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# Comprehensive Predictive Control Method for Automated Vehicles With Delays

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**ABSTRACT** This paper considers the control problem of automated vehicles with delays. We develop a comprehensive control method within the framework of model predictive control (MPC) with compensation for the input delay and the state delay for automated vehicles on the multi-lane road, where the road boundaries and lane markings are modeled as potential functions. The original vehicle dynamics model is modified to formulate the augmented prediction model to compensate for delays. We also present a parallel strategy for the executions of the perception system, the controller, and the actuator, which can directly cut delays in the loop. The simulations show that this method can greatly reduce the impact of delays on the performance of the controller and, therefore, enhance its capabilities.

**INDEX TERMS** Automated vehicles, time delays, model predictive control.

## I. INTRODUCTION

Researchers and engineers are tempting to reduce traffic accidents, congestion, transport time and energy consumption for the future transportation system by developing automated vehicles (AVs) [1], even though there will be a lot of challenging issues standing in their way. The time delay in the control system of automated vehicles is one of the most crucial problems which need to be solved properly.

The control system and the perception system are two general subsystems in the complex system of an automated vehicle [2]. Typically, the control system is hierarchically decomposed into four different layers, namely, the route planning, the behavioral layer, the motion planning, and the local feedback control [3] when people try to design the architecture of the control system for the AV. Using the idea of decomposition is an alternative way to deal with a knotty problem at the beginning when it is not crystal clear or there are no tools which are powerful enough, but it might induce more unanticipated issues. Communication protocols should be established for the exchange of information between those different layers at the design phase, which definitely increases the workload for building a fully autonomous vehicle and the complexity of the system [4]. In addition, the control

problem in the dynamical traffic environment is generally studied case by case. People tend to propose a decision, planning or control method to solve one specific problem in a specific traffic scenario. For instance, as for the decision problem, a Bayesian network is used to make the decision about lane-change maneuvers in [5]; a fuzzy logic controller is studied for making the decision of overtaking in [6]. As for the path planning problem, the piecewise quadratic Bezier curve is used in a path planning method for lane changing in [7]. A modified Rendezvous Guidance method is used to plan the longitudinal and lateral velocity configuration for overtaking in [8]. Those methods can only cover the task in a certain scenario in a single layer of the decomposed control system. In other words, many different methods with a limited capability should be implemented and combined together at the same time in the control system in order to realize an automated control system, which would inevitably make the system more cumbersome [4] and therefore cause significant delays for actuation.

A comprehensive predictive control method for automated vehicles was first proposed in [4], which shows that we can cope with the complex dynamical traffic environment and the tasks in the decision, path planning and control layers at the same time by only using one single method. That would greatly reduce the complexity of the control system and make it easier to implement it on board. However, the time

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delay induced by generating the control commands using that method is not trivial as presented in [4]. A small time delay in the control system may significantly degrade the performance of a controller originally designed for the ideal system without any delays. For example, the string stability may be lost due to the delay induced by wireless communication among a platoon of connected and automated vehicles [9]–[13]. The in-vehicle communication via the Controller Area Network (CAN) also imposes network-induced time delays which may cause the control system unstable [14].

The time delay which is a source of instability in many situations often appears in real-world control engineering systems in the control input or in the state [15]. In the control system of an automated vehicle, the input delay is usually caused by the computing time cost to generate the control command in the controller and the time spent on the communication via the CAN bus. The computing time of the controller is rarely taken into consideration when people design the controller for the AV, e.g. [4], [16]–[21]. The authors in [4] have mentioned that the computing time of the controller is less than its cycle, which is actually nontrivial in the control system of an automated vehicle. However, the authors did not propose a method which can compensate for the input delay. On the other hand, communications via the CAN bus contribute more input delays to the control loop. The delay bounds for all CAN priorities can be determined by an analytical method with a closed formula [22]. Time-varying delays induced by CAN communication are assumed to be uniformly distributed on the interval between zero and the upper bound [14].

The control problem is always studied based on the assumption that the information of the environment and the state of the plant can be acquired in real time. Unfortunately, the perception system which gets the knowledge of the traffic environment and the state of the vehicle always induces significant time delays in reality due to the computer's limited comprehension of the complicated and volatile traffic environment. The object scene flow is one of the state-of-the-art methods for automated vehicles to understand the environment, but the runtime it takes to process one scene with four frames captured by two cameras is pretty large [23]. A novel 3D object detector proposed in [24] that can exploit both LIDAR as well as cameras to perform a high detection accuracy can run at more than 15 frames per second. We can see that the delay caused by the perception system is non-negligible. However, it is commonly neglected by the researchers who intend to propose novel control methods for the automated vehicle.

In order to make the control system of the automated vehicle simpler, more compact and more capable and reduce the adverse effect imposed by the time delay in the control loop, we would like to propose a comprehensive predictive control method with compensation for the input delay and the state delay. The contribution of this work is threefold. First, it clarifies the input delay and the state delay in the control system of an automated vehicle. Second, it proposes a predictive control method which can deal with different kinds

of traffic environment and integrates the tasks of decision, planning, and control, and meanwhile significantly reduce the effect of time delays. Third, it employs a parallel control scheme which can execute the control commands at a higher frequency and cut the delay in the loop directly.

The remainder of this paper is structured as follows. Section II formulates the problem and models the traffic, vehicle dynamics, and delays. Section III describes the comprehensive predictive control method with compensation for delays. Section IV presents some simulations. Finally, Section V briefly concludes our work.

## II. PROBLEM FORMULATION

This section formulates the control problem of automated vehicles which we are focusing on. First, the traffic environment is introduced and modeled with potential functions. Then, a vehicle dynamics model for the controlled automated vehicle is presented. At last, the input delay and state delay in the automated vehicle system are explained.

### A. TRAFFIC ENVIRONMENT

The automated vehicle runs on the flat multi-lane road with different kinds of vehicles no matter whether they are automated or human-driven. The perception system can obtain real-time information about the traffic environment, such as the road boundaries, lane markers, the velocities of the surrounding vehicles and the shortest distances between the automated vehicle with them. The traffic environment around the automated vehicle is demonstrated in Fig. 1, where the sedan icon represents the automated vehicle which is being controlled by our method, and the gray rectangles represent the surrounding vehicles. The solid black lines are the road boundaries and the dashed black lines are the crossable lane markers. The width of the lane is  $D$ . The distance from the center of gravity (CG) of the automated vehicle to the left and right road boundary are  $d_l$  and  $d_r$ , respectively. The distance between the CG of the automated vehicle and a certain lane marker is  $d_i$ , where  $i$  is the index of that lane marker. The distance from the CG of the automated vehicle to the one of the surrounding vehicles is denoted as  $d_j$ . The position, the longitudinal and lateral velocity of a certain surrounding vehicle are  $(X_j, Y_j)$ ,  $v_{jx}$  and  $v_{jy}$ , respectively. The variable mentioned above can be calculated by the perception system based on the data gathered from cameras, radars, LIDARs or other sensors by using different kinds of perception and estimation techniques, such as Deep Learning [25], [26] and SLAM [27], [28].

