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Underdetermined Blind Source Separation for Heart Sound Using Higher-Order Statistics and Sparse Representation

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ABSTRACT Underdetermined blind source separation (UBSS) is a hot and challenging problem in signal processing. In the traditional UBSS algorithm, the number of source signals is often assumed to be known, which is very inconvenient in practice. In addition, it is more difficult to obtain the accurate estimation of mixing matrix in the underdetermined case. However, this information has a great influence on the source separation results, which can easily lead to poor separation performance. In this paper, a novel UBSS algorithm is presented to carry out a combined source signal number estimation and source signal separation task. First, in the proposed algorithm, we design a gap-based detection method to detect the number of source signals by eigenvalue decomposition. Then, the estimation of the mixing matrix is processed using a higher-order cumulant-based method so that the uniqueness of the estimated mixing matrix is guaranteed. Furthermore, an improved l_1-norm minimization algorithm is proposed to estimate the source signals. Meanwhile, the pre-conditioned conjugate gradient technology is employed to accelerate the convergence rate such that the computational load is reduced. Finally, a series of simulation experiments with synthetic heart sound data and image reconstruction results demonstrate that the proposed algorithm achieves better separating property than the state-of-the-art algorithms.

INDEX TERMS Underdetermined blind source separation, heart sound signals, higher-order statistics, sparse representation.

I. INTRODUCTION

Blind source separation (BSS) is to separate the unknown source signals from the mixture signals with no information about the mixing matrix, which has been applied in various fields, such as speech processing, image processing [1], [2]. Most of BSS algorithms have been proposed by exploiting some assumptions about the source signals. For example, the number of source signals is known, the source signals are mutually independent or assumed to be sparse. However, in practice, these assumptions are difficult to be satisfied. In particular, for the underdetermined case, i.e., the number of sensors is less than the number of source signals, which is a more challenging problem. For this reason, underdetermined BSS faces three challenging issues. The first is how to detect the number of source signals, the second is how to estimate the mixing matrix correctly, and the third is how to separate the source signals.

To solve the source number estimation problem, the classical method is based on information theoretic criteria for model selection [3]. For example, the Parallel Factor Analysis (PARAFAC) model order selection [4], [5], higher-order tensors method [6]. He *et al.* [7] proposed a method to detect the number of clusters in N-way probabilistic clustering, as well as the automatic order selection [8]. Chen *et al.* [9] developed a new source counting algorithm based on sparse modeling of direction-of-arrival histogram. However, these methods are prone to underestimate or overestimate the number of source signals, leading to error source number estimation.

To estimate precisely the mixing matrix, some clustering algorithms have been developed. For example, K-means clustering algorithm [10], which is easy to use but sensitive

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to the initialization. Zhen *et al.* [11] proposed a novel hierarchical clustering algorithm to estimate the mixture time-frequency (TF) vectors at signal source points based on sparse cording and obtain an accurate mixing matrix estimation. However, it is difficult to estimate the mixing matrix when the source signals are insufficiently sparse. Zhou *et al.* [12] proposed a mixing matrix estimation algorithm from sparse mixtures without knowing the number of source signals, and it can perform well even if the source signals are of quite less sparsity. Sun *et al.* [13] proposed a novel approach to enhance the estimation accuracy of the mixing matrix by using the density-based spatial clustering of application with noise and the Hough transform. In addition, the simultaneous matrix diagonalization-based method is also an efficient algorithm to estimate the mixing matrix for the underdetermined case [14], [15]. The PARAFAC algorithms are combined with a dimensionality reduction step so that the computational complexity is reduced [16]–[18]. However, it takes more time consumption.

To separate the source signals, many traditional methods rely on some hypothesis about the origin sources. For instance, exploiting the assumption that the source signals are mutually independent, i.e., independent component analysis (ICA) methods [19]. Non-negative matrix factorization (NMF) is a well-known matrix decomposition approach [20], [21], which has been employed for signal processing [22]–[24]. However, the initialization is an essential part for the signal separation since NMF is very sensitive to the initialization. In recent years, the sparsity assumption of the source signals is largely utilized for source separation problem. Sparse representation has been proved to be a useful tool in many application, such as deep learning [25], speech signal processing [26], especially in the BSS [27], [28]. Xie *et al.* [29] proposed a new UBSS algorithm with free active sources based on Wigner-Ville distribution and Khatri-Rao product, which further relax the sparsity constraint. A novel robust sparse BSS has been proposed to retrieve spares sources in the presence of outliers [30]. In addition, Orthogonal Matching Pursuit (OMP) is a sparse approximation algorithm to handle the source signal recovery problem [31], [32]. More and more attentions have been paid to l_1-regularization methods for the sparse signal reconstruction [33]. However, the objective function in the l_1-regularization least-squares programs is convex but not differentiable, leading to a computational challenge. A fast and efficient algorithm has been proposed to learn an overcomplete dictionary for the sparse representation of signals [34]. Meanwhile, to obtain better sparse solutions, Li *et al.* [35] presents a novel and efficient method for a manifold optimization-based analysis dictionary learning with the $l_{1/2}$ as a regularizer such that the solutions can give sparse results than the l_1-norm method. But how to choose a suitable sparsity constraint to obtain the sparse solutions, and how to avoid trivial solutions for the analysis dictionary are two challenge problem in the analysis dictionary learning.

In this paper, a novel UBSS algorithm is proposed to separate the mixed heart sound signals by using higher-order statistics and sparse representation. First of all, to detect the number of source signals, a Second ORder sTatistic of the Eigenvalues (SORTE) method is presented to search the significant gap in the ordered eigenvalues of the covariance matrix, which is easy to implement and can work well for the underdetermined case. Then, using the decomposition of a higher-order symmetric tensor, a higher-order cumulant-based approach is employed to estimate the mixing matrix in the underdetermined mixture case. Meanwhile, by determining the maximum number of terms, the uniqueness of the estimated mixing matrix is proved. At the signal separation stage, an l_1-norm-like diversity measure is used as a measure of sparsity for the sparse signal reconstruction. Additionally, the pre-conditioned conjugate gradient technology is used to pre-process the estimated mixing matrix. The pre-processing is based on the eigenvalue decomposition (EVD) of matrix, which can reduce the computational load of the l_1-norm regularized method. Furthermore, the source signals are estimated using the sparse algorithm.

