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Methods for Evaluating the Technological Innovation Capability for the High-Tech Enterprises With Generalized Interval Neutrosophic Number Bonferroni Mean Operators

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ABSTRACT As we all know, the Bonferroni mean (BM) operator has the advantage of considering interrelationships between parameters. In this paper, we combine the generalized weighted BM (GWBM) operator and generalized weighted Bonferroni geometric mean (GWGBM) operator with interval neutrosophic numbers (INNs) to develop the generalized interval neutrosophic number weight BM (GINNWBM) operator and generalized interval neutrosophic numbers weighted GBM (GINNWGBM) operator which consider the relationship among three aggregated arguments, then the MADM methods are developed with these operators. Finally, we use an example for evaluating the technological innovation capability for the high-tech enterprises to illustrate the proposed methods.

INDEX TERMS Multiple attribute decision making (MADM), interval neutrosophic numbers (INNs), generalized weighted BM (GWBM) operator, generalized weighted geometric Bonferroni mean (GWGBM) operator, technological innovation capability, high-tech enterprises.

I. INTRODUCTION

Neutrosophic sets (NSs), which were proposed originally by Smarandache [1], [2], have been attracted the attention of many scholars, and NSs have been acted as a workspace in depicting indeterminate and inconsistent information. A NS has a more potential power than other modeling mathematical tools, such as fuzzy set [3], IFS [4] and IVIFS [5]. But, it is difficult to apply NSs in solving of real life problems. Therefore, Wang *et al.* [6], [7] defined a single valued neutrosophic set (SVNS) and an interval neutrosophic set (INS), which are characterized by a truth-membership, an indeterminacy membership and a falsity membership. Hence, SVNSs and INSs can express much more information (truth-membership degree, indeterminacy-membership degree, and falsity-membership degree information) than fuzzy sets (only membership degree information), IFSs and IVIFSs (both membership degree and non-membership degree information). Ye [8] proposed a MADM method with correlation coefficient of SVNSs. Broumi and Smarandache [9] extended the correlation coefficient to INSs. Biswas *et al.* [10] developed the TOPSIS method with SVNNs. Liu *et al.* [11] defined the generalized neutrosophic number Hamacher aggregation for SVNSs. Sahin and Liu [12] defined the maximizing deviation model under neutrosophic environment. Ye [13] developed some similarity measures of INS. Zhang *et al.* [14] defined some interval neutrosophic information aggregating operators. Ye [15] proposed a simplified neutrosophic set (SNS). Peng *et al.* [16] developed simplified neutrosophic information aggregation operators. Additionally, Peng *et al.* [17] studied an outranking approach for handling SNS, and then Zhang *et al.* [18] gave an extended

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version of Peng's approach. Liu and Xi [19] proposed generalized weighted power averaging operator with SVNNs. Deli and Subas [20] discussed a method to rank single valued neutrosophic numbers. Peng *et al.* [21] proposed multi-valued neutrosophic sets. Zhang *et al.* [22] gave the improved weighted correlation coefficient for interval neutrosophic sets. Chen and Ye [23] proposed Dombi operations for neutrosophic sets. Liu and Wang [24] proposed the MADM method based on SVN normalized weighted Bonferroni mean. Wu *et al.* [25] proposed cross-entropy and prioritized aggregation operator with SNSs in MADM problems. Li *et al.* [26] developed the some SVNN Heronian mean operators in MADM problems. Wei and Wei [27] proposed some single-valued neutrosophic dombi prioritized weighted aggregation operators in MADM. Broumi and Smarandache [9] proposed the correlation coefficient of INS. Zhang *et al.* [22] defined the improved weighted correlation coefficient of INNs for MADM. Zhang *et al.* [18] defined an outranking approach for INN MADM. Tian *et al.* [28] developed a cross-entropy in INN MADM. Some other operators are defined in [29]–[33]. Ye [13] defined two similarity measures between INNs.

Obviously, these established INN aggregation operators cannot be utilized to aggregate the arguments which are correlated [34]. Meanwhile, the Bonferroni mean (BM) [35]–[42] is a very useful tool to deal with the arguments which are correlated. How to effectively expand the traditional generalized weighted BM (GWBM) operator and generalized weighted Bonferroni geometric mean (GWGBM) operator to INN environment is a significant research task which the focus of this paper.

The organization of this manuscript is given as follows. Section 2 reviews INSs and some other basic definitions. Section 3 introduces the extended GWBM and GWGBM which can be used to fuse the INNs which consider the relationship among three aggregated arguments, and gives some properties of these operators. Section 4 illustrates the effectiveness of the proposed operators with an application for evaluating the technological innovation capability for the high-tech enterprises. Section 5 concludes the paper.

II. BASIC CONCEPTS

A. NSs AND INSs

Smarandache [1], [2] proposed Neutrosophic sets (NSs). Wang *et al.* [7] further proposed the interval neutrosophic sets (INSs).

Definition 1 [7]: Let X be a space of points (objects) with a generic element in fix set X, denoted by x. An interval neutrosophic sets (INSs) A in X is characterized as following:

$$\tilde{A} = \left\{ \left(x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x) \right) | x \in X \right\} \quad (1)$$

where the truth-membership function $T_{\tilde{A}}(x)$, indeterminacy-membership $I_{\tilde{A}}(x)$ and falsity-membership function $F_{\tilde{A}}(x)$ are interval values, that is, $T_{\tilde{A}}(x) : X \to [0,1], I_{\widehat{A}}(x) : X \to [0,1]$ and $F_{\tilde{A}}(x) : X \to [0,1]$.

and $0 \le \sup\left(\mathrm{T}_{\tilde{A}}(\mathrm{x})\right) + \sup\left(\mathrm{I}_{\tilde{A}}(\mathrm{x})\right) + \sup\left(\mathrm{F}_{\tilde{A}}(\mathrm{x})\right) \le 3$. Then a simplification of A is denoted by $\tilde{A} = \left(T_{\tilde{A}}, I_{\tilde{A}}, F_{\tilde{A}}\right) = \left(\left[T_{\tilde{A}}^L, T_{\tilde{A}}^R\right], \left[I_{\tilde{A}}^L, I_{\tilde{A}}^R\right], \left[F_{\tilde{A}}^L, F_{\tilde{A}}^R\right]\right)$, which is a INN, where $\left[T_{\tilde{A}}^L, T_{\tilde{A}}^R\right] \subseteq [0,1], \left[I_{\tilde{A}}^L, I_{\tilde{A}}^R\right] \subseteq [0,1], \left[F_{\tilde{A}}^L, F_{\tilde{A}}^R\right] \subseteq [0,1]$ and $0 \le T_{\tilde{A}}^R + I_{\tilde{A}}^R + F_{\tilde{A}}^R \le 3$.

Definition 2 [43]: Let $\tilde{A} = \left(T_{\tilde{A}}, I_{\tilde{A}}, F_{\tilde{A}}\right) = \left(\left[T_{\tilde{A}}^L, T_{\tilde{A}}^R\right], \left[I_{\tilde{A}}^L, I_{\tilde{A}}^R\right], \left[F_{\tilde{A}}^L, F_{\tilde{A}}^R\right]\right)$ be an INN, a score function is defined:

$$s\left(\tilde{A}\right) = \frac{\left(2 + T_{\tilde{A}}^L - I_{\tilde{A}}^L - F_{\tilde{A}}^L\right) + (2 + T_{\tilde{A}}^R - I_{\tilde{A}}^R - F_{\tilde{A}}^R)}{6}$$
$$s\left(\tilde{A}\right) \in [0,1], \quad (2)$$

Definition 3 [43]: Let $\tilde{A} = \left(T_{\tilde{A}}, I_{\tilde{A}}, F_{\tilde{A}}\right) = \left(\left[T_{\tilde{A}}^L, T_{\tilde{A}}^R\right], \left[I_{\tilde{A}}^L, I_{\tilde{A}}^R\right], \left[F_{\tilde{A}}^L, F_{\tilde{A}}^R\right]\right)$ be an INN, an accuracy function $H\left(\tilde{A}\right)$ is proposed:

$$H\left(\tilde{A}\right) = \frac{\left(T_{\tilde{A}}^L + T_{\tilde{A}}^R\right) - \left(F_{\tilde{A}}^L + F_{\tilde{A}}^R\right)}{2}, \quad H\left(\tilde{A}\right) \in [-1,1] \quad (3)$$

Definition 4 [43]: Let $\tilde{A} = \left(\left[T_{\tilde{A}}^L, T_{\tilde{A}}^R\right], \left[I_{\tilde{A}}^L, I_{\tilde{A}}^R\right], \left[F_{\tilde{A}}^L, F_{\tilde{A}}^R\right]\right)$ and $\tilde{B} = \left(\left[T_{\tilde{B}}^L, T_{\tilde{B}}^R\right], \left[I_{\tilde{B}}^L, I_{\tilde{B}}^R\right], \left[F_{\tilde{B}}^L, F_{\tilde{B}}^R\right]\right)$ be two INNs, $s\left(\tilde{A}\right) = \frac{\left(2+T_{\tilde{A}}^L-I_{\tilde{A}}^L-F_{\tilde{A}}^L\right)+(2+T_{\tilde{A}}^R-I_{\tilde{A}}^R-F_{\tilde{A}}^R)}{6}$, $s\left(\tilde{A}\right) \in [0,1]$, and $s\left(\tilde{B}\right) = \frac{\left(2+T_{\tilde{B}}^L-I_{\tilde{B}}^L-F_{\tilde{B}}^L\right)+(2+T_{\tilde{B}}^R-I_{\tilde{B}}^R-F_{\tilde{B}}^R)}{6}$, $s\left(\tilde{B}\right) \in [0,1]$ be the scores, and

$$H\left(\tilde{A}\right) = \frac{\left(T_{\tilde{A}}^L + T_{\tilde{A}}^R\right) - \left(F_{\tilde{A}}^L + F_{\tilde{A}}^R\right)}{2}, \quad H\left(\tilde{A}\right) \in [-1,1],$$

$$H\left(\tilde{B}\right) = \frac{\left(T_{\tilde{B}}^L + T_{\tilde{B}}^R\right) - \left(F_{\tilde{B}}^L + F_{\tilde{B}}^R\right)}{2}, \quad H\left(\tilde{B}\right) \in [-1,1]$$

be the accuracy function, then if $S\left(\tilde{A}\right) < S\left(\tilde{B}\right), \tilde{A} < \tilde{B}$; if $S\left(\tilde{A}\right) = S\left(\tilde{B}\right)$, then

(1) *if* $H\left(\tilde{A}\right) = H\left(\tilde{B}\right), \tilde{A} = \tilde{B}$;

(2) *if* $H\left(\tilde{A}\right) < H\left(\tilde{B}\right), \tilde{A} < \tilde{B}$.

