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Cable Routing Design and Performance Evaluation for Multi-Link Cable-Driven Robots With Minimal Number of Actuating Cables

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ABSTRACT A multi-link cable-driven robot (MCDR) usually has a large number of redundant actuating cables due to its modular cable routing scheme. To reduce the number of actuating cables while keeping the advantages of the modular MCDRs, a hybrid modular cable routing method is proposed, in which some actuating cables are co-shared by adjacent cable-driven joints. Consequently, the total number of actuating cables can be reduced to $n + 1$ for an n -degree-of-freedom (n -DOF) MCDR. Focusing on MCDRs composed of identical 2-DOF cable-driven universal joint modules, the performance of the MCDR with the hybrid modular cable routing scheme is evaluated. It is concluded that: 1) the wrench-closure workspace of an MCDR with the hybrid modular cable routing scheme remains unchanged compared to the conventional modular cable routing scheme; 2) the maximal joint speed is inversely proportional to the total number of joint modules that co-share one actuating cable; and 3) the loading capability of an MCDR is a monotone decreasing function of the number of co-shared actuating cables. To verify the conclusions obtained, computer simulations are conducted on an MCDR with different cable routing schemes. Besides, the hybrid modular cable routing with alternatively co-shared actuating cables is an ideal cable routing scheme as it has the minimum loss of performance on motion speed and loading capability.

INDEX TERMS Cable-driven robot, cable routing, workspace analysis, motion speed, loading capability.

I. INTRODUCTION

Cable-driven robots (CDRs) are a type of mechanisms driven by a set of cables that are typically arranged in parallel. CDRs usually have lightweight structures with low weight and inertia because the motors are installed on the base instead of the links of the robots. Such advantages enable the CDRs a large workspace, large payload and high motion speed. Therefore, CDRs have drawn a lot of attentions among researchers due to their unique advantages and pervasive applications, such as high speed manufacturing [1], building construction [2], large-scale manipulation [3] and rehabilitation [4].

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Present research about CDRs mostly focuses on **cable-driven parallel robots** (CDPRs) which have a single moving platform [5]–[8], where all cables connect the base to the moving platform directly. CDPRs usually have a large reachable workspace but limited rotational capability due to their parallel configuration. To increase the dexterity of CDRs, **multi-link cable-driven robots** (MCDRs) are designed which have a hybrid parallel-serial configuration [9]–[12]. Such MCDRs have the advantages of large rotational workspace like serial robots while retaining the lightweight structure of CDRs.

For an MCDR, the cable routing scheme (the arrangement of how cables route through the links of the robot) is an important issue since it significantly affects the performance of the MCDR. One challenge is that various cable routing schemes

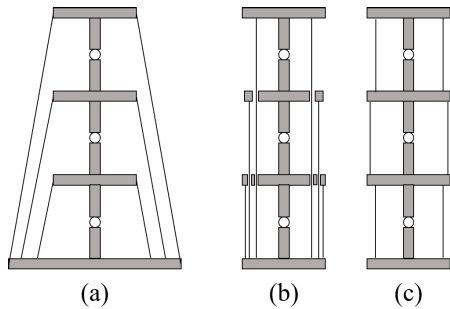


FIGURE 1. Three types of cable routing schemes.

bring difficulties in kinematics and dynamics modeling. To solve the problem, the Cable-Routing Matrix was proposed to represent arbitrary cable routing within a generalized MCDR kinematics and dynamics model [9]. However, such a general analysis approach is difficult to figure out the performance difference among various cable routing schemes.

In previous studies, cable routing schemes can be broadly classified into three categories. The first category is called **direct-connecting cable routing scheme** as shown in Fig. 1a, where each of the actuating cables directly connect one of the links to the base [13], [14]. An MCDR with such a cable arrangement is simple in structure but the wrench-closure workspace is limited and the kinematics of different joints are coupled. The second category of cable routing are **passing-through cable routing scheme** as shown in Fig. 1b, where each link is driven by cables that pass through some guide points on other links that are closer to the base [15]–[17]. This cable arrangement reduces cable interference and hence increases the workspace. However, the motions of each joint are also coupled and the number of required cables are usually large. The third category is called the **modular cable routing scheme** as shown in Fig. 1c, where each joint is an independent module driven by a set of independent cables [18]. Bowden cables are usually employed in MCDRs with such a cable routing so that each joint can be controlled independently. Since each joint is an independent cable-driven module, the motions of each joint are decoupled, simplifying the kinematics and dynamics, and also increasing the workspace of the robot. In this cable routing arrangement, since each joint is essentially a fully constrained cable-driven mechanism which requires at least one redundant cable, the minimum number of cables is $n + k$ where n is the total degrees of freedom of the MCDR and k is the total number of the joint modules. A larger number of actuating cables will increase the hardware cost and system complexity.

To overcome the drawbacks of modular MCDRs, it is necessary to investigate how to reduce the number of the actuating cables. It has been proven that for an n -DOF CDR, at least $n + 1$ cables are required to be capable of sustaining any arbitrary external wrench [19]. Redesigning the cable routing is a practical approach to reduce the number of cables. An optimization method to reduce the number of actuators for a planar cable-driven continuum manipulator is presented in [20]. However, such a method is not suitable for modular

MCDRs because the multi-segment cables pass through all links which increases the complexity in kinematic modeling. Few works investigate how to reduce the number of cables while maintaining the advantage of modular cable routing.

In this paper, a **hybrid modular cable routing scheme** for MCDRs is proposed to reduce the cable number of a modular MCDR. This hybrid modular cable routing scheme is obtained by co-sharing cables between two adjacent modules such that the total number of cable is reduced to $n + 1$. By the proposed new method, the equilibrium equations of each module are still satisfied after co-sharing some cables, and the kinematic and dynamic models are similar to the conventional modular MCDRs. The wrench-closure workspace, motion speed and loading capability of the hybrid modular cable routing scheme are analyzed. It is shown that the wrench-closure workspace of the hybrid modular cable routing scheme is identical to the conventional modular method. It is also shown that the joint speed is inversely proportional to the total number of joints that co-share one cable while the loading capabilities of each joint reduce when the number of co-shared cables increases. Finally, inverse dynamics simulation for a 4U (universal joint) MCDR shows that the relationships of motion speed and loading capability obtained are satisfied. Besides, the hybrid modular cable routing with alternatively co-shared cable is an optimal cable routing.