The main contributions of this paper are summarized as follows:

- We design a gap-based detection method to detect the number of source signals by eigenvalue decomposition, which can work well to detect the source signal number for the underdetermined case, avoiding any prior knowledge on the source number.
- We employ the higher-order tensor blind identification of underdetermined mixture approach to estimate the mixing matrix. Meanwhile, we determine the maximum number of source signals to ensure that the uniqueness of the estimated mixing matrix is guaranteed.
- We propose an improved l_1-norm regularized algorithm to estimate the source signals. Meanwhile, the pre-conditioned conjugate gradient technology is used to compute the search step such that the convergence rate is accelerated. Additionally, it is conducive to reduce the computational cost.

The public heart sound signal data are used in the simulation. Experimental results show the effectiveness and competitiveness of the proposed algorithm.

The structure of the remaining of this paper is organized as follows. The model description and outline of the proposed method will be demonstrated in Section II. In Section III, the proposed UBSS algorithm will be given including the source number estimation and the mixing matrix estimation. Experiment results and analysis will be shown in Section IV. Finally, the conclusion and future work will be drawn in Section V. In addition, some meaning of notations used in this paper are demonstrated in Table 1.

II. PROBLEM FORMULATION

A. MODEL DESCRIPTION

we consider the linear instantaneous mixture model:

$$\mathbf{x} = \mathbf{H}\mathbf{s} \tag{1}$$

TABLE 1. Meaning of notations.

Notation	Meaning
$(\cdot)^T$	Transpose
$(\cdot)^*$	Complex conjugate
$(\cdot)^H$	Complex conjugate transpose
$\|\cdot\|_2$	l_2 norm
$\|\cdot\|_F$	Frobenius norm
$E[\cdot]$	Expectation operator
vec$(\cdot)$	Vectorization
unvec$(\cdot)$	Inverse of vectorization
diag$(\cdot)$	Diagonal elements
$\odot$	The Khatri-Rao product
$*$	The Hadarmard product
$\otimes$	The Kronecker product
R	Number of source signals
M	Number of sensors
N	Data length

where $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_M]^T \in \mathbb{R}^{M\times N}$ is the observed mixture signals, $\mathbf{s} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_R]^T \in \mathbb{R}^{R\times N}$ denotes the source matrix of R sparse sources or hidden sparse components, and $\mathbf{H} \in \mathbb{R}^{M\times R}$ represents the unknown mixing matrix. The linear instantaneous mixture model (1) is an underdetermined case, i.e., $M < R$. The main goal of this paper is to separate the sources $\mathbf{s}$ from the observed mixture signals $\mathbf{x}$ without prior information on the mixing matrix $\mathbf{H}$.

B. OUTLINE

In this paper, a new UBSS algorithm is proposed using the higher-order statistics and sparse representation. The block diagram of the UBSS algorithm is given in Figure 1. First, the number of source signals is estimated based on the observed mixture signals. Then, the mixing matrix is estimated using higher-order statistics. In the source separation stage, the estimated mixing matrix is pre-processed by adding a pre-conditioner. The function of the pre-conditioner is to accelerate the convergence rate. Finally, the source signals are estimated using sparse representation. Detailed descriptions of the proposed algorithm will be given in the following section.

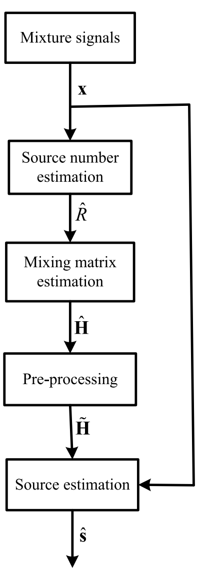


FIGURE 1. Block diagram of the proposed UBSS algorithm.

III. PROPOSED UNDERDETERMINED BSS ALGORITHM

A. SOURCE NUMBER ESTIMATION BASED ON SORTE

First of all, using the observed data matrix $\mathbf{x} \in \mathbb{R}^{M\times N}$, we consider the quadri-covariance $\mathcal{Q}^{\mathbf{x}} = \mathrm{Cum}\{\mathbf{x}, \mathbf{x}^*, \mathbf{x}^*, \mathbf{x}\} \in \mathbb{R}^{M\times M\times M\times M}$. According to the multi-linearity property of cumulant tensors, we obtain

$$\mathcal{Q}^{\mathbf{x}}_{ijkl} = \sum_{r=1}^{R} \kappa_r h_{ir} h^*_{jr} h^*_{kr} h_{lr} \tag{2}$$

where κ_r is the kurtosis of the rth source, defined as [36]

$$\kappa_r = \frac{E(\mathbf{x}_r - \mu)^4}{(E(\mathbf{x}_r - \mu)^2)^2} = \frac{\mu_4}{\sigma^4} \tag{3}$$

where $\mathbf{x}_r$ denotes the rth observed vector, μ is the mean, μ_4 is the fourth moment about the mean, and σ is the standard deviation.

Then, in term of matrix representation of tensor, (2) can be expressed as

$$\mathbf{Q} = \mathrm{mat}(\mathcal{Q}^{\mathbf{x}}_{ijkl}) = (\mathbf{H} \odot \mathbf{H}^*)\mathbf{K}(\mathbf{H} \odot \mathbf{H}^*)^H \tag{4}$$

where $\mathbf{Q} \in \mathbb{R}^{M^2\times M^2}$, and $\mathbf{K} = \mathrm{diag}(\kappa_1, \ldots, \kappa_I) \in \mathbb{R}^{R\times R}$.

Perform EVD of matrix $\mathbf{Q}\mathbf{Q}^T$ as follows:

$$\mathrm{EVD}\left(\frac{1}{M^2}\mathbf{Q}\mathbf{Q}^T\right) = \mathbf{U}\Lambda\mathbf{U}^T \tag{5}$$

where $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_{M^2})$. Suppose that the M^2 eigenvalues are sorted to be $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{M^2}$. The $M^2 - R$ smallest eigenvalues are all equal to σ^2, i.e.,

$$\lambda_1 \geq \cdots \geq \lambda_R > \lambda_{R+1} = \ldots = \lambda_{M^2} = \sigma^2 \tag{6}$$

where σ^2 is an unknown scalar constant. If λ_R is significantly larger than λ_{R+1}, there will exist a gap between λ_R and λ_{R+1}. Thus, we compute the differences of eigenvalues

$$\Delta\lambda_m = \lambda_m - \lambda_{m+1} \tag{7}$$

where $m = 1, 2, \ldots, M^2 - 1$. Based on (6), we can obtain

$$\Delta\lambda_{R+1} = \Delta\lambda_{R+2} = \cdots = \Delta\lambda_{M^2-1} \tag{8}$$

