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Fault-Tolerant Containment Control for Linear Multi-Agent Systems: An Adaptive Output Regulation Approach

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ABSTRACT This paper addresses the problem of fault-tolerant containment control (FTCC) for linear multi-agent systems (MASs) with process faults. First, distributed observers for followers are designed by utilizing their relative output estimation errors. Then, a new distributed adaptive FTCC law based upon the output regulation theory is proposed, where a distributed adaptive observer is implemented to estimate the synthesized information for leaders. It is shown that no matter whether there exist faults or not, all the followers can asymptotically move into a convex hull formed by the leaders. Finally, an example is given to illustrate the effectiveness of the designed distributed control law.

INDEX TERMS Fault-tolerant containment control, multi-agent systems, process faults, output regulation.

I. INTRODUCTION

Containment control can be seen as a consensus problem in MASs with multiple leaders, aiming at driving all followers to a smallest convex region shaped by multiple leaders, e.g., a safe zone for working followers. In the past several years, the issue of containment control of MASs has been widely studied, e.g., [1]–[3]. Some typical consensus protocols, e.g., [4]–[6], have been extended to solve the containment control problems. Heterogeneity [7], time delay [8] and packet dropouts [9] are also considered in the containment control problems.

With the urgent demand of safety and reliability in MASs, fault-tolerant control (FTC) problems of MASs have received much attention recently. In general, faults in a dynamic system can be classified into three types: process faults, actuator faults, and sensor faults [10]. Process faults [11]–[13] represent gross parameter changes in a model, component faults or even a special kind of additive actuator faults. When a process fault occurs in MASs, some model parameters and state variables of subsystems may change abruptly, which will further affect the cooperative behavior of MASs. Classical fault diagnosis (FD) methods, especially observer-based approaches [14]–[17], provide a feasible way to solve the FTC problems of MASs. For example, Menon and Edwards [15] presented a fault estimation observer via the sliding mode technique. Zhang *et al.* [16], [17] proposed distributed fault estimation observers for MASs under directed topology. It should be noted that these fault estimation observers are only for MASs with no leader or single leader, which are not applicable to MASs with multiple leaders. In particular, Ye e*t al.* [18] solved the containment control problem of MASs with multiple leaders and actuator faults by designing a fault estimation observer.

It should be noticed that the output regulation framework [19] provides a new method for solving the consensus problem, which is also known as the cooperative output regulation problem (CORP). Different from classical FTC approaches [20]–[22], some FTC approaches based on the output regulation theory have been proposed for MASs, e.g., [23], [24]. Qin e*t al.* [23] constructed a unified framework with FT and FTC for linear MASs with sensor faults. Deng and Yang [24] solved a CORP of linear MASs with actuator faults via a distributed finite-time observer. Note that there exists only single leader in the MASs in [23], [24]. As far as we know, few studies aim at solving the FTCC problem

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of MASs with multiple leaders and process faults based upon the output regulation theory. This motivates the present work.

Different from the existing results [18], [23], [24], an adaptive output regulation approach is proposed to solve the FTCC problem of MASs with multiple leaders and process faults. The salient features of this paper are as follows:

(1) To compensate for process faults existing in the followers, state observers and fault estimation observers are designed simultaneously by embedding a new measurement error, which is based on the interaction gains among leaders and followers. The FT process is not needed due to the fact that the FTCC scheme depends upon real-time fault estimation.
(2) To overcome a global condition, a new adaptive observer is designed such that the synthesized information of leaders can be estimated. By introducing adaptive feedback gains to replace fixed gains subject to the eigenvalues of Laplacian matrices, this global information of underlying topology is no longer needed.
(3) For the MASs with multiple leaders, we transform the FTCC problem into the CORP in the presence of process faults. The proposed FTCC strategy can be simply extended to solve some general containment control/trajectory tracking problems.

The reminder of this paper can be summarized as: Section II gives some preliminaries and the FTCC problem formulation for linear MASs with faults. In Section III, a distributed observer design method is first proposed. Then a distributed adaptive FTCC law is developed to achieve the output containment control. Section IV presents simulation results to illustrate the validity of analytical results. Finally, we draw a conclusion for this paper in Section V.

Notations: Through this paper, $\mathcal{R}^p$ and $\mathcal{R}^{n\times m}$ represent the real sets of p-vectors and $n \times m$ matrices, respectively. The n-vectors $\mathbf{1}_n$ and $\mathbf{0}_n$ have the same elements being 1 and 0, respectively. I_n is the real identity matrix with n-dimensional. $diag(a_1, \cdots, a_n)$ is the diagonal matrix composed by the entries $a_i, i = 1, \cdots, n$. The convex hull is represented by $Co(X)$, which is the smallest convex polygon formed by a finite point sets X. The term $||\cdot||$ represents the Euclidean 2-norm. And $dist(x, \mathcal{C}) = \inf_{y\in\mathcal{C}} ||x - y||$ denotes the minimal distance from $x \in \mathcal{R}^n$ to the set $\mathcal{C}$ with respect to Euclidean norm. The symbol $\otimes$ denotes the Kronecker product. The term $\lambda(A)$ denotes the spectrum of a square matrix A. In particular, $\lambda_{\min}(A)$ is the minimum eigenvalue of A.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. PRELIMINARIES

To analyze interactions between agents in MASs, we introduce a communication graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ to model the network topology. The node set $\mathcal{V} = \{1, \cdots, N\}$ contains all the agents, and the corresponding edge set $\mathcal{E} \subseteq \{(i,j)|i,j \in \mathcal{V}, i \neq j\}$ describes the communication links among them. An edge $(i,j) \in \mathcal{E}$ represents that agent i is able to obtain the information of agent j. If the agent j can also receive the information of agent i, then $\mathcal{G}$ is called undirected graph. For a graph $\mathcal{G}$, if there exists a directed path which is composed by a sequence of edges of $(i_1, i_2), (i_2, i_3), \cdots, (i_{k-1}, i_k)$, then agent i_1 is said to be reachable from agent i_k. We can also say that $\mathcal{G}$ contains a spanning tree with agent i_k as its root node. The adjacency matrix $\mathcal{A} = [a_{ij}]$ can be defined by selecting the elements with $a_{ij} = 1$ if $(j,i) \in \mathcal{E}$, while $a_{ij} = 0$ if $(j,i) \notin \mathcal{E}$. Further, the Laplacian matrix of graph $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}] \in \mathcal{R}^{n\times n}$, where $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \neq j$. And the corresponding weighted Laplacian matrix can be written as $\mathcal{L}_w = [l_{ij}w_{ij}] \in \mathcal{R}^{n\times n}$ with weight values $w_{ij} > 0$. Then the following lemma for Laplacian matrix $\mathcal{L}$ of undirected graph $\mathcal{G}$ can be obtained:

Lemma 1 ([25]): The Laplacian matrix $\mathcal{L}$ has an eigenvalue $\lambda_1 = 0$, and the remaining eigenvalues have positive real parts if the graph $\mathcal{G}$ contains a spanning tree. For the weighted Laplacian matrix $\mathcal{L}_w$ with $w_{ij} = w_{ji}$, the following property can be satisfied:

$$x^T(\mathcal{L}_w \otimes I_m)x = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}w_{ij}\left\|x_j - x_i\right\|^2 .$$

The Kronecker product of $M \in \mathcal{R}^{a\times b}$ and $N \in \mathcal{R}^{c\times d}$ is represented by $M \otimes N \in \mathcal{R}^{ac\times bd}$, which has the following properties:

1) $(M \otimes N)(Q \otimes T) = (MQ) \otimes (NT)$,
2) $(M \otimes N)^T = M^T \otimes N^T$,
3) $(M + N) \otimes Q = (M \otimes Q) + (N \otimes Q)$.