The remainder of this paper is organised as follows: section II introduces the design of the hybrid modular cable routing scheme and statics model of the MCDRs; section III introduces the analysis of the force closure workspace; section IV introduces the analysis of the motion speed; section V introduces the analysis of the loading capability; section VI is a computer simulation to verify the analysis results of the motion speed and loading capability of MCDRs with the hybrid modular cable routing scheme. Section VII is the conclusion of this paper.

II. STATICS OF MULTI-LINK CABLE-DRIVEN ROBOTS WITH HYBRID MODULAR CABLE ROUTING SCHEME

Without loss of generality, a modular MCDR with a set of identical joints (2-DOF universal joint driven by 3 cables) will be studied in this paper. As shown in Fig. 2, for a k -joint MCDR with the conventional modular cable routing scheme, the total number of cables is $3k$.

The hybrid modular cable routing scheme for MCDRs is obtained by letting each two adjacent joints co-share one cable. In this arrangement, the total number of cables can be reduced from $3k$ to $2k + 1$. Since each joint has 3 cables, by choosing different cables to be co-shared, there are 3^{k-1} specific cable routings using the hybrid method. A specific cable routing scheme can be represented by a cable connecting matrix C .

$$C = \begin{bmatrix} c_{11}, & c_{12}, & c_{13} \\ c_{21}, & c_{22}, & c_{23} \\ \vdots & \vdots & \vdots \\ c_{k1}, & c_{k2}, & c_{k3} \end{bmatrix}$$

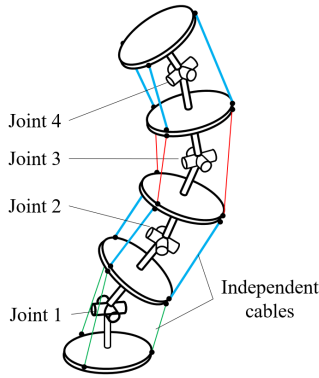


FIGURE 2. The modular MCDR composed of 2-DOF joints.

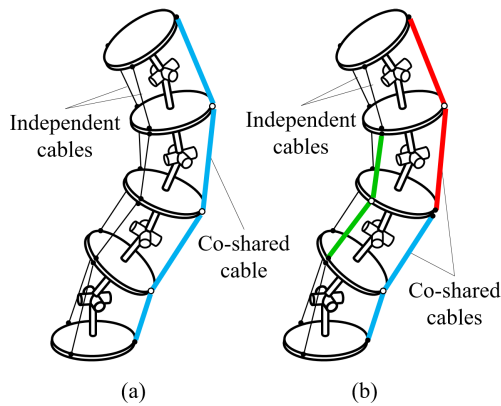


FIGURE 3. Examples of the hybrid modular cable routing schemes. a) one co-share cable; b) alternatively co-share cables.

where $c_{ij} = 0$ if the j^{th} cable in joint i does not co-share a cable in joint $i + 1$ while $c_{ij} = 1$ if the j^{th} cable in joint i is co-shared by the j^{th} cable in joint $i + 1$.

For example, two specific cable routings based the proposed cable routing scheme are shown in Fig.3. In the first cable routing example (Fig.3a), all joints in the MCDR co-share one cable and the cable connecting matrix of the MCDR is

$$C = \begin{bmatrix} 0, & 0, & 1 \\ 0, & 0, & 1 \\ 0, & 0, & 1 \\ 0, & 0, & 0 \end{bmatrix}$$

In the second cable routing (Fig.3b), the cables are alternately co-shared and the cable connecting matrix of the MCDR is

$$C = \begin{bmatrix} 0, & 0, & 1 \\ 0, & 1, & 0 \\ 0, & 0, & 1 \\ 0, & 0, & 0 \end{bmatrix}$$

For joint i , the equilibrium equation is given as

$$J_i^T [t_{i1} \ t_{i2} \ t_{i3}]^T = \mathbf{w}_i \tag{1}$$

where $J_i^T = [\mathbf{a}_{i1}, \mathbf{a}_{i2}, \mathbf{a}_{i3}]$ represents the transpose of the Jacobian matrix of joint i which is also called the structure matrix, and t_{ij} represents the tension of the j^{th} cable in MCDR i , and $\mathbf{w}_i = [w_{i1}, w_{i2}]^T$ represents the external wrenches on joint i .

Based on the cable connecting matrix, the structure matrix of the MCDR with hybrid modular cable routing can be obtained. Firstly, let the structure matrix of each joint diagonally construct a matrix. If $c_{ij} = 1$, add the $(3i + j)^{th}$ to $(3i + j - 3)^{th}$ column and delete the $(3i + j)^{th}$ column. Then, the structure matrix of the MCDR is obtained. As a example, the structure matrix for the MCDR in Fig.3b can be expressed as

$$J^T = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{a}_{23} & \mathbf{a}_{21} & \mathbf{a}_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{a}_{32} & \mathbf{a}_{31} & \mathbf{a}_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{a}_{43} & \mathbf{a}_{41} & \mathbf{a}_{42} \end{bmatrix}$$

III. WRENCH-CLOSURE WORKSPACE ANALYSIS OF MCDRS WITH THE HYBRID CABLE ROUTING METHOD

An MCDR satisfies the **wrench closure condition** (WCC) at a particular pose if there exists a set of positive cable tensions that can sustain any external wrench. The set of all the poses where the MCDR satisfies the WCC is called the **wrench-closure workspace**. To study how the wrench-closure workspace changes when using the hybrid modular cable routing scheme in any MCDR, a 2-joint MCDR is firstly investigated.

