IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received May 27, 2019, accepted June 10, 2019, date of publication June 19, 2019, date of current version July 10, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2923789

Sliding Mode Fault Tolerant Tracking Control for a Single-Link Flexible Joint Manipulator System

YONG CHEN, (Senior Member, IEEE), AND BIN GUO
School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China
Institute of Electric Vehicle Driving System and Safety Technology, University of Electronic Science and Technology of China, Chengdu 611731, China

Corresponding author: Yong Chen (ychencd@uestc.edu.cn)

This work was supported in part by the National Key Research and Development Plan Programs of China under Grant 2018YFB0106101, in part by the National Natural Science Foundation of China under Grant 51607027, in part by the Scientific and Technical Supporting Programs of Sichuan Province of China under Grant 2016GZ0395, Grant 2017GZ0394, and Grant 2017GZ0395, and in part by the Fundamental Research Funds for the Central Universities under Grant ZYGX2016J140 and Grant ZYGX2016J146.

ABSTRACT In this paper, the fault-tolerant tracking control problem for a class of single-link flexible joint manipulator (SFJM) system with uncertainty, fault, nonlinear function, and unmatched disturbance is investigated. An observer-based sliding mode control approach is designed. Concretely, first of all, the SFJM dynamic system with uncertainty, fault, and unmatched disturbance is established. Then, by transforming the system into two subsystems, a novel composite observer is proposed to estimate the fault and disturbance, respectively. Furthermore, a robust sliding mode controller, which contains a third-order sliding mode surface, a continuous control strategy, and a visual estimated fault signal, is constructed. In the control scheme, an adaptive law is also concluded to compensate for the estimation error. Finally, the proposed method is applied to the SFJM system and the simulation results illustrate the effectiveness of the proposed method.

INDEX TERMS SFJM, actuator fault, sensor fault, unmatched disturbance, composite observer, fault-tolerant tracking control, sliding mode control.

I. INTRODUCTION

The flexible joint manipulator has become an indispensable part in the increasing automation system especially in the dynamic assembly line, to avoid danger working condition and reduce labor cost [1]. For Flexible joint manipulator system, on the one hand, if the robot has n links, then one need $2n$ generalized coordinates to describe the whole dynamic behavior when taking the joint flexibility into consideration [4]. On the other hand, the flexible joint has the characteristics of nonlinearity and coupling. Therefore, the modeling of a flexible joint is complex. In addition, the fault and disturbance signals exist widely in the practical system due to the flexible structure, which makes the manipulator system more complicated.

Some achievements have been studied about the flexible manipulator system. For example, in [2], a class of flexible joint robots with uncertainty was developed; an adaptive controller was designed to guarantee a high precision position. In [3], the finite-time optimal control approach using the state-dependent Riccati equation was studied for a class of flexible joint manipulator systems. An industrial flexible joint was investigated in [4], where the full state was used to maintain the tracking ability. In [5], the parameter identification technique was adopted for flexible joint system without the angular acceleration information; an adaptive control method was presented. In [6], a full state feedback neural network (NN) control was proposed for a class of robotics with flexible joints, the robustness was enhanced and the stability was guaranteed. In [7], a fractional order sliding mode controller for a flexible link manipulator based on fractional calculus was proposed, by adding an extra degree of freedom, the control performance was guaranteed, the application to a single-link flexible manipulator robot signify the effectiveness of the proposed controller. These achievements have studied the control strategies for flexible manipulator system, however, the fault and external disturbance are not considered. This remains one of the motivations of this study.

It is worth to point out that the full states of the SFJM system are also hard to be obtained because of the high

The associate editor coordinating the review of this manuscript and approving it for publication was Sing Kiong Nguang.

cost and the flexible joint structure, in some conditions, it is impossible to install sensor because of the volume limitation. Observer design becomes a possible way to reconstruct the states and other unknown variables from the input and output information. For example, in [8], an extended state observer (ESO) was proposed to estimate the states and uncertainties for the trajectory tracking control of a flexible-joint robotic system. In [9], a disturbance observer (DO) was designed for a class of two link flexible joint manipulator systems to decouple the joint interactions. These achievements have studied the observer design for flexible joint manipulator system; however, the actuator fault and sensor fault have not been discussed. For fault or disturbance, the observer is also of significance in the controller design framework. Sliding mode observer (SMO), which forces the trajectories to stay on the sliding surface, shows excellent performance, such as disturbance rejection [11]–[14]. For instance, in [11], a class of stochastic nonlinear system was considered; the SMO technique was addressed to estimate the fault and state, a passive fault tolerant control scheme was constructed. An adaptive gain super-twisting sliding mode observer was proposed for fault reconstruction in electro-hydraulic servo systems with unknown bounded perturbations in [12]. For descriptor systems, SMO has also received fruitful results. In [13], the fault tolerant control problem was investigated for a class of Lipschitz nonlinear systems, a descriptor sliding mode observer was presented and the controller was constructed to achieve a satisfactory fault reconstruction performance. In [14], a class of multi-area power systems with sensor fault and disturbance was investigated, by introducing an argument system, a descriptor SMO was constructed for sensor fault reconstruction.

The trajectory tracking performance in the flexible-joint robot is of significant importance when designing the controller. Some works of literature related to the trajectory tracking control of the flexible-joint robot have been developed. In [15], an extended Kalman filter (EKF) observer was presented to estimate the manipulator states, by using these estimated values; an adaptive rigid-link flexible joint controller was proposed to guarantee the tracking performance. In [16], the tracking control problem of flexible-joint robots with unknown dynamics and variable elasticity was addressed; a full state feedback control technique was used to guarantee the tracking error within a neighborhood of zero. In [17], a quadruped robot system with a compliant joint under the perturbing external forces was investigated; the fuzzy approximation method was used to keep the tracking accuracy. In [18], the tracking control problem of a class of flexible joint robot was addressed, the feedback linearization methodology was used to model the system and a sliding mode control method was constructed to guarantee the tracking performance. However, the fault in the output channel is not considered, which motivates the composite observer design of this study.

Motived by [10], [13], [16], this paper addresses the problem of robust fault-tolerant tracking control for SFJM system with actuator fault, sensor fault, external disturbance, and parameter uncertainties. The main contributions are highlighted as follows. 1) The SFJM system with parameter uncertainty, fault, and unmatched disturbance model is established. 2) Different from the results in [17] and [18], where the disturbance and fault in the output channel were not considered. In this study, both the actuator fault and sensor fault are considered. A coordinate transformation is introduced to decompose the system into two subsystems, in which the fault and disturbance are separated. By introducing two new auxiliary variables, a novel composite observer is presented. 3) A novel observer-based sliding mode control scheme is designed to guarantee the tracking performance. In the sliding mode approach, an adaptive law is contained to compensate for the estimation error and enhance the robustness of the system.

The rest of this paper is organized as follows. In section II, the SFJM system model with actuator fault, sensor fault, disturbance, and parameter uncertainty is established. In section III, the construction of the observer and the observer-based controller are designed. In section IV, the proposed method is applied to the SFJM system. Section V concludes the study.

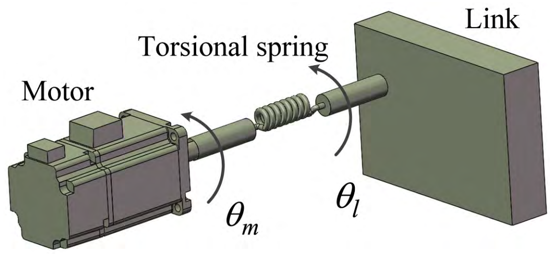

FIGURE 1. Schematic of SFJM system.

II. PROBLEM FORMULATION

A. SFJM SYSTEM NORMAL MODEL

Generally, the SFJM system, which is actuated by a DC motor [1], can be shown in Fig.1. The normal dynamic system can be presented as (1)

$$\begin{cases} \dot{\theta}_m = w_m \\ \dot{w}_m = \dfrac{k}{J_m}(\theta_l - \theta_m) - \dfrac{G}{J_m} w_m + \dfrac{k_\tau}{J_m} u \\ \dot{\theta}_l = w_l \\ \dot{w}_l = -\dfrac{k}{J_l}(\theta_l - \theta_m) - \dfrac{mgq}{J_l}\sin(\theta_l) \end{cases} \tag{1}$$

where J_m is the inertia of the DC motor, J_l is the inertia of the link. θ_m and θ_l denote the rotation angles of the motor and link, respectively. w_m and w_l represent the angular velocities of the motor and link, respectively. k is the torsional spring constant, k_τ is the amplifier gain, G is the viscous friction, m is the pointer mass, g is the gravity constant, q is the distance from the rotor to the center of the gravity of the link, u is the control input delivered by the motor.