As for the lanes and road boundaries, we would like to introduce the potential functions [16], [29], [30] to construct the potential field of the road, which can be used to regulate the automated vehicle to drive in the center of its current lane if it is free from other constraints.

The overall potential function of the road boundaries is formulated as follows.

$$U_R(X, Y) = k_R \left( \frac{1}{d_l(X, Y)} + \frac{1}{d_r(X, Y)} \right), \quad (1)$$

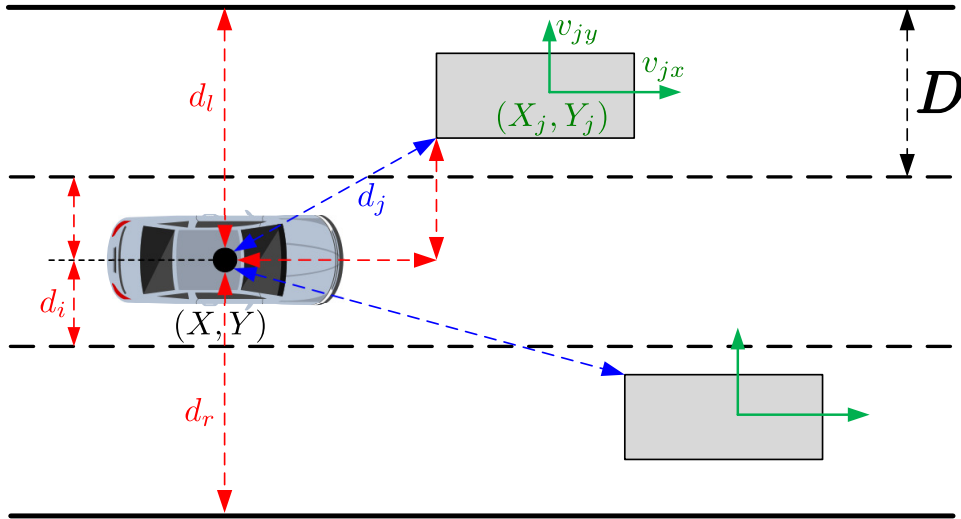


FIGURE 1. Traffic environment around the automated vehicle.

where  $k_R$  is a constant,  $(X, Y)$  is the world coordinate of the automated vehicle. The overall potential function of the road boundaries increases dramatically when the AV is close to the boundaries, while it is almost flat and close to zero when the AV is in the middle of the road.

The potential function of the crossable lane marker is written as follows.

$$U_i(X, Y) = \alpha e^{-\frac{d_i^2(X, Y)}{2\sigma^2}}, \quad (2)$$

where the  $\alpha$  and  $\sigma$  are constants,  $d_i$  is the shortest distance from the AV to the  $i$ -th lane marker. It forms a small bump on the crossable lane marker in the potential field, which can prevent the AV from riding on the marker for a long time, but it will not cost too much energy to cross it if the AV cannot continue to run in its current lane. And we treat the non-crossable lane markers in the same way as we treat the road boundaries in this work.

By synthesizing the potential functions of the road boundaries and the lane markers, we can obtain the overall potential field of the road configurations. The function of the overall potential field can be written as follows.

$$U(X, Y) = U_R(X, Y) + \sum_i U_i(X, Y). \quad (3)$$

The overall potential field of the road is an important part of our method to solve the control problem of automated vehicles.

Except for the regulations of the road, the vehicles surrounding the AV also restrict its actions. The elliptical constraints are introduced to restrict the safety distance between the AV and its surrounding vehicles. The safety constraint imposed by one of the surrounding vehicles is formulated as follows.

$$\frac{(X - X_j)^2}{P} + \frac{(Y - Y_j)^2}{Q} - 1 \geq 0, \quad (4)$$

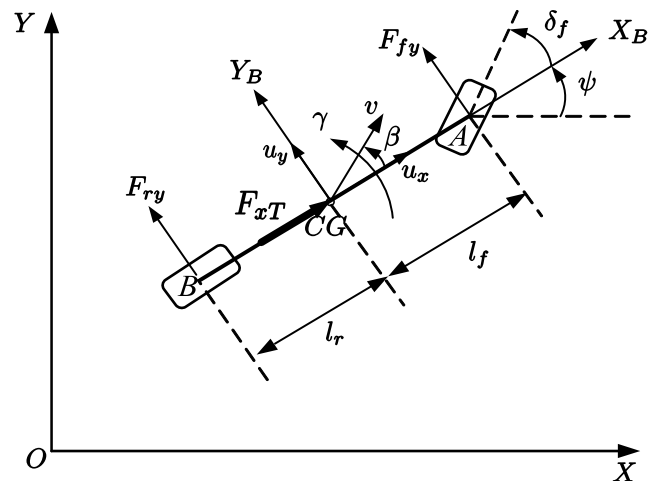


FIGURE 2. Vehicle dynamics model.

where  $P > Q > 0$ . This is consistent with the intuition and the experience in the normal driving that we do not need to care about other vehicles when they are far away from our vehicle and the longitudinal safety distance should be larger than the lateral one.

### B. VEHICLE DYNAMICS

A simple nonlinear vehicle dynamics model [16] is adopted for the control of the AV in this work. As shown in Fig. 2,  $X - Y$  is the world coordinate system,  $X_B - Y_B$  is the body-fixed coordinate of the AV, CG is the center of gravity of the AV, point A and B are the center of the front and rear tire, respectively. Other variables in the vehicle dynamic system are presented in the Table 1.

The differential equations of the vehicle dynamics are written as follows.

$$m_0 \dot{u}_x = F_{xT} + m_0 u_y \gamma \quad (5)$$

$$m_0 \dot{u}_y = F_{fy} + F_{ry} - m_0 u_x \gamma \quad (6)$$

TABLE 1. Variables in the vehicle dynamics system.

