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Hybrid Reliability-Based Design Optimization of Complex Structures With Random and Interval Uncertainties Based on ASS-HRA

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ABSTRACT In this paper, an efficient hybrid reliability analysis (HRA) method and a hybrid reliability-based design optimization (HRBDO) approach are proposed for realistic complex engineering structures with random and interval uncertainties. First, the HRBDO model for complex engineering structures is constructed with its objective and performance functions described as the implicit functions of design variables and random and interval parameters. Then, an efficient HRA method based on adaptive step size (ASS-HRA) is put forward to calculate the minimum reliability of the structure's performance function under the influences of both random and interval uncertainties, the computational efficiency and accuracy of which are verified by a benchmark test. Subsequently, an efficient HRBDO approach integrating the proposed ASS-HRA method with the polynomial response surface model (PRSM) is developed for solving the HRBDO problems of complex engineering structures, the effectiveness of which is demonstrated by a numerical example. Finally, the HRBDO of a high-speed press slider demonstrates the efficiency, effectiveness, and versatility of the proposed HRBDO approach based on the ASS-HRA in the design of realistic complex engineering structures.

INDEX TERMS Hybrid reliability analysis (HRA), adaptive step size (ASS), hybrid reliability-based design optimization (HRBDO), random and interval uncertainties, complex structure.

I. INTRODUCTION

Uncertain factors in material properties, load conditions, geometrical dimensions, and so on are unavoidable in practical engineering. These uncertain factors must be taken into consideration in the design process of practical engineering structures since they will result in the fluctuations of the mechanical properties of structures [1]–[4]. Reliability-based design optimization (RBDO) is regarded as one of the most powerful non-deterministic optimization methods [5]–[8] which simultaneously considers both the performance and reliability of uncertain structures in design optimization.

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The RBDO problems of engineering structures are usually described as either probabilistic or non-probabilistic models [9]–[11]. Specifically, uncertain parameters are usually described as random variables or random fields based on the theory of probability and statistics when the sample data of uncertain factors are abundant [12]–[14], and then the failure probability of an engineering structure can be precisely calculated based on reliability analysis [15], [16]. The probabilistic reliability analysis has two characteristics: 1) It depends on the probability density function (PDF) which relies on a large amount of statistical data; 2) It is very sensitive to the variations of the parameters' probabilistic distributions. Whereas the uncertainties are described as non-probabilistic models such as the interval model when there are insufficient sample data [17]. The non-probabilistic reliability analysis is an effective method to deal with the reliability problem that has few or insufficient statistical data. The non-probabilistic reliability index based on the interval model is actually the minimum norm of the coordinate vector in the standardized space while the solution of non-probabilistic reliability index is actually an optimization problem with equality constraint [18]. Non-probabilistic RBDO approaches have also been put forward based on imperialistic competitive algorithm and interval model [19], enhanced chaos control method [20], and so on [21], [22].

For practical engineering structures, it is probable that there are abundant sample data for some uncertain factors but insufficient sample data for others. Thus, a lot of scholars have devoted to the investigation on the reliability analysis and RBDO approaches for engineering structures based on hybrid probabilistic and non-probabilistic models in recent years [23]. In the aspect of reliability analysis (RA), Guo and Du [24] proposed a new reliability sensitivity analysis method considering random and interval variables. Chen and Qiu [25] proposed a novel uncertainty analysis method based on polynomial chaos expansion for composite structures with random and interval variables. Li *et al.* [26] developed a precise and efficient univariate method for the mixed reliability evaluation of composite laminates with random and interval parameters. Zhang *et al.* [27] proposed a hybrid reliability analysis (HRA) method by combining the projection outline based active learning method with Kriging model. Brevault *et al.* [28] proposed the reliability analysis method in the presence of aleatory and epistemic uncertainties, and applied it to the prediction of a launch vehicle fallout zone. Zhang *et al.* [29] presented a HRA method for spacecraft docking lock with random and interval variables. Liu *et al.* [30] proposed a new HRA method based on probability and probability box models. Bai *et al.* [31] proposed a probabilistic and non-probabilistic HRA method based on dynamic substructural extreme response surface decoupling. In the aspect of RBDO, Wang *et al.* [32], [33] investigated the hybrid time-variant reliability estimation for active control structures under aleatory and epistemic uncertainties, and proposed the structural design optimization approach based on hybrid time-variant reliability measure under non-probabilistic convex uncertainties. Wu *et al.* [34] proposed a hybrid uncertain design optimization method for structures with both random and interval variables utilizing orthogonal series expansion. Huang *et al.* [35] established a hybrid RBDO (HRBDO) model which included interval variables in the probability distribution functions of random parameters, and developed an efficient decoupling algorithm to solve the model. Keshtegara and Hao [36] proposed a hybrid descent mean value for accurate and efficient performance measure in RBDO. Kang and Luo [37] presented a hybrid reliability index for uncertain structures based on the probabilistic and multi-ellipsoid convex set hybrid model, and solved the optimization problem including reliability constraints based on the linearization of the performance function.

However, most of the present researches on structures with random and interval uncertainties focused on the hybrid reliability analyses of relatively simple structures with explicit performance functions, whereas the research topic of HRBDO of uncertain structures receives much less attention. The linearization-based HRBDO approach [37] is obviously inapplicable to realistic engineering structures because their performance functions with respect to uncertain variables are usually nonlinear. Although several HRA approaches have been proposed in recent years for structures with no explicit performance functions [25]–[28], the HRA of practical engineering structures with high geometrical complexity, especially those with strong nonlinearity, needs to be further investigated. Meanwhile, the computational efficiency of the HRA with random and interval uncertainties needs to be enhanced in order to realize the HRBDO of realistic engineering structures since a huge amount of HRA computations are involved in the HRBDO process. Therefore, it is necessary and meaningful to develop an efficient and versatile HRA method and the built on HRBDO approach for realistic complex engineering structures with the consideration of both random and interval uncertainties.

This paper systematically investigates the HRA and HRBDO of complex engineering structures with random and interval uncertainties. After the construction of the HRBDO model for complex engineering structures considering the influences of random and interval uncertainties on their performance indices, a new HRA method based on adaptive step size (ASS-HRA) is proposed to efficiently calculate the minimum reliability of structural performance under random and interval uncertainties. Then a new HRBDO approach integrating polynomial response surface model (PRSM), ASS-HRA and genetic algorithm (GA) is developed for complex practical engineering structures with implicit performance functions of strong nonlinearity. The computational efficiency and effectiveness of the proposed ASS-HRA and the built on HRBDO approach as well as their applicability and effectiveness in realistic complex engineering problems are demonstrated by illustrative examples. Therefore, the proposed HRBDO approach based on ASS-HRA can provide an efficient and versatile design optimization method for realistic complex engineering structures with probabilistic and non-probabilistic uncertainties.

This rest of the paper is organized as follows. Firstly, the HRBDO model of complex engineering structures is proposed in Section II. Then an efficient ASS-HRA method is presented for analyzing the minimum reliability of structures with random and interval uncertainties in Section III. Subsequently, a new HRBDO approach integrating the ASS-HRA method and PRSM is proposed in Section IV. Then an engineering example is investigated in Section V to demonstrate the effectiveness and applicability of the proposed approach in solving realistic complex nonlinear engineering problems. Finally, conclusions are summarized in Section VI.

