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An ELECTRE-Based Multiple Criteria Decision Making Method for Supplier Selection Using Dempster-Shafer Theory

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ABSTRACT The selection of the optimal supply is an open and crucial issue in supply chain management (SCM), which can be considered as a multi-criteria decision-making (MCDM) problem where the expression and processing of uncertain information could be involved. The purpose of this paper is to develop an elimination and choice translating reality (ELECTRE)-based MCDM method where the evaluation information is expressed and handled by a Dempster–Shafer theory (DST). DST is a primary methodology for uncertainty modeling. In this paper, the weight of the criteria and the performance of each alternative are expressed by linguistic terms and confidence levels, which are then converted to basic probability assignment (BPA) representations. To aggregate evaluations of different experts more rationally and efficiently, a discounting method in DST is presented based on the proposed concept of evidential reliability. In addition, as one family of MCDM models, the ELECTRE method is famous for its outranking relations to rank a set of alternatives. As an extension, synthetic weight, including subjective and objective weights, is applied to determine the concepts of concordance and discordance. The proposed DS-ELECTRE approach not only maintains the advantage of the DST that directly represents and handles uncertainty but also can play the role of the ELECTRE method in analyzing outranking relations among alternatives. An illustrative numerical example is conducted to demonstrate the effectiveness of the DS-ELECTRE method.

INDEX TERMS Dempster-Shafer evidence theory, ELECTRE method, reliability, discounting, multiple criteria decision making, supplier selection.

I. INTRODUCTION

Supplier selection has recently emerged as an active research field, which plays a crucial role in supply chain management (SCM). The primary task of supplier selection is to obtain judgments of experts, deal with evaluation information and select the optimal alternative from a group of potential suppliers under different criteria. A large amount of research papers have been published around this area within the last few years [1]–[3]. It has been pointed out that appropriate supplier selection can improve customer satisfaction and business performance [4]. Supplier selection can be regarded as a typical multi-criteria decision making problem where the expression and processing of experts' evaluations are two crucial components.

In the literatures, numerous mathematical methods have been undertaken to provide the supplier selection problems with sufficient and effective solutions, such as the analytic hierarchy process (AHP) [5]–[7], fuzzy set theory [8]–[12], hierarchical ranking method [13]–[15], Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method [16]–[19], D numbers method [20], [21], Dempster-Shafer evidence theory [22]–[25], rough set theory [26], [27] and others [28]–[31]. Among these approaches, DST is popular and extensively employed to deal with complex decision problem due to its advantages in representing uncertain information. A PROMETHEE-based approach was proposed in [32], which can effectively model the conflict in DST as multi-criteria decision making problem. An evidential

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supplier selection method based on interval data fusion within the framework of DST was developed in [18], which was proposed aiming at solving multi-criterion decision-making problem. A DS-VIKOR approach was presented as a multi-criteria decision making method for supplier selection in [33]. In addition, a few of multi-criteria discounting approaches in DST were developed, such as [34], [35] and [36]. In this paper, a reliability-based discounting method in DST was proposed to improve the accuracy of evidence combination. All these MCDM methods proposed under the framework of DST take the advantages of DST in expressing uncertain information and the fusion rules it provides. These applications demonstrate the effectiveness of DST in addressing supplier selection issues.

In the supplier selection model based on DST developed in this paper, the experts' evaluation of criteria weights and alternatives are expressed by basic probability assignment (BPA), and the reliability-based discounting method we proposed is employed to obtain the decision matrix in the process of information fusion. The following step is to rank all the alternatives based on the resulting decision matrix. The common method to solve MCDM problems is the relational method, which starts from the priority order among criteria and employs the outranking relation or priority function to select, sort or classify the alternatives. The representative methods are Elimination and choice translating reality (ELECTRE) method [37], [38], QUALIFLEX method [39] and ORESTE method [40], [41]. Due to the superiority of ELECTRE method in ranking the alternatives in terms of priorities among the criteria, it has been applied to problems in many fields. The following is a brief introduction of ELECTRE method.

ELECTRE method was first presented by Benayoun *et al.* [37], which was extended subsequently as ELECTRE I, II, III, IV and IS method [42]–[45]. ELECTRE methods have been successfully employed in a wide variety of fields including risk evaluation [46], supplier selection [47]–[50] and multiple criteria decision making [51], [52]. As an illustration, ELECTRE TRI-nC method was presented in [48] and applied to classify the suppliers of a manufacturing industry from an emerging economy. In [52], a method for comparing multi-hesitant fuzzy numbers was presented, and an enhanced version of the ELECTRE method, called ELECTRE III, was extended under the interval-valued intuitionistic fuzzy environment in [53]. A review of the existing literature suggests that previous studies of ELECTRE approaches have focused more on certain information. However, in real applications, to obtain the criteria values precisely is difficult, and in most cases the information that can be collected is uncertain. Therefore, to model MCDM problem for supplier selection, the study performed in this paper focuses on information characterized by uncertainty that is expressed by BPA in DST. Based on the decision matrix obtained by the improved combination rule in DST, in the novel ELECTRE method, the subjective and objective weights are calculated as the synthesis weight, which will be employed in the ranking process.

On these bases, the classical ELECTRE method is ameliorated by using novel expressions within the framework of DST and by improving the ranking process of ELECTRE, so the primary motivations of this paper are summarized below.

(1) To effectively deal with various uncertainties involved in the supplier selection problem, evaluation information of experts is expressed by linguistic terms and confidence levels, and which will be converted to BPA representation in DST. Evaluations of different experts can be aggregated based on the combination rule in DST. To reduce the impact of uncertainty associated with evaluation information on fusion results, a reliability-based evidential discounting method is proposed in this paper.

(2) Considering the different knowledge background and experience of decision makers, they may have different views on criteria, so the importance weight should be assigned to each attribute. Criteria weights are almost determined by decision makers in the traditional decision-making process, which is widely criticized by scholars because it is not objective enough. So in this study, to construct concordance and discordance matrices more reasonable, the comprehensive weight, including subjective weight and objective weight, is defined in the process of ELECTRE method. The subjective weight is obtained only based on the preference or judgments of decision makers, and objective weight is determined using entropy weight method.

(3) The final decision matrix can be obtained based on fusion results, which will be employed later in the ELECTRE method. In this stage, the outranking relations between all the pairs of alternatives are constructed as in ELECTRE method using the notions of concordance and discordance, which are represented by some related matrix. A decision graph will be drawn based on the resulting global matrix, and the ranking results can be analyzed to determine the appropriate supplier.

In summary, a new ELECTRE-based method is developed for solving MCDM problems in DST environments, in which decision makers can consider the evaluation itself without formality and can also employ imperfect or insufficient knowledge of data.

The remainder of this paper is organized as follows. Section II introduces the concept of DST, the entropy functions of BPAs, and ELECTRE method. Section III presents the reliability-based discounting method in DST, including the definition of inner and outer reliability. Section IV describes the specific steps of the proposed ELECTRE-based outranking method for MCDM. Section V illustrates the procedures followed in the proposed method by using a numerical example for supplier selection. In Section VI, we present our conclusions and future research directions.

II. PRELIMINARIES

A. DEMPSTER-SHAFER THEORY

Dempster-Shafer theory (DST) was proposed by Dempster [54] and developed by Shafer [55]. It works on the numerical treatment of the "probabilities" of events without sharply defined bounds. In DST, the elementary events are no longer single points in the universe of all admissible events. The base set and its subsets are defined in DST named frame of discernment (FOD) and denoted as $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$. All events are included in the power set of Θ, denoted as $2^\Theta = \{\phi, \theta_1, \theta_2, \ldots, \theta_n, \theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \ldots, \Theta\}$. Each elementary event will be assigned a value like probability, and these values sum to one. For a FOD $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, a mass function can be defined as a mapping $2^\Theta \rightarrow [0, 1]$ which satisfies the following condition:

$$m(\phi) = 0, \quad \sum_{A \in 2^\Theta} m(A) = 1 \tag{1}$$

The mass function is also called belief function, basic probability assignment (BPA) or a piece of evidence in DST. A denotes one of the propositions in 2^Θ and is called a focal element if $m(A) > 0$.

When using DST, multiple evidence may be collected from different sources. To combine these evidence, the concept of orthogonal sum was defined by Dempster [54] as follows.

Definition 1: (Dempster's rule of combination) Let m_1 and m_2 be two mass functions, the Dempster's rule of combination denoted by $m = m_1 \oplus m_2$ is defined as:

$$m(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K} \tag{2}$$

with

$$K = \sum_{B \cap C = \phi} m_1(B) m_2(C) \tag{3}$$

Note that the Dempster's rule is only applicable to such two BPAs which satisfy the condition $K < 1$.

In real applications, uncertainty is inevitably introduced because of different sources of evidence, which needs to be eliminated as much as possible before information fusion. Therefore, the evidence discount factor was proposed [55] to discount the evidence sources, as defined below.

Definition 2: Given a BPA m and a discounting coefficient α, the discounted BPA m^α on Θ is defined as:

$$\begin{aligned} m^\alpha(A) &= \alpha \times m(A), \quad \forall A \subset \Theta,\ A \neq \Theta \\ m^\alpha(\Theta) &= (1 - \alpha) + \alpha \times m(\Theta) \end{aligned} \tag{4}$$

where $m(\Theta)$ denotes the vacuous BPA. The discounting operation can be employed in such a situation where the reliability of evidence sources is measured by α [56].

To make decisions, the fusion results of multiple pieces of evidence usually need to be transformed into probability distribution based on pignistic probability [57] which can be defined as follows.

Definition 3: (Pignistic probability) Let m be a BPA, the pignistic probability function was defined as:

$$BetP_m(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\phi)}, \quad \forall A \subseteq \Theta \tag{5}$$

where $|A|$ is the cardinality of focal element A.

To apply DST in more fields, the distance measure between two pieces of evidence proposed by Jousselme *et al.* [58] is often employed, which can be defined as follows.

