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# Log Likelihood Monitoring for Multimode Process Using Variational Bayesian Mixture Factor Analysis Model

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**ABSTRACT** When a traditional mixture factor analysis (MFA) model is used for multimode process monitoring, the determination of parameter is complex, and the construction of monitoring statistics only considers the expectation in probability distributions of factor space and residual space. In this paper, a novel fault detection method based on a variational Bayesian MFA model for multimode process is introduced. The parameters of the MFA model structure, namely the number of local factor analyzer and the reduced dimensionality inside each factor analyzer, can be easily obtained through the birth-and-death Markov chain Monte Carlo algorithm and the variational inference technique. After parameter estimation for the Bayesian MFA model is done, a new monitoring index called negative variational log likelihood is developed by utilizing the whole information in probability distribution functions of all parameters. At last, two case studies, including a numerical example and the Tennessee Eastman (TE) process, verify the effectiveness and feasibility of the proposed monitoring scheme.

**INDEX TERMS** Fault detection, multimode process, mixture factor analysis.

## I. INTRODUCTION

Process monitoring, including fault detection and fault diagnosis, plays critical role in the successful and safe operation of complex chemical processes. Because accurate first-principles dynamic models are hard to obtain for most industrial processes, existing process monitoring approaches are popularly based on measurement data acquired from operating processes [1]. Among these "data-based" methods, most popular ones are principal component analysis (PCA) [2], partial least-squares (PLS) [3] and independent component analysis (ICA) [4]. But the above conventional monitoring approaches have no probabilistic interpretation of measurement data for the reason that the latent variable model is constructed in the deterministic way. Compared to traditional techniques, probabilistic methods such as probabilistic principal component analysis (PPCA) [5], maximum likelihood PCA (MLPCA) [6] and factor analysis (FA) [7] demonstrate more satisfactory monitoring performance. This is because the probabilistic manner has following advantages: (1) the latent variable model is achieved in probability density space so that statistical decisions can be made; (2) random noises existing in process variables are considered; (3) missing values in data set can be handled. PPCA, MLPCA and FA have been employed for process monitoring and good monitoring performance has been gained [8]–[15].

When it comes to multimode process, measurement data of multiple modes are comprised of several data clusters, which need different treatment. A few mixture models are proposed to deal with this problem. Traditional latent variable models are extended to its "mixture" form, leading to MixPCA [16], multiple PCA [17], multiple PLS [18] and their variants. In addition, some probabilistic mixture models are utilized for multimode process monitoring, for example, finite Gaussian mixture model (GMM) [19], [20] and hidden Markov model (HMM) [21]–[23]. The above methods achieved satisfactory monitoring results but PCA-kind approaches lack probabilistic expression while probabilistic mixture models

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(such as GMM) pay no attention to dimensionality reduction and random noise of process variables. As a comparison, the mixture form of probabilistic latent variable model has its own merits. Choi *et al.* [24] used MLPCA mixture to model normal data and two monitoring statistics are developed to detect faults in the principal component space and the residual space through the conventional way. Ge and Song employed Bayesian inference to integrate fault detection results of local models based on Mixture PPCA [25] and Mixture FA [26]. Ma and Shi [27] utilized aligned mixture factor analysis (AMFA) to array the parted sub-models together and preserve both within-mode and cross-mode correlations, which improves monitoring performance through the global model. Zhu *et al.* [28] proposed a Bayesian robust mixture factor analyzer to characterize multimode process data with outliers by using student distribution but fault detection part is not given. Xiao *et al.* [29], [30] developed fault diagnosis approaches based on variational Bayesian mixture factor analysis and applied them to wastewater treatment. Jiang and Yan [31] constructed an effective multimode process monitoring method by integrating a variational Bayesian Gaussian mixture model with canonical correlation analysis (VBGMM-CCA). Khodabakhsh *et al.* [32] proposed a new approach for real-time data validation, gross error detection and classification over multivariate sensor data streams in multimode processes. Wang *et al.* [33] integrated multi-subspace factor analysis and support vector data description for multimode process. They used FA to derive statistical indices in each subspace and each mode. Yang *et al.* [34] developed multimode process monitoring method based on dictionary learning. Their robust approach could deal with outliers and noise.

In this manuscript, the mixture of factor analysis (MFA) is chose to build a novel monitoring scheme because factor analysis (FA) is a more general probabilistic latent variable model. The noise levels are presumed to be different for distinct process variables while they are the same in PPCA situation. However, there are still some flaws when the mixture form of FA is carried out on multimode process monitoring. First, model selection is not well handled. When MFA is used to model data from multimode process, the number of factor analyzer and the number of factors in each factor analyzer need to be given before the implementation of EM algorithm for parameter learning. Previous literatures provide solutions such as Bayesian Ying-Yang system and variance explanation ratio to determine these two parameters, which is inconvenient and complex. Second, as reference [15] points out, probabilistic monitoring statistics computed in the $T^2$ and $SPE$ way only consider expectations of factor space and residual space. This index may result in misleading outcomes due to the overlook of other useful information.

This work is motivated to deal with the above stated problems. First of all, instead of EM algorithm, the variational approximation technique is employed to achieve parameter estimation for Bayesian treatment of MFA. The optimal number of components can be automatically obtained via one birth and death type algorithm. The reduced dimensionality inside every local factor analyzer can be gotten spontaneously during parameter learning process. Then, the negative log likelihood probability of monitored data over parameter's variational distribution is developed as monitoring index. This new fault detection index contains all information of parameter's probability distribution, which is helpful to improve monitoring performance. In summary, the advantages of the proposed method are as follows: (1) instead of EM algorithm, the variational approximation technique is used for parameter estimation. (2) the number of local factor analyzers can be automatically obtained. (3) the whole information in probabilistic distribution function is utilized to construct a new index named negative variational log likelihood.