II. HRBDO MODEL OF COMPLEX STRUCTURE WITH RANDOM AND INTERVAL UNCERTAINTIES

The reliability index of a complex structure under random and interval uncertainties is an interval number due to the influence of interval uncertainties on structural performance. The lower bound of the interval reliability index, which are named as the minimum reliability index herein, is chosen as the measure for evaluating the reliability index of the performance function for the complex uncertain structure in order to ensure the satisfaction of the reliability requirement on a performance function in its worst case under random and interval uncertainties. Thus, the HRBDO model considering the influences of both random and interval uncertainties can be described as:

$$\begin{aligned} &\min_{\boldsymbol{d}} f(\boldsymbol{d}) \\ &\text{s.t. } R_{i\min}\left[g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U}) \geq 0\right] \geq \eta_i, i = 1, 2, \ldots, p. \\ &\quad \boldsymbol{d} = (d_1, d_2, \ldots, d_l), \boldsymbol{X} = (X_1, X_2, \ldots, X_m), \\ &\quad \boldsymbol{U} = (U_1, U_2, \ldots, U_n); \boldsymbol{d}^L \leq \boldsymbol{d} \leq \boldsymbol{d}^U. \end{aligned} \tag{1}$$

where $\boldsymbol{d}$ is the l-dimensional design vector of a complex uncertain structure while $\boldsymbol{d}^L$ and $\boldsymbol{d}^U$ are its allowable minimum and maximum values; $\boldsymbol{X}$ is an m-dimensional random parameter vector; $\boldsymbol{U}$ is an n-dimensional interval parameter vector; $f(\boldsymbol{d})$ is the objective function; $g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U})$ is the i-th performance function; p is the number of performance functions; $R_{i\min}$ is the minimum reliability of the i-th performance function; η_i is the i-th desired value of the minimum reliability index prescribed according to the reliability requirement on the uncertain structure.

III. A NEW ASS-HRA METHOD FOR STRUCTURES WITH RANDOM AND INTERVAL UNCERTAINTIES

This section firstly introduces the fundamentals of HRA with random and interval variables, and then proposes a new ASS-HRA method for improving the convergence efficiency of HRA. A cantilever beam with random geometrical variables and interval external loads is utilized to verify the efficiency and accuracy of the proposed ASS-HRA method.

A. FUNDAMENTALS OF HRA

The failure probability of the i-th performance function $g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U})$ in the HRBDO model (1) can be computed by:

$$p_{if} \in \Pr\left\{g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U}) < 0\right\} \tag{2}$$

which generally belongs to a limit state zone $p_{if} \in [p_{if}^L, p_{if}^R]$ composed of two boundary limit state surfaces $\max_{\boldsymbol{U}} g_i(\boldsymbol{d}, X, \boldsymbol{U}) = 0$ and $\min_{\boldsymbol{U}} g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U}) = 0$ due to the existence of interval parameter vector $\boldsymbol{U}$. Geometrically, two boundary limit state surfaces have the nearest and furthest distances to the origin among all the limit state functions with different values of interval parameter vector $\boldsymbol{U}$, see Fig. 1 for the illustration of a 2D problem [38].

According to the first order second moment (FOSM) method [39], the random parameter vector can be transformed from $\boldsymbol{X}$ space into the standard normal space $\boldsymbol{V}$, and the limit state space $g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U})$ can also be transformed into the standard limit state space $G_i(\boldsymbol{d}, \boldsymbol{V}, \boldsymbol{U}) = 0$, as shown in Fig. 2. Then the reliability index is defined as the minimum distance from the origin to the failure surface in the standard normal space. The reliability index corresponding to a design vector $\boldsymbol{d}$ under the influences of both random and interval uncertainties is an interval number, the minimum and maximum values of which can be calculated by

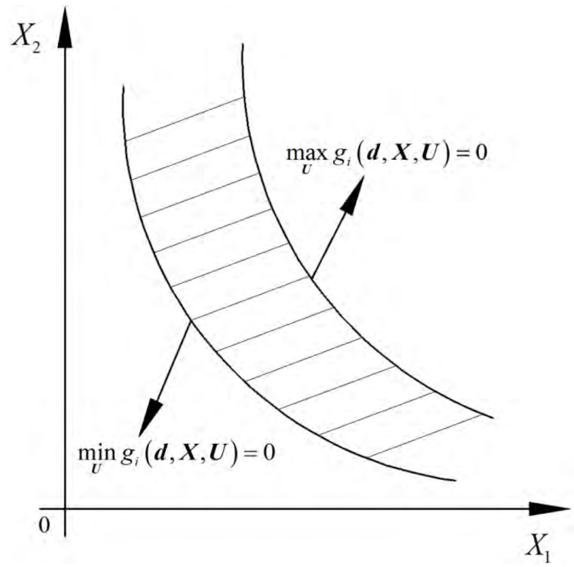


FIGURE 1. Limit-state strip for a 2D problem.

$$\begin{cases} \beta_{i\min} = \min_{\boldsymbol{V}} \|\boldsymbol{V}\| \\ \text{s.t.} \min_{\boldsymbol{U}} G_i(\boldsymbol{d}, \boldsymbol{V}, \boldsymbol{U}) = 0 \end{cases} \tag{3}$$

$$\begin{cases} \beta_{i\max} = \max_{\boldsymbol{V}} \|\boldsymbol{V}\| \\ \text{s.t.} \max_{\boldsymbol{U}} G_i(\boldsymbol{d}, \boldsymbol{V}, \boldsymbol{U}) = 0 \end{cases} \tag{4}$$

where $\boldsymbol{V}$ represents the standard normal parameter vector transformed from $\boldsymbol{X}$; $G_i(\boldsymbol{d}, \boldsymbol{V}, \boldsymbol{U}) = 0$ represents the standard limit state space transformed from $g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U}) = 0$; $\beta_{i\max}$ and $\beta_{i\min}$ represent the i-th maximum and minimum reliability indices, respectively.

In order to guarantee the satisfaction of reliability requirement on the uncertain structure in the worst case, the minimum reliability index $\beta_{i\min}$ is chosen as the hybrid reliability index, which can be calculated by:

$$R_{i\min}=1 - \Phi(-\beta_{i\min}) \tag{5}$$

where Φ is the standard normal cumulative probability function.

B. THE PROPOSED ASS-HRA METHOD

The HL-RF method was firstly proposed by Hasofer-Lind and Rackwitz-Fiessler based on the idea of expanding the performance function in Taylor's series [40], [41]. It is widely utilized for HRA due to its advantages of simplicity and high efficiency. Later, the improved HL-RF (iHL-RF) method was developed to quantify the effects of random and interval inputs on reliability associated with performance characteristics [42]. However, for highly complex and nonlinear limit-state functions, the iHL-RF method may also converge slowly or even result in divergence. Guo and Du [24] developed an efficient sequential single-loop (SSL) method for structural reliability analysis, in which the reliability in terms of random variables was decoupled from the interval analysis in terms of interval variables. However, a rigorous mathematical foundation is absent for this decoupling strategy, and it is difficult to ensure the robust convergence of this reliability analysis method. Jiang *et al.* [43] proposed a new HRA method through the construction of an equivalent model with only random variables, in which the maximum failure probability was evaluated by solving the equivalent model. However, their approach involves the minimization of a merit function to determine the iterative step size of iHL-RF method in solving the equivalent model, which increases the computational cost of HRA.

In order to overcome the shortcomings of the above HRA methods and improve the computational efficiency of HRA, a new ASS-HRA method is proposed in this section which decouples the HRA process into two layers of relatively independent iterations based on HL-RF method [44], [45] and efficiently locates the most probable point (MPP) by avoiding zigzagging iteration with the introduction of a correction angle. Specifically, the inner layer RA iteration takes random parameters as variables, in which the correction angle is utilized to avoid zigzagging iteration. The outer layer RA iteration takes interval parameters as variables. The HRA process terminates when reaching the prescribed convergence threshold.