Definition 4: (Jousselme's distance) Let m_1 and m_2 be two pieces of evidence on FOD Θ, the Jousselme's distance can be calculated as:

$$d_J(m_1, m_2) = \sqrt{\frac{1}{2} \cdot (\vec{m_1} - \vec{m_2})^T \underline{\underline{D}}\ (\vec{m_1} - \vec{m_2})} \tag{6}$$

where $\vec{m_1}$ and $\vec{m_2}$ are the vector representations of BPAs m_1 and m_2, and $\underline{\underline{D}}$ is a $2^\theta \times 2^\theta$-dimensional matrix with elements

$$\underline{\underline{D}}(A, B) = \frac{|A \cap B|}{|A \cup B|} \tag{7}$$

Note that Jousselme's distance strictly satisfies the distance axioms [58].

B. ENTROPY FUNCTIONS OF BASIC PROBABILITY ASSIGNMENT

To measure the uncertainty of a mass function, quite a few entropy functions have been developed [24], [59]–[63]. Recently, a new definition of entropy has been proposed by *Jiroušek* and *Shenoy* [64] with some positive properties, which can be described as follows.

Definition 5: (Belief entropy [64]) Let m be a BPA, the entropy function of it was defined as:

$$\begin{aligned} H(m) &= H_s(Pl\_P_m) + H_d(m) \\ &= \sum_{A \subseteq \Theta} Pl\_P_m(A) log(\frac{1}{Pl\_P_m(A)}) \\ &\quad + \sum_{A \subseteq \Theta} m(A) log(|A|) \end{aligned} \tag{8}$$

where Pl_P_m is the plausibility transform defined as:

$$Pl\_P_m(A) = K^{-1} Pl_m(A) \tag{9}$$

where $K = \sum_{A \subseteq \Theta} Pl_m(A)$ and Pl_m is the plausibility function of m defined as:

$$Pl_m(A) = \sum_{B \cap A \neq \phi} m(B), \quad \forall A \subseteq \Theta \tag{10}$$

It is obvious that the definition of belief entropy is made of two components. The first one is the conflict measure, which is the Shannon entropy of an equivalent probability mass function obtained using the plausibility transform. And the second one is the Dubois-Prade's definition of entropy in DST, which denotes the non-specificity measure.

C. ELECTRE I METHOD

A prominent role in MCDM methods is played by the ELECTRE approach (ELECTRE I method is collectively called ELECTRE method in this study) [37], [65]. The main idea of this method is the proper utilization of ''outranking relations'' to rank a set of alternatives. Several significant definitions are described below.

Definition 6: Preference in ELECTRE method is modeled based on binary outranking relations, S, whose meaning is "at least as good as". Considering two alternatives μ and ν, four cases could arise:

(i) $\mu S\nu$ and not $\nu S\mu$, i.e., $\mu P\nu$ (μ is strictly preferred to ν),
(ii) $\nu S\mu$ and not $\mu S\nu$, i.e., $\nu P\mu$ (ν is strictly preferred to μ),
(iii) $\mu S\nu$ and $\nu S\mu$, i.e., $\mu I\nu$ (μ is indifferent to ν), and
(iv) not $\mu S\nu$ and not $\nu S\mu$ (μ is incomparable to ν).

Note that the incomparability preference is a significant relation to account for cases in which decision makers cannot compare two alternatives.

Definition 7: According to ELECTRE method, for given two alternatives μ and ν, their outranking relation depends on two major aspects, namely, the concordance and discordance. The following statements provide insights into these concepts.

1) ***In the concept of concordance, a sufficient majority of the criteria should be in favor of the assertion for an outranking $\mu S\nu$ to be validated.***
2) ***In the concept of discordance, when the concordance condition holds, none of the criteria in the minority should oppose too strongly to the assertion $\mu S\nu$.***

These two situations must be implemented for validating the assertion $\mu S\nu$.

III. RELIABILITY-BASED DISCOUNTING METHOD IN DST

A. INNER RELIABILITY MEASURES

In our previous study [66], an evidential reliability indicator has been proposed to measure the reliability of a BPA from the perspective of entropy. In this paper, the reliability indicator will be redefined to overcome some existing shortcomings. The novel definition will be employed as an indicator to measure the inner reliability of a mass function.

As described above, in DST, entropy function is employed to measure the uncertainty of BPAs. For a BPA, it is considered that the greater its uncertainty is, the lower its reliability will be; on the contrary, the smaller its uncertainty is, the higher its reliability will be. Therefore, the concepts of positive and negative ideal BPAs are defined when a BPA takes its minimum entropy and maximum entropy. The concepts of positive and negative ideal BPAs can be defined as follows.

Definition 8: (Positive ideal BPA) Assume that the FOD is Θ, the positive ideal BPA m^* on Θ is defined as

$$m^* = \arg\min_m (E(\cdot)) \tag{11}$$

where $E(\cdot)$ is an entropy function in DST, and min *denotes m^* is the BPA with minimum entropy on Θ.*

Definition 9: (Negative ideal BPA) Assume that the FOD is Θ, the negative ideal BPA m_* on Θ can be defined as

$$m_* = \arg\max_m (E(\cdot)) \tag{12}$$

where max *represents m_* is one of BPAs on Θ with the maximum entropy.*

Then the distance between m and the positive ideal BPA m^*, and the distance between m the negative ideal BPA m_* can be calculated based on distance measure in DST, which can be denoted as $d(m, m^*)$ and $d(m, m_*)$. Note that $d(\cdot)$ is a distance function employed to measure the distance between two mass functions. Apparently, for the given BPA m, the smaller the distance $d(m, m^*)$, the closer it is to the positive ideal BPA, and thereby the higher its reliability. On the contrary, the smaller the distance $d(m, m_*)$, the closer it is to the negative ideal BPA, and thereby the lower its reliability. Therefore, inspired by TOPSIS method [67], the concept of inner reliability associated with mass functions can be defined as follows.

Definition 10: Let m be the mass function defined on FOD Θ, the inner reliability of m can be calculated as

$$\mathcal{I}(m) = \frac{d(m, m_*)}{d(m, m_*) + d(m, m^*)} \tag{13}$$

where m^* and m_* are the positive and negative ideal BPAs respectively, and $d(\cdot)$ is a distance measure between mass functions. In addition, $\mathcal{I}(m) = 0$ iff $m = m_*$, and $\mathcal{I}(m) = 1$ iff $m = m^*$, so $\mathcal{I}(\cdot) \in [0, 1]$ for all mass functions.

The concept of inner reliability has been proposed, and we will discuss subsequently how to determine the positive and negative ideal BPA for a given FOD Θ.

1) Determine negative ideal BPA. From the perspective of belief entropy, the mass function on Θ with the maximum entropy is the negative ideal BPA. For different definitions of belief entropy, the method to determine the mass function with the maximum entropy is different. In this study, taking the belief entropy introduced in Definition 5 as an example, for the first and second components (i.e. conflict measure and non-specificity measure), their entropy is maximized when all the belief is assigned to Θ, thereby the negative ideal BPA can be determined as $m_* = \{(\Theta, 1)\}$. Note that the negative ideal BPA may be different if other forms of belief entropy are employed.
2) Determine positive ideal BPA. From the perspective of belief entropy, the mass function on Θ with the minimum entropy is the positive ideal BPA. Apparently, a mass function takes the minimum entropy when the belief is all distributed over a single set, e.g. $m^* = \{(A, 1)\}$ where A is an element within Θ. However, how to determine the single set A is a challenging problem. Consequently, the definition of the deterministic A is given as follows.

Definition 11: Let m be a BPA on Θ, the single set A, which produces the positive ideal BPA, can be calculated as

$$A = \arg\max_{B}(BetP_m(\cdot)), \forall B \subseteq \Theta \tag{14}$$

where $BetP_m(\cdot)$ is the pignistic probability function introduced in Definition 3.

Inner reliability measures the reliability of mass functions from the perspective of belief entropy, which only investigates the mass function itself without considering the relationship between other mass functions. Therefore, the concept of outer reliability will be studied further as follows.

B. OUTER RELIABILITY MEASURES

The outer reliability of mass functions primarily considers the conflict relation between mass functions to be aggregated. According to the framework of multi-source data fusion proposed by *Yager* [68], the outer reliability of a BPA can be achieved through three measures: compatibility, support degree and credibility weight. These three measures can be defined as follows.

Definition 12: Let $S = \{m_1, m_2, \ldots, m_n\}$ be n independent sources of evidence, the support degree of mass function m_i can be defined as

$$Sup(m_i) = \sum_{j=1, j\neq i}^{n} (1 - d(m_i, m_j)) \tag{15}$$

where $d(\cdot)$ is a distance measure that calculates the dissimilarity between mass functions m_i and m_j. For mass function m_i, the smaller the distance between it and other sources of evidence, the greater their compatibility, consequently, the larger the $Sup(m_i)$, the greater the support degree of other sources of evidence for m_i. The credibility degree of mass function m_i can be calculated subsequently as

$$Crd(m_i) = \frac{Sup(m_i)}{\sum_{i=1}^{n} Sup(m_i)} \tag{16}$$

based on which, the outer reliability of mass function m_i can be denoted as $\mathcal{O}(m_i) = Crd(m_i)$.

Combined with the concepts of inner and outer reliability, the overall reliability can be defined as follows.

Definition 13: Let $S = \{m_1, m_2, \ldots, m_n\}$ be n independent sources of evidence, the reliability of mass function m_i can be defined as

$$\mathcal{R}(m_i) = \beta\mathcal{I}(m_i) + (1-\beta)\mathcal{O}(m_i) \tag{17}$$

where $\beta \in [0, 1]$ is a tuning parameter employed to control the influence of inner and outer reliability on the overall measure.