B. PROBLEM FORMULATION

In this paper, a linear MAS with process faults is considered, which is composed of n follower agents and m leader agents. The terms $\mathcal{V}_f$ and $\mathcal{V}_l$ are used to describe the sets of follower agents and leader agents, respectively. A graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ with $\bar{\mathcal{V}} = (\mathcal{V}_l, \mathcal{V}_f)$ is introduced to describe the communication network, and the corresponding Laplacian matrix is $\bar{\mathcal{L}} \in \mathcal{R}^{(n+m)\times(n+m)}$. Moreover, an induced subgraph $\mathcal{G}_1$ can be obtained by n follower agents and the edges between them, and the Laplacian matrix of $\mathcal{G}_1$ is $\mathcal{L}_1 \in \mathcal{R}^{n\times n}$.

The n followers are subject to process faults, which can be expressed as

$$\begin{cases} \dot{x}_i(t) = & Ax_i(t) + Bu_i(t) + Ef_i(t) \\ y_i(t) = & Cx_i(t) \end{cases}, \quad i \in \mathcal{V}_f, \tag{1}$$

where $x_i \in \mathcal{R}^d, u_i \in \mathcal{R}^q, y_i \in \mathcal{R}^p$ are the state, the control input and the measured output of follower i, respectively. $f_i \in \mathcal{R}^r$ denotes the process fault of follower i. A, B, C and E are constant matrices. In particular, $f_i(t)$ represents the additive actuator fault when $E = B$. In this paper, it is assumed that the process faults are constant vectors with slowly varying rate (i.e. $\dot{f}_i(t) \backsimeq 0$) and only occur in the followers.

The dynamics of m leaders with linear dynamics can be described as follows:

$$\begin{cases} \dot{v}_k(t) = Sv_k(t) \\ w_k(t) = C_r v_k(t) \end{cases}, \quad k \in \mathcal{V}_l, \tag{2}$$

where $v_k \in \mathcal{R}^h$ and $w_k \in \mathcal{R}^p$ are the state and the measured output of leader k, respectively. S and C_r are system matrices with appropriate dimensions.

The main objective of the fault-tolerant containment control problem is to develop a distributed adaptive FTCC law such that follower agents will move into the convex space spanned by leader agents. This space is also called convex hull, and its definition can be described as:

Definition 1 ([26]): A set $\mathcal{C} \subseteq \mathcal{R}^n$ is convex if $(1-\lambda)x + \lambda y \in \mathcal{V}$ for any $x, y \in \mathcal{C}$ and any $\lambda \in [0, 1]$. The convex hull $Co(X)$ of a finite set of points $X = \{x_1, \ldots, x_n\}$ is the minimal convex set containing all points in X, i.e., $Co(X) = \{\sum_{i=1}^{n} \alpha_i x_i | x_i \in X, \alpha_i \in R^+, \sum_{i=1}^{n} \alpha_i = 1\}$.

Then the definition of the FTCC problem in this paper is given as follows:

Definition 2: Consider a linear MAS (1) and (2) under a interaction topology determined by $\bar{\mathcal{G}}$. When some process faults occur in follower agents i, $i \in \mathcal{V}_f$, the FTCC problem is solved under a distributed FTCC law u_i, $i \in \mathcal{V}_f$ such that follower-outputs y_i, $i \in \mathcal{V}_f$ will move into the convex hull formed by leader-outputs w_k, $k \in \mathcal{V}_l$ as time goes to infinity, i.e.,

$$\lim_{t\to\infty} dist(y_i(t), Co(w)) = 0, \tag{3}$$

where $w = \{w_1, \cdots, w_m\}$.

To reflect the real-time distances between agents, we give the following output error vector

$$e_i = \sum_{j=1}^{n} a_{ij}(y_i - y_j) + \sum_{k=1}^{m} b_{ik}(y_i - w_k), \quad i \in \mathcal{V}_f,\ k \in \mathcal{V}_l, \tag{4}$$

where $b_{ik} = 1$ if agent $i, i \in \mathcal{V}_f$ is reachable from the leader $k, k \in \mathcal{V}_l$, and $b_{ik} = 0$ otherwise. Then the following lemma can be obtained.

Lemma 2: With Assumption 5, the term $\lim_{t\to\infty} e_i = 0$ implies $\lim_{t\to\infty} dist(y_i(t), Co(w)) = 0$.

Proof: The proof is much similar to that of Lemma 2 in [7] and thus is omitted here. ■

Next, the following standard assumptions for the solvability of FTCC problem are given:

Assumption 1: Matrix S has no eigenvalues with negative real parts.

Assumption 2: The pair (A, B) is stabilizable and $rank(B, E) = rank(B)$.

Assumption 3: The pair (A, C) are observable.

Assumption 4: For all $\sigma \in \lambda(S)$, the following equation holds

$$rank \begin{bmatrix} A - \sigma I_n & B \\ C & D \end{bmatrix} = d + p. \tag{5}$$

Assumption 5: The induced subgraph $\mathcal{G}_1$ is undirected and each leader $k \in \mathcal{V}_l$ has at least one directed path to each follower $i \in \mathcal{V}_f$.

Remark 1: Assumptions 1−5 are quite standard to solve the CORP [19]. Assumption 1 is made for convenience and without loss of generality. If the CORP is solvable by the designed controller, then it is also solvable by the same controller even if Assumption 1 is violated. This is because the stability of the closed-loop system has nothing to do with the exosystem and the output errors are only concerned with asymptotic property of the closed-loop system. Assumption 2 guarantees that the system can be locally stabilized by a state feedback control. $rank(B, E) = rank(B)$ means that we can find a matrix B^* such that $(I - BB^*)E = 0$, and the detailed proof is given in [11]. Moreover, under a special case of additive actuator fault with $E = B$, the term $rank(B, E) = rank(B, B) = rank(B)$ still satisfies Assumption 2. Assumption 3 is used to design the state observer for each follower. Assumption 4 is called the transmission zero condition and this is a necessary condition to guarantee the existence of the regulator equation, which will be elaborated in the next section. Assumption 5 is a common condition to solve the CORP, which is equivalent to the assumption of spanning forest in [7], [27].