Definition 5 [7], [13]: Let $\tilde{A} = \left(\left[T_{\tilde{A}}^L, T_{\tilde{A}}^R\right], \left[I_{\tilde{A}}^L, I_{\tilde{A}}^R\right], \left[F_{\tilde{A}}^L, F_{\tilde{A}}^R\right]\right)$ and $\tilde{B} = \left(\left[T_{\tilde{B}}^L, T_{\tilde{B}}^R\right], \left[I_{\tilde{B}}^L, I_{\tilde{B}}^R\right], \left[F_{\tilde{B}}^L, F_{\tilde{B}}^R\right]\right)$ be two INNs and λ be a positive real number. The basic operations of INNs are:

$$\tilde{A} \oplus \tilde{B} = \begin{pmatrix} \left[T_{\tilde{A}}^L + T_{\tilde{B}}^L - T_{\tilde{A}}^L T_{\tilde{B}}^L, T_{\tilde{A}}^R + T_{\tilde{B}}^R - T_{\tilde{A}}^R T_{\tilde{B}}^R\right], \\ \left[I_{\tilde{A}}^L I_{\tilde{B}}^L, I_{\tilde{A}}^R I_{\tilde{B}}^R\right], \left[F_{\tilde{A}}^R F_{\tilde{B}}^R, F_{\tilde{A}}^R F_{\tilde{B}}^R\right] \end{pmatrix};$$

$$\tilde{A} \otimes \tilde{B} = \left(\begin{array}{l} \left[T_{\tilde{A}}^L T_{\tilde{B}}^L, T_{\tilde{A}}^R T_{\tilde{B}}^R\right], \left[I_{\tilde{A}}^L + I_{\tilde{B}}^L - I_{\tilde{A}}^L I_{\tilde{B}}^L, I_{\tilde{A}}^R + I_{\tilde{B}}^R - I_{\tilde{A}}^R T_{\tilde{B}}^R\right], \\ \left[F_{\tilde{A}}^L + F_{\tilde{B}}^L - F_{\tilde{A}}^L F_{\tilde{B}}^L, F_{\tilde{A}}^R + F_{\tilde{B}}^R - F_{\tilde{A}}^R F_{\tilde{B}}^R\right] \end{array} \right);$$

$$\lambda\tilde{A} = \left(\begin{array}{l} \left[1 - \left(1 - T_{\tilde{A}}^L\right)^{\lambda}, 1 - \left(1 - T_{\tilde{A}}^R\right)^{\lambda}\right], \\ \left[(I_{\tilde{A}}^L)^{\lambda}, (I_{\tilde{A}}^R)^{\lambda}\right], \left[(F_{\tilde{A}}^L)^{\lambda}, (F_{\tilde{A}}^R)^{\lambda}\right] \end{array} \right), \quad \lambda > 0;$$

$$(\tilde{A})^{\lambda} = \left(\begin{array}{l} \left[\left(T_{\tilde{A}}^L\right)^{\lambda}, \left(T_{\tilde{A}}^R\right)^{\lambda}\right], \left[1 - \left(1 - I_{\tilde{A}}^L\right)^{\lambda}, 1 - \left(1 - I_{\tilde{A}}^L\right)^{\lambda}\right], \\ \left[1 - \left(1 - F_{\tilde{A}}^L\right)^{\lambda}, 1 - \left(1 - F_{\tilde{A}}^L\right)^{\lambda}\right] \end{array} \right), \quad \lambda > 0.$$

B. GBM OPERATORS

Beliakov *et al.* [35] further extended the BM operator by considering the correlations of any three aggregated arguments instead of any two.

Definition 6 [35]: Let $p, q, r \geq 0$ and $a_i (i = 1, 2, \ldots, n)$ be a set of nonnegative crisp numbers. The generalized BM (GBM) is defined as follows:

$$\mathrm{GBM}^{p,q,r}(a_1, a_2, \ldots, a_n) = \left(\frac{1}{n(n-1)(n-2)} \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} a_i^p a_j^q a_k^r \right)^{1/(p+q+r)} \tag{4}$$

Xia *et al.* [37] also introduced the generalized geometric Bonferroni mean (GGBM) operator.

$$\begin{aligned} &\mathrm{GINNBM}^{s,t,r}(a_1, a_2, \cdots, a_n) \\ &= \left(\frac{1}{n(n-1)(n-2)} \bigoplus_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} \left(a_i^s \otimes a_j^t \otimes a_k^r\right) \right)^{1/(s+t+r)} \\ &= \left(\begin{array}{l} \left[\begin{array}{l} \left(1 - \prod\limits_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} \left(1 - \left(T_i^L\right)^s \left(T_j^L\right)^t \left(T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\ \left(1 - \prod\limits_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} \left(1 - \left(T_i^R\right)^s \left(T_j^R\right)^t \left(T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \end{array} \right], \\ [60pt] \left[\begin{array}{l} 1 - \left(1 - \prod\limits_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} \left(1 - \left(1 - I_i^L\right)^s \left(1 - I_j^L\right)^t \left(1 - I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\ 1 - \left(1 - \prod\limits_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} \left(1 - \left(1 - I_i^R\right)^s \left(1 - I_j^R\right)^t \left(1 - I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \end{array} \right], \\ \left[\begin{array}{l} 1 - \left(1 - \prod\limits_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} \left(1 - \left(1 - F_i^L\right)^s \left(1 - F_j^L\right)^t \left(1 - F_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\ 1 - \left(1 - \prod\limits_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} \left(1 - \left(1 - F_i^R\right)^s \left(1 - F_j^R\right)^t \left(1 - F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \end{array} \right]. \end{array} \right) \end{aligned} \tag{7}$$

Definition 7 [37]: Let $p, q, r \geq 0$ and $a_i (i = 1, 2, \ldots, n)$ be a set of nonnegative crisp numbers, if

$$\mathrm{GGBM}^{p,q,r}(a_1, a_2, \ldots, a_n) = \frac{1}{p+q+r} \prod_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} \left(pa_i + qa_j + ra_k\right)^{\frac{1}{n(n-1)(n-2)}} \tag{5}$$

III. GINNBM AND GINNWBM OPERATORS

A. GINNBM OPERATOR

This section expands GBM to fuse the INNs and develops some generalized interval neutrosophic number BM operators (GINNBM).

Definition 8: Let $s, t, r > 0$ and $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right) (i = 1, 2, \ldots, n)$ be a set of INNs. If

$$\mathrm{GINNBM}^{s,t,r}(a_1, a_2, \cdots, a_n) = \left(\frac{1}{n^3} \bigoplus_{i,j,k=1}^{n} \left(a_i^s \otimes a_j^t \otimes a_k^r\right)\right)^{1/(s+t+r)} \tag{6}$$

We can obtain the following theorem 1 according to definition 5.

Theorem 1: Let $s, t, r > 0$ and $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right) (i = 1, 2, \ldots, n)$ be a set of INNs. The aggregated value by GINNBM is also an INN and (7), as shown at the bottom of the previous page.

Proof: According to definition 5, we can obtain (8), as shown at the bottom of this page.

Thus, (9), as shown at the bottom of this page.

Thereafter, (10), as shown at the bottom of the next page.

Furthermore, (11), as shown at the top of the next page.

Therefore, (12), as shown at the top of the next page.

Hence, (7) is maintained.

Thereafter, (13)–(15), as shown at the top of the page 6.

Thereafter, (16), as shown at the top of the page 6.

Thereby the (7) is also an INN.

Moreover, GINNBM has the following properties.

Property 1 (Idempotency): If a_i $(i = 1, 2, \ldots, n)$ are equal, that is, $a_i = a = \left(\left[T^L, T^R\right], \left[I^L, I^R\right], \left[F^L, F^R\right]\right)$, then

$$\mathrm{GINNBM}^{s,t,r}(a_1, a_2, \cdots, a_n) = a \tag{17}$$

Proof: Let $T_1, T_2, I_1, I_2, F_1, F_2$, as shown at the top of the page 7.

