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Finite-Time Stability of a Time-Delay Fractional-Order Hydraulic Turbine Regulating System

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ABSTRACT Finite-time terminal sliding mode control of a time-delay fractional-order hydraulic turbine regulating system (HTRS) is studied. First, an improved Adams–Bashforth–Moulton algorithm is introduced to solve the fractional-order nonlinear system with a time delay. Then, given the unique advantage of fractional calculus and the great influence that time delay has on system stability, a time-delay fractional-order HTRS is introduced. Moreover, by means of a frequency distribution model, the transformation of the fractional-order HTRS is realized. To stabilize the system under the influence of a random disturbance, a novel terminal sliding surface and a controller are proposed, and the detailed mathematical deduction for system stability is given. Finally, the simulation results, compared with traditional proportional-integral-derivative control and the conventional sliding mode control in the existing literature, demonstrate the validity and significant advantages of the proposed finite-time control scheme.

INDEX TERMS Finite-time stability, fractional-order stability theorem, hydraulic turbine regulating system, time delay, frequency distribution model.

I. INTRODUCTION

The renewability of water resources is very environmentally friendly, and people have reached a consensus on vigorously developing hydropower, especially in China with its abundant water resources [1], [2]. As a hub for the conversion of water energy into electrical energy, the hydroelectric generating unit is responsible for supplying power to the vast number of electric users. The HTRS is a complex system integrating hydraulic, mechanical, and electrical subsystems that can operate under a variety of conditions [3]–[5]. The HTRS plays a significant role in the safe and reliable operation of hydro-generator units, and affects the quality and reliability of the power system that supplies power to users directly [6]–[10]. Therefore, with the continuous increase of high-head, large-capacity hydropower stations, high-quality control of the HTRS is crucial for the stable and efficient operation of the whole system [11]–[14].

HTRS modeling is usually based on integral calculus [15]–[17]. In recent years, it has been found that, for complex systems with memory, heredity, historical dependence, soft properties, and multiple agents, fractional-order calculus can describe such physical properties more accurately [18]–[24]. Consequently, relevant scholars have established fractional dynamic models for HTRSs [25], [26]. Moreover, HTRSs exhibit strong nonlinear and time-delay characteristics [27]. Time lag can significantly affect not only the nonlinear dynamic behavior of HTRSs but also the stability and controllability of the system. Accordingly, in this study, we introduce a time-delay fractional-order HTRS that is more in line with actual conditions.

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Generally, proportional–integral–derivative (PID) control is a traditional and effective control scheme used for HTRSs. An adaptive fast fuzzy fractional-order PID (AFFFOPID) control method for pumped storage hydro units (PSHUs) was proposed in Ref. [28]. In Ref. [13], a fractional-order PID controller was designed that uses a gravitational search algorithm based on the Cauchy and Gaussian mutation for PSHUs. However, the control effect cannot meet the expected requirements usually because of the defects of PID control itself [29]. The regulation time is long, and the oscillations are serious, which is not conducive to stable operation of the unit. Moreover, the PID parameters are set only for several working conditions. The adaptability of working conditions is weak, especially in light of today's increasing high demand for flexible operation. Hence, many new intelligent control strategies have been applied to the HTRS [30]–[32]. Numerous related experiments have been conducted, but the results are not very satisfactory. The learning speed of neural networks is slow, and the network decision time is long [30]; the adaptive method cannot meet the requirements of real-time control [31]; fuzzy control offers low efficiency and accuracy [32]. Therefore, the application of the above methods is limited.

Sliding mode variable structure control (SMVSC) is a nonlinear control strategy [33]–[35] that can well address the uncertainty of the system. In the actual process, the control parameters will be automatically adjusted with changes of the current state, thereby moving the system to the predetermined state trajectory [36], [37]. SMVSC has prominent advantages such as strong anti-interference ability, fast response, and simple physical implementation. However, the traditional linear sliding mode control is based on asymptotic stability theory, so the steady-state error cannot converge to zero in a finite time [38]. At the end of 1980s, the terminal sliding mode (TSM) concept was proposed; it replaced the traditional linear sliding mode with a nonlinear sliding mode, enabling the system state to converge to the equilibrium point in a finite time [39], [40]. It can greatly improve the dynamic transition process of the system and has great potential to ensure stable and efficient operation of the HTRS. Moreover, time-delay in the HTRS has a great impact on the stability of the system. Can finite-time stability be combined with TSM for stabilizing the HTRS with a time delay? If it can, what are the synthetic controller form and detailed mathematical derivation? They have not been given yet. This is a challenging problem that deserves further study.

The main contributions of this study are as follows. First, by considering the historical dependence of the relay components and the time delay caused by the mechanical inertia of the hydraulic servo system, a fractional-order HTRS with a time delay is introduced. Second, a novel terminal sliding surface is designed for the system, and its stability is guaranteed based on the frequency distribution model (FDM) and Lyapunov stability theory. It provides a new approach for stability analysis of the system. Then, based on finite-time stability theory, a finite-time TSM control is proposed for the time-delay fractional-order HTRS, and the upper limit of the stability time can be calculated. Finally, simulation results verify the validity of the proposed scheme. It also provides a reference for the stability of other relevant hydropower systems.

The rest of this paper is organized as follows. The definition of the Caputo fractional derivative and gamma function and a method for solving systems with a time delay are given in Sect. 2. In Sect. 3, mathematical model of a time-delay fractional-order HTRS is introduced. The FDM and controller design are presented in Sect. 4. Simulation results are shown in Sect. 5. In Sect. 6, the conclusion is given.

II. PRELIMINARIES

A. RIEMANN-LIOUVILLE AND CAPUTO FRACTIONAL CALCULUS

The βth fractional-order Riemann-Liouville integration of function $f(t)$ is defined as:

$$ {}_{t_0}I_t^{\beta} f(t) = {}_{t_0}D_t^{-\beta} f(t) = \frac{1}{\Gamma(\beta)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\beta}} d\tau, \quad (1) $$

where $t_0 \in R$, $\beta > 0$, $f(t)$ is an arbitrary integrable function, ${}_{t_0}D_t^{-\beta}$ is the fractional integral of order β on $[t_0, t]$, and $\Gamma(.)$ denotes the Gamma function.