If Assumption 5 holds, the Laplacian matrix $\bar{\mathcal{L}}$ of the graph $\bar{\mathcal{G}}$ can be partitioned as follows

$$\bar{\mathcal{L}} = \begin{bmatrix} \mathbf{0}_{m\times m} & \mathbf{0}_{m\times n} \\ \bar{\mathcal{L}}_2 & \bar{\mathcal{L}}_1 \end{bmatrix}, \tag{6}$$

where $\bar{\mathcal{L}}_2 \in \mathcal{R}^{n\times m}$ and $\bar{\mathcal{L}}_1 \in \mathcal{R}^{n\times n}$. Then define a square matrix $H_k = \frac{1}{m}\mathcal{L}_1 + \Lambda_k, k \in \mathcal{V}_l$ with $\Lambda_k = diag(b_{i1}, \cdots, b_{nk}), k \in \mathcal{V}_l$. It should be noted that $\bar{\mathcal{L}}_1 = \sum_{k=1}^{m} H_k = \mathcal{L}_1 + \sum_{k=1}^{m} \Lambda_k, k \in \mathcal{V}_l$.

In order to analyze the properties of these matrices, the definition of a well-known M-matrix is given.

Definition 3 ([28]): Given a matrix $A = [a_{ij}] \in \mathcal{R}^{n\times n}$, if all the eigenvalues of A have positive real parts and its off-diagonal elements are nonpositive, i.e., $A = [a_{ij}], a_{ij} \leq 0$, $i \neq j$, then matrix A can be called an M-matrix.

It is obvious that $\bar{\mathcal{L}}_1$ belongs to symmetric positive definite and nonsingular M-matrix. Moreover, some properties of H_k are given as follows:

Lemma 3 [29]: The matrices H_k and $\sum_{k=1}^{m} H_k$ are symmetric positive definite and nonsingular M-matrix if Assumption 5 is satisfied. These matrices have the following properties:

1) The eigenvalues of H_k and $\sum_{k=1}^{m} H_k$ have positive real parts.
2) $(H_k)^{-1}$ and $(\sum_{k=1}^{m} H_k)^{-1}$ exist and both are nonnegative.

III. MAIN RESULTS

In this section, a new distributed adaptive FTCC law is designed based upon the output regulation theory. Fig 1 shows the distributed FTCC scheme. In Fig 1, we can see that ith follower can communicate with jth follower and receive the information from kth leader. The state and fault of each follower can be estimated based on the information-exchange between itself and its neighboring agents. Then the estimation information can be embedded into the FTCC law to compensate for the faults in the followers.

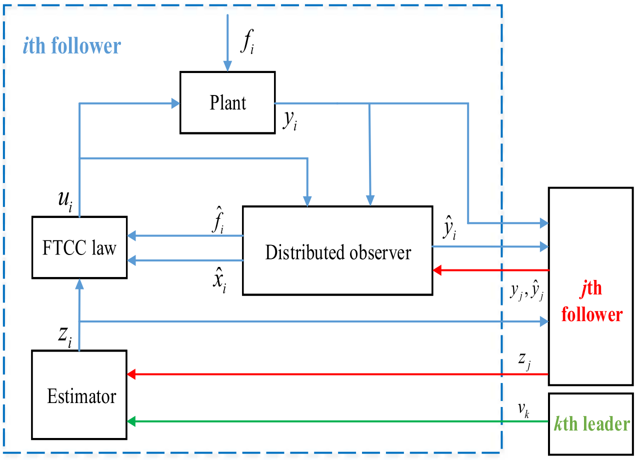

FIGURE 1. Distributed FTCC scheme.

A. OBSERVER DESIGN

Inspired by the fault estimation algorithm [14], [16], distributed state observers and fault estimation observers for followers are designed as follows:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + E\hat{f}_i(t) - H\bar{e}_i(t) \\ \hat{y}_i(t) = C\hat{x}_i(t) \\ \dot{\hat{f}}_i(t) = -\Gamma F\left(\bar{e}_i(t) + \dot{\bar{e}}_i(t)\right) \end{cases}, \quad i \in \mathcal{V}_f, \tag{7}$$

where $\hat{x}_i(t) \in \mathcal{R}^d$ and $\hat{f}_i(t) \in \mathcal{R}^r$ are the state estimation and the fault estimation of follower $i, i \in \mathcal{V}_f$ respectively. $H \in \mathcal{R}^{d\times p}, \Gamma \in \mathcal{R}^{r\times r}$ and $F \in \mathcal{R}^{r\times p}$ are observer gain matrices to be designed. It is noted that Γ is a positive definite symmetric matrix, i.e., $\Gamma = \Gamma^T > 0$. The measurement error between followers can be defined as follows:

$$\begin{aligned} \bar{e}_i &= \sum_{j=1}^{n} a_{ij}\left(\left(\hat{y}_i - y_i\right) - \left(\hat{y}_j - y_j\right)\right) + \sum_{k=1}^{m} b_{ik}\left(\hat{y}_i - y_i\right) \\ &= C\left(\sum_{j=1}^{n} a_{ij}\left(e_{xi} - e_{xj}\right) + \sum_{k=1}^{m} b_{ik} e_{xi}\right), i, \quad j \in \mathcal{V}_f,\ k \in \mathcal{V}_l, \end{aligned} \tag{8}$$

where a_{ij} and b_{ik} are entries of $\mathcal{L}_1$ and Λ_k defined in the last section. Further, denote the error vectors of state observers and fault observers as:

$$e_{xi} = \hat{x}_i - x_i, \quad e_{fi} = \hat{f}_i - f_i. \tag{9}$$

Then the derivatives of these error vectors can be obtained respectively

$$\begin{aligned} \dot{e}_{xi} &= \dot{\hat{x}}_i - \dot{x}_i \\ &= Ae_{xi} + Ee_{fi} \\ &\quad - HC\left[\sum_{j=1}^{n} a_{ij}\left(e_{xi} - e_{xj}\right) + \sum_{k=1}^{m} b_{ik} e_{xi}\right], \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{e}_{fi} &= \dot{\hat{f}}_i - \dot{f}_i \\ &= -\Gamma FC\sum_{j=1}^{n} a_{ij}\left(e_{xi} - e_{xj}\right) \\ &\quad -\Gamma FC\left[\sum_{j=1}^{n} a_{ij}\left(\dot{e}_{xi} - \dot{e}_{xj}\right) + \sum_{k=1}^{m} b_{ik}\dot{e}_{xi}\right] - \dot{f}_i. \end{aligned} \tag{11}$$

Let $e_x = \left[e_{x1}^T, \cdots, e_{xn}^T\right]^T, e_f = \left[e_{f1}^T, \cdots, e_{fn}^T\right]^T$ and $f = \left[f_1^T, \cdots, f_n^T\right]^T$. The derivatives of these compact error vectors can be written as

$$\begin{aligned} \dot{e}_x &= \left(I_n \otimes A - \bar{\mathcal{L}}_1 \otimes HC\right) e_x + \left(I_n \otimes E\right) e_f, \\ \dot{e}_f &= -\bar{\mathcal{L}}_1 \otimes (\Gamma FC)\left(e_x + \dot{e}_x\right) - \dot{f}. \end{aligned} \tag{12}$$

We have the following theorem:

Theorem 1: Under Assumption 3, if there exist a symmetric positive matrix $P \in \mathcal{R}^{d\times d}$ and a matrices $Y = PH$ satisfying the following conditions:

$$E^T P = FC \tag{13}$$

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0 \tag{14}$$

where

$$\begin{aligned} \Sigma_{11} &= I_n \otimes \left(A^T P + PA\right) - \bar{\mathcal{L}}_1 \otimes \left(YC + C^T Y^T\right) \\ \Sigma_{12} &= -I_n \otimes A^T PE + \bar{\mathcal{L}}_1^T \otimes C^T Y^T E \\ \Sigma_{22} &= -2I_n \otimes E^T PE \end{aligned}$$

then the estimation errors (10) and (11) of each follower will converge to 0 asymptotically as time approaches infinity.