Variable	Definition
$\beta$	The vehicle sideslip angle which is between the heading of the vehicle and the velocity of its CG
$\psi$	The angle from the X-axis to the longitudinal axis of the vehicle body AB.
$\gamma$	The yaw rate of the vehicle, $\gamma = \dot{\psi}$
$\delta_f$	The front steering angle
$m_0$	The mass of the vehicle
$I_z$	The moment of inertia around the vertical axis through CG
$l_f$	The distance from point A and point CG
$l_r$	The distance from point B and point CG
$v$	The velocity of the vehicle
$u_x$	The longitudinal velocity of the vehicle in the body-fixed coordinate system
$u_y$	The lateral velocity of the vehicle in the body-fixed coordinate system
$F_{xT}$	The longitudinal force of the front wheel in its own coordinate system
$F_{fy}$	The lateral force of the front wheel in its own coordinate system
$F_{ry}$	The lateral force of the rear wheel in its own coordinate system
$C_f$	Stiffness coefficients of front tires
$C_r$	Stiffness coefficients of rear tires

$$I_z \dot{\gamma} = F_{fy} l_f - F_{ry} l_r \quad (7)$$

$$\dot{\psi} = \gamma \quad (8)$$

$$\dot{X} = u_x \cos \psi - u_y \sin \psi \quad (9)$$

$$\dot{Y} = u_x \sin \psi + u_y \cos \psi. \quad (10)$$

In the above differential equations, the total lateral forces of the front and rear tires can be calculated by the following equations according to the dynamics of tires [16].

$$F_{fy} = C_f \left( \delta_f - \frac{u_y + l_f \gamma}{u_x} \right) \quad (11)$$

$$F_{ry} = C_r \left( -\frac{u_y - l_r \gamma}{u_x} \right) \quad (12)$$

where  $C_f$  and  $C_r$  are the cornering stiffness coefficients of the front and rear tires, respectively.

Therefore, the inputs of the dynamic system are the steering angle  $\delta_f$  and the total longitudinal force  $F_{xT}$ . Normalizing the inputs with  $[\delta_f, F_{xT}]^T = [C_\delta u_\delta, C_F u_F]^T$ , where  $C_\delta$  and  $C_F$  are normalizing constants which can regulate dimensionless variables  $u_\delta$  and  $u_F \in [-1, 1]$  in consideration of mechanical constraints of the steering and the engine, then we can rewrite the differential equations of the vehicle dynamical

system in a compact and general formal as follows.

$$\dot{\mathbf{x}} = f_0(\mathbf{x}, \mathbf{u}). \quad (13)$$

where  $\mathbf{x} = [u_x, u_y, \gamma, \psi, X, Y]^T \in \mathbb{R}^m$  is the state vector of the system, and  $\mathbf{u} = [u_\delta, u_F]^T \in \mathbb{R}^n$  is the control vector of the dynamic system. Here,  $m = 6, n = 2$ .

In order to make the problem more feasible and reduce the computation burden, we would like to linearize the nonlinear dynamic system around its operating point. Assume that the vehicle runs straight at a steady longitudinal speed of  $u_x \neq 0$ , and  $u_y = \gamma = \psi = 0$  at time  $t$ . The linearized system can be written as follows.

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)\mathbf{u}. \quad (14)$$

where

$$\mathbf{A}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{C_f + C_r}{m u_x} & -\frac{C_f l_f + C_r l_r}{m u_x} & -u_x & 0 & 0 \\ 0 & -\frac{C_f l_f + C_r l_r}{I_z u_x} & -\frac{C_f l_f^2 + C_r l_r^2}{I_z u_x} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & u_x & 0 & 0 \end{bmatrix} \quad (15)$$

$$\mathbf{B}(t) = \begin{bmatrix} 0 & \frac{C_F}{m} \\ \frac{C_f C_\delta}{m} & 0 \\ \frac{C_f l_f C_\delta}{I_z} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (16)$$

Then, discretizing the state space system by using the zero-order hold (ZOH), we can get the discrete system as follows which can be adopted to the digital control system.

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k, \quad (17)$$

where  $\mathbf{A}_k = e^{\mathbf{A}(t)\Delta t}$ ,  $\mathbf{B}_k = \int_0^{\Delta t} e^{\mathbf{A}(t)\tilde{t}} \tilde{t} \mathbf{B}(t) dt$ , and  $\Delta t$  is the sample time of the digital system.

### C. INPUT DELAYS AND STATE DELAYS

In the automated driven vehicles, the communication between the control unit and the actuator is connected through the CAN bus. Network communications always cause information delays. The delays of all messages induced by the CAN bus are assumed to be time-varying and uniformly distributed in the interval from zero to the upper bound in [14], and the upper bound can be known in advance by experimental tests [31] or a theoretical analysis [22].

In addition to the delays induced by the network communications, solving the control problem in the controller unit will

inevitably cost some extra input delays, especially when some advanced control methods are employed. The computing time for solving the model predictive control problem for the control of the automated vehicle in a normal laptop is analyzed in [4], from which we can see that the delays induced by the control unit are nontrivial.

In this work, we only focus on the compensation for the maximum delays. If our method has the capacity to deal with the maximum input delay, it is easy to adapt it to deal with the case with varying delays which are not greater than the maximum one. Assuming that the maximum input delay is  $\tau_{in} \geq 0$ , then in the discrete system, the time steps of the input delay can be denoted as  $d = \lceil \frac{\tau_{in}}{\Delta t} \rceil$ . Therefore, we can write the discrete-time linear control system with the input delay as follows which represents the actual dynamics of the controlled vehicle.

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_{k-d}. \tag{18}$$

Researchers work on the control problems of automated vehicles rarely consider issues induced by the perception system, and usually propose their control methods basically based on the assumption that states of the AV and the environment are estimated instantly. However, the perception system, which in general has the functionalities of environment recognition, ego-vehicle localization, and obstacle detection & classification [1], is always pretty time-consuming. The estimated state fed into the controller is actually out of date. The state delay can be denoted as  $b = \lceil \frac{\tau_{st}}{\Delta t} \rceil$  in the discrete system, where  $\tau_{st}$  is the maximum time delay of the state.

### III. METHOD

In this section, we would like to introduce our comprehensive predictive control method with compensation for the input delay and the state delay.

The comprehensive predictive control method for automated vehicles was first proposed in [4]. This method eliminates the divisions of driving behavior decisions, trajectory planning, and motion control. In other words, there is no need to design those modules in the control system for automated vehicles, which greatly reduces the design complexity of the control system and avoids the communications between different modules [4]. And at the same time, this method has the strong capability to deal with different kinds of traffic environments without modifying the method to adjust to the changing traffic and classify them into different scenarios. However, the authors did not consider those problems induced by the nontrivial computing time and the communication delay between the controller and the actuators.

