), and (31), respectively.

From the above analysis, it is known that $f_i(\mathbf{x}(k)) - g_i(\mathbf{x}(k)) \leq 0$ is always satisfied. This indicates that

$\mathbf{x} = \mathbf{x}(k)$ satisfies all the constraints of the approximately convex problem at iteration k, i.e., $f_i(\mathbf{x}(k)) - \left[g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}(k) - \mathbf{x}(k))\right] = f_i(\mathbf{x}(k)) - g_i(\mathbf{x}(k)) \leq 0$, thus it is a feasible solution to this problem at iteration k. On the other hand, the obtained optimal solution to the approximately convex problem at iteration k is $\mathbf{x}(k+1)$. Therefore, we obtain that $J(\mathbf{x}(k+1)) \geq J(\mathbf{x}(k))$, that is because the objective function at the optimal solution is always not worse than that at the feasible solution. $J(\mathbf{x}(k+1)) \geq J(\mathbf{x}(k))$ reflects that the objective function keeps non-decreasing as the iteration number k increases. In addition, the constraint is non-empty and bounded by the total power constraint and $0 < \rho < 1$. Therefore, the objective function has a limit point, which is the convergence point (Corollary 3.2 in [31]).

Then we will analyze that the convergence point is a stationary point by adopting the techniques in [31]. The DC problem can be written in the following form,

$$\begin{aligned} &\min f_0(\mathbf{x}) - g_0(\mathbf{x}) \\ &\text{s.t. } f_i(\mathbf{x}) - g_i(\mathbf{x}) \leq 0, \quad i = 1, \cdots, m, \end{aligned} \tag{47}$$

where $f_i(\mathbf{x})$, $g_i(\mathbf{x})$, $\forall i \in 0, 1, \cdots, m$, are convex functions. From the considered DC problems (14), (16), (29), and (31), it is easy to get that $f_i(\mathbf{x})$, $g_i(\mathbf{x})$, $\forall i \in 0, 1, \cdots, m$ are also differentiable. By using the first-order Taylor series expansion, problem (47)'s approximated convex optimization problem at a given point $\mathbf{x}(k)$ can be written as

$$\begin{aligned} &\min f_0(\mathbf{x}) - \left(g_0(\mathbf{x}(k)) + \nabla g_0(\mathbf{x}(k))^T(\mathbf{x} - \mathbf{x}(k))\right) \\ &\text{s.t. } f_i(\mathbf{x}) - \left(g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x} - \mathbf{x}(k))\right) \\ &\quad \leq 0, i = 1, \cdots, m, \end{aligned} \tag{48}$$

We can easily to check that the convex optimization problem (48) satisfies the Slater's condition and its objective function and constraints are differential functions, hence the Karush-Kuhn-Tucker (KKT) condition is the sufficient and necessary condition for optimality [29]. Define λ_i, $i = 1, \cdots, m$ as the nonnegative Lagrange multiplier associated to the ith constraint in problem (48). Let $(\mathbf{x}^*, \lambda_i^*)$ be the optimal point, then $\forall i = 1, \cdots, m$, the optimal point satisfies the following KKT condition,

$$f_i(\mathbf{x}^*) - \left\{g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}^* - \mathbf{x}(k))\right\} \leq 0, \tag{49}$$

$$\lambda_i^* \geq 0, \tag{50}$$

$$\lambda_i^* \left\{f_i(\mathbf{x}^*) - \left(g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}^* - \mathbf{x}(k))\right)\right\} = 0, \tag{51}$$

$$\begin{aligned} &\nabla \left\{f_0(\mathbf{x}^*) - \left(g_0(\mathbf{x}(k)) + \nabla g_0(\mathbf{x}(k))^T(\mathbf{x}^* - \mathbf{x}(k))\right)\right\} \\ &\quad + \lambda_i^* \nabla \left\{f_i(\mathbf{x}^*) - \left(g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}^* - \mathbf{x}(k))\right)\right\} \\ &= 0. \end{aligned} \tag{52}$$

On the convergence of the algorithm ($k \to \infty$), $\mathbf{x}(k) \to \mathbf{x}^*$ holds, hence

$$\begin{aligned} &\forall i = 0, 1, \cdots, M, \\ &g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}^* - \mathbf{x}(k)) \to g_i(\mathbf{x}^*) \end{aligned} \tag{53}$$

$$\begin{aligned} &\nabla \left(f_i(\mathbf{x}^*) - \left[g_i(\mathbf{x}(k)) + \nabla g_i(\mathbf{x}(k))^T(\mathbf{x}^* - \mathbf{x}(k))\right]\right) \\ &\quad \to \nabla \left(f_i(\mathbf{x}^*) - g_i(\mathbf{x}^*)\right). \end{aligned} \tag{54}$$