B. B SFJM SYSTEM MODEL WITH FAULT, DISTURBANCE AND PARAMETER UNCERTAINTY

Note that the SFJM system is inevitable to suffer from external noise and load torque fluctuation, which means that the external disturbance $d(t)$ exists in the system. For example, when the load torque increases, the mechanical speed will decrease, which in turn affect the angular velocity. In addition, the link is driven by a motor through a torsional spring, which is a nonlinear and coupled process. Thus, the dynamic SFJM system (1) can be regarded as a simplified form of the actual system, which means that the uncertainty may be contained. For example, the spring stiffness may be affected by temperature or humidity, i.e., Δk should be considered in the system [1]. What's more, actuator fault is easy to occur because of the mechanical problems. Consequently, the uncertainty, external unmatched disturbance, and fault should be considered in the scheme of system analysis.

Integrated the above analysis, the dynamic SFJM system (1) can be rewritten as

$$\begin{cases} \dot{\theta}_m = w_m \\ \dot{w}_m = \dfrac{k+\Delta k}{J_m+\Delta J_m}(\theta_l-\theta_m) - \dfrac{G}{J_m+\Delta J_m}w_m \\ +\dfrac{k_\tau}{J_m+\Delta J_m}\left(u+u_f(t)\right) + d_m(t) \\ \dot{\theta}_l = w_l \\ \dot{w}_l = -\dfrac{k+\Delta k}{J_l+\Delta J_l}(\theta_l-\theta_m) - \dfrac{mgq}{J_l+\Delta J_l}\sin(\theta_l) + d_l(t) \end{cases} \tag{2}$$

where $d_i(t)$ $(i=m,l)$ is the external unmatched disturbance, $u_f(t)$ represents the bias actuator fault and $\|u_f(t)\| \leq \alpha_1$, where α_1 is a positive scalar. Δk, ΔJ_m and ΔJ_1 are small fluctuations of the torsional spring constant, motor inertia and link inertia, respectively.

As the motor angle, link angle, motor angular velocity, and link angular velocity can better represent the working condition of the SFJM system, in this paper, the output of the system can be chosen as the four variables. In addition, the sensor fault and disturbance in the output channel are also considered. Let $x(t) = \begin{bmatrix}\theta_m & \theta_l & w_m & w_l\end{bmatrix}^T$, then the flexible joint manipulator system with fault and disturbance can be rewritten in the form of

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + E_m d_m(t) + E_l d_l(t) \\ +Bu_f(t) + M\Delta f(x,t) \\ y = Cx(t) + D_s u_s(t) + D_N d_s(t) \end{cases} \tag{3}$$

where $x(t) \in R^4$ is the state vector, $y(t) \in R^4$ is the output of the system, $u(t) \in R^1$ is the control input, $u_s(t) \in R^1$ is the sensor fault signal, $d_s(t) \in R^1$ is the mismatched external disturbance in the output channel. D_s, D_N are known constant coefficient matrices with appropriate dimensions. The other parameters are given as follows

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k}{J_m} & \dfrac{k}{J_m} & -\dfrac{G}{J_m} & 0 \\ \dfrac{k}{J_1} & -\dfrac{k}{J_1} & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{k_\tau}{J_m} \\ 0 \end{bmatrix}, E_m = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$El = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Delta f(x,t) = \begin{bmatrix} 0 \\ 0 \\ \Delta f_m \\ -\Delta f_l - \dfrac{mgq}{J_l+\Delta J_l}\sin(x_2(t)) \end{bmatrix}.$$

where

$$\Delta f_m = \frac{\Delta k}{J_m}(x_2(t)-x_1(t)) + \Delta J'_m\left[Gx_3(t) + k_\tau\left(u+u_f(t)\right)\right]$$

$$\Delta f_l = \left(\frac{\Delta k}{J_l} + \Delta J'_l(k+\Delta k)\right)(x_2(t)-x_1(t))$$

$$\Delta J'_m = \frac{\Delta J_m}{J_m(J_m+\Delta J_m)}, \Delta J'_l = \frac{\Delta J_l}{J_l(J_l+\Delta J_l)}$$

Similar to [13], by simple simplification, (3) can be rewritten as (4),

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + E_l d_l(t) + B_a f_a(t) \\ +M\Delta f(x,t) \\ y = Cx(t) + D_s u_s(t) + D_N d_s(t) \end{cases} \tag{4}$$

where $B_a = \begin{bmatrix} E_m & B \end{bmatrix}, f_a(t) = \begin{bmatrix} d_m(t) \\ u_f(t) \end{bmatrix} f_a(t) = \begin{bmatrix} d_m(t) \\ u_f(t) \end{bmatrix}$.

Remark1: $d_s(t)$ can be caused by unmeasured outputs and noises in the output channel. $u_s(t)$ can represent the intermittent sensor connection, the bias in sensor measurement, and sensor gain drop, etc. In addition, assume that $\|u_s(t)\| \leq \alpha_2$ and α_2 is a positive constant.

Remark2: The actuator fault, sensor fault, and disturbance are different. From (2) and (3), one can check that the actuator fault and disturbance enters the system in different channels, these factors are unknown, which increases the complexity of the controller design.

Remark3: [13] From (4), one can check that $B_a f_a = E_m \begin{bmatrix} 1 & \frac{k_\tau}{J_m} \end{bmatrix} \begin{bmatrix} d_m(t) \\ u_f(t) \end{bmatrix} = E_m \left[d_m(t) + \frac{k_\tau}{J_m} u_f(t)\right]$.

Remark 4: In (3), one can check that the nonlinear function satisfies $\Delta\tilde{f}^T R\Delta\tilde{f} \leq \tilde{x}^T Q\tilde{x}$, where $\Delta\tilde{f} = \Delta f(x_a, t) - \Delta f(x_b, t)$, $\tilde{x} = x_a - x_b$, R and Q are two positive symmetry matrices [19].

III. MAIN RESULTS

In this part, an observer-based fault tolerant control scheme is designed to keep the tracking performance, which contains three steps. Step one is to design a composite observer to estimate the unknown variables in the system. In step two, by using the estimated values, a proper observer-based sliding mode surface is constructed. A sliding mode controller is given in step three to meet the reaching condition. In step three, the stability is also verified.

A. SYSTEM DECOMPOSITION

Theorem 1: For the SFJM system (4), there exist two transform coordinate matrices ϕ, ψ, such that

$$\phi E_l = \begin{bmatrix} E_2 \\ 0 \end{bmatrix}, \psi C\phi^{-1} = \begin{bmatrix} C_{22} & 0 \\ 0 & C_3 \end{bmatrix}, \quad \phi A\phi^{-1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

Proof: Note that $rank\,(CE_l) = rank\,(E_l) < 4$, one can partition E_l as $E_l = \begin{bmatrix} E_1^T & E_2^T \end{bmatrix}^T$, where $E_1 \in R^{3\times1}$, $E_2 \in R^{1\times1}$ with $rank\,(E_2) = 1$. In this paper, ϕ is computed as $\phi = \phi_2\phi_1$, in which ϕ_1 is constructed directly as

$$\phi_1 = \begin{bmatrix} I_3 & -E_1E_2^{-1} \\ 0 & I_1 \end{bmatrix} \tag{5}$$

then, it can be obtained that

$$\phi_1 E = \begin{bmatrix} I_3 & -E_1E_2^{-1} \\ 0 & I_1 \end{bmatrix}\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_2 \end{bmatrix} \tag{6}$$

In order to simplify the design procedure, partition $C\phi_1^{-1} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$. Therefore, it can be checked that

$$CE = C\phi_1^{-1}\phi_1 E = \begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} 0 \\ E_2 \end{bmatrix} = C_2E_2 \tag{7}$$

As a result, one can directly infer that E_2 is non-singular, and the following conditions hold,

$$rank\,(C_2E_2) = 1 \tag{8}$$

$$rank\,(C_2) = 1 \tag{9}$$

Therefore, a similar approach can be adopted, one can partition $C_2 = \begin{bmatrix} C_{21}^T & C_{22}^T \end{bmatrix}^T$, where $C_{22} \in R^{1\times1}$ and $rank\,(C_{22}) = 1$. Consequently, C_{22} is non-singular, i.e. $\det(C_{22}) \neq 0$. Then $C\phi_1^{-1}$ can be rewritten as $C\phi_1^{-1} = \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix}$, where $C_{11} \in R^{3\times3}$, $C_{12} \in R^{1\times3}$, $C_{21} \in R^{3\times1}$, $C_{22} \in R^{1\times1}$. Construct ψ as

$$\psi = \begin{bmatrix} 0 & I_1 \\ I_3 & -C_{21}C_{22}^{-1} \end{bmatrix} \tag{10}$$

Then, one can check that

$$\psi C\phi_1^{-1} = \begin{bmatrix} 0 & I_1 \\ I_3 & -C_{21}C_{22}^{-1} \end{bmatrix}\begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \begin{bmatrix} C_{12} & C_{22} \\ C_3 & 0 \end{bmatrix} \tag{11}$$

where $C_3 = C_{11} - C_{21}C_{22}^{-1}C_{12}$. Let $\phi_2^{-1} = \begin{bmatrix} 0 & I_3 \\ I_1 & -C_{22}^{-1}C_{12} \end{bmatrix}$, thus the following equation holds

$$\begin{aligned} \psi C\phi^{-1} &= \psi C\phi_1^{-1}\phi_2^{-1} \\ &= \begin{bmatrix} C_{12} & C_{22} \\ C_3 & 0 \end{bmatrix}\begin{bmatrix} 0 & I_3 \\ I_1 & -C_{22}^{-1}C_{12} \end{bmatrix} \\ &= \begin{bmatrix} C_{22} & 0 \\ 0 & C_3 \end{bmatrix} \end{aligned} \tag{12}$$

and one can check that

$$\phi_2 = \begin{bmatrix} C_{22}^{-1}C_{12} & I_1 \\ I_3 & 0 \end{bmatrix} \tag{13}$$

