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Performance Analysis of Linear Precoding in Massive MIMO Systems With Finite-Alphabet Inputs

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ABSTRACT This paper investigates the performance of linear precoders in massive multiple-input multiple-output (MIMO) systems. Different from the existing research, in this paper, we consider a more realistic scenario, where the input signals are taken from finite-alphabet constellation sets, such as phase shift keying (PSK) or quadrature amplitude modulation (QAM), instead of Gaussian signals. The expressions are derived for the achievable mutual information with two commonly known linear precoders, i.e., zero forcing (ZF) and matched filter (MF), in the scenarios were perfect and imperfect channel state information (CSI) is known at the base station (BS). Also, the performance upper bound of mutual information with precoding techniques is analyzed. Both the theoretical analysis and simulation results show that ZF and MF precoders are near optimal when the number of antennas equipped at the BS is much larger than the number of users, which is similar to the case of Gaussian inputs. However, different from the Gaussian inputs, for the case of finite-alphabet inputs, the increase in the number of antennas does not always mean the improvement of performance; specifically, after the number of antennas at the BS, reaches a certain value, more antennas actually almost have no help for the performance improvement of mutual information, which is true whether the CSI is perfect or imperfect.

INDEX TERMS Finite-alphabet inputs, linear precoding, massive MIMO.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) has been widely studied during the last two decades. It has been adopted in many wireless standards due to the improved capacity and reliability. In recent years, multi-user MIMO (MU-MIMO) systems have received much attention, where typically a base station (BS) with multiple antennas simultaneously serves a set of single-antenna users. To improve the performance of MU-MIMO systems, transmit precoding at the BS is usually necessary. Precoding techniques include non-linear and linear methods. As a non-linear method, dirty-paper-coding (DPC) [1] can achieve the optimal spectral efficiency, but at the cost of high complexity; on the other hand, although linear precoders usually have low complexity, the performance loss can be significant. Additionally, in MU-MIMO systems, only a few antennas are equipped at the BS, so the improvement of spectral efficiency is actually still relatively modest.

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In order to further improve the spectral efficiency, massive MIMO systems have been proposed [2], [3], where the BS is equipped with orders of magnitude more antennas. Theoretical analysis and simulation results demonstrate that, as the number of BS antennas in a MIMO cell grows to infinity, the effects of uncorrelated noise and small-scale fading are eliminated; as a result, massive MIMO exhibits dramatic performance gains [4], and is also regarded as a key technology for 5G wireless systems [5]. Extensive research has shown that, in massive MIMO systems, signal processing techniques can be greatly simplified. Specifically, simple linear precoding methods at the BS, such as zero forcing (ZF) [6], [7] and matched filter (MF) [8], can achieve near optimal spectral efficiency when the number of BS antennas is much larger than the number of users [9]. This is an appealing advantage because excessive numbers of antennas lead to huge complexity when nonlinear precoding is employed. Therefore, linear precoder design with low complexity in massive MIMO systems has always been a hot research topic. For example, linear precoders based on Chebyshev iteration and hybrid precoding have been reported in [10] and [11], respectively.

Note that research results of massive MIMO mentioned above are obtained based on the system with Gaussian-input assumption. Although Gaussian signals can achieve the information-theoretic optimality, the input signals in practical systems are often taken from finite-alphabet constellation sets, such as phase shift keying (PSK), pulse amplitude modulation (PAM), and quadrature amplitude modulation (QAM), which are non-Gaussian distributed. Recent works have shown that optimal algorithms designed for Gaussian inputs do not achieve optimal performance for finite-alphabet inputs [12]–[15]. To maximize the mutual information over independent parallel Gaussian channels with finite-alphabet inputs, [12] provided an optimal algorithm called mercury-waterfilling. For general MIMO systems with perfect channel state information (CSI) known at the transmitter, [13] presented an iterative algorithm to yield a real-valued precoder for real-valued signals and channels; [14] investigated the precoder design in the complex field and presented a near optimal algorithm. When the transmitter only knows the statistical CSI, the optimal precoder designs were studied in [15] and [16] for MIMO broadcast and interference channels, respectively. Recently, the research on precoding for finite-alphabet inputs has been expanded to the cognitive radio networks [17], wireless power transfer systems [18], and spatial modulation MIMO systems [19]. All these works show that there is a clear difference between the mutual information with Gaussian inputs and that with finite-alphabet inputs; as a result, the same precoder exhibits different performance when the inputs are Gaussian and finite-alphabet signals, respectively. This means that the performance of a precoder needs to be re-evaluated when the inputs to a system change from Gaussian signals to finite-alphabet signals.

In [20], the authors analyzed the structure of the precoder for MIMO channels with finite-alphabet inputs, and developed a near-optimal precoder design. Moreover, when it was applied to massive MIMO systems, the proposed algorithm can be further simplified by exploiting the characteristics of physical channels. However, in [20], the investigated scenario is a single-user MIMO system, and the proposed precoder is obtained by using an iterative algorithm. So far, to the best of our knowledge, no research has been reported on the performance of simple linear precoding techniques, such as ZF and MF, in a more realistic massive MIMO system, i.e., the multi-user massive system with finite-alphabet inputs. This paper focuses on the performance of ZF and MF precoders in multi-user massive MIMO systems. Unlike existing research, this paper assumes the inputs to systems to be practical finite-alphabet signals instead of Gaussian signals. We derive the expressions of achievable mutual information with ZF and MF precoders in this case, where the scenarios with perfect and imperfect CSI known at the BS are both considered. To evaluate the performance of linear precoders, the theoretical upper bound of mutual information with precoding is also derived. Simulation results verify the correctness of theoretical analysis. Both theoretical and simulated results demonstrate that ZF and MF precoders are asymptotically optimal as the number of antennas at the BS grows. However, different from the Gaussian input case, the mutual information with finite-alphabet inputs exhibits an interesting property; that is, whether the CSI is perfect or imperfect, when the number of antennas reaches some certain value, further increasing the number of antennas can hardly improve the performance in spectral efficiency.

This paper is organized as follows. Section II describes the mutual information of finite-alphabet inputs. Section III analyzes the performance of linear precoders in massive MIMO systems, including the performance upper bound and the achievable performance of ZF and MF precoding in the cases of perfect and imperfect CSI. Section IV provides the simulation results and Section V concludes the paper.

