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Research on the Optimal Aggregation Method of Decision Maker Preference Judgment Matrix for Group Decision Making

WEI LIU[1] **AND LEI LI**[2]

[1]School of Internet of Things Engineering, Jiangnan University, Wuxi, 214122, China
[2]Business School, Jiangnan University, Wuxi, 214122, China

Corresponding author: Lei Li (lilei59@jiangnan.edu.cn)

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ABSTRACT The main focus of this paper is to present a new aggregation method of judgment matrices, which is based on the optimal aggregation model and the efficient aggregation algorithm. The reciprocal elements in the decision maker judgment matrices are mapped into the corresponding points on the two-dimensional coordinates. We can express the differences between different decision makers' preferences by the Euclidean distance among these points. We use the plant growth simulation algorithm (PGSA) to obtain the optimal aggregation points which can reflect the opinions of the entire decision makers group. The aggregation matrix of decision maker preference is composed of these optimal aggregation points and the consistency test has been passed. Compared with the weighted geometric mean method (WGMM) and minimum distance method (MDM), the sum of Euclidean distances from the aggregation points to other given points in this paper is minimal. The validity and rationality of this method are also verified by the analysis and comparison of examples, which provides a new idea to solve the group decision making (GDM) problems.

INDEX TERMS Group decision making, analytic hierarchy process, judgment matrices, aggregation, plant growth simulation algorithm (PGSA).

I. INTRODUCTION

Analytic Hierarchy Process (AHP) is a hierarchical weight decision analysis method, which was proposed by Saaty to solve the GDM problems [1]. In past decades, the aggregation method of group AHP judgment matrix has been widely concerned by researchers and practitioners. This method has developed rapidly and has been widely used in many fields, such as government, industry, management, education, and engineering, etc. [2]–[6].

For the aggregation method of group AHP judgment matrices, researchers have come up with numerous methods. Harsanyi [7] provided in-depth analysis of the weighted average (WA). However, Dong and Cheng [8] indicated that the WA operator relies on the constant weights given by the decision maker. Yager [9] offered a new aggregation operator named ordered weighted average (OWA) to integrate the experts' preference, which drew researchers' attention to further research on the aggregation method. Previous research has indicated that the OWA operator adaptively selects the weights of experts according to the preferences of decision makers. Crawford and Williams [10] derived a comparable estimate, the geometric mean (GM) vector that can be applied to solve the hierarchical problems.

Compared with the WA and OWA operators, Saaty and Kearns [11] argued that the WG operator is more suitable for the assembly of the AHP judgment matrix. He discussed several aggregation methods for group AHP opinions, one of which was the weighted geometric mean method (WGMM). Xu [12] pointed out that the weighted geometric mean complex judgment matrix (WGMCJM) is of acceptable consistency, which developed a theoretic basis for the WGMM. Similarly, Krejčí and Stoklasa [13] showed that the weighted arithmetic mean (WAM) did not reflect the preference information of alternatives properly. In contrast, some researchers showed that the weighted geometric mean (WGM) aggregation method can properly reflect the preference of alternatives. They strongly discouraged the use of the WAM and

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advocated the use of the WGM for the aggregation in AHP. Under row geometric mean prioritization method (RGMM), Dong *et al.* [14] presented two AHP consensus models, which improved the consensus indexes of AHP judgment matrices. Huang and Li [15] calculated the weight of the judgment matrix by the geometric mean over a super-transitive approximate method, which synthesized the opinions of various experts to a certain extent.

However, some researchers did not consider the information about the relationship between fusion values in the aggregation process. Yager [16] provided more versatility in the information aggregation, such as power average (PA) and power OWA (POWA) operators, which can be seen in MAGDM, information intelligence fusion, and data mining.

Based on the WG operator, Xu and Yager [17] proposed a power-geometric (PG) operator, which introduced a similarity between the assembly elements into the assembly process. However, the PG operator did not consider the consistency between the elements of the individual judgment matrix, and the inconsistent individual matrix may not be able to obtain the aggregated result with satisfactory consistency [18]. On the basis of the similarity and consistency index of the assembly elements, Huang *et al.* [18] used the DS evidence theory to calculate the rationality index of the contrast value of different attributes in the assembly matrix. Yu [19], Cook and Kress [20] proposed the distance concept to aggregate group judgments. Based on the basic principles and methods of AHP, Zhou and Kocaoglu [21] designed a new aggregation method named minimum distance method(MDM), which can help the decision makers achieve consensus. It is effective that they evaluated the performance of aggregation methods by using accuracy and group disagreement criteria.