Consequently, from (5), one can obtain that

$$\phi E_l = \phi_2\phi_1 E_l = \begin{bmatrix} C_{22}^{-1}C_{12} & I_1 \\ I_3 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ E_2 \end{bmatrix} = \begin{bmatrix} E_2 \\ 0 \end{bmatrix} \tag{14}$$

Assume D_s and D_N has the following structure $D_N = \begin{bmatrix} C_{21}C_{22}^{-1}D_\delta \\ D_\delta \end{bmatrix}$, $D_s = \begin{bmatrix} D_{s1} \\ 0 \end{bmatrix}$, where $D_\delta \in R^{1\times1}$, $D_{s1} \in R^{3\times1}$. Then it can be deduced that

$$\psi D_N = \begin{bmatrix} 0 & I_1 \\ I_3 & -C_{21}C_{22}^{-1} \end{bmatrix}\begin{bmatrix} C_{21}C_{22}^{-1}D_\delta \\ D_\delta \end{bmatrix} = \begin{bmatrix} D_\delta \\ 0 \end{bmatrix} \tag{15}$$

$$\psi D_s = \begin{bmatrix} 0 & I_1 \\ I_3 & -C_{21}C_{22}^{-1} \end{bmatrix}\begin{bmatrix} D_{s1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ D_{s1} \end{bmatrix} \tag{16}$$

This completes the proof.

Remark 5: In this study, the formulated SFJM system in (3) is a general form. Additionally, the sensor fault and disturbance in the output channel are investigated, which is different from the results in [17], [18], where the output channel fault is not considered. In order to estimate the disturbance and fault separately, the disturbance and fault matrices in the output channel are assumed to satisfy the conditions in (15) and (16). This is the inadequacy of this study, we are spared our efforts to find some new methods without the assumptions in our future study.

From the thorem1, it is easy to check that

$$\begin{aligned} \phi B_a &= \begin{bmatrix} C_{22}^{-1}C_{12} & -C_{22}^{-1}C_{12}E_1E_2^{-1} + I_1 \\ I_3 & -E_1E_2^{-1} \end{bmatrix}\begin{bmatrix} B_{a1} \\ B_{a2} \end{bmatrix} \\ &= \begin{bmatrix} C_{22}^{-1}C_{12}B_{a1} + \left(C_{22}^{-1}C_{12}E_1E_2^{-1} + I_1\right)B_{a2} \\ B_{a1} - E_1E_2^{-1}B_{a2} \end{bmatrix} \end{aligned} \tag{17}$$

where $B_{a1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & \frac{k_\tau}{J_m} \end{bmatrix}^T$, $B_{a2} = \begin{bmatrix} 0 & 0 \end{bmatrix}$.

From the definition of C_{12}, E_1 and B_{a2}, one can check that C_{12} and E_1 are two zero matrices, then (17) can be reduced to $\phi B_a = \begin{bmatrix} 0 & B_{a1}^T \end{bmatrix}^T$.

Similarly, one can check that

$$\phi B=\begin{bmatrix}0\\B_1\end{bmatrix},\phi M=\begin{bmatrix}M_2\\M_1\end{bmatrix},B_1=\begin{bmatrix}0 & 0 & \frac{k_\tau}{J_m}T\end{bmatrix}^T,$$

$$M_1=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix},M_2=\begin{bmatrix}0&0&0&1\end{bmatrix}.$$

Define $z=\phi x=\phi\begin{bmatrix}x_1\\x_2\end{bmatrix}$, $s=\psi y=\psi\begin{bmatrix}y_1\\y_2\end{bmatrix}$, from theorem1, (4) can be rewritten as the following two subsystems

$$\begin{cases}\dot z_1=A_1z_1+A_2z_2+E_2d_l+M_2\Delta f\left(\phi^{-1}z,t\right)\\ s_1=C_{22}z_1+D_\delta d_s\end{cases}\tag{18}$$

$$\begin{cases}\dot z_2=A_3z_1+A_4z_2+B_1u+B_{a1}f_a+M_1\Delta f\left(\phi^{-1}z,t\right)\\ s_2=C_3z_2+D_{s1}u_s\end{cases}\tag{19}$$

where $s_1\in R^1, M_1\in R^{3\times4}, B_{a1}\in R^{3\times2}, s_2\in R^3, z_1\in R^1, z_2\in R^3, M_2\in R^{1\times4}, B_1\in R^3, A_i(i=1,\ldots,4)$ are defined in theorem1.

Remark 6: It can be seen that (18) has the effects of disturbances without any faults and (19) includes faults but free from disturbance.

Remark 7: In [14], a descriptor system was constructed in order to estimate disturbance and fault, however, only sensor and output disturbance are considered. In [20], the ESO technique was designed for the disturbance and fault estimation, however, the fault and disturbance cannot be decoupled. In [21], the unknown input observer (UIO) was designed and the disturbance was extracted from the argument state; by proper assumption, the disturbance was partial decoupled. Compared with these results, in this paper, the actuator fault, sensor fault, disturbance in the input and output channels are considered, by defining a virtual actuator fault, a partial disturbance is decoupled.

B. B OBSERVER DESIGN

In this subsection, a composite observer is proposed for (18) and (19). Before designing the observer, two auxiliary variables are introduced. Define $\dot s_3=C_{22}z_1+D_\delta d_s$, $s_p=s_3$ and $z_p=\begin{bmatrix}z_1^T & s_3^T\end{bmatrix}^T$, then the following equation holds

$$\begin{cases}\dot z_p=A_pz_p+A_{2p}z_q+E_pd_p+\overline{M}_2\Delta f\left(\phi^{-1}z,t\right)\\ s_p=C_pz_p\end{cases}\tag{20}$$

where $z_p\in R^2, s_p\in R^1, A_p=\begin{bmatrix}A_1&0\\C_{22}&0\end{bmatrix}, d_p=\begin{bmatrix}d_l\\d_s\end{bmatrix}$, $A_{2p}=\begin{bmatrix}A_2&0\\0&0\end{bmatrix}, E_p=\begin{bmatrix}E_2&0\\0&D_\delta\end{bmatrix}, \overline{M}_2=\begin{bmatrix}M_2\\0\end{bmatrix}, C_p=\begin{bmatrix}0 & I_1\end{bmatrix}$, z_q is a vector to be designed later.

Analogously, define $\dot z_3=C_3z_2+D_{s1}u_s$, $s_q=z_3$ and $z_q=\begin{bmatrix}z_2^T & z_3^T\end{bmatrix}^T$. then a new argument system is formulated as

$$\begin{cases}\dot z_q=A_qz_q+A_{1q}z_p+B_qu+B_ff_q+\overline{M}_1\Delta f\left(\phi^{-1}z,t\right)\\ s_q=C_qz_q\end{cases}\tag{21}$$

where $z_q\in R^6, s_q\in R^3, A_q=\begin{bmatrix}A_4&0\\C_3&0\end{bmatrix}, A_{1q}=\begin{bmatrix}A_3&0\\0&0\end{bmatrix}$, $B_f=\begin{bmatrix}B_{a1}&0\\0&D_{s1}\end{bmatrix}, B_q=\begin{bmatrix}B_1\\0\end{bmatrix}, f_q=\begin{bmatrix}f_a\\u_s\end{bmatrix}, \overline{M}_1=\begin{bmatrix}M_1\\0\end{bmatrix}$, $C_q=\begin{bmatrix}0 & I_3\end{bmatrix}$. Before the observer design, the following assumption is needed

Assumption 1 [22]: Assume that the derivative of the disturbance signals d_l $d_l(t), d_m(t)$ and $d_s(t)$ in the system are bounded, i.e., $\left\|\dot d_l(t)\right\|\le\varpi_1$, $\left\|\dot d_m(t)\right\|\le\varpi_2$, $\left\|\dot d_s(t)\right\|\le\varpi_3$, where ϖ_1, ϖ_2 and ϖ_2 are three positive constants.

Remark 8: From (4) and assumption 1, one can check that there exist two positive constants f^* and d^*, such that $\|f_q\|\le f^*$ and $\|dp\|\le d^*$.