Notation: Boldface uppercase and lowercase letters denote matrices and column vectors, respectively, and italics denote scalars. The superscript $(\cdot)^H$ and $(\cdot)^T$ represents Hermitian and transpose operations, respectively; $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix; $\mathrm{E}_x(\cdot)$ represents the expectation over x; $\exp(\cdot)$ denotes the exponential function, i.e., $\exp(x) = e^x$; $\|\cdot\|$ denotes the Euclidean norm; $\mathbb{C}$ and $\mathbb{R}$ stand for the complex and real spaces, respectively; log denotes the base two logarithm; $\mathbf{I}_{L\times L}$ denotes an identity matrix of dimension L; $\mathbf{A}^\dagger$ denotes the pseudo-inverse of matrix $\mathbf{A}$, i.e., $\mathbf{A}^\dagger = \mathbf{A}^H(\mathbf{A}\mathbf{A}^H)^{-1}$.

II. MUTUAL INFORMATION WITH FINITE-ALPHABET INPUTS

Consider an MIMO system with N_t transmit antennas and N_r receive antennas. Let $\mathbf{H} \in \mathbb{C}^{N_r\times N_t}$ denote the channel from the transmitter to the receiver; let the inputs $\mathbf{x} \in \mathbb{C}^{N_t\times 1}$ be zero mean with covariance $\mathrm{E}(\mathbf{x}\mathbf{x}^H) = \mathbf{I}_{N_t\times N_t}$. Then the received signal $\mathbf{y} \in \mathbb{C}^{N_r\times 1}$ at the receiver is given by

$$\mathbf{y} = \mathbf{H}\mathbf{W}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{W} \in \mathbb{C}^{N_t\times N_t}$ denotes the precoder; $\mathbf{n} \in \mathbb{C}^{N_r\times 1}$ is the additive white Gaussian noise (AWGN) with zero mean and covariance $\mathrm{E}(\mathbf{n}\mathbf{n}^H) = \sigma^2\mathbf{I}_{N_r\times N_r}$.

Instead of the traditional assumption of Gaussian signal, we let the signal $\mathbf{x}$ be equiprobably drawn from discrete constellations, such as PSK, PAM, or QAM, with cardinality M. Assuming that perfect CSI is known at the receiver, the mutual information between the channel input $\mathbf{x}$ and output $\mathbf{y}$ is given by [14]

$$\bar{\mathcal{I}}(\mathbf{H}) = N_t \log M - \frac{1}{M^{N_t}} \sum_{i=1}^{M^{N_t}} \mathrm{E}_{\mathbf{n}}\Bigg\{ \log \sum_{j=1}^{M^{N_t}}$$

$$\exp\Big[-(\| \mathbf{HWb}_{ij}+\mathbf{n}^2 \| - \| \mathbf{n}^2 \|)/\sigma^2\Big]\Big\} \quad (2)$$

where $\mathbf{b}_{ij}$ is equal to $(\mathbf{x}_i - \mathbf{x}_j)$ in which $\mathbf{x}_i$ and $\mathbf{x}_j$ are input signals containing N_t symbols taken from the M-ary constellation; $\mathrm{E}_{\mathbf{n}}$ stands for the expectation over noise $\mathbf{n}$. In (2), $\bar{\mathcal{I}}(\mathbf{H})$ is used to emphasize the dependence of instantaneous mutual information on channel $\mathbf{H}$.

Expression (2) involves the expectation over $\mathbf{n}$, which implies the evaluation of $2N_r$ integrals; so it is very difficult to directly compute the mutual information with finite-alphabet inputs according to (2). In [21], the authors derived the lower bound of $\bar{\mathcal{I}}(\mathbf{H})$; also, theoretical analysis and numeric results have shown that the lower bound plus a constant is a very accurate approximation to the mutual information. According to [21], the approximation expression of the mutual information is given by

$$\mathcal{I}(\mathbf{H}) = N_t \log M - \frac{1}{M^{N_t}} \sum_{i=1}^{M^{N_t}} \log \sum_{j=1}^{M^{N_t}} \exp\left(-\frac{\mathbf{c}_{ij}^H \mathbf{c}_{ij}}{2\sigma^2}\right) \quad (3)$$

with $\mathbf{c}_{ij} = \mathbf{HWb}_{ij}$.

From (3), $\mathcal{I}(\mathbf{H})$ is a closed form expression, and it provides a very good approximation to the mutual information $\bar{\mathcal{I}}(\mathbf{H})$. So, for simplicity, in this paper, we use (3) to compute the mutual information with finite-alphabet inputs.

When channels change slowly enough to be reliably fed back to the transmitter with negligible delay, the ergodic limit on mutual information is the average of $\mathcal{I}(\mathbf{H})$ over channel realizations, i.e.,

$$\mathcal{I} = \mathrm{E}_{\mathbf{H}} \{\mathcal{I}(\mathbf{H})\}. \quad (4)$$

III. PERFORMANCE OF LINEAR PRECODERS IN MASSIVE MIMO SYSTEMS

To improve the performance, transmit precoding is commonly required in the downlink of an MU-MIMO system. Generally speaking, precoding techniques can be classified into two categories, i.e., nonlinear and linear techniques. The former includes DPC and vector perturbation (VP) [22], while the latter includes MF and ZF. For regular MIMO systems, nonlinear precoders have better performance, but the cost is high complexity. However, with the increase of the number of transmit antennas, it is shown that linear precoders can achieve near optimal rate performance, which makes low-complexity linear precoding very attractive for massive MIMO systems. This conclusion results from the assumption of Gaussian inputs. We next investigate the performance of linear precoders in realistic massive MIMO systems with finite-alphabet inputs.

Consider an MU-MIMO system with a single cell, where the BS is equipped with N antennas and serves K single-antenna users. Let $\mathbf{s} \in \mathbb{C}^{K\times 1}$ denote the information vector intended for K users, and each component of $\mathbf{s}$ has average power ρ/K, in which ρ denotes the downlink transmit power. The received signal vector $\mathbf{z} \in \mathbb{C}^{K\times 1}$ at all K users can be expressed as

$$\mathbf{z} = \mathbf{GPs} + \mathbf{v} \quad (5)$$

where $\mathbf{P} \in \mathbb{C}^{N\times K}$ denotes a linear precoder; $\mathbf{G} \in \mathbb{C}^{K\times N}$ is the downlink channel matrix; $\mathbf{v} \in \mathbb{C}^{K\times 1}$ is a zero-mean noise vector with complex Gaussian distribution and covariance $\sigma^2 \mathbf{I}_{K\times K}$. Note that, in this paper, the channel is assumed to be uncorrelated, and thus the entries of $\mathbf{G}$ are independently identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance.