Remark 1: Note that the inner reliability of mass function is measured based on the uncertainty, so it is crucial to choose an appropriate entropy function. The first entropy function was presented by Hohle [69] in 1982, and then a number of other functions were proposed [59], [70], [71]. In practical applications, the suitable entropy function can be selected according to the specific environment. Moreover, distance measures are crucial to calculate the outer reliability, which denotes the degree of dissimilarity between mass functions. A variety of distance measures were developed, such as Jousselme's distance [58], belief interval-based distance [13], and other measures [72], [73]. An appropriate dissimilarity measure could also be selected according to the actual situation. In this paper, we employ the entropy function by Jiroušek and Shenoy [64] and the distance measure by Jousselme et al. [58] because of their positive properties.

TABLE 1. Linguistic terms and corresponding value.

Importance	Abbreviation	Linguistic Judgment	Corresponding Values
Very Low	VL	Almost no recognition to the performance	1
Low	L	Low evaluation to the performance	2
Medium Low	ML	A low and middle level of performance	3
Medium	M	The level of the performance is medium	4
Medium High	MH	A middle and high level of performance	5
High	H	High evaluation to the performance	6
Very High	VH	Almost fully recognized this performance	7

The reliability measure of mass functions defined in this paper can be considered as a discounting coefficient in DST. Therefore, the reliability-based discounting formula can be presented as follows.

Definition 14: Suppose m is a mass function whose reliability (discounting coefficient) is $\mathcal{R}(m)$, the discounted mass function $m^{\mathcal{R}}$ can be defined as

$$\begin{aligned} m^{\mathcal{R}}(A) &= \mathcal{R} \times m(A), \quad \forall A \subset \Theta, \ A \neq \Theta \\ m^{\mathcal{R}}(\Theta) &= (1-\mathcal{R}) + \mathcal{R} \times m(\Theta) \end{aligned} \tag{18}$$

IV. THE PROPOSED ELECTRE-BASED OUTRANKING METHOD FOR MCDM USING DST

In this section, the DS-ELECTRE method is proposed, and the detailed steps of it will be introduced. The problem of supplier selection in supply chain system could be treated as a multiple criteria decision making problem, which could be denoted by the sets as follows:

1. A set of potential alternatives denoted as: $A = \{A_1, A_2, \ldots, A_m\}$;
2. A set of decision criteria denoted as: $C = \{C_1, C_2, \ldots, C_n\}$;
3. A set of decision experts denoted as: $E = \{E_1, E_2, \ldots, E_p\}$;
4. A set of evaluation values denoted as: $X = \{x_{ij}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\}$, where x_{ij} represents the rating for *ith* alternative under *jth* criterion.

The main steps of the proposed DS-ELECTRE method for MCDM have been described as the following steps.

A. STEP 1: DETERMINE THE LINGUISTIC TERMS

In the decision-making process for supplier selection, the alternatives will be evaluated by decision experts in the field considering different criteria. It is paramount to define the reasonable evaluation level and corresponding values. In Table 1, the linguistic terms, abbreviation, linguistic judgment and their corresponding values are given to better express the evaluation information of decision experts.

TABLE 2. Scale of the confidence level.

Specification of the confidence level	Scale
Fully convinced	1.0
Almost convinced	0.8
Properly convinced	0.6
Some convinced	0.4
Almost not convinced	0.2
Completely not convinced	0.0
Intermediate values between two adjacent levels	0.9, 0.7, 0.5, 0.3, 0.1

B. STEP 2: DEFINE THE SCALE OF THE CONFIDENCE LEVEL

In the process of supplier selection, decision experts provide evaluation information based on their experience. Subjective judgments inevitably introduce uncertainty (e.g. ambiguity and incompleteness). To express the uncertainty reasonably, in this study, a more flexible method is presented for the judgments of decision experts. A numerical scale [0, 1] is employed to represent the confidence levels of experts' evaluation, which is given in Table 2. Number 1 denotes the complete confidence of experts' judgments while 0 represents no confidence with his/her judgment.

C. STEP 3: OBTAIN THE EVALUATION INFORMATION

Based on the defined linguistic terms and confidence level above, the evaluation results and corresponding confidence level will be provided associated with each alternative under different criteria by decision experts. With respect to the evaluation information of expert E_k, the decision matrix can be denoted as:

$$DM_k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \left[\begin{array}{cccc} \{\xi_{11}\}, \gamma_{11} & \{\xi_{12}\}, \gamma_{12} & \cdots & \{\xi_{1n}\}, \gamma_{1n} \\ \{\xi_{21}\}, \gamma_{21} & \{\xi_{22}\}, \gamma_{22} & \cdots & \{\xi_{2n}\}, \gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \{\xi_{m1}\}, \gamma_{m1} & \{\xi_{m2}\}, \gamma_{m2} & \cdots & \{\xi_{mn}\}, \gamma_{mn} \end{array} \right] \end{array}, \tag{19}$$

where $\{\xi_{ij}\}$ means the evaluation result provided by expert E_k for alternative A_i under criterion C_j, and γ_{ij} denotes its confidence level. As an example, $\{\xi_{12}\} = \{M\}$ means that the expert evaluates alternative A_1 under criterion C_2 by "The level of the performance is medium", and $\gamma_{12} = 0.8$ indicates the confidence level of $\{\xi_{12}\}$ is 0.8.

Similarity, the subjective importance weights provided by experts for different criteria could be represented in the form manifested in Tables 1 and 2. Therefore, the subjective weight matrix can be denoted as follows.

$$W^s = \begin{array}{c} \\ E_1 \\ E_2 \\ \vdots \\ E_p \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \left[\begin{array}{cccc} \{\psi_{11}\}, \vartheta_{11} & \{\psi_{12}\}, \vartheta_{12} & \cdots & \{\psi_{1n}\}, \vartheta_{1n} \\ \{\psi_{21}\}, \vartheta_{21} & \{\psi_{22}\}, \vartheta_{22} & \cdots & \{\psi_{2n}\}, \vartheta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \{\psi_{m1}\}, \vartheta_{p1} & \{\psi_{p2}\}, \vartheta_{p2} & \cdots & \{\psi_{pn}\}, \vartheta_{pn} \end{array} \right] \end{array}, \tag{20}$$

where $\{\psi_{kj}\}$ represents the evaluation result of expert E_k on criterion C_j, and ϑ_{kj} denotes its confidence level.

D. STEP 4: CONSTRUCT THE EVIDENTIAL REPRESENTATIONS OF THE OBTAINED EVALUATION INFORMATION

The elements of linguistic terms manifested in Table 1 could be considered as the frame of discernment in DST, which can be utilized to indicate the proposition that the expert's evaluation belongs to a certain level, such as "VL" and "VH". Consequently, the frame of discernment of the proposed DS-ELECTRE method can be denoted as $\Theta = \{VL, L, ML, M, MH, H, VH\}$. In addition, the mass of belief associated with the elements in FOD can be determined by the corresponding confidence level. Note that if the confidence level of evaluation level is less than 1, it means that the evaluation information is incomplete, in which case, the remaining mass of belief can be assigned to the universal set. It manifests the advantages of DST as a method of uncertain information expression in supplier selection. Then the evaluation information of experts can be converted into evidential representations by using BPAs. As an example, the evaluation information of alternative A_1 under criterion C_2 "$\{M\}$, 0.8" can be represented as $m(\{M\}) = 0.8$, $m(\Theta) = 0.2$.

E. STEP 5: AGGREGATE EVALUATION INFORMATION OF DIFFERENT EXPERTS

The decision panel is composed of multiple experts. Considering the differences in knowledge background and experience, it is necessary to aggregate the evaluation information of different experts. First, evaluation information represented by BPAs will be discounted based on the proposed reliability-based discounting method, and then the discounted BPAs from different experts could be aggregated by using Dempster's rule of combination. The following is an example to illustrate the process of evidence aggregation.

Example 1: Alternative A is evaluated by three experts and the results can be represented by BPAs as

$m_1 : m_1(\{ML\}) = 1,$

$m_2 : m_2(\{ML\}) = 0.8, m_2(\Theta) = 0.2,$

$m_3 : m_3(\{ML\}) = 0.2, m_3(\{M\}) = 0.4, m_3(\Theta) = 0.2.$

First, the inner reliability of m_1, m_2 *and* m_3 *can be calculated based on Section III-A as* $\mathcal{I}(m_1) = 1$, $\mathcal{I}(m_2) = 0.7446$, $\mathcal{I}(m_3) = 0.4020$, *and then the outer reliability of* m_1, m_2 *and* m_3 *can be calculated based on Section III-B as* $\mathcal{O}(m_1) = 0.3436$, $\mathcal{O}(m_2) = 0.3640$, $\mathcal{O}(m_3) = 0.2923$. *We assume that the tuning parameter* $\beta = 0.5$, *then the reliability measures can be calculated based on Definition 13 as* $\mathcal{R}(m_1) = 0.6718$, $\mathcal{R}(m_2) = 0.5543$, $\mathcal{R}(m_3) = 0.3472$. *The discounted BPAs can be obtained based on Definition 14 as*

$m_1^{\mathcal{R}} : m_1^{\mathcal{R}}(\{ML\}) = 0.6718, m_1^{\mathcal{R}}(\Theta) = 0.3282,$

$m_2^{\mathcal{R}} : m_2^{\mathcal{R}}(\{ML\}) = 0.4434, m_2^{\mathcal{R}}(\Theta) = 0.5566$

$m_3^{\mathcal{R}} : m_3^{\mathcal{R}}(\{ML\}) = 0.0694, m_3^{\mathcal{R}}(\{M\}) = 0.1389, m_3^{\mathcal{R}}(\Theta) = 0.7917.$

then the aggregated BPA representation of alternative A can be determined based on Dempster's rule of combination as $m_{1\oplus2\oplus3}^{\mathcal{R}} : m_{1\oplus2\oplus3}^{\mathcal{R}}(\{ML\}) = 0.8082, m_{1\oplus2\oplus3}^{\mathcal{R}}(\{M\}) = 0.0286, m_{1\oplus2\oplus3}^{\mathcal{R}}(\Theta) = 0.1631.$

F. STEP 6: DETERMINE THE DECISION MATRIX

The aggregated BPAs for each alternative under different criteria have been obtained above. To determine the decision matrix, the pignistic probability function $BetP_m$ is employed to transform BPA into probability distribution by using Definition 3. Further, an aggregate function could be defined to integrate probability distributions into numerical values as follows.