Then, the composite observer for subsystems (20) and (21) can be designed as (22) and (23)

$$\begin{cases}\dot{\hat z}_p=A_p\hat z_p+A_{2p}\hat z_q+E_p\hat d_p+\overline{M}_2\Delta f\left(\phi^{-1}\hat z,t\right)\\ +H_1\left(s_p-\hat s_p\right)\\ \hat s_p=C_p\hat z_p\\ \hat d_p=g+Ls_p\\ \dot v=-LC_pE_pg-LC_pE_pLs_p-LC_pA_p\hat z_p\\ -LC_p\left(A_{2p}\hat z_q+\overline{M}_1\Delta f\left(\phi^{-1}\hat z,t\right)\right)+\delta(t)\end{cases}\tag{22}$$

$$\begin{cases}\dot{\hat z}_q=A_q\hat z_q+A_{1q}\hat z_p+B_f\hat f_q+H_2\left(s_q-\hat s_q\right)+B_qu\\ +\overline{M}_1\Delta f\left(\phi^{-1}\hat z,t\right)\\ \hat s_q=C_q\hat z_q\end{cases}\tag{23}$$

where $\hat z_p,\hat z_q,\hat s_p,\hat s_q,\hat d_p$ and $\hat f_q$ are the estimations of z_p, z_q, s_p, s_q, d_p and f_q, v is an intermediate state, H_1, H_2 and L are three observer gains need to be designed, $\delta(t)$ is a compensator. $\hat f_q$ and $\delta(t)$ are given by

$$\hat f_q=\frac{B_f^TP_ae_q}{\left\|e_q^TP_aB_f\right\|}\left(f^*+\tau\right)\tag{24}$$

$$\delta(t)=\frac{P_w^Te_w}{\left\|e_w^TP_w\right\|}\left(d^*+w_\delta\right)\tag{25}$$

where P_a and P_w are two positive symmetry matrices, τ and w_δ are two positive scalars. $e_q=z_q-\hat z_q$ and $e_w=d_p-\hat d_p$

Remark 9: From the definitions of A, B and C of the SFJM system in (3), one can check that

$$rank\begin{bmatrix}\lambda I-A & B\\ C & 0\end{bmatrix}=rank\,(B)+4\tag{26}$$

where λ is any positive scalar. Then one can obtain that the detectable condition is satisfied.

C. STABILITY ANALYSIS

Define $e_p=z_p-\hat z_p$, then the error systems can be obtained in the form of

$$\dot e_p=\left(A_p-H_1C_p\right)e_p+A_{2p}e_q+E_pe_d+\overline{M}_2\Delta\tilde f\tag{27}$$

$$\dot e_q=\left(A_q-H_2C_q\right)e_q+A_{1q}e_p+B_f\left(f_q-\hat f_q\right)+\overline{M}_1\Delta\tilde f\tag{28}$$

$$\begin{aligned}\dot e_w=&-LC_pE_pe_w+\dot d_p-LC_pA_pe_p-\delta(t)\\&-LC_pA_{2p}e_q-LC_p\overline{M}_1\Delta\tilde f\end{aligned}\tag{29}$$

where $\Delta\tilde f=\Delta f\left(\phi^{-1}z,t\right)-\Delta f\left(\phi^{-1}\hat z,t\right)$.

Define $e = \left[\, e_p^T \;\; e_q^T \,\right]$, one can obtain that

$$\dot{e} = A_e e + B_e\left(f_q - \hat{f}_q\right) + E_e e_w + M_e \Delta\tilde{f} \tag{30}$$

where

$$A_e = \begin{bmatrix} A_p - H_1 C_p & A_{2p} \\ A_{1q} & A_q - H_2 C_q \end{bmatrix}, B_e = \begin{bmatrix} 0 \\ B_f \end{bmatrix},$$
$$E_e = \begin{bmatrix} E_p \\ 0 \end{bmatrix}, M_e = \begin{bmatrix} \overline{M}_2 \\ \overline{M}_1 \end{bmatrix}.$$

Theorem 2: With assumption 1 and the observer (22) and (23) for the SFJM system (4), the dynamic error system(29) and (30) are stable if there exist two positive symmetry matrices P, P_w, , and matrices Q, Q_1, R and R_1 such that the following (31) holds.

$$\begin{bmatrix} \Xi_1 & -\Gamma^T P_w & PM_e & 0 \\ * & \Xi_{w1} & 0 & P_w LC_p \overline{M}_1 \\ * & * & -R & 0 \\ * & * & * & -R_1 \end{bmatrix} < 0 \tag{31}$$

where

$$\begin{aligned} \Xi_1 &= PA_e + A_e^T P + Q + Q_1, \\ \Gamma &= \left[\, LC_p A_p \quad LC_p A_{2p} \,\right], \\ \Xi_{w1} &= -P_w LC_p E_p - E_p^T C_p^T L^T P_w. \end{aligned}$$

Proof: Consider the following Lyapunov function as

$$V = e^T P e + e_w^T P_w e_w \tag{32}$$

where $P = \begin{bmatrix} P_m & 0 \\ 0 & P_a \end{bmatrix}$, P_m and P_a are two positive matrices.

The derivative of (32) can be obtained as

$$\begin{aligned} \dot{V} = &\, 2e^T P\left(A_e e + B_e\left(f_q - \hat{f}_q\right) + M_e \Delta\tilde{f} + E_e e_w\right) \\ &+ 2e_w^T P_w\left(-LC_p E_p e_w + \dot{d}_p - LC_p A_p e_p\right) \\ &- 2e_w^T P_w\left(LC_p A_{2p} e_q + LC_p \overline{M}_1 \Delta\tilde{f}\right) - 2e_w^T P_w \delta(t) \end{aligned} \tag{33}$$

From (24), it follows that

$$\begin{aligned} & e^T P B_e\left(f_q - \hat{f}_q\right) \\ &= e_q^T P_a B_f f_q - e_q^T P_a B_f \hat{f}_q \\ &\le \left\| e_q^T P_a B_f f_q \right\| - e_q^T P B_f \frac{B_f^T P_a e_q}{\left\| e_q^T P_a B_f \right\|}\left(f^* + \tau\right) \\ &= -\tau \left\| e_q^T P_a B_f \right\| < 0 \end{aligned} \tag{34}$$

From (25), one can obtain that

$$\begin{aligned} & e_w^T P_w\left(\dot{d}_p + \delta(t)\right) \\ &= e_w^T P_w \dot{d}_p - e_w^T P_w \delta(t) \\ &\le \left\| e_w^T P_w \right\| \left\| \dot{d}_p \right\| - e_w^T P_w \frac{P_w e_w}{\left\| e_w^T P_w \right\|}\left(d^* + w_\delta\right) \\ &= -w_\delta \left\| e_w^T P_w \right\| < 0 \end{aligned} \tag{35}$$

Note that

$$\begin{aligned} 2e^T PM_e \Delta\tilde{f} &\le e^T PM_e R^{-1} M_e^T Pe + \Delta\tilde{f}^T R \Delta\tilde{f} \\ &\le e^T PM_e R^{-1} M_e^T Pe + e^T Qe \end{aligned} \tag{36}$$

$$\begin{aligned} & 2e_w^T P_w LC_p \overline{M}_1 \Delta\tilde{f} \\ &\le e_w^T P_w LC_p \overline{M}_1 R_1^{-1} \overline{M}_1^T C_p^T L^T P_w e_w + \Delta\tilde{f}^T R_1 \Delta\tilde{f} \\ &\le e_w^T P_w LC_p \overline{M}_1 R_1^{-1} \overline{M}_1^T C_p^T L^T P_w e_w + e^T Q_1 e \end{aligned} \tag{37}$$

Consequently, (33) can be rewritten as

$$\begin{aligned} \dot{V} &\le e^T \Xi e + e_w^T \Xi_w e_w + 2e_w^T P_w \Gamma e \\ &= \left[\, e^T \quad e_w^T \,\right] \begin{bmatrix} \Xi & -\Gamma^T P_w + PE_e \\ * & \Xi_w \end{bmatrix} \begin{bmatrix} e \\ e_w \end{bmatrix} \end{aligned} \tag{38}$$

where $\Gamma = \left[\, LC_p A_p \quad LC_p A_{2p} \,\right]$,

$$\begin{aligned} \Xi &= PA_e + A_e^T P + PM_e R^{-1} M_e^T P + Q + Q_1, \\ \Xi_w &= -P_w LC_p E_p - E_p^T C_p^T L^T P_w \\ &\quad + P_w LC_p \overline{M}_1 R_1^{-1} \overline{M}_1^T C_p^T L^T P_w. \end{aligned}$$

According to the Schur complement, it follows that the estimation error will converge to zero if (31) holds. The proof is completed.

Remark 10: From the condition (31), it is a sufficient condition of the convergence of the estimation error, and the matrix may not unique, the optimal matrix will help to increase the estimation performance, and this will extend to our future work.

D. FAULT RECONSTRUCTION

In this part, the virtual actuator fault, sensor fault of the SFJM system will be reconstructed through the sliding mode technology.

Define the sliding mode surface as follows

$$\mathsf{S} = \left\{\mathsf{S}(t) \,\middle|\, \mathsf{S}(t) = e_q\right\} \tag{39}$$

From theorem 1, it can be deduced that there exist two scalars μ and μ_1 , such that $\|e\| \le \mu and \left\|\Delta\tilde{f}\right\| \le \mu_1$.