For the scenario described by (5), assume that the inputs in $\mathbf{s}$ are finite-alphabet signals and that the number of BS antennas is much larger than the number of users and grows to infinity, i.e., $N \gg K$ and $N \to \infty$. Based on these premises, we next derive the mutual information upper bound of MU-MIMO systems and achievable mutual information with ZF and MF precoders.

A. PERFORMANCE UPPER BOUND

For MU-MIMO systems, the best performance is achieved if all the channel energy to user k is delivered to user k without any inter-user interference [23]. In this case, the received signal at user k is given by

$$z_k = \sqrt{\sum_{q=1}^{N} |g_{qk}|^2} s_k + v_k \quad (6)$$

where g_{qk} denotes the channel gain from the q-th antenna to user k, and v_k is the additive noise at user k.

Actually, for user k, (6) describes a single-input single-output (SISO) communication scenario. According to (3), the mutual information upper bound of user k is given by

$$\mathcal{I}_{UB}^{(k)}(\mathbf{G}) = \log M - \frac{1}{M} \sum_{i=1}^{M} \log \sum_{j=1}^{M} \exp\left(-\frac{|e_{ij}|^2 \cdot \sum_{q=1}^{N} |g_{qk}|^2}{2\sigma^2}\right) \quad (7)$$

where e_{ij} is equal to $(s_i - s_j)$ in which s_i and s_j denote input symbols taken from the M-ary constellation.

As N grows, we have $(\sum_{q=1}^{N} |g_{qk}|^2)/N \to 1$. For massive MIMO systems with large antenna arrays at BS, substituting $\sum_{q=1}^{N} |g_{qk}|^2 = N$ into (7) yields the expression of mutual information upper bound per user, which is given by

$$\mathcal{I}_{UB}^{(\text{Massive})} = \log M - \frac{1}{M} \sum_{i=1}^{M} \log \sum_{j=1}^{M} \exp\left(-\frac{|e_{ij}|^2 \cdot N}{2\sigma^2}\right). \quad (8)$$

B. PERFORMANCE OF LINEAR PRECODERS WITH PERFECT CSI

Now we move on to practical precoding methods. Let us first investigate the performance of ZF and MF precoders in the scenario where perfect CSI is available at the BS.

1) ZF PRECODER

ZF is a popular linear precoding method, which inverts the downlink channel matrix by means of the pseudo-inverse. When ZF is used, the precoder is expressed as

$$\mathbf{P} = \frac{1}{\sqrt{\beta}}\mathbf{G}^{\dagger} = \frac{1}{\sqrt{\beta}}\mathbf{G}^{H}(\mathbf{G}\mathbf{G}^{H})^{-1} \tag{9}$$

where $\beta = \mathrm{Tr}(\mathbf{G}\mathbf{G}^{H})^{-1}/K$ is the power normalization factor.

With the ZF precoder, the received signal at user k is

$$z_k = \frac{s_k}{\sqrt{\beta}} + v_k. \tag{10}$$

Now the mutual information of user k is given by

$$\mathcal{I}_{ZF}^{(k)}(\mathbf{G}) = \log M - \frac{1}{M}\sum_{i=1}^{M}\log\sum_{j=1}^{M}\exp\left(-\frac{|e_{ij}|^2}{2\sigma^2\cdot\beta}\right). \tag{11}$$

Let $\mu = N/K$. Then, when N grows large, $\mathrm{Tr}(\mathbf{G}\mathbf{G}^{H})^{-1}$ converges to a fixed deterministic value $1/(\mu-1)$, i.e., $\mathrm{Tr}(\mathbf{G}\mathbf{G}^{H})^{-1} \to 1/(\mu-1)$ [24]. Based on this, for massive MIMO, we have $\beta \to 1/(N-K)$, and thus (11) can be rewritten as

$$\mathcal{I}_{ZF}^{(\mathrm{Massive})} = \log M - \frac{1}{M}\sum_{i=1}^{M}\log\sum_{j=1}^{M}\exp\left(-\frac{|e_{ij}|^2\cdot(N-K)}{2\sigma^2}\right). \tag{12}$$

2) MF PRECODER

MF is another popular linear precoding method, which can be expressed as

$$\mathbf{P} = \frac{1}{\sqrt{\eta}}\mathbf{G}^{H} \tag{13}$$

with $\eta = \mathrm{Tr}(\mathbf{G}\mathbf{G}^{H})/K$ denoting the power normalization factor. When the MF precoder is used, (5) can be rewritten as

$$\mathbf{z} = \frac{1}{\sqrt{\eta}}\mathbf{G}\mathbf{G}^{H}\mathbf{s} + \mathbf{v}. \tag{14}$$

The received signal at user k is given by

$$z_k = \frac{1}{\sqrt{\eta}}\mathbf{g}_k\mathbf{g}_k^{H}s_k + \frac{1}{\sqrt{\eta}}\sum_{j=1,j\neq k}^{K}\mathbf{g}_k\mathbf{g}_j^{H}s_j + v_k \tag{15}$$

where $\mathbf{g}_k \in \mathbb{C}^{1\times N}$ is the k-th row of $\mathbf{G}$, i.e., the channel vector from the BS antennas to the k-th user.

$$\mathcal{I}_{MF}^{(k)}(\mathbf{G}) = \log M - \frac{1}{M}\sum_{i=1}^{M}\log\sum_{j=1}^{M} \exp\left(-\frac{|e_{ij}|^2\cdot\frac{1}{\eta}|\mathbf{g}_k\mathbf{g}_k^{H}|^2}{2\left(\sigma^2+\frac{1}{\eta}\sum_{j=1,j\neq k}^{K}|\mathbf{g}_k\mathbf{g}_j^{H}|^2|s_j|^2\right)}\right) \tag{16}$$

where $|s_j|^2 = \rho/K$.