Definition 15: Suppose the importance is denoted as $I = \{I_1, I_2, \ldots, I_\kappa\}$ ($\kappa = 7$ in this study) in the linguistic terms, and their corresponding values are $V = \{v_1, v_2, \ldots, v_\kappa\}$. And the probability distribution of mass function m by the function $BetP_m$ is denoted as $P = \{p_1, p_2, \ldots, p_\kappa\}$. The numerical value of m can be calculated by

$$F(p_1, p_2, \ldots, p_\kappa) = PV^T = p_1v_1 + p_2v_2 + \cdots + p_\kappa v_\kappa \tag{21}$$

Finally, the decision matrix $F = (f_{ij})_{m\times n}$ will be determined by Eq. (21).

G. STEP 7: OBTAIN SUBJECTIVE AND OBJECTIVE WEIGHTS

Both subjective and objective weights are taken into account with regard to decision criteria. Subjective weights of criteria will be provided by decision makers and can be obtained by using the subjective matrix constructed in Step 3. In Eq. (20), the subjective importance weight of different criteria is given by decision experts, which will be aggregated based on the approaches introduced in Step 5. First, the BPA representations will be obtained based on Step 4. Second, the discounted BPAs will be calculated and denoted as m^w_{kj} which represents the evaluation result of criterion j by expert k. Third, the aggregated BPAs will be combined and denoted as m^w_j which represents the overall evaluation of criterion j. Then, the BPA representations of criteria will be converted into numerical values based on Definition 15 and denoted as f^s_j, which should be normalized to obtain the subjective weights $W^s = [w^s_1, w^s_2, \ldots, w^s_n]$.

Objective weights will be obtained by employing Shannon entropy based on the determined decision matrix F. First, the normalization of the decision matrix f_{ij} is performed as follows

$$\widetilde{f}_{ij} = \frac{f_{ij}}{\sqrt{\sum_{i=1}^m f_{ij}^2}}, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{22}$$

where $\widetilde{f}_{ij}$ is the normalized decision matrix, which will be employed for ELECTRE method.

To obtain objective weight by using entropy weight method, matrix $\widetilde{f}_{ij}$ needs to be further normalized as follows

$$r_{ij} = \frac{\widetilde{f}_{ij}}{\sum_{i=1}^m \widetilde{f}_{ij}}, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{23}$$

Then, the entropy value of each criterion can be calculated as follows

$$e_j = -k\sum_{j=1}^n r_{ij} ln(r_{ij}) = -\frac{1}{ln(m)}\sum_{j=1}^n r_{ij} ln(r_{ij}) \tag{24}$$

Afterward, the divergence degree of the intrinsic information of each criterion $C_j (j = 1, 2, \ldots, n)$ can be calculated as follows

$$div_j = 1 - e_j \tag{25}$$

where div_j represents the inherent contrast intensity of each criterion C_j, and the higher the div_j is, the more important criterion C_j becomes for the decision problem. The objective weight of criteria can be determined as follows

$$w^o_j = \frac{div_j}{\sum_{j=1}^n div_j} \tag{26}$$

The overall weight can be defined as a combination of subjective weight and objective weight, which can be calculated as follows

$$w_j = \lambda w^s_j + (1-\lambda) w^o_j, \quad j = 1, 2, \ldots, n \tag{27}$$

where $\lambda \in [0, 1]$ is a tuning parameter employed to adjust the ratio between subjective weight and objective weight. If $\lambda \geq 0.5$, subjective weight plays a major role, and if $\lambda < 0.5$, objective weight plays a major role.

H. STEP 8: WEIGHT THE NORMALIZED DECISION MATRIX

The weighted normalized matrix v_{ij} will be obtained by multiplying decision criteria weights w_j as follows

$$v_{ij} = w_j \widetilde{f}_{ij}, \quad i = 1, \ldots, m,\ j = 1, \ldots, n \tag{28}$$

where $\widetilde{f}_{ij}$ is the normalized decision matrix based on Eq. (22) and w_j is the overall weight based on Eq. (27).

I. STEP 9: DETERMINE THE CONCORDANCE SET AND DISCORDANCE SET

For each pair of alternatives A_i and $A_{\bar{i}}$ in the alternative set, the criteria set J = {1,2,…,n} is divided into two unintersected subsets $C_{i\bar{i}}$ and $D_{i\bar{i}}$. The former is composed of criteria that A_i is not inferior to $A_{\bar{i}}$ (i.e. $A_i\ S\ A_{\bar{i}}$) and becomes the concordance set, while the latter is composed of criteria that A_i is inferior to $A_{\bar{i}}$ and is called the discordance set, which is respectively defined as follows

$$C_{i\bar{i}} = \{j | v_{ij} \geq v_{\bar{i}j}\} \tag{29}$$

$$D_{i\bar{i}} = \{j | v_{ij} < v_{\bar{i}j}\} = J - C_{i\bar{i}}, \tag{30}$$

J. STEP 10: CONSTRUCT THE CONCORDANCE MATRIX AND DISCORDANCE MATRIX

The concordance matrix for each pairwise comparison of the alternative can be defined as

$$C = \begin{bmatrix} - & c_{12} & \cdots & c_{1m} \\ c_{21} & - & \cdots & c_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & - \end{bmatrix}, \tag{31}$$

where $c_{i\bar{i}} = \sum_{j\in C_{i\bar{i}}} w_j$ reflects the importance of one alternative relative to another, and the larger the value of $c_{i\bar{i}}$ is, the greater the degree to which alternative A_i is superior to alternative $A_{\bar{i}}$. It is obvious that $c_{i\bar{i}} \in [0, 1]$, and in general the matrix C is asymmetric.

The discordance matrix can be defined as

$$D = \begin{bmatrix} - & d_{12} & \cdots & d_{1m} \\ d_{21} & - & \cdots & d_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \cdots & - \end{bmatrix}, \tag{32}$$

where $d_{i\bar{i}} = \frac{max_{j \in D_{i\bar{i}}} |v_{ij} - v_{\bar{i}j}|}{max_{j \in J} |v_{ij} - v_{\bar{i}j}|}$ reflects the degree to which one alternative is inferior to another, and the greater the value of $d_{i\bar{i}}$ is, the greater the degree to which alternative A_i is inferior to alternative $A_{\bar{i}}$. It is obvious that $d_{i\bar{i}} \in [0, 1]$.

The information can be analyzed to be complementary between concordance matrix and discordance matrix. Specifically, the difference between weights is represented by concordance matrix, while the difference between criteria is represented by discordance matrix. Both concordance and discordance indices have to be calculated for each pair of alternatives $(A_i, A_{\bar{i}})$, where $i \neq \bar{i}$.

K. STEP 11: DETERMINE THE CONCORDANCE DOMINANCE MATRIX AND DISCORDANCE DOMINANCE MATRIX

The concordance dominance matrix B is a Boolean matrix which represents the dominance of one alternative over another. The elements in B can be determined as follows

$$b_{i\bar{i}} = \begin{cases} 0 & c_{i\bar{i}} < \alpha \\ 1 & c_{i\bar{i}} \geq \alpha \end{cases} \tag{33}$$

where α is a threshold of concordance index. Generally, α can take the average of concordance index, that is, $\alpha = \sum_{i=1}^{n} \sum_{\bar{i}=1}^{n} c_{i\bar{i}}/n(n-1)$. $b_{i\bar{i}} = 1$ represents alternative A_i dominating alternative $A_{\bar{i}}$.

The discordance dominance matrix H is also defined as a Boolean matrix. H is measured by a minimum discordance level as

$$h_{i\bar{i}} = \begin{cases} 0 & d_{i\bar{i}} > \beta \\ 1 & d_{i\bar{i}} \leq \beta \end{cases} \tag{34}$$

where β is a threshold of discordance index representing the discordance level and can be defined as $\beta = \sum_{i=1}^{n} \sum_{\bar{i}=1}^{n} d_{i\bar{i}}/n(n-1)$. The elements in H measure the utility of the discordant coalition, meaning that if its value exceeds the given level β, the assertion will no longer function. $h_{i\bar{i}} = 1$ manifests dominance relations among the alternatives.

L. STEP 12: DETERMINE THE COMPREHENSIVE DOMINANCE MATRIX AND IDENTIFY THE BEST COMPROMISE ALTERNATIVE

The comprehensive dominance matrix Z will be obtained by peer to peer multiplication of the elements of the matrices B and H as follows

$$Z = B * H \tag{35}$$

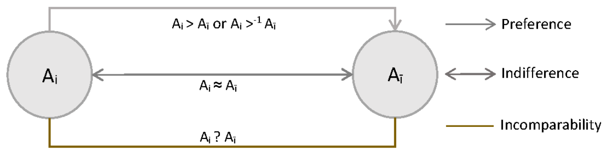

FIGURE 1. The graphical representation of the binary relations $(>, >^{-1}, \approx, ?)$.

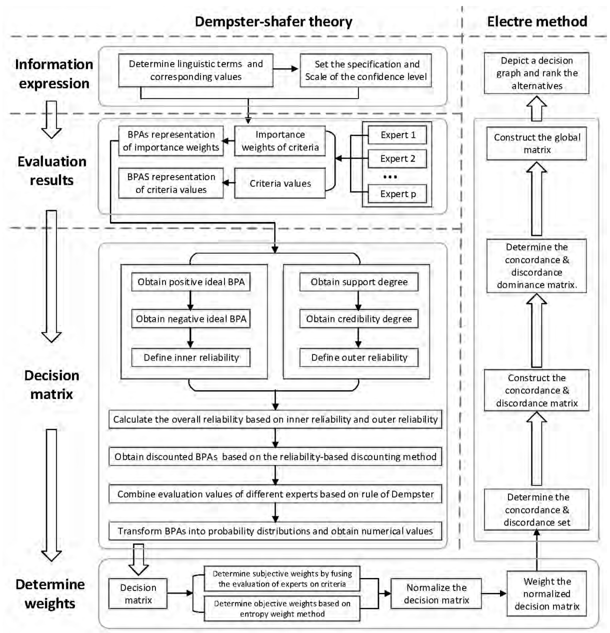

FIGURE 2. Flow chart of the proposed DS-ELECTRE method.

where each element $z_{i\bar{i}}$ in matrix Z can be calculated as $z_{i\bar{i}} = b_{i\bar{i}} h_{i\bar{i}}$.