Given that $T_i^L = T_j^L = T_k^L = T^L$, then

$$\begin{aligned} T_1 &= \left(1 - \prod_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} \left(1 - \left(T_i^L\right)^s \left(T_j^L\right)^t \left(T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \\ &= \left(1 - \prod_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} \left(1 - \left(T^L\right)^{s+t+r}\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \\ &= \left(1 - \left(\left(1 - \left(T^L\right)^{s+t+r}\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{n(n-1)(n-2)}\right)^{1/(s+t+r)} \\ &= T^L \end{aligned} \tag{18}$$

$$\begin{aligned} a_i^s &= \left(\left[\left(T_i^L\right)^s, \left(T_i^R\right)^s\right], \left[1-(1-I_i^L)^s, 1-(1-I_i^R)^s\right], \left[1-(1-F_i^L)^s, 1-(1-F_i^R)^s\right]\right); \\ a_j^t &= \left(\left[\left(T_j^L\right)^t, \left(T_j^R\right)^t\right], \left[1-(1-I_j^L)^t, 1-(1-I_j^R)^t\right], \left[1-(1-F_j^L)^t, 1-(1-F_j^R)^t\right]\right); \\ a_k^r &= \left(\left[\left(T_k^L\right)^r, \left(T_k^R\right)^r\right], \left[1-(1-I_k^L)^r, 1-(1-I_k^R)^r\right], \left[1-(1-F_k^L)^r, 1-(1-F_k^R)^r\right]\right). \end{aligned} \tag{8}$$

$$a_i^s \otimes a_j^t \otimes a_k^r = \left(\begin{array}{l} \left[(T_i^L)^s \left(T_j^L\right)^t (T_k^L)^r, (T_i^R)^s \left(T_j^R\right)^t (T_k^R)^r\right], \\ \left[1-(1-I_i^L)^s \left(1-I_j^L\right)^t (1-I_k^L)^r, 1-(1-I_i^R)^s \left(1-I_j^R\right)^t (1-I_k^R)^r\right], \\ \left[1-(1-F_i^L)^s \left(1-F_j^L\right)^t (1-F_k^L)^r, 1-(1-F_i^R)^s \left(1-F_j^R\right)^t (1-F_k^R)^r\right]. \end{array}\right) \tag{9}$$

$$\begin{aligned} &\bigoplus_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} (a_i^s \otimes a_j^k \otimes a_k^r) \\ &= \left(\begin{array}{l} \left[1 - \prod_{i,j,k=1}^{n} \left(1 - \left(T_i^L\right)^s \left(T_j^L\right)^t \left(T_k^L\right)^r\right), 1 - \prod_{i,j,k=1}^{n} \left(1 - \left(T_i^R\right)^s \left(T_j^R\right)^t \left(T_k^R\right)^r\right)\right], \\ \left[\prod_{i,j,k=1}^{n} \left(1 - \left(1 - I_i^L\right)^s \left(1 - I_j^L\right)^t \left(1 - I_k^L\right)^r\right), \prod_{i,j,k=1}^{n} \left(1 - \left(1 - I_i^R\right)^s \left(1 - I_j^R\right)^t \left(1 - I_k^R\right)^r\right)\right], \\ \left[\prod_{i,j,k=1}^{n} \left(1 - \left(1 - F_i^L\right)^s \left(1 - F_j^L\right)^t \left(1 - F_k^L\right)^r\right), \prod_{i,j,k=1}^{n} \left(1 - \left(1 - F_i^R\right)^s \left(1 - F_j^R\right)^t \left(1 - F_k^R\right)^r\right)\right]. \end{array}\right) \end{aligned} \tag{10}$$

$$\frac{1}{n(n-1)(n-2)} \mathop{\oplus}_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(a_i^s \otimes a_j^k \otimes a_k^r\right)$$

$$= \left(\begin{array}{l} \left[1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(T_i^L\right)^s \left(T_j^L\right)^t \left(T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}, 1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(T_i^R\right)^s \left(T_j^R\right)^t \left(T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}} \right], \\ \left[\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(1-I_i^L\right)^s \left(1-I_j^L\right)^t \left(1-I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}, \prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(1-I_i^R\right)^s \left(1-I_j^R\right)^t \left(1-I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}} \right], \\ \left[\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(1-F_i^L\right)^s \left(1-F_j^L\right)^t \left(1-F_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}, \prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(1-F_i^R\right)^s \left(1-F_j^R\right)^t \left(1-F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}} \right]. \end{array} \right) \tag{11}$$

$$= \left(\begin{array}{l} \left[\begin{array}{l} \left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(T_i^L\right)^s \left(T_j^L\right)^t \left(T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\ \left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(T_i^R\right)^s \left(T_j^R\right)^t \left(T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \end{array} \right], \\ \left[\begin{array}{l} 1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(1-I_i^L\right)^s \left(1-I_j^L\right)^t \left(1-I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\ 1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(1-I_i^R\right)^s \left(1-I_j^R\right)^t \left(1-I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \end{array} \right], \\ \left[\begin{array}{l} 1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(1-F_i^L\right)^s \left(1-F_j^L\right)^t \left(1-F_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\ 1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} \left(1-\left(1-F_i^R\right)^s \left(1-F_j^R\right)^t \left(1-F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \end{array} \right]. \end{array} \right) \tag{12}$$

Similarly, we can get $T_2 = T^R, I_1 = I^L, I_2 = I^R$, $F_1 = F^L$ *and* $F_2 = F^R$. that means

$$\text{GINNBM}^{s,t,r}\left(a_1, a_2, \cdots, a_n\right) = a \tag{19}$$

Property 2 (Monotonicity): Let $a_i = \left(\left[T_{a_i}^L, T_{a_i}^R\right], \left[I_{a_i}^L, I_{a_i}^R\right], \left[F_{a_i}^L, F_{a_i}^R\right]\right) (i = 1, 2, \ldots, n)$ and $b_i = \left(\left[T_{b_i}^L, T_{b_i}^R\right], \left[I_{b_i}^L, I_{b_i}^R\right], \left[F_{b_i}^L, F_{b_i}^R\right]\right) (i = 1, 2, \ldots, n)$ be two sets of INNs. If $T_{a_i}^L \leq T_{b_i}^L, T_{a_i}^R \leq T_{b_i}^R$ *and* $I_{a_i}^L \geq I_{b_i}^L, I_{a_i}^R \geq_{b_i}^R$, *and* $F_{a_i}^L \geq F_{b_i}^L$, $F_{a_i}^R \geq F_{b_i}^R$ holds for all i, then

$$\text{GINNBM}^{s,t,r}(a_1, a_2, \cdots, a_n) \leq \text{GINNBM}^{s,t,r}(b_1, b_2, \cdots, b_n). \tag{20}$$

Proof: Let $\text{GINNBM}^{s,t,r}\left(a_1, a_2, \cdots, a_n\right) = \left(\left[T_a^L, T_a^R\right], \left[I_a^L, I_a^R\right], \left[F_a^L, F_a^R\right]\right)$ and $\text{GINNBM}^{s,t,r}\left(b_1, b_2, \cdots, b_n\right) = \left(\left[T_b^L, T_b^R\right], \left[I_b^L, I_b^R\right], \left[F_b^L, F_b^R\right]\right)$. Given that $T_{a_i}^L \leq T_{b_i}^L$,we

$$\left[\begin{array}{l}\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(T_i^L\right)^s\left(T_j^L\right)^t\left(T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\ \left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right]\in[0,1] \quad (13)$$

$$\left[\begin{array}{l}1-\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-I_i^L\right)^s\left(1-I_j^L\right)^t\left(1-I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\ 1-\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right]\in[0,1] \quad (14)$$

$$\left[\begin{array}{l}1-\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-I_i^L\right)^s\left(1-I_j^L\right)^t\left(1-I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\ 1-\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right]\in[0,1] \quad (15)$$

$$\begin{aligned}0\leq&\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\\ &+1-\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\\ &+1-\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-F_i^R\right)^s\left(1-F_j^R\right)^t\left(1-F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\leq 3 \quad (16)\end{aligned}$$

can obtain

$$\left(T_{a_i}^L\right)^s\left(T_{a_j}^L\right)^t\left(T_{a_k}^L\right)^r \leq \left(T_{b_i}^L\right)^s\left(T_{b_j}^L\right)^t\left(T_{b_k}^L\right)^r \quad (21)$$

$$\left(1-\left(T_{a_i}^L\right)^s\left(T_{a_j}^L\right)^t\left(T_{a_k}^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}} \geq \left(1-\left(T_{b_i}^L\right)^s\left(T_{b_j}^L\right)^t\left(T_{b_k}^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}} \quad (22)$$

Therefore,

$$\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(T_{a_i}^L\right)^s\left(T_{a_j}^L\right)^t\left(T_{a_k}^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}} \geq \prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(T_{b_i}^L\right)^s\left(T_{b_j}^L\right)^t\left(T_{b_k}^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}} \quad (23)$$

$$T_1=\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(T_i^L\right)^s\left(T_j^L\right)^t\left(T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}$$

$$T_2=\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}$$

$$I_1=1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-I_i^L\right)^s\left(1-I_j^L\right)^t\left(1-I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}$$

$$I_2=1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}$$

$$F_1=1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-F_i^L\right)^s\left(1-F_j^L\right)^t\left(1-F_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}$$

$$F_2=1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-F_i^R\right)^s\left(1-F_j^R\right)^t\left(1-F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}$$

Thus,

$$\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(T_{a_i}^L\right)^s\left(T_{a_j}^L\right)^t\left(T_{a_k}^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \leq \left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(T_{b_i}^L\right)^s\left(T_{b_j}^L\right)^t\left(T_{b_k}^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)} \quad (24)$$

Which means $T_a^L \leq T_a^L$ .Similarly, we can obtain $T_a^R \leq T_b^R, I_a^L \geq I_b^L,\ I_a^R \geq I_b^R, F_a^L \geq F_b^L \text{ and } F_a^R \geq F_b^R$.