According to the law of large numbers, we have

$$\mathrm{E}\left\{|\mathbf{g}_k\mathbf{g}_k^{H}|^2\right\} = N(N+1) \tag{17}$$

and

$$\mathrm{E}\left\{\sum_{j=1,j\neq k}^{K}|\mathbf{g}_k\mathbf{g}_j^{H}|^2\right\} = N(K-1). \tag{18}$$

In addition, as N grows, we have $\mathrm{Tr}(\mathbf{G}\mathbf{G}^{H}) \to NK$, which means

$$\eta \to N. \tag{19}$$

By approximating $|\mathbf{g}_k\mathbf{g}_k^{H}|^2$ and $\sum_{j=1,j\neq k}^{K}|\mathbf{g}_k\mathbf{g}_j^{H}|^2$ in (16) with their expectations, respectively, and substituting (19) into (16), we finally obtain the mutual information per user in massive MIMO systems with MF precoding, i.e.,

$$\mathcal{I}_{MF}^{(\mathrm{Massive})} = \log M - \frac{1}{M}\sum_{i=1}^{M}\log\sum_{j=1}^{M}\exp\left(-\frac{|e_{ij}|^2\cdot(N+1)}{2\left(\sigma^2+\frac{K-1}{K}\rho\right)}\right). \tag{20}$$

From (8), (12), and (20), for massive MIMO systems with finite-alphabet inputs and perfect CSI, we have remarks as follows:

(i) $\mathcal{I}_{UB}^{(\mathrm{Massive})}$, $\mathcal{I}_{ZF}^{(\mathrm{Massive})}$, and $\mathcal{I}_{MF}^{(\mathrm{Massive})}$ are related to the number N of antennas. As $N \gg K$ and $N \to \infty$, the second term on the right-hand side of (8), (12), and (20) approaches zero. Therefore, when the number of BS antennas goes to infinity, linear precoders ZF and MF show asymptotic optimality; in other words, in this case, the achievable mutual information with ZF and MF precoders almost reaches the performance upper bound, which is actually similar to the case of Gaussian inputs.

(ii) Although the growth of the number N of BS antennas can lead to the increase of mutual information, the maximum of mutual information does not exceed $\log M$ for each user. According to (8), (12), and (20), when N grows to a certain level, more antennas will not lead to obvious increase of mutual information. In comparison, when the inputs are Gaussian signals, the mutual information keeps increasing without limit as the number of BS antennas grows.

C. PERFORMANCE OF LINEAR PRECODERS WITH IMPERFECT CSI

Now we turn our attention to the scenarios in which the BS only has imperfect CSI. Let $\hat{\mathbf{G}} \in \mathbb{C}^{K\times N}$ denote the minimum mean square error (MMSE) channel estimation of channel $\mathbf{G}$, and then $\hat{\mathbf{G}}$ is given by [23], [27]

$$\hat{\mathbf{G}} = \sqrt{1-\tau^2}\mathbf{G} + \tau\mathbf{U} \tag{21}$$

where $0 \le \tau \le 1$ represents the estimation error parameter, and $\mathbf{U} \in \mathbb{C}^{K\times N}$ denotes an error matrix whose entries are i.i.d. complex Gaussian random variables with zero mean and unit variance. In this case, the received signal vector at K users, i.e., (5), can be rewritten as

$$\mathbf{z} = \mathbf{G}\hat{\mathbf{P}}\mathbf{s} + \mathbf{v} \tag{22}$$

where $\hat{\mathbf{P}}$ denotes the precoder designed based on the estimated channel $\hat{\mathbf{G}}$. Thus, the received signal at user k can be given by

$$z_k = \mathbf{g}_k\hat{\mathbf{p}}_k s_k + \sum_{j=1, j\neq k}^{K} \mathbf{g}_k\hat{\mathbf{p}}_j s_j + v_k \tag{23}$$

where $\hat{\mathbf{p}}_k$ denotes the k-th column of $\hat{\mathbf{P}}$, i.e., the precoding vector for user k. In (23), the first term of the received signal z_k is the desired signal for user k, and the second term represents the inter-user interference from other $K-1$ users.

1) ZF PRECODER

When the channel estimate $\hat{\mathbf{G}}$ is exploited for ZF precoding in the downlink transmission, the precoding matrix can be written as

$$\hat{\mathbf{P}}_{ZF} = \frac{1}{\sqrt{\hat{\beta}}}\hat{\mathbf{G}}^{\dagger} = \frac{1}{\sqrt{\hat{\beta}}}\hat{\mathbf{G}}^{H}(\hat{\mathbf{G}}\hat{\mathbf{G}}^{H})^{-1}. \tag{24}$$

Here, to remain the transmit power after precoding unchanged, the power normalization factor satisfies $\hat{\beta} = \mathrm{Tr}(\hat{\mathbf{G}}\hat{\mathbf{G}}^{H})^{-1}/K$.

According to (23) and (3), the mutual information of user k achieved by ZF precoding with imperfect CSI is given by

$$\hat{\mathcal{I}}_{ZF}^{(k)}(\hat{\mathbf{G}}) = \log M - \frac{1}{M}\sum_{i=1}^{M}\log\sum_{j=1}^{M} \exp\left(-\frac{|e_{ij}|^2\cdot|\mathbf{g}_k\hat{\mathbf{p}}_{ZF,k}|^2}{2(\sigma^2+\sum_{j=1,j\neq k}^{K}|\mathbf{g}_k\hat{\mathbf{p}}_{ZF,j}|^2|s_j|^2)}\right) \tag{25}$$

where $\hat{\mathbf{p}}_{ZF,k}$ denotes the k-th column of $\hat{\mathbf{P}}_{ZF}$.