The inner RA iteration searches for the optimal random parameter vector $\boldsymbol{V}^*$

$$\begin{cases} \left\|\boldsymbol{V}^*\right\| = \min\limits_{V} \|\boldsymbol{V}\| \\ \text{s.t. } G_i(\boldsymbol{d}, \boldsymbol{V}, \boldsymbol{U}) = 0 \end{cases} \tag{6}$$

The outer RA iteration searches for the optimal interval parameter vector $\boldsymbol{U}^*$

$$\begin{cases} G_i(\boldsymbol{d}, \boldsymbol{V}^*, \boldsymbol{U}^*) = \min\limits_{\boldsymbol{U}} G_i(\boldsymbol{d}, \boldsymbol{V}^*, \boldsymbol{U}) \\ \text{s.t. } U_j^L \le U_j \le U_j^U, j = 1, 2, \ldots, n \end{cases} \tag{7}$$

Assuming that the random vector $\boldsymbol{V}^k$ and interval vector $\boldsymbol{U}^k$ have been obtained by the kth iteration, the interval vector $\boldsymbol{U}^k$ is fixed for the next inner RA iteration, and then the random vector $\boldsymbol{V}^{k+1}$ can be obtained by (8) based on the iHL-RF method.

$$\boldsymbol{V}^{k+1} = \frac{\nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)^{\mathrm{T}} \boldsymbol{V}^k - G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)}{\left\|\nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)\right\|^2} \nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right) \tag{8}$$

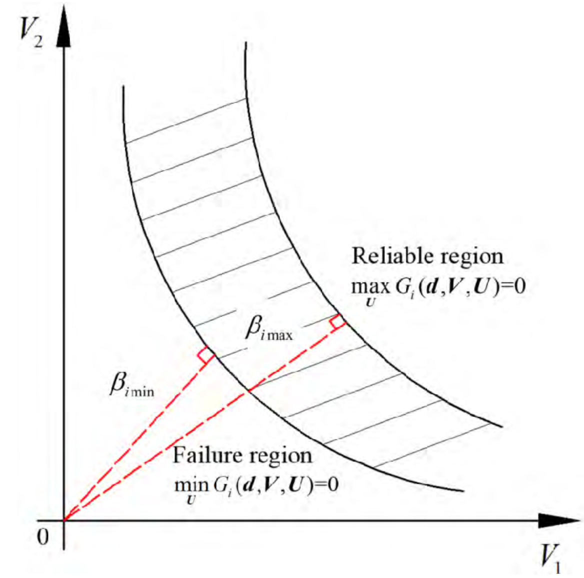


FIGURE 2. Limit-state strip in standard normal space for a 2D problem.

where $\nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)$ is the gradient of $G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)$, $\left\|\nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)\right\|$ is the Euclidean norm of $\nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)$.

The ideal iteration process for HRA would converge rapidly without zigzagging iteration. To achieve the ideal iteration for HRA, a correction angle (denoted as θ herein) between the random vector and the gradient direction of the standard normal limit-state space is introduced for a point on the standard normal limit-state space, see Fig. 3 for illustration. For an ideal HRA process, the correction angle θ should decrease gradually and finally approach zero, namely, the following condition should be satisfied

$$\theta^{k+1} \le \theta^k \tag{9}$$

where θ^{k+1} at the $(k+1)$th iteration can be calculate by

$$\theta^{k+1} = \operatorname{arccos}\left(\frac{\nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^{k+1}, \boldsymbol{U}^k\right)^{\mathrm{T}} \nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)}{\left\|\nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^{k+1}, \boldsymbol{U}^k\right)\right\| \left\|\nabla G_i\left(\boldsymbol{d}, \boldsymbol{V}^k, \boldsymbol{U}^k\right)\right\|}\right) \tag{10}$$

The zigzagging iteration will occur when the convergence condition in (9) cannot be satisfied. Thus, an adaptive step size $\lambda(\theta)$ calculated by (11) is introduced herein to adjust the iterative random vector $\boldsymbol{V}$ when (9) cannot be satisfied. And the new iterative random vector $\boldsymbol{V}_{\text{new}}^{k+1}$ is calculated by (12).

$$\lambda(\theta) = \frac{1}{1 + \left(\theta^{k+1}/\theta^k\right)^2} \tag{11}$$

$$\boldsymbol{V}_{\text{new}}^{k+1} = \boldsymbol{V}^k + \lambda(\theta)\left(\boldsymbol{V}^{k+1} - \boldsymbol{V}^k\right) \tag{12}$$

As demonstrated in Fig. 4, the adaptive step size $\lambda(\theta)$ approaches 0 when the inner RA iteration deviates far from the MPP and the angle ratio θ^{k+1}/θ^k is large. The adaptive step size approaches 1 when the inner RA iteration approaches the MPP and the angle ratio θ^{k+1}/θ^k is small.

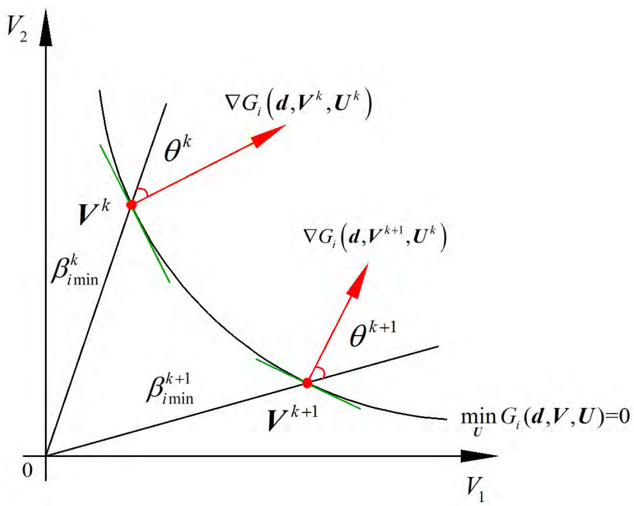


FIGURE 3. Schematic diagram of HRA iteration process in 2D condition.

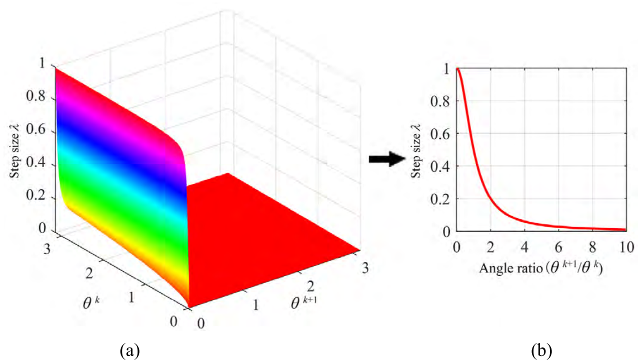


FIGURE 4. The relation of adaptive step size with regard to the correction angle.

As for the outer RA iteration, the interval vector U^{k+1} can be obtained by (13) through interval analysis based on the random vector V^{k+1} obtained by the inner RA iteration. In this paper, the MATLAB build-in function *fmincon* that applies the trust region reflective method is utilized to determine U^{k+1} efficiently.

$$U^{k+1} = \min G_i\left(V^{k+1}, U^k\right) \tag{13}$$

The above two-layer RA iteration process continues until (14) and (15) are satisfied.

$$\left\|V^{k+1} - V^k\right\| / \left\|V^k\right\| \le \varepsilon_1 \tag{14}$$

$$\left|G_i\left(d, V^{k+1}, U^{k+1}\right)\right| \le \varepsilon_2 \tag{15}$$

where ε_1 and ε_2 are the prescribed convergence tolerances.

And finally, the minimum reliability index $\beta_{i\min}$ can be calculated as the shortest distance from the MPP to the origin. Consequently, the minimum reliability $R_{i\min}$ of the HRBDO problem can be calculated by (5). The proposed ASS-HRA algorithm is provided as follows while its flowchart is embedded in Fig. 6:

Algorithm 1 The Proposed ASS-HRA for Structures With Random and Interval Uncertainties

Step 1: Initialize the random variables and interval variables with their means, prescribe the convergent tolerances of ε_1 and ε_2, and set k as 1.

Step 2: Calculate the random vector V^{k+1} and angle θ^{k+1} by (8) and (10).

Step 3: If $\theta^{k+1} > \theta^k$, calculate the adaptive step size λ by (11) and adjust the random vector V^{k+1} by (12), then go back to Step 2. Otherwise, go to Step 4.

Step 4: Calculate the interval vector U^{k+1} by (13) based on the V^{k+1} obtained in Step3.