The Caputo definition of the fractional derivative can be written as

$$ {}_a^C D_t^{\beta} f(t) = \frac{1}{\Gamma(n-\beta)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\beta-n+1}} d\tau. \quad (2) $$

The Caputo fractional derivative has the following property:

$$ {}^C D^{\alpha}({}^C D^{-\beta} f(t)) = {}^C D^{\alpha-\beta} f(t). \quad (3) $$

The form of the Gamma function is as follows:

$$ \Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt \quad (Re(z) > 0). \quad (4) $$

The Gamma function has the following property:

$$ \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z} \quad (0 < Re(z) < 1). \quad (5) $$

B. A SOLUTION FOR TIME-DELAY FRACTIONAL-ORDER SYSTEMS

An improved Adams-Bashforth-Moulton algorithm is presented to solve the fractional-order nonlinear system with a time delay.

The following nonlinear system is given:

$$ \begin{cases} {}_0^c D_t^{\alpha} x(t) = f(t, x(t), x(t-\tau)), t \in [0, T], 0 < \alpha \le 1 \\ x(t) = g(t), t \in [-\tau, 0], \end{cases} \quad (6) $$

Letting $\{t_n = nh : n = -k, -k+1, \cdots, N\}$, where k and N are integers satisfying $k = \tau/h$ and $N = T/h$, and $x_h(t_j)$ be the approximate value of $g(t_j)$, where $j = -k, -k+1, \cdots, -1, 0$. One obtains:

$$ x_h(t_j - \tau) = x_h(jh - kh) = x_h(t_{j-k}), j = 0, 1, \cdots, N, \quad (7) $$

under the assumption that $x_h\left(t_j\right) \approx x\left(t_j\right)$, where $j = -k, -k+1, \cdots, -1, 0, 1, \cdots, n$. To obtain $x_h\left(t_{n+1}\right)$, $x\left(t_{n+1}\right)$ is

$$x\left(t_{n+1}\right) = g(0) + \frac{1}{\Gamma(\alpha)} \int_0^{t_{n+1}} \left(t_{n+1} - \xi\right)^{\alpha-1} f\left(\xi, x(\xi), x(\xi - \tau)\right) d\xi. \tag{8}$$

It is obvious that (8) can be obtained by multiplying both sides of (6) by ${}_0D_{t_{n+1}}^{-\alpha}$. In (8), we can replace $x\left(t_n\right)$ with the approximate value $x_h\left(t_n\right)$. By using the trapezoidal rule, (8) can be solved approximately by the following formula:

$$\begin{aligned} x_h\left(t_{n+1}\right) &= g(0) + \frac{h^\alpha}{\Gamma(\alpha+2)} f\left(t_{n+1}, x_h\left(t_{n+1}\right), x_h\left(t_{n+1} - \tau\right)\right) \\ &\quad + \frac{h^\alpha}{\Gamma(\alpha+2)} \sum_{j=0}^{n} a_{j,n+1} f\left(t_j, x_h\left(t_j\right), x_h\left(t_j - \tau\right)\right) \\ &= g(0) + \frac{h^\alpha}{\Gamma(\alpha+2)} f\left(t_{n+1}, x_h\left(t_{n+1}\right), x_h\left(t_{n+1-k}\right)\right) \\ &\quad + \frac{h^\alpha}{\Gamma(\alpha+2)} \sum_{j=0}^{n} a_{j,n+1} f\left(t_j, x_h\left(t_j\right), x_h\left(t_{j-k}\right)\right), \end{aligned} \tag{9}$$

where

$$a_{j,n+1} = \begin{cases} n^{\alpha+1} - (n-\alpha)(n+1)^\alpha, j = 0 \\ (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1}, \\ 1 \le j \le n1, j = n+1. \end{cases}$$

Because there exist the unknown term $x_h\left(t_{n+1}\right)$ and the nonlinear function $f(x)$ in (9), it is difficult to solve $x_h\left(t_{n+1}\right)$ precisely. Therefore, we can replace $x_h\left(t_{n+1}\right)$ with the initial approximate predicted value $x_h^P\left(t_{n+1}\right)$, and (9) can be rewritten as:

$$\begin{aligned} x_h\left(t_{n+1}\right) &= g(0) + \frac{h^\alpha}{\Gamma(\alpha+2)} f\left(t_{n+1}, x_h^P\left(t_{n+1}\right), x_h\left(t_{n+1-k}\right)\right) \\ &\quad + \frac{h^\alpha}{\Gamma(\alpha+2)} \sum_{j=0}^{n} a_{j,n+1} f\left(t_j, x_h\left(t_j\right), x_h\left(t_{j-k}\right)\right). \end{aligned} \tag{10}$$

The predicted item $x_h^P\left(t_{n+1}\right)$ in (10) is given as:

$$x_h^P\left(t_{n+1}\right) = g(0) + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f\left(t_j, x_h\left(t_j\right), x_h\left(t_j - \tau\right)\right), \tag{11}$$

where $b_{j,n+1} = \frac{h^\alpha}{\alpha}\left((n+1-j)^\alpha - (n-j)^\alpha\right)$.

III. MATHEMATICAL MODEL OF A TIME-DELAY FRACTIONAL-ORDER HTRS

By modeling each part of the integer-order HTRS separately and integrating the transfer function according to each subsystem, the following mathematical model of the nonlinear HTRS under small fluctuations can be established [41]:

$$\begin{cases} \dot{\delta} = \omega_0 \omega \\ \dot{\omega} = \dfrac{1}{T_{ab}} \left[m_t - D\omega - \dfrac{E_q' V_s}{x_{d\Sigma}'} \sin\delta - \dfrac{V_s^2}{2} \dfrac{x_{d\Sigma}' - x_{q\Sigma}}{x_{d\Sigma}' x_{q\Sigma}} \sin 2\delta \right] \\ \dot{m}_t = \dfrac{1}{e_{qh} T_w} \left[-m_t + e_y y + \dfrac{e e_y T_w}{T_y} y \right] \\ \dot{y} = -\dfrac{1}{T_y} y, \end{cases} \tag{12}$$

In the HTRS (12), δ, ω, m_t, and y are the generator rotor angle deviation, rotational speed relative deviation of the generator, hydro-turbine output incremental torque deviation, and incremental deviation of the guide vane opening, respectively; T_{ab} is the inertia time constant of the rotating part; D is the damping coefficient of the generator; E_q' is the transient electromotive force of the q axis; V_s is the infinite system bus voltage of the power system; $x_{d\Sigma}'$ is the transient reactance of the d axis and $x_{q\Sigma}$ is the synchronous reactance of the q axis; e_{qh} is the transfer coefficient of turbine flow on the head; T_w is the water inertia time constant; e_y is the transfer coefficient of turbine torque on the main servomotor stroke; e is the transfer coefficient; and T_y is the transfer time of turbine servomotor stroke.