Using the derivations similar to Section III in [28], for massive MIMO, we have

$$\hat{\beta} \approx \frac{1}{N-K} \tag{26}$$

$$E\left\{|\mathbf{g}_k\hat{\mathbf{p}}_{ZF,k}|^2\right\} \approx (N-K)(1-\tau^2) \tag{27}$$

and

$$E\left\{\sum_{j=1,j\neq k}^{K}\left|\mathbf{g}_k\hat{\mathbf{p}}_{ZF,j}\right|^2\right\} \approx (K-1)\tau^2. \tag{28}$$

Substituting (27) and (28) into (25), that is, replacing the related terms with their expected values, and noting $|s_j|^2 = \rho/K$, we can obtain the mutual information per user in massive MIMO systems with ZF precoder and imperfect CSI, which is given by

$$\hat{\mathcal{I}}_{ZF}^{(\text{Massive})} = \log M - \frac{1}{M}\sum_{i=1}^{M}\log\sum_{j=1}^{M} \exp\left(-\frac{|e_{ij}|^2\cdot(N-K)(1-\tau^2)}{2(\sigma^2+\rho(K-1)\tau^2/K)}\right). \tag{29}$$

2) MF PRECODER

When only the imperfect CSI is known at the BS, the precoding matrix for the MF precoder is written as

$$\hat{\mathbf{P}}_{MF} = \frac{1}{\sqrt{\hat{\eta}}}\hat{\mathbf{G}}^{H} \tag{30}$$

where the power normalization factor is $\hat{\eta} = \mathrm{Tr}(\hat{\mathbf{G}}\hat{\mathbf{G}}^{H})/K$.

In this case, according to (23) and (3), the mutual information of user k is expressed as

$$\hat{\mathcal{I}}_{MF}^{(k)}(\hat{\mathbf{G}}) = \log M - \frac{1}{M}\sum_{i=1}^{M}\log\sum_{j=1}^{M} \exp\left(-\frac{|e_{ij}|^2\cdot|\mathbf{g}_k\hat{\mathbf{p}}_{MF,k}|^2}{2(\sigma^2+\sum_{j=1,j\neq k}^{K}|\mathbf{g}_k\hat{\mathbf{p}}_{MF,j}|^2|s_j|^2)}\right) \tag{31}$$

where $\hat{\mathbf{p}}_{MF,k}$ denotes the k-th column of $\hat{\mathbf{P}}_{MF}$.

Similarly, by using the derivations in [28], for massive MIMO systems with MF precoding, we can obtain

$$\hat{\eta} \approx N \tag{32}$$

$$E\left\{|\mathbf{g}_k\hat{\mathbf{p}}_{MF,k}|^2\right\} \approx N(1-\tau^2)+1 \tag{33}$$

and

$$E\left\{\sum_{j=1,j\neq k}^{K}\left|\mathbf{g}_k\hat{\mathbf{p}}_{MF,j}\right|^2\right\} \approx K-1. \tag{34}$$

By introducing (33) and (34) into (31) and recalling $|s_j|^2 = \rho/K$, the mutual information per user in massive MIMO systems with MF precoder and imperfect CSI can be calculated as

$$\hat{\mathcal{I}}_{MF}^{(\text{Massive})} = \log M - \frac{1}{M}\sum_{i=1}^{M}\log\sum_{j=1}^{M} \exp\left(-\frac{|e_{ij}|^2\cdot\left(N(1-\tau^2)+1\right)}{2(\sigma^2+\rho(K-1)/K)}\right). \tag{35}$$

Based on (8), (29), and (35), for the case of imperfect CSI, we have remarks as follows:

(i) The imperfect CSI has a negative impact on the performance of achievable mutual information. However, given τ, $\hat{\mathcal{I}}_{ZF}^{(\text{Massive})}$ and $\hat{\mathcal{I}}_{MF}^{(\text{Massive})}$ can approach the performance upper bound $\mathcal{I}_{UB}^{(\text{Massive})}$ as $N \to \infty$, which means that ZF and MF precoders still show asymptotic optimality even in the imperfect CSI case. On other hand, according to Table 1 in [23], for the systems with Gaussian inputs and imperfect CSI, the signal-to-interference-plus-noise ratio (SINR) expressions for each user in the cases of upper bound, ZF and MF precoding are given, respectively, by

$$\mathrm{SINR}_{UB} = \rho N/K \tag{36}$$

$$\mathrm{SINR}_{ZF} = \frac{(1-\tau^2)\rho(N-K)/K}{\tau^2\rho+1} \tag{37}$$

and

$$\mathrm{SINR}_{MF} = \frac{(1-\tau^2)\rho N/K}{\rho+1}. \tag{38}$$

From the expressions above, it can be concluded that, in the case of Gaussian inputs, due to the presence of τ, even as N grow very large, there is still a performance gap between the upper bound and the mutual information achieved by ZF and MF precoders.

(ii) Similar to the perfect CSI case, it can be seen from (29) and (35) that more antennas will not bring obvious performance gain in mutual information when the number N of antennas increases to a certain level.

So far, for a single user under the assumptions of perfect and imperfect CSI, we have derived the mutual information upper bound and achievable mutual information with ZF and MF precoders. Adding up all the mutual information of K users in the three cases yields the throughput upper bound and achievable throughput with the two linear precoders, respectively.

IV. SIMULATION RESULTS

In this section, simulation results are provided to verify the theoretical analysis above. In the simulations, MIMO channels are assumed to be complex Gaussian random matrix; that is, each entry satisfies the complex Gaussian distribution with zero mean and unit variance. We random generated 10^4 channel realizations. For each realization, we calculated the mutual information upper bound by using (7), the achievable mutual information with perfect CSI by using (11) and (16), and the achievable mutual information with imperfect CSI by using (25) and (31), respectively. The averages of mutual information over these channel realizations (refer to (4)) are used as numerical results of mutual information. The theoretical mutual information in different cases is calculated according to (8), (12), (20), (29), and (35), respectively.

A. PERFECT CSI CASE

Figs. 1-3 provide the results under the assumption of perfect CSI. Fig. 1 compares the mutual information per user with finite-alphabet inputs achieved by using theoretical analysis and numerical simulations, respectively, where a single-cell MU-MIMO system is considered with $K = 5$ single-antenna users and binary phase shift keying (BPSK) modulation (i.e., modulation order $M = 2$). The lines represent the theoretical results while the markers denote the simulation results. As shown in Fig. 1, the simulation results are consistent with the theoretical analysis. In the derivation of expressions (8), (12), and (20), we assume the number N of antennas to be large. Therefore, when $N = 20$, there is a slight difference between the theoretical results and simulation results. However, when N grows to 30 and 50, the theoretical results and simulation results become almost identical, which shows that our analysis using large N provides a tight approximation for finite N.