To detect the cluster gap, the variance of the sequence $\{\Delta\lambda_m\}_{m=r}^{M^2-1}$ is computed as follows:

$$\sigma_r^2 = \frac{1}{M^2 - r} \sum_{m=r}^{M^2-1} \left(\Delta\lambda_m - \frac{1}{M^2 - r}\Delta\lambda_m\right)^2 \tag{9}$$

where $r = 1, 2, \ldots, M^2 - 1$. From (8) and (9), we have

$$\begin{cases} \sigma_r^2 > 0, & r = 1, \ldots, R, \\ \sigma_r^2 = 0, & r = R+1, \ldots, M^2 - 1 \end{cases} \tag{10}$$

Then we define a SORTE as follows:

$$\mathrm{SORTE}(r) = \begin{cases} \frac{\sigma_{r+1}^2}{\sigma_r^2}, & \sigma_r^2 > 0, \\ +\infty, & \sigma_r^2 = 0 \end{cases} \tag{11}$$

where $r = 1, 2, \ldots, M^2 - 2$. Then, we obtain

$$\begin{cases} \mathrm{SORTE}(r) > 0, & r = 1, \ldots, R-1, \\ \mathrm{SORTE}(r) = 0, & r = R. \\ \mathrm{SORTE}(r) = +\infty, & r = R+1, \ldots, M^2-3, \\ \mathrm{SORTE}(r) = 0, & r = M^2-2. \end{cases} \tag{12}$$

Thus, we can perform the source number estimation criterion:

$$\hat{R} = \arg \min_{r=1,\ldots,M^2-3} \mathrm{SORTE}(r). \tag{13}$$

B. MIXING MATRIX ESTIMATION USING HIGHER-ORDER STATISTICS

In the following, we estimate the mixing matrix $\mathbf{H}$. The higher-order statistics is a new attractive blind identification method to solve the underdetermined mixture problem. The mixing matrix $\mathbf{H}$ can be estimated by exploiting the information contained in the higher-order data statistics. The detailed process is explained as follows:

Denote $u_r = \mathrm{vec}(\mathbf{U}_r)$, $r = 1, \ldots, R$, $\bar{\mathbf{U}} = (u_1, \ldots, u_R) \in \mathbb{R}^{M^2 \times R}$, and $\bar{\Lambda} = \mathrm{diag}(\lambda_1, \ldots, \lambda_R) \in \mathbb{R}^{R \times R}$. Thus, (4) is equivalent to

$$(\mathbf{H} \odot \mathbf{H}^*)\mathbf{K}(\mathbf{H} \odot \mathbf{H}^*)^H = \bar{\mathbf{U}} \cdot \bar{\Lambda} \cdot \bar{\mathbf{U}}^H \tag{14}$$

Then we have

$$(\mathbf{H} \odot \mathbf{H}^*)\mathbf{K}^{\frac{1}{2}} = \bar{\mathbf{U}} \cdot \bar{\Lambda}^{\frac{1}{2}} \cdot \mathbf{A} \tag{15}$$

where $\mathbf{A}$ is a real orthogonal matrix. $\bar{\mathbf{U}}$ and $\bar{\Lambda}^{\frac{1}{2}}$ can be computed based on (5). To compute the matrix $\mathbf{A}$, we define $\mathbf{B} = \bar{\mathbf{U}} \cdot \bar{\Lambda}^{\frac{1}{2}}$, and $\mathbf{B}_s = \mathrm{unvec}(\mathbf{b}_s)$.

Consider the mapping: $\Phi : (\mathbf{X}, \mathbf{Y}) \in \mathbb{R}^{M \times M} \times \mathbb{R}^{M \times M} \longmapsto \Phi(\mathbf{X}, \mathbf{Y}) \in \mathbb{R}^{M \times M \times M \times M}$ defined by

$$\Phi(\mathbf{X}, \mathbf{Y})_{ijkl} = x_{ij}y_{kl} + y_{ij}x_{kl} - x_{il}y_{kj} - y_{il}x_{kj} \tag{16}$$

Using the bilinearly of Φ, it is found that

$$\sum_{s,t=1}^{R} (\mathbf{W}_r)_{st} \Phi(\mathbf{B}_s, \mathbf{B}_t) = 0 \tag{17}$$

Thus, given $\Phi[\mathbf{B}_s, \mathbf{B}_t]$, $1 \leq s, t \leq R$, R linearly independent real symmetric matrices $\mathbf{W}_r \in \mathbb{R}^{R \times R}$ can be computed based on a set of linear equations. The detailed calculation process of $\mathbf{W}_r$ is summarized as follows:

Due to the symmetry of $\mathbf{W}_r$, and $\Phi(\mathbf{B}_s, \mathbf{B}_t) = \Phi(\mathbf{B}_t, \mathbf{B}_s)$, (17) can be rewritten as

$$\sum_{s=1}^{R} (\mathbf{W}_r)_{ss} \Phi(\mathbf{B}_s, \mathbf{B}_s) + 2 \sum_{\substack{s,t=1 \\ s<t}}^{R} (\mathbf{W}_r)_{st} \Phi(\mathbf{B}_s, \mathbf{B}_t) = 0 \tag{18}$$

Thus, we obtain

$$\mathbf{C} \cdot \begin{bmatrix} (\mathbf{W}_r)_{1,1}, \ldots, (\mathbf{W}_r)_{R,R} \\ 2(\mathbf{W}_r)_{1,2}, 2(\mathbf{W}_r)_{1,3}, \ldots, 2(\mathbf{W}_r)_{R-1,R} \end{bmatrix} = 0 \tag{19}$$

$$\mathbf{C} = \begin{bmatrix} \mathrm{vec}(\Phi(\mathbf{B}_1, \mathbf{B}_1)), \ldots, \mathrm{vec}(\Phi(\mathbf{B}_R, \mathbf{B}_R)) \\ \mathrm{vec}(\Phi(\mathbf{B}_1, \mathbf{B}_2)), \ldots, \mathrm{vec}(\Phi(\mathbf{B}_{R-1}, \mathbf{B}_R)) \end{bmatrix}^T \tag{20}$$

Then, compute the singular value decomposition (SVD) of $\mathbf{C}$, and obtain the R right singular vectors $\mathbf{c}_r, r = 1, \ldots, R$. Stack these vectors in upper triangular matrix $\mathbf{C}_r \in \mathbb{R}^{R \times R}$. Therefore, the matrix $\mathbf{W}_r$ can be computed as follows:

$$\mathbf{W}_r = \frac{1}{2}(\mathbf{C}_r + \mathbf{C}_r^T) \tag{21}$$

Thus, we find $\mathbf{A}$ as a common eigenmatrix that jointly diagonalizes each of these matrices $\mathbf{W}_r$ by congruence, i.e.,

$$\begin{cases} \mathbf{W}_1 & = \mathbf{A} \cdot \mathbf{D}_1 \cdot \mathbf{A}^T \\ & \vdots \\ \mathbf{W}_R & = \mathbf{A} \cdot \mathbf{D}_R \cdot \mathbf{A}^T \end{cases} \tag{22}$$

where $\mathbf{D}_1, \ldots, \mathbf{D}_R \in \mathbb{R}^{R \times R}$ are any diagonal matrices.