Given that the hydraulic servo system has strong historical dependence and mechanical inertia, a fractional-order hydraulic servo system with a time lag is introduced:

$$D_t^\alpha y(t) = -\frac{1}{T_y} y(t-\tau). \tag{13}$$

By combining (12) and (13), a time-delay fractional-order HTRS is presented as

$$\begin{cases} \dot{\delta}(t) = \omega_0 \omega \\ \dot{\omega}(t) = \dfrac{1}{T_{ab}} \\ \quad \left[m_t - D\omega - \dfrac{E_q' V_s}{x_{d\sum}'} \sin\delta(t) - \dfrac{V_s^2}{2} \dfrac{x_{d\sum}' - x_{q\sum}}{x_{d\sum}' x_{q\sum}} \sin 2\delta(t) \right] \\ \dot{m}_t(t) = \dfrac{1}{e_{qh} T_w} \left[-m_t + e_y y(t-\tau) + \dfrac{e e_y T_w}{T_y} y(t-\tau) \right] \\ D_t^\alpha y(t) = -\dfrac{1}{T_y} y(t-\tau). \end{cases} \tag{14}$$

Because the actual operation of the HTRS is always affected by uncertain changes of the power load, the following random disturbances are considered:

$$d_1(t) = d_3(t) = d_2(t) = d_4(t) = 0.01\text{rand}(1).$$

To facilitate the analysis, let $[x_1, x_2, x_3, x_4]^{\mathrm{T}} = [\delta, \omega, m_t, y]^{\mathrm{T}}$, then select the parameters as follows: $\omega_0 = 314\text{rad/s}, T_{ab} = 19.0\text{s}, D = 2.0, E_q' = 1.35, T_w = 0.8\text{s}, T_y = 0.1\text{s}, x_{d\sum}' = 1.15, x_{q\sum} = 1.474, V_s = 1.0, e = 0.7, e_{qh} = 0.5, e_y = 1.0$, and $\alpha = 0.98$. State trajectories of the time-delay fractional-order HTRS (14) are

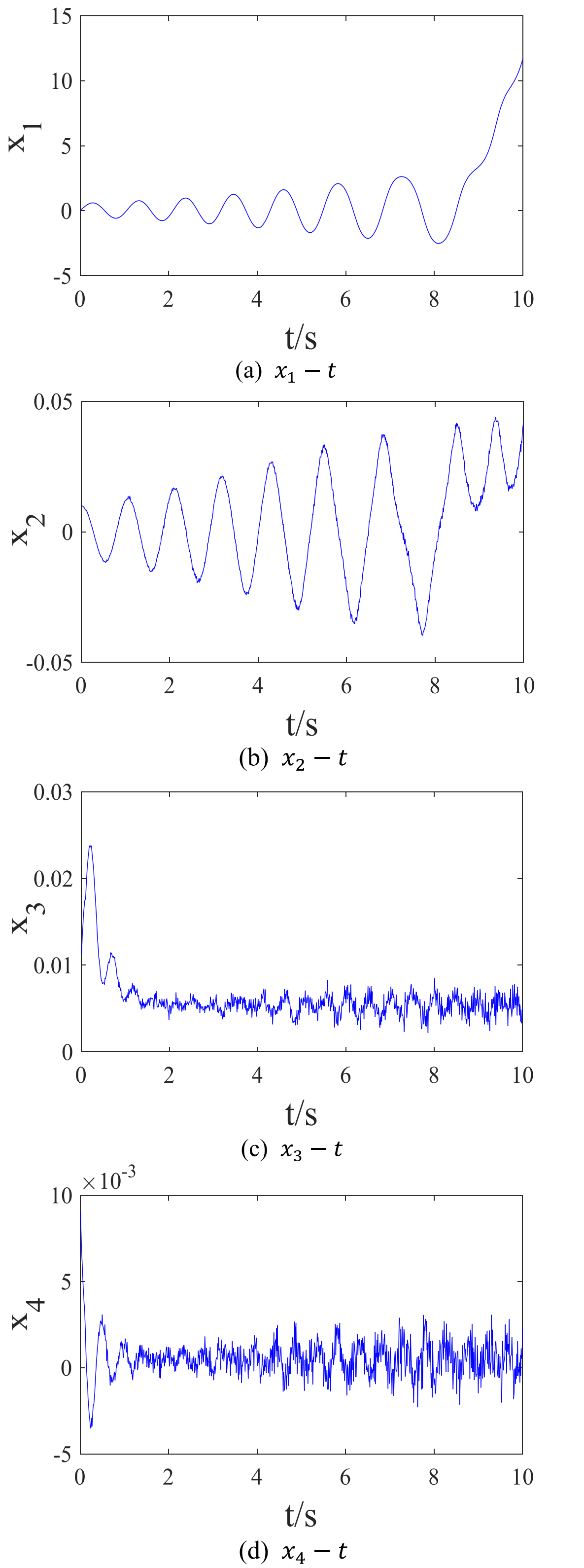

FIGURE 1. State trajectories of the time-delay fractional-order HTRS (14).

shown in Figure 1 with initial value [0.01, 0.01, 0.01, 0.01]. It can be seen that the state variables of the time-delay fractional-order HTRS (14) vibrate irregularly and unstable operation ensues at the starting condition, which needs to be controlled.

IV. FINITE-TIME CONTROLLER DESIGN

A. FREQUENCY DISTRIBUTION MODEL

With the help of a frequency distribution model, we can realize the transformation of the fractional-order system and simplify the stability analysis of fractional-order systems.

To facilitate the mathematical analysis and calculation, we first define a fractional-order system in the following form:

$$D^{\beta}X(t) = F(X), \tag{15}$$

where $F(X)$ represents the nonlinear term of a fractional-order system and β represents the fractional order.

Define an auxiliary function based on the time and frequency domains as follows:

$$\phi(\omega, t) = \int_0^t e^{-\omega^2(t-\tau)}F(\tau)d(\tau). \tag{16}$$

Theorem 1: Based on Equation (16), the fractional-order system (15) can be expressed as

$$\begin{cases} \dfrac{\partial\phi(\omega, t)}{\partial t} = -w^2\varphi(\omega, t) + F(X, t) \\ X(t) = \int_0^{\infty} u(\omega)\varphi(\omega, t)d\omega, \end{cases} \tag{17}$$

where $u(\omega) = \frac{2\sin(\pi\beta)}{\pi}\omega^{1-2\beta}$, $\beta \in (0, 1)$.