Proof: We can construct the following Lyapunov function candidate

$$V_1(t) = e_x^T\left(I_n \otimes P\right) e_x + e_f^T\left(\bar{\mathcal{L}}_1^{-1} \otimes \Gamma^{-1}\right) e_f. \tag{15}$$

With (12), we can obtain the time derivative form of (15):

$$\begin{aligned} &\dot{V}_1(t) \\ &= \dot{e}_x^T\left(I_n \otimes P\right) e_x + e_x^T\left(I_n \otimes P\right)\dot{e}_x \\ &\quad + \dot{e}_f^T\left(\bar{\mathcal{L}}_1^{-1} \otimes \Gamma^{-1}\right) e_f + e_f^T\left(\bar{\mathcal{L}}_1^{-1} \otimes \Gamma^{-1}\right)\dot{e}_f \\ &= \left(\left(I_n \otimes A - \bar{\mathcal{L}}_1 \otimes HC\right) e_x + \left(I_n \otimes E\right) e_f\right)^T\left(I_n \otimes P\right) e_x \\ &\quad + e_x^T\left(I_n \otimes P\right)\left(\left(I_n \otimes A - \bar{\mathcal{L}}_1 \otimes HC\right) e_x + \left(I_n \otimes E\right) e_f\right) \\ &\quad + \left(-\bar{\mathcal{L}}_1 \otimes (\Gamma FC)\left(e_x + \dot{e}_x\right) - \dot{f}\right)^T\left(\bar{\mathcal{L}}_1^{-1} \otimes \Gamma^{-1}\right) e_f \\ &\quad + e_f^T\left(\bar{\mathcal{L}}_1^{-1} \otimes \Gamma^{-1}\right)\left(-\bar{\mathcal{L}}_1 \otimes (\Gamma FC)\left(e_x + \dot{e}_x\right) - \dot{f}\right) \\ &= e_x^T\left(I_n \otimes \left(A^T P + PA\right) - \bar{\mathcal{L}}_1 \otimes \left(YC + C^T Y^T\right)\right) e_x \\ &\quad + e_f^T\left(I_n \otimes E^T P\right) e_x + e_x^T\left(I_n \otimes PE\right) e_f \\ &\quad - 2\left(e_x + \dot{e}_x\right)^T\left(I_n \otimes (FC)^T\right) e_f - 2e_f^T\left(\bar{\mathcal{L}}_1^{-1} \otimes \Gamma^{-1}\right)\dot{f}. \end{aligned} \tag{16}$$

Simple calculation yields

$$
\begin{aligned}
&-\dot{e}_x^T\left(I_n\otimes(FC)^T\right)e_f\\
&\quad=-e_x^T\left(I_n\otimes A-\bar{\mathcal{L}}_1\otimes HC\right)^T\left(I_n\otimes(FC)^T\right)e_f\\
&\qquad-e_f^T\left(I_n\otimes E\right)^T\left(I_n\otimes(FC)^T\right)e_f\\
&\quad=-e_x^T\left(I_n\otimes A^TC^TF^T-\bar{\mathcal{L}}_1\otimes C^TH^TC^TF^T\right)e_f\\
&\qquad-e_f^T\left(I_n\otimes E^TC^TF^T\right)e_f\\
&\quad=-e_x^T\left(I_n\otimes A^TC^TF^T-\bar{\mathcal{L}}_1\otimes C^TY^TE\right)e_f\\
&\qquad-e_f^T\left(I_n\otimes E^TPE\right)e_f.
\end{aligned}
\tag{17}
$$

Let $\zeta=\left[e_x^T,e_f^T\right]^T$, and time derivative of (15) can be unified as

$$
\begin{aligned}
\dot{V}(t)=&\ e_x^T\left(I_n\otimes\left(A^TP+PA\right)-\bar{\mathcal{L}}_1\otimes\left(YC+C^TY^T\right)\right)e_x\\
&+e_x^T\left(I_n\otimes PE-I_n\otimes\left(A^TPE+PE\right)\right.\\
&\left.+\bar{\mathcal{L}}_1^T\otimes C^TY^TE\right)e_f\\
&+e_f^T\left(I_n\otimes E^TP-I_n\otimes\left(E^TPA+E^TP\right)\right.\\
&\left.+\bar{\mathcal{L}}_1\otimes E^TY^TC\right)e_x\\
&+e_f^T\left(-2I_n\otimes E^TPE\right)e_f\\
=&\ \zeta^T\Sigma\zeta,
\end{aligned}
\tag{18}
$$

where $\Sigma=\begin{bmatrix}\Sigma_{11}&\Sigma_{12}\\ *&\Sigma_{22}\end{bmatrix}$. We can see that if condition (14) holds, the time derivative of (15) satisfies $\dot{V}_1(t)<-\mu\|\zeta\|^2<0$, where $\mu=\lambda_{\min}(-\Sigma)$. The vector ζ is composed of the estimation vectors e_x and e_f, which shows that these estimation errors can guarantee asymptotic convergence under the conditions (13) and (14). ■

Remark 2: The condition (13) can be transformed into the following optimization problem by selecting a small enough positive constant φ, which can also be found in [16], [30]:

$$
\min\varphi,\ s.t.\begin{bmatrix}\varphi I & E^TP-FC\\ * & \varphi I\end{bmatrix}>0. \tag{19}
$$

Remark 3: The faults considered in this section can also be time-varying faults, i.e., $\dot{f}_i\neq 0, i\in\mathcal{V}_f$. However, the derivatives of faults should be bounded with a positive upper bound $\bar{f}_i$, i.e., $\left\|\dot{f}_i(t)\right\|\le\bar{f}_i$. This assumption is quite general in the classical observer-based approaches [11], [14]. The time-varying faults will make the term $\dot{f}_i(t)$ remain in (12), and thus we have $\dot{V}_1(t)<-\mu\|\zeta\|^2+\delta$ if $\Sigma=\begin{bmatrix}\Sigma_{11}&\Sigma_{12}\\ *&\Sigma_{22}+I_n\otimes M\end{bmatrix}<0$, where $M=M^T>0$ and δ is a positive constant depending on $\left\|\dot{f}_i(t)\right\|$. Then the error system (12) is uniformly ultimately bounded.