A significant delay in the control system of an automated vehicle is a crucial and challenging issue which will degrade the performance of the controller or even cause traffic accidents. Therefore, we would like to modify the original comprehensive predictive control method for automated vehicles to compensate for delays inspired by the work [32] and make it more practical to implement it in the on-board automated driving system.

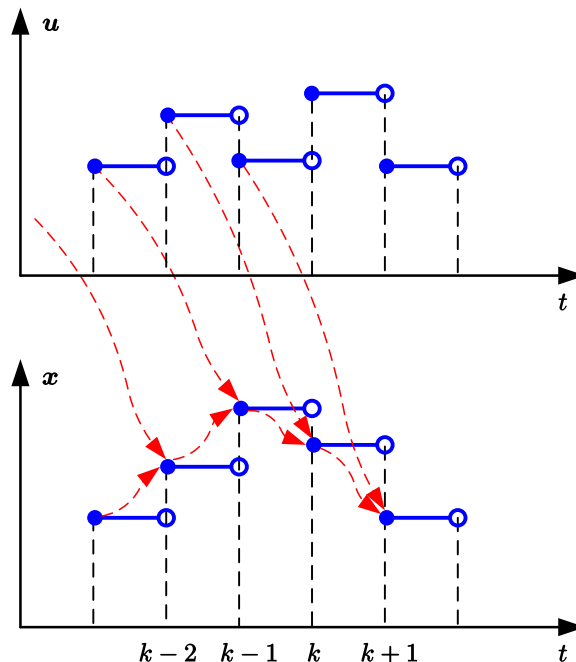


FIGURE 3. A control system with an input delay  $d = 1$ .

At the beginning of the method, we assume that the control system knows the traffic environment in a certain horizon from the current position to a point ahead of the automated vehicle along the road. This assumption is reasonable because the perception system can obtain the environment information in front of the AV within sight.

#### A. COMPENSATING FOR THE INPUT DELAY

Except for the communication delays, in the implementation of the controller, the computing time for solving the control input is often nontrivial for the methods within the framework of MPC. At first, we only consider the case with delays induced by the computation of the controller and the network communications while neglecting the computing time of the perception system. We call the sum of the former two as the input delay. In this case, the prediction model is denoted as (17).

We assume that the cycle of the controller and the cycle of the actuator are aligned. The actuator cycle  $T_a = \Delta t$ , the controller cycle is  $T_c$ , and  $T_c = aT_a$ ,  $a \in \mathbb{Z}^+$ . And the beginnings of the first cycle of the actuator and the controller are exactly at  $t = 0$ . Fig. 3 shows a control system with the input delay  $d = 1$ , and the controller cycle equals to the actuator cycle ( $a = 1$ ).

Defining the augmented state as

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \\ \mathbf{u}_{k+1} \\ \vdots \\ \mathbf{u}_{k+d-1} \end{bmatrix}, \tag{19}$$

$\mathbf{x}_k \in \mathbb{R}^{m+n \times d}$ , then we can write an augmented system as

$$\mathbf{x}_{k+1} = \underbrace{\begin{bmatrix} A_k & B_k & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & & \mathbf{0} \\ \vdots & \vdots & & \ddots & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & & I \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}}_{A_k} \mathbf{x}_k + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ I \end{bmatrix}}_{B_k} \mathbf{u}_{k+d} \quad (20)$$

The augmented system (20) is regarded as the prediction model in the model predictive control framework. Combining the overall potential function, the safety constraints, the vehicle dynamics, state and control constraints, the comprehensive predictive control method with compensation for input delays can be written as follows.

$$\min_{\mathbf{U}} \sum_{k=0}^{N_p-1} \phi(\mathbf{r}_{k+1}, \mathbf{x}_{k+1}, \mathbf{u}_k) \quad (21a)$$

$$\text{s.t. } \mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k, \quad k = 0, 1, \dots, N_p - 1 \quad (21b)$$

$$\mathbf{x}_{\min} \leq \mathbf{x}_k \leq \mathbf{x}_{\max}, \quad k = 1, 2, \dots, N_p \quad (21c)$$

$$\mathbf{u}_{\min} \leq \mathbf{u}_k \leq \mathbf{u}_{\max}, \quad k = 0, 1, \dots, N_p - 1 \quad (21d)$$

$$\Delta \mathbf{u}_{\min} \leq \mathbf{u}_{-1} - \mathbf{u}_0 \leq \Delta \mathbf{u}_{\max} \quad (21e)$$

$$\Delta \mathbf{u}_{\min} \leq \mathbf{u}_k - \mathbf{u}_{k+1} \leq \Delta \mathbf{u}_{\max}, \quad k = 0, 1, \dots, N_p - 2 \quad (21f)$$

$$\frac{(X_k - X_{j,k})^2}{P} + \frac{(Y_k - Y_{j,k})^2}{Q} - 1 \geq 0, \quad k = 1, 2, \dots, N_p, j = 1, 2, \dots, N_o \quad (21g)$$

$$\mathbf{x}_0 = \mathbf{x}(t). \quad (21h)$$

In (21a),  $\mathbf{U} = [\mathbf{u}_0^T, \mathbf{u}_1^T, \dots, \mathbf{u}_{N_p-1}^T]^T$ ,  $N_p \in \mathbb{Z}^+$  is the prediction horizon, the objective function

$$\phi(\mathbf{r}_{k+1}, \mathbf{x}_{k+1}, \mathbf{u}_k) = U(X_{k+1}, Y_{k+1}) + \mathbf{u}_k^T \mathbf{S} \mathbf{u}_k + (\mathbf{r}_{k+1} - \mathbf{C} \mathbf{x}_{k+1})^T \mathbf{R} (\mathbf{r}_{k+1} - \mathbf{C} \mathbf{x}_{k+1}). \quad (22)$$

where  $U(X_{k+1}, Y_{k+1})$  is the overall potential when the AV is at the position  $(X_{k+1}, Y_{k+1})$ ,  $R$  and  $S$  are semi-definite matrixes,  $\mathbf{r}_{k+1}$  is the reference at time  $k + 1$ , and  $\mathbf{C} \in \mathbb{R}^{1 \times (m+n \times d)}$ . The reference in the objective function only refers to the speed limit posted by the traffic sign along the road or input by the manipulator for some specific reasons. There is no need to plan the speed profiles by using extra methods in advance. Defining  $\mathbf{e}_i \in \mathbb{R}^{m+n \times d}$  as a vector with only the  $i$ -th element equal to 1, and all others equal to 0, we can get

$$\mathbf{C} = \mathbf{e}_1^T, \quad (23)$$

$$X_k = \mathbf{e}_5^T \mathbf{x}_k, \quad (24)$$

$$Y_k = \mathbf{e}_6^T \mathbf{x}_k. \quad (25)$$

The equality constraints in (21b) are derived from the augmented system. Equations (21c) -(21f) are constraints of states and control inputs due to the mechanical limitations.