The remainder of this article is organized as follows. In Section II, an introduction to variational inference for Bayesian mixtures of factor analyzers is presented. In Section III, the novel monitoring index is constructed and the process monitoring flow is reported. After that, applications to a simulation example and the Tennessee Eastman (TE) process demonstrate the effectiveness of the proposed monitoring scheme in Section IV. In the end, the conclusion of this manuscript is given in Section V.

## II. VARIATIONAL INFERENCE FOR PAEAMETER ESTIMATION

### A. FA AND MFA

Given the data set $\boldsymbol{Y} = [y_1, y_2, \ldots, y_N] \in \boldsymbol{R}^{M\times N}$, through factor analysis, the measurement matrix can be presented as [7]

$$\boldsymbol{Y} = \boldsymbol{\Lambda Z} + \boldsymbol{\mu} + e \tag{1}$$

where $\boldsymbol{\Lambda} \in \boldsymbol{R}^{M\times K}$ is the linear transformation called the factor loading matrix, the elements of $\boldsymbol{Z} \in \boldsymbol{R}^{K\times N}$ are hidden independent zero-mean unit-variance Gaussian sources (factors), $\boldsymbol{\mu}$ is the mean of the analyzer and $e$ is the noise following the distribution $\boldsymbol{e} \sim \boldsymbol{N}(0, \psi)$. $\boldsymbol{\psi}$ is a diagonal matrix and the elements in it represent different noise levels of process variable.

Similar to conventional PCA approach, two statistics ($T^2$ and SPE) are constructed for fault detection. Compared with PCA, the score vector in FA should be represented by the expectation of factors. Therefore, the monitoring statistics of factor space and residual space are defined as following equations.

$$T^2 = \|\bar{z}\| \tag{2}$$

$$SPE = \left\|\psi^{-1/2}\bar{e}\right\| \tag{3}$$

However, a single factor analyzer is not enough to model data from several operating conditions. Therefore, when the data set $\boldsymbol{Y}$ is multimode process measurement data, a mixture of factor analyzers is utilized. The density for $\boldsymbol{Y}$ is calculated as a weighted average of a few local factor analyzer densities [35]:

$$p(Y|\mu, \Lambda, \Psi, \pi) = \sum_{s=1}^{S} p(s|\pi)p(y|s, \mu^s, \Lambda^s, \psi^s) \tag{4}$$

where $\pi$ is the vector of mixing proportions and $S$ is the number of mixture component. The local dimensionality reduction $K_s$ is embodied in loading matrix $\Lambda^s$. The parameter set can be organized as $\theta = (\pi, \mu, \Lambda, \psi)$.

When MFA is used for fault detection, the monitoring results in several local FA can be combined together in different ways, such as hard assignment and soft assignment based on probability. In addition, it should be noted that between-mode transition exists in multimode processes. Transition happens when production status changes from one mode to another one. However, in this work, transitional process is removed and we only pay attention to stable modes.

Given training data set $Y$, the parameters of MFA model can be estimated via maximizing the log likelihood function:

$$L(\theta|Y) = \ln \prod_{i=1}^{N} p(y_i|\theta) \tag{5}$$

However, in this case, it is difficult to deal with the complex nonlinear optimization problem. Therefore, previous researches apply EM algorithm to maximizing the expected complete-data log likelihood iteratively. But EM algorithm is easy to get caught in local maxima and very sensitive to initialization. In addition, the number of components and their local dimensionalities need to be known before the computation. In order to overcome those drawbacks, variational Bayesian approach is introduced in this work.

### B. BAYESIAN TREATMENT FOR PARAMETERS

The so-called Bayesian treatment is regarding the parameters of the MFA model as unknown quantities. Then the probability of the data set over priors for those parameters should be given as:

$$\begin{aligned} p(Y|\theta) &= \int d\theta p(\theta)p(Y|\theta) = \int d\theta p(\theta) \prod_{i=1}^{N} p(y_i|\theta) \\ &= \int d\pi p(\pi) \int d\Lambda p(\Lambda) \int d\mu p(\mu) \prod_{i=1}^{N} [\sum_{s_i=1}^{S} p(s_i|\pi) \\ &\quad \times \int dz_i p(z_i) p(y_i|s_i, z_i, \Lambda, \mu, \psi)] \end{aligned} \tag{6}$$

This equation is known as the marginal likelihood. Maximizing the marginal likelihood leads to parameter estimation.

First of all, different priors are designed for various parameters. For the simplicity of inference and calculation, conjugate priors are preferred. The choices of priors are demonstrated by the following equations [35].

$$p(\Lambda|v) = \prod_{s=1}^{S} \prod_{j=1}^{K_s} p(\Lambda_j^s|v_j^s) = \prod_{s=1}^{S} \prod_{j=1}^{K_s} N(\Lambda_j^s|0, I/v_j^s) \tag{7}$$

$$p(v|a^*, b^*) = \prod_{s=1}^{S} \prod_{j=1}^{K_s} p(v_j^s|a^*, b^*) = \prod_{s=1}^{S} \prod_{j=1}^{K_s} Ga(v_j^s|a^*, b^*) \tag{8}$$

$$\begin{aligned} p(\mu|\mu^*, v^*) &= \prod_{s=1}^{S} p(\mu^s|\mu^*, v^*) \\ &= \prod_{s=1}^{S} N(\mu^s|\mu^*, \mathrm{diag}(v^*)^{-1}) \end{aligned} \tag{9}$$

Besides, the mixing proportion $\pi$ can be initialized as $[\frac{1}{S}, \frac{1}{S}, \ldots, \frac{1}{S}]$ for simplicity, meaning that every component has the same chance to produce one data sample. Then the hyper-parameters of the Bayesian MFA model can be formed as $\Theta = (a^*, b^*, \mu^*, v^*, \psi)$.