Based on (53), the conditions (49) and (51) on the convergence can be written as

$$f_i(\mathbf{x}^*) - g_i(\mathbf{x}^*) \leq 0, \tag{55}$$

$$\lambda_i^* \left\{f_i(\mathbf{x}^*) - g_i(\mathbf{x}^*)\right\} = 0, \tag{56}$$

and on the basis of (54), the condition (52) at the convergence point can be replaced by

$$\nabla \left\{f_0(\mathbf{x}^*) - g_0(\mathbf{x}^*)\right\} + \lambda_i^* \nabla \left\{f_i(\mathbf{x}^*) - g_i(\mathbf{x}^*)\right\} = 0. \tag{57}$$

It is readily to get that (50), (55)-(57) are the KKT condition of the original DC problem (47). Hence, the convergence point satisfies the KKT condition of the original DC problem, i.e., the convergence point is a stationary point of the original DC problem. □

It should be noted that, there are no theoretical results demonstrating that the DC algorithm can finally converge to the global optimal solution. However, the DC algorithm has been widely verified to often achieve the global optimal solution (in the cases the global optimal solution can be obtained) in different practical applications, such as in wireless relay networks [32], [33], femtocell networks [34], and energy harvesting networks [7].

VI. NUMERICAL SIMULATION

Consider a three-node relay network, where the source, the relay, and the destination are located on the same line. The wireless channel experiences large scale fading with path loss factor 3 and small scale fading, which is modeled as a frequency selective channel consisting of six independent Rayleigh multipaths. The power delay profile is exponentially decaying with e^{-2l}, where l is the multipath index [35]. The parameters are set as $N_a = N_b = 10^{-12}$W/Hz, $T = 10$ms, $P_{s,tot} = 5$W, $P_{s,c} = 0.1$W, and $P_{r,c} = 0.05$W. The distance from the source to the relay and that from the relay to the destination are 0.6m and 2.4m, respectively. The total bandwidth is $B = 10MHz$, and the number of subcarriers is $N = 64$. The other parameters are $P_{s,max} = P_{r,max} = 2$W, and $\eta_{s,max} = \eta_{r,max} = 0.38$, which is within the reasonable range of maximum PA efficiency [36], [37]. The results of interests are obtained by averaging 200 independent channel realizations. CVX toolbox is adopted for solving the convex problems in all the algorithms [38].

For performance comparison, both the symmetric DF relay transmission with $t_1 = t_2 = 0.5$ (denoted as DF in all the figures), and the baseline algorithm with fixed value of ρ for both DF and AF relay transmissions (denoted as baseline in all the figures) are simulated. For the symmetric DF relay transmission, since it is a simplified version of the asymmetric DF relay transmission studied in the paper, thus the proposed methods and algorithm can efficiently solve this problem by some modifications. For the baseline algorithms with fixed value of ρ, their solutions can be easily obtained by modifying the proposed algorithm. In all the simulations, the value of ρ in the baseline algorithms is set as $\rho = 0.15$ and $\rho = 0.25$. That is because the value of ρ has a direct effect on the amount of harvested energy at the relay, which is given in equation (6). If the value of ρ is very large, the harvested energy at the relay may become too small, which will result in the failure of the relay transmission. Hence, $\rho = 0.15$ and $\rho = 0.25$ are adopted. For clarity, the AF and DF algorithms based on the proposed method 1 and method 2 are denoted by 'log' and 'exp' respectively in all the figures.

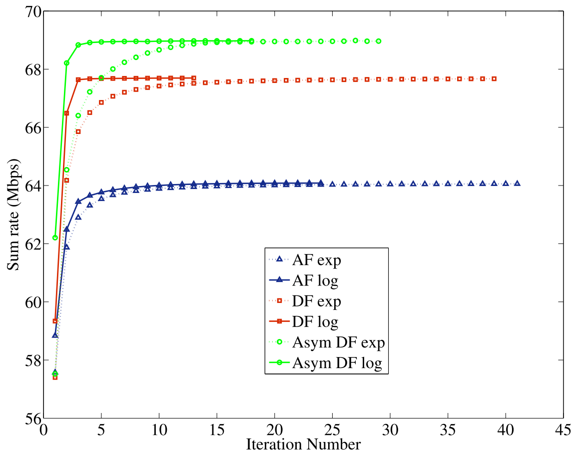

FIGURE 2. Convergence of the proposed algorithms for both DF and AF relay transmissions ($Q_{ini} = 0$, $\kappa = 0.5$mW/Gbps).