The constraints of the state in (21c) can be written as

$$\underbrace{\begin{bmatrix} I \\ -I \end{bmatrix}}_{A_x} \mathbf{x}_k \leq \underbrace{\begin{bmatrix} \mathbf{x}_{\max} \\ -\mathbf{x}_{\min} \end{bmatrix}}_{B_x} \quad (26)$$

Substituting the augmented system (21b) into (26), one can transform the constraints of the states to those of the inputs as follows.

$$A_e \mathbf{U} \leq B_e \quad (27)$$

in which,

$$A_e = \begin{bmatrix} A_x \mathbf{B}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ A_x \mathbf{A}_1 \mathbf{B}_0 & A_x \mathbf{B}_1 & & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ A_x \prod_{i=1}^{N_p-1} \mathbf{A}_i \mathbf{B}_0 & A_x \prod_{i=2}^{N_p-1} \mathbf{A}_i \mathbf{B}_1 & \cdots & A_x \mathbf{B}_{N_p-1} \end{bmatrix} \quad (28)$$

$$B_e = \begin{bmatrix} B_x - A_x \mathbf{A}_0 \mathbf{x}_0 \\ \vdots \\ \vdots \\ B_x - A_x \prod_{i=0}^{N_p-1} \mathbf{A}_i \mathbf{x}_0 \end{bmatrix} \quad (29)$$

A special constraint for the first control vector showed in (21e), where  $\mathbf{u}_{-1}$  is the control input generated in the last control cycle and stored in the memory, can be rewritten as

$$\Delta \mathbf{u}_{\min} \leq \mathbf{u}_{-1} - \mathbf{u}_0 \leq \Delta \mathbf{u}_{\max}. \quad (30)$$

Then, the constraints for the first control vector when  $k = 0$  in (21d) should be modified to

$$\tilde{\mathbf{u}}_{\min} \leq \mathbf{u}_0 \leq \tilde{\mathbf{u}}_{\max}, \quad (31)$$

where

$$\tilde{\mathbf{u}}_{\max} = \min(\mathbf{u}_{\max}, \mathbf{u}_{-1} - \Delta \mathbf{u}_{\min}), \quad (32)$$

$$\tilde{\mathbf{u}}_{\min} = \max(\mathbf{u}_{\min}, \mathbf{u}_{-1} - \Delta \mathbf{u}_{\max}). \quad (33)$$

Therefore, the constraints of the inputs derived from (21d) and (21e) can be written as

$$\underbrace{\begin{bmatrix} I & & & \\ & \ddots & & \\ & & I & \\ -I & & & \\ & & & \ddots \\ & & & & -I \end{bmatrix}}_{A_u} \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{N_p-1} \end{bmatrix} \leq \underbrace{\begin{bmatrix} \tilde{\mathbf{u}}_{\max} \\ \mathbf{u}_{\max} \\ \vdots \\ \mathbf{u}_{\max} \\ -\tilde{\mathbf{u}}_{\min} \\ -\mathbf{u}_{\min} \\ \vdots \\ -\mathbf{u}_{\min} \end{bmatrix}}_{B_u} \quad (34)$$

The constraints in (21f) can also be transformed as follows.

$$\underbrace{\begin{bmatrix} I & -I & & & & \\ & \ddots & \ddots & & & \\ & & I & -I & & \\ -I & I & & & & \\ & \ddots & \ddots & & & \\ & & -I & I & & \end{bmatrix}}_{A_d} \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{N_p-1} \end{bmatrix} \preceq \underbrace{\begin{bmatrix} \Delta \mathbf{u}_{\max} \\ \vdots \\ \Delta \mathbf{u}_{\max} \\ -\Delta \mathbf{u}_{\min} \\ \vdots \\ -\Delta \mathbf{u}_{\min} \end{bmatrix}}_{B_d} \quad (35)$$

Finally, we can obtain all the linear inequality constraints in a compact form as follows.

$$\underbrace{\begin{bmatrix} A_e \\ A_u \\ A_d \end{bmatrix}}_{A_{in}} \mathbf{U} \preceq \underbrace{\begin{bmatrix} B_e \\ B_u \\ B_d \end{bmatrix}}_{B_{in}} \quad (36)$$

As for the nonlinear inequality constraints in (21g) which originate from the safety distance between the AV with other  $N_o$  traffic participants, we rewrite them here in a simpler form as

$$\mathbf{f}_{\text{non}}(\mathbf{X}, \mathbf{Y}) \leq \mathbf{0}. \quad (37)$$

where  $\mathbf{X} = [X_1, X_2, \dots, X_{N_p}]^T$ ,  $\mathbf{Y} = [Y_1, Y_2, \dots, Y_{N_p}]^T$ ,

$$\mathbf{X} = \begin{bmatrix} \mathbf{e}_5^T & & & \\ & \mathbf{e}_5^T & & \\ & & \ddots & \\ & & & \mathbf{e}_5^T \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_{N_p} \end{bmatrix} \quad (38)$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{e}_6^T & & & \\ & \mathbf{e}_6^T & & \\ & & \ddots & \\ & & & \mathbf{e}_6^T \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_{N_p} \end{bmatrix} \quad (39)$$

and

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_{N_p} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{A}_1 \mathbf{B}_0 & \mathbf{B}_1 & & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \prod_{i=1}^{N_p-1} \mathbf{A}_i \mathbf{B}_0 & \prod_{i=2}^{N_p-1} \mathbf{A}_i \mathbf{B}_1 & \cdots & \mathbf{B}_{N_p-1} \end{bmatrix} \mathbf{U} + \begin{bmatrix} \mathbf{A}_0 \mathbf{x}_0 \\ \mathbf{A}_1 \mathbf{A}_0 \mathbf{x}_0 \\ \vdots \\ \prod_{i=0}^{N_p-1} \mathbf{A}_i \mathbf{x}_0 \end{bmatrix} \quad (40)$$

Therefore, the nonlinear inequality constraints can be written as

$$\tilde{\mathbf{f}}_{\text{non}}(\mathbf{U}) \leq \mathbf{0}. \quad (41)$$

As a result, the Model Predictive Control problem for automated vehicles with compensation for input delays can be transformed into an optimization problem with linear and nonlinear inequality constraints as follows.

$$\min_{\mathbf{U}} \sum_{k=0}^{N_p-1} \tilde{\phi}(\mathbf{r}_{k+1}, \mathbf{u}_k) \quad (42a)$$

$$\text{s.t. } A_{in} \mathbf{U} \leq B_{in} \quad (42b)$$

$$\tilde{\mathbf{f}}_{\text{non}}(\mathbf{U}) \leq \mathbf{0}. \quad (42c)$$

This kind of optimization problem in (42) is consistent with the functional interfaces of many widely used optimization techniques, such as *fmincon* [33], so it is easy to be implemented. The MPC problem for automated vehicles without any consideration of input delays can also be transformed into this kind of optimization problem in a similar way.