To select the best compromise alternative, the outranking relation (i.e. matrix Z) will be exploited to identify as small as possible a subset of alternatives. Consequently, it is extremely effective to construct an illustrative graph $G = (V, J)$, where V is the set of vertices and J is the set of arcs. Each alternative is treated as a vertex, and an arc exists between alternatives A_i and $A_{\bar{i}}$ if either A_i is preferred to $A_{\bar{i}}$ or A_i is indifferent to $A_{\bar{i}}$. Alternative A_i outranks $A_{\bar{i}}$ if an arc exists between A_i and $A_{\bar{i}}$ and the arrow goes from A_i to $A_{\bar{i}}$ (in which case, $z_{i\bar{i}} = 1$). The relationship is incomparable if no arc exists between A_i outranks $A_{\bar{i}}$ (in which case, $z_{i\bar{i}} = 0$). They are indifferent if an arc exists between A_i and $A_{\bar{i}}$ and an arrow exists in both directions (in which case, $z_{i\bar{i}} = 1$ and $z_{\bar{i}i} = 1$). To represent the relationship between two vertices more clearly, a graphical expression of the binary relations $(>, >^{-1}, \approx, ?)$ is manifested in Figure 1 [74].

In summary, the DS-ELECTRE method proposed in this study can be described in the above 12 steps depicted in Figure 2.

V. NUMERICAL EXAMPLE

In this section, a numerical example employed in [75] and [33] will be given to demonstrate the details of the proposed DS-ELECTRE method. In [33] and [75], the proposed method is applied to a middle-sized petrochemical factory

TABLE 3. Importance weight of criteria assessed by decision makers.

	C_1	C_2	C_3	C_4	C_5
E_1	{ML}, 1	{M}, 0.8	{H}:{VH}=6:1, 0.7	{VH}, 1	{MH}:{H}=1:4, 1
E_2	{ML}, 0.8	{M}:{MH}=1:4, 1	{VH}, 0.8	{VH}, 1	{MH}, 0.7
E_3	{ML}:{M}=1:3, 0.8	{M}, 0.7	{VH}, 1	{VH}, 1	{MH}, 0.9

to evaluate and select its suppliers, which is a multi-criteria decision making problem. The problem description is given in Section V-A, and the detailed steps to select the optimal alternative are traced in Section V-B.

A. PROBLEM DESCRIPTION

The middle-sized petrochemical factory desires to select a suitable supplier among five alternatives, A_1, A_2, A_3, A_4 and A_5, who should be evaluated by a committee of three experts E_1, E_2 and E_3 against five criteria, namely, product quality (C_1), difficulty to establish cooperation (C_2), service performance (C_3), risk factor (C_4) and price/cost (C_5). Supplier selection in this part is a cross functional, MCDM problem, which can be solved by the proposed DS-ELECTRE method according to the following specific steps.

B. THE SOLUTION

Steps 1-3: The importance weights of the five criteria, and the performance ratings (i.e. criteria values) are expressed using the following linguistic terms: very low, low, medium low, medium, medium high, high and very high, which are introduced in Table 1. The reliability of experts' judgment is described based on the scale of the confidence level shown in Table 2. The importance weights of criteria assessed by decision experts are manifested in Table 3, and the evaluation results of alternatives on different criteria from domain expert are shown in Table 4. Several representative cases are conducted to demonstrate the significance of the evaluation information. Case 1: "$\{ML\}$, 1" expresses that expert E_1 evaluates the importance weight of criterion C_1 as "medium low" with confidence level "1". Case 2: "$\{MH, H\}, 0.2$" represents that expert E_1 evaluates alternative A_1 under criterion C_1 as the combination of "medium high" and "high" with confidence level "0.2". Case 3: "$\{H\} : \{VH\} = 6 : 1, 0.7$" means that the importance weight of criterion C_3 evaluated by expert E_1 is "high" and "very high" with a ratio of 6 : 1, and the confidence level is "0.7".

Step 4: In Steps 1-3, the importance weights of criteria and evaluation ratings of alternatives are obtained, then linguistic evaluations manifested in Tables 3 and 4 will be transformed into evidential representations in the form of basic probability assignment. The rules for the transformation are given in Section IV, and based on which, the transformed BPA representations are manifested in Tables 5 and 6.

Step 5: The BPA representations of evaluations shown in Tables 5 and 6 will be discounted based on the proposed reliability-based discounting method, and then the discounted BPAs from different experts could be aggregated by using Dempster's rule of combination. Note that in this example we take the tuning parameters $\beta = 0.5$ in Eq. (17) and $\lambda = 0.5$ in Eq. (27). The discounted

TABLE 4. Evaluation results of alternatives from domain experts.

		C_1	C_2	C_3	C_4	C_5
E_1	A_1	$\{MH, H\}$, 0.2	{M}, 0.4	{MH}, 0.5	{H, VH}, 1	{VH}, 0.1
	A_2	{MH}, 0.3	{MH}, 0.3	$\{H\} : \{VH\} = 1 : 3$, 0.4	{H}, 0.2	{MH, H}, 0.2
	A_3	{H, VH}, 0.5	{MH, H}, 0.3	{M}, 0.3	{MH}:{H,VH} = 3:2, 0.5	{VH}, 0.5
	A_4	{H, VH}, 1	{H, VH}, 0.2	{M, H}, 1	{MH}, 0.2	{H}:{VH} = 4:1, 0.5
E_2	A_1	{H, VH}, 0.5	{MH}:{H} = 4:1, 0.5	{H, VH}, 0.2	{H, VH}, 0.2	{H, VH}, 0.2
	A_2	{MH, H}, 0.3	{H}, 0.2	{MH}, 0.3	{MH}, 0.2	{M, MH}, 1
	A_3	{H}, 0.2	{H, VH}, 0.2	{M}:{MH} = 4:1, 0.5	{MH, H}, 0.3	{H}:{VH} = 1:4, 0.5
	A_4	{H, VH}, 1	{MH}, 0.3	{M, H}, 1	{H}, 0.3	{H}, 0.3
E_3	A_1	{MH, H}, 0.3	{VH}, 0.3	{H, VH}, 0.3	{M, MH}, 0.2	{M, MH}, 1
	A_2	{MH}:{VH} = 3:1, 0.4	{H, VH}, 0.3	{H}:{VH} = 1:4, 0.5	{M, H}, 0.3	{H}, 0.3
	A_3	{H, VH}, 0.2	{MH, H}:{VH} = 4:1, 0.5	{M, MH}, 0.3	{MH, H}, 0.3	{MH, H}, 0.2
	A_4	{H, VH}, 1	{MH}, 0.2	{H}, 1	{MH}, 0.2	{MH}, 0.3

TABLE 5. The BPA representations of importance weight of criteria.

	C_1	C_2	C_3	C_4	C_5
E_1	m({ML})=1	m({M})=0.8, m(Θ)=0.2	m({H})=0.6, m({VH})=0.1 m(Θ)=0.3	m({VH})=1	m({MH})=0.2, m({H})=0.8
E_2	m({ML})=0.8, m(Θ)=0.2	m({M})=0.2, m({MH})=0.8	m({VH})=0.8, m(Θ)=0.2	m({VH})=1	m({MH})=0.7, m(Θ)=0.3
E_3	m({ML})=0.2, m({M})=0.6 m(Θ)=0.2	m({M})=0.7, m(Θ)=0.3	m({VH})=1	m({VH})=1	m({MH})=0.9, m(Θ)=0.1

TABLE 6. The BPA representations of evaluation results of alternatives.

		C_1	C_2	C_3	C_4	C_5
E_1	A_1	m({MH, H})=0.2, m(Θ)=0.8	m({M})=0.4, m(Θ)=0.6	m({MH})=0.5,m(Θ)=0.5	m({H, VH})=1	m({VH})=0.1, m(Θ)=0.9
	A_2	m({MH})=0.3, m(Θ)=0.7	m({MH})=0.3, m(Θ)=0.7	m({H})=0.1, m({VH})=0.3	m({H})=0.2, m(Θ)=0.8	m({MH, H})=0.2, m(Θ)=0.8
	A_3	m({H, VH})=0.5, m(Θ)=0.5	m({MH, H})=0.3, m(Θ)=0.7	m({M})=0.3, m(Θ)=0.7	m({MH})=0.3, m({H,VH})=0.2 m(Θ)=0.5	m({VH})=0.5, m(Θ)=0.5
	A_4	m({H, VH})=1	m({H, VH})=0.2, m(Θ)=0.8	m({M, H})=1	m({MH})=0.2, m(Θ)=0.8	m({H})=0.4, m({VH})=0.1 m(Θ)=0.5
E_2	A_1	m({H, VH})=0.5, m(Θ)=0.5	m({MH})=0.4, m({H})=0.1, m(Θ)=0.5	m({H, VH})=0.2, m(Θ)=0.8	m({H, VH})= 0.2, m(Θ)=0.8	m({H, VH})=0.2, m(Θ)=0.8
	A_2	m({MH, H})=0.3, m(Θ)=0.7	m({H})=0.2, m(Θ)=0.8	m({MH})=0.3, m(Θ)=0.7	m({MH})=0.2, m(Θ)=0.8	m({M, MH})=1
	A_3	m({H})=0.2, m(Θ)=0.8	m({H, VH})=0.2, m(Θ)=0.8	m({M})=0.4, m({MH})=0.1 m(Θ)=0.5	m({MH, H})=0.3, m(Θ)=0.7	m({H})=0.1, m({VH})=0.4 m(Θ)=0.5
	A_4	m({H, VH}=1	m({MH})=0.3, m(Θ)=0.7	m({M, H})=1	m({H})=0.3, m(Θ)=0.7	m({H})=0.3, m(Θ)=0.7
E_3	A_1	m({MH, H})=0.3, m(Θ)=0.7	m({VH})=0.3, m(Θ)=0.7	m({H, VH})=0.3, m(Θ)=0.7	m({M, MH})=0.2, m(Θ)=0.8	m({M, MH})=1
	A_2	m({MH})=0.3, m({VH})=0.1 m(Θ)=0.6	m({H, VH})=0.3, m(Θ)=0.7	m({H})=0.1, m({VH})=0.4 m(Θ)=0.5	m({M, H})=0.3, m(Θ)=0.7	m({H})=0.3, m(Θ)=0.7
	A_3	m({H, VH})=0.2, m(Θ)=0.8	m({MH, H})=0.4, m({VH})=0.1 m(Θ)=0.5	m({M, MH})=0.3, m(Θ)=0.7	m({MH, H})=0.3, m(Θ)=0.7	m({MH, H})=0.2, m(Θ)=0.8
	A_4	m({H, VH})=1	m({MH})=0.2, m(Θ)=0.8	m({H})=1	m({MH})=0.2, m(Θ)=0.8	m({MH})=0.3, m(Θ)=0.7

TABLE 7. The discounted BPA representations of importance weight of criteria.