B. FAULT-TOLERANT CONTROLLER DESIGN

Now we are ready to design a new distributed adaptive FTCC law

$$
\begin{cases}
u_i(t)=K_1\hat{x}_i+K_2z_i-B^*E\hat{f}_i(t)\\
\dot{z}_i(t)=Sz_i+\sum\limits_{j=1}^{n}a_{ij}k_{ij}\Psi(z_j-z_i)+\sum\limits_{k=1}^{m}b_{ik}k_{ik}\Psi(v_k-z_i)\\
\delta_i(t)=z_i-\xi_i\\
\dot{k}_{ij}(t)=a_{ij}\gamma_{ij}(\delta_j-\delta_i)^T\Psi(\delta_j-\delta_i)\\
\dot{k}_{ik}(t)=b_{ik}\gamma_{ik}\delta_i^T\Psi\delta_i
\end{cases}
\tag{20}
$$

where $\hat{x}_i(t)$ and $\hat{f}_i(t)$ are generated by the observer (7). $z_i(t)\in\mathcal{R}^{n_z}$ is the vector associated with the information of leaders. $\xi_i(t)\in\mathcal{R}^{n_z}$ is generated by $\dot{\xi}=(I_n\otimes S)\xi$ with its initial value $\xi(0)=\sum_{k=1}^m\left(\left(\left(\sum_{j=1}^m H_{jw}\right)^{-1}H_{kw}\mathbf{1}_n\right)\otimes v_k(0)\right)$, where $\xi=\left[\xi_1^T,\cdots,\xi_n^T\right]^T$ and $v_k(0)$ is the initial value of $v_k, k\in\mathcal{V}_l$. Detailed descriptions for $\xi_i(t)$ and H_{kw} will be given later. K_1, K_2 and B^* are the control gains satisfying Assumption 2 and Assumption 3. Ψ is a positive definite matrix. γ_{ij}, $k_{ij}(t)$, and $k_{ik}(t)$ are defined to satisfy $\gamma_{ij}=\gamma_{ji}>0$, $k_{ij}(t)=k_{ji}(t)>0$, and $k_{ik}(t)>0$ for $i,j\in\mathcal{V}_f$, $k\in\mathcal{V}_l$.

Consider the following weighted Laplacian matrix $\bar{\mathcal{L}}_w$, which can be transformed from the Laplacian matrix $\mathcal{L}$ of the graph $\bar{\mathcal{G}}$ by regarding the adaptive gains $k_{ij}(t)$ and $k_{ik}(t)$ as weight values

$$
\bar{\mathcal{L}}_w=\begin{bmatrix}\mathbf{0}_{m\times m}&\mathbf{0}_{m\times n}\\ \bar{\mathcal{L}}_{2w}&\bar{\mathcal{L}}_{1w}\end{bmatrix}, \tag{21}
$$

where $\bar{\mathcal{L}}_{1w}=\sum_{k=1}^m H_{kw}$ with $H_{kw}=\frac{1}{m}\mathcal{L}_{1w}+\Lambda_{kw}, k\in\mathcal{V}_l$. Obviously, the matrices H_{kw} and $\sum_{k=1}^m H_{kw}$ still satisfy the properties in Lemma 3.

Next, the main theorem is given to deal with the FTCC problem.

Theorem 2: Given a MAS governed by (1) and (2) satisfying Assumptions 1–5 and conditions (13)–(14). Then for any initial values $x_i(0), i\in\mathcal{V}_f$ and $v_k(0), k\in\mathcal{V}_k$, the FTCC problem is solved under the control protocol (20) if the following regulator equation

$$
\begin{cases}
XS=AX+BU\\
\mathbf{0}=CX-C_r
\end{cases}
\tag{22}
$$

has a unique solution (X,U) with $K_2=U-K_1X$.

Proof: Let $x=[x_1^T,\cdots,x_n^T]^T$, $z=[z_1^T,\cdots,z_n^T]^T$, $\delta=\left[\delta_1^T,\cdots,\delta_n^T\right]^T$, $x_c=\left[x^T,z^T\right]^T$, $\chi=\left[x_c^T,\zeta^T\right]^T$. With the FTCC law (20), we have the following closed-loop system in block matrix form

$$
\begin{aligned}
\dot{\chi}&=A_c\chi+B_c\\
&=\begin{bmatrix}\Pi_{11}&\Pi_{12}\\ \mathbf{0}&\Pi_{14}\end{bmatrix}\begin{bmatrix}x_c\\ \zeta\end{bmatrix}+\begin{bmatrix}B_{c_1}\\ B_{c2}\end{bmatrix},
\end{aligned}
\tag{23}
$$

where

$$\Pi_{11} = \begin{bmatrix} I_n \otimes (A+BK_1) & I_n \otimes BK_2 \\ \mathbf{0} & I_n \otimes S - \sum_{k=1}^{m} H_{kw} \otimes \Psi \end{bmatrix},$$

$$\Pi_{12} = \begin{bmatrix} I_n \otimes BK_1 & E \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, B_{c1} = \begin{bmatrix} \mathbf{0} \\ \sum_{k=1}^{m} H_{kw}\mathbf{1}_n \otimes \Psi v_k \end{bmatrix},$$

$$\Pi_{14} = \begin{bmatrix} I_n \otimes A - \bar{\mathcal{L}}_1 \otimes HC & I_n \otimes E \\ \mathbf{0} & -\bar{\mathcal{L}}_1 \otimes (\Gamma FC) \end{bmatrix},$$

$$B_{c2} = \begin{bmatrix} \mathbf{0} \\ -\left(\bar{\mathcal{L}}_1 \otimes (\Gamma FC)\right)\dot{e}_x \end{bmatrix}.$$

Under Assumption 2, $A+BK_1$ can be designed to be Hurwitz by the pole placement technique. The stability of system (23) is trivially equivalent to the stability of the following system based on the fact that the system $\dot{\zeta} = \Pi_{14}\zeta + B_{c2}$ is stable

$$\dot{z} = \left(I_n \otimes S - \sum_{k=1}^{m} H_{kw} \otimes \Psi\right) z + \sum_{k=1}^{m} H_{kw}\mathbf{1}_n \otimes \Psi v_k. \tag{24}$$

The compact vector form of $\xi_i(t)$ is given by:

$$\xi = \sum_{k=1}^{m}\left(\left(\left(\sum_{j=1}^{m} H_{jw}\right)^{-1} H_{kw}\mathbf{1}_n\right) \otimes v_k\right). \tag{25}$$

Let $\bar{v}_k = \mathbf{1}_n \otimes v_k$. One can deduce that ξ can be generated by $\dot{\xi} = (I_n \otimes S)\xi$ with the initial value being $\xi(0) = \sum_{k=1}^{m}\left(\left(\left(\sum_{j=1}^{m} H_{jw}\right)^{-1} H_{kw}\mathbf{1}_n\right) \otimes v_k(0)\right)$, which has the same dynamics as $\dot{\bar{v}}_k = (I_n \otimes S)\bar{v}_k$ but with the different initial value $\mathbf{1}_n \otimes v_k(0)$. Let $\delta = z - \xi$, then we have

$$\dot{\delta} = \left(I_n \otimes S - \sum_{k=1}^{m} H_{kw} \otimes \Psi\right)\delta. \tag{26}$$