So, we can obtain

$$\mathbf{E} = \bar{\mathbf{U}} \cdot \bar{\Lambda}^{\frac{1}{2}} \cdot \mathbf{A} \tag{23}$$

Thus, the mixing matrix $\mathbf{H}$ can be estimated using (15) and (23), i.e.,

$$(\mathbf{H} \odot \mathbf{H}^*)\mathbf{K}^{\frac{1}{2}} = \mathbf{E} \tag{24}$$

Then, compute the SVD of $\mathbf{E}$, we can estimate the mixing vector $\mathbf{h}_r$ as the left singular vector of $\mathrm{unvec}(\mathbf{e}_r)$, $r = 1, \ldots, R$. In this case, the estimated mixing matrix $\hat{\mathbf{H}}$ is obtained.

In addition, the uniqueness of the estimated mixing matrix is proved in an underdetermined case. For any given sensor number M, the upper bound of source number R need to satisfy

$$\frac{R(R-1)}{2} \leq \frac{M(M-1)}{4}\left(\frac{M(M-1)}{2} + 1\right) - \frac{M!}{(M-4)!4!} 1_{\{M \geq 4\}} \tag{25}$$

where

$$1_{\{M \geq 4\}} = \begin{cases} 0, & \text{if} \quad M < 4 \\ 1, & \text{if} \quad M \geq 4 \end{cases}$$

For example, for several different values of M, the maximum value of R is shown in Table 2.

TABLE 2. The upper bound of source number.

M	2	3	4	5	6	7	8	9	10
R_{max}	2	4	6	10	15	20	26	33	41

C. SOURCE SIGNAL SEPARATION USING SPARSE REPRESENTATION

Using the estimated mixing matrix $\hat{\mathbf{H}}$, we can separate the source signals using sparse representation. First of all, the pre-conditioned conjugate gradient (PCG) method is employed. By computing the EVD on matrix $\hat{\mathbf{H}}\hat{\mathbf{H}}^T$,

$$\hat{\mathbf{H}}\hat{\mathbf{H}}^T = \mathbf{F}\tilde{\Lambda}\mathbf{F} \tag{26}$$

Then, multiplying the matrix $\tilde{\Lambda}^{-1}\mathbf{F}^{-1}$ on both of the mixture model (1), we can obtain

$$\tilde{\mathbf{x}} = \tilde{\mathbf{H}}\mathbf{s} \tag{27}$$

where $\tilde{\mathbf{x}} = \tilde{\Lambda}^{-1}\mathbf{F}^{-1}\mathbf{x}$ and $\tilde{\mathbf{H}} = \tilde{\Lambda}^{-1}\mathbf{F}^{-1}\mathbf{H}$.

Then the sparse representation for model (27) can be cast into the following optimization problem:

$$\begin{cases} \min_{\mathbf{s}} \mathbf{J}(\mathbf{s}) = \sum_{r=1}^{R} |\mathbf{s}_r| \\ \text{subject to} : \tilde{\mathbf{H}}\mathbf{s} = \tilde{\mathbf{x}} \end{cases} \tag{28}$$

Define the Lagrange function:

$$L(\mathbf{s}, \alpha) = \mathbf{J}(\mathbf{s}) + \alpha^T(\tilde{\mathbf{H}}\mathbf{s} - \tilde{\mathbf{x}}) \tag{29}$$

where α is a Lagrange multiplier. A necessary condition for a sparse solution $\mathbf{s}^*$ to exist is that $(\mathbf{s}^*, \alpha^*)$ be stationary points of the Lagrange function $L(\mathbf{s}, \alpha)$, i.e.,

$$\begin{cases} \dfrac{\partial L(\mathbf{s}^*, \alpha^*)}{\partial \mathbf{s}^*} = \dfrac{\partial \mathbf{J}(\mathbf{s}^\star)}{\partial \mathbf{s}^*} + \tilde{\mathbf{H}}^T\alpha^* = 0 \\ \dfrac{\partial L(\mathbf{s}^*, \alpha^*)}{\partial \alpha^*} = \tilde{\mathbf{H}}\mathbf{s}^* - \tilde{\mathbf{x}} = 0 \end{cases} \tag{30}$$

where

$$\frac{\partial \mathbf{J}(\mathbf{s}^\star)}{\partial \mathbf{s}^*} = \begin{bmatrix} \partial|\mathbf{s}_1^*| \\ \vdots \\ \partial|\mathbf{s}_R^*| \end{bmatrix} = \Pi(\mathbf{s}^*)\mathbf{s}^* \tag{31}$$

in which

$$\begin{cases} \partial|\mathbf{s}_r^*| = 1, & \mathbf{s}_r^* > 0 \\ -1 \le \partial|\mathbf{s}_r^*| \le 1, & \mathbf{s}_r^* = 0 \\ \partial|\mathbf{s}_r^*| = -1, & \mathbf{s}_r^* < 0 \end{cases} \tag{32}$$

$\partial|\mathbf{s}_r^*|$ denotes the subderivative of the function $f(\mathbf{s}_r^*) = |\mathbf{s}_r^*|$, $r = 1, \cdots, R$, and

$$\Pi(\mathbf{s}^*) = \begin{pmatrix} |\mathbf{s}_1^*|^{-1} & 0 & \cdots & 0 \\ 0 & |\mathbf{s}_2^*|^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & |\mathbf{s}_R^*|^{-1} \end{pmatrix} \tag{33}$$

Then, based on (30) and (31),

$$\Pi(\mathbf{s}^*)\mathbf{s}^* + \tilde{\mathbf{H}}^T\alpha^* = 0 \tag{34}$$

we obtain

$$\mathbf{s}^* = -\Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T \cdot \alpha^* \tag{35}$$

Submit (35) to (27),

$$\tilde{\mathbf{x}} = -\tilde{\mathbf{H}} \cdot \Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T \cdot \alpha^* \tag{36}$$

Thus

$$\alpha^* = -(\tilde{\mathbf{H}} \cdot \Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T)^{-1} \cdot \tilde{\mathbf{x}} \tag{37}$$

Then, submit (37) to (35), we obtain

$$\mathbf{s}^* = \Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T \cdot [\tilde{\mathbf{H}} \cdot \Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T]^{-1} \cdot \tilde{\mathbf{x}} \tag{38}$$