Theorem 3: With the observer (22)-(23), for two given positive matrices R_a and Q_a , the sliding mode motion in (39) will take place on the hyper plane S if the parameter τ satisfies the following condition

$$\begin{aligned} \tau > \left\|B_f\right\|^{-1}\Big(&\left\|A_{1\,q}\right\| + \frac{1}{2}\left\|P_a^{-1} Q_a\right\| \mu \\ &+ \frac{1}{2}\left\|\overline{M}_1 R_a^{-1} \overline{M}_1^T P_a\right\| \mu + 1\Big) \end{aligned}$$

Proof: Consider the Lyapunov function $V_1 = e_q^T P_a e_q$, where P_a comes from the component of P in the theorem 1. Then it can be obtained that

$$\begin{aligned} \dot{V}_1 = &\, 2e_q^T P_a\left(\left(A_q - H_2 C_q\right) e_q + B_f\left(f_q - \hat{f}_q\right)\right) \\ &+ 2e_a^T P_a\left(\overline{M}_1 \Delta\tilde{f} + A_{1q} e_p\right) \end{aligned} \tag{40}$$

Due to $2e_q^T P_a (A_q - H_2 C_q) e_q$ is a part of Ξ, it can be obtained that $2e_q^T P_a (A_q - H_2 C_q) e_q < 0$, then (40) is equal to (41)

$$\begin{aligned}\dot{V}_1 &\le 2e_q^T P_a \left(A_{1q} e_p + B_f \left(f_q - \hat{f}_q\right)\right) + 2e_q^T P_a \overline{M}_1 \Delta\tilde{f} \\ &\le \left\|2e_q^T P_a\right\| \left\|A_{1q} e_p\right\| + 2e_q^T P_a B_f \left(f_q - \hat{f}_q\right) \\ &\quad + \left\|e_q^T P_a\right\| \left(\left\|\overline{M}_1 R_a^{-1} \overline{M}_1^T P_a e_q\right\| + \left\|P_a^{-1} Q_a e_q\right\|\right) \\ &\le \left\|2e_q^T P_a\right\| \left\{\left\|A_{1q}\right\| \mu + \frac{1}{2}\left\|P_a^{-1} Q_a\right\| \mu \right. \\ &\quad \left. + \frac{1}{2}\left\|\overline{M}_1 R_a^{-1} \overline{M}_1^T P_a\right\| \mu\right\} - \left\|2e_a^T P_a\right\| \left\|B_f\right\| \tau \end{aligned} \tag{41}$$

then (41) can be rewritten as

$$\begin{aligned}\dot{V}_1 &\le -\left\|2e_a^T P_a\right\| \left\{\left\|B_f\right\| \varpi - \left\{\left\|A_{1q}\right\| + \frac{1}{2}\left\|P_a^{-1} Q_a\right\| \right.\right. \\ &\quad \left.\left. + \frac{1}{2}\left\|\overline{M}_1 R_a^{-1} \overline{M}_1^T P_a\right\|\right\} \mu\right\} \\ &\le -\left\|2e_a^T P_a\right\| \le -2\sqrt{\lambda_{\min}(P_a)}\sqrt{V_1} \end{aligned} \tag{42}$$

This shows the reachability is satisfied and the sliding motion will take place in finite time. This completes the proof.

Consequently, when the sliding motion takes place on S, the error dynamic system can be rewritten as

$$\left(A_q - H_2 C_q\right) e_q + A_{1q} e_p + B_f \left(f_q - \hat{f}_q\right) + \overline{M}_1 \Delta\tilde{f} = 0 \tag{43}$$

Theorem 4: With the Theorem 1 and Theorem 2, the following equation holds

$$f_q = \frac{B_f^T P_a e_q}{\left\|e_q^T P_a B_f\right\|}\left(f^* + \tau\right) \tag{44}$$

Proof: From the SFJM system (4) and (43), it can be obtained that

$$f_q - \hat{f}_q = B_f^{\dagger}\left(\left(A_q - H_2 C_q\right) e_q + A_{1q} e_p + \overline{M}_1 \Delta\tilde{f}\right) \tag{45}$$

where $B_f^{\dagger} = \left(B_f^T B_f\right)^{-1} B_f^T$.

Then it follows that

$$\begin{aligned} f_q - \hat{f}_q &\le \left\|B_f^{\dagger}\right\| \left(\left\|A_q - H_2 C_q\right\| \mu + \left\|A_{1q}\right\| e_p + \overline{M}_1 \Delta\tilde{f}\right) \\ &\le \left\|B_f^{\dagger}\right\| \left(\left\|A_q - H_2 C_q\right\| \mu + \left\|A_{1q}\right\| \mu + \overline{M}_2 \mu_1\right) \\ &\le \vartheta \end{aligned} \tag{46}$$

where

$$\vartheta \ge \left\|B_f^{\dagger}\right\| \left(\left\|A_q - H_2 C_q\right\| \mu + \left\|A_{1\,q}\right\| \mu + \overline{M}_2 \mu_1\right)$$

From theorem 1, one can deduce that ϑ is small enough, then (44) holds. This completes the proof.

Remark 11: It is worth noting that (46) do not reconstruct f_q precisely due to the nonlinear function, in order to overcome the discontinuous problem, (44) can also be designed as

$$f_q = \frac{B_f^T P_a e_q}{\left\|e_q^T P_a B_f\right\| + \alpha_c}\left(f^* + \tau\right) \tag{47}$$

where α_c is any small positive scalar. Note that the two parameters α_c and τ are important, one can adjust these two parameters so that the reconstructed signal f_q achieves satisfactory accuracy.

Remark 12: It is worthing to point out that (24) is a part of the observer, i.e., the disturbance is estimated by the observer (22) and (23), and the fault signal can be obtained by (24). Simultaneously and separately estimating the faults and disturbances is one of the characteristics of this study. Based on the estimated information, the fault and disturbance compensators can be designed to compensate for the influences of the two factors, respectively, this provides the flexibility to design the controller.

E. FAULT TOLERANT CONTROLLER DESIGN

In this part, the main purpose is to design a robust fault tolerant controller to track the desired trajectories.

Define the following tracking errors $e_1 = \hat{z}_p - z_{pr}$, $e_2 = \hat{z}_q - z_{qr}$, where z_{pr} and z_{qr} are the reference signals. And define the following variable

$$s(t) = \lambda_1 e_1 + \lambda_2 e_2 \tag{48}$$

where $\lambda_1 = \left[\lambda_{11}\ \lambda_{12}\right]$, and $\lambda_2 = [\lambda_{21}, \ldots, \lambda_{26}]$ are two matrices.

The sliding mode surface is designed as

$$\sigma(t) = \beta_1 s(t) + \beta_2 \dot{s}(t) + \beta_3 \int s(t)dt - \beta_2 \lambda_2 B_f \hat{f}_q \tag{49}$$

where β_1, β_2 and β_3 are three positive parameters.

From (22), (23) and (48), (49) can be rewritten as

$$\begin{aligned}\sigma(t) &= \beta_1 s(t) + \beta_2 \dot{s}(t) + \beta_3 \int s(t)dt \\ &= \beta_1 \lambda_1 e_1 + \beta_1 \lambda_2 e_2 + \beta_2 \lambda_1 \dot{e}_1 + \beta_3 \int (\lambda_1 e_1 + \lambda_2 e_2)dt \\ &\quad + \beta_2 \lambda_2 \left(A_q \hat{z}_q + A_{1q} \hat{z}_p + B_f \hat{f}_q + \overline{M}_1 \Delta f\left(\phi^{-1}\hat{z}, t\right)\right) \\ &\quad - \beta_2 \lambda_2 \dot{z}_{qr} + \beta_2 \lambda_2 B_q u + \beta_2 \lambda_2 H_2 C_q e_q - \beta_2 \lambda_2 B_f \hat{f}_q \end{aligned} \tag{50}$$

Remark 13: Different from results in the method in [18], where the derivation information is needed in the sliding mode surface, which may cause instability of the system. Without the acceleration signal, in this study, only the tracking error information is needed, thus the computation complexity is reduced. Additionally, the estimated fault is utilized to design the sliding mode surface to enhance the robustness of the proposed method.

From theorem 1, $\|e\| \le \mu$, it can be assumed that $\left\|\frac{d}{dt}\left(\beta_2 \lambda_2 H_2 C_q e_q\right)\right\| \le k_\xi$, where $k_\xi > 0$ is a constant.

In this paper, the controller is designed as

$$\begin{aligned} u = &-\left(\beta_2 \lambda_2 B_q\right)^{-1} \{\beta_1 \lambda_1 e_1 + \beta_1 \lambda_2 e_2 + \beta_2 \lambda_1 \dot{e}_1 \\ &+ \beta_3 \int (\lambda_1 e_1 + \lambda_2 e_2)dt - \beta_2 \lambda_2 \dot{z}_{qr} - u_n \\ &+ \beta_2 \lambda_2 \left(A_q \hat{z}_q + A_{1q} \hat{z}_p + B_f \hat{f}_q + \overline{M}_1 \Delta f\left(\phi^{-1}\hat{z}, t\right)\right)\} \end{aligned} \tag{51}$$

$$\dot{u}_n = -\left(\hat{k}_\xi + \eta\right) sign(\sigma(t)) - k_a\sigma(t) \tag{52}$$

$$\dot{\hat{k}}_\xi = \gamma\left|\sigma(t)\right| \tag{53}$$

The following theorem gives the convergence of tracking performance.