### C. VARIATIONAL INFERENCE

Directly calculating the marginal likelihood of data set in (6) is not tractable so the variational approximation method [35] is utilized to find the lower bound of this log marginal likelihood. The variational approximation is conducted through employing a variational distribution over the parameters in set $\theta$. The detailed steps of variational Bayesian derivation and the corresponding equations can be found in reference [35]. The results from literature are used directly here.

### D. MODEL SELECTION

The model selection here is about how to decide two parameters of MFA model structure, namely the number of local factor analyzers $S$ and their reduced dimensionality $K_s$. The latter one has been dealt with in variational inference procedure.

In order to acquire the quantity $S$ easily, birth-and-death type methods are helpful. In this work, the idea from birth-and-death Markov Chain Monte Carlo (BDMCMC) [36] is selected because of its low computation and easy interpretation. To the best of our knowledge, no one has introduced BDMCMC into variational Bayesian MFA (VBMFA) yet.

The core idea of the method is to view the parameters of the MFA model as a point process where each point represents a component's parameter set $\theta_s$, and then use theory of point process simulation to build a Markov chain with the posterior distribution of the parameters as its stationary distribution. Our contribution here is, under variational inference framework, using variational distribution of the parameters as the stationary distribution, shown as (10). The other parts remain the same as reference [36]. We borrow the idea but construct new stationary distribution.

After the point of convergence for the variational inference is reached, this birth-and-death algorithm is conducted to give out the proper number of factor analyzers.

The variational distribution of one component's parameter set $\theta_s$, namely the novel stationary distribution, is stated as:

$$p(\theta_s|Y) = \int dv^s q(v^s)\left[\ln\frac{p(v^s|a^*, b^*)}{q(v^s)} + \int d\tilde{\Lambda}^s q(\tilde{\Lambda}^s)\frac{p(\tilde{\Lambda}^s|v^s, \mu^*, v^*)}{q(\tilde{\Lambda}^s)}\right] + \sum_{i=1}^{N} q(s_i)\left[\int d\pi q(\pi)\ln\frac{p(s_i|\pi)}{q(s_i)} + \int dz_i q(z_i|s_i)\ln\frac{p(z_i)}{q(z_i|s_i)} + \int d\tilde{\Lambda}^s q(\tilde{\Lambda}^s) \times \int dz_i q(z_i|s_i)\ln p(y_i|s_i, z_i, \tilde{\Lambda}^s, \psi)\right] \tag{10}$$

In addition, it should be noted that the number of components (namely local factor analyzers) $S$ is not necessarily the number of operational modes. The quantity $S$ expresses how many factor analyzers are needed to well describe the training data set $Y$. According to (1), a factor analyzer equals one Gaussian component which follows a unimodal Gaussian distribution. Therefore, $S$ means the number of working conditions only under the condition that the measurement data from each mode is presumed to obey a unimodal Gaussian distribution.

## III. NEGATIVE VARIATIONAL LOG LIKELIHOOD FOR FAULT DETECTION

In this part, a new monitoring index called negative variational log likelihood (NVLL) is proposed for multimode process monitoring. The procedure of monitoring scheme is described in detail subsequently.

### A. THE PROPOSE OF NEGATIVE VARIATIONAL LOG LIKELIHOOD

In statistics, the likelihood function is a function of the parameters of a statistical model given data. Therefore, this likelihood needs to be maximized in order to obtain the best parameters for specific training data set during parameter estimation phase. On the contrary, when the parameters of a statistical model are known, this quantity represents the probability of observed measurement given those parameters. In other words, it indicates the extent that a measured sample accords with the trained statistical model.

Thus, this likelihood can act as monitoring indicator of process state. More specifically, we use normal data to train a MFA model and get its parameter set. When it comes to process monitoring, calculate NVLL of monitored sample in the trained MFA model. For normal samples, the value of NVLL should be low because they conform to the trained MFA. But for faulty samples, the value of NVLL is supposed to be large for the reason that they deviate from the trained model a lot.

After the training of variational Bayesian MFA is done, a variant likelihood function named negative variational log likelihood is constructed in similar way as ordinary likelihood function. The difference lies in the use of variational distribution. Ordinary likelihood function is constituted via common distribution functions. However, NVLL is developed by variational distribution of parameters.

$$NVLL = -\ln p(y_{\text{test}}|\theta) = -\sum_{s_i=1}^{S} q(s_i)\left[\int d\pi q(\pi)\ln\frac{p(s_i|\pi)}{q(s_i)} + \int dz_i q(z_i|s_i)\ln\frac{p(z_i)}{q(z_i|s_i)} + \int d\tilde{\Lambda} q(\tilde{\Lambda}) \times \int dz_i q(z_i|s_i)\ln p(y_{\text{teet}}|s_i, z_i, \tilde{\Lambda}, \psi)\right] \tag{11}$$

The value of normal data sample for this index should be in some range and below a threshold. But for a faulty sample, the value of this monitoring index should be larger than the threshold because it deviates from the trained MFA model.