Define

$$\mathbf{v}_r(\mathbf{s}^*) \doteq \mathbf{h}_r^T \cdot [\tilde{\mathbf{H}} \cdot \Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T] \cdot \tilde{\mathbf{x}} \tag{39}$$

we have

$$\mathbf{V}(\mathbf{s}^*) = \begin{bmatrix} \mathbf{v}_1(\mathbf{s}^*) \\ \vdots \\ \mathbf{v}_R(\mathbf{s}^*) \end{bmatrix} = \tilde{\mathbf{H}}^T \cdot [\tilde{\mathbf{H}} \cdot \Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T]^{-1} \cdot \tilde{\mathbf{x}} \tag{40}$$

According to (38) to (40), we obtain

$$\begin{aligned} \mathbf{s}^* &= \Pi(\mathbf{s}^*)^{-1} \cdot \mathbf{V}(\mathbf{s}^*) \\ &= \begin{bmatrix} |\mathbf{s}_1^*| & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & |\mathbf{s}_R^*| \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_1(\mathbf{s}^*) \\ \vdots \\ \mathbf{v}_R(\mathbf{s}^*) \end{bmatrix} \\ &= \begin{bmatrix} |\mathbf{s}_1^*| \cdot \mathbf{v}_1(\mathbf{s}^*) \\ \vdots \\ |\mathbf{s}_R^*| \cdot \mathbf{v}_R(\mathbf{s}^*) \end{bmatrix} \end{aligned} \tag{41}$$

Meanwhile, for obtaining the sparse solution of (27), we have the following iterative formula:

$$\mathbf{s}^{k+1} = \left[|\mathbf{s}_1^k| \cdot \mathbf{v}_1(\mathbf{s}^k), \quad \cdots \quad, |\mathbf{s}_R^k| \cdot \mathbf{v}_R(\mathbf{s}^k) \right]^T \tag{42}$$

Therefore, the proposed UBSS algorithm is summarized in Algorithm 1.

Algorithm 1 Proposed UBSS Algorithm for Mixture Heart Sound Signals

Input: Mixture heart sound signals $\mathbf{x}(t) \in \mathbb{R}^{M \times N}$.
1: Compute the quadricovariance $\mathcal{Q}^{\mathbf{x}}$ of the matrix $\mathbf{x}(t)$ and obtain $\mathbf{Q} = \text{mat}(\mathcal{Q}_{ijkl}^{\mathbf{x}})$,
2: Perform EVD of matrix $\mathbf{Q}\mathbf{Q}^T$ using (5) and estimate the source number based on (13),
3: Estimate the mixing vector $\mathbf{h}_r$ as the left singular vector of unvec($\mathbf{e}_r$), $r = 1, \ldots, R$ based on (14)–(24),
4: Estimate the source signals $\mathbf{s}$ using (41), meanwhile update $\mathbf{s}$ based on (42) until the convergence.
Output: Estimated heart sound source signals $\hat{\mathbf{s}}$.

IV. SIMULATIONS

In the following, we test a series of simulation experiments to evaluate the separation performance of our proposed algorithm. The experiment datasets come from public heart and lung sounds data sets.[1] The first experiment is considered to separate a set of heart sound mixture signals. In the second experiment, we test two sets of heart sound mixture signals in the underdetermined case. Finally, we investigate the effectiveness and feasibility of the proposed algorithm in the task of image restoration.

A. PERFORMANCE CRITERIA

For evaluating the source signal separation performance, we select the signal-to-distortion ratio (SDR), signal-to-interference ratio (SIR), and signal-to-artifact ratio (SAR) as the evaluation criteria, which are defined in [37]. The larger they are, the better their performance will be.

For measuring the quality of the estimated mixing matrix, we consider the mixing error ratio (MER) as the evaluation, which is defined in [38]. The estimated r-th column vector of mixing matrix $\mathbf{H}$ can be decomposed as

$$\hat{\mathbf{h}}_r = \mathbf{h}_r^{\text{coll}} + \mathbf{h}_r^{\text{orth}} \tag{43}$$

where $\mathbf{h}_r^{\text{coll}}$ represents the collinear component of $\mathbf{h}_r$, and $\mathbf{h}_r^{\text{orth}}$ represents the orthogonal component of $\mathbf{h}_r$. Thus, the r-th MER is defined as

$$\text{MER}_r = 10 \log_{10} \frac{\| \mathbf{h}_r^{\text{coll}} \|_2^2}{\| \mathbf{h}_r^{\text{orth}} \|_2^2} \tag{44}$$

In addition, the average mean square error (MSE) is also used as the performance measure, which is defined as [39]

$$\text{MSE} = \min_{\substack{\pi \in \Pi \\ c_1, \ldots, c_R \in \{\pm 1\}}} \frac{1}{R} \sum_{r=1}^{R} \left\| \frac{h_r}{\|h_r\|_2} - c_r \frac{\hat{h}_{\pi_r}}{\|\hat{h}_{\pi_r}\|_2} \right\|_2^2 \tag{45}$$

where Π is the set of all permutations of $\{1, 2, \ldots, R\}$, h_r and $\hat{h}_r$ are the ground truth of the kth column of the mixing matrix and the corresponding estimated mixing matrix, respectively.

B. SEPARATION OF TWO HEART SOUND MIXTURE SIGNALS

In the first experiment, we test a set of heart sound mixing signals. The time of heart sound is 10 second, and the sampling frequency is 2000 Hz. The mixing matrix is randomly selected as follows:

$$\mathbf{H} = \begin{pmatrix} 0.9085 & -0.2391 \\ -2.2207 & 0.0687 \end{pmatrix} \tag{46}$$

To evaluate the mixing matrix estimation performance, the PARAFAC algorithm [16] is compared with the proposed algorithm. The MER and MSE of the algorithms is shown in Table 3. As shown, the average MER improves 31.6630 dB, and the MSE performance of the proposed algorithm is better than the PARAFAC algorithm.