Theorem 5: For the system (20) and (21), with the controller in (51-53), the convergence of the tracking error to a neighborhood of zero can be guaranteed.

Proof: The Lyapunov candidate function is designed as

$$V_2(t) = \frac{1}{2}\sigma(t)^T\sigma(t) + \frac{1}{2\gamma}\left(k_\xi - \hat{k}_\xi\right)^2 \tag{54}$$

From the definition of (52) and controller (53), one has

$$\sigma(t) = u_n + \beta_2\lambda_2 H_2 C_q e_q \tag{55}$$

The derivative of (54) can be obtained as

$$\begin{aligned}\dot{V}_2(t) &= \sigma(t)^T\dot{\sigma}(t) - \left(k_\xi - \hat{k}_\xi\right)\left|\sigma(t)\right| \\ &= \sigma(t)^T\left(-\left(\hat{k}_\xi + \eta\right) sign(\sigma(t)) - k_a\sigma(t)\right) \\ &\quad + \sigma(t)^T\frac{d}{dt}\left(\beta_2\lambda_2 H_2 C_q e_q\right) - \left(k_\xi - \hat{k}_\xi\right)\left|\sigma(t)\right| \\ &\le \left|\sigma(t)^T\right| k_\xi - \sigma(t)^T\left(\hat{k}_\xi + \eta\right) sign(\sigma(t)) \\ &\quad - k_a\sigma(t)^T\sigma(t) - \left(k_\xi - \hat{k}_\xi\right)\left|\sigma(t)\right| \\ &\le -k_a\left\|\sigma(t)^T\right\|^2 - \eta\left|\sigma(t)\right| \end{aligned} \tag{56}$$

According to the Lyapunov criterion, the stability of the tracking error is guaranteed even in the presence of disturbances and faults. This completes the proof.

Remark 14: From [23], one can check that the estimation of k_ξ is bounded, then the overestimation problem can be avoided. In addition, in order to overcome the parameter drift case, the adaptive law (53) can be reconfigured as

$$\dot{\hat{k}}_\xi = \begin{cases}\gamma\left|\sigma(t)\right| & (\left|\sigma(t)\right| > \rho) \\ 0 & (\left|\sigma(t)\right| < \rho)\end{cases} \tag{57}$$

where ρ is a parameter should be chosen such that the desired tracking performance can be obtained.

Remark 15: From (52), it can be observed the function $sign(\sigma(t))$ is contained in the controller, one can use a sigmoid function or saturation function to replace it, however, the accuracy and response speed may be decreased.

Remark 16: It is also noting that the proposed method is not only suitable for the SFJM system, it can also be applied to the general manipulator system such as the investigated results in [9] and [10], the scalability of this method is also one of the highlights of this study.

IV. RESULTS ANALYSIS

In this section, the performance of the proposed controller is verified for the SFJM system. The parameters of the link robot are given as follows [5] From the definition of the SFJM system (3), one has

TABLE 1. Manipulator parameters.

Symbol	System parameters	Values
J_m	Motor inertia	3.7×10^{-3} kg.m^2
J_l	Link inertia	9.3×10^{-3} kg.m^2
m	Pointer mass	2.1×10^{-1} kg
b	Link length	3.1×10^{-1} m
k	Torsional spring constant	1.8×10^{-1}Nm.rad^{-1}
G	Viscous friction coefficient	4.6×10^{-2} Nm.V^{-1}
k_τ	Amplifier gain	8×10^{-2} Nm.V^{-1}

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -48.6 & 48.6 & -1.25 & 0 \\ 1.95 & -1.95 & 0 & 0 \end{bmatrix},$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 21.6 \\ 0 \end{bmatrix}, E_m = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, E_l = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$D_N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, D_s = \begin{bmatrix} 1 \\ -2 \\ 0.5 \\ 0 \end{bmatrix}, B_a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 21.6 \\ 0 & 0 \end{bmatrix}.$$

From (5), (10) and (13), ϕ and ψ can be computed as

$$\phi = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \psi = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Then one has

$$\phi M = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \phi B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 21.6 \end{bmatrix},$$

$$\psi D_s = \begin{bmatrix} 0 \\ 1 \\ -2 \\ 0.5 \end{bmatrix},$$

$$\phi A\phi^{-1} = \left[\begin{array}{c|ccc} 0 & 1.9500 & -1.9500 & 0 \\ \hline 0 & 0 & 0 & 1.000 \\ 1.000 & 0 & 0 & 0 \\ 0 & -48.6000 & 48.6000 & -1.2500 \end{array}\right],$$

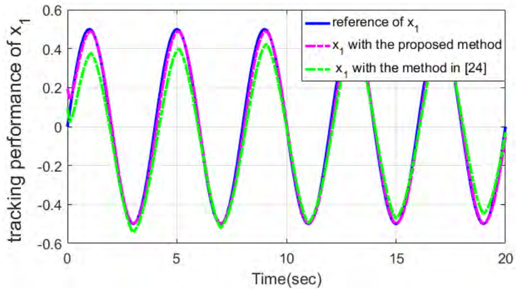

FIGURE 2. Responses of $x_1(t)$ with two methods.

$$\phi E_l = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \psi D_N = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$
$$\psi C \phi^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{bmatrix}.$$

The parameters are given as follows $\varpi_\delta = 1$, $w_\delta = 1$, $\mu = 1$, $\beta_1 = 0.5$, $\beta_2 = 0.45$, $\beta_3 = 0.5$.

Case 1: Fault and disturbance-free case.

In this case, the normal model is considered, in order to illustrate the effectiveness of the proposed method, the method in [24] is also take into consideration. The desired references are given as

$$\theta_{mr} = 0.5 \sin(0.5\pi t) \ (0 \le t \le 20),$$
$$\theta_{lr} = \begin{cases} 0.6 \sin(0.5\pi t) & (0 \le t < 8) \\ 0.4 \sin(0.5\pi t) & (8 \le t \le 20) \end{cases},$$
$$w_{mr} = -0.25\pi \sin(0.5\pi t) \ \ (0 \le t \le 20),$$
$$w_{lr} = \begin{cases} 0.3\pi \cos(0.5\pi t) & (0 \le t < 8) \\ 0.2\pi \cos(0.5\pi t) & (8 \le t \le 20) \end{cases}.$$

By solving (31), the parameter matrices are obtained as P_a, H_1, P_m, P_w H_2, and L are shown at the bottom of this page.

The simulations are shown as follows

Figs. 2-3 show the response of motor angle and link angle with the proposed method and the method in [24].

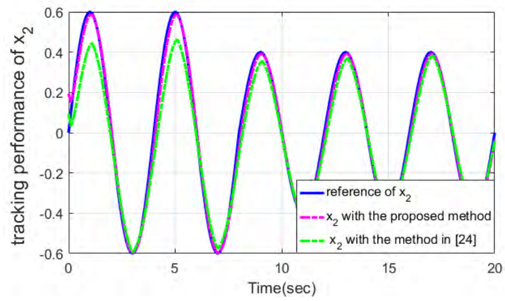

FIGURE 3. Responses of $x_2(t)$ with two methods.

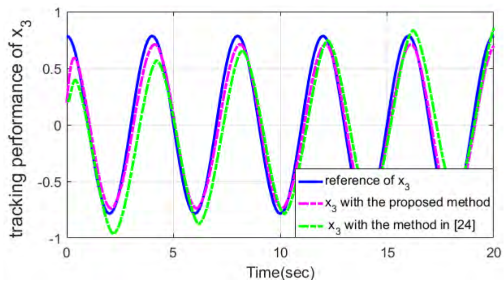

FIGURE 4. Responses of $x_3(t)$ with two methods.

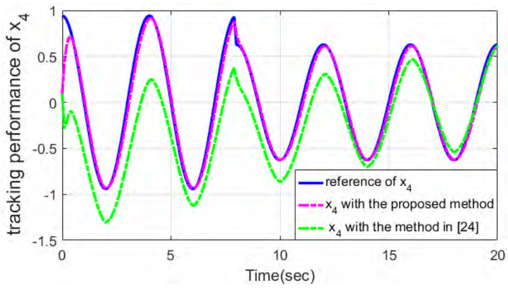

FIGURE 5. Responses of $x_4(t)$ with two methods.