Since the distribution of NVLL is not known, the kernel-based density estimation (KDE) technique is employed to decide the control limit. The NVLLs of all normal data are firstly calculated. Then, the threshold can be estimated through KDE. According to reference [37], the type of alarm system in this paper is uni-threshold alarm system. It means that a single threshold is set for every operating mode.

This novel monitoring index is developed under variational inference framework. According to (11), it contains all parameters of the variational Bayesian MFA model. That is to say, all information in probabilistic distributions of those parameters is used to construct this monitoring indicator. As a comparison, the statistics $T^2$ and $SPE$ developed under EM algorithm framework are designed to monitor factor space and residual space. To be specific, those two statistics are responsible for watching over probability distributions of factors and residuals. But only expectations in distributions are considered when calculating these two monitoring indices. The lack of other information may result in dissatisfied fault detection performance.

### B. THE PROCEDURE OF MULTIMODE PROCESS MONITORING

In this section, step by step introduction of fault detection method for process with multiple modes is outlined below. The procedure includes offline modeling and online monitoring. The schematic diagram of presented process monitoring approach is exhibited in Fig. 1. The left side illustrates the offline modeling part while the right one corresponds to online monitoring.

#### 1) STAGE 1: OFFLINE MODELING

1) The training data set is collected from several normal working conditions in the historical database. Standardize the dataset to get rid of the difference in scales. In addition, mean and variance of training data can be obtained.

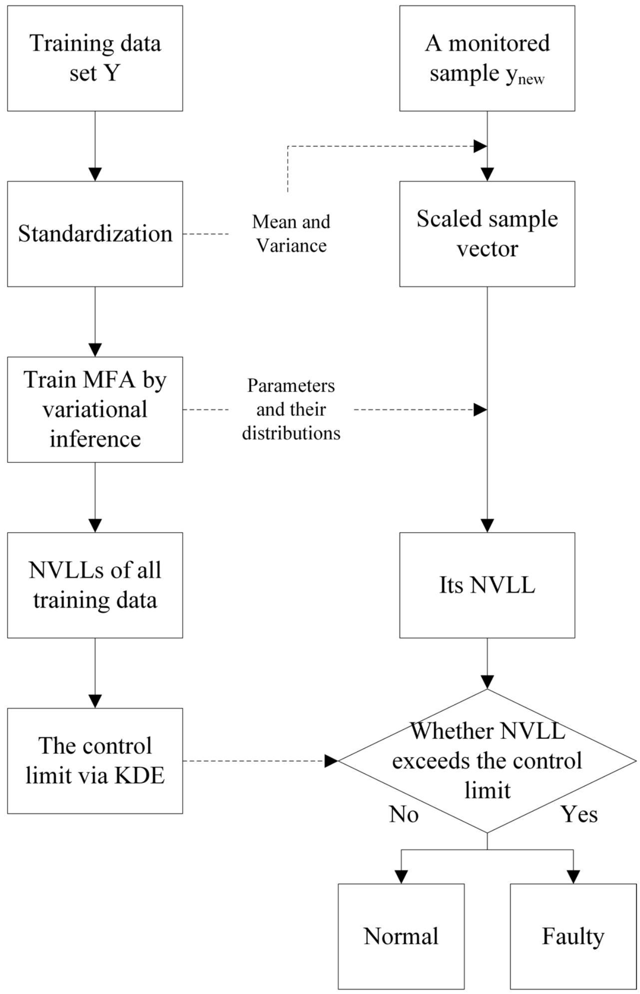


**FIGURE 1.** **Schematic diagram of the proposed method.**

2) This normal dataset is used to train Bayesian MFA model through variational inference technique.
3) After convergence of variational inference is guaranteed, the birth-and-death Markov Chain Monte Carlo method is utilized to determine the number of local factor analyzers.
4) After the above steps are done, all parameters of Bayesian MFA model and their probabilistic distributions are gotten.
5) Calculate negative variational log likelihood index of every normal data sample using (11). The control limit can be calculated by kernel density estimation (KDE) approach.

#### 2) STAGE 2: ONLINE MONITORING

1) For a monitored sample, standardize it by utilizing the mean and variance gotten in the first step of offline modeling.
2) After the scaled sample vector is calculated, its negative variational log likelihood index should be computed via (11). The trained model parameters and their probability distributions are all used during the computation process.
3) In the end, compare the value of NVLL with control limit to decide whether this sample is in normal state or not. If its NVLL is smaller than control limit, it indicates that this monitored sample comes from normal production status. Otherwise, this monitored sample is considered faulty.

## IV. CASE STUDIES

### A. A NUMERICAL EXAMPLE

In order to show that the proposed method is useful when data of some modes does not obey Gaussian distribution, this section applies our approach to a numerical simulation which owns five process variables. They are constituted via two source variables and a few Gaussian distributed observation noises. This numerical example is explained clearly by Ma *et al.* [38]. This multivariate system is described through the following five equations.

$$y_1 = 0.5768x_1 + 0.3766x_2 + e_1 \tag{12}$$
$$y_2 = 0.7382x_1^2 + 0.0566x_2 + e_2 \tag{13}$$
$$y_3 = 0.8291x_1 + 0.4009x_2^2 + e_3 \tag{14}$$
$$y_4 = 0.6519x_1x_2 + 0.2070x_2 + e_4 \tag{15}$$
$$y_5 = 0.3972x_1 + 0.8045x_2 + e_5 \tag{16}$$