If

$$T_a^L < T_a^L, T_a^R < T_b^R \text{ and } I_a^L \geq I_b^L, I_a^R \geq I_b^R \text{ and } F_a^L \geq F_b^L, F_a^R \geq F_b^R.$$

then

$$\text{GINNBM}^{s,t,r}(a_1,a_2,\cdots,a_n) < \text{GINNBM}^{s,t,r}(b_1,b_2,\cdots,b_n);$$

If

$$T_a^L = T_a^L, T_a^R = T_b^R \text{ and } I_a^L > I_b^L, I_a^R > I_b^R \text{ and } F_a^L > F_b^L, F_a^R > F_b^R.$$

then

$$\text{GINNBM}^{s,t,r}(a_1,a_2,\cdots,a_n) < \text{GINNBM}^{s,t,r}(b_1,b_2,\cdots,b_n);$$

If

$$T_a^L = T_a^L, T_a^R = T_b^R \text{ and } I_a^L = I_b^L, I_a^R = I_b^R \text{ and } F_a^L = F_b^L, F_a^R = F_b^R.$$

then

$$\text{GINNBM}^{s,t,r}(a_1,a_2,\cdots,a_n) = \text{GINNBM}^{s,t,r}(b_1,b_2,\cdots,b_n).$$

Therefore, the proof of Property 2 is completed.

Property 3 (Boundedness): Let $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right) (i = 1, 2, \ldots, n)$ be a set of INNS. If $a^+ = (\max_i(T_i), \min_i(I_i), \min_i(F_i))$ and $a^- = (\min_i(T_i), \max_i(I_i), \max_i(F_i))$, then

$$a^- \leq \text{GINNBM}^{s,t,r}(a_1,a_2,\cdots,a_n) \leq a^+ \quad (25)$$

Proof: From property 2, we can obtain

$$\text{GINNBM}^{s,t,r}(a^-,a^-,\ldots,a^-) = a^-,$$
$$\text{GINNBM}^{s,t,r}(a^+,a^+,\ldots,a^+) = a^+.$$

From property 3, we can obtain

$$\begin{aligned}\mathrm{GINNBM}^{s,t,r}(a^-, a^-, \ldots, a^-) &\le \mathrm{GINNBM}^{s,t,r}(a_1, a_2, \ldots, a_n \\ &\le \mathrm{GINNBM}^{s,t,r}(a^+, a^+, \ldots, a^+).\end{aligned}$$

Therefore, $a^- \le \mathrm{GINNBM}^{s,t,r}(a_1, a_2, \cdots, a_n) \le a^+$.

B. GINNWBM OPERATOR

In actual MADM, it's important to consider attribute weights. This section will propose the generalized INN weighted BM (GINNWBM) operator as follows.

Definition 9: Let $s, t, r > 0$ and $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be a set of INNs with their weight vector being $w_i = (w_1, w_2, \ldots, w_n)^T$, thereby satisfying $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. If

$$\mathrm{GINNWBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) = \left(\mathop{\oplus}_{i,j,k=1}^{n} w_i w_j w_k \left(a_i^s \otimes a_j^t \otimes a_k^r\right)\right)^{1/(s+t+r)} \tag{26}$$

then $\mathrm{GINNWBM}_w^{s,t,r}$ is called the generalized interval neutrosophic number weight Bonferroni mean (GINNWBM) operator.

We can obtain the following theorem 2 according to definition 5.

Theorem 2: Let $s, t, r > 0$ and $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be a set of INNs. The aggregated value by GINNWBM is also an INN and (27), as shown at the top of the next page.

Proof: According to definition 5, we can obtain (28), as shown at the top of the next page.

Thus, (29), as shown at the top of the next page.

Thereafter, (30), as shown at the top of the next page.

Furthermore, (31), as shown at the top of page 10.

Therefore, (32), as shown at the top of page 10.

Hence, (27) is maintained.

Thereafter, (33)–(35), as shown at the top of page 10.

Thereafter, (36), as shown at the top of page 11.

Thereby the (27) is also an INN.

Moreover, GINNWBM has the following properties.

Property 4 (Idempotency): If a_i $(i = 1, 2, \ldots, n)$ are equal, that is, $a_i = a = \left(\left[T^L, T^R\right], \left[I^L, I^R\right], \left[F^L, F^R\right]\right)$, then

$$\mathrm{GINNWBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) = a \tag{37}$$

Proof: Let $T_1, T_2, I_1, I_1, F_1, F_1$, as shown at the top of the page 11

Given that $T_i^L = T_j^L = T_k^L = T^L$, then

$$\begin{aligned}T_1 &= \left(1 - \prod_{i,j,k=1}^{n}\left(1 - \left(T_i^L\right)^s \left(T_j^L\right)^t \left(T_k^L\right)^r\right)^{w_i w_j w_k}\right)^{1/(s+t+r)} \\ &= \left(1 - \prod_{i,j,k=1}^{n}\left(1 - \left(T^L\right)^{s+t+r}\right)^{w_i w_j w_k}\right)^{1/(s+t+r)} \\ &= \left(1 - \left(1 - \left(T^L\right)^{s+t+r}\right)^{\sum_1^n w_i \sum_1^n w_j \sum_1^n w_k}\right)^{1/(s+t+r)} \\ &= T^L \end{aligned} \tag{38}$$

Similarly, we can get $T_2 = T^R, I_1 = I^L, I_2 = I^R$, $F_1 = F^L$ *and* $F_2 = F^R$. that means

$$\mathrm{GINNWBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) = a \tag{39}$$

Property 5 (Monotonicity): Let $a_i = \left(\left[T_{a_i}^L, T_{a_i}^R\right], \left[I_{a_i}^L, I_{a_i}^R\right], \left[F_{a_i}^L, F_{a_i}^R\right]\right)$ $(i = 1, 2, \ldots, n)$ and $b_i = \left(\left[T_{b_i}^L, T_{b_i}^R\right], \left[I_{b_i}^L, I_{b_i}^R\right], \left[F_{b_i}^L, F_{b_i}^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be two sets of INNs. If $T_{a_i}^L \le T_{b_i}^L, T_{a_i}^R \le T_{b_i}^R$ *and* $I_{a_i}^L \ge I_{b_i}^L, I_{a_i}^R \ge_{b_i}^R$ *and* $F_{a_i}^L \ge F_{b_i}^L, F_{a_i}^R \ge F_{b_i}^R$ holds for all i, then

$$\mathrm{GINNWBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) \le \mathrm{GINNWBM}_w^{s,t,r}(b_1, b_2, \cdots, b_n). \tag{40}$$

Proof: Let $\mathrm{GINNWBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) = \left(\left[T_a^L, T_a^R\right], \left[I_a^L, I_a^R\right], \left[F_a^L, F_a^R\right]\right)$ and $\mathrm{GSVNNWBM}_w^{s,t,r}(b_1, b_2, \cdots, b_n) = \left(\left[T_b^L, T_b^R\right], \left[I_b^L, I_b^R\right], \left[F_b^L, F_b^R\right]\right)$. Given that $T_{a_i}^L \le T_{b_i}^L$, we can obtain

$$\left(T_{a_i}^L\right)^s \left(T_{a_j}^L\right)^t \left(T_{a_k}^L\right)^r \le \left(T_{b_i}^L\right)^s \left(T_{b_j}^L\right)^t \left(T_{b_k}^L\right)^r \tag{41}$$

$$\left(1 - \left(T_{a_i}^L\right)^s \left(T_{a_j}^L\right)^t \left(T_{a_k}^L\right)^r\right)^{w_i w_j w_k} \ge \left(1 - \left(T_{b_i}^L\right)^s \left(T_{b_j}^L\right)^t \left(T_{b_k}^L\right)^r\right)^{w_i w_j w_k} \tag{42}$$

Therefore,

$$\prod_{i,j,k=1}^{n}\left(1 - \left(T_{a_i}^L\right)^s \left(T_{a_j}^L\right)^t \left(T_{a_k}^L\right)^r\right)^{w_i w_j w_k} \ge \prod_{i,j,k=1}^{n}\left(1 - \left(T_{b_i}^L\right)^s \left(T_{b_j}^L\right)^t \left(T_{b_k}^L\right)^r\right)^{w_i w_j w_k} \tag{43}$$

Thus,

$$\begin{aligned}&\left(1 - \prod_{i,j,k=1}^{n}\left(1 - \left(T_{a_i}^L\right)^s \left(T_{a_j}^L\right)^t \left(T_{a_k}^L\right)^r\right)^{w_i w_j w_k}\right)^{1/(s+t+r)} \\ &\le \left(1 - \prod_{i,j,k=1}^{n}\left(1 - \left(T_{b_i}^L\right)^s \left(T_{b_j}^L\right)^t \left(T_{b_k}^L\right)^r\right)^{w_i w_j w_k}\right)^{1/(s+t+r)}\end{aligned} \tag{44}$$

Which means $T_a^L \le T_a^L$. Similarly, we can obtain $T_a^R \le T_b^R, I_a^L \ge I_b^L$, $I_a^R \ge I_b^R, F_a^L \ge F_b^L$ *and* $F_a^R \ge F_b^R$.