At each system time step  $k$ ,  $k = \lceil \frac{t}{\Delta t} \rceil$ , the controller generates the control sequence  $\mathbf{U}$ , and sends the first control vector  $\mathbf{u}_0$  which is the prediction control input  $\mathbf{u}_{k+d}$ , to the actuator via the CAN bus. The actuator receives and executes the control command at the time step  $(k + d)$  after a delay of  $d$  time steps.

### B. COMPENSATING FOR THE STATE DELAY

In addition to the computing time of the controller and the communication delays, the perception system needs time, which is always more significant compared to the time used for the controller and communications, to analyze the captured raw data from the traffic environment before transiting the processed information of the traffic to the controller. In general, we assume that the controller generates the control at time step  $k$  based on the traffic environment and the state of the AV at time  $(k - b)$ . The prediction model with state delays can be written as

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_{k-b} + \mathbf{B}_k \mathbf{u}_k. \quad (43)$$

Defining the augmented state as

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_{k-1} \\ \vdots \\ \mathbf{x}_{k-b} \\ \mathbf{u}_k \\ \mathbf{u}_{k+1} \\ \vdots \\ \mathbf{u}_{k+d-1} \end{bmatrix} \in \mathbb{R}^{m \times (b+1) + n \times d}, \quad (44)$$

similarly, one can get the augmented system as follows which will be used as the predictive model in the proposed method to compensate for the state delay and the input delay.

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k \quad (45)$$

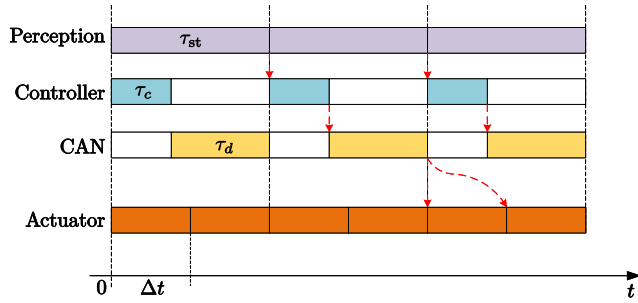


FIGURE 4. Sequence diagram.

where

$$A_k = \begin{bmatrix} A_k & \mathbf{0} & \dots & \mathbf{0} & B_k & & \\ I & & \mathbf{0} & \mathbf{0} & \mathbf{0} & & \mathbf{0} \\ & \ddots & & \vdots & \vdots & & \\ \mathbf{0} & & I & \mathbf{0} & \mathbf{0} & & \\ & & & & & I & \mathbf{0} \\ & & \mathbf{0} & & & & \ddots \\ & & & & & \mathbf{0} & I \\ & & & & & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix}, \quad (46)$$

and  $B_k$  has the same structure as in (20),  $u_k$  is the same as in (20). In a similar way, we can also obtain the comprehensive predictive control method with compensation for the delays induce by the perception system, the controller and the communication as in section III-A.

For the discrete control system, the smaller the sample time is, the smaller the discretization error it has. Assume that the maximum processing time for the perception system, the controller and the communication delay are  $\tau_{st}$ ,  $\tau_c$  and  $\tau_d$ , respectively. If these three events run serially, the cycle of the control system  $\Delta t$  should be no shorter than  $\max(\tau_{st}, \tau_c + \tau_d)$ , which is usually not a short period. In order to solve this problem, we propose a parallel scheme to implement the control system with a higher frequency based on the advantages of the model predictive control method. The frequency of the controller and the frequency of the actuator in this parallel control scheme are different, see Fig. 4 for an illustration, in which the frequency of the actuator is as twice as the one of the controller. Therefore, the nominal control system can be discretized with a sufficient small sample time. Then, the plant can be controlled more smoothly at a higher control frequency.

In this case, we assume that the cycle of the perception system is  $T_b$ ,  $T_b \geq \tau_{st}$  and  $T_b = T_c = aT_a$ ,  $T_a = \Delta t$ ,  $T_c \geq \tau_c + \tau_d$ ,  $a \in \mathbb{Z}^+$  and  $a > 1$ . By solving the similar optimization problem as in (42), the control sequence  $U$  can be obtained and the first  $a$  elements of which will be sent to the actuator in every control cycle. The actuator will store the control sequence in the buffer and execute them one by one from the first to the last in every its own cycle.

The updates of the control input received by the actuator can be described as a buffer policy [34]. Defining the command buffer of the actuator is  $b_k$  ( $b_k \in \mathbb{R}^{a \times n}$ ) at the system

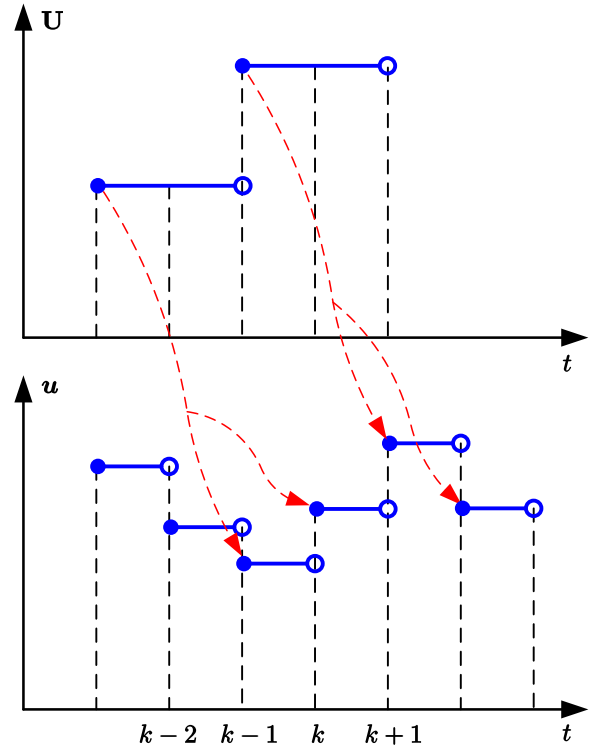


FIGURE 5. The output of the controller and the command executed by the actuator.