[1] https://www.welchallyn.com/en.html

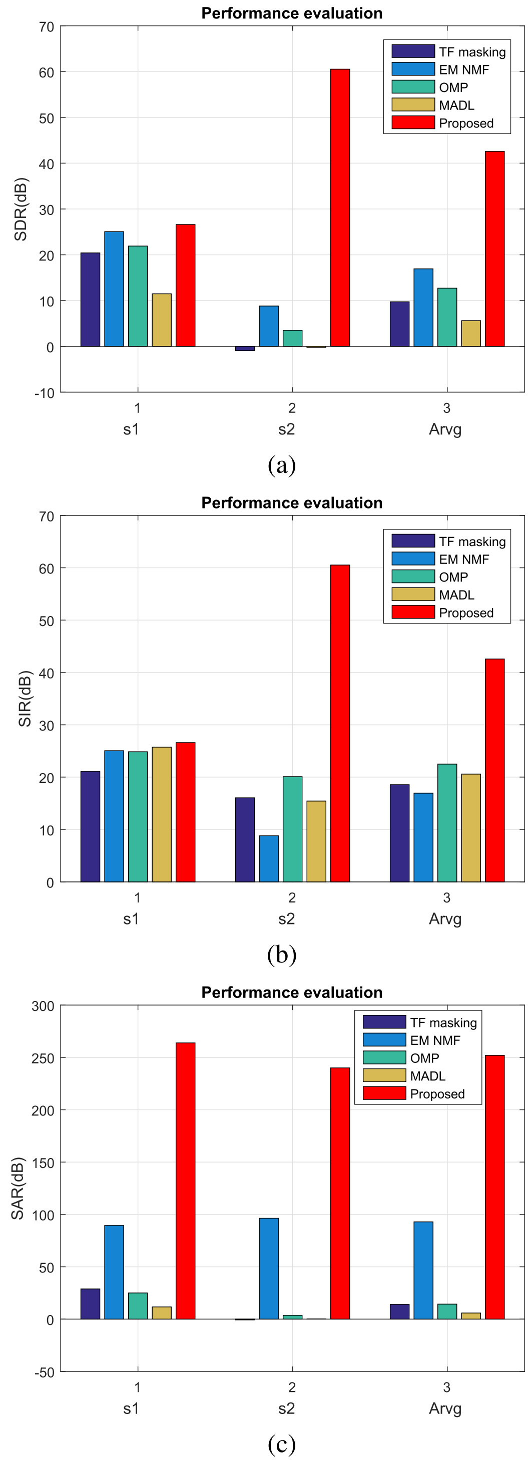


FIGURE 2. BSS performance over heart sound signal mixtures, measured in terms of (a) SDR, (b) SIR, (c) SAR.

TABLE 3. MER and MSE for two heart sound signals.

Algorithm	MER			MSE
	s_1	s_2	Avrg	
PARAFAC	22.3938	-0.1742	11.1098	0.3029
Proposed	**81.7361**	**3.8095**	**42.7728**	**0.1596**

In the stage of source separation, to better illustrate the superiority of our proposed algorithm, we compare with the TF masking algorithm [40], EM NMF algorithms [41],

TABLE 4. Source separation results for heart sound mixing signals in the underdetermined case (all units are dB).

Algorithm	SDR					SIR					SAR				
	s_1	s_2	s_3	s_4	Avrg	s_1	s_2	s_3	s_4	Avrg	s_1	s_2	s_3	s_4	Avrg
l_0-norm min [42]	0.04	5.58	8.64	11.80	6.51	5.89	8.65	9.96	12.41	9.23	2.34	9.08	14.89	20.86	11.79
TF masking [40]	-0.50	-1.97	3.33	3.42	1.06	2.39	7.66	8.44	10.83	7.33	4.59	-0.78	5.51	4.63	3.48
OMP [31]	4.05	4.33	5.94	9.06	5.85	11.23	13.58	12.51	13.50	12.71	5.29	4.91	6.74	9.64	6.65
MADL [35]	4.63	6.41	7.62	11.4	7.53	10.62	12.57	12.28	14.04	12.63	5.40	9.85	9.20	12.17	9.15
Proposed	**5.17**	**9.59**	**11.36**	**13.61**	**9.93**	**12.28**	**14.09**	**12.86**	**14.15**	**13.35**	**6.36**	**11.66**	**16.95**	**23.06**	**14.51**

orthogonal matching pursuit (OMP) algorithm [31], and manifold analysis dictionary learning (MADL) [35]. The separation results are demonstrated in Figure 2. It can be seen that the SDR, SIR, and SAR of the proposed algorithm are better than the compared algorithms, and the average SDR, SIR, SAR improvements of 29.8604 dB, 20.0844 dB, 237.5788 dB over the best current results, respectively.

In addition, to better demonstrate the separation results, we compare the spectrums of the source signals and the separated signals. The original heart sound signals along with their spectrograms are shown in Figure 3(a), the waveforms of mixture heart sound signals are shown in Figure 3(b), and the reconstructed heart sound signals along with their spectrograms are shown in Figure 3(c). According to Figure 3(a) and Figure 3(c), it is shown that the mixture heart sound signals are separated well.

C. SEPARATION OF HEART SOUND MIXTURE SIGNALS IN THE UNDERDETERMINED CASE

In the second experiment, we consider a set of heart sound mixing signals, which is consisted of three sensors and four heart sound signals. The mixing matrix is randomly selected as follows:

$$\mathbf{H} = \begin{pmatrix} 0.9085 & 0.0687 & -0.0813 & 0.7366 \\ -2.2207 & -2.0202 & -1.9797 & 0.9553 \\ -0.2391 & -0.3641 & 0.7882 & 1.9295 \end{pmatrix} \quad (47)$$

First of all, the number of source signals is estimated using the minimum SORTE criterion. The estimated results is shown in Figure 4. The eigenvalues are sorted in descending order seen in Figure 4(a). By searching the minimum SORTE-value, it is seen that the estimated source number is $\hat{R} = 4$ in Figure 4(b). Therefore, the minimum SORTE criterion is effective to detect the number of source signals in the underdetermined case.

TABLE 5. MER and MSE of the algorithms for four heart sound signals.

Algorithm	MER					MSE
	s_1	s_2	s_3	s_4	Avrg	
PARAFAC	14.46	-1.91	16.16	9.91	9.55	0.2303
Proposed	**21.41**	**26.26**	**25.50**	**31.09**	**26.17**	**0.0032**

In Table 5, the MER and MSE performance of the proposed algorithm are comparable to the PARAFAR algorithm. The MESs of proposed algorithm are larger, and the average MES improves 16.62 dB. Meanwhile, the MSE of proposed algorithm is around 0.2271 dB lower than the PARAFAC algorithm. It is shown that the proposed algorithm obtains better estimation of the mixing matrix than the PARAFAC algorithm.