	C_1	C_2	C_3	C_4	C_5
E_1	m({ML})=0.6749, m(Θ)=0.3251	m({M})=0.4439, m(Θ)=0.5561	m({H})=0.2492, m({VH})=0.0415 m(Θ)=0.7093	m({VH})=1	m({MH})=0.0992, m({H})=0.3970 m(Θ)=0.5038
E_2	m({ML})=0.4456, m(Θ)=0.5544	m({M})=0.0987, m({MH})=0.3948 m(Θ)=0.5065	m({VH})=0.4482, m(Θ)=0.5518	m({VH})=1	m({MH})=0.3543 m(Θ)=0.6457
E_3	m({ML})=0.0831, m({M})=0.2494 m(Θ)=0.6675	m({M})=0.3541, m(Θ)=0.6459	m({VH})=0.6765, m(Θ)=0.3235	m({VH})=1	m({MH})=0.5507 m(Θ)=0.4493

TABLE 8. The discounted BPA representations of evaluation results of alternatives.

		C_1	C_2	C_3	C_4	C_5
E_1	A_1	m({MH, H})=0.0752 m(Θ)=0.9248	m({M})=0.1585 m(Θ)=0.8415	m({MH})=0.2001 m(Θ)=0.7999	m({H, VH})=0.3748 m(Θ)=0.6252	m({VH})=0.0395 m(Θ)=0.9605
	A_2	m({MH})=0.1163 m(Θ)=0.8837	m({MH})=0.1139 m(Θ)=0.8861	m({H})=0.0383 m({VH})=0.1149 m(Θ)=0.8468	m({H})=0.0760 m(Θ)=0.9240	m({MH, H})=0.0776 m(Θ)=0.9224
	A_3	m({H, VH})=0.1836 m(Θ)=0.8164	m({MH, H})=0.1124 m(Θ)=0.8876	m({M})=0.1165 m(Θ)=0.8835	m({MH})=0.1074, m({H,VH})=0.0716 m(Θ)=0.8210	m({VH})=0.2107 m(Θ)=0.7893
	A_4	m({H, VH})=0.4070 m(Θ)=0.5930	m({H, VH})=0.0728 m(Θ)=0.9272	m({M, H})=0.4261 m(Θ)=0.5739	m({MH})=0.0766 m(Θ)=0.9234	m({H})=0.1560, m({VH})=0.0390 m(Θ)=0.8050
E_2	A_1	m({H, VH})=0.1807 m(Θ)=0.8193	m({MH})=0.1546, m({H})=0.0387 m(Θ)=0.8067	m({H, VH})=0.0757 m(Θ)=0.9243	m({H, VH})= 0.0784 m(Θ)=0.9216	m({H, VH})=0.0782 m(Θ)=0.9218
	A_2	m({MH, H})=0.1091 m(Θ)=0.8909	m({H})=0.0762 m(Θ)=0.9238	m({MH})=0.1118 m(Θ)=0.8882	m({MH})=0.0751 m(Θ)=0.9249	m({M, MH})=0.3721 m(Θ)=0.6279
	A_3	m({H})=0.0757 m(Θ)=0.9243	m({H, VH})=0.0730 m(Θ)=0.9270	m({M})=0.1553, m({MH})=0.0388 m(Θ)=0.8059	m({MH, H})=0.1123 m(Θ)=0.8877	m({H})=0.0353, m({VH})=0.1414 m(Θ)=0.8233
	A_4	m({H, VH}=4070 m(Θ)=0.5930	m({MH})=0.1155 m(Θ)=0.8845	m({M, H})=0.4261 m(Θ)=0.5739	m({H})=0.1119 m(Θ)=0.8881	m({H})=0.1173 m(Θ)=0.8827
E_3	A_1	m({MH, H})=0.1127 m(Θ)=0.8873	m({VH})=0.1158 m(Θ)=0.8842	m({H, VH})=0.1132 m(Θ)=0.8868	m({M, MH})=0.0760 m(Θ)=0.9240	m({M, MH})=0.3653 m(Θ)=0.6347
	A_2	m({MH})=0.1131, m({VH})=0.0377 m(Θ)=0.8492	m({H, VH})=0.1110 m(Θ)=0.8890	m({H})=0.0392, m({VH})=0.1568 m(Θ)=0.8040	m({M, H})=0.1124 m(Θ)=0.8876	m({H})=0.1202 m(Θ)=0.8798
	A_3	m({H, VH})=0.0751 m(Θ)=0.9249	m({MH, H})=0.1442, m({VH})=0.0361 m(Θ)=0.8197	m({M, MH})=0.1097 m(Θ)=0.8903	m({MH, H})=0.1106 m(Θ)=0.8894	m({MH, H})=0.0712 m(Θ)=0.9288
	A_4	m({H, VH})=0.4070 m(Θ)=0.5930	m({MH})=0.0765 m(Θ)=0.9235	m({H})=0.6285 m(Θ)=0.3715	m({MH})=0.0765 m(Θ)=0.9235	m({MH})=0.1127 m(Θ)=0.8873

BPA representations of importance weight of criteria and the discounted BPAs of evaluation results of alternatives are shown in Tables 7 and 8, respectively. The combined BPAs of importance weight of criteria can be obtained as m_1 : $m_1(ML) = 0.7923, m_1(M) = 0.0565, m_1(\Theta) = 0.1512$, m_2 : $m_2(M) = 0.5666, m_2(MH) = 0.1898, m_2(\Theta) = 0.2435$, $m_3 : m_3(H) = 0.0559, m_3(VH) = 0.7849, m_3(\Theta) = 0.1592$, $m_4 : m_4(VH) = 1$ and $m_5 : m_5(MH) = 0.6361, m_5(H) = 0.1604, m_5(\Theta) = 0.2035$, where m_i denotes the importance weight of C_i. The combined BPAs of criteria values are shown in Table 9.

Step 6: To determine the decision matrix, the pignistic probability function $BetP_m$ is employed to transform BPA into probability distribution, and the results are shown in Table 10. Then the probability distributions will be integrated into numerical values based on Definition 15, and the results are shown in Table 11.

Step 7: First, the subjective weights will be obtained based on the combined BPAs of importance weight of criteria in Step 5. The numerical values can be calculated as in the previous step and denoted as $f_1^s = 3.2077, f_2^s = 4.1898, f_3^s = 6.4653, f_4^s = 7$ and $f_5^s = 4.9577$. The subjective weights can

TABLE 9. The combined BPAs of criteria values.

Criteria	Alternatives	Combined BPAs
C_1	A_1	m({MH, H})=0.1470, m({M})=0.0324, m({H, VH})=0.1483, m({Θ})=0.6723
	A_2	m({MH})=0.2136, m({VH})=0.0299, m({MH, H})=0.0825, m({Θ})=0.6740
	A_3	m({H})=0.0757, m({H,VH})=0.2264, m({Θ})=0.6979
	A_4	m({H, VH})=0.7915, m({Θ})=0.2085
C_2	A_1	m({M})=0.1208, m({MH})=0.1229, m({H})=0.0308, m({VH})=0.0840, m({Θ})=0.6415
	A_2	m({MH})=0.0955, m({H})=0.0689, m({MH, H})=0.0928, m({Θ})=0.7428
	A_3	m({H})=0.0173, m({MH, H})=0.2200, m({VH})=0.0322, m({H,VH})=0.0533, m({Θ})=0.6772
	A_4	m({H, VH})=0.0603, m({MH})=0.1721, m({Θ})=0.7676
C_3	A_1	m({MH})=0.1702, m({H, VH})=0.1496, m({Θ})=0.6802
	A_2	m({H})=0.0609, m({VH})=0.2262, m({MH})=0.0797, m({Θ})=0.6332
	A_3	m({H})=0.2503, m({MH})=0.0344, m({M,MH})=0.0785, m({Θ})=0.6368
	A_4	m({H})=0.6285, m({M,H})=0.2491, m({Θ})=0.1224
C_4	A_1	m({H,VH})=0.4046 m({M,MH})=0.0452, m({Θ})=0.5501
	A_2	m({H})=0.0713 m({MH})=0.0624, m({M,H})=0.0974, m({Θ})=0.7689
	A_3	m({H})=0.0151, m({MH})=0.1074, m({H,VH})=0.0565, m({MH,H})=0.1728, m({Θ})=0.6482
	A_4	m({MH})=0.1330, m({H})=0.0970, m({Θ})=0.7700
C_5	A_1	m({VH})=0.0262, m({H,VH})=0.0498 m({M,MH})=0.3376, m({Θ})=0.5865
	A_2	m({H})=0.0790, m({MH})=0.0266, m({MH,H})=0.0449, m({M,H})=0.3161, m({Θ})=0.5334
	A_3	m({H})=0.0287, m({VH})=0.3015, m({MH,H})=0.0477, m({Θ})=0.6221
	A_4	m({MH})=0.0831, m({VH})=0.0317, m({H})=0.2307, m({Θ})=0.6545