time step  $k$ , the indicator of receiving a new sequence is  $w_k$ , and the time step when the last most recent sequence received is  $\hat{k}$ , then the dynamics of the buffer can be written as

$$b_k = w_k \hat{U}_{\hat{k}} + (1 - w_k) \hat{S} b_{k-1} \quad (47)$$

where  $w_k$  equals to 1 if a new sequence was received at time step  $k$ , otherwise it equals to 0.  $\hat{U}_{\hat{k}}$  is the sequence received,  $\hat{S} \in \mathbb{R}^{(a \times n) \times (a \times n)}$  is a shift matrix defined as

$$\hat{S} = \begin{bmatrix} \mathbf{0} & I & & \mathbf{0} \\ \vdots & & \ddots & \\ \mathbf{0} & \mathbf{0} & & I \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix} \quad (48)$$

Therefore, the command will be executed at time step  $k$  can be described as

$$\hat{u}_k = [I \quad \mathbf{0} \quad \dots \quad \mathbf{0}] b_k \quad (49)$$

Fig. 5 shows the trajectories of the output of the controller and the command executed by the actuator. A closed-loop control block diagram is given in Fig. 6 for detailed design analysis.

#### IV. SIMULATIONS

In order to demonstrate the performance of the proposed method and investigate the compensation for time delays, this section presents some numerical simulations performed in MATLAB.



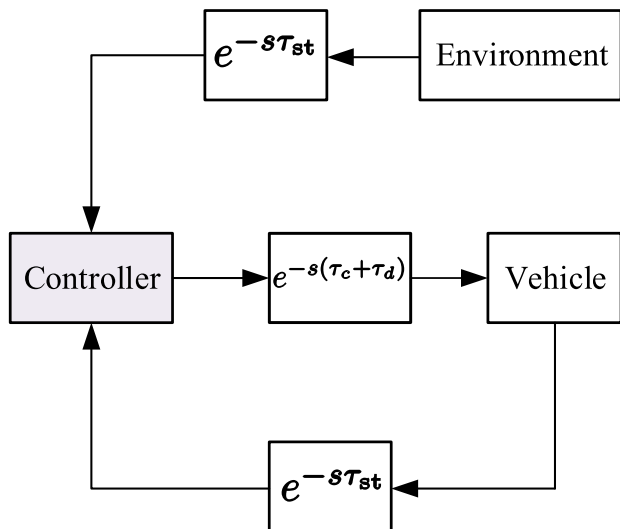


FIGURE 6. The control block diagram.

TABLE 2. Parameters.

Parameter	Value	Parameter	Value
$m$ (kg)	1450	$I_z$ (kg · m <sup>2</sup> )	2740
$l_f$ (m)	1.1	$l_r$ (m)	1.6
$C_f$ (N/rad)	65000	$C_r$ (N/rad)	65000
$C_\delta$ (rad)	1.0	$C_F$ (N)	8000
$\mathbf{u}_{\min}$	$[-1.0, -1.0]^T$	$\mathbf{u}_{\max}$	$[1.0, 1.0]^T$
$\Delta \mathbf{u}_{\min}$	$[-0.2, -0.5]^T$	$\Delta \mathbf{u}_{\max}$	$[0.2, 0.5]^T$
$k_R$	2.0	$D$ (m)	3.5
$\alpha$	1.0	$\sigma$	1.0
$P$	25	$Q$	4
$S$	$\text{diag}(2, 1)$	$R$	10

The automated vehicle runs on a straight road which stretches along with the longitudinal axis of the world coordinate system. The width of each lane of the road is 3.5 [m]. The sample time of the discrete system  $\Delta t = 0.05$  [s]. The total steps of the prediction horizon  $N_p = 20$ . Most of the parameters related to the traffic model, the vehicle model and the proposed controller are listed in Table 2. The constraints of states  $\mathbf{x}_{\min}$  and  $\mathbf{x}_{\max}$  can be derived from the limitations in (50) with other elements limited in a relative large reasonable domain instead of the whole infinite space for the sake of the computational feasibility.

$$\begin{aligned}
 -1.5 \text{ [rad/s]} &\leq \gamma \leq 1.5 \text{ [rad/s]} \\
 -2.0 \text{ [rad]} &\leq \psi \leq 2.0 \text{ [rad]}
 \end{aligned}
 \tag{50}$$

Note that the parameters of the controller are roughly set and fixed for all the simulations in this work. Better performance of the controller may be achieved by finely tuning some parameters.

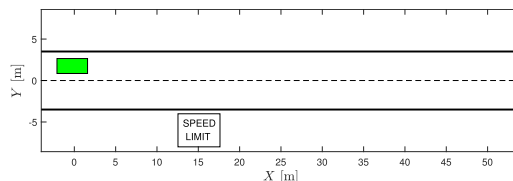


FIGURE 7. Speed control simulation.

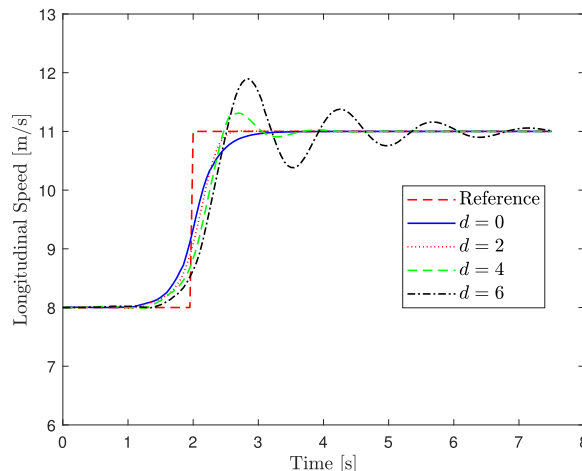


FIGURE 8. Speed control without compensation for different input delays.

### A. COMPENSATING FOR THE INPUT DELAY

In this case, we first investigate the impact of the input delay on the control performance. The AV starts from the center of the second lane (from the right to the left) at an initial longitudinal speed of 8 [m/s] towards to the direction of the X-axis and there are no other vehicles on the road, see Fig. 7. A speed limit sign of 11 [m/s] is posted on the road in front of the automated vehicle. The comprehensive predictive control method without compensations for the input delay is implemented at first to control the vehicle on this road. The performance of the control system with different input time delays is showed in Fig. 8, in which we can see that the trajectory of the longitudinal speed starts to fluctuate as the input delay increases. The larger the input delay is, the more time the controller needs to make the response of the speed converge to the target speed limit.