If $T_a^L < T_a^L, T_a^R < T_b^R$*and* $I_a^L \ge I_b^L$, $I_a^R \ge I_b^R$ *and* $F_a^L \ge F_b^L, F_a^R \ge F_b^R$. then

$$\mathrm{GINNWBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) < \mathrm{GINNWBM}_w^{s,t,r}(b_1, b_2, \cdots, b_n);$$

$$
\begin{aligned}
&\mathrm{GINNWBM}_w^{s,t,r}(a_1,a_2,\cdots,a_n)\\
&=\left(\begin{array}{l}
\left[\begin{array}{l}
\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^L\right)^s\left(T_j^L\right)^t\left(T_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right],\\
\left[\begin{array}{l}
1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^L\right)^s\left(1-I_j^L\right)^t\left(1-I_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right],\\
\left[\begin{array}{l}
1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^L\right)^s\left(1-F_j^L\right)^t\left(1-F_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^R\right)^s\left(1-F_j^R\right)^t\left(1-F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right].
\end{array}\right)
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
a_i^s&=\left(\left[\left(T_i^L\right)^s,\left(T_i^R\right)^s\right],\left[1-(1-I_i^L)^s,1-(1-I_i^R)^s\right],\left[1-(1-F_i^L)^s,1-(1-F_i^R)^s\right]\right);\\
a_j^t&=\left(\left[\left(T_j^L\right)^t,\left(T_j^R\right)^t\right],\left[1-(1-I_j^L)^t,1-(1-I_j^R)^t\right],\left[1-(1-F_j^L)^t,1-(1-F_j^R)^t\right]\right);\\
a_k^r&=\left(\left[\left(T_k^L\right)^r,\left(T_k^R\right)^r\right],\left[1-(1-I_k^L)^r,1-(1-I_k^R)^r\right],\left[1-(1-F_k^L)^r,1-(1-F_k^R)^r\right]\right).
\end{aligned}
\tag{28}
$$

$$
a_i^s\otimes a_j^t\otimes a_k^r=\left(\begin{array}{l}
\left[(T_i^L)^s\left(T_j^L\right)^t(T_k^L)^r,(T_i^R)^s\left(T_j^R\right)^t(T_k^R)^r\right],\\
\left[1-(1-I_i^L)^s\left(1-I_j^L\right)^t(1-I_k^L)^r,1-(1-I_i^R)^s\left(1-I_j^R\right)^t(1-I_k^R)^r\right],\\
\left[1-(1-F_i^L)^s\left(1-F_j^L\right)^t(1-F_k^L)^r,1-(1-F_i^R)^s\left(1-F_j^R\right)^t(1-F_k^R)^r\right].
\end{array}\right)
\tag{29}
$$

$$
\begin{aligned}
&w_iw_jw_k(a_i^s\otimes a_j^k\otimes a_k^r)\\
&=\left(\begin{array}{l}
\left[\left(1-(T_i^L)^s\left(T_j^L\right)^t(T_k^L)^r\right)^{w_iw_jw_k},\left(1-(T_i^R)^s\left(T_j^R\right)^t(T_k^R)^r\right)^{w_iw_jw_k}\right],\\
\left[\left(1-(1-I_i^L)^s\left(1-I_j^L\right)^t(1-I_k^L)^r\right)^{w_iw_jw_k},\left(1-(1-I_i^R)^s\left(1-I_j^R\right)^t(1-I_k^R)^r\right)^{w_iw_jw_k}\right],\\
\left[\left(1-(1-F_i^L)^s\left(1-F_j^L\right)^t(1-F_k^L)^r\right)^{w_iw_jw_k},\left(1-(1-F_i^R)^s\left(1-F_j^R\right)^t(1-F_k^R)^r\right)^{w_iw_jw_k}\right].
\end{array}\right)
\end{aligned}
\tag{30}
$$

If $T_a^L = T_a^L, T_a^R = T_b^R$ *and* $I_a^L > I_b^L$, $I_a^R > I_b^R$ *and* $F_a^L > F_b^L, F_a^R > F_b^R$. then

$$
\begin{aligned}
&\mathrm{GINNWBM}_w^{s,t,r}(a_1,a_2,\cdots,a_n)\\
&\qquad<\mathrm{GINNWBM}_w^{s,t,r}(b_1,b_2,\cdots,b_n);
\end{aligned}
$$

If $T_a^L = T_a^L, T_a^R = T_b^R$ *and* $I_a^L > I_b^L$, $I_a^R > I_b^R$ *and* $F_a^L > F_b^L, F_a^R > F_b^R$. then

$$
\begin{aligned}
&\mathrm{GINNWBM}_w^{s,t,r}(a_1,a_2,\cdots,a_n)\\
&\qquad=\mathrm{GINNWBM}_w^{s,t,r}(b_1,b_2,\cdots,b_n).
\end{aligned}
$$

Therefore, the proof of Property 2 is completed.

Property 6 (Boundedness): Let $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be a set of SVNNS. If $a^+ = (\max_i(T_i), \min_i(I_i), \min_i(F_i))$ and $a^- = (\min_i(T_i), \max_i(I_i), \max_i(F_i))$, then

$$
a^- \le \mathrm{GINNWBM}_w^{s,t,r}(a_1,a_2,\cdots,a_n) \le a^+ \tag{45}
$$

Proof: From property4, we can obtain

$$
\begin{aligned}
\mathrm{GINNWBM}_w^{s,t,r}(a^-,a^-,\ldots,a^-)&=a^-,\\
\mathrm{GINNWBM}_w^{s,t,r}(a^+,a^+,\ldots,a^+)&=a^+.
\end{aligned}
$$

From property5, we can obtain

$$
\mathrm{GINNWBM}_w^{s,t,r}(a^-,a^-,\ldots,a^-)
$$

$$\mathop{\oplus}_{i,j,k=1}^{n} w_iw_jw_k(a_i^s \otimes a_j^k \otimes a_k^r)$$
$$= \left(\begin{array}{l} \left[1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^L\right)^s\left(T_j^L\right)^t\left(T_k^L\right)^r\right)^{w_iw_jw_k}, 1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{w_iw_jw_k} \right], \\ \left[\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^L\right)^s\left(1-I_j^L\right)^t\left(1-I_k^L\right)^r\right)^{w_iw_jw_k}, \prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{w_iw_jw_k} \right], \\ \left[\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^L\right)^s\left(1-F_j^L\right)^t\left(1-F_k^L\right)^r\right)^{w_iw_jw_k}, \prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^R\right)^s\left(1-F_j^R\right)^t\left(1-F_k^R\right)^r\right)^{w_iw_jw_k} \right]. \end{array} \right) \tag{31}$$

$$\text{GINNWBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n)$$
$$= \left(\begin{array}{l} \left[\begin{array}{l} \left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^L\right)^s\left(T_j^L\right)^t\left(T_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}, \\ \left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} \end{array} \right], \\ \left[\begin{array}{l} 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^L\right)^s\left(1-I_j^L\right)^t\left(1-I_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}, \\ 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} \end{array} \right], \\ \left[\begin{array}{l} 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^L\right)^s\left(1-F_j^L\right)^t\left(1-F_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}, \\ 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^R\right)^s\left(1-F_j^R\right)^t\left(1-F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} \end{array} \right]. \end{array} \right) \tag{32}$$

$$\left[\begin{array}{l} \left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^L\right)^s\left(T_j^L\right)^t\left(T_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}, \\ \left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} \end{array} \right] \in [0, 1] \tag{33}$$

$$\left[\begin{array}{l} 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^L\right)^s\left(1-I_j^L\right)^t\left(1-I_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}, \\ 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} \end{array} \right] \in [0, 1] \tag{34}$$

$$\left[\begin{array}{l} 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^L\right)^s\left(1-F_j^L\right)^t\left(1-F_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}, \\ 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^R\right)^s\left(1-F_j^R\right)^t\left(1-F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} \end{array} \right] \in [0, 1] \tag{35}$$

$$0 \leq \left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} + 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} + 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^R\right)^s\left(1-F_j^R\right)^t\left(1-F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)} \leq 3 \tag{36}$$

$$T_1=\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^L\right)^s\left(T_j^L\right)^t\left(T_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}$$

$$T_2=\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(T_i^R\right)^s\left(T_j^R\right)^t\left(T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}$$

$$I_1=1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^L\right)^s\left(1-I_j^L\right)^t\left(1-I_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}$$

$$I_2=1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-I_i^R\right)^s\left(1-I_j^R\right)^t\left(1-I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}$$

$$F_1=1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^L\right)^s\left(1-F_j^L\right)^t\left(1-F_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}$$

$$F_2=1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-F_i^R\right)^s\left(1-F_j^R\right)^t\left(1-F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}$$

$$\leq \mathrm{GINNWBM}_w^{s,t,r}(a_1, a_2, \ldots, a_n$$
$$\leq \mathrm{GINNWBM}_w^{s,t,r}(a^+, a^+, \ldots, a^+).$$

Therefore, $a^- \leq \mathrm{GINNWBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) \leq a^+$.

C. GINNGBM OPERATOR

Thereafter, we extend GGBM to INNS and introduce the generalized interval neutrosophic numbers geometric Bonferroni mean (GINNGBM) operator.

Definition 10: Let $s, t, r > 0$ and $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be a set of INNs. If

$$\mathrm{GINNGBM}^{s,t,r}(a_1, a_2, \cdots a_n) = \frac{1}{s+t+r} \underset{\substack{i,j,k=1\\ i\neq j\neq k}}{\overset{n}{\otimes}} (sa_i \oplus ta_j \oplus ra_k)^{\frac{1}{n(n-1)(n-2)}} \tag{46}$$

Then $\mathrm{GINNGBM}^{s,t,r}$ is called GINNGBM.

Theorem 3: Let $s, t, r > 0$ and $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be a set of INNs. The aggregated value by GINNGBM is also an INN and (47), as shown at the top of the next page.

Proof: Though definition 3, we can obtain (48)-(49), as shown at the top of the next page.

Thereafter, (50), as shown at the top of the next page.

Therefore, (51), as shown at the top of the page 13.

Thus, (52), as shown at the top of the page 13.

Hence, (47) is maintained.

Thereafter, (53)–(55), as shown at the top of the page 14.

Therefore, (56), as shown at the top of the page 14.

Thereby completing the proof.

The GINNGBM has the following properties.