Figs. 4-5 illustrate the motor angular velocity and link angular velocity with the two methods. From the Figs. 2-3, it can be observed that the states response faster than the method in [24]. The reason can be explained as follows, the proposed method conclude the integration of the tracking error,

$$P_a = \begin{bmatrix} 1.7679 & 0.0411 & -0.4405 & -0.9936 & -0.0487 & -0.8086 \\ 0.0411 & 0.0418 & -0.0418 & 0.0561 & -0.5110 & -0.0305 \\ -0.4405 & -0.0418 & 1.8637 & -0.7566 & -1.0038 & -0.9244 \\ -0.9936 & 0.0561 & -0.7566 & 3.8561 & 0.2808 & 1.0039 \\ -0.0487 & -0.5110 & -1.0038 & 0.2808 & 8.4776 & 1.1102 \\ -0.8086 & -0.0305 & -0.9244 & 1.0039 & 1.1102 & 3.8593 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 6.7407 \\ 3.4757 \end{bmatrix}, P_m = \begin{bmatrix} 1.5463 & -1.8281 \\ -1.8281 & 4.8055 \end{bmatrix}, P_w = 9.5154,$$
$$H_2 = \begin{bmatrix} 8.8956 & 3.2595 & -56.3453 & 3.3579 & 0.9777 & -2.2376 \\ 5.5479 & 5.0979 & 26.1486 & 1.2821 & 3.0946 & 0.7931 \\ -13.3484 & 8.2420 & 459.1866 & 3.0946 & 3.6586 & 17.7683 \end{bmatrix}^T,$$
$$L = 1.2.$$

which contributes the convergence performances. Seen from Figs. 4-5, the tracking performance is also better than the method in [24]. The simulation results show the capacity and feasibility of the proposed method.

Case 2: SFJM system with faults, disturbances and parameter uncertainties

In this case, the actuator fault, sensor fault, unmatched disturbance, and parameter uncertainties are considered. Additionally, the faults and unmatched external disturbances are supposed to be as follows

$$f_a(t) = \begin{cases} 0 & (0 \le t < 2) \\ 0.6\sin(\pi t) & (2 \le t < 3) \\ 0 & (3 \le t < 14) \\ 0.6\sin(\pi t) & (14 \le t < 20) \end{cases},$$

$$f_s(t) = \begin{cases} 0 & (0 \le t < 4) \\ 2\cos(3t) & (4 \le t < 10) \\ 1 & (10 \le t \le 20) \end{cases}$$

$$d_m(t) = \begin{cases} 2\sin(3t) + e^{0.1t} & (0 \le t < 8) \\ e^{0.1t} & (8 \le t < 10) \\ 2 & (10 \le t < 20) \end{cases},$$

$$d_l(t) = \begin{cases} 5\sin(2t) & (0 \le t < 8) \\ 0.6\sin(t) & (8 \le t < 13) \\ 0.002t & (13 \le t < 15) \\ 0 & (15 \le t < 20) \end{cases},$$

$$d_s(t) = \begin{cases} 3\sin(\pi t) & (0 \le t < 9) \\ t & (9 \le t < 14) \\ 1 & (14 \le t < 20) \end{cases}, \Delta k(t) = 0.25e^{-t}.$$

Given $x_0 = \left[0\ 0.01\ 0.01\ 0.01\right]^T$, $\tau = 7$, and the desired reference signals are given as

$$\theta_{mr} = \begin{cases} 0.5\sin(t) & (0 \le t < 6) \\ 0.5 & (6 \le t < 13) \\ 0.5\,s\cos(t) & (13 \le t \le 20) \end{cases},$$

$$\theta_{lr} = \begin{cases} 0.6\sin(t) & (0 \le t < 8) \\ 0.8 & (8 \le t \le 20) \end{cases},$$

$$w_{mr} = \begin{cases} 0.5\cos(t) & (0 \le t < 6) \\ 0 & (6 \le t < 13) \\ -0.5\sin(t) & (13 \le t \le 20) \end{cases},$$

$$w_{lr} = \begin{cases} 0.6\cos(t) & (0 \le t < 8) \\ 0 & (8 \le t \le 20) \end{cases}.$$

The simulation results are shown as follows

Figs. 6-9 show the tracking performances of the motor angle, link angle, motor angular velocity and link angular velocity with the proposed method and the methods in [5] and [18]. As depicted in the Figs.6-9, both the three methods

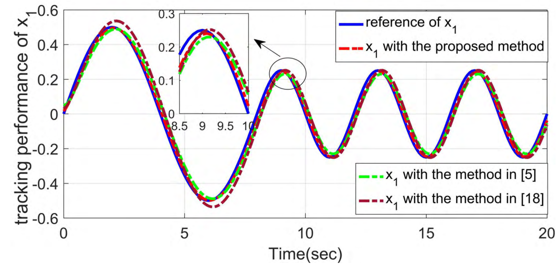

FIGURE 6. Trajectories of $x_1(t)$ and its estimation.

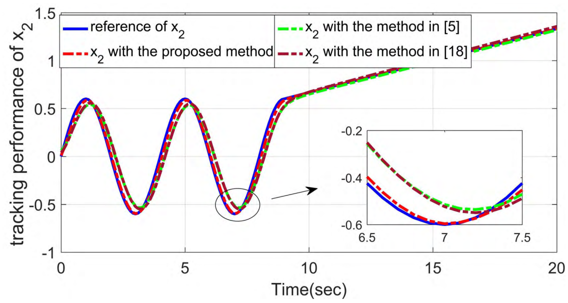

FIGURE 7. Trajectories of $x_2(t)$ and its estimation.

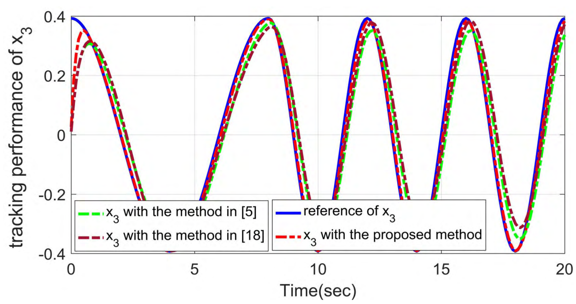

FIGURE 8. Trajectories of $x_3(t)$ and its estimation.

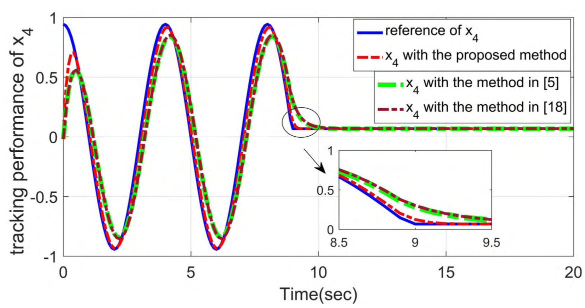

FIGURE 9. Trajectories of $x_4(t)$ and its estimation.

can track the desired trajectories, however, on the one hand, the integration term of tracking error is included in this study, the response is faster than the methods in [5] and [18]; on the other hand, the estimated information is used to design the controller, the robustness of the method is also better.

Figs. 10-11 illustrate the actuator fault and sensor fault and their estimations. Seen from these two figures, the observer has a good performance for the fault estimation by choosing

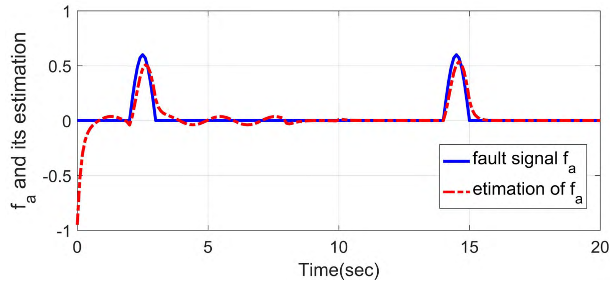

FIGURE 10. Trajectories of the actuator $f_a(t)$ and its estimation.

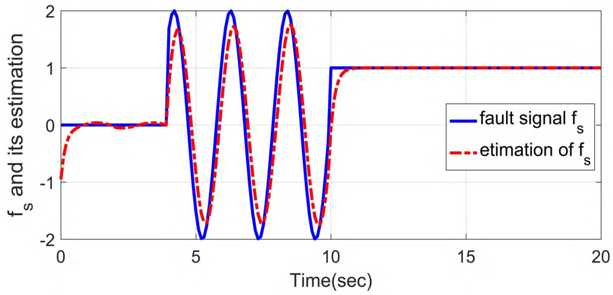

FIGURE 11. Trajectories of the sensor fault $f_s(t)$ and its estimation.

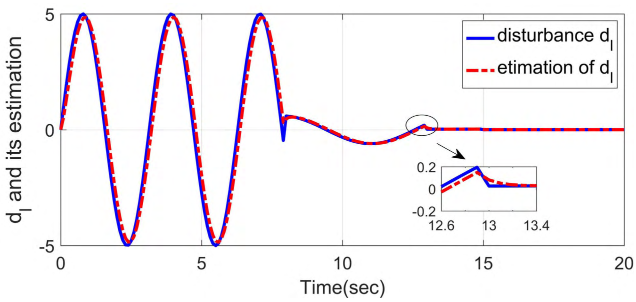

FIGURE 12. Trajectories of disturbance $d_l(t)$ and its estimation.