Lemma 1: If a 2-joint modular MCDR with independent cable-driven joints satisfies the WCC at a particular pose, the WCC will also be satisfied for the MCDR with one co-shared cable.

Proof: According to [21], since joint i satisfies the WCC, J_i^T is full-ranked and there exists a positive tension vector $\mathbf{t}_i^* = [t_{i1}^*, t_{i2}^*, t_{i3}^*]^T$ such that

$$J_i^T \mathbf{t}_i^* = 0 \tag{2}$$

where $i = 1, 2$.

Without loss of generality, assume that the third cable is the co-shared cable between joints 1 and 2. Hence, the structure matrix of the MCDR can be expressed as

$$J^T = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & 0 & 0 & \mathbf{a}_{13} \\ 0 & 0 & \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} \end{bmatrix} \tag{3}$$

For any arbitrary external wrench \mathbf{w}_i , there will be a solution $\mathbf{t}_i = [t_{i1}, t_{i2}, t_{i3}]^T$ such that

$$J_i^T \mathbf{t}_i = \mathbf{w}_i \tag{4}$$

Let $\lambda = -t_{i3}/t_{i3}^*$. Multiplying λ on (2) and adding to (4), it yields

$$[\mathbf{a}_{i1} \ \mathbf{a}_{i2} \ \mathbf{a}_{i3}] \begin{bmatrix} t_{i1} + \lambda t_{i1}^* \\ t_{i2} + \lambda t_{i2}^* \\ t_{i3} + \lambda t_{i3}^* \end{bmatrix} = \mathbf{w}_i \tag{5}$$

Since $t_{i3} + \lambda t_{i3}^* = t_{i3} + (-t_{i3}/t_{i3}^*)t_{i3}^* = 0$, it yields

$$[\mathbf{a}_{i1} \ \mathbf{a}_{i2}] \begin{bmatrix} t_{i1} + \lambda t_{i1}^* \\ t_{i2} + \lambda t_{i2}^* \end{bmatrix} = \mathbf{w}_i \quad (6)$$

Since \mathbf{w}_i is an arbitrary wrench, $[\mathbf{a}_{i1} \ \mathbf{a}_{i2}]$ is a full-rank matrix and thus J^T is full-ranked.

It is easy to verify that $\mathbf{t}^* = [t_{11}^*, t_{13}^*, t_{21}^*, t_{23}^*, 1]^T$ is a positive solution for

$$J^T \mathbf{t}^* = 0 \quad (7)$$

Therefore, \mathbf{t}^* is positive and J^T is full-ranked. Then, the WCC of the MCDR with one co-shared cable is satisfied. ■

This proof is conducted at an arbitrary pose, and thus the wrench-closure workspace of the MCDR does not reduce when one cable is co-shared.

To prove that the wrench-closure workspace of the MCDR does not increase, it is equivalent to prove that the wrench-closure workspace does not reduce when dividing a co-shared cable into two independent cables. If the MCDR with co-shared cable satisfies the WCC, it is easily verified that each joint also satisfies the WCC. Giving an arbitrary wrench to the MCDR, the MCDR will be in equilibrium and thus each joint is in equilibrium. Therefore, the conventional modular MCDR also satisfies the WCC at the position. This can prove that the wrench-closure workspace of the MCDR does not increase when one cable is co-shared.

Therefore, the wrench-closure workspace of an modular MCDR does not change when one cable is co-shared. In this proof, the 2-DOF joints is employed for the sake of example. The result of Lemma 1 also applies to any MCDR with any type and number of joints.

IV. MOTION SPEED ANALYSIS

This section analyzes the motion speed of MCDRs with the proposed hybrid modular cable routing scheme based on the assumption that the maximum speed of all cables is a const. The relationship between the joint velocities in an MCDR and cable speeds is given as:

$$J\dot{\mathbf{q}} = \mathbf{s} \quad (8)$$

where $J \in \mathbb{R}^{n \times m}$ is the Jacobian matrix of the MCDR, and $\mathbf{s} = [s_1, s_2, \dots, s_m]^T$ is the vector of cable speeds, and $\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \dots, \dot{q}_k]^T$ is the vector of joint velocities.

From [22], it is known that given the lower and upper bounds of the cable speed, the set of the feasible joint velocity of the MCDR is a convex polyhedron (zonotope).

Considering a 2-DOF joint, the set of its joint velocity is a convex polygon as shown in Fig.4. To conveniently describe the size of the polygon with a single scalar value, let B be the maximally inscribed square of the polygon with center at the origin. The half of the edge length of B is v , termed as the **maximal joint velocity** (MJV), which represents the maximal joint velocity that every degree of freedom can simultaneously produce.

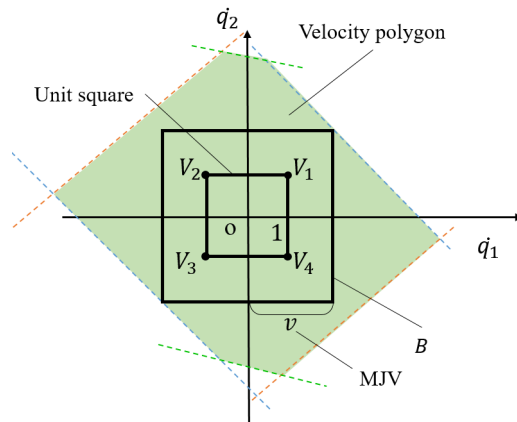


FIGURE 4. The feasible polyhedron of joint velocity.

To compute the MJV of a joint, the scaling factor method is applied and the algorithm is given as:

- 1) Generate a unit square with center at the origin. Let the vertexes be represented by $V_1 = [1, 1]^T, V_2 = [-1, 1]^T, V_3 = [-1, -1]^T$ and $V_4 = [1, -1]^T$.
- 2) Let $s_i = \|JV_i\|_\infty$ where $\|\bullet\|_\infty$ represents the infinity norm of a vector and $i = 1, 2, 3, 4$.
- 3) The **maximal cable speed under unit joint velocity** is given as

$$s_{unit} = \max(s_1, s_2, s_3, s_4) \quad (9)$$

- 4) The MJV v is obtained using:

$$v = \frac{s_{max}}{s_{unit}} \quad (10)$$

where s_{max} is the maximal allowable cable speed which is a constant.