Proof:

Equation (16) is rewritten as:

$$\varphi(\omega, t) = e^{-\omega^2 t}\int_0^t e^{\omega^2\tau}F(\tau)d(\tau). \tag{18}$$

The first derivative of Equation (18) is

$$\begin{aligned} \frac{\partial\varphi(\omega, t)}{\partial t} &= -\omega^2 e^{-\omega^2 t}\int_{t_0}^t e^{\omega^2\tau}F(\tau)d(\tau) + e^{-\omega^2 t}e^{\omega^2 t}F(t) \\ &= -\omega^2\int_{t_0}^t e^{-\omega^2(t-\tau)}F(\tau)d(\tau) + F(t) \\ &= -\omega^2\varphi(\omega, t) + F(t). \end{aligned} \tag{19}$$

Combining the definitions of fractional-order Riemann–Liouville integration (1) and the Gamma function (4) gives the following equation:

$$\begin{aligned} &{}_{t_0}I_t^{\beta}F(t) \\ &= \frac{1}{\Gamma(\beta)\Gamma(1-\beta)}\int_{t_0}^t \frac{F(\tau)}{(t-\tau)^{1-\beta}}d\tau\int_0^{\infty} e^{-m}m^{-\beta}dm \\ &= \frac{1}{\Gamma(\beta)\Gamma(1-\beta)}\int_0^{\infty}\int_{t_0}^t \frac{F(\tau)}{(t-\tau)^{1-\beta}}e^{-m}m^{-\beta}d\tau dm \\ &= \frac{1}{\Gamma(\beta)\Gamma(1-\beta)}\int_0^{\infty}\int_{t_0}^t F(\tau)(\frac{m}{t-\tau})^{-\beta}\frac{1}{(t-\tau)}e^{-m}d\tau dm. \end{aligned} \tag{20}$$

Define a variable

$$m = \omega^2(t-\tau), \tag{21}$$

so $dm = 2\omega(t-\tau)d\omega$.

According to Equations (20) and (21), one can obtain

$$\begin{aligned} {}_{t_0}I_t^{\beta}F(t) &= \frac{2}{\Gamma(\beta)\Gamma(1-\beta)}\int_0^{\infty}\int_{t_0}^{t}F(\tau)\omega^{1-2\beta}e^{-\omega^2(t-\tau)}d\tau d\omega \\ &= \frac{2}{\Gamma(\beta)\Gamma(1-\beta)}\int_0^{\infty}\omega^{1-2\beta}\int_{t_0}^{t}F(\tau)e^{-\omega^2(t-\tau)}d\tau d\omega. \end{aligned} \tag{22}$$

By introducing the auxiliary function, we can obtain

$${}_{t_0}I_t^{\beta}F(t) = \frac{2}{\Gamma(\beta)\Gamma(1-\beta)}\int_0^{\infty}\omega^{1-2\beta}\varphi(\omega,t)d\omega. \tag{23}$$

Now let

$$\mu(\omega) = \frac{2}{\Gamma(\beta)\Gamma(1-\beta)}\omega^{1-2\beta}. \tag{24}$$

According to Equation (5), one gets

$$\mu(\omega) = \frac{2\sin\beta\pi}{\pi}\omega^{1-2\beta}. \tag{25}$$

Equation (23) can be represented as

$${}_{t_0}I_t^{\beta}F(t) = \int_0^{\infty}\mu(\omega)\varphi(\omega,t)d\omega. \tag{26}$$

According to Equations (1) and (26), one obtains

$$X(t) = D^{-\beta}F(t) = \int_0^{\infty}\mu(\omega)\varphi(\omega,t)d\omega. \tag{27}$$

The proof is completed.

B. SLIDING SURFACE DESIGN

To stabilize the fractional-order HTRS (14) in a finite time, a suitable sliding surface and controller for the system were needed. A sliding surface $s(x)$ with progressive stability and good dynamic quality was designed and then a suitable controller was designed. The controller $u(t)$ needed to have a good control effect, so that the system can form a sliding mode area on the sliding surface under the action of the controller.

Once the sliding surface $s(x)$ and the controller $u(t)$ are determined, under the action of the controller, the controlled system can reach the predetermined sliding surface. That is, the system can reach the stable state.

The following terminal sliding surface was introduced:

$$s_i(t) = D^{\alpha-1}x_i + D^{-1}\left(\lambda_1 x_i + \lambda_2 \ln^u(|x_i|+1)sat(x_i)\right), \tag{28}$$

where x_i, $i = 1, 2, 3, 4$ are the state variables of the system, λ_1, λ_2, and u are the parameters related to the above sliding surface, and the conditions $\lambda_1 > 0, \lambda_2 > 0, 0 < u < 1$ are met.

When the state trajectory of the fractional-order HTRS (14) reached the predetermined sliding surface, then

$$s_i(t) = 0. \tag{29}$$

According to Equations (28) and (29),

$$s_i(t) = D^{\alpha-1}x_i + D^{-1}\left(\lambda_1 x_i + \lambda_2 \ln^u(|x_i|+1)sat(x_i)\right) = 0. \tag{30}$$

Then,

$$D^{\alpha-1}x_i = -D^{-1}(\lambda_1 x_i + \lambda_2 \ln^u(|x_i|+1)sat(x_i)). \tag{31}$$

According to the property of Caputo fractional calculus (3),

$$D^{\alpha}x_i = -(\lambda_1 x_i + \lambda_2 \ln^u(|x_i|+1)sat(x_i)). \tag{32}$$

Theorem 2: Under the selected terminal sliding surface (28), the fractional-order sliding mode dynamic system (32) will be asymptotically stable and eventually converge to zero.