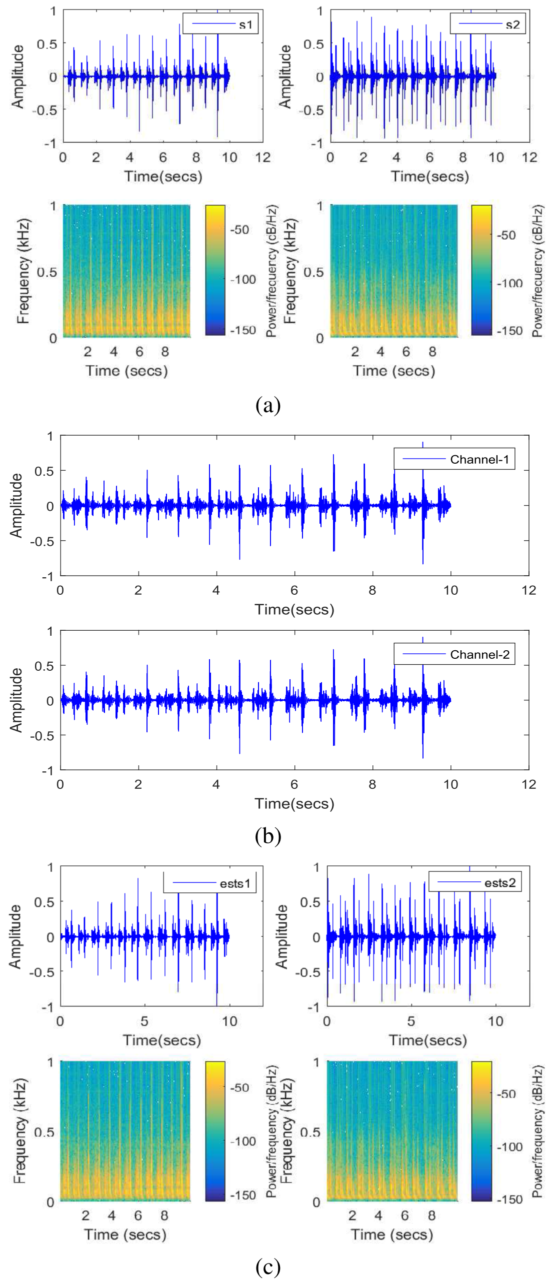


FIGURE 3. A typical 10s heart sound signal: (a) Original heart sound signals along with their spectrograms, (b) mixture heart sound signals with two channels, and (c) estimated heart sound signals along with their spectrograms.

Then, the source separation results are tabulated in Table 4. It is shown that the average SDR, SIR, and SAR of the

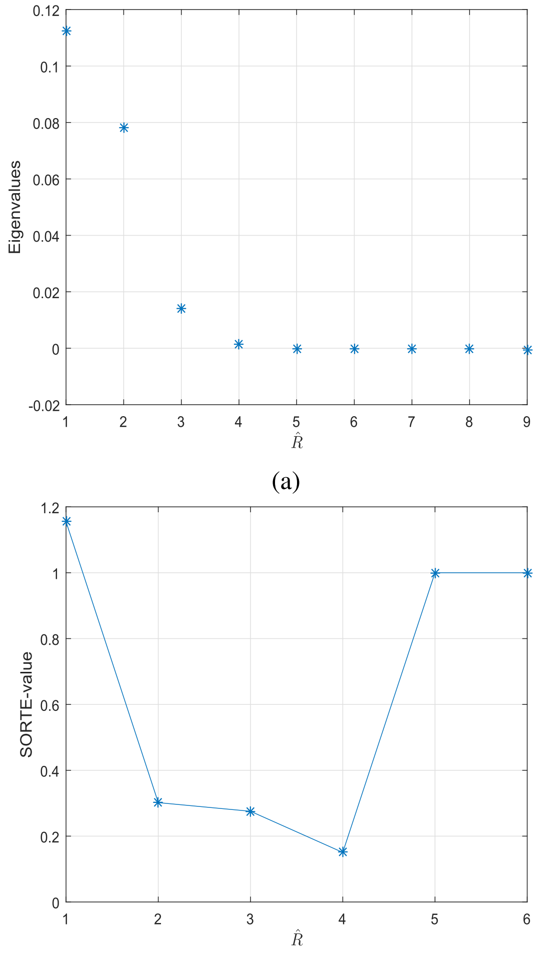


FIGURE 4. Source signal number estimation (a) the eigenvalues are sorted in descending order and (b) the SORTE cure.

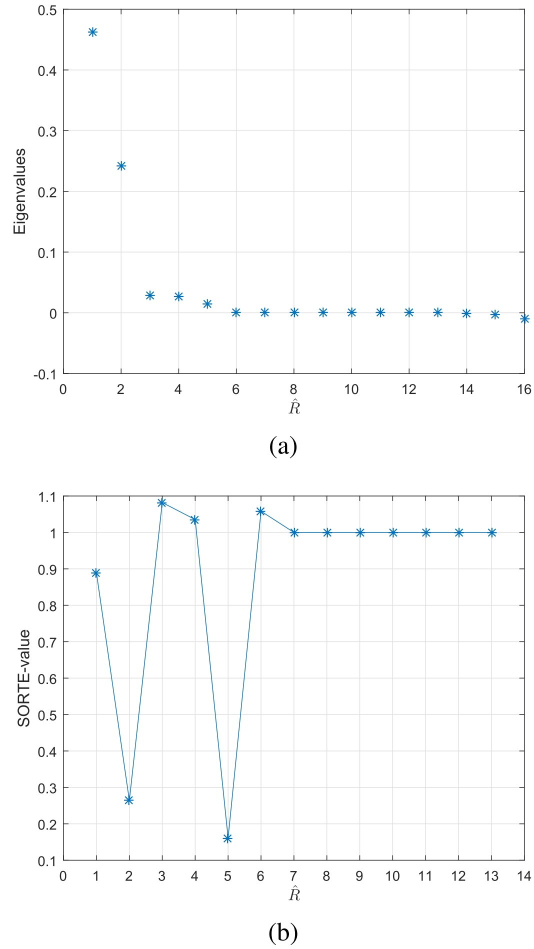


FIGURE 5. Source signal number estimation (a) the ordered eigenvalue sequence and (b) the SORTE sequence.

proposed algorithm are improved 2.40 dB, 0.64 dB, and 2.72 dB compared with the best current results, respectively. Therefore, the proposed algorithm obtains better separation performance than the compared methods.

In the third experiment, we test a set of underdetermined mixture signals, which is composed of four sensors and five heart sound signals. We randomly select the mixing matrix, which is expressed as follows:

$$\mathbf{H}=\begin{pmatrix} 0.9085 & -2.0202 & 0.7882 & -0.7453 & 0.6134 \\ -2.2207 & -0.3641 & 0.7366 & -0.8984 & 1.0446 \\ -0.2391 & -0.0813 & 0.9553 & -3.2625 & -0.8073 \\ 0.0687 & -1.9797 & 1.9295 & -0.0300 & 0.2059 \end{pmatrix} \tag{48}$$

First, the source number estimation is shown in Figure 5. The eigenvalues are sorted in descending order seen in Figure 5(a). By searching the minimum SORTE-value, it is clear that the estimated source number is $\hat{R} = 5$ in Figure 5(b).