Property 7: Let $s, t, r > 0$ and $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be a set of INNs. Then

(1) (Idempotency). If $a_i (i = 1.2, \ldots, n)$ are equal, that is $a_i = a = \left(\left[T^L, T^R\right], \left[I^L, I^R\right], \left[F^L, F^R\right]\right)$, then

$$\mathrm{GINNGBM}^{s,t,r}(a_1, a_2, \cdots, a_n) = a \tag{57}$$

(2) (Monotonicity). Let $a_i = \left(\left[T_{a_i}^L, T_{a_i}^R\right], \left[I_{a_i}^L, I_{a_i}^R\right], \left[F_{a_i}^L, F_{a_i}^R\right]\right)$ $(i = 1, 2, \ldots, n)$ and $b_i = \left(\left[T_{b_i}^L, T_{b_i}^R\right], \left[I_{b_i}^L, I_{b_i}^R\right], \left[F_{b_i}^L, F_{b_i}^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be two sets of INNs. If $T_{a_i}^L \leq T_{b_i}^L, T_{a_i}^R \leq T_{b_i}^R$ *and* $I_{a_i}^L \geq I_{b_i}^L, I_{a_i}^R \geq_{b_i}^R$ *and* $F_{a_i}^L \geq F_{b_i}^L, F_{a_i}^R \geq F_{b_i}^R$ holds

$$
\begin{aligned}
&\mathrm{GINNGBM}^{s,t,r}(a_1,a_2,\cdots a_n) \\
&= \frac{1}{s+t+r} \bigotimes_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} (sa_i \oplus ta_j \oplus ra_k)^{\frac{1}{n(n-1)(n-2)}} \\
&= \left(\begin{array}{l}
\left[\begin{array}{l}
1-\left(1-\prod\limits_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-T_i^L\right)^s\left(1-T_j^L\right)^t\left(1-T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\
1-\left(1-\prod\limits_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}
\end{array} \right], \\
\left[\begin{array}{l}
\left(1-\prod\limits_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(I_i^L\right)^s\left(I_j^L\right)^t\left(I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\
\left(1-\prod\limits_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}
\end{array} \right], \\
\left[\begin{array}{l}
\left(1-\prod\limits_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(F_i^L\right)^s\left(F_j^L\right)^t\left(F_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}, \\
\left(1-\prod\limits_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}
\end{array} \right].
\end{array} \right)
\end{aligned} \tag{47}
$$

$$
\begin{aligned}
sa_i &= \left(\left[1-\left(1-T_i^L\right)^s, 1-\left(1-T_i^R\right)^s\right], \left[\left(I_i^L\right)^s, \left(I_i^R\right)^s\right], \left[\left(F_i^L\right)^s, \left(F_i^R\right)^s\right]\right), \\
ta_j &= \left(\left[1-\left(1-T_j^L\right)^t, 1-\left(1-T_j^R\right)^t\right], \left[\left(I_j^L\right)^t, \left(I_j^R\right)^t\right], \left[\left(F_j^L\right)^t, \left(F_j^R\right)^t\right]\right), \\
ra_k &= \left(\left[1-\left(1-T_k^L\right)^r, 1-\left(1-T_k^R\right)^r\right], \left[\left(I_k^L\right)^r, \left(I_k^R\right)^r\right], \left[\left(F_k^L\right)^r, \left(F_k^R\right)^r\right]\right).
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
&sa_i \oplus ta_j \oplus ra_k \\
&= \left(\begin{array}{l}
\left[1-(1-T_i^L)^s\left(1-T_j^L\right)^t(1-T_k^L)^r, 1-(1-T_i^R)^s\left(1-T_j^R\right)^t(1-T_k^R)^r\right], \\
\left[(I_i^L)^s\left(I_j^L\right)^t(I_k^L)^r, (I_i^R)^s\left(I_j^R\right)^t(I_k^R)^r\right], \\
\left[(F_i^L)^s\left(F_j^L\right)^t(F_k^L)^r, (F_i^R)^s\left(F_j^R\right)^t(F_k^R)^r\right]
\end{array} \right)
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
&(sa_i \oplus ta_j \oplus ra_k)^{\frac{1}{n(n-1)(n-2)}} \\
&= \left(\begin{array}{l}
\left[\left(1-(1-T_i^L)^s\left(1-T_j^L\right)^t(1-T_k^L)^r\right)^{\frac{1}{n(n-1)(n-2)}}, \left(1-(1-T_i^R)^s\left(1-T_j^R\right)^t(1-T_k^R)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right], \\
\left[1-\left(1-(I_i^L)^s\left(I_j^L\right)^t(I_k^L)^r\right)^{\frac{1}{n(n-1)(n-2)}}, 1-\left(1-(I_i^R)^s\left(I_j^R\right)^t(I_k^R)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right], \\
\left[1-\left(1-(F_i^L)^s\left(F_j^L\right)^t(F_k^L)^r\right)^{\frac{1}{n(n-1)(n-2)}}, 1-\left(1-(F_i^R)^s\left(F_j^R\right)^t(F_k^R)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right].
\end{array} \right)
\end{aligned} \tag{50}
$$

$$\begin{aligned}&\mathop{\otimes}_{i,j,k=1}^{n}(sa_i\oplus ta_j\oplus ra_k)^{\frac{1}{n(n-1)(n-2)}}\\&=\left(\begin{array}{l}\left[\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-T_i^L\right)^s\left(1-T_j^L\right)^t\left(1-T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}},\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right],\\\left[1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(I_i^L\right)^s\left(I_j^L\right)^t\left(I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}},1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right],\\\left[1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(F_i^L\right)^s\left(F_j^L\right)^t\left(F_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}},1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right].\end{array}\right)\end{aligned}\tag{51}$$

$$\begin{aligned}&\text{GINNGBM}^{s,t,r}(a_1,a_2,\cdots a_n)\\&=\frac{1}{s+t+r}\mathop{\otimes}_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}(sa_i\oplus ta_j\oplus ra_k)^{\frac{1}{n(n-1)(n-2)}}\\&=\left(\begin{array}{l}\left[\begin{array}{l}1-\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-T_i^L\right)^s\left(1-T_j^L\right)^t\left(1-T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\1-\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right],\\\left[\begin{array}{l}\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(I_i^L\right)^s\left(I_j^L\right)^t\left(I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right],\\\left[\begin{array}{l}\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(F_i^L\right)^s\left(F_j^L\right)^t\left(F_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\\left(1-\prod_{\substack{i,j,k=1\\i\neq j\neq k}}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right].\end{array}\right)\end{aligned}\tag{52}$$

for all i, then

$$\text{GINNGBM}^{s,t,r}(a_1,a_2,\cdots,a_n)\leq\text{GINNGBM}^{s,t,r}(b_1,b_2,\cdots,b_n).\tag{58}$$

(3) (Boundedness). Let $a_i=\left(\left[T_i^L,T_i^R\right],\left[I_i^L,I_i^R\right],\left[F_i^L,F_i^R\right]\right)$ $(i=1,2,\ldots,n)$ be a set of INNS. If $a^+=(\max_i(T_i),\min_i(I_i),\min_i(F_i))$ and $a^-=(\min_i(T_i),\max_i(I_i),\max_i(F_i))$, then

$$a^-\leq\text{GINNGBM}^{s,t,r}(a_1,a_2,\cdots,a_n)\leq a^+.\tag{59}$$

D. GINNWGBM OPERATOR

In actual MADM, it's important to consider attribute weights. Thereafter, we extend GWGBM to INNS and introduce the

$$\left[\begin{array}{l}1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-T_i^L\right)^s\left(1-T_j^L\right)^t\left(1-T_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\ 1-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right]\in[0,1] \tag{53}$$

$$\left[\begin{array}{l}\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(I_i^L\right)^s\left(I_j^L\right)^t\left(I_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\ \left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right]\in[0,1] \tag{54}$$

$$\left[\begin{array}{l}\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(F_i^L\right)^s\left(F_j^L\right)^t\left(F_k^L\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)},\\ \left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\end{array}\right]\in[0,1] \tag{55}$$

$$\begin{aligned}0\leq 1&-\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\\ &+\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\\ &+\left(1-\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{\frac{1}{n(n-1)(n-2)}}\right)^{1/(s+t+r)}\leq 3\end{aligned} \tag{56}$$

generalized interval neutrosophic numbers weighted geometric Bonferroni mean (GINNWGBM) operator.

Definition 11: Let $s,t,r>0$ and $a_i=\left(\left[T_i^L,T_i^R\right],\left[I_i^L,I_i^R\right],\left[F_i^L,F_i^R\right]\right)$ $(i=1,2,\ldots,n)$ be a set of INNs with their weight vector being $w_i=(w_1,w_2,\ldots,w_n)^T$, thereby satisfying $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$. If

$$\text{GINNWGBM}_w^{s,t,r}(a_1,a_2,\cdots a_n)=\frac{1}{s+t+r}\underset{i,j,k=1}{\overset{n}{\otimes}}(sa_i\oplus ta_j\oplus ra_k)^{w_iw_jw_k} \tag{60}$$

Then $\text{GINNWGBM}_w^{s,t,r}$ is called GINNWGBM.

Theorem 4: Let $s,t,r>0$ and $a_i=\left(\left[T_i^L,T_i^R\right],\left[I_i^L,I_i^R\right],\left[F_i^L,F_i^R\right]\right)$ $(i=1,2,\ldots,n)$ be a set of INNs. The aggregated value by GINNWGBM is also an INN and (61), as shown at the top of the next page.

Proof: Though definition 5, we can obtain (62), (63), as shown at the top of the next page.

Thereafter, (64), as shown at the top of the next page.

Therefore, (65), as shown at the top of the next page.

Thus, (66), as shown at the top of the page 16.