Proof:

According to Theorem 1, the sliding mode dynamic system (32) is written as

$$\begin{cases} \dfrac{\partial\varphi(\omega,t)}{\partial t} = -w^2\varphi(\omega,t) - (\lambda_1 x_i + \lambda_2 \ln^u(|x_i|+1)\mathrm{sat}(x_i)) \\ x_i(t) = \int_0^{\infty} u(\omega)\varphi(\omega,t)d\omega. \end{cases} \tag{33}$$

Select the Lyapunov function:

$$V_1(t) = \frac{1}{2}\int_0^{\infty}\mu(\omega)\phi^2(\omega,t)d\omega. \tag{34}$$

Taking the first derivative with respect to time of the above Equation (34) gives

$$\begin{aligned} \frac{dV_1}{dt} &= \int_0^{\infty}\mu(\omega)\varphi(\omega,t)\frac{\partial\varphi(\omega,t)}{\partial t}d\omega \\ &= \int_0^{\infty}\mu(\omega)\varphi(\omega,t)[-w^2\varphi(\omega,t) \\ &\quad -(\lambda_1 x_i + \lambda_2 \ln^u(|x_i|+1)\mathrm{sat}(x_i))]d\omega \\ &= -\int_0^{\infty}\mu(\omega)\omega^2\varphi^2(\omega,t)d\omega \\ &\quad -\int_0^{\infty}\mu(\omega)\varphi(\omega,t)(\lambda_1 x_i + \lambda_2 \ln^u(|x_i|+1)\mathrm{sat}(x_i))d\omega \\ &= -\int_0^{\infty}\mu(\omega)\omega^2\varphi^2(\omega,t)d\omega \\ &\quad -(\lambda_1 x_i + \lambda_2 \ln^u(|x_i|+1)\mathrm{sat}(x_i))\int_0^{\infty}\mu(\omega)\varphi(\omega,t)d\omega \\ &= -\int_0^{\infty}\mu(\omega)\omega^2\phi^2(\omega,t)d\omega \\ &\quad -\lambda_1 x_i^2 - \lambda_2 \ln^u(|x_i|+1)x_i\mathrm{sat}(x_i). \end{aligned} \tag{35}$$

According to the definition of the saturation function sat(·), the following two cases are discussed:

Case 1: When $|x_i| > \rho$,

$$x_i\mathrm{sat}(x_i) = x_i\mathrm{sign}(x_i/\rho). \tag{36}$$

Because $x_i \times \mathrm{sign}(x_i) = |x_i|$ and ρ is a positive real number, then

$$x_i\mathrm{sat}(x_i) = |x_i| > 0. \tag{37}$$

TABLE 1. Comparison between the proposed method and the existing method.

Time delay	Existing method			Proposed method	
τ	State variable	Maximum deviation	Stability time(s)	Maximum deviation	Stability time(s)
0.1	δ	-7.834	1.4	-0.83	0.195
	ω	-7.834	1.32	-0.83	0.2
	m_t	-7.834	1.36	-0.83	0.198
	y	-8.039	1.32	-0.84	0.21
0.3	δ	-23.06	1.8	-2.383	0.85
	ω	-23.06	1.92	-2.383	0.865
	m_t	-23.06	2.02	-2.383	0.845
	y	-23.07	1.9	-2.343	0.85

Therefore,

$$\frac{dV_1}{dt} \le 0. \tag{38}$$

Case 2: When $|x_i| \le \rho$,

$$x_i \mathrm{sat}(x_i) = x_i \mathrm{sign}(x_i/\rho) = x_i^2/\rho > 0. \tag{39}$$

It is obvious that

$$\frac{dV_1}{dt} \le 0. \tag{40}$$

Based on the above two cases, one has

$$\frac{dV_1}{dt} \le 0. \tag{41}$$

That is, the state variables of the sliding mode dynamic system (32) should be asymptotically stable and eventually converge to zero.

The proof is completed.

C. CONTROLLER DESIGN

For the time-delay fractional-order HTRS (14), its controlled form is written as follows:

$$D^\alpha x_i = f_i(x) + d_i(t) + u_i(t). \tag{42}$$

In the above formula, the system state variables are $x_i = [x_1, x_2, x_3, x_4]^T$, the linear and nonlinear terms of the system are $f_i(x) = [f_1(x), f_2(x), f_3(x), f_4(x)]^T$, the random disturbances are $d_i(t) = [d_1, d_2, d_3, d_4]^T$, the control input parameters are $u_i = [u_1, u_2, u_3, u_4]^T$, and the orders are $\alpha = [1, 1, 1, 0.98]$.

For the time-delay fractional-order HTRS (42) and sliding surfaces (28), the TSM controller can be designed as

$$u_i(t) = -(f_i(x) + \lambda_1 x_i + \lambda_2 \ln^u(|x_i| + 1)\mathrm{sat}(x_i) + \left(L\,|s_i|^r + \xi\right)\mathrm{sign}(s_i) + \xi_1 s_i), \tag{43}$$

where L and r are the known controller parameters and meet the conditions $L > 0$, $0 < r < 1$, and ξ_i denotes the bounded range of an external disturbance.

Theorem 3: Under the action of the controller (43), the state trajectories of the time-delay fractional-order HTRS (14) will be stable in a finite time.

Proof:

Select the Lyapunov function $V_2(t) = |s_i|$, such that

$$\dot{V}_2(t) = \mathrm{sign}(s_i)\dot{s}_i. \tag{44}$$

Substituting $\dot{s}_i(t) = D^\alpha x_i + (\lambda_1 x_i + \lambda_2 \ln^u(|x_i| + 1)sat(x_i))$ into (44) gives

$$\dot{V}_2(t) = \mathrm{sign}(s_i)(D^\alpha x_i + (\lambda_1 x_i + \lambda_2 \ln^u(|x_i| + 1)sat(x_i))). \tag{45}$$

Because $D^\alpha x_i = f_i(x) + d_i(t) + u_i(t)$, one gets

$$\begin{aligned}\dot{V}_2(t) &= \mathrm{sign}(s_i)(f_i(x) + d_i(t) + u_i(t) \\ &\quad + (\lambda_1 x_i + \lambda_2 \ln^u(|x_i| + 1)sat(x_i))) \\ &\le \mathrm{sign}(s_i)(f_i(x) + u_i(t) \\ &\quad + (\lambda_1 x_i + \lambda_2 \ln^u(|x_i| + 1)sat(x_i))) + |\xi|\,. \end{aligned} \tag{46}$$

Substituting the controller (43) into (46), one obtains

$$\begin{aligned}\dot{V}_2(t) &\le \mathrm{sign}(s_i)(f_i(x) - (f_i(x) + \lambda_1 x_i + \lambda_2 \ln^u(|x_i| + 1)\mathrm{sat}(x_i) \\ &\quad + \left(L\,|s_i|^r + \xi\right)\mathrm{sign}(s_i) \\ &\quad + \xi_1 s_i) + (\lambda_1 x_i + \lambda_2 \ln^u(|x_i| + 1)sat(x_i))) + \xi \\ &= \mathrm{sign}(s_i)(-(L\,|s_i|^u + \xi)\mathrm{sign}(s_i)) - \xi_1 s_i) + \xi \\ &= -(L\,|s_i|^u + \xi + \xi_1\,|s_i|) + \xi \\ &= -L\,|s_i|^u - \xi_1\,|s_i| \le 0. \end{aligned} \tag{47}$$