In Table 6, we show the MER and MSE of the algorithms for five heart sound signals. The average MES of proposed algorithm improves 21.4 dB compared with the PARAFAC algorithm. Meanwhile, the MSE of proposed algorithm is around 0.3140 dB lower than the PARAFAC algorithm. Thus, our proposed algorithm shows better performance for the estimation of the mixing matrix than the PARAFAC algorithm.

The source separation results are demonstrated in Figure 6. It can be shown that the proposed algorithm has superior separation performance than the compared algorithms.

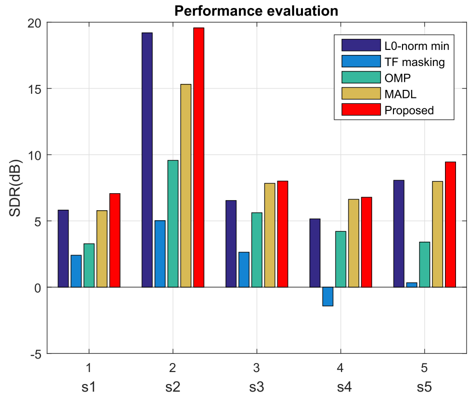
D. IMAGE RESTORATION

Finally, we consider image restoration as a further evaluation of the performance of the proposed algorithm. The peak signal-to-noise radio (PSNR) is used as the quality measures, which is defined as

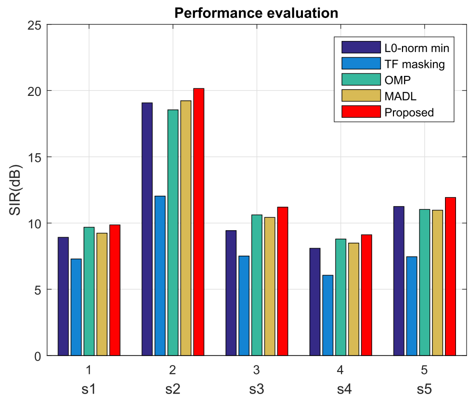
$$\mathrm{PSNR}(\mathbf{s}) = 10 \cdot \log_{10} \frac{255^2}{\|\mathbf{s}-\mathbf{s}_0\|_F^2} \tag{49}$$

where $\mathbf{s}$ is the recovered signal and $\mathbf{s}_0$ is the source signal.

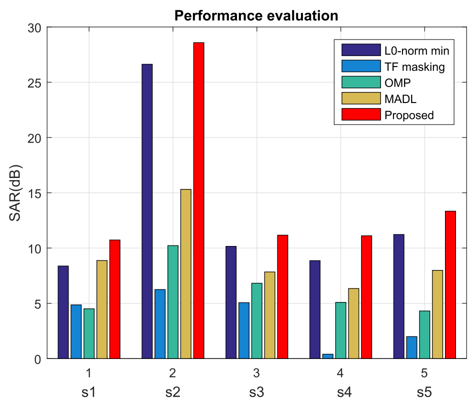
In the experiments, we consider three sets of test images including "Lena" (256 × 256), "Peppers" (256 × 256), "House" (256 × 256). The mixing matrix $\mathbf{H} \in \mathbb{R}^{190\times 256}$ is randomly generated. The PSNR values and runtime of all



(a)



(b)



(c)

FIGURE 6. BSS performance over heart sound signal mixtures, measured in terms of (a) SDR, (b) SIR, and (c) SAR.

TABLE 6. MER and MSE of the algorithms for five heart sound signals.

Algorithm	MER						MSE
	s_1	s_2	s_3	s_4	s_5	Avrg	
PARAFAC	-2.6	2.5	7.9	7.7	9.1	4.9	0.3194
Proposed	**18.7**	**33.8**	**35.5**	**21.8**	**21.8**	**26.3**	**0.0054**

algorithms are shown in Table 7. It is clear that the PSNR results of the proposed algorithm are improved well. Meanwhile, it is faster than the FOCUSS (focal underdetermined system solver) algorithm. Additionally, the image restoration results are shown in Figure 7. It is shown that our proposed algorithm obtain better image restoration than the typical algorithms.

TABLE 7. Peak signal-to-noise radio (dB) and runtime (sec.) for the image restoration.

Algorithm	PSNR			Runtime
	Lena	Peppers	House	
OMP [31]	30.7884	30.0573	31.6264	14.5003
FOCUSS [27]	18.7932	29.0586	31.2788	27.4243
Proposed	**35.3713**	**34.9902**	**36.7109**	**16.4040**

FIGURE 7. Restored images by different algorithms. Left-right: original images, restored images by OMP algorithm, restored images by FOCUSS algorithm, and restored images by proposed algorithm.

Discussion: We carefully consider the computational complexity of the proposed algorithm, one of the most time-consuming calculations is the matrix-matrix multiplication $\tilde{\mathbf{H}} \cdot \Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T$, whose computational complexity is $\mathcal{O}(M^3)$, which is controlled by the number of sensors M. For this reason, our proposed preconditioner is an identity matrix so that the matrix-matrix multiplication $\tilde{\mathbf{H}} \cdot \Pi(\mathbf{s}^*)^{-1} \cdot \tilde{\mathbf{H}}^T$ can be efficiently done by fast Fourier transform. In this case, the computational complexity can be reduced to $\mathcal{O}(M^2 \log M)$.

V. CONCLUSION AND FUTURE WORK

In this paper, a novel algorithm has been presented based on higher-order statistics and sparse representation to solve the UBSS problem. First, to detect the number of source signals, the minimum SORTE method was presented by searching the gap in the ordered eigenvalues of the covariance matrix, which worked well for the underdetermined case. Second, for the estimation of the mixing matrix, we employed the higher-order cumulant-based approach to obtain the unique estimated mixing matrix. Then, using the sparsity of source signals, an improved l_1-norm regularized method was developed based on sparse representation to obtain the sparse solutions. In the simulation experiments, we tested the heart sound mixture signals in the underdetermined case and the image restoration. Experimental results have demonstrated that the separation performance of the proposed algorithm is better than the state-of-the-art algorithms.

A quote from the French novelist Marcel Proust: "The voyage of discovery is not in seeking new landscapes but in having new eyes." In the future work, we hope that our proposed algorithm could be used to deal with the convolutive mixture signals. Especially, the blind separation of moving sources is a challenging issue. Additionally, it is a meaningful topic to perform the image processing using deep learning and sparse representations.

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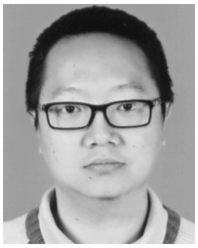

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