The definition of MJV can be expanded to an MCDR with multiple joints. The MJV of an MCDR represents the maximal joint velocity that every joint can simultaneously produce. The method to obtain the MJV of an MCDR with hybrid modular cable-routing scheme will be illustrated through a 2-joint MCDR.

Lemma 2: For a 2-joint modular MCDR with two independent joints, let v_1 and v_2 be the MJVs of joint 1 and joint 2, respectively. When the two joints co-share one cable, the MJV of the 2-joint MCDR satisfies

$$v \geq \frac{v_1 v_2}{v_1 + v_2} \quad (11)$$

Proof Let $s_{i,unit}$ represent the maximal cable speed in joint i under unit joint velocity. Then,

$$s_{1,unit} = \frac{s_{max}}{v_1} \quad (12)$$

and

$$s_{2,unit} = \frac{s_{max}}{v_2} \quad (13)$$

For the special case where the fastest cables in joint 1 and joint 2 are co-shared, the maximal cable speed of in the

MCDR under unit joint velocity will be that of the co-shared cable which is given as

$$s_{unit} = s_{1,unit} + s_{2,unit} \quad (14)$$

Therefore, the MJV of the MCDR is given as

$$v = \frac{s_{max}}{(s_{1,unit} + s_{2,unit})} = \frac{s_{max}}{\frac{s_{max}}{v_1} + \frac{s_{max}}{v_2}} = \frac{v_1 v_2}{v_1 + v_2} \quad (15)$$

For the general case where two arbitrary cables in joint 1 and joint 2 are co-shared, then

$$s_{unit} \leq s_{1,unit} + s_{2,unit} \quad (16)$$

Therefore,

$$v = \frac{s_{max}}{s_{unit}} \geq \frac{s_{max}}{s_{1,unit} + s_{2,unit}} = \frac{v_1 v_2}{v_1 + v_2} \quad (17)$$

Combining (15) and (17), the MJV of the MCDR always satisfies

$$v \geq \frac{v_1 v_2}{v_1 + v_2} \quad (18)$$

This result is appropriate for any k -joint MCDR with the hybrid modular cable routing scheme. As a result extending from (18), the MJV for any MCDR with hybrid modular cable routing scheme satisfies

$$v \geq \left(\sum_{i=1}^k \frac{1}{v_i} \right)^{-1} \quad (19)$$

The MJV can describe the motion speed of an MCDR at a particular pose. To evaluate the motion speed of an MCDR in the entire workspace, it is necessary to find the maximal velocity that the MCDR can produce at any pose in the workspace.

Definition 1: Let U represent the workspace of a cable-driven joint described in joint space and $s_{unit}(\mathbf{q})$ be the maximal cable speed of the MCDR under unit joint velocity at a particular pose \mathbf{q} . Let $s_{g,unit}$ represent the largest value of s_{unit} within U . Then, the **global maximal joint velocity** (GMJV) of the cable-driven joint is given as

$$v_g = \frac{s_{max}}{s_{g,unit}} \quad (20)$$

The GMJV of a joint represents the maximal joint velocity that the joint can simultaneously produce at any pose. The definition of the GMJV can be extended to an MCDR with multiple joints. The GMJV of an MCDR represents the maximal joint velocity that every axis in the MCDR can simultaneously produce at any pose.

Lemma 3: For a 2-joint modular MCDR with independent joints, let v_{g1} and v_{g2} represent the GMJVs of the joint 1 and joint 2, respectively. When the two joints co-share one cable, let v_g represent the GMJV of the MCDR and then,

$$v_g \geq \frac{v_{g1} v_{g2}}{v_{g1} + v_{g2}} \quad (21)$$

Proof: Let $s_{gi,unit}$ represent the maximal cable speed of the joint i under unit joint velocity for all poses in the workspace. Let $s_{g,unit}$ be the maximal cable speed of the MCDR under unit joint velocity for all poses.

If the fastest cables in joint 1 and joint 2 are co-shared, the co-shared cable has the maximal speed in the MCDR. Then,

$$s_{g,unit} = s_{g1,unit} + s_{g2,unit} \quad (22)$$

Therefore,

$$v_g = \frac{s_{max}}{s_{g,unit}} = \frac{s_{max}}{s_{g1,unit} + s_{g2,unit}} = \frac{v_{g1} v_{g2}}{v_{g1} + v_{g2}} \quad (23)$$

If two arbitrary cables in joint 1 and joint 2 are co-shared,

$$s_{g,unit} \leq s_{g1,unit} + s_{g2,unit} \quad (24)$$

Therefore,

$$v_g = \frac{s_{max}}{s_{g,unit}} \geq \frac{s_{max}}{s_{g1,unit} + s_{g2,unit}} = \frac{v_{g1} v_{g2}}{v_{g1} + v_{g2}} \quad (25)$$

For an MCDR with multiple identical joints, all the joints has the same GMJV. Then,

$$v_{gi} = v_{gj} \quad (26)$$

and

$$s_{gi,unit} = s_{gj,unit} \quad (27)$$

Let k_c represent the maximal number of joints that one cable is co-shared by. Then, the maximal cable speed in the MCDR under unit joint velocity satisfies

$$s_{g,unit} \leq \sum_{i=1}^{k_c} s_{gi,unit} = k_c s_{g1,unit} \quad (28)$$

Therefore, the GMJV of the MCDR satisfies

$$v_g = \frac{s_{max}}{s_{g,unit}} \geq \frac{v_{g1}}{k_c} \quad (29)$$

Particularly, for an MCDR with alternatively co-shared cables (Fig.2b), there must be $k_c = 2$, and hence

$$v_g \geq \frac{v_{g1}}{2} \quad (30)$$

From the above analysis, the GMJV of an MCDR with hybrid modular cable routing scheme is determined by the maximal number of joints that one cable connects. One important property is that an MCDR with alternatively co-shared cables has a constant GMJV when the number of joints increases.