According to Lyapunov stability theorem, the system (42) will be stable asymptotically. Taking the integral of both sides of (47) gives

$$\frac{dV}{dt} = \frac{ds}{dt} \le -L\,|s_i|^u - \xi_1\,|s_i|\,, \tag{48}$$

$$dt \le \frac{ds}{-L\,|s_i|^u - \xi_1\,|s_i|}, \tag{49}$$

$$\int_0^T dt \le \int_0^T \frac{ds}{-L\,|s_i|^u - \xi_1\,|s_i|}. \tag{50}$$

The finite time can be obtained by calculation of $T \leq \frac{1}{\xi_1(1-u)} \ln\left(\frac{\xi_1|s_i(0)|^{1-u}+L}{L}\right)$, so the state trajectories of the time-delay fractional-order HTRS will be stabilized in a finite time. The proof is completed.

V. SIMULATION RESULTS

The terminal sliding surface parameters were selected as follows: $\lambda_1 = 50, \lambda_2 = 1$, and $u = 0.2$; the controller parameters were $\lambda_1 = 50, \lambda_2 = 1, L = 0.01, u = r = 0.2$, and $\xi_1 = 10$.

When the values of the given parameters were substituted into Equation (28), the terminal sliding surface of the time-delay fractional-order HTRS can be obtained as

$$s_i(t) = D^{0.98-1}x_i + D^{-1}(50x_i + \ln^{0.2}(|x_i| + 1)sat(x_i)), \quad (i = 1, 2, 3, 4). \tag{51}$$

According to Equation (43), the controller is given as

$$\begin{aligned} u_1(t) &= -(f(x_1) + 50x_1 + \ln^{0.2}(|x_1| + 1)sat(x_1) \\ &\quad + \left(0.01\,|s_1|^{0.2} + 0.1\right) sign(s_1) + 10s_1) \\ u_2(t) &= -(f(x_2) + 50x_2 + \ln^{0.2}(|x_2| + 1)sat(x_2) \\ &\quad + \left(0.01\,|s_1|^{0.2} + 0.1\right) sign(s_1) + 10s_2) \\ u_3(t) &= -(f(x_3) + 50x_3 + \ln^{0.2}(|x_3| + 1)sat(x_3) \\ &\quad + \left(0.01\,|s_1|^{0.2} + 0.1\right) sign(s_1) + 10s_3) \\ u_4(t) &= -(f(x_4) + 50x_4 + \ln^{0.2}(|x_4| + 1)sat(x_4) \\ &\quad + \left(0.01\,|s_1|^{0.2} + 0.1\right) sign(s_1) + 10s_4). \end{aligned} \tag{52}$$

The existing TSM control method in [42] was also applied to the HTRS. Simulation results for time delays of 0.1 and 0.3 s are shown in Figures 2 and 3, respectively. A quantitative comparison of the proposed method and the existing method is presented in Table 1.

It can be seen that the time-delay fractional-order HTRS can be stabilized in a finite time. The result illustrates the effectiveness of the designed terminal sliding surface and the controller. Compared with the control method in [42], obviously, the TSM control method designed here has lesser overshoot and a shorter control time. Moreover, there is less system chattering after stabilization.

Three-dimensional diagrams of the effect of a time delay increase are shown in Figure 4. It is obvious that, with the increase of time delay, the overshoot of the system state variable trajectory increases. The stabilization time of the system also increases gradually.

Traditional PID control was also applied to the HTRS. Simulation results with different time delays are given in Figures 5 and 6. Compared with the proposed TSM control, the stabilization time of the PID is longer and the oscillations are more frequent, demonstrating the robustness and superiority of the proposed scheme.

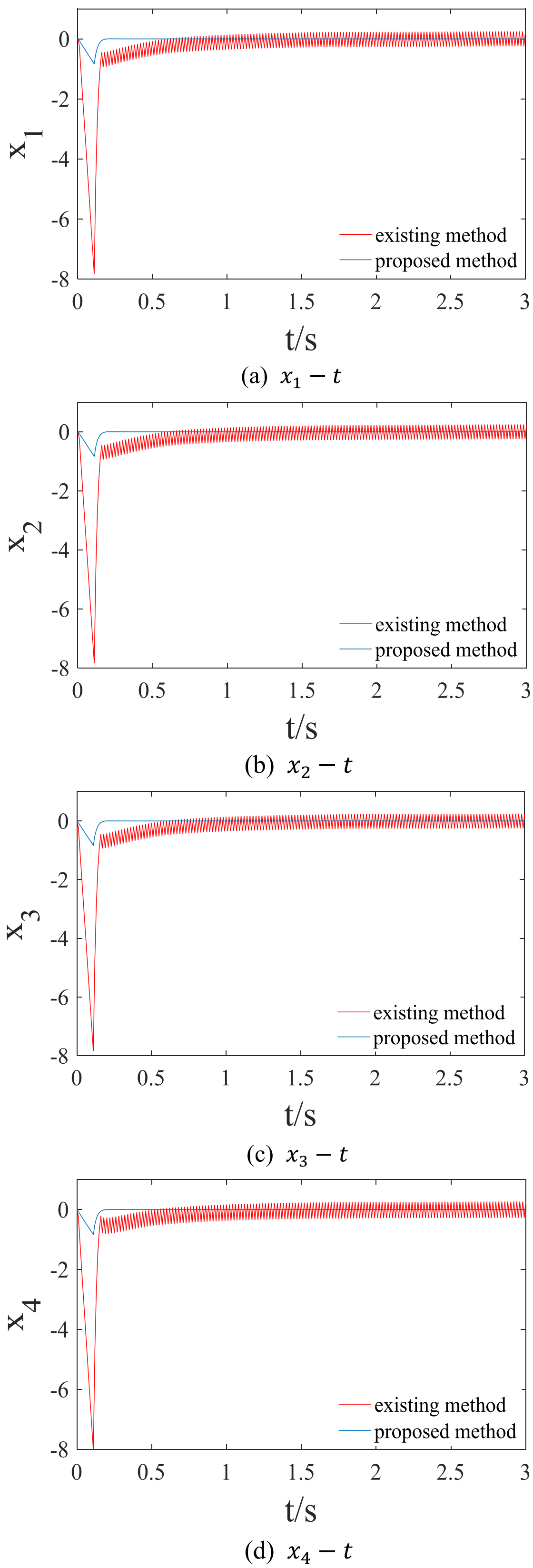

FIGURE 2. Control effect diagram of TSM control when the time delay is 0.1s.