TABLE 10. BetP representation of criteria values.

adf	asdf	adsf
C_1	A_1	BetP(VL)=0.0960, BetP(L)=0.0960, BetP(ML)=0.0960, BetP(M)=0.0960, BetP(MH)=0.1695, BetP(H)=0.2761, BetP(VH)=0.1702
	A_2	BetP(VL)=0.0963, BetP(L)=0.0963, BetP(ML)=0.0963, BetP(M)=0.0963, BetP(MH)=0.3511, BetP(H)=0.1375, BetP(VH)=0.1262
	A_3	BetP(VL)=0.0997, BetP(L)=0.0997, BetP(ML)=0.0997, BetP(M)=0.0997, BetP(MH)=0.0997, BetP(H)=0.2886, BetP(VH)=0.2129
	A_4	BetP(VL)=0.0298, BetP(L)=0.0298, BetP(ML)=0.0298, BetP(M)=0.0298, BetP(MH)=0.0298, BetP(H)=0.4255, BetP(VH)=0.4255
C_2	A_1	BetP(VL)=0.0916, BetP(L)=0.0916, BetP(ML)=0.0916, BetP(M)=0.2124, BetP(MH)=0.2145, BetP(H)=0.1224, BetP(VH)=0.1756
	A_2	BetP(VL)=0.1061, BetP(L)=0.1061, BetP(ML)=0.1061, BetP(M)=0.1061, BetP(MH)=0.2016, BetP(H)=0.2214, BetP(VH)=0.1525
	A_3	BetP(VL)=0.0967, BetP(L)=0.0967, BetP(ML)=0.0967, BetP(M)=0.0967, BetP(MH)=0.2067, BetP(H)=0.2507, BetP(VH)=0.1556
	A_4	BetP(VL)=0.1097, BetP(L)=0.1097, BetP(ML)=0.1097, BetP(M)=0.1097, BetP(MH)=0.2818, BetP(H)=0.1398, BetP(VH)=0.1398
C_3	A_1	BetP(VL)=0.0972, BetP(L)=0.0972, BetP(ML)=0.0972, BetP(M)=0.0972, BetP(MH)=0.2674, BetP(H)=0.1720, BetP(VH)=0.1720
	A_2	BetP(VL)=0.0960, BetP(L)=0.0960, BetP(ML)=0.0960, BetP(M)=0.0960, BetP(MH)=0.1695, BetP(H)=0.2761, BetP(VH)=0.1702
	A_3	BetP(VL)=0.0910, BetP(L)=0.0910, BetP(ML)=0.0910, BetP(M)=0.3805, BetP(MH)=0.1646, BetP(H)=0.0910, BetP(VH)=0.0910
	A_4	BetP(VL)=0.0175, BetP(L)=0.0175, BetP(ML)=0.0175, BetP(M)=0.1420, BetP(MH)=0.0175, BetP(H)=0.7705, BetP(VH)=0.0175
C_4	A_1	BetP(VL)=0.0786, BetP(L)=0.0786, BetP(ML)=0.0786, BetP(M)=0.1012, BetP(MH)=0.1012, BetP(H)=0.2809, BetP(VH)=0.2809
	A_2	BetP(VL)=0.1098, BetP(L)=0.1098, BetP(ML)=0.1098, BetP(M)=0.1585, BetP(MH)=0.1722, BetP(H)=0.2298, BetP(VH)=0.1098
	A_3	BetP(VL)=0.0926, BetP(L)=0.0926, BetP(ML)=0.0926, BetP(M)=0.0926, BetP(MH)=0.2864, BetP(H)=0.2223, BetP(VH)=0.1209
	A_4	BetP(VL)=0.1100, BetP(L)=0.1100, BetP(ML)=0.1100, BetP(M)=0.1100, BetP(MH)=0.2430, BetP(H)=0.2070, BetP(VH)=0.1100
C_5	A_1	BetP(VL)=0.0838, BetP(L)=0.0838, BetP(ML)=0.0838, BetP(M)=0.2526, BetP(MH)=0.2526, BetP(H)=0.1087, BetP(VH)=0.1349
	A_2	BetP(VL)=0.0762, BetP(L)=0.0762, BetP(ML)=0.0762, BetP(M)=0.2343, BetP(MH)=0.2833, BetP(H)=0.1777, BetP(VH)=0.0762
	A_3	BetP(VL)=0.0889, BetP(L)=0.0889, BetP(ML)=0.0889, BetP(M)=0.0889, BetP(MH)=0.1127, BetP(H)=0.1414, BetP(VH)=0.3904
	A_4	BetP(VL)=0.0935, BetP(L)=0.0935, BetP(ML)=0.0935, BetP(M)=0.0935, BetP(MH)=0.1766, BetP(H)=0.3242, BetP(VH)=0.1252

TABLE 11. Decision matrix F.

	A_1	A_2	A_3	A_4
C_1	4.6555	4.4269	4.7174	5.9785
C_2	4.4353	4.4649	4.5939	4.3234
C_3	4.5450	4.8813	4.0740	5.5060
C_4	5.0341	4.3012	4.5381	4.3270
C_5	4.3727	4.4105	5.0337	4.6396

be calculated by normalized as $W^s = [w_1^s, w_2^s, w_3^s, w_4^s, w_5^s] = [0.1242, 0.1623, 0.2504, 0.2711, 0.1920]$. Second, the objective weights can be calculated based on the entropy weight method introduced in Section IV. The normalization of the decision matrix is performed based on Eq. (22) and the results are shown as follows

$$\widetilde{F} = \begin{bmatrix} 0.4673 & 0.4977 & 0.4754 & 0.5520 & 0.4731 \\ 0.4443 & 0.5011 & 0.5106 & 0.4717 & 0.4772 \\ 0.4735 & 0.5155 & 0.4261 & 0.4976 & 0.5446 \\ 0.6001 & 0.4852 & 0.5759 & 0.4745 & 0.5019 \end{bmatrix} \tag{36}$$

TABLE 12. Calculated entropy measure, divergence and objective weights of criteria.

	C_1	C_2	C_3	C_4	C_5
e_j	0.9988	0.9998	0.9956	0.9985	0.9988
div_j	0.0012	0.0002	0.0044	0.0015	0.0012
w_j^o	0.1412	0.0235	0.5176	0.1765	0.1412

TABLE 13. The concordance set.

	A_1	A_2	A_3	A_4
A_1	-	{1,4}	{3,4}	{2,4}
A_2	{2,3,5}	-	{3}	{2}
A_3	{1,2,5}	{1,2,4,5}	-	{2,4}
A_4	{1,3,5}	{1,3,4,5}	{1,3,5}	-

TABLE 14. The discordance set.

	A_1	A_2	A_3	A_4
A_1	-	{2,3,5}	{1,2,5}	{1,3,5}
A_2	{1,4}	-	{1,2,4,5}	{1,3,4,5}
A_3	{3,4}	{3}	-	{1,3,5}
A_4	{2,4}	{2}	{2,4}	-

Then the matrix $\widetilde{F}$ needs to be further normalized based on Eq. (23) and the results are shown as follows

$$r = \begin{bmatrix} 0.2354 & 0.2489 & 0.2391 & 0.2766 & 0.2369 \\ 0.2238 & 0.2506 & 0.2568 & 0.2363 & 0.2390 \\ 0.2385 & 0.2578 & 0.2143 & 0.2493 & 0.2727 \\ 0.3023 & 0.2427 & 0.2897 & 0.2377 & 0.2514 \end{bmatrix} \quad (37)$$

The relevant results of objective weights are presented in Table 12. Third, the overall weight can be defined as a combination of subjective weight and objective weight based on Eq. (27) and the results are denoted as $W = [w_1, w_2, w_3, w_4, w_5] = [0.1327, 0.0929, 0.3840, 0.2238, 0.1666]$.

Step 8-12: First, the weighted normalized matrix will be obtained based on the criteria weights and criteria values by using Eq. (28), and the results are shown as follows

$$v = \begin{bmatrix} 0.0312 & 0.0231 & 0.0918 & 0.0619 & 0.0395 \\ 0.0297 & 0.0233 & 0.0986 & 0.0529 & 0.0398 \\ 0.0316 & 0.0239 & 0.0823 & 0.0558 & 0.0454 \\ 0.0401 & 0.0225 & 0.1112 & 0.0532 & 0.0419 \end{bmatrix} \quad (38)$$

Second, the concordance set and discordance set can be determined and shown in Tables 13 and 14. Third, the concordance matrix and discordance matrix can be constructed and shown in Tables 15 and 16. Moreover, the Boolean matrix B based on the minimum concordance level, the Boolean matrix H based on the minimum discordance level and the global matrix Z can be calculated and shown in Tables 17, 18 and 19, respectively.

TABLE 15. The concordance matrix.

	A_1	A_2	A_3	A_4
A_1	-	0.4125	0.4013	0.3849
A_2	0.5875	-	0.1958	0.1794
A_3	0.5987	0.8042	-	0.3849
A_4	0.6151	0.8206	0.6151	-

TABLE 16. The discordance matrix.

	A_1	A_2	A_3	A_4
A_1	-	0.4278	1.0000	1.0000
A_2	1.0000	-	0.8536	1.0000
A_3	0.7309	1.0000	-	1.0000
A_4	0.5536	0.0844	0.1788	-

TABLE 17. Boolean matrix *B* based on the minimum concordance level.

	A_1	A_2	A_3	A_4
A_1	-	0	0	0
A_2	1	-	0	0
A_3	1	1	-	0
A_4	1	1	1	-

TABLE 18. Boolean matrix *H* based on the minimum discordance level.

	A_1	A_2	A_3	A_4
A_1	-	1	0	0
A_2	0	-	0	0
A_3	1	0	-	0
A_4	1	1	1	-

TABLE 19. The global matrix Z.