Step 5: Output the minimum reliability $R_{i\min}$ calculated by (5) based on the minimum reliability index $\beta_{i\min}$ calculated by (3) if (14) and (15) are satisfied. Otherwise, increase k by 1 and go to Step 2.

C. BENCHMARK TEST

The cantilever beam from the literature [43] is utilized as a benchmark example to verify the feasibility of the proposed ASS-HRA method. As illustrated in Fig. 5, the cantilever beam is subjected to a vertical load Y_1(N) and a lateral load Y_2(N). The width w(mm) and thickness t(mm) of the cross section as well as the length l(mm) are treated as random variables while the vertical load Y_1(N) and lateral load Y_2(N) are interval variables. All the statistics of these uncertain variables are listed in Table 1. According to the failure mode of the cantilever beam, the maximum stress at the fixed end of the beam should be less than a yield stress $S = 320$MPa. Thus, the HRA problem of the cantilever beam can be described as follows:

$$\begin{aligned} &\min \; R\left[g\left(w,t,l,Y_1,Y_2\right) \ge 0\right] \\ &\text{s.t. } g\left(w,t,l,Y_1,Y_2\right) = S - \left(\frac{6Y_1 l}{wt^2} + \frac{6Y_2 l}{w^2 t}\right) \\ &\quad X = (w,t,l)\,, U = (Y_1, Y_2) \end{aligned} \tag{16}$$

where X is a 3-dimensional random parameter vector, U is a 2-dimensional interval parameter vector, $g(w,t,l,Y_1,Y_2)$ is the performance function of the cantilever beam, $R[g(w,t,l,Y_1,Y_2) \ge 0]$ is the reliability of the cantilever beam under random and interval uncertainties.

The minimum reliability of the cantilever beam is analyzed by the proposed ASS-HRA method as well as the HRA method based on the equivalent transformation of interval variables into random ones (ET-HRA) [43] and the HRA method based on the sequential single-loop (SSL-HRA) [24], the results of which are listed in Table 2. The HRA result based on 10^7 groups of Monte Carlo simulations (MCSs) [46] is utilized as the reference value for verifying the accuracy of three different HRA methods. The minimum reliability index calculated by the proposed ASS-HRA converges at the 6th iteration with the minimum reliability achieved as $R_{\min} = 0.9172$, the corresponding random and interval variables are $X = (75.60, 188.90, 1043.00)$mm and $U = (27000.00, 53000.00)$ N, respectively.

TABLE 1. Statistics of the uncertain variables for the cantilever beam.

Uncertain parameter	Random variable			Interval variable	
	Mean value	Standard deviation	Distribution type	Lower bound	Upper bound
w (mm)	100	15	Normal	N/A	N/A
t (mm)	200	20	Normal	N/A	N/A
l (mm)	1000	100	Normal	N/A	N/A
Y_1 (N)	N/A	N/A	N/A	23000	27000
Y_2 (N)	N/A	N/A	N/A	47000	53000

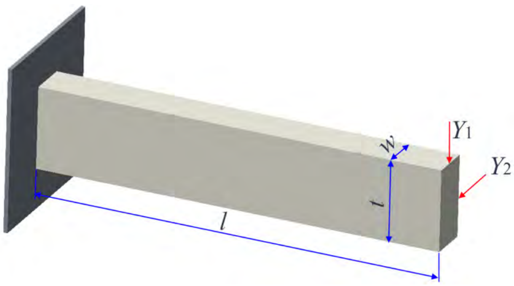


FIGURE 5. A cantilever beam.

As listed in Table 2, the relative errors of the minimum reliabilities of the cantilever beam calculated by both the proposed ASS-HRA method and the SSL-HRA method are only around 0.5% with regard to the reference value calculated by the MCS-HRA method, demonstrating that the ASS-HRA and SSL-HRA methods can achieve accurate HRA results. Whereas the ET-HRA method has much lower analysis precision since the minimum reliability calculated by the ET-HRA method has much larger relative error of 2.48% with regard to the reference value of 0.9125 calculated by the MCS-HRA method.

Meanwhile, the proposed ASS-HRA method is very efficient in comparison with both the ET-HRA and SSL-HRA methods since it only needs a total of 36 functional evaluations and 6 iterations. Here the iteration refers to the outer RA iteration that described between Step 2 and Step 5 in the HRA algorithm. The SSL-HRA method has much lower efficiency since it needs a total of 198 functional evaluations and 18 iterations although it can also achieve the accurate HRA results. Therefore, the proposed ASS-HRA method has advantages in both the HRA accuracy and computational efficiency over the other two HRA methods.

IV. AN EFFICENT HRBDO APPROACH BASED ON ASS-HRA

In this section, an efficient HRBDO approach integrating the proposed ASS-HRA method with the PRSM technique is developed for the HRBDO of complex engineering structures with random and interval uncertainties. The effectiveness of the proposed HRBDO approach is verified by a numerical example.

A. CONSTRUCTION OF PRSMS

The RBDO of complex engineering structures involves the massive calculation of structural performance indices which are implicit functions with high nonlinearity. To avoid the computationally intensive finite element analyses (FEAs), surrogate models are often utilized to compute the performance functions of complex uncertain structures. Considering that the derivatives of the performance functions with respect to the design vector need to be calculated in the RBDO process, the performance functions in the HRBDO model are approximated by the PRSMs [47] in this paper. In order to improve the prediction accuracy, the PRSMs are constructed based on the determination of the basis functions according to the physical property of the optimization problem [48].

Taking the objective function in (1) as an example, the complete PRSM with the unknown highest order is constructed as:

$$f(\boldsymbol{d}) = \varphi_0 + \sum_{i=1}^{l} \varphi_i \cdot d_i + + \sum_{i=1}^{l} \varphi_{i+l} \cdot d_i^a + \ldots + \sum_{i=1}^{l}\sum_{j\geq i}^{l} \varphi_{ij} \cdot d_i d_j + \sum_{i=1}^{l}\sum_{j\geq i}^{l} \varphi_{ij+x} \cdot d_i^a d_j^b + \ldots + \sum_{i=1}^{l}\sum_{j\geq i}^{l}\sum_{k\geq j}^{l} \varphi_{ijk} \cdot d_i d_j d_k + \sum_{i=1}^{l}\sum_{j\geq i}^{l}\sum_{k\geq j}^{l} \varphi_{ijk+y} \cdot d_i^a d_j^b d_k^c \tag{17}$$

where d_i is the i-th variable of the l-dimensional vector $\boldsymbol{d}$, φ_0, φ_i, φ_{i+l}, φ_{ij}, φ_{ij+x}, φ_{ijk}, φ_{ijk+y} are the unknown coefficients, a, b, c are the highest order of variables. As for the performance functions $g_i(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U})$, the corresponding PRSMs can be constructed by substituting vector $\boldsymbol{d}$ with $(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U})$.

The unknown coefficients and the orders of every basis function in (17) are determined by the reverse design method. Specifically, the coupling effects among different variables are analyzed and the corresponding coupling item is removed if there is no coupling effect. Then the highest order of every variable in every basis function is determined by the control variable method. And finally, the coefficients of a PRSM can be determined by the least square polynomial regression method based on the sample data arranged by Latin hypercube sampling (LHS). The prediction capability of a PRSM is evaluated by the root mean squared error $RMSE$, fitting determination coefficient R^2 and correlation coefficient CC calculated by (18), (19), (20), respectively [49].

$$RMSE = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(f^k - \hat{f}^k\right)^2} \tag{18}$$

where f^k and $\hat{f}^k$ denote the actual performance value and the predicted performance value of an engineering structure at the kth test point respectively; n is the number of test points.

$$R^2 = 1 - \frac{\sum_{k=1}^{n}\left(f^k - \hat{f}^k\right)^2}{\sum_{k=1}^{n}\left(f^k - \bar{\hat{f}}^k\right)^2} \tag{19}$$

TABLE 2. HRA results of the cantilever beam obtained by different methods.