Hence, (61) is maintained.

Thereafter, (67)–(69), as shown at the top of the page 16.

Therefore, (70), as shown at the top of the page 16.

The GINNWGBM has the following properties.

Property 8: Let $s,t,r>0$ and $a_i=\left(\left[T_i^L,T_i^R\right],\left[I_i^L,I_i^R\right],\left[F_i^L,F_i^R\right]\right)$ $(i=1,2,\ldots,n)$ be a set of INNs. Then

$$\begin{aligned}
&\mathrm{GINNWGBM}_w^{s,t,r}(a_1,a_2,\cdots a_n)\\
&=\frac{1}{s+t+r}\mathop{\otimes}_{i,j,k=1}^{n}(sa_i\oplus ta_j\oplus ra_k)^{w_iw_jw_k}\\
&=\left(\begin{array}{l}
\left[\begin{array}{l}1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-T_i^L\right)^s\left(1-T_j^L\right)^t\left(1-T_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\ 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}\end{array}\right],\\
\left[\begin{array}{l}\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(I_i^L\right)^s\left(I_j^L\right)^t\left(I_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\ \left(1-\prod_{i,j,k=1}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}\end{array}\right],\\
\left[\begin{array}{l}\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(F_i^L\right)^s\left(F_j^L\right)^t\left(F_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\ \left(1-\prod_{i,j,k=1}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}\end{array}\right].
\end{array}\right)
\end{aligned}\tag{61}$$

$$\begin{aligned}
sa_i&=\left(\left[1-\left(1-T_i^L\right)^s,1-\left(1-T_i^R\right)^s\right],\left[\left(I_i^L\right)^s,\left(I_i^R\right)^s\right],\left[\left(F_i^L\right)^s,\left(F_i^R\right)^s\right]\right),\\
ta_j&=\left(\left[1-\left(1-T_j^L\right)^t,1-\left(1-T_j^R\right)^t\right],\left[\left(I_j^L\right)^t,\left(I_j^R\right)^t\right],\left[\left(F_j^L\right)^t,\left(F_j^R\right)^t\right]\right),\\
ra_k&=\left(\left[1-\left(1-T_k^L\right)^r,1-\left(1-T_k^R\right)^r\right],\left[\left(I_k^L\right)^r,\left(I_k^R\right)^r\right],\left[\left(F_k^L\right)^r,\left(F_k^R\right)^r\right]\right).
\end{aligned}\tag{62}$$

$$\begin{aligned}
&sa_i\oplus ta_j\oplus ra_k\\
&=\left(\begin{array}{l}\left[1-(1-T_i^L)^s\left(1-T_j^L\right)^t(1-T_k^L)^r,1-(1-T_i^R)^s\left(1-T_j^R\right)^t(1-T_k^R)^r\right],\\ \left[(I_i^L)^s\left(I_j^L\right)^t(I_k^L)^r,(I_i^R)^s\left(I_j^R\right)^t(I_k^R)^r\right],\left[(F_i^L)^s\left(F_j^L\right)^t(F_k^L)^r,(F_i^R)^s\left(F_j^R\right)^t(F_k^R)^r\right]\end{array}\right)
\end{aligned}\tag{63}$$

$$\begin{aligned}
&(sa_i\oplus ta_j\oplus ra_k)^{w_iw_jw_k}\\
&=\left(\begin{array}{l}\left[\left(1-(1-T_i^L)^s\left(1-T_j^L\right)^t(1-T_k^L)^r\right)^{w_iw_jw_k},\left(1-(1-T_i^R)^s\left(1-T_j^R\right)^t(1-T_k^R)^r\right)^{w_iw_jw_k}\right],\\ \left[1-\left(1-(I_i^L)^s\left(I_j^L\right)^t(I_k^L)^r\right)^{w_iw_jw_k},1-\left(1-(I_i^R)^s\left(I_j^R\right)^t(I_k^R)^r\right)^{w_iw_jw_k}\right],\\ \left[1-\left(1-(F_i^L)^s\left(F_j^L\right)^t(F_k^L)^r\right)^{w_iw_jw_k},1-\left(1-(F_i^R)^s\left(F_j^R\right)^t(F_k^R)^r\right)^{w_iw_jw_k}\right].\end{array}\right)
\end{aligned}\tag{64}$$

$$\begin{aligned}
&\mathop{\otimes}_{i,j,k=1}^{n}(sa_i\oplus ta_j\oplus ra_k)^{w_iw_jw_k}\\
&=\left(\begin{array}{l}\left[\prod_{i,j,k=1}^{n}\left(1-\left(1-T_i^L\right)^s\left(1-T_j^L\right)^t\left(1-T_k^L\right)^r\right)^{w_iw_jw_k},\prod_{i,j,k=1}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{w_iw_jw_k}\right],\\ \left[1-\prod_{i,j,k=1}^{n}\left(1-\left(I_i^L\right)^s\left(I_j^L\right)^t\left(I_k^L\right)^r\right)^{w_iw_jw_k},1-\prod_{i,j,k=1}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{w_iw_jw_k}\right],\\ \left[1-\prod_{i,j,k=1}^{n}\left(1-\left(F_i^L\right)^s\left(F_j^L\right)^t\left(F_k^L\right)^r\right)^{w_iw_jw_k},1-\prod_{i,j,k=1}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{w_iw_jw_k}\right].\end{array}\right)
\end{aligned}\tag{65}$$

(1) (Idempotency). If $a_i (i = 1.2, \ldots, n)$ are equal, that is $a_i = a = \left(\left[T^L, T^R\right], \left[I^L, I^R\right], \left[F^L, F^R\right]\right)$, then

$$\mathrm{GINNWGBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) = a \tag{71}$$

(2) (Monotonicity). Let $a_i = \left(\left[T_{a_i}^L, T_{a_i}^R\right], \left[I_{a_i}^L, I_{a_i}^R\right], \left[F_{a_i}^L, F_{a_i}^R\right]\right)$ $(i = 1, 2, \ldots, n)$ and $b_i = \left(\left[T_{b_i}^L, T_{b_i}^R\right], \left[I_{b_i}^L, I_{b_i}^R\right], \left[F_{b_i}^L, F_{b_i}^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be two sets of INNs. If $T_{a_i}^L \leq T_{b_i}^L$,

$$\begin{aligned}
&\mathrm{GINNWGBM}_w^{s,t,r}(a_1,a_2,\cdots a_n)\\
&\quad=\frac{1}{s+t+r}\underset{i,j,k=1}{\overset{n}{\otimes}}\left(sa_i\oplus ta_j\oplus ra_k\right)^{w_iw_jw_k}\\
&\quad=\left(\begin{array}{l}
\left[\begin{array}{l}
1-\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(1-T_i^L\right)^s\left(1-T_j^L\right)^t\left(1-T_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
1-\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right],\\
\left[\begin{array}{l}
\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(I_i^L\right)^s\left(I_j^L\right)^t\left(I_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right],\\
\left[\begin{array}{l}
\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(F_i^L\right)^s\left(F_j^L\right)^t\left(F_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right].
\end{array}\right)
\end{aligned}\tag{66}$$

$$\left[\begin{array}{l}
1-\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(1-T_i^L\right)^s\left(1-T_j^L\right)^t\left(1-T_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
1-\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right]\in[0,1]\tag{67}$$

$$\left[\begin{array}{l}
\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(I_i^L\right)^s\left(I_j^L\right)^t\left(I_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right]\in[0,1]\tag{68}$$

$$\left[\begin{array}{l}
\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(F_i^L\right)^s\left(F_j^L\right)^t\left(F_k^L\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)},\\
\left(1-\prod\limits_{i,j,k=1}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}
\end{array}\right]\in[0,1]\tag{69}$$

$$\begin{aligned}
0\leq\ &1-\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(1-T_i^R\right)^s\left(1-T_j^R\right)^t\left(1-T_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}\\
&+\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(I_i^R\right)^s\left(I_j^R\right)^t\left(I_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}\\
&+\left(1-\prod_{i,j,k=1}^{n}\left(1-\left(F_i^R\right)^s\left(F_j^R\right)^t\left(F_k^R\right)^r\right)^{w_iw_jw_k}\right)^{1/(s+t+r)}\leq 3
\end{aligned}\tag{70}$$

TABLE 1. INN decision making.

	G_1	G_2	G_3	G_4
A_1	([0.6,0.8],[0.7,0.8],[0.3,0.4])	([0.7,0.8],[0.4,0.5],[0.2,0.3])	([0.4,0.5],[0.7,0.8],[0.3,0.4])	([0.6,0.7],[0.5,0.8],[0.3,0.5])
A_2	([0.8,0.9],[0.5,0.7],[0.1,0.3])	([0.7,0.8],[0.4,0.6],[0.2,0.3])	([0.7,0.8],[0.1,0.2],[0.6,0.7])	([0.6,0.8],[0.4,0.5],[0.3,0.5])
A_3	([0.6,0.7],[0.5,0.7],[0.3,0.4])	([0.5,0.6],[0.8,0.9],[0.4,0.6])	([0.4,0.5],[0.6,0.7],[0.3,0.4])	([0.7,0.8],[0.4,0.5],[0.3,0.5])
A_4	([0.6,0.8],[0.3,0.4],[0.5,0.6])	([0.5,0.7],[0.4,0.5],[0.5,0.6])	([0.4,0.6],[0.5,0.7],[0.3,0.5])	([0.6,0.7],[0.5,0.7],[0.2,0.3])
A_5	([0.7,0.8],[0.5,0.6],[0.4,0.5])	([0.3,0.4],[0.7,0.8],[0.3,0.4])	([0.7,0.8],[0.5,0.6],[0.1,0.2])	([0.3,0.4],[0.7,0.8],[0.2,0.3])

TABLE 2. The calculating results of the high-tech enterprises by the GINNWBM and GINNWGBM ($s = t = r = 1$).