In this simulation, the elements of $e = [e_1, e_2, e_3, e_4, e_5]$ are zero-mean white noises with a standard deviation of 0.01. Three operational modes are generated via alternating two source variables. In mode 1, $x_1 \sim N(10, 0.64)$ $x_2 \sim N(12, 1.69)$; in mode 2, $x_1 \sim N(5, 0.36)$ $x_2 \sim N(20, 0.49)$; in mode 3, $x_1 \sim N(16, 2.25)$ $x_2 \sim N(30, 6.25)$. In order to attain the training data set, 400 normal data samples are produced form each operating mode. In addition, two kinds of fault situations are designed: for situation 1, the system firstly work in mode 2, then it changes to mode 1 from the $401^{st}$ sample, finally a step bias with the magnitude of 5 is introduced to variable $y_5$ from the $801^{st}$ sample. In the end, a total of 400 samples are collected after the fault happens; for situation 2, the system initially operates in mode 3, then it switches to mode 2 from $401^{st}$ sample, at last a drift error with the slope of 0.02 is added to $y_1$ from the $801^{st}$ sample. Similarly, 400 samples are gathered after the occurrence of fault. Therefore, the dimension of both the training data set and the test data set should be $5 \times 1200$.

As a comparison, both the maximum-likelihood mixture factor analysis (MLMFA) based process monitoring method in reference [26] and aligned mixture factor analysis (AMFA) based fault detection approach [27] are used. MLMFA employs EM algorithm to calculate parameters of MFA model and uses Bayesian inference combination strategy to integrate local monitoring results into $\boldsymbol{CT}^2$ and $\boldsymbol{CSPE}$. AMFA also uses EM algorithm for parameter estimation and aligns separated local models together for process monitoring. For these three methods, the reduced dimension of local factor analyzer is 3 and the confidence level is set as 99% when computing the control limit. Because the data of some modes does not obey a unimodal Gaussian distribution, the number of local factor analyzer is determined as 10 through birth-and-death technique. At last, the fault detection results for two specific fault cases are illustrated in Table 1. In case 1, the false rates of MLMFA($CT^2$), MLMFA(CSPE), AMFA and NVLL are 0%, 0%, 0.25% and 0.25%, respectively. In case 2, the false rates of MLMFA($CT^2$), MLMFA(CSPE), AMFA and NVLL are 0%, 0%, 0.5% and 0%, respectively. Monitoring results of the proposed method are demonstrated in Fig. 2 and 3. As is shown in Table 1, the MLMFA fault detection method can only achieve general monitoring outcomes. AMFA is able to achieve good fault detection results for step fault. But it is still not good enough for slope fault. As an obvious contrast, satisfactory monitoring performance is obtained by using ***NVLL*** index. Our proposed fault detection indicator works well for both step failure and slope problem. Conventional monitoring statistics only consider expectations of factor and residual space, which causes some misses during monitoring process. The novel index ***NVLL*** with more information included leads to better results.

**TABLE 1. Miss alarm rates of the numerical example.**

| Case | MLMFA($CT^2$) | MLMFA (CSPE) | AMFA | NVLL |
|---|---|---|---|---|
| 1 | 9.75% | 8.5% | 0.5% | 0% |
| 2 | 12.5% | 9.25% | 8.25% | 1.5% |

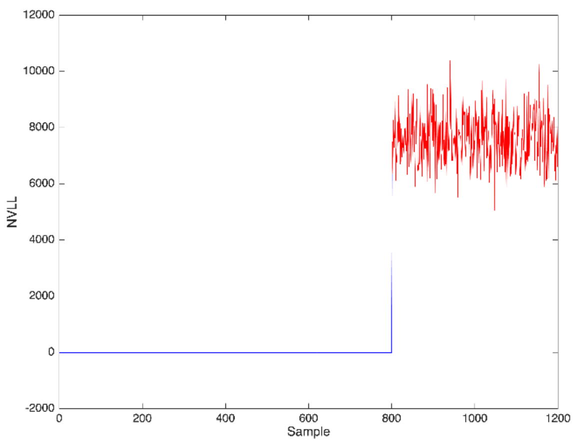


**FIGURE 2. Fault detection results of the proposed method in case 1.**

### B. THE TENNESSEE EASTMAN PROCESS

In this part, the ***NVLL*** based monitoring scheme for multimode process is tested by the Tennessee Eastman (TE) process [39]. There are total of 53 process variables in this chemical process. 41 of them are measured variables and the other 12 ones are manipulated variables. And the process data is produced through a control strategy developed by Richer [40]. Besides, there are six different kinds of process working conditions which own various G/H mass ratio or production rate.

In the manuscript, observed data from mode No. 1 and mode No. 3 is utilized. The normal data set and test data set are generated using Simulink and MATLAB codes from the