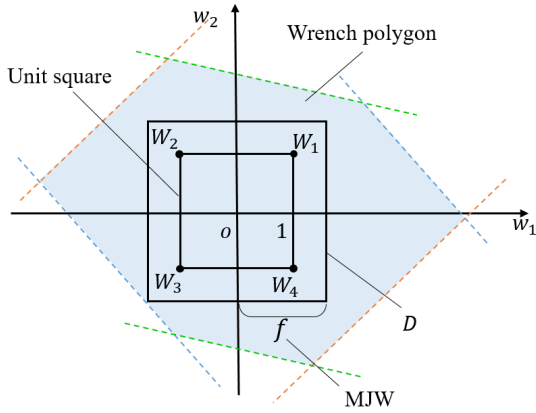


FIGURE 5. The joint wrench value domain of an MCDR.

V. LOADING CAPABILITY ANALYSIS OF MCDRS WITH CO-SHARED CABLES

The analysis of loading capability of an MCDR is based on the assumption that the maximal tensions of all cables are equal while the lower bound of cable forces is zero. For an n -DOF MCDR driven by m cables, the equilibrium equation is given as:

$$J^T \mathbf{t} = \mathbf{w} \quad (31)$$

where $J \in \mathbb{R}^{m \times n}$ is the Jacobian matrix of the MCDR, and $\mathbf{w} \in \mathbb{R}^{n \times 1}$ represents the joint wrenches.

If the MCDR is driven by the minimal number of cables, i.e. $m = n + 1$. The cable tensions solution can be decomposed into two terms.

$$\mathbf{t} = \mathbf{t}^\dagger + \mathbf{t}^* \quad (32)$$

where $\mathbf{t}^\dagger \in \mathbb{R}^{n+1}$ is the lowest cable tension solution in which at least one element is zero, and $\mathbf{t}^* \in \mathbb{R}^{n+1}$ is the internal cable tension in which all the elements are positive and $J^T \mathbf{t}^* = 0$.

Particularly, for a 2-DOF joint driven by three cables, when the maximal cable tension is determined, the set of \mathbf{w} is a convex polygon (zonotope) as shown in Fig.5.

To conveniently describe the size of a zonotope with a single scalar value, let D represent the maximally inscribed square of the wrench polygon with center at the origin. The half of the edge length of D is f , termed as the **maximal joint wrench** (MJW) which represents the maximal wrench that every degree of freedom can sustain at the same time.

To compute the MJW of a joint, a similar scaling factor method is applied [23]. The algorithm to compute the MJW is given as:

- 1) Generate a unit square with center at the origin. Let the vertexes be represented by $W_1 = [1, 1]^T, W_2 = [-1, 1]^T, W_3 = [-1, -1]^T$ and $W_4 = [1, -1]^T$.
- 2) Find the lowest tension solution \mathbf{t}_i^\dagger for $J^T \mathbf{t}_i^\dagger = W_i$ where $i = 1, 2, 3, 4$.
- 3) The **maximal tension under unit joint wrench** is given as

$$t_{unit}^\dagger = \max(\|\mathbf{t}_1^\dagger\|_\infty, \|\mathbf{t}_2^\dagger\|_\infty, \|\mathbf{t}_3^\dagger\|_\infty, \|\mathbf{t}_4^\dagger\|_\infty) \quad (33)$$

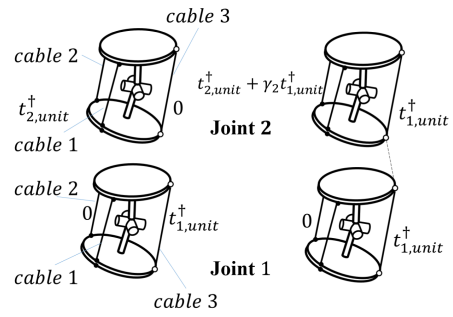


FIGURE 6. A 2-joint MCDR co-sharing a cable.

- 4) The the MJW of the joint is obtained using

$$f = \frac{t_{max}}{t_{unit}^\dagger} \quad (34)$$

where t_{max} is the maximal allowable cable tension which is a constant.

The definition of the MJW can be expanded to an MCDR with multiple joints. The MJW of an MCDR also represents the maximal wrench that every degree of freedom can sustain.

The MJW of an MCDR with hybrid modular cable routing scheme can be derived from the MJWs of each joint. Considering a 2-joint MCDR as shown in Fig.6. Let $T_i^\dagger = [t_{i1}^\dagger, t_{i2}^\dagger, t_{i3}^\dagger]^T$ represent the lowest tension solution under unit joint wrench for joint i . Let $T_i^* = [t_{i1}^*, t_{i2}^*, t_{i3}^*]^T$ represent the internal cable tensions for joint i . Without loss of generality, assume cable 3 in joint 1 and cable 1 in joint 2 have the largest tensions. Since the lowest cable tension solutions have at least one zero element, assume cable 1 in joint 1 and cable 3 in joint 2 have zero tension. Then,

$$t_{13}^\dagger = t_{1,unit}^\dagger = \frac{t_{max}}{f_1} \quad (35)$$

$$t_{21}^\dagger = t_{2,unit}^\dagger = \frac{t_{max}}{f_2} \quad (36)$$

$$t_{11}^\dagger = t_{23}^\dagger = 0 \quad (37)$$

where $t_{i,unit}^\dagger$ represents the maximal tension under unit joint wrench in joint i without internal cable tension.