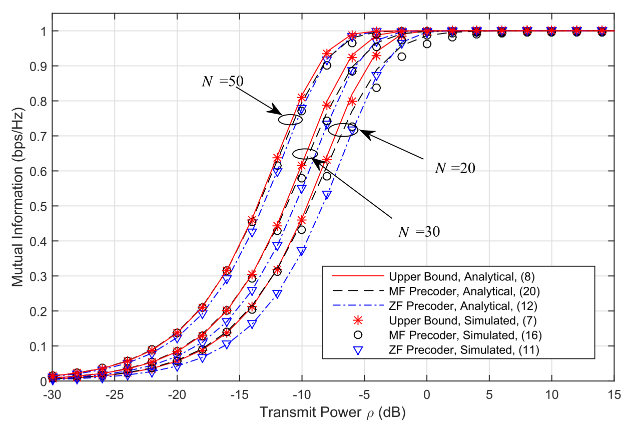


FIGURE 1. Mutual information per user comparison between theoretical analysis and simulation results in single-cell MU-MIMO systems with finite-alphabet inputs ($K = 5$ users, BPSK modulation).

Fig. 1 proves the correctness of the theoretical analysis in Sections III-A and III-B.

Different from Fig. 1, where mutual information per user is considered, Fig. 2 provides the theoretical and simulation results of the throughput in a single-cell MU-MIMO system with $K = 10$ users and quadrature phase shift keying (QPSK) signals (modulation order $M = 4$) as the finite-alphabet inputs. Again, the theoretical results show good agreement with the simulation results. Note that, in Figs. 1 and 2, it can be observed that there is a performance gap between linear precoders and upper bound for a small value of N. Especially for the MF precoders in Fig. 2, when $N = 30$, the performance gap is very obvious. This is because the effect of interference power $(K-1)\rho/K$ in (20) can not be ignored for a small value of N. However, as the number of BS antennas grows, the performance difference between the linear precoders and upper bound decreases. When $N = 50$ in Fig. 1 or $N = 100$ in Fig. 2, the performance of ZF and MF precoders is very close to the performance upper bound. This property is similar to the case of Gaussian inputs; that is, in massive systems, simple linear precoding can achieve near optimal mutual information.

Another observation from Figs. 1 and 2 is that the mutual information (or throughput) with finite-alphabet inputs is bounded, which approaches $\log M$ per user (or $K\log M$ for K users) as the transmit power increases. This is because raising the transmit power at the BS can lead to the increase of $|e_{ij}|^2$, which finally improves the mutual information. However, according to (8), (12), and (20), the mutual information does not exceed the bound $\log M$ (or $K\log M$ for throughput).

Fig. 3 shows the relation between the throughput and the number N of antennas, where a single cell serving $K = 5$ users is considered with QPSK inputs and transmit power $\rho = 0$ dBW. As shown in Fig. 3, for the finite-alphabet input case, the achievable throughput with ZF and MF precoders steadily increases and asymptotically approaches the performance upper bound as N increases. However, the throughput still does not exceed the limit $K\log M$. For ZF and MF precoders,

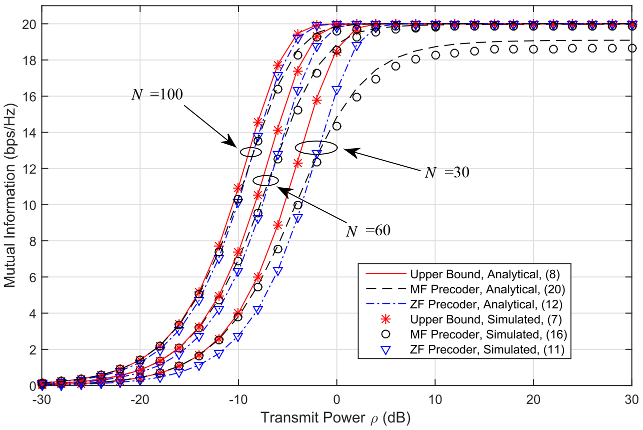


FIGURE 2. Throughput performance of linear precoders in single-cell MU-MIMO systems with finite-alphabet inputs ($K = 10$ users, QPSK modulation).

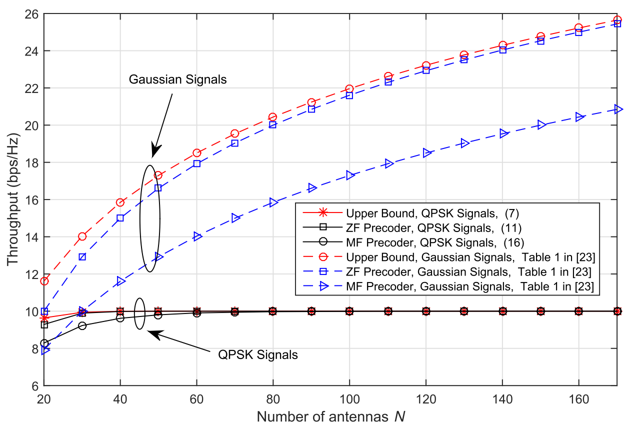


FIGURE 3. Throughput versus the number N of antennas in single-cell MU-MIMO systems with finite-alphabet inputs ($K = 5$ users, QPSK modulation, $\rho = 0$ dBW).

when N increases to 30 and 60, respectively, the corresponding achievable throughput almost reach the limit 10 bps/Hz for QPSK; in this case, more antennas at the BS actually can not lead to obvious performance improvement unless higher modulation order is used. This property is distinctly different from the case of Gaussian inputs. In Fig. 3, the performance upper bound and achievable throughput with ZF and MF precoders for Gaussian inputs are also presented, which are obtained by using the expressions in Table 1 in [23]. It is easy to find that, for Gaussian inputs, the increase of the number N of antennas causes continuous improvement in the throughput performance. Also, it is can be seen that, in the Gaussian input case, the ZF precoder shows a significant superiority over the MF precoder, whose performance is very close to the upper bound as N grows.