Method	Iteration	Functional evaluation	R_{min}	Relative error	Random variable (w, t, l) (mm)	Interval variable (Y_1,Y_2) (N)
ASS-HRA	6	36	0.9172	0.52%	(75.60,188.90,1043.00)	(27000.00,53000.00)
ET-HRA	16	128	0.9351	2.48%	(73.70,188.30,1046.80)	(25090.79,50454.08)
SSL-HRA	18	198	0.9171	0.50%	(75.70,188.80,1047.70)	(27000.00,53000.00)
MCS-HRA	/	10^7	0.9125	/	/	/ /

where $\bar{\hat{f}}^k$ denotes the mean of predicted performance value.

$$CC = \frac{\sum_{k=1}^{n}\left(f^k - \bar{f}^k\right)\left(\hat{f}^k - \bar{\hat{f}}^k\right)}{\sqrt{\sum_{k=1}^{n}\left(f^k - \bar{f}^k\right)^2}\sqrt{\sum_{k=1}^{n}\left(\hat{f}^k - \bar{\hat{f}}^k\right)^2}} \quad (20)$$

where $\bar{f}^k$ denotes the mean of actual performance value.

For a PRSM with high-fidelity, the *RMSE* should be close to 0, while the R^2 and *CC* should be close to 1.

B. DESCRIPTION OF THE PROPOSED HRBDO APPROACH

The HRBDO process for solving the optimization model (1) is decoupled into the optimization solution and reliability analysis, which forms double loops. In the inner loop, the reliability analysis of an engineering structure at a design point is implemented based on the ASS-HRA method proposed in Section III. In the outer loop, the HRBDO model is solved by GA for locating the optimal design with the minimum objective function under the condition that the reliability requirement on the structure is satisfied.

The fitness value of an individual is settled as 0 in GA when its reliability index does not meet the reliability requirement. The individuals whose reliability indices meet reliability requirement are ranked according to their objective function value. As a result, design vector $\boldsymbol{d}_i$ corresponding to the i-th individual in the current population of GA is assigned a rank number $Rank_i$. And the smaller rank number means the better design vector. The fitness value of design vector $\boldsymbol{d}_i$ can be calculated by

$$Fit\left(\boldsymbol{d}_i\right) = 1/Rank_i \quad (21)$$

The flowchart of the proposed HRBDO approach is shown in Fig. 6, the implementation of which proceeds as follows.

Step 1: Construct the HRBDO model of the uncertain structure with random and interval variables.

Step 2: Construct the PRSMs for calculating the objective and performance functions.

Step 3: Prescribe ε_1, ε_2 and initialize GA parameters, including the population size, maximum iteration number, crossover and mutation probabilities, and the convergence threshold. Set the iteration number as 1 and generate the initial population.

Step 4: Set the initial random and interval vectors to their means. Calculate the minimum reliabilities of all individuals in the current population based on the proposed ASS-HRA method.

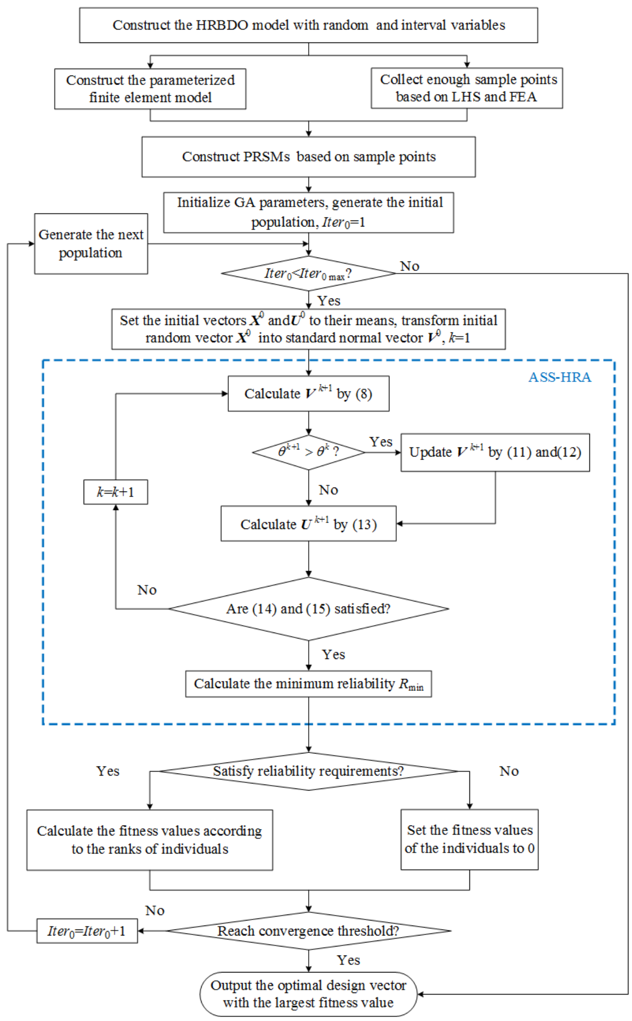


FIGURE 6. Flowchart of the proposed HRBDO approach.

Step 5: Rank the individuals in the current population of GA according to their objective values and calculate their fitness values according to their ranks if they satisfy reliability requirements. Otherwise, set the fitness values to 0.

Step 6: Output the design vector with the largest fitness value when reaching the maximum iteration number or the convergence threshold of GA. Otherwise, increase the iteration number of GA by 1 and go to Step 4.

TABLE 3. Statistics of uncertain variables for the numerical example.

Uncertain parameter	Random variable			Interval variable	
	Mean value	Standard deviation	Distribution type	Lower bound	Upper bound
x_1	d_1	$0.1d_1$	Normal	N/A	N/A
x_2	d_2	$0.1d_2$	Normal	N/A	N/A
U_1	N/A	N/A	N/A	0	0.5
U_2	N/A	N/A	N/A	0.5	2.5

C. A NUMERICAL EXAMPLE

The numerical HRBDO problem with two random variables and two interval variables in (22) is utilized to verify the efficiency of the proposed HRBDO approach based on ASS-HRA. Both the convergence values for ε_1 and ε_2 in ASS-HRA process are prescribed as 0.001 for this numerical example. The population size, maximum iteration number, crossover and mutation probabilities are prescribed as 50, 50, 0.90 and 0.01 respectively. Besides the maximum iteration number prescribed as the stop criterion, the GA evolution terminates when the difference between the objective value at the optimal solution of current GA generation and that of the last GA generation is less than 10^{-4}.

$$\begin{aligned}
&\min_{\boldsymbol{d}} f(\boldsymbol{d}) = (d_1+3)^2 + (d_2+3)^2 \\
&\text{s.t. } R_{i\min}\left[g_i(\boldsymbol{X},\boldsymbol{U}) \geq 0\right] \geq \eta_i, i = 1, 2. \\
&\quad g_1(\boldsymbol{X},\boldsymbol{U}) = x_1\left(x_2+U_1\right) - U_2 \\
&\quad g_2(\boldsymbol{X},\boldsymbol{U}) = x_1 - \left(x_2+U_1\right)^2 U_2 \\
&\quad \mu_X = \boldsymbol{d}, \sigma_X = 0.1\boldsymbol{d} \\
&\quad \boldsymbol{d} = \left(d_1, d_2\right), \boldsymbol{X} = \left(x_1, x_2\right), \boldsymbol{U} = \left(U_1, U_2\right) \\
&\quad 0.01 \leq d_1 \leq 10, 0.01 \leq d_2 \leq 10, \eta_1 = \eta_2 = 0.9956
\end{aligned} \tag{22}$$

where $\boldsymbol{d}$ is the design vector, which is the mean of random parameter vector $\boldsymbol{X} = (x_1, x_2)$; the standard deviation of random parameter vector $\boldsymbol{X}$ is $0.1\boldsymbol{d}$; $\boldsymbol{U}$ is the interval parameter vector; $f(\boldsymbol{d})$ is the objective function; $g_i(\boldsymbol{d}, X, U)$ is the i-th performance function; $R_{i\min}[g_i(\boldsymbol{X},\boldsymbol{U}) \geq 0]$ is the minimum reliability for $g_i(\boldsymbol{X},\boldsymbol{U}) \geq 0$; η_i is the desired reliability for the i-th performance function. All the statistics of uncertain variables are listed in Table 3.