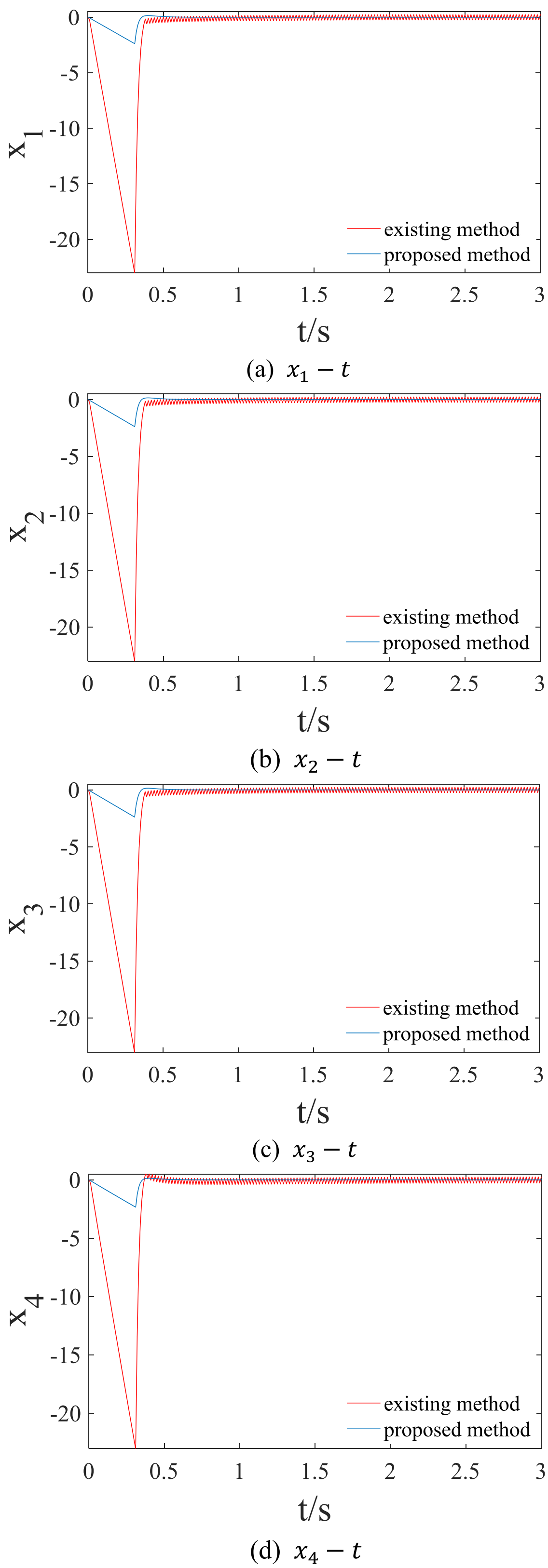

FIGURE 3. Control effect diagram of TSM control when the time delay is 0.3s.

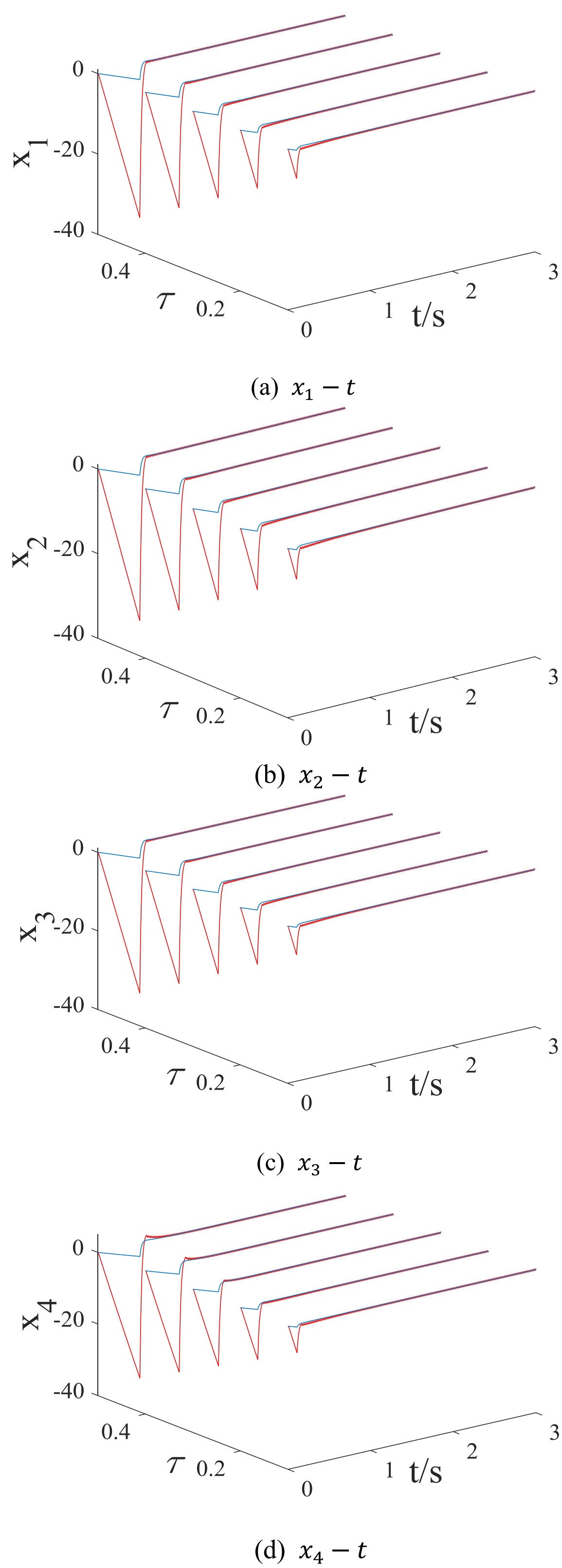

FIGURE 4. Three-dimensional diagram of the system state variable trajectory.