B. IMPERFECT CSI CASE

Now we turn to the case of imperfect CSI. Throughout the simulations, the estimation error parameter τ is assumed to be 0.1. Fig. 4 provides the theoretical and simulated mutual information per user with imperfect CSI, where a single-cell

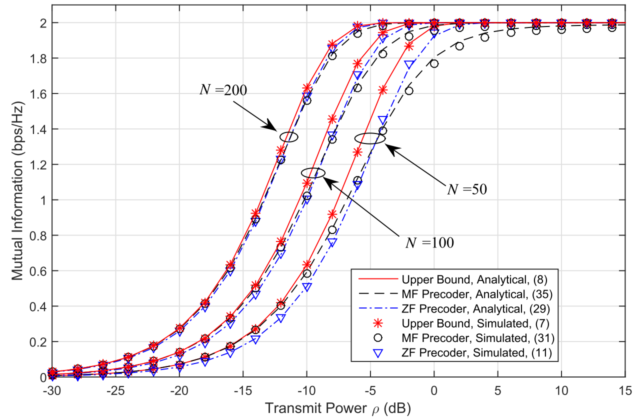


FIGURE 4. Mutual information per user comparison between theoretical analysis and simulation results in single-cell MU-MIMO systems with imperfect CSI and finite-alphabet inputs ($K = 10$ users, QPSK modulation, $\tau = 0.1$).

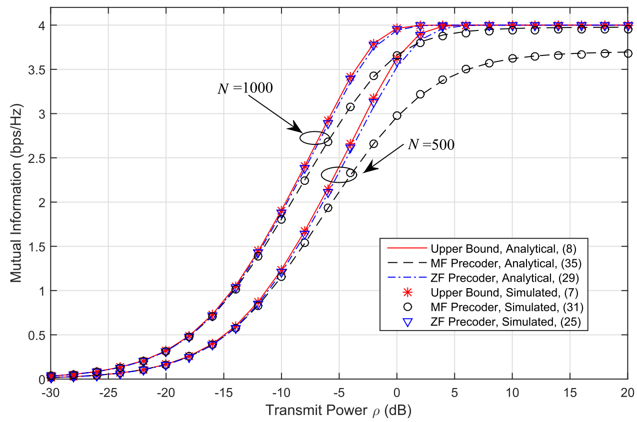


FIGURE 5. Mutual information per user comparison between theoretical analysis and simulation results in single-cell MU-MIMO systems with imperfect CSI and finite-alphabet inputs ($K = 10$ users, 16QAM modulation, $\tau = 0.1$).

MU-MIMO system serving $K = 10$ users is considered. The inputs are QSPK signals. It can been observed that, whether for MF or ZF precoder, the simulated results are in very good agreement with the analytical results, even for a small value of N, for example, $N = 50$. Fig. 4 validates that, for the case of imperfect CSI, our theoretical analysis in Section III-C is correct. Besides, from Fig. 4, we can see that, as the number of BS antennas grows, the performance of MF and ZF precoders gradually approaches the performance upper bound, which is similar to the case of perfect CSI.

In Fig. 5, the setup of simulation scenario is the same as that in Fig. 4 except that the modulation scheme is changed to 16 quadrature amplitude modulation (16QAM) in which the modulation order is $M = 16$. Similar results can be seen in Fig. 5. Specifically, for the 16QAM modulation, the simulation results are still consistent with our theoretical analysis; moreover, the performance gap between the linear precoders and the upper bound decreases with the increase of N.

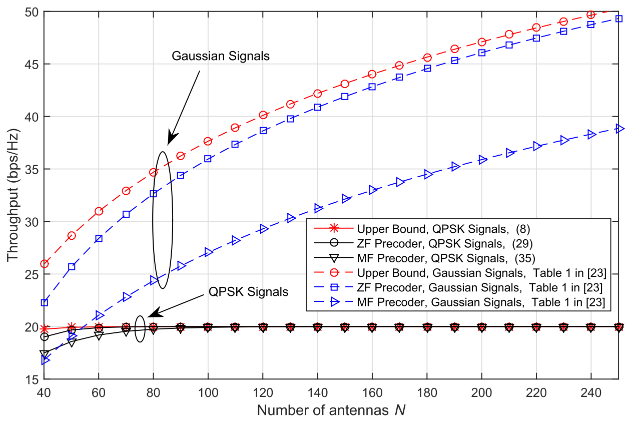


FIGURE 6. **Throughput versus the number N of antennas in single-cell MU-MIMO systems with imperfect CSI and finite-alphabet inputs ($K = 10$ users, QPSK modulation, $\rho = 1$ dBW, $\tau = 0.1$).**

Fig. 6 shows the throughput versus the number of BS antennas in a single-cell MU-MIMO system with $K = 10$ users and imperfect CSI, where QPSK modulation is employed and the transmit power is assumed to be $\rho = 1$ dBW. As shown in Fig. 6, when N equals 50 for ZF precoder and 90 for MF precoder, respectively, the corresponding achievable mutual information almost reaches the upper bound, which means that, in this case, increasing the number N of antennas can not cause more performance gain. For comparison, the case of Gaussian inputs is also illustrated, which is obtained according to Table 1 in [23]. As seen in this figure, with Gaussian inputs, the upper bound and the mutual information achieved by MF and ZF precoders keep growing with the increase of the number of BS antennas. Moreover, as mentioned earlier, due to the presence of of τ, even when N grows very large, there is still a significant performance gap between the upper bound and the achievable mutual information.

V. CONCLUSION

In this paper, the mutual information in realistic massive MIMO systems with finite-alphabet inputs has been investigated. We have derived the mutual information expression for massive MIMO with finite-alphabet inputs when there is no interference across the users, and used it as the performance upper bound. Further, the achievable mutual information expressions for two typical linear precoders, i.e., ZF and MF precoders, have also been derived. Both the scenarios with perfect and imperfect CSI have been considered. Both the theoretical analysis and simulation results have shown that, for massive MIMO systems with finite-alphabet inputs, simple linear precoders also offer asymptotically optimal performance, which is similar to the Gaussian input case. However, different from Gaussian inputs, the mutual information with finite-alphabet inputs can not grow infinitely large with the increase of the number of BS antennas, which means that, in realistic massive MIMO systems with finite-alphabet inputs, excessive number of antennas actually can not bring about obvious performance improvement. Note that, in this paper, the antennas at the BS are assumed to be uncorrelated. The performance of linear precoders in massive MIMO systems with correlated antennas is a very interesting research topic left for further studies.

REFERENCES

[1] M. H. M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. IT-29, no. 3, pp. 439–441, May 1983.