Firstly, the convergence of the algorithms for DF and AF relay transmissions is plotted in Fig. 2. It is clear to see from Fig. 2 that all the algorithms converge in several steps of iterations. For either DF or AF relay transmission, algorithms with both method 1 and method 2 can converge to almost the same objective function. DF relay transmission achieves much higher sum rate than AF relay transmission, and the asymmetric DF relay transmission achieves much better sum rate than the symmetric DF relay transmission. That is because the asymmetric DF relay transmission has more flexible time allocation than the symmetric DF relay transmission, thus much more sum rate can be achieved. From the convergence of Fig. 2, we can see that to achieve the same objective function, method 1 with logarithmic operation on constraints is always much faster than method 2 with logarithmic change of variables for all the three kinds of different transmissions. By doing simulations based on a large number of channel realizations, it verifies that this result is very common, and the algorithm with logarithmic operation on constraints is often much faster than that with logarithmic change of variables for most of the realizations.

Secondly, the effect of κ on the sum rate is described in Fig. 3. It is shown from Fig. 3 that as κ increases the achieved sum rates of all the algorithms except the baseline algorithms gradually decrease. That is because the increase of κ indicates the increase of the circuit power consumption, then the remaining energy used for data transmission decreases, thus the sum rate decreases. Among the three algorithms, the asymmetric DF relay transmission always admits the highest sum rate under different κ, the AF relay transmission achieves the lowest sum rate, and the symmetric DF relay transmission is in between of the other two transmissions. This reveals that the DF relay transmission has a much higher sum rate performance than the AF relay transmission under the same network scenario. Generally, the baseline algorithms have a much lower sum rate performance than the corresponding proposed algorithms, as Fig. 3 shows. That is because the baseline algorithms do not consider the effect of ρ on the sum rate performance, but the proposed algorithms obtain the sum rate with the optimal ρ. The sum rate performance gap between the proposed algorithm and the corresponding baseline algorithm varies as ρ changes. If ρ is appropriately chosen for the baseline algorithm, the performance gap can be very small, e.g, the asymmetric DF baseline algorithm with $\rho = 0.15$ achieves sum rate that is approaching to that of the proposed asymmetric DF algorithms when $\kappa \geq 3$. Fig. 3 also shows that the proposed algorithms with method 1 and method 2 achieve nearly the same sum rate.

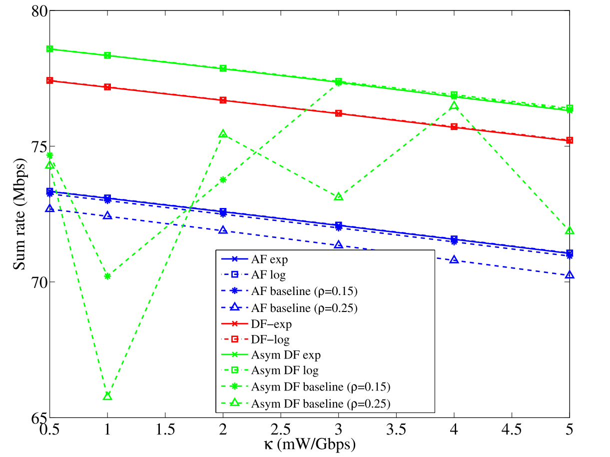

FIGURE 3. Sum rate versus κ for DF and AF relay algorithms ($Q_{ini} = 0$).

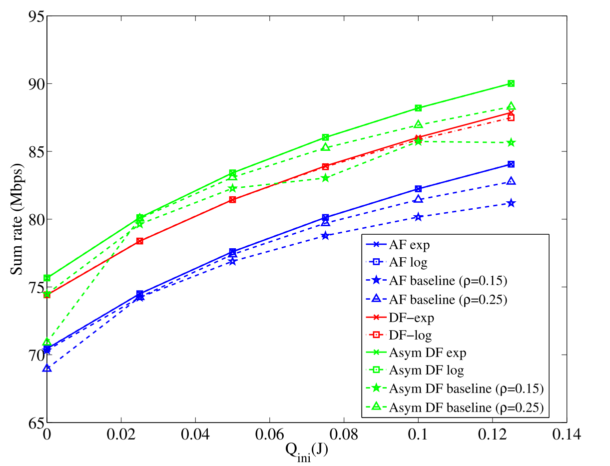

FIGURE 4. Sum rate versus Q_{ini} for DF and AF relay algorithms ($\kappa = 0.5$mW/Gbps).