The convergence curve of the optimal objective value obtained by the proposed HRBDO approach based on ASS-HRA is shown in Fig. 7. The optimal objective value converges at the 27th generation of GA, the minimum reliabilities of performance functions g_1 and g_2 at the optimal solution are 0.9983 and 0.9995, respectively. Thus, the reliability requirements are satisfied. To verify the accuracy of the minimum reliabilities calculated by the proposed ASS-HRA, the minimum reliabilities of performance functions g_1 and g_2 at the optimal solution (4.8459, 0.7613) are also calculated by the MCS-HRA involving 10^7 sample points, which are 0.9994 and 0.9984, respectively. As can be seen, the HRA results at the optimal solution calculated by the proposed ASS-HRA are very close to those calculated by the MCS-HRA. Specifically, the relative errors of the minimum reliabilities corresponding to performance functions g_1 and g_2 are only 0.1101% and 0.1102%, respectively. However, the computational time of the proposed ASS-HRA is much smaller than that of MCS-HRA. This further proves the accuracy and efficiency of the proposed ASS-HRA as well as its applicability in the GA-based HRBDO algorithm.

The solution of the HRBDO model in (22) is also carried out by replacing the ASS-HRA with previous HRA approaches, namely SSL-HRA and ET-HRA, in the identical GA-based optimization algorithm illustrated in Fig. 6. The convergence curves are demonstrated in Fig. 7 while the performance comparison of different HRBDO approaches and the HRA results at the optimal solutions are listed in Table 4 and Table 5 along with the results of the HRBDO based on the proposed ASS-HRA. All the HRA results of their corresponding optimal solutions are verified by MCS-HRA. As demonstrated in Table 4, all three approaches achieve similar optimal solutions as well as minimum objective function values, further demonstrating the correctness of HRBDO results. The slight difference in GA generation among three HRBDO approaches based on different HRA methods is considered normal due to the randomness of the GA framework. Neglecting this difference, it is evident that the application of the proposed ASS-HRA in the GA-based HRBDO process dramatically reduces the total and average HRA iterations, especially for the case of g_2 where the decreases of average HRA iterations are 59.33% and 58.20% compared with SSL-HRA and ET-HRA, respectively. The results demonstrate that the proposed ASS-HRA can achieve accurate optimal solutions and improve the computational efficiency in solving HRBDO problems. Consequently, the good convergence performance and high efficiency of the proposed HRBDO approach based on ASS-HRA are demonstrated.

V. APPLICATION IN ENGINEERING

To demonstrate the versatility and efficiency of the proposed HRBDO approach based on ASS-HRA in the design of complex engineering structures with strong nonlinearity, it is applied to the design optimization of a realistic complex engineering structure (namely, the slider of an ultra-precision high-speed stamping press with random and interval uncertainties), the objective of which is to realize the reliability-based lightweight design. With the PRSMs for efficiently calculating the performance functions of the slider constructed based on sample points, the optimal design of the slider satisfying reliability requirement is achieved utilizing the proposed HRBDO approach, which are compared in detail with those achieved by substituting ASS-HRA with SSL-HRA.

A. DESCRIPTION OF THE HRBDO PROBLEM

The slider is the most important part of a high-speed stamping press, which rapidly moves up and down along the guide rails of high-speed press in the stamping process. The slider frequently suffers heavy impact forces in the stamping

TABLE 4. Performance comparison of three HRBDO approaches based on different HRA methods.

HRA method	GA generation	Average HRA iterations for each generation		Total HRA iterations		Optimal solution	Objective function value
		g_1	g_2	g_1	g_2	(d_1, d_2)	$f_{\min}$
ASS-HRA	27	212.964	249.750	5963	6993	(4.8459, 0.7613)	75.7059
SSL-HRA	24	362.125	614.125	8691	14739	(4.8512, 0.6728)	75.1407
ET-HRA	28	330.833	597.541	7940	14341	(4.9463, 0.6569)	76.5164

TABLE 5. HRA results comparison among the optimal solutions obtained by different HRBDO approaches.

HRA method	$R_{\min}$ at the optimal solutions		$R_{\min}$ calculated by MCS-HRA		Relative errors compared with MCS-HRA	
	g_1	g_2	g_1	g_2	g_1	g_2
ASS-HRA	0.9983	0.9995	0.9994	0.9984	0.1101%	0.1102%
SSL-HRA	0.9988	0.9995	0.9994	0.9984	0.1069%	0.1102%
ET-HRA	0.9976	0.9989	0.9995	0.9982	0.1901%	0.0713%

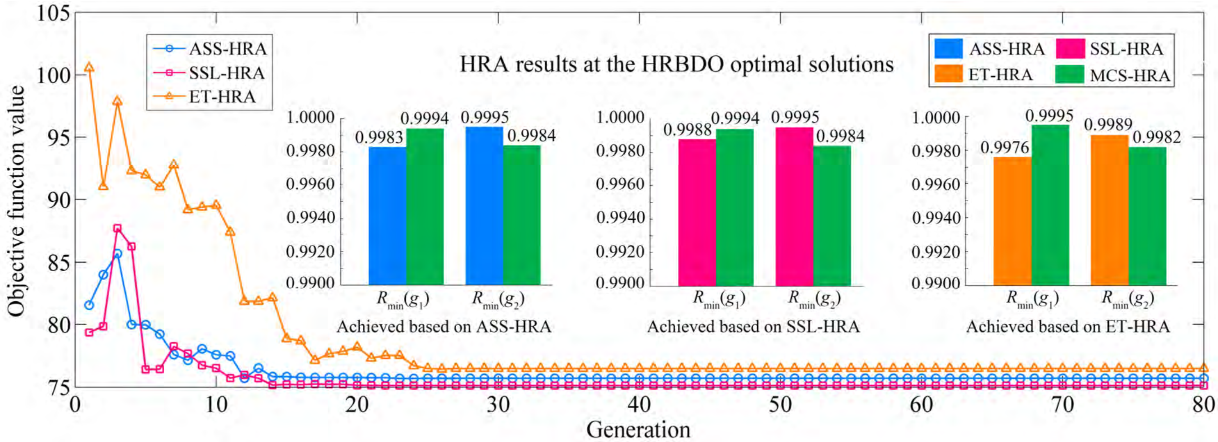


FIGURE 7. Convergence curves of the objective function of the numerical example utilizing ASS-HRA, SSL-HRA and ET-HRA.

process, which makes it vulnerable to damage [50]. Thus, it is necessary to optimize the slider's structure to ensure its performance reliability and the safe operation of high-speed press.