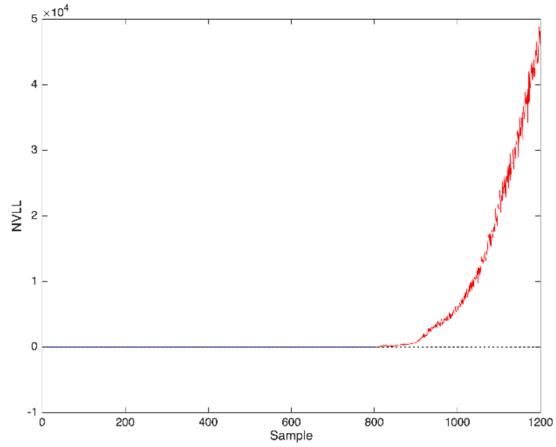


**FIGURE 3. Fault detection results of the proposed method in case 2.**

website (http://depts.washington.edu/control/LARRY/TE/download.html). When producing process data of mode No.1, use the Simulink model of mode No.1 and corresponding initialization m file. It is the same procedure when obtaining process data of mode No.3. Normal observed measurements of 60 hours in total are generated from each operational mode. A measured sample is observed from the system every 0.03h. There are total of thirty-one monitored process variables. Therefore, the dimension of training data set is 4000 × 31. In addition, there are 14 kinds of particular pre-defined process faults. Specific information regarding monitored process variables and detailed description about fault types can be found in reference [39] and [40]. Every test data set contains 1000 data samples and the specific fault happens from the $201^{st}$ sample. Thus, the dimension of each test data set is 1000 × 31. As is well known, the fault detection for fault No.3 and No.9 is very hard for the reason that there are no apparent changes in the mean, variance or the higher orders of the data. Therefore, no statistical monitoring methods are capable of detecting these faults. Therefore, they are not considered in this paper. The other twelve kinds of process faults of mode No.1 and No.3 are utilized.

The same training data set and test data set are employed for the comparative methods, namely maximum-likelihood mixture factor analysis (MLMFA) based process monitoring method [26], aligned mixture factor analysis (AMFA) based fault detection approach [27] and HMM-based NLLP [41]. For fair competition, parameters of MFA model structure are also the same. The number of local factor analyzers $\boldsymbol{S}$ is 2, which is determined by the birth-and-death algorithm. The reduced dimensionality $\boldsymbol{K}_s$ inside each factor analyzer is 15. The confidence level is set as 99% when calculating the control limit of monitoring index. At last, the multimode process monitoring results of three approaches are presented in Table 2 and 3. Several typical fault detection results are shown in Fig. 4-11.

False rate is not listed in Table 2 and 3 because the value of it is always below 1.5% during the testing procedure. For simplified presentation, they are removed from the above

**TABLE 2.** Monitoring results for 12 faults of TE mode 1 (miss rate).

| FAULT NO. | MLMFA ($CT^2$) | MLMFA (CSPE) | AMFA | NLLP | NVLL |
|---|---|---|---|---|---|
| 1 | 0.25% | 0.125% | 0.125% | 0.375% | 0 |
| 2 | 1.25% | 1.125% | 0.875% | 0.75% | 0.25% |
| 4 | 0.25% | 0 | 0 | 0 | 0 |
| 5 | 99.75% | 99.25% | 98.125% | 98.875% | 96.125% |
| 6 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 |
| 8 | 2.25% | 3.875% | 2.375% | 6.75% | 2.125% |
| 10 | 19.875% | 19.25% | 11.75% | 94.25% | 6.5% |
| 11 | 9.375% | 6.125% | 1.5% | 7.125% | 0.625% |
| 12 | 75.875% | 92% | 56.125% | 79.875% | 44.75% |
| 13 | 3.75% | 4.125% | 3.125% | 7.125% | 2.5% |
| 14 | 1.125% | 0 | 0 | 23.25% | 0 |

**TABLE 3.** Monitoring results for 12 faults of TE mode 3 (miss rate).

| FAULT NO. | MLMFA ($CT^2$) | MLMFA (CSPE) | AMFA | NLLP | NVLL |
|---|---|---|---|---|---|
| 1 | 0.25% | 0.375% | 0.125% | 1.75% | 0.125% |
| 2 | 5.125% | 12.5% | 2.75% | 93.375% | 1.25% |
| 4 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 90.125% | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 89.75% | 0 |
| 8 | 2.875% | 2.625% | 2.5% | 7.75% | 2.125% |
| 10 | 19.375% | 20.125% | 11.5% | 88.25% | 3.5% |
| 11 | 11.75% | 7.125% | 2% | 11.375% | 0.75% |
| 12 | 1.875% | 1.25% | 1% | 4.625% | 0.5% |
| 13 | 18.75% | 28.25% | 12.375% | 27.25% | 8.5% |
| 14 | 1% | 0.75% | 0 | 31% | 0 |

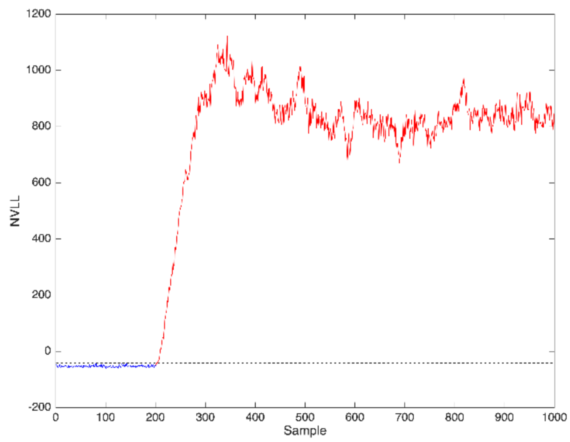


**FIGURE 4.** Fault detection results of NVLL for fault No.2 in Mode No.1.

two tables. From the above two tables, it is clear that ***NVLL*** achieves lower fault miss rate than the other three methods when they are carried out for detecting twelve distinct kinds of faults. The improvement of monitoring ability is significant