We can further construct a Lyapunov function candidate as follows:

$$V_2 = \frac{1}{2}\delta^T\delta + \frac{1}{4m}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(k_{ij}-c)^2}{\gamma_{ij}} + \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{m}\frac{(k_{ik}-c)^2}{\gamma_{ik}}, \tag{27}$$

where constant c is determined later. Then we can obtain time derivative of (27):

$$\begin{aligned}
\dot{V}_2(t) &= \delta^T\dot{\delta} + \frac{1}{2m}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(k_{ij}-c)\dot{k}_{ij}}{\gamma_{ij}} + \sum_{i=1}^{n}\sum_{k=1}^{m}\frac{(k_{ik}-c)\dot{k}_{ik}}{\gamma_{ik}} \\
&= \delta^T\left(I_n \otimes S - \sum_{k=1}^{m} H_{kw} \otimes \Psi\right)\delta \\
&\quad + \frac{1}{2m}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}k_{ij}(\delta_j-\delta_i)^T\Psi(\delta_j-\delta_i) \\
&\quad - \frac{c}{2m}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}k_{ij}(\delta_j-\delta_i)^T\Psi(\delta_j-\delta_i) \\
&\quad + \sum_{i=1}^{n}\sum_{k=1}^{m} k_{ik}b_{ik}\delta_i^T\Psi\delta_i - \sum_{i=1}^{n}\sum_{k=1}^{m} ck_{ik}b_{ik}\delta_i^T\Psi\delta_i \\
&= \delta^T\left(I_n \otimes S - \sum_{k=1}^{m} H_{kw} \otimes \Psi\right)\delta \\
&\quad + \frac{1}{m}\delta^T\left(L_{1w} \otimes \Psi\right)\delta - \frac{c}{m}\delta^T\left(L_1 \otimes \Psi\right)\delta \\
&\quad + \sum_{k=1}^{m}\delta^T\left(\Lambda_{kw} \otimes \Psi\right)\delta - c\sum_{k=1}^{m}\delta^T\left(\Lambda_k \otimes \Psi\right)\delta \\
&= \delta^T\left(I_n \otimes S - c\sum_{k=1}^{m} H_k \otimes \Psi\right)\delta.
\end{aligned} \tag{28}$$

Since matrices H_k and Ψ are positive definite, we can choose a sufficiently large constant c such that $\dot{V}_2(t) < 0$ for any $\delta \neq 0$. Thus the system (23) is asymptotically stable. When time goes to infinity, $\delta_i(t) = z_i(t) - \xi_i(t) \to 0$. So adaptive parameters $k_{ij}(t)$ and $k_{ik}(t)$ are bounded. Moreover, $\lim_{t\to\infty}\dot{k}_{ij}(t) = 0$ and $\lim_{t\to\infty}\dot{k}_{ik}(t) = 0$ imply that the adaptive parameters $k_{ij}(t)$ and $k_{ik}(t)$ will tend to some constants when time goes to infinity.

Let $\tilde{x}_i = x_i - X\xi_i$, $\hat{e}_i = y_i - C_r\xi_i$. Define the compact vectors $\hat{e} = \left[\hat{e}_1^T, \cdots, \hat{e}_n^T\right]^T$, $\tilde{x}_c = \left[\tilde{x}^T, \delta^T\right]^T$, $\tilde{\chi} = \left[\tilde{x}_c^T, \delta^T\right]^T$. The term $\lim_{t\to\infty}\hat{e} = 0$ implies $\lim_{t\to\infty} dist(y_i(t), Co(w)) = 0$ by replacing H_w by H_{kw} in Lemma 2. Then the system (23) can be rewritten as:

$$\begin{aligned}
\dot{\tilde{\chi}} &= A_c\tilde{\chi} + \tilde{B}_c \\
&= \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \mathbf{0} & \Pi_{14} \end{bmatrix}\begin{bmatrix} \tilde{x}_c \\ \zeta \end{bmatrix} + \begin{bmatrix} \tilde{B}_{c_1}\xi \\ \mathbf{0} \end{bmatrix}, \\
\hat{e} &= (I_n \otimes C)\tilde{x}_c + (I_n \otimes (CX - C_r))\xi,
\end{aligned} \tag{29}$$

where

$$\tilde{B}_{c1} = \begin{bmatrix} I_n \otimes (-XS + AX + BK_1X + BK_2) \\ \mathbf{0} \end{bmatrix}. \tag{30}$$

If conditions (13) and (14) hold, we have $\lim_{t\to\infty}\zeta(t) = 0$ for $i \in \mathcal{V}_f$. If the regulator equation (22) has a unique solution (X, U) with $K_2 = U - K_1X$, we can further deduce that $\lim_{t\to\infty}\tilde{x}_c = 0$, and thus $\lim_{t\to\infty}\hat{e} = 0$. The proof is completed. ■

Remark 4: The distributed observer for each follower proposed in [31] is designed as follows:

$$\dot{z}_i(t) = Sz_i + \alpha\left(\sum_{j=1}^{n} a_{ij}(z_j - z_i) + \sum_{k=1}^{m} b_{ik}(v_k - z_i)\right). \tag{31}$$

It should be pointed out that the constant should satisfy $\alpha > \mathfrak{R}_{\max}(S)/\mathfrak{R}_{\min}\left(\sum_{k=1}^{m} H_k\right)$, and the term $\mathfrak{R}(X)$ represents the real part of matrix X. So α depends on the eigenvalues of matrices S and H_k, which contains a global nature of interconnection topology. The distributed observer given in the second equation of (20) in this paper can provide the synthesized information of multiple leaders. The adaptive laws in this distributed observer can adaptively tend to some appropriate values. Therefore, we overcome a global condition by replacing a large constant α by adaptive gains.

Remark 5: The matrix equation (22) is called regulator equation in output regulation theory [19]. The solvability of the regulation equation is necessary for the solvability of the output regulation problem. Assumption 4 gives the existence condition for the solution of regulator equation (22). We can see that the first equation of (22) is a Sylvester equation, which has a unique solution if S and $A+BK_1$ have no common eigenvalues. Under Assumption 2, we select an appropriate gain matrix K_1 by the pole placement technique such that $A+BK_1$ is Hurwitz. Thus Assumption 1 and the exponential stability of $A+BK_1$ guarantee the existence of X, satisfying the first equation of (22). Then the gain matrix K_2 can be further obtained by solving the equation (22).

Remark 6: The proposed control law (20) can be extended to solve some general CORPs. For example, for the CORP of MAS with single leader in the presence of process/additive actuator fault, our controller is still effective by defining the number of leaders as $m = 1$. More generally, if there is no fault in followers, i.e., $f(t) = \dot{f}(t) = 0$, we can simply obtain a new state feedback controller by replacing $\hat{x}_i$ by x_i, which can be further applied to solve the CORP of MASs with single/multiple leaders.

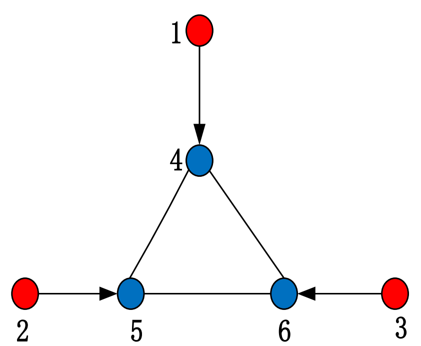

FIGURE 2. Topology of the multi-vehicle system.