The press slider illustrated in Fig. 8 is utilized to demonstrate the feasibility and effectiveness of the proposed HRBDO approach in the optimization of complex engineering structures. Figure 8(a) illustrates the location of the slider in the actuating mechanism of a high-speed stamping press while Figs. 8(b) and 8(c) illustrate the 3D solid model and the cross section of the slider respectively. The geometrical parameters h, b_1, b_2, b_3(mm) in the cross section of the slider shown in Fig. 8(c) are chosen as the design variables while the elastic modulus E(MPa), Poisson's ratio υ and admissible stress Q(MPa) are described as interval variables considering that it is difficult to determine their probability distributions. The distance l(mm) between the inner and outer linkages in Fig. 8(a) is described as a random variable considering that there are sufficient data to determine its probability distribution. The distance between two outer linkages is fixed as 1900mm. The initial design is $\boldsymbol{d}_0 = (h, b_1, b_2, b_3) = (820, 70, 35, 40)$ mm, the corresponding weight of which is $w_0 = [1125.6, 1139.5]$kg. The maximum equivalent stress is the most important mechanical performance index according to the performance requirement of the investigated slider. Thus, the difference between the admissible stress and the actual maximum equivalent stress of the slider is described as the performance function. To realize the lightweight design of the slider, the minimization of the weight is described as the objective function. Consequently, the HRBDO model of the slider is constructed as

$$\begin{aligned} &\min_{\boldsymbol{d}} \ w(\boldsymbol{d}, \boldsymbol{X}) \\ &\text{s.t. } R_{\min}\left[g(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U}) \geq 0\right] \geq 0.98 \\ &\quad g(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U}) = Q - s\left(h, b_1, b_2, b_3, l, E, \upsilon\right) \\ &\quad \boldsymbol{d} = (h, b_1, b_2, b_3), \boldsymbol{X} = l, \boldsymbol{U} = (E, \upsilon, Q) \\ &\quad 700\text{mm} \leq h \leq 910\text{mm}, 54\text{mm} \leq b_1 \leq 120\text{mm}, \\ &\quad 16\text{mm} \leq b_2 \leq 40\text{mm}, 16\text{mm} \leq b_3 \leq 48\text{mm} \qquad (23) \end{aligned}$$

where $\boldsymbol{d} = (h, b_1, b_2, b_3)$ (mm) is the design vector; $\boldsymbol{X} = l$(mm) is the random vector (variable); $\boldsymbol{U} = (E, \upsilon, Q)$ is the interval parameter vector; $w(\boldsymbol{d}, \boldsymbol{X})$ is the weight of the slider; $g(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U})$ is the performance

TABLE 6. The statistics of uncertain variables of the slider.

Uncertain parameters	Random variable			Interval variable	
	Mean value	Standard deviation	Distribution type	Lower bound	Upper bound
l (mm)	590	30	Normal	N/A	N/A
E (MPa)	N/A	N/A	N/A	1.26x10^5	1.54 x10^5
υ	N/A	N/A	N/A	0.23	0.27
Q (MPa)	N/A	N/A	N/A	55	65

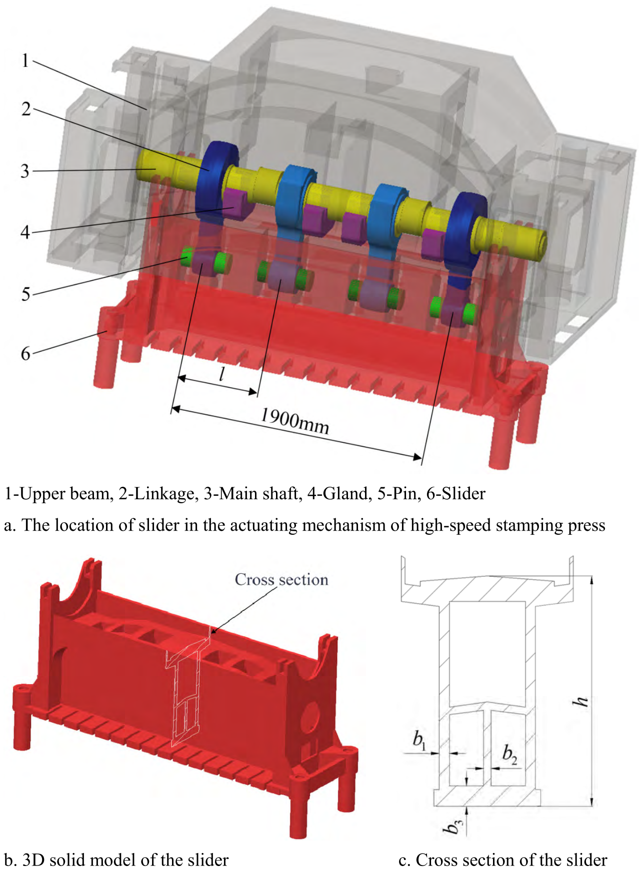


FIGURE 8. The slider in an ultra-precision high-speed press.

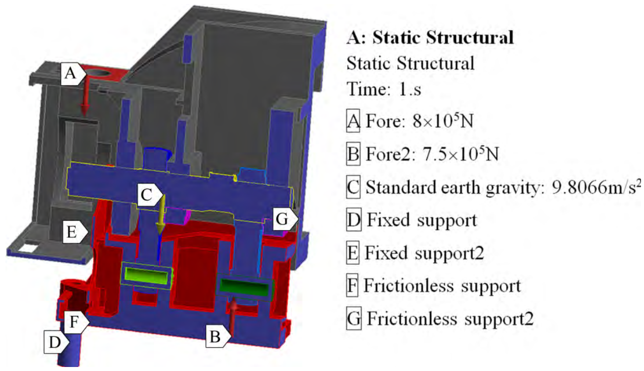


FIGURE 9. 1/4 FE model of the actuating mechanism: Loads and constrains.

function; $s(h, b_1, b_2, b_3, l, E, \upsilon)$ is the actual maximum equivalent stress of the slider; $R_{\min}$ is the minimum reliability of the slider under the influences of random and interval uncertainties. All the statistics of these uncertain variables are listed in Table 6.

B. CONSTRUCTION OF THE PRSMS

A total of 60 sample points is generated within the 8-dimensional space defined by 4 design variables, 1 random variable and 3 interval variables utilizing the LHS method, 55 points of which are chosen for generating the PRSMs while the others are utilized as test points. The LHS procedure applies uniform distribution and Gaussian distribution, respectively, for producing sample points of interval and probabilistic uncertain variables. Then the maximum equivalent stress and weight corresponding to the 60 sample points are calculated by finite element analysis (FEA). The 1/4 FE model of the actuating mechanism shown in Fig. 9 is utilized for FEA considering its symmetry. A uniformly distributed load of 750kN is exerted on the lower surface of the slider since the nominal force of the investigated press is 3000kN while a pressure of 800kN is exerted at the connection of the upper beam with the oil cylinder. The mesh model of the slider includes 64439 solid187 elements and 117603 nodes.

TABLE 7. Statistics of PRSMs for calculating the objective and constraint functions.

Function	*RMSE*	R^2	*CC*
$w(\boldsymbol{d}, \boldsymbol{X})$	0.03863	0.99999	1
$g(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U})$	0.03396	0.99982	0.99998

Considering the influences of design variables h, b_1, b_2, b_3, random variable l and interval variables E, υ, Q on the maximum equivalent stress s and the weight w, the PRSMs for computing $w(\boldsymbol{d}, \boldsymbol{X})$ and $g(\boldsymbol{d}, \boldsymbol{X}, \boldsymbol{U})$ are constructed as:

$$\begin{aligned} w(\boldsymbol{d}, \boldsymbol{X}) &= 74.65634 + 0.89488 \cdot h + 3.18046 \cdot b_1 \\ &\quad + 11.66997 \cdot b_2 + 1.76934 \cdot b_3 + 0.86049 \cdot l \\ &\quad - 0.01077 \cdot h \cdot b_2 - 0.00124 \cdot h \cdot b_3 \\ &\quad - 0.00131 \cdot h \cdot l + 0.00026 \cdot b_1 \cdot l \\ &\quad - 0.02353 \cdot b_2 \cdot l - 0.01188 \cdot b_3 \cdot l \\ &\quad + 0.00003 \cdot h \cdot b_2 \cdot l + 0.00002 \cdot h \cdot b_3 \cdot l \end{aligned} \tag{24}$$

TABLE 8. Comparison of the computational efficiency between two different HRBDO approaches.

HRBDO approach	GA generation	Total HRA iterations	Average HRA iterations for each generation
ASS-HRA-based HRBDO	196	365706	1865.847
SSL-HRA-based HRBDO	198	601762	3038.747

TABLE 9. Comparison of the optimal designs located by two different HRBDO methods.