[2] G. Fodor, N. Rajatheva, W. Zirwas, L. Thiele, M. Kurras, K. Guo, A. Tolli, J. H. Sorensen, and E. de Carvalho, "An overview of massive MIMO technology components in METIS," *IEEE Commun. Mag.*, vol. 55, no. 6, pp. 155–161, Jun. 2017.

[3] S. Yuan and Q. Liang, "Game theoretical method for sum-rate maximization in full-duplex massive MIMO heterogeneous networks," *Signal Process.*, vol. 126, pp. 4–11, Sep. 2016.

[4] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[5] V. W. Wong, R. Schober, D. W. K. Ng, and L.-C. Wang, *Key Technologies for 5G Wireless Systems*. Cambridge, U.K.: Cambridge Univ. Press, 2017.

[6] L. Zhao, Y. Wang, and P. Chargé, "Zero-forcing DPC beamforming design for multiuser MIMO broadcast channels," *Signal Process.*, vol. 108, pp. 489–497, Mar. 2015.

[7] L. Wang, J. Li, J.-K. Zhang, Z. Dong, and J. Huang, "Sum rate analysis for precoder design in distributed MIMO systems over composite fading channels," *IEEE Access*, vol. 6, pp. 27480–27490, 2018.

[8] S. Atapattu, P. Dharmawansa, C. Tellambura, and J. Evans, "Exact outage analysis of multiple-user downlink with MIMO matched-filter precoding," *IEEE Commun. Lett.*, vol. 21, no. 1, pp. 2754–2757, Dec. 2017.

[9] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 742–758, Oct. 2014.

[10] C. Zhang, Z. Li, L. Shen, F. Yan, M. Wu, and X. Wang, "A low-complexity massive MIMO precoding algorithm based on Chebyshev iteration," *IEEE Access*, vol. 5, pp. 22545–22551, Oct. 2017.

[11] M. Cui, W. Zou, Y. Wang, and R. Zhang, "Low complexity joint hybrid precoding algorithm for millimeter wave MIMO systems," *IEEE Access*, vol. 6, pp. 56423–56432, 2018.

[12] A. Lozano, A. M. Tulino, and S. Verdú, "Optimum power allocation for parallel Gaussian channels with arbitrary input distributions," *IEEE Trans. Inf. Theory*, vol. 52, no. 7, pp. 3033–3051, Jul. 2006.

[13] M. Lamarca, "Linear precoding for mutual information maximization in MIMO systems," in *Proc. Int. Symp. Wireless Commun. Syst.*, Sep. 2009, pp. 26–30.

[14] C. Xiao, Y. R. Zheng, and Z. Ding, "Globally optimal linear precoders for finite alphabet signals over complex vector Gaussian channels," *IEEE Trans. Signal Process.*, vol. 59, no. 7, pp. 3301–3314, Jul. 2011.

[15] W. Zeng, C. Xiao, M. Wang, and J. Lu, "Linear precoding for finite-alphabet inputs over MIMO fading channels with statistical CSI," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 3134–3148, Jun. 2012.

[16] Y. Zhang, J. Xu, and S. Xu, "Linear precoding for finite-alphabet inputs over mimo interference channels by exploiting SCSI," in *Proc. IEEE Int. Conf. Commun. Syst. (ICCS)*, Dec. 2016, pp. 1–6.

[17] W. Zeng, Y. R. Zheng, and C. Xiao, "Multiantenna secure cognitive radio networks with finite-alphabet inputs: A global optimization approach for precoder design," *IEEE Trans. Wireless Commun.*, vol. 15, no. 4, pp. 3044–3057, Apr. 2016.

[18] X. Zhu, W. Zeng, and C. Xiao, "Precoder design for simultaneous wireless information and power transfer systems with finite-alphabet inputs," *IEEE Trans. Veh. Technol.*, vol. 66, no. 10, pp. 9085–9097, Oct. 2017.

[19] X. Guan, Y. Cai, and W. Yang, "On the mutual information and precoding for spatial modulation with finite alphabet," *IEEE Wireless Commun. Lett.*, vol. 2, no. 4, pp. 383–386, Aug. 2013.

[20] Y. Wu, D. W. K. Ng, C. W. Wen, R. Schober, and A. Lozano, "Low-complexity MIMO precoding for finite-alphabet signals," *IEEE Trans. Wireless Commun.*, vol. 16, no. 7, pp. 4571–4584, Jul. 2017.

[21] W. Zeng, C. Xiao, and J. Lu, "A low-complexity design of linear precoding for MIMO channels with finite-alphabet inputs," *IEEE Wireless Commun. Lett.*, vol. 1, no. 1, pp. 38–41, Feb. 2012.

[22] B. M. Hochwald, C. B. Peel, and A. L. Swindlehust, "A vector-perturbation technique for near-capacity multiantenna multiuser communication—Part II: Perturbation," *IEEE Trans. Commun.*, vol. 53, no. 5, pp. 537–544, May 2005.

[23] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, pp. 40–46, Jan. 2013.

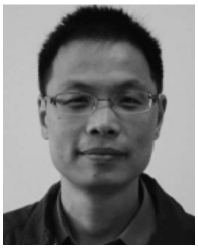
[24] L.-U. Choi and R. D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 20–24, Jan. 2004.

[25] W. Wu, C. Xiao, and X. Gao, "Multicell massive MIMO multicast transmission with finite-alphabet inputs," *IEEE Trans. Veh. Technol.*, to be published, doi: 10.1109/TVT.2019.2917069.

[26] W. Wu, K. Wang, W. Zeng, Z. Ding, and C. Xiao, "Cooperative multicell MIMO downlink precoding with finite-alphabet inputs," *IEEE Trans. Commun.*, vol. 63, no. 3, pp. 766–779, Mar. 2015.

[27] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1811–1822, May 2010.

[28] D. Mi, M. Dianati, L. Zhang, S. Muhaidat, and R. Tafazolli, "Massive MIMO performance with imperfect channel reciprocity and channel estimation error," *IEEE Trans. Commun.*, vol. 65, no. 9, pp. 3734–3748, Sep. 2017.



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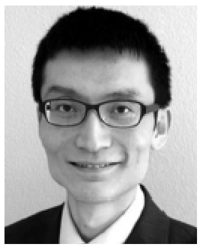

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