Then, the method with compensation for the input delay which is introduced in section III-A is utilized to control the automated vehicle with the maximum input delay  $d = 10$  under the same traffic situation. Fig. 9 shows the trajectory of the longitudinal speed generated by the controller with compensation for the input delay along with those without compensation or without input delays. We can see that the fluctuation of the trajectory is very large and the amplitude decreases very slowly in the case  $d = 10$  but without compensation. There is no doubt that the input delay can significantly reduce the passenger comfort and may cause serious traffic accidents if there are other vehicles running ahead of or behind the AV. Fortunately, the method with compensation for the input delay greatly improves the control performance. The trajectory generated by the controller with compensation for

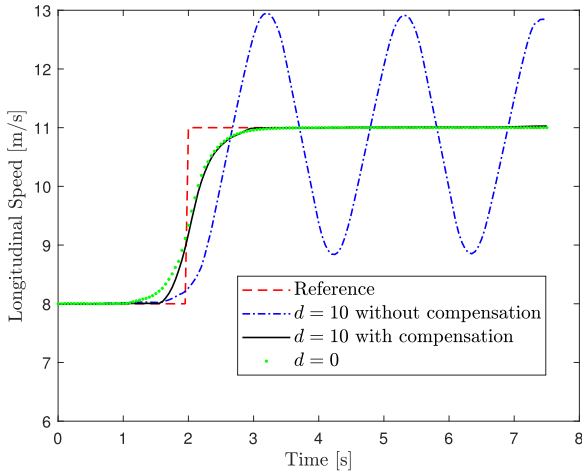


FIGURE 9. Speed control with compensation for the input delay.

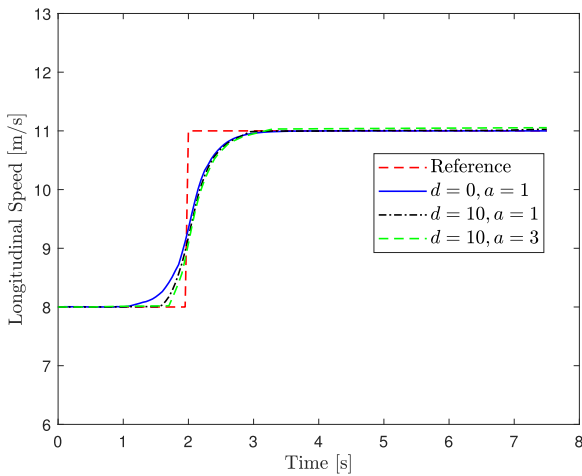


FIGURE 10. Parallel control strategy with different frequency ratios of the actuator to the controller.

the maximum input delay is almost following the trajectory without any input delay. This case shows that the proposed method is capable of handling the maximum input delay in the control system of the automated vehicle.

**B. COMPENSATING FOR THE STATE DELAY**

Fig. 10 presents the performance of the parallel control strategy in which the actuator frequency is several times of the controller frequency. The simulation shows that the controller can still have good performance compared to the case in which the frequencies of the controller and the actuator are the same. But actually, the controller cannot be successfully implemented within such a short cycle which is equal to the sample time of the discrete system  $\Delta t = 0.05$  [s] here. This case proves that we can use this strategy to deploy the control system which requires higher actuation frequency with limit computation source for the controller.

**C. COMPREHENSIVE PREDICTIVE CONTROL WITH COMPENSATION FOR DELAYS**

The initial settings in this scenario are the same as the case in section IV-A, except that there is a fixed vehicle at the

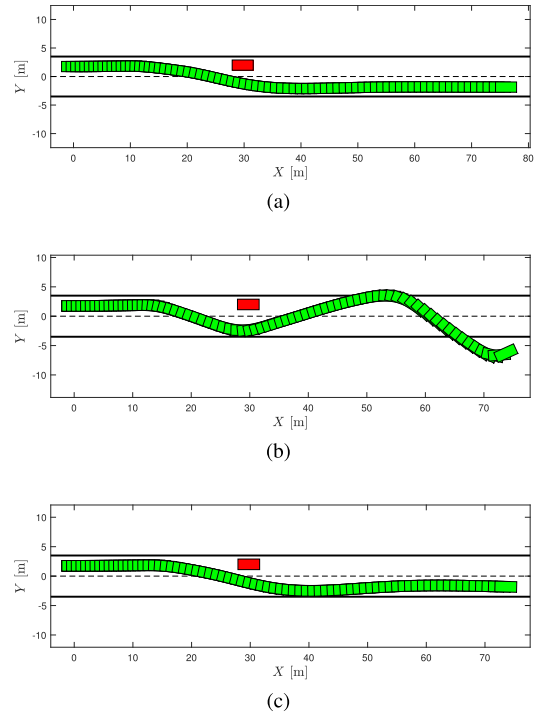


FIGURE 11. Comprehensive predictive control: (a) the input delay  $d = 0$ ; (b) the input delay  $d = 6$ , using the controller without compensation for delays; (c) the input delay  $d = 6$ , using the controller with compensation for delays.

position (30.0, 2.0). The automated vehicle can keep going at the center of its current lane if there is no vehicle ahead. When there is a vehicle ahead which may block its way or cause a collision, the automated vehicle can change to the available adjacent lane without giving any driving decision and any planned trajectory in advance by only using the comprehensive predictive control method [4]. In other words, the automated vehicle controlled with this method is not aware of the idea of decision making and trajectory planning. Fig. 11(a) demonstrates the capability of this advanced method for automated vehicles when there are no delays in the system.

However, serious problems may be caused due to the delays in the system. Fig. 11(b) shows that the automated vehicle rushes out of the road when there is a significant input delay  $d = 6$ , while the speed response is similar to the corresponding trajectory presented in Fig. 8. We can see that, even though the fluctuation of the speed is not quite large, it still may lead to dangerous consequences if there are obstacles around. Interestingly, the trajectory of the automated vehicle is much better if using the controller with compensation for delays, see Fig. 11(c), in which the vehicle changes to the adjacent lane smoothly and keeps running in that lane without conflicting with the road boundaries.

**V. CONCLUSIONS**

Time delays are unavoidable and cannot be ignored in the control system of an automated vehicle which has the essential requirements of safety, robustness, and stability. Thus, a comprehensive predictive control method with

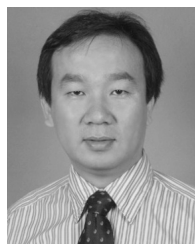
compensation for time delays including the state delay and the control input delay was proposed here for automated vehicles. The traffic environment, vehicle dynamics, and delays in the control loop of an automated vehicle were analyzed to formulate the model of this problem. The comprehensive predictive control method with compensation for delays was then constructed and transformed into an optimization problem. A parallel control scheme was also proposed to directly reduce the time delay in the loop and make the sample time of the discrete system much smaller. Simulations were conducted to show the strengths of this method.

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