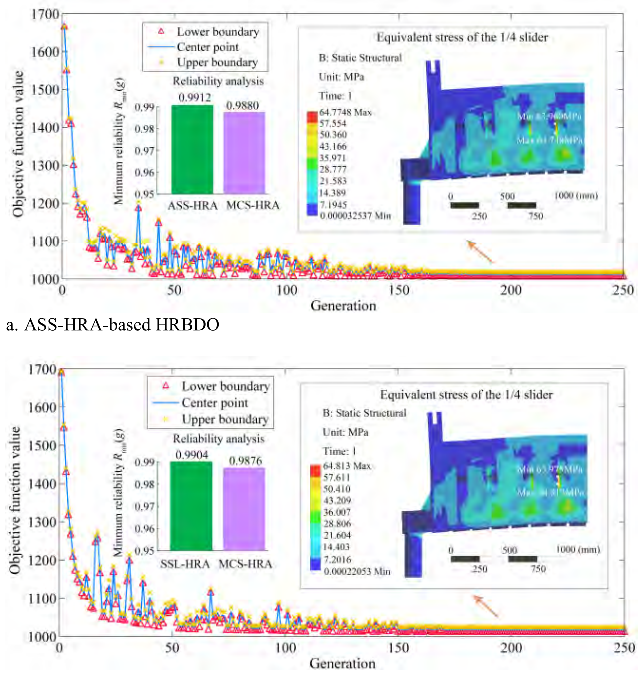
Design scheme	Design vector (h,b_1,b_2,b_3)(mm)	Weight w (kg)	Minimum Reliability $R_{min}(g(\boldsymbol{d},\boldsymbol{X},\boldsymbol{U}))$		Maximum equivalent stress (MPa)
			Applied HRA	MCS-HRA	
Initial design	(820,70,35,40)	[1125.6,1139.5]	0.9732	0.9685	[67.521,68.213]
ASS-HRA-based design	(700.9,54.1,16.0,17.0)	[1005.1,1018.0]	0.9912	0.9880	[63.960,64.748]
SSL-HRA-based design	(702.2,54.2,16.5,17.2)	[1011.1,1024.2]	0.9904	0.9876	[63.975,64.813]

$$
\begin{aligned}
g(\boldsymbol{d},\boldsymbol{X},\boldsymbol{U}) &= Q-(1.084\times 10^{-7}\cdot E\cdot \nu-2.697\times 10^{8}\cdot E \\
&\quad +0.00003\cdot \nu-5.136\times 10^{6})\cdot(1.10804\cdot h \\
&\quad +0.78354\cdot b_1+1.91884\cdot b_2+0.93603\cdot b_3 \\
&\quad +1.36302\cdot l+0.75433\cdot h\cdot b_2 \\
&\quad +0.75657\cdot h\cdot b_3+0.75611\cdot h\cdot l \\
&\quad +0.75657\cdot b_1\cdot l+0.75256\cdot b_2\cdot l \\
&\quad +0.75491\cdot b_3\cdot l+0.75433\cdot h\cdot b_1 \\
&\quad +0.75687\cdot h\cdot b_2\cdot l+0.75687\cdot h\cdot b_3\cdot l+4.14164)
\end{aligned} \tag{25}
$$

The prediction errors of the resulting PRSMs for $w(\boldsymbol{d},\boldsymbol{X})$ and $g(\boldsymbol{d},\boldsymbol{X},\boldsymbol{U})$ are listed in Table 7, which demonstrates that both of their *RMSE* values are less than 0.04 and both of their R^2 and *CC* values are greater than 0.99. Thus the PRSMs in (24), (25) are accurate enough to be utilized as substitutes for FEAs in the optimization of the press slider.

C. OPTIMIZATION RESULTS AND DISCUSSIONS

Based on the PRSMs in (24), (25), the HRBDO model in (23) is solved by the proposed HRBDO approach based on ASS-HRA. The convergence values of ε_1 and ε_2 in the ASS-HRA method are 0.001. The population size, maximum iteration number, crossover and mutation probabilities of GA are prescribed as 200, 250, 0.90 and 0.01, respectively. Besides the maximum iteration number, the GA evolution terminates when the difference between the objective value at the optimal solution of current GA generation and that of the last GA generation is less than 10^{-4}. The convergent curve of weight obtained by the proposed approach is shown in Fig. 10, which converges at the 196th generation of GA evolution. The optimal solution is obtained as $\boldsymbol{d} = (700.9, 54.1, 16.0, 17.0)$ (mm), the corresponding weight and the maximum equivalent stress of which are $w(\boldsymbol{d},\boldsymbol{X}) = [1005.1, 1018.0]$kg and $s(\boldsymbol{d},\boldsymbol{X},\boldsymbol{U}) = [63.960, 64.748]$MPa, respectively. The HRA and HRBDO results are also provided in Table 8 and Table 9 with the corresponding HRA results obtained by 10^7 groups of MCSs utilized as the reference values. As can be observed from Table 9, the HRA results of the optimal solution obtained by



a. ASS-HRA-based HRBDO

b. SSL-HRA-based HRBDO

FIGURE 10. Convergence curves of the objective function value for the engineering example.

the proposed ASS-HRA are very close to the values obtained by MCS-HRA with the relative error of only 0.4853%. The relative HRA error at the initial design is 0.3239%, which further demonstrates the accuracy of the proposed ASS-HRA approach.

As can be seen from the results of the benchmark test in Table 2 of Section III.C, the proposed ASS-HRA has the similar accuracy as the SSL-HRA while the ET-HRA may yield relatively large error although it can reduces the iteration and functional evaluation in the HRA process for each design vector. Therefore, the HRBDO model (23) is solved herein by an identical GA framework utilizing the SSL-HRA instead of the ASS-HRA for comparison so as to ensure the reliable comparison of the solution efficiency. The computational efficiency of the HRBDO approaches based on ASS-HRA and SSL-HRA are compared in Table 8 while the performance of the optimal designs located by two different HRBDO approaches are compared in Table 9. It is obvious from Table 9 that the initial design cannot satisfy the reliability requirement (namely, the minimum reliability of the performance function at the initial design is less than 0.98) while both the optimal designs achieved by two HRBDO approaches can satisfy the reliability requirement. However, the ASS-HRA-based HRBDO involves fewer HRA iterations with an evident drop of 39.227% (total) and 38.598% (average for each GA generation), respectively, compared to the SSL-HRA-based HRBDO. Moreover, the ASS-HRA-based HRBDO produces the optimal design with the higher minimum reliability of the constraint function, the smaller weight, and the smaller maximum equivalent stress. That is, the optimal design achieved by the ASS-HRA-based HRBDO has better performance than that achieved by the SSL-HRA-based HRBDO. Consequently, the applicability and effectiveness of the proposed approach in the HRBDO of realistic complex engineering structures with random and interval uncertainties are demonstrated.

VI. CONCLUSIONS

To realize the HRBDO of complex engineering structures with random and interval uncertainties, a HRBDO model is constructed with implicit objective and constraint performance functions. For the purpose of efficiently calculating the minimum reliability index of an engineering structure with random and interval variables, a new ASS-HRA method is proposed which decouples the HRA process into two layers of iterations and efficiently locates the MPP with the introduction of a correction angle to avoid zigzagging iteration. The benchmark example of a cantilever beam demonstrates that the proposed ASS-HRA method can efficiently achieve the HRA results of high accuracy in comparison with the ET-HRA and SSL-HRA methods.

Subsequently, an efficient GA-based HRBDO approach integrating the proposed ASS-HRA with PRSM is developed to solve the HRBDO problem for complex engineering structures with random and interval uncertainties. The results of a numerical example demonstrate that the proposed HRBDO approach based on ASS-HRA produces good convergence performance and high efficiency. The HRBDO results of a high-speed press slider with random geometrical parameter and interval material parameters demonstrate the efficiency and versatility of the proposed approach for the design optimization of realistic complex engineering structures.

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