If cable 3 in joint 1 and cable 3 in joint 2 are co-shared, there will be additional internal cable tensions in joint 2. Obviously, the maximal cable tension of the MCDR will be on cable 1 in joint 2 which is given as

$$t_{21} = t_{2,unit}^\dagger + \frac{t_{21}^*}{t_{23}^*} t_{1,unit}^\dagger \quad (38)$$

Then, the MJW of the entire MCDR f is obtained using

$$f = \frac{t_{max}}{t_{21}} \quad (39)$$

In (38), $\frac{t_{21}^*}{t_{23}^*}$ depends on the particular cable routing scheme (which cables in joint 1 and joint 2 are co-shared). To study a

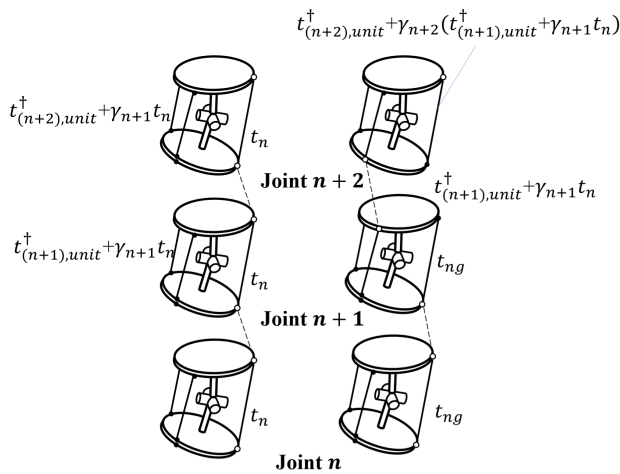


FIGURE 7. Two cable routing scheme among three joints.

general case, an **internal force ratio index** of joint i is defined

$$\gamma_i = \max\left(\frac{t_{i1}^*}{t_{i2}^*}, \frac{t_{i2}^*}{t_{i1}^*}, \frac{t_{i1}^*}{t_{i3}^*}, \frac{t_{i3}^*}{t_{i1}^*}, \frac{t_{i2}^*}{t_{i3}^*}, \frac{t_{i3}^*}{t_{i2}^*}\right) \quad (40)$$

such that (39) becomes

$$f \geq (f_1^{-1} + \gamma_2 f_2^{-1})^{-1} \quad (41)$$

Considering a k -joint MCDR, as shown in Fig.7, if joint $n+2$, joint $n+1$ and joint n co-share the same cable, the maximal tension on joint $n+2$ is

$$t_{n+2} = t_{(n+2),unit}^\dagger + \gamma_{n+2} t_n \quad (42)$$

where t_i is the maximal tension in joint i .

If joint $n+2$, joint $n+1$ and joint n co-share different cables, the maximal tension on joint is

$$t_{n+2} = t_{(n+2),unit}^\dagger + \gamma_{n+2}(t_{(n+1),unit}^\dagger + \gamma_{n+1} t_n) \quad (43)$$

Obviously, the maximal tension in the MCDR will be in the k^{th} joint. Using (42) and (43), the maximal tension in the k^{th} joint t_k can be obtained with recursion method. Then, the MJW of the MCDR is

$$f = \frac{t_{max}}{t_k} \quad (44)$$

The MJW can only describe the loading capability of the MCDR at a particular pose. To evaluate the loading capability of an MCDR at the entire workspace, a global maximal joint wrench (GMJW) is defined.

Definition 2: Let U represent the workspace of a cable-driven joint described in joint space and $t_{unit}^\dagger(\mathbf{q})$ be the maximal cable tension of the joint under unit joint wrench at a particular pose \mathbf{q} . Let $t_{g,unit}^\dagger$ represent the largest $t_{unit}^\dagger(\mathbf{q})$ within U . Then, the **global maximal joint wrench** (GMJW) of the cable-driven joint is given as

$$f_g = \frac{t_{max}}{t_{g,unit}^\dagger} \quad (45)$$

The GMJW of a joint represents the maximal joint velocity that the joint can simultaneously produce at any pose. The definition of the GMJV can be extended to an MCDR with multiple joints. The GMJV of an MCDR represents the maximal joint velocity that every axis in the MCDR can simultaneously produce at any pose.

For an MCDR with identical joints, the GMJWs of each joint are equaled. Then,

$$t_{gi,unit}^\dagger = t_{gj,unit}^\dagger \quad (46)$$

and

$$f_{gi} = f_{gj} \quad (47)$$

where $t_{gi,unit}^\dagger$ represents the maximal tension under unit joint wrench in the entire workspace for the i^{th} joint, and f_{gi} represents the GMJW of the i^{th} joint.

Similarly, if all joints are identical, the largest internal tension ratio indexes of each joint in the workspace should be equaled. Let γ_g represent the largest value of $\gamma_i(\mathbf{q})$ within the workspace.

To study the loading capability of an MCDR with the hybrid modular cable routing scheme, the extreme case should be considered where all co-shared cables transmit the largest tension to the next joint. Therefore, both $t_{i,unit}^\dagger$ and γ_i should take the largest value in the workspace. Then, the upper bound of the maximal tension of the k^{th} joint is given as

$$\begin{aligned} t_{kg} &\leq t_{g1,unit}^\dagger + \gamma_g \{t_{g1,unit}^\dagger + \gamma_g [t_{g1,unit}^\dagger + \dots]\} \\ &= \sum_{j=0}^{m_c} \gamma_g^j t_{g1,unit}^\dagger \end{aligned} \quad (48)$$

where m_c is the total number of co-shared cables.

Then, the GMJW of the MCDR f_g is

$$f_g = \frac{t_{max}}{t_{kg}} \geq \frac{1}{\sum_{j=0}^{m_c} \gamma_g^j} f_{g1} \quad (49)$$

Particularly, for an MCDR with one co-shared cable, it has $m_c = 1$. Therefore, for an MCDR one co-shared cable, the GMJV always satisfies

$$f_g \geq \frac{f_{g1}}{1 + \gamma_g} \quad (50)$$

From the above analysis, it can be observed that the GMJW of an MCDR with hybrid modular cable routing schemes is a function of the number of co-shared cables. One important property is that for the cable routing with one co-shared cable (Fig.2a), the GMJW of the MCDR is a constant value regardless of the total number of joints.