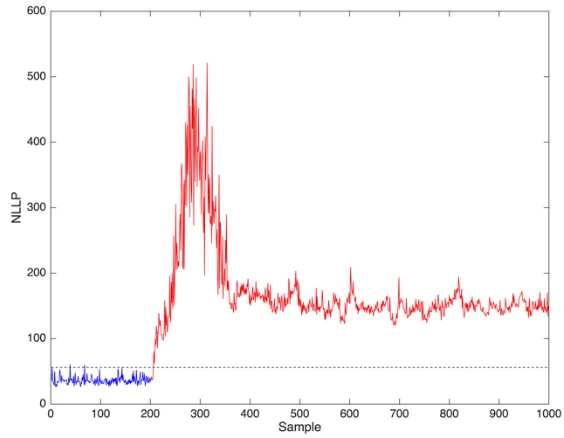


**FIGURE 5.** Fault detection results of NLLP for fault No.2 in Mode No.1.

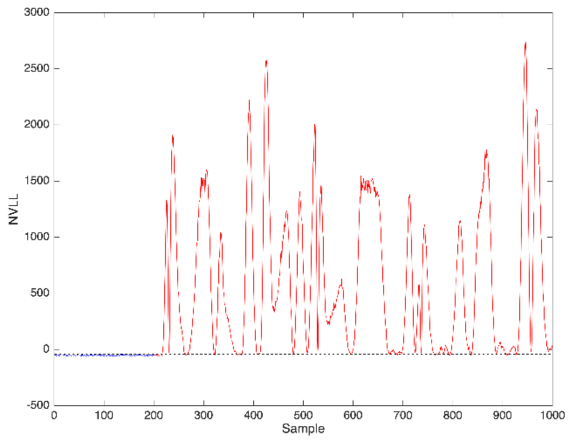


**FIGURE 6.** Fault detection results of NVLL for fault No.10 in Mode No.1.

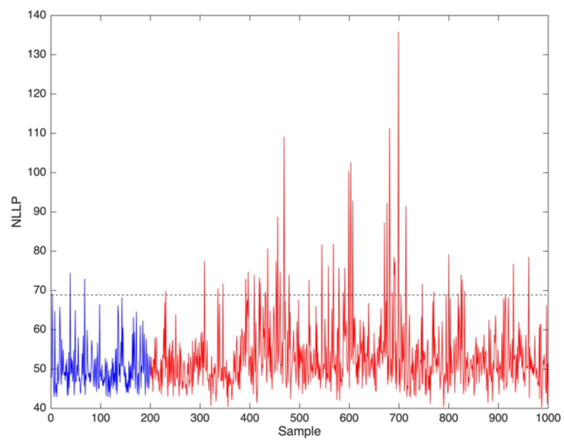


**FIGURE 7.** Fault detection results of NLLP for fault No.10 in Mode No.1.

in these situations which are fault No.10 of mode No.1, fault No.11 of mode No.1, fault No.10 of mode No.3 and fault No.11 of mode No.3.

Then take fault No.10 as an example, this fault is random variation in C feed temperature (stream 4). After the variation is introduced, most monitored variables remain

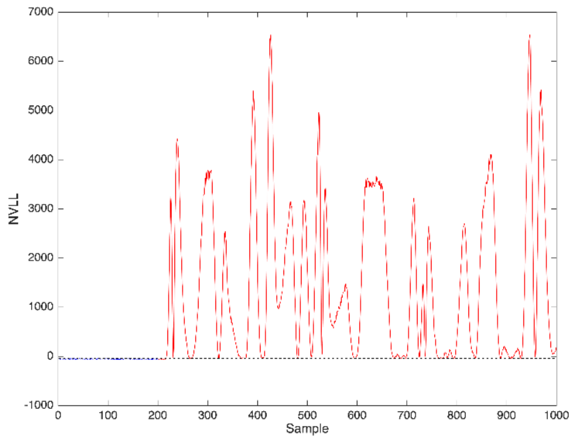


**FIGURE 8.** Fault detection results of NVLL for fault No.10 in Mode No.3.

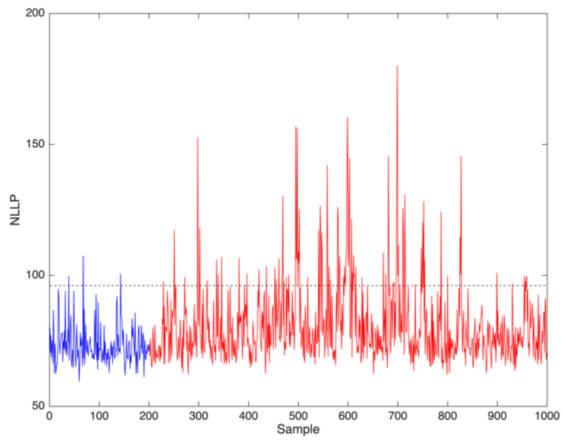


**FIGURE 9.** Fault detection results of NLLP for fault No.10 in Mode No.3.

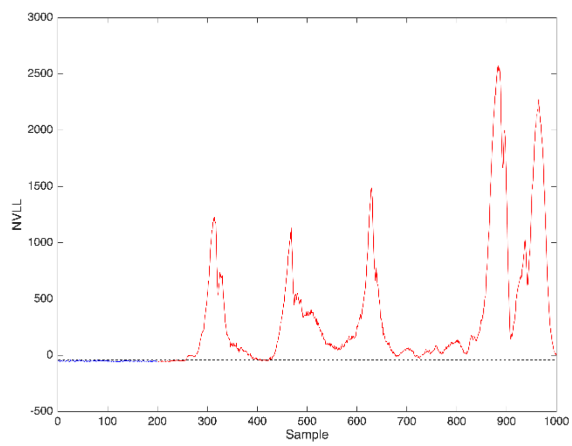


**FIGURE 10.** Fault detection results of NVLL for fault No.13 in Mode No.3.

nearly the same as normal working state. Only three variables, namely No.18 stripper temperature, No.28 separator valve and No.29 stripper valve, deviate from normal operating condition due to the impact. The fluctuations of these variables are shown in the following figures. The values of variables in graph are calculated through standardization. From Fig. 12 and 13, the change of values for variable