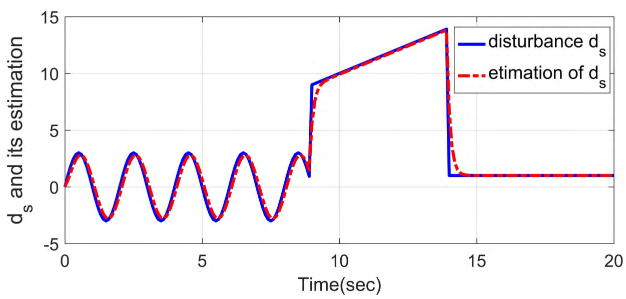

FIGURE 13. Trajectories of disturbance $d_s(t)$ and its estimation.

a proper positive scalar α_c. Figs. 12-14 show the estimation results of the disturbances. From Figs. 12-14, the estimated disturbances are smooth with the proposed approach, which indicates that the estimated values can be further utilized to design the controller. In addition, Fig. 15 exhibits the response of control input of the SFJM system. It can be observed that the bounded control input can be guaranteed. The simulations verify the fault tolerance and robustness abilities of the proposed method.

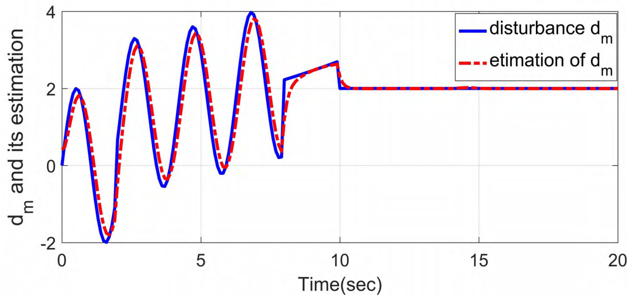

FIGURE 14. Trajectories of disturbance $d_m(t)$ and its estimation.

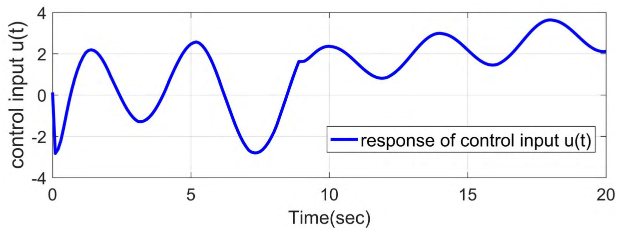

FIGURE 15. Trajectories of control input $u(t)$.

V. CONCLUSION

In this paper, the problem of fault-tolerant tracking control for a class of SFJM systems with uncertainty, actuator fault, sensor fault, and unmatched disturbance is addressed. A robust sliding mode control scheme is proposed. Firstly, the SFJM with fault, disturbance, and parameter uncertainty model is established. Then, by introducing two new variables, the system is divided into two subparts, such that fault and disturbance are separated. Subsequently, a novel composite observer is proposed to estimate the unknown variables. Furthermore, a sliding mode approach is designed based on the estimated values to keep the tracking performance of the manipulator system. Finally, the results of the two examples verify the effectiveness of the proposed method.

Note that only the single joint manipulator is considered in this study, the general mechanical manipulator system tracking control is very important in our future study. In addition, the result of this work is illustrated by two simulation cases; the application of our method to the real system will also be extended to our future work.

REFERENCES

[1] Y.-C. Chang and H.-M. Yan, "Design of a robust position feedback tracking controller for flexible-joint robots," *IET Control Theory Appl.*, vol. 5, no. 2, pp. 351–363, Jan. 2011.
[2] L. Le-Tien and A. Albu-Schäffer, "Robust adaptive tracking control based on state feedback controller with integrator terms for elastic joint robots with uncertain parameters," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 6, pp. 2259–2267, Nov. 2018.
[3] M. H. Korayem and S. R. Nekoo, "Finite-time state-dependent Riccati equation for time-varying nonaffine systems: Rigid and flexible joint manipulator control," *ISA Trans.*, vol. 54, pp. 125–144, Jan. 2015.
[4] J. Kim and E. A. Croft, "Full-state tracking control for flexible joint robots with singular perturbation techniques," *IEEE Trans. Control Syst. Technol.*, vol. 27, no. 1, pp. 63–73, Jan. 2019.
[5] H. Jin, Z. Liu, H. Zhang, Y. Liu, and J. Zhao, "A dynamic parameter identification method for flexible joints based on adaptive control," *IEEE/ASME Trans. Mechatronics*, vol. 23, no. 6, pp. 2896–2908, Dec. 2018.

[6] W. He, Z. Yan, Y. Sun, Y. Ou, and C. Sun, "Neural-learning-based control for a constrained robotic manipulator with flexible joints," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 12, pp. 5993–6003, Dec. 2018.

[7] H. Delavari, P. Lanusse, and J. Sabatier, "Fractional order controller design for a flexible link manipulator robot," *Asian J. Control*, vol. 15, no. 3, pp. 783–795, May 2013.

[8] S. E. Talole, J. P. Kolhe, and S. B. Phadke, "Extended-state-observer-based control of flexible-joint system with experimental validation," *IEEE Trans. Ind. Electron.*, vol. 57, no. 4, pp. 1411–1419, Apr. 2010.

[9] J. N. Yun and J.-B. Su, "Design of a disturbance observer for a two-link manipulator with flexible joints," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 2, pp. 809–815, Mar. 2014.

[10] L. Zhao, Q. Li, B. Liu, and H. Cheng, "Trajectory tracking control of a one degree of freedom manipulator based on a switched sliding mode controller with a novel extended state observer framework," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 6, pp. 1110–1118, Jun. 2019. doi: 10.1109/TSMC.2017.2719057.

[11] H. Yang, Y. Jiang, and S. Yin, "Fault-tolerant control of time-delay Markov jump systems with *It*ô stochastic process and output disturbance based on sliding mode observer," *IEEE Trans. Ind. Informat.*, vol. 14, no. 12, pp. 5299–5307, Dec. 2018.

[12] M. Bahrami, M. Naraghi, and M. Zareinejad, "Adaptive super-twisting observer for fault reconstruction in electro-hydraulic systems," *ISA Trans.*, vol. 76, pp. 235–245, May 2018.

[13] Y.-D. Song, Y. Lu, and Z.-X. Gan, "Descriptor sliding mode approach for fault/noise reconstruction and fault-tolerant control of nonlinear uncertain systems," *Inf. Sci.*, vols. 367–368, pp. 194–208, Nov. 2016.

[14] X. Su, X. Liu, and Y.-D. Song, "Fault-tolerant control of multiarea power systems via a sliding-mode observer technique," *IEEE/ASME Trans. Mechatronics*, vol. 23, no. 1, pp. 38–47, Feb. 2018.

[15] C. A. Lightcap and S. A. Banks, "An extended Kalman filter for real-time estimation and control of a rigid-link flexible-joint manipulator," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 1, pp. 91–103, Jan. 2010.

[16] A. K. Kostarigka, Z. Doulgeri, and G. A. Rovithakis, "Prescribed performance tracking for flexible joint robots with unknown dynamics and variable elasticity," *Automatica*, vol. 49, no. 5, pp. 1137–1147, May 2013.

[17] Z. Li, Q. Ge, W. Ye, and P. Yuan, "Dynamic balance optimization and control of quadruped robot systems with flexible joints," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 10, pp. 1338–1351, Oct. 2016.

[18] M. Ramírez-Neria, G. Ochoa-Ortega, N. Lozada-Castillo, M. A. Trujano-Cabrera, J. P. Campos-López, and A. Luviano-Juárez, "On the robust trajectory tracking task for flexible-joint robotic arm with unmodeled dynamics," *IEEE Access*, vol. 4, pp. 7816–7827, 2016.

[19] M. Ekramian, F. Sheikholeslam, S. Hosseinnia, and M. J. Yazdanpanah, "Adaptive state observer for Lipschitz nonlinear systems," *Syst. Control. Let*, vol. 62, no. 4, pp. 319–323, Apr. 2013.

[20] Z. Gao and S. X. Ding, "State and disturbance estimator for time-delay systems with application to fault estimation and signal compensation," *IEEE Trans. Signal Process.*, vol. 55, no. 12, pp. 5541–5551, Dec. 2007.

[21] Z. Gao, X. Liu, and M. Z. Q. Chen, "Unknown input observer-based robust fault estimation for systems corrupted by partially decoupled disturbances," *IEEE Trans. Ind. Electron.*, vol. 63, no. 4, pp. 2537–2547, Apr. 2016.

[22] J. Yang, W.-H. Chen, and S. Li, "Non-linear disturbance observer-based robust control for systems with mismatched disturbances/uncertainties," *IET Control Theory Appl.*, vol. 5, no. 18, pp. 2053–2062, Dec. 2011.

[23] F. Plestan, Y. Shtessel, V. Brégeault, and A. Poznyak, "New methodologies for adaptive sliding mode control," *Int. J. Control*, vol. 83, no. 9, pp. 1907–1919, Jul. 2010.

[24] X. Yin and L. Pan, "Direct adaptive robust tracking control for 6 DOF industrial robot with enhanced accuracy," *ISA Trans.*, vol. 72, pp. 178–184, Jan. 2018.