VI. SIMULATION OF THE MCDR WITH HYBRID MODULAR CABLE ROUTING SCHEME

To verify the resulting relationships in Section IV and V, and also investigate the actual speeds and wrenches, kinematics

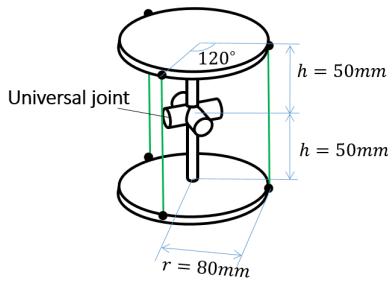


FIGURE 8. The design of the single joint.

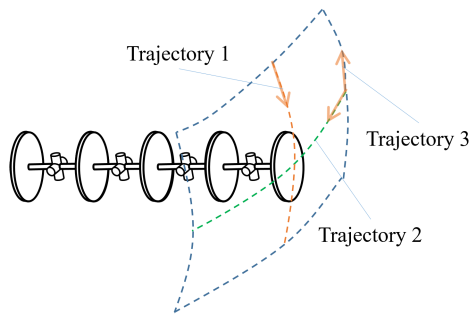


FIGURE 9. The trajectories of the simulation.

and dynamics simulations are performed for different trajectories.

In the simulation, a model of a 4-joint modular MCDRs is constructed within the open-source CASPR software [24]. Three different cable routings were considered for comparison. Cable routing 1 is conventional modular cable routing scheme (Fig.2). Cable routing 2 is the hybrid modular cable routing scheme with one co-shared cable (Fig.3a). Cable routing 3 is the hybrid modular cable routing scheme with alternatively co-shared cables(Fig.3b).

The design of a cable-driven joint is given in Fig.8. Each link of a module has a mass of 0.27243kg and the moment of inertia 0.00039596kgm², 0.00039596kgm² and 0.0006393kgm² in the x, y and z direction, respectively. The joint displacement range of each degree of freedom is set as $\pm\pi/4$. Given the workspace of a joint, the maximal internal cable tension index can be computed which is $\gamma_g = 1.9$.

Three trajectories are considered as shown in Fig.9. Let the MCDR with different cable routings be executed for the same trajectories under the same external loadings. All the cables are assumed to be ideal cables with the maximum tension of 200N and the minimum tension of 10N. The quadratic program algorithm is applied to compute the cable tensions solution with the minimum tension sum.

Figure 10 and Fig.11 shows the resulting speeds and tensions of the cables in the MCDR while executing trajectory 1. The maximal cable speeds and tensions of the MCDR with three cable routings during trajectory 1 are obtained from the figures. Similarly, the results of trajectory 2 and 3 are also obtained, which are given in Table 1.

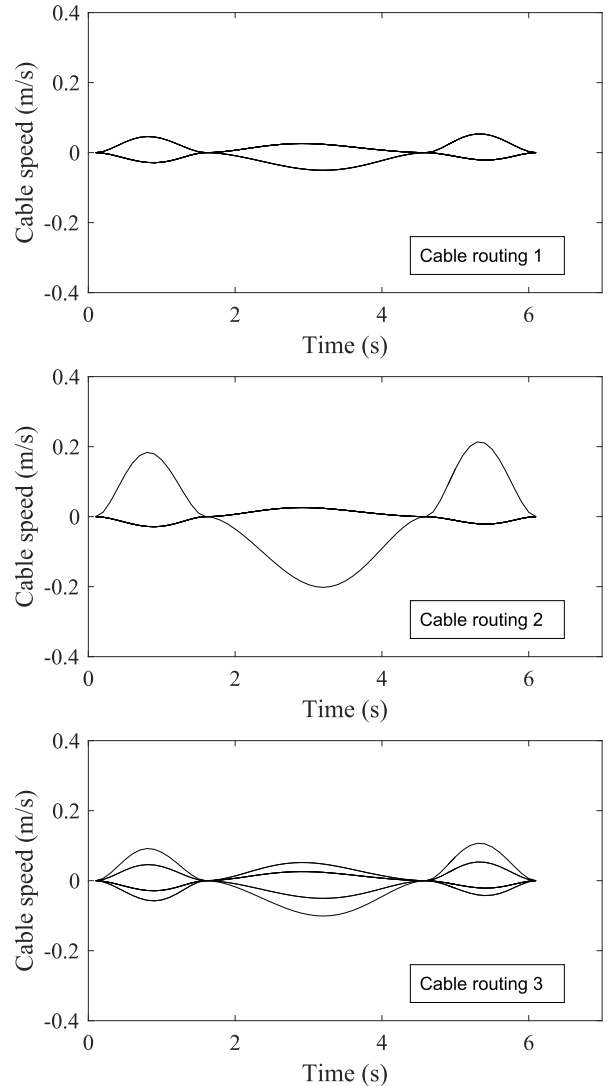


FIGURE 10. The speeds of cables during trajectory 1.

In Fig.10, each curve represents the speed of a cable during the motion. It can be seen that the cable speed will increase when co-sharing cables in the MCDR. Let s_1, s_2 and s_3 be the maximal cable speed of the MCDR with cable routing 1, 2 and 3. From the results, it has $s_1 = 0.0534$, and $s_2 = 0.2135$, and $s_3 = 0.1067$. For cable routing 2, one cable is co-shared by 4 joints, and it can be observed that

$$s_2 \leq 4s_1 \tag{51}$$

For cable routing 3, one cable is co-shared by 2 joints, and it can be observed that

$$s_3 \leq 2s_1 \tag{52}$$

It is easily verified that trajectory 2 and trajectory 3 also satisfy the relationship. Therefore, the MJV of the hybrid modular cable routing method the motion speed of the MCDR with hybrid modular cable routing satisfies

$$v_g \geq \frac{v_{g1}}{k_c} \tag{53}$$

which verified the results in section IV.