	A_1	A_2	A_3	A_4
A_1	-	0	0	0
A_2	0	-	0	0
A_3	1	0	-	0
A_4	1	1	1	-

The aggregation matrix Z is obtained including all the indispensable information for constructing the decision graph from matrices B and H. Finally, the decision graph is constructed and presented in Figure 3. The decision graph, derived from a large amount of uncertain information, manifests which alternative is preferable, incomparable or indifferent. There are eight relationships between A_1, A_2, A_3 and A_4. A_3 is preferred to A_1, and A_4 is preferred to A_1, A_2 and A_3.

There are actually three relationships that can be derived from this decision graph. It can be clearly inferred that A_4 is the optimal alternative from the following relationship: A_4 is preferred to A_1, A_2 and A_3. The relation that A_4 is preferred to A_3 is preferred to A_1 can be obtained from the following two relationships: A_4 is preferred to A_3 and A_3 is preferred to A_1. In addition, A_2 is inferior to A_4 but its relationship with A_1 and A_3 cannot be inferred.

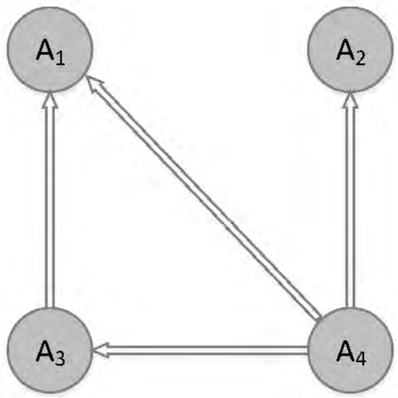

FIGURE 3. The decision graph for the numerical example.

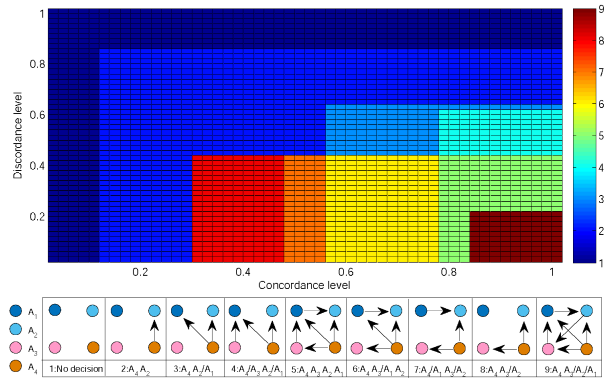

FIGURE 4. Sensitivity analysis of concordance and discordance levels.

C. RESULT ANALYSES AND DISCUSSIONS

With regard to the solving process and obtained results in the previous section, some analyses and discussions have been carried out from different perspectives.

(1) The proposed DS-ELECTRE framework is feasible and effective for MCDM problems in different fields, such as supplier selection and purchasing decision.

For the supplier selection problem in the previous section, the rankings of Fei *et al.* [33] and Shemshadi *et al.* [75] are $A_4 = A_3 \succ A_1 \succ A_2$ and $A_4 \succ A_3 \succ A_1 \succ A_2$ respectively. It can be seen that they are not completely consistent with the ranking of DS-ELECTRE method. For the optimal alternative, they reached the consistent result as A_4. So the inconsistency has no influence on the final decision. More method comparisons are presented in the next section. In practical applications, all these frameworks would guide decision makers to determine the optimal solution A_4. Therefore, DS-ELECTRE method is reliable to solve the problem of supplier selection.

(2) Both subjective weight and objective weight are considered in DS-ELECTRE framework, leading a reasonable direction to selecting appropriate suppliers.

For MCDM problem, each attribute should be given a specific importance weight due to different insights and understandings of criteria. As for how to determine the weight of criteria, in the literature, most typical MCDM methods leave this problem to decision makers, but this approach has been criticized as too subjective. To get a more robust weight system, in the framework of DS-ELECTRE, we divide the weight into two categories: subjective weight and objective weight. While subjective weight is determined solely based on the preference or judgments of decision makers, objective weight employed entropy weight method.

(3) The evaluation information of experts represented by BPAs is more reliable using the evidential discounting method based on the reliability measure.

The proposed DS-ELECTRE framework provides innovations in the use of DST to deal with uncertain information. Preprocessing is a key step before information fusion, which can reduce the negative impact of information uncertainty. Positive ideal BPA and negative ideal BPA are defined to propose the concept of evidence reliability, based on which, the evidence discounting algorithm is defined. It has been demonstrated to effectively reduce the uncertainty of information sources.

D. SENSITIVITY ANALYSIS

In this study, the concordance level is defined as the average of the elements in the concordance matrix, and the discordance level is defined as the average of the elements in the discordance matrix. The selection of consistency level and discordance level has a great impact on the global matrix and will eventually affect the ranking of alternatives. Therefore, how to determine the consistency and discordance levels is an open issue. A sensitivity analysis is conducted in order to monitor the robustness of the preference ranking among the alternatives to changes in the concordance and discordance levels. Figure 4 manifests the ranking of the alternatives at different consistency and discordance levels. Admissible levels for the concordance index and the discordance index (varying by 2%) have 9 types of outcomes. According to Figure 4, it can be concluded that A_4 is the best choice in all cases except the first case which is invalid. Therefore, this analysis indicates that the recommendation for alternative A_4 is sufficiently robust, regarding the limits of variation mentioned above.

E. COMPARATIVE ANALYSIS

Scholars have done a lot of work on supplier selection in uncertain environment. In this section, DS-ELECTRE method will be compared with other existing methods in terms of application environment, criteria types, problem types and main ideas. Results are manifested in Table 20, based on which the conclusion can be drawn that:

(1) DS-ELECTRE method considers the uncertain, imprecise and linguistic evaluation given by a group of decision makers. Linguistic variables and confidence levels are employed to evaluate the weights of criteria and the performance of alternatives. These evaluation information would

TABLE 20. Comparison analysis between DS-ELECTRE approach and existing methods.

	Application environment	Criteria types	Problem types	Main Idea
Method [33]	Basic probability assignment	Subjective and objective	Supplier selection	DST & VIKOR
Method [75]	Trapezoidal fuzzy numbers	Subjective and objective	Supplier selection	FS & VIKOR
Method [53]	Interval-valued intuitionistic fuzzy numbers	Subjective	Investment project selection	IVIFS & ELECTRE III
Method [71]	Fuzzy uncertainty &D number theory	Subjective	Supplier selection	FS & D number theory
Method [74]	Fuzzy numbers	Subjective	Supplier selection	FS & ELECTRE
Our method	Basic probability assignment	Subjective and objective	Supplier selection	DST & ELECTRE

TABLE 21. Ranking comparisons between DS-ELECTRE approach and existing methods.

Alternative	DS-VIKOR [33]	F-VIKOR [75]	IVIFS-ELECTRE III [53]	F-D numbers [71]	F-ELECTRE [74]	DS-ELECTRE
A_1	3	3	4	4	4	4
A_2	4	4	2	3	2	2
A_3	1	2	2	2	3	2
A_4	1	1	1	1	1	1

be represented by BPAs in DST. But other methods cannot do this except for method [33].

(2) DS-ELECTRE method not only makes use of the advantages of DST in dealing with uncertain problems, but also gives play to the superiority of the ELECTRE method in multiple criteria outranking. So the developed method uses the idea of outranking relationship, while other existing methods don't, except for methods [53] and [74].

(3) With regard to criteria weights, DS-ELECTRE method considers both subjective and objective aspects, which can undoubtedly reduce the decision errors caused by the subjectivity of decision makers. Methods [33] and [75] also consider this problem, but they cannot have the advantages of both (1) and (2).

In summary, DS-ELECTRE method innovatively combines the ELECTRE method with DST, and considers the subjective and objective weight of criteria. Compared with other existing methods, it is an effective alternative in MCDM problems.

In order to further compare the performance of different methods in practical applications, the suppliers selection problem in Section V is addressed using existing methods, including DS-VIKOR, F-VIKOR, IVIFS-ELECTRE III, F-D numbers and F-ELECTRE. The ranking results are shown in Table 21.

Results indicate that all the six methods report suppliers A_4 is the best alternative, which shows the feasibility of our method. In addition, IVIFS-ELECTRE III, F-D numbers, F-ELECTRE and our method get that A_1 is the worst supplier, while DS-VIKOR and F-VIKOR show that A_2 is the worst one. In general, methods IVIFS-ELECTRE III, F-D numbers, F-ELECTRE have similar ranking results to DS-ELECTRE, and methods DS-VIKOR and F-VIKOR have similar results. There may be several reasons for the inconsistency of ranking results: (1) different ways of information expression (2) different types of criteria weight, and (3) different ranking methods. The combination of DST and ELECTRE method not only retains their respective advantages, but also exerts greater potential, which provides an effective methodology for MCDM problems.

VI. CONCLUSION

ELECTRE method has already been applied to improve performance in multiple criteria outranking problem. As our studies demonstrate for the first time, ELECTRE method can be combined with Dempster-Shafer theory. The proposed DS-ELECTRE framework is demonstrated to be suitable for addressing situations in which decision makers present evaluation information in uncertain environments. Moreover, both the subjective and objective weights are considered in decision process, which allows for the fuzzy expression of criteria evaluation.

The DS-ELECTRE method has been applied to an illustrative example of provider selection for a petrochemical factory. Results manifest its feasibility in addressing supplier selection problems. Sensitivity analysis and comparative analysis are carried out in this study. Several MCDM methods are compared with the proposed DS-ELECTRE method in the comparative analysis. Results indicate the advantages of DS-ELECTRE method. From a management perspective, our approach can help enterprises choose suitable suppliers, which can further improve business performance.

In future research, we may explore several interesting directions. The theoretical framework of the DS-ELECTRE method could be increasingly perfected, and the extensions of ELECTRE methods (e.g. ELECTRE II, III, IV, IS and TRI) can be added to the framework. The proposed methodology should be employed in a wider range of applications, such as purchasing decision.

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