IV. SIMULATION EXAMPLE

Consider a multi-vehicle system composed by three leader vehicles and three follower vehicles. Fig. 2 shows the communication topology among agents. Clearly, the leaders are labeled as nodes 1, 2 and 3 and the followers are labeled as other nodes. Three followers can be described as follows:

$$\begin{cases} \dot{x}_{1i} = x_{2i} \\ m\dot{x}_{2i} = u_i + f_i \\ y_i = x_{1i} \end{cases}, \quad i \in 4, 5, 6, \tag{32}$$

where $x_{1i} \in \mathcal{R}^2$ and $x_{2i} \in \mathcal{R}^2$ represent the position and velocity of vehicle i, respectively. $y_i \in \mathcal{R}^2$ is the position output. $u_i \in \mathcal{R}^2$ and $m \in \mathcal{R}$ are the torque input and the mass of vehicle i, respectively. $f_i \in \mathcal{R}^2$ represents the faults occurring in the followers. In this example we choose $m = 1$ such that $E = B = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$, which means additive actuator fault occurs in the followers.

Three leader vehicles with autonomous linear dynamics are given as follows:

$$\begin{cases} \dot{v}_{1k} = v_{2k} \\ \dot{v}_{2k} = 0 \\ w_k = v_{1k} \end{cases}, \quad k \in 1, 2, 3, \tag{33}$$

where $v_{1k} \in \mathcal{R}^2$ and $v_{2k} \in \mathcal{R}^2$ represent the position and velocity of vehicle k, respectively. $w_k \in \mathcal{R}^2$ is the position output.

The faults of each follower are given as follows:

$$f_{14} = f_{24} = \begin{cases} 0, & 0 \le t < 5 \\ 2, & t \ge 5 \end{cases},$$
$$f_{15} = f_{25} = \begin{cases} 0, & 0 \le t < 15 \\ 0.5, & t \ge 15 \end{cases},$$
$$f_{16} = f_{26} = \begin{cases} 0, & 0 \le t < 25 \\ 1.5, & t \ge 25 \end{cases}. \tag{34}$$

Obviously, the Laplacian matrix $\bar{\mathcal{L}}$ is

$$\bar{\mathcal{L}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & -1 & 3 & -1 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{bmatrix}. \tag{35}$$

Choose the following initial values of all the vehicles

$$v_{11}(0) = [2, 0.1]^T, \quad v_{21}(0) = [0.21, 0.21]^T,$$
$$v_{12}(0) = [3, 0.3]^T,$$
$$v_{22}(0) = [0.23, 0.23]^T, \quad v_{13}(0) = [4, 0.2]^T,$$
$$v_{23}(0) = [0.20, 0.20]^T,$$
$$x_{14}(0) = [0.3, -0.8]^T, \quad x_{24}(0) = [0.12, 0.18]^T,$$
$$x_{15}(0) = [1, -0.6]^T,$$
$$x_{25}(0) = [0.23, 0.34]^T, \quad x_{16}(0) = [1.6, 0.8]^T,$$
$$x_{26}(0) = [0.27, 0.42]^T.$$

For each follower, the gain matrices of (7) are obtained by Theorem 1:

$$H = \begin{bmatrix} 1.6065 \\ 1.8938 \end{bmatrix}, \quad F = 0.1259, \ \Gamma = 6.5.$$

The feedback gain matrices in the controller (20) can be designed as follows:

$$K_1 = \begin{bmatrix} -10 & 0 & -8 & 0 \\ 0 & -10 & 0 & -8 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 10 & 0 & 8 & 0 \\ 0 & 10 & 0 & 8 \end{bmatrix},$$
$$B^* = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