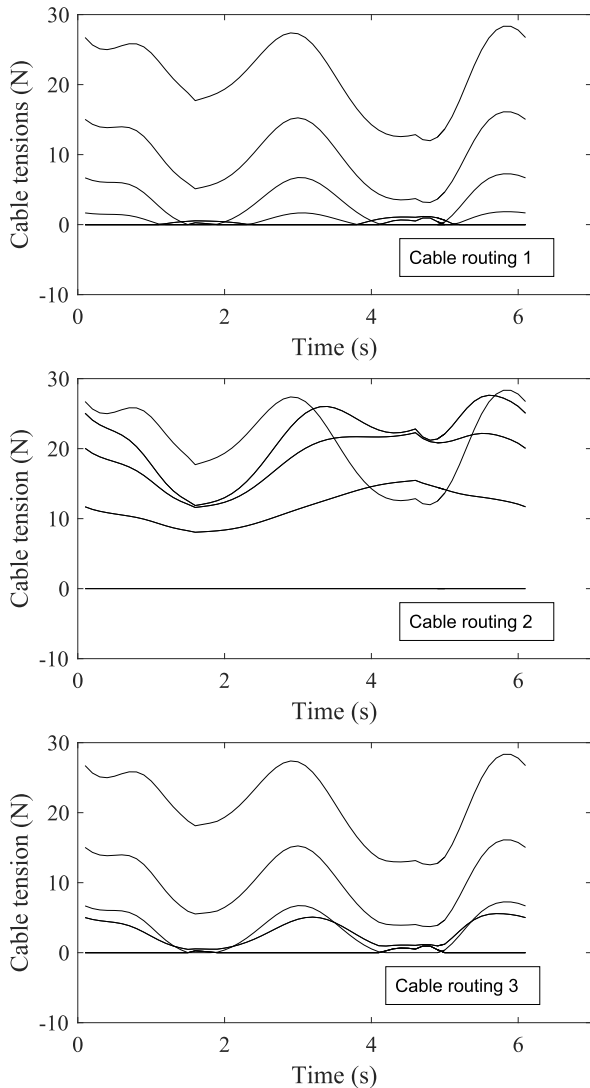


FIGURE 11. The tensions of cables during trajectory 1.

In Fig.11, each curve represents the tension of a cable during the motion. It can be seen that the cable tensions will change when the hybrid modular cable routing scheme is applied in the MCDR. Let t_1 , t_2 and t_3 be the maximal cable speed of the MCDR with cable routing 1, 2 and 3. From the results, it has $t_1 = 38.9N$, and $t_2 = 38.9N$, and $t_3 = 38.9N$. For cable routing 2, there are one co-shared cables, and it can be observed that

$$t_2 \leq (1 + \gamma_g)t_1 \tag{54}$$

For cable routing 3, there are three co-shared cables, and it can be observed that

$$t_3 \leq \sum_{j=0}^3 \gamma_g^j t_1 \tag{55}$$

It is easily verified that trajectory 2 and trajectory 3 also satisfy the relationship. Therefore, the MJW of the hybrid

TABLE 1. The maximal cable speeds and tensions of the MCDRs with different cable routing schemes.

	Modular cable routing	One co-shared cable	Alternatively co-shared cables
Number of cables	12	9	9
Maximal cable speed of trajectory 1	0.0534 m/s	0.2135 m/s	0.1067 m/s
Maximal cable speed of trajectory 2	0.0351 m/s	0.0351 m/s	0.0702 m/s
Maximal cable speed of trajectory 3	0.0366 m/s	0.1462 m/s	0.0731 m/s
Maximal cable tension of trajectory 1	38.9 N	38.9 N	38.9 N
Maximal cable tension of trajectory 2	37.3 N	39.7 N	37.3 N
Maximal cable tension of trajectory 3	33.2 N	39.4 N	35.1 N

modular cable routing method the motion speed of the MCDR with hybrid modular cable routing satisfies

$$f_g \geq \frac{1}{\sum_{j=0}^{m_c} \gamma_g^j} f_{g1} \tag{56}$$

which verified the results in section V.

The simulation results indicate that the hybrid modular cable routing scheme satisfies the analysis in section IV and V. Particularly, for the hybrid modular cable routing with alternatively co-shared cables, although the GMJW is low according to section V, the cable tensions does not increase too much compared with conventional modular cable routing scheme. This is a significant advantage for MCDRs with alternatively co-shared cables. Since the adjacent joints usually have similar cable tensions, when co-sharing cables, the maximal tension will not increase significantly. This indicates that the actual loading capability does not reduce as the number of joints increases. Therefore, such a hybrid modular cable routing with alternatively co-shared cable is a ideal cable routing scheme compared with the other two cable routing schemes.

VII. CONCLUSION

In this paper, a novel hybrid modular cable routing scheme is proposed for MCDRs with identical 2-DOF universal joint modules, in which one co-shared actuating cable is employed between two adjacent joint modules. With the hybrid modular cable routing scheme, the number of actuating cables of a k -joint MCDR is reduced to $2k + 1$. The wrench-closure workspace, motion speed and loading capability have been investigated. It is found that the wrench-closure workspaces of the MCDR with the hybrid cable routing scheme and conventional modular cable routing scheme are equal to each other. The GMJV of the MCDR with the hybrid cable routing scheme is inversely proportional to the number of joints that co-share one cable. The alternatively co-shared cable routing has the maximal GMJV which is a constant value regardless of the number of joints, while the GMJV of the

MCDR is determined by the number of co-shared cables. The cable routing where all joints co-shared one cable has the maximal GMJW which is a constant value regardless of the number of joints. To verify such findings, the model of a 4-joint MCDR is built in the CASPR software. Three different cable routing methods are employed for the MCDR, i.e. the modular method, the hybrid method with one co-shared cable and the hybrid method with alternatively co-shared cables. Simulation results validate the correctness of the proposed performance evaluation method. In conclusion, the hybrid modular cable routing scheme with alternatively co-shared cable is an ideal cable routing for a MCDR as it can reduce the number of cables to the minimum without significantly reducing the motion speed and loading capability. The analysis in this paper is conducted without considering the frictions between the cables and links. Future works will consider non-negligible frictions in an MCDR system.

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