Finally, the effect of the initial energy Q_{ini} at the relay on the sum rate is shown in Fig. 4. It is revealed from

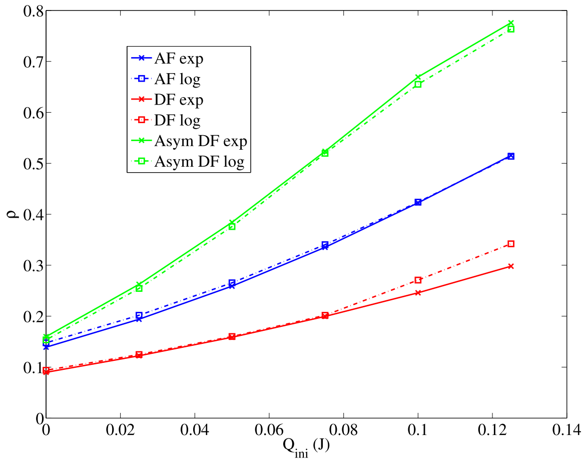

FIGURE 5. ρ **versus** Q_{ini} **for DF and AF relay algorithms (**$\kappa = 0.5$**mW/Gbps).**

Fig. 4 that Q_{ini} has a large effect on the sum rate. The larger Q_{ini} is, the higher the obtained sum rates of the proposed algorithms are. Under different values of Q_{ini}, the proposed algorithms that achieve sum rate from the highest to the lowest are asymmetric DF relay, symmetric DF relay, and AF relay, respectively. For the baseline algorithms, different values of ρ lead to different values of sum rate when all the other parameters are the same, but baseline algorithms achieve much or a little lower sum rate than the corresponding proposed algorithms given different values of Q_{ini}. To explore the property of the algorithms, the effect of Q_{ini} on ρ is plotted in Fig. 5. As it is shown in Fig. 5, the value of ρ increases as Q_{ini} increases. That is because the larger Q_{ini}, the more available energy at the relay, thus less harvested energy is needed, which makes the power splitting ratio ρ increase. Comparing all these algorithms, under different values of Q_{ini}, asymmetric DF relay has the highest ρ, symmetric DF relay has the lowest ρ, and AF relay transmission has the value of ρ in between. Combining Fig. 4 and Fig. 5, we get that although AF relay transmission and the symmetric DF relay transmission use the equal transmission time, but the symmetric DF relay transmission harvests more energy (a much less power splitting ratio ρ) to achieve a much higher sum rate than the AF relay transmission.

VII. CONCLUSION

In this paper, the transmission rate maximization problem in multi-carrier relay networks with PS scheme has been studied, with the consideration of the non-ideal power consumption. The investigated problem is formulated as a non-convex optimization problem for both the asymmetric DF relay and AF relay transmission, respectively. Since this kind of problem is generally difficult to solve, we propose two methods to transform these problems into DC optimization problems by exploiting their special structures. To solve the DC problems, we propose an efficient algorithm and prove the convergence of the algorithm as well. Simulations verify that the asymmetric DF relay transmission always achieves the highest sum rate comparing with the AF relay and symmetric DF relay transmissions, and the AF relay transmission has the least sum rate. The proposed algorithms for the asymmetric DF and AF relay transmissions obtain a much better sum rate than the corresponding baseline algorithms. Overall, the proposed algorithm in this work provides a guideline for transmission parameter setting to realize sum rate maximization in multi-carrier energy harvesting relay networks under the consideration of more realistic power consumption. Particularly, our work performs comprehensive study for both the asymmetric DF relay and AF relay transmission and demonstrate their performance comparisons by numerical simulation under different conditions.

APPENDIX

DC OPTIMIZATION PROBLEM

DC optimization problem is with the following form [39],

$$\begin{aligned}&\text{minimize } f_0(\mathbf{x}) - g_0(\mathbf{x})\\&\text{subject to } f_i(\mathbf{x}) - g_i(\mathbf{x}) \leq 0,\ i = 1, \cdots, m,\end{aligned}$$

where $\mathbf{x} \in \Re^n$ is the optimization variable and $f_i : \Re^n \to \Re$ and $g_i : \Re^n \to \Re$ for $i = 0, \cdots, m$, are convex. And the function $f_i - g_i$ is called the DC function.

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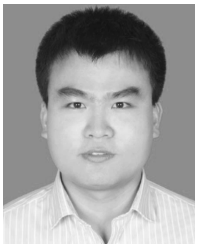

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