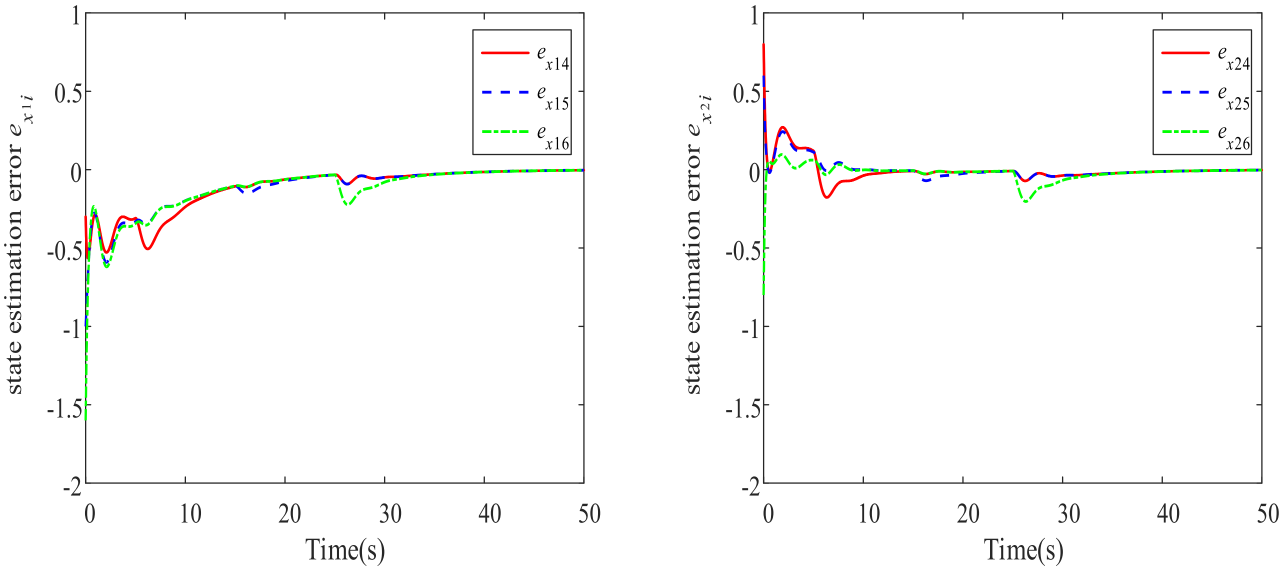

FIGURE 3. **State estimation errors.**

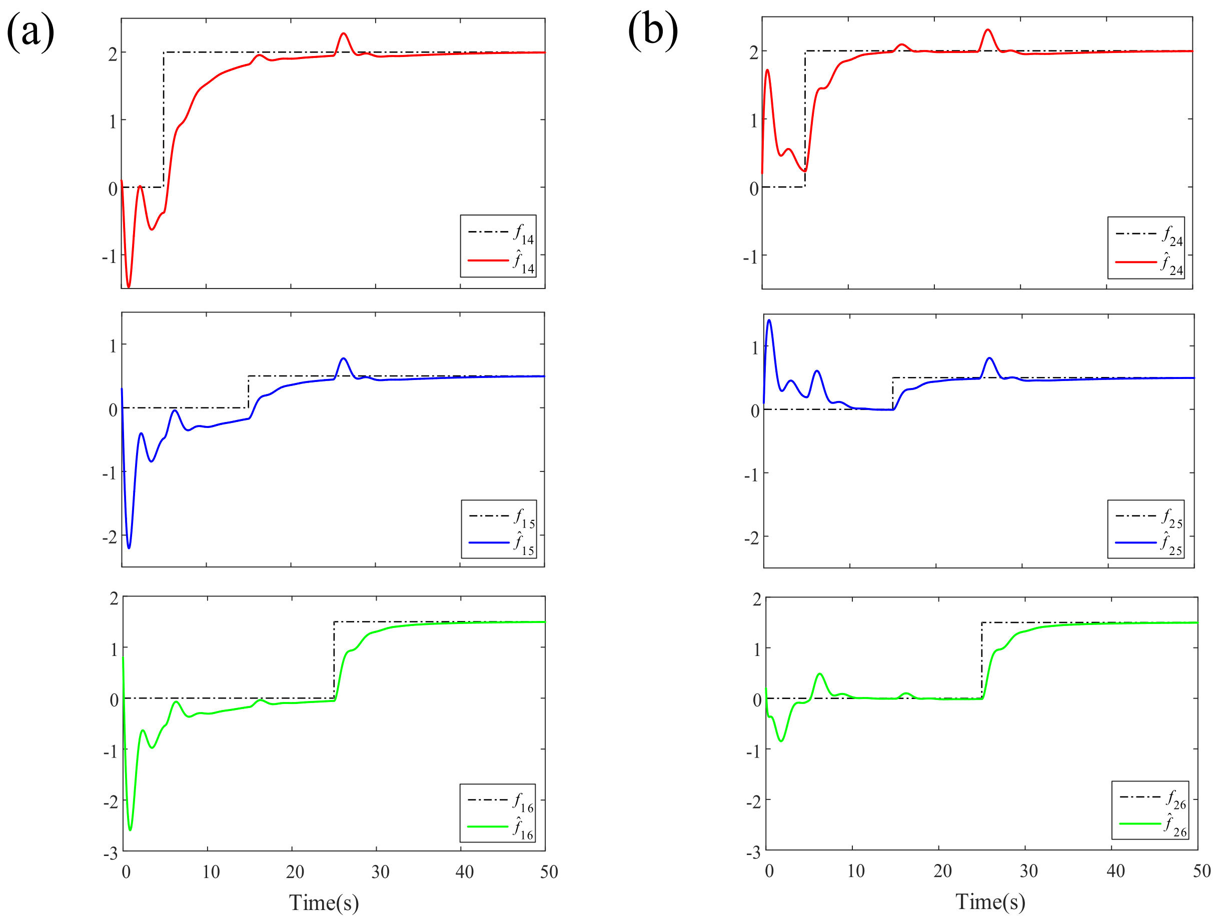

FIGURE 4. **Faults and fault estimation curves.**

Fig. 3–Fig. 6 show the simulation results. Fig. 3 shows two components of the state estimation errors of follower i, $i \in$ 4, 5, 6. Fig. 4 shows two components of faults and fault estimation curves. It is easily from Fig. 3 and Fig. 4 that the proposed distributed state observer and fault estimation observer are effective. The trajectory formed by all the

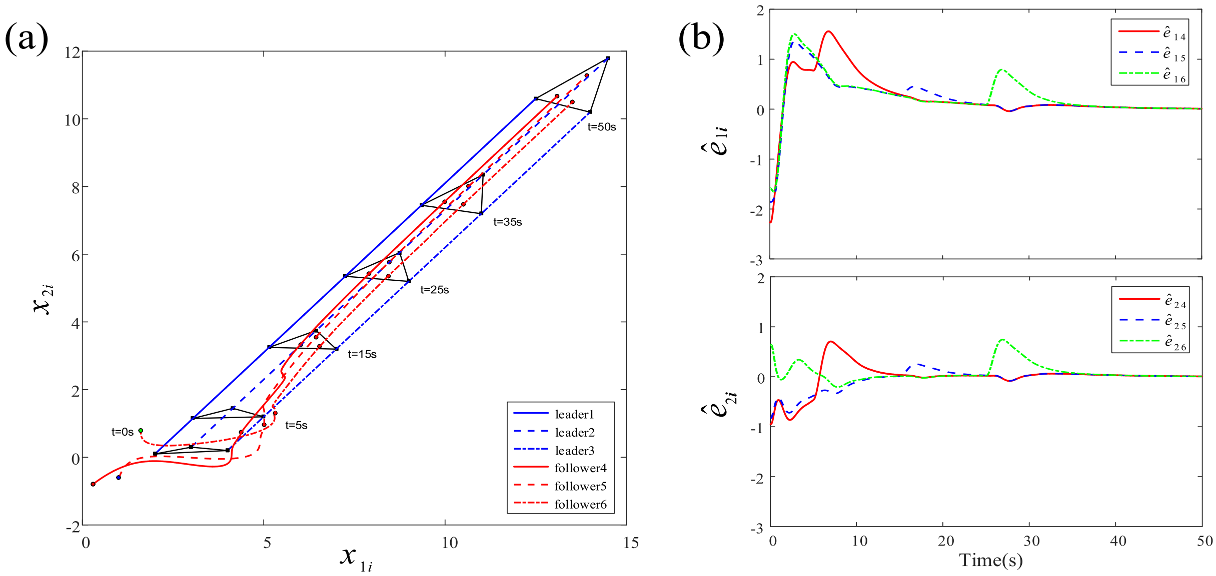

FIGURE 5. Trajectories of all vehicles under FTCC law.

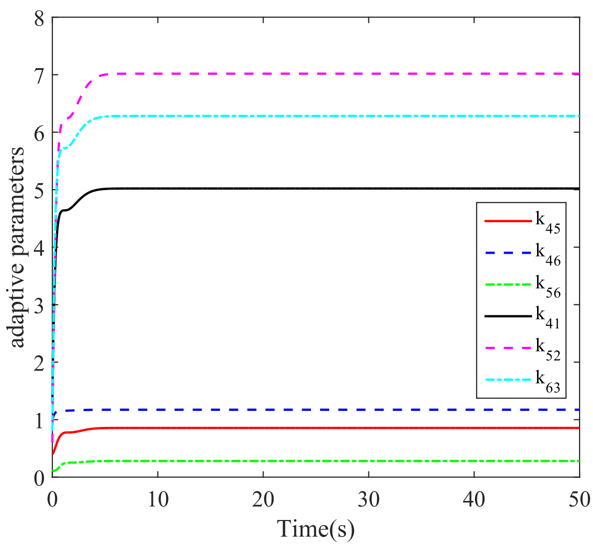

FIGURE 6. Adaptive parameters.

vehicles is presented in Fig. 5 (a). It is observed that with the time goes to infinity, all the follower vehicles (red nodes) will move into the triangular convex region spanned by the three leader vehicles (black nodes). The corresponding two components of all containment errors $\hat{e}_i$, $i = 4, 5, 6$, as shown in Fig. 5 (b), will go to 0 asymptotically. Moreover, the adaptive parameters $k_{ij}(t)$, $i = 4, 5, 6; j = 1, 2, 3, 4, 5, 6$ in (20) are shown in Fig. 6, which will approach some constants as time goes to infinity. The simulation results show that all the vehicles can achieve containment control regardless of faults, which demonstrate that the proposed FTCC law (20) is effective.

V. CONCLUSION

In this paper, the FTCC problem of linear MASs with process faults has been investigated based upon the output regulation theory. Distributed observers have been introduced to estimate the state and fault for each follower. The corresponding existence conditions of the observers have been presented in terms of linear matrix inequalities. The estimations have been embedded into a new distributed adaptive FTCC law to compensate for the performance degradation caused by process faults. We have proved that if the regulator equation has a unique solution, the FTCC problem can be solved under the designed control protocol. In the future, we will take into account the fault-tolerant control problems of more complicated heterogeneous MASs associated with the directed communication graphs.

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