	GINNWBM	GINNWGBM
A1	([0.6073,0.7348],[0.5428,0.6937],[0.2639,0.3885])	([0.5009,0.6249],[0.6373,0.7679],[0.3694,0.4940])
A2	([0.7019,0.8266],[0.3751,0.5362],[0.2475,0.4048])	([0.5944,0.7294],[0.4853,0.6341],[0.3655,0.5146])
A3	([0.5621,0.6634],[0.5922,0.7187],[0.3343,0.4939])	([0.4587,0.5541],[0.6836,0.7913],[0.4405,0.5911])
A4	([0.5359,0.7116],[0.4136,0.5531],[0.3902,0.5075])	([0.4363,0.6046],[0.5176,0.6458],[0.4998,0.6058])
A5	([0.4660,0.5698],[0.6192,0.7196],[0.2650,0.3671])	([0.3630,0.4550],[0.7008,0.7850],[0.3750,0.4752])

$T_{a_i}^R \leq T_{b_i}^R$ *and* $I_{a_i}^L \geq I_{b_i}^L, I_{a_i}^R \geq_{b_i}^R$ *and* $F_{a_i}^L \geq F_{b_i}^L, F_{a_i}^R \geq F_{b_i}^R$ holds for all i, then

$$\text{GINNWGBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) \leq \text{GINNWGBM}_w^{s,t,r}(b_1, b_2, \cdots, b_n). \quad (72)$$

(3) (Boundedness). Let $a_i = \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right)$ $(i = 1, 2, \ldots, n)$ be a set of INNS. If $a^+ = (\max_i(T_i), \min_i(I_i), \min_i(F_i))$ and $a^- = (\min_i(T_i), \max_i(I_i), \max_i(F_i))$, then

$$a^- \leq \text{GINNWGBM}_w^{s,t,r}(a_1, a_2, \cdots, a_n) \leq a^+. \quad (73)$$

IV. NUMERICAL EXAMPLE AND COMPARATIVE ANALYSIS

A. NUMERICAL EXAMPLE

As the knowledge-based economy emergence and innovation country building presentation, High-tech enterprises increase the innovation activities, so innovation capability become very important and innovation management should be put emphasis on. Although many companies have developed the standard of innovation management in practice, the level of innovation management capability and innovation performance should be improved. High-tech enterprise allocation resources rationally for improving innovation management capability attract people's attention from both theoretical and practical perspective. For innovation research, technology innovation is rich, but innovation management is still scarcity, while the theory of innovation management is lacking of empirical support, lacking of concerning on how to build and update it. Meanwhile, application modern project management theory on R&D management and innovative management practice and theory have been focused on. With the modern project management developing, it needs to combine with innovation management theory from both organizational strategy management theory and project management theory for in-depth studying on independence innovation management capacity. Thus, we give an example for evaluating the technological innovation capability for the high-tech enterprises with INNs. There are five possible high-tech enterprises A_i $(i = 1, 2, 3, 4, 5)$ to assess. The experts use the four attributes to assess the five high-tech enterprises: ①G_1 is the innovative culture; ②G_2 is the infrastructure and support for industry development; ③G_3 is the knowledge management & organizational learning; ④G_4 is the funding on technological innovation. The five possible high-tech enterprises A_i $(i = 1, 2, 3, 4, 5)$ are to be evaluated with the INNs by the DMs under the above four attributes (whose weighting vector $\omega = (0.3, 0.20, 0.10, 0.40)^T$), as listed in the Table 1.

TABLE 3. The score functions of the high-tech enterprises.

	GINNWBM	GINNWGBM
A1	0.5755	0.4762
A2	0.6608	0.5541
A3	0.5144	0.4177
A4	0.5639	0.4620
A5	0.5108	0.4137

In the following, we use the approach developed to select the best high-tech enterprises.

Step 1: According to w and INNs A_{ij} $(i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4)$, we can fuse all SVNNs A_{ij} by using the GINNWBM (GINNWGBM) operator to get the overall INNs A_i $(i = 1, 2, 3, 4, 5)$ of the high-tech enterprise A_i. The calculating results are shown in Table 2.

Step 2: According to the calculating results in Table 2, the score values of the high-tech enterprises are shown in Table 3.

TABLE 4. Ordering of the high-tech enterprises.

	Ordering
GINNWBM	$A_2 > A_1 > A_4 > A_3 > A_5$
GINNWGBM	$A_2 > A_1 > A_4 > A_3 > A_5$

TABLE 5. Ranking results for different operational parameters of the GINNWBM operator.

(s,t,r)	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	$S(A_5)$	Ordering
(1,1,1)	0.5755	0.6608	0.5144	0.5639	0.5108	$A_2 > A_1 > A_4 > A_3 > A_5$
(2,2,2)	0.7965	0.8593	0.7542	0.7930	0.7430	$A_2 > A_1 > A_4 > A_3 > A_5$
(3,3,3)	0.8556	0.9023	0.8252	0.8487	0.8156	$A_2 > A_1 > A_4 > A_3 > A_5$
(4,4,4)	0.8772	0.9154	0.8529	0.8666	0.8472	$A_2 > A_1 > A_4 > A_3 > A_5$
(5,5,5)	0.8872	0.9206	0.8664	0.8739	0.8643	$A_2 > A_1 > A_4 > A_3 > A_5$
(6,6,6)	0.8927	0.9232	0.8742	0.8775	0.8749	$A_2 > A_1 > A_4 > A_5 > A_3$
(7,7,7)	0.8961	0.9247	0.8794	0.8796	0.8820	$A_2 > A_1 > A_5 > A_4 > A_3$
(8,8,8)	0.8985	0.9258	0.8831	0.8810	0.8870	$A_2 > A_1 > A_5 > A_3 > A_4$
(9,9,9)	0.9002	0.9267	0.8860	0.8821	0.8907	$A_2 > A_1 > A_5 > A_3 > A_4$
(10,10,10)	0.9016	0.9275	0.8884	0.8830	0.8936	$A_2 > A_1 > A_5 > A_3 > A_4$

TABLE 6. Ranking results for different operational parameters of the GINNWGBM operator.

(s,t,r)	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	$S(A_5)$	Ordering
(1,1,1)	0.4762	0.5541	0.4177	0.4620	0.4137	$A_2 > A_1 > A_4 > A_3 > A_5$
(2,2,2)	0.3522	0.4121	0.2984	0.3395	0.3092	$A_2 > A_1 > A_4 > A_5 > A_3$
(3,3,3)	0.3091	0.3497	0.2599	0.2985	0.2795	$A_2 > A_1 > A_4 > A_5 > A_3$
(4,4,4)	0.2893	0.3162	0.2424	0.2799	0.2665	$A_2 > A_1 > A_4 > A_5 > A_3$
(5,5,5)	0.2780	0.2953	0.2322	0.2696	0.2590	$A_2 > A_1 > A_4 > A_5 > A_3$
(6,6,6)	0.2707	0.2810	0.2252	0.2629	0.2537	$A_2 > A_1 > A_4 > A_5 > A_3$
(7,7,7)	0.2655	0.2706	0.2198	0.2582	0.2497	$A_2 > A_1 > A_4 > A_5 > A_3$
(8,8,8)	0.2615	0.2627	0.2156	0.2546	0.2465	$A_2 > A_1 > A_4 > A_5 > A_3$
(9,9,9)	0.2584	0.2564	0.2120	0.2519	0.2438	$A_1 > A_2 > A_4 > A_5 > A_3$
(10,10,10)	0.3214	0.2513	0.2090	0.2496	0.2416	$A_1 > A_2 > A_4 > A_5 > A_3$

Step 3: According to the score functions shown in Table 3 and the comparison formula of score functions, the ordering is shown in Table 4. As we can see, the ordering of the high-tech enterprises is the same, and the best high-tech enterprises is A_2.

B. INFLUENCE OF THE PARAMETER ON THE FINAL RESULT

In order to show the effects on the ranking results by changing parameters of $(s, t, r) \in [1, 10]$ in the GINNWBM (GINNWGBM) operators, all the results are shown in Tables 5 and 6.

TABLE 7. Ordering of the high-tech enterprises.

	Ordering
INWA[14]	$A_2 > A_1 > A_4 > A_3 > A_5$
INWG[14]	$A_2 > A_1 > A_4 > A_5 > A_3$
Similarity degree[49]	$A_2 > A_1 > A_4 > A_5 > A_3$

C. COMPARATIVE ANALYSIS

Then, we compare our methods with INWA operator, INWG operator [14] and similarity degree [44]. The results are shown in Table 7.

From above, we can that we get the same results to show the practicality and effectiveness of the proposed approaches. However, INWA and INWG operators, do not consider the interrelationship between aggregated arguments, and thus cannot eliminate the influence of unfair arguments on decision result. The GINNWBM and GINNWGBM operators consider the relationship among three aggregated arguments.

V. CONCLUSION

In this paper, we investigate the aggregation operators with INN and their application in MADM. In order to fuse the INNs, the GINNWBM and GINNWGBM operators which consider the relationship among three aggregated arguments have been developed. We have studied these two operator's desirable properties. Furthermore, we also show the effectiveness of the GINNWBM and GINNWGBM operators with practical MADM problems. Finally, we give an example for evaluating the technological innovation capability for the high-tech enterprises to show applicability of these two operators, meanwhile, the comparison analysis and influence analysis have been studied. In the future works, we shall expand the proposed methods to other fuzzy MADM problems [45]–[58] and uncertain MADM problems [59]–[73].

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