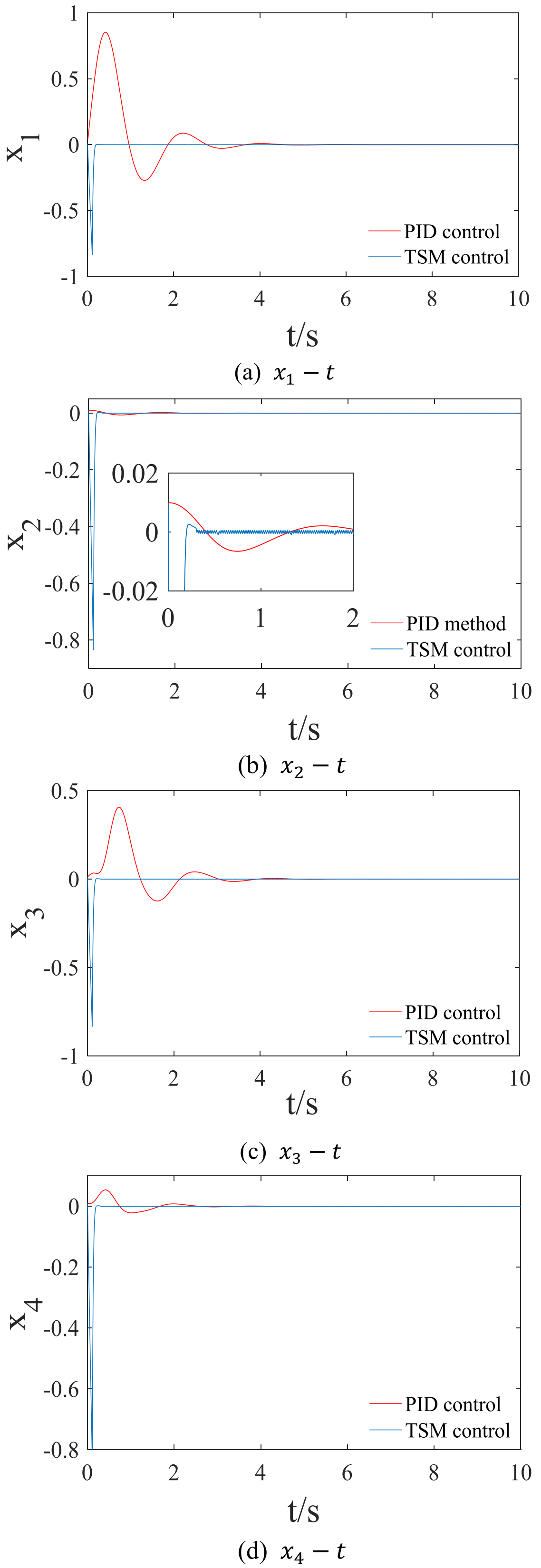

FIGURE 5. Control effect diagram of PID and TSM control when the time delay is 0.1s.

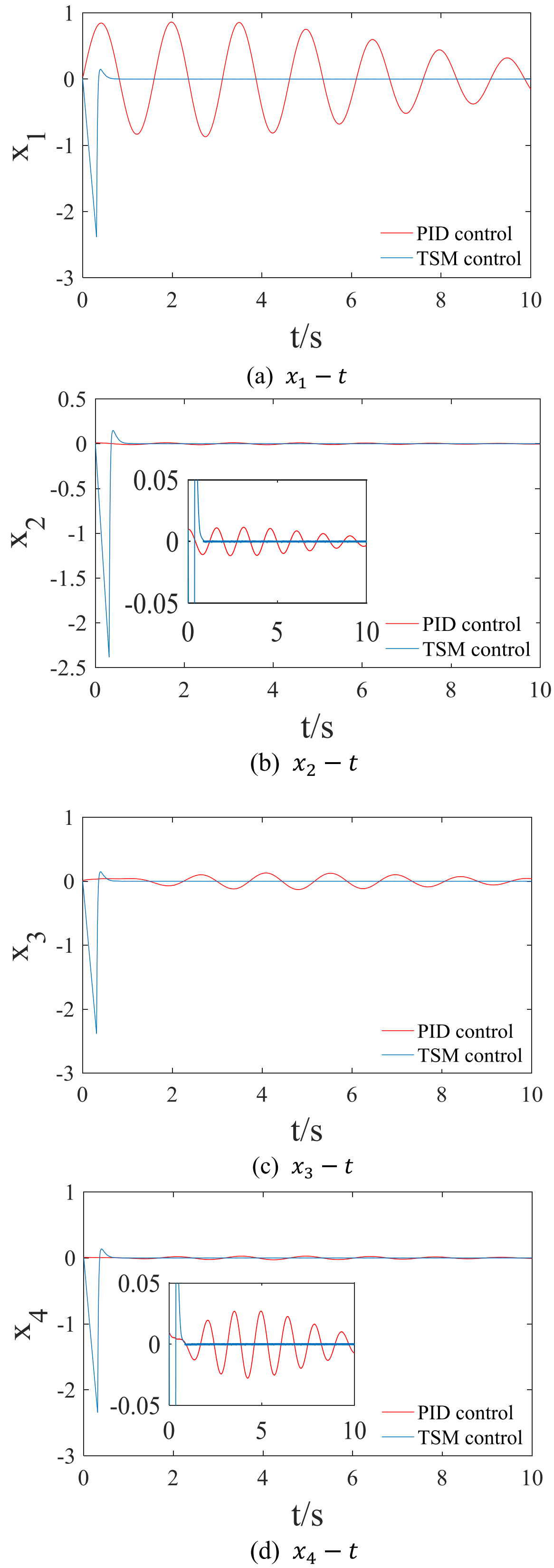

FIGURE 6. Control effect diagram of PID and TSM control when the time delay is 0.3s.

VI. CONCLUSION

Finite-time control of a time-delay fractional-order HTRS was studied. The mathematical model of a time-delay fractional-order HTRS was presented first. By application of the FDM and Lyapunov stability theorem, the fractional-order HTRS was transformed and the stability of the sliding mode was guaranteed. Furthermore, a new finite-time control scheme was proposed for the stability of HTRSs. Experimental results verified the effectiveness and theoretical analysis.

Though the proposed finite-time control scheme exhibits certain superiority, there are some limitations in using the FDM to analyze HTRS stability. First, the FDM is an indirect approach for proving the stability of the time-delay fractional-order HTRS. Moreover, the model transformation and proof are complicated. In the future, we will explore more direct and convenient control schemes for hydropower systems with a time delay.

VII. CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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