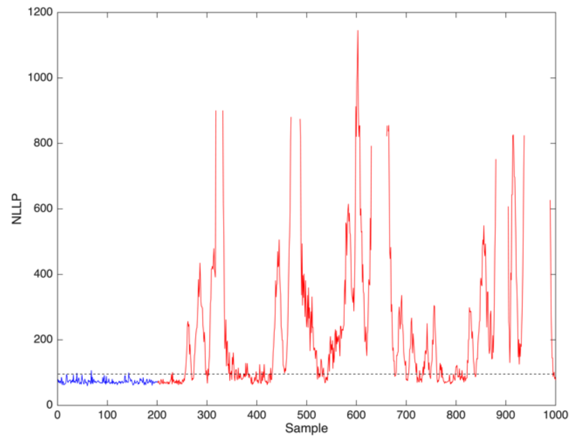


**FIGURE 11.** Fault detection results of NLLP for fault No.13 in Mode No.3.

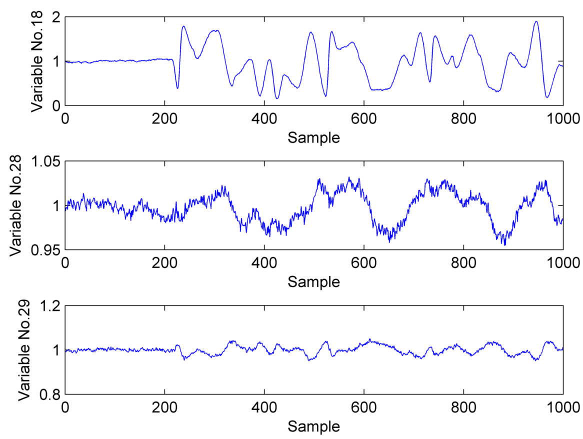


**FIGURE 12.** Trend plots of variable No.18, No.28 and No.29 in fault No.10 situation of mode No.1.

No.28 and No.29 is relatively small in magnitude. Thus, the change of monitoring statistics mainly relies on the value of variable No.18. It can be seen from Fig. 12 and 13, the value of variable No.18 waves wildly around the normal value. When the value is far away from the normal one, the change of monitoring index is obvious so that fault detection is easy. But process monitoring becomes difficult when the value is near the normal condition. It is for the reason that the change of monitoring statistics is small and then the faulty sample is regarded as normal one. Conventional statistics $T^2$ and ***SPE*** only exploits expectation of probability distribution and the lack of other information results in the insensitive change for this kind of fault. Therefore, in mode No.1 situation, MLMFA based monitoring approach misses nearly 20% faulty samples and AMFA based method loses 11.75% failure data. For mode No.3 case, the former one reaches approximately 20% miss rate and the latter one improves the rate to 11.5%. Taking more information in distributions of model parameters into account, the novel monitoring index ***NVLL*** cuts the fault miss rate to 6.5% and 3.5% respectively for fault No.10 of mode No.1 and No.3.

To conclude, based on the same mixture form of probabilistic latent variable model MFA, three monitoring methods

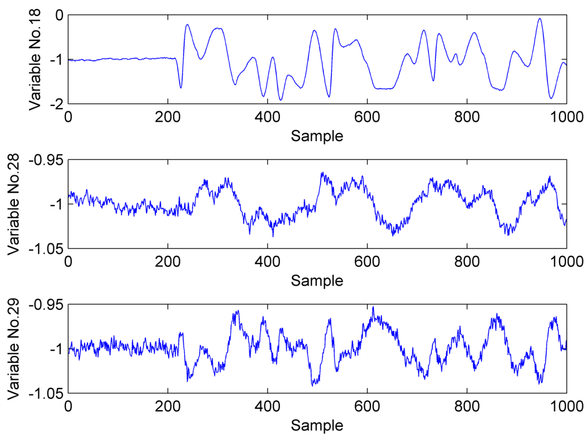


**FIGURE 13. Trend plots of variable No.18, No.28 and No.29 in fault No.10 situation of mode No.3.**

construct different fault detection indices. MLMFA develops traditional statistics $CT^2$ and $CSPE$ by using the expectations of factor space and residual space. AMFA adds the information of cross-mode correlations into conventional statistic $T^2$ to improve process monitoring ability. The proposed index $NVLL$ exploits all information in the distributions of model parameters, which leads to better monitoring performance.

## V. SUMMARY

In this brief, a new monitoring scheme based on variational Bayesian mixture factor analysis is introduced for multimode process fault detection. The selection of model structure parameters is well handled through variational inference technique and birth-and-death Markov Chain Monte Carlo method. Additionally, taking advantage of more information of parameter distributions, a new monitoring indicator called negative variational log likelihood is constructed for process monitoring. The proposed approach can be applied to multimode process whose mode data of every working condition follows non-Gaussian or other complex distributions because these data distributions can be adequately dealt with by using appropriate number of local factor analyzers. But this method is not suitable when facing dynamic or time-variant processes. Fixing this problem can be the future work. Besides, it should be noted that the proposed monitoring index has chattering alarms [42] when the process variables are operating close to their alarm points. Fault No.10 is an example. According to reference [42], chattering alarms frequently occur due to noise/disturbance. This is the drawback of the proposed method.

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