A number of studies had been focused on fuzzy AHP, Öztaysi *et al.* [22] developed a hesitant fuzzy AHP method aggregated by OWA operator, which was applied in a multi-criteria supplier selection problem. Meng and Chen [23] proposed a method of incomplete fuzzy information for group decision making. Krejci and Ishizaka [24] presented a fuzzy extension of the AHPSort method, which was suitable for the aggregation method of decision-making problems. Büyüközkan and Çifçi [25] proposed the methodology containing fuzzy AHP and fuzzy TOPSIS. For an efficient selection, Kahraman *et al.* [26] developed a hesitant fuzzy linguistic AHP method. Zhang and Guo [27] mainly discussed the intuitionistic multiplicative preference relation (IMPR) for GDM problems. In order to improve the additive consistency and make up for the missing elements in incomplete HFPR, Zhang *et al.* [28], [29] proposed the new approach and models to solve these problems.

However, most of these methods are extended or mixed by using WA,OWA,WG,PG operators, etc. The methods mentioned above only can obtain approximate aggregation points, yet not suitable to solve the optimal aggregation points, which caused big deviation. Thus it is indispensable to improve the performance of the aggregation method.

Therefore, the aggregation problem of the decision maker judgment matrix requires further research. The major objective of this paper is to present a new aggregation method of decision maker judgment matrices, which is based on the optimal aggregation model and the efficient aggregation algorithm. The research idea of this paper can be divided into three parts. First, the evaluation values of attributes by experts are mapped into points on the plane. Second, the "Steiner point" is introduced into the construction of the optimal aggregation model. Third, the PGSA is used to solve the optimal aggregation points. Because this method determines the direction through the morphological concentration on the branches and its operation speed is fast. It avoids the construction of new computational target functions with low dependency on parameter settings. In order to achieve the goals, we map the reciprocal elements of each decision maker judgment matrix in the AHP into the corresponding points on the two-dimensional coordinates. These two-dimensional points are the individual preference points of decision makers. Then, PGSA is used to find the aggregation points, whose weighted Euclidean distance to other given points is minimal. The optimal aggregation matrix obtained by this method can reflect the consensus of all decision maker preferences in the group.

In order to do this, we organize the rest of the paper in four sections. Section 2 introduces the preliminary knowledge, including the mapping relationship of the expert judgment matrices, the optimal aggregation model, the principle of PGSA and the probabilistic model of PGSA. Section 3 elaborates the aggregation idea and core process for searching the optimal aggregation points based on PGSA. In section 4, two numerical examples are given to illustrate the validity and rationality of the proposed method in this paper. Section5 summarizes conclusions and some suggestions for further research.

II. PRELIMINARY KNOWLEDGE

A. THE MAPPING OF EXPERT JUDGMENT MATRICES IN TWO-DIMENSIONAL PLANE

In order to facilitate the expression of the expert judgment matrices, it is advisable to set the comparison judgment matrices of m indicators by p experts as follows:

$$
\begin{aligned}
A^{(1)} &= \begin{bmatrix} 1 & \cdots & a_{1m}^{1} \\ \vdots & 1 & \vdots \\ a_{m1}^{1} & \cdots & 1 \end{bmatrix} \\
A^{(2)} &= \begin{bmatrix} 1 & \cdots & a_{1m}^{2} \\ \vdots & 1 & \vdots \\ a_{m1}^{2} & \cdots & 1 \end{bmatrix} \\
&\quad\vdots \\
A^{(p)} &= \begin{bmatrix} 1 & \cdots & a_{1m}^{p} \\ \vdots & 1 & \vdots \\ a_{m1}^{p} & \cdots & 1 \end{bmatrix}
\end{aligned} \tag{1}
$$

The corresponding points in formula (1) are mapped to the two-dimensional coordinates: $A^{(i)} \subseteq R^2 (i = 1, 2, \cdots, p; p \geq 2)$, which can obtain the following set of plane points.

$$\left(a_{12}^1, a_{21}^1\right), \left(a_{13}^1, a_{31}^1\right), \cdots, \left(a_{1m}^1, a_{m1}^1\right)$$
$$\left(a_{12}^2, a_{21}^2\right), \left(a_{13}^2, a_{31}^2\right), \cdots, \left(a_{1m}^2, a_{m1}^2\right)$$
$$\vdots$$
$$\left(a_{12}^p, a_{21}^p\right), \left(a_{13}^p, a_{31}^p\right), \cdots, \left(a_{1m}^p, a_{m1}^p\right) \quad (2)$$

The expert judgment matrices of (1) are mapped to the set of plane points of (2), which signifies that the aggregation of the expert judgment matrices is transformed into the aggregation of the two-dimensional coordinates points.

B. THE OPTIMAL AGGREGATION MODEL

Definition1: Let $P_1, P_2, \ldots, P_n (n \geq 2)$ be the n decision preference points on the two-dimensional plane. These points with the corresponding positive weights are $\mu_i \in [0, 1] (1 \leq i \leq n)$, and $\sum_{i=1}^{n} \mu_i = 1$. Suppose there is a point P^* on the plane, and the sum of weighted Euclidean distance from this point to other given points satisfies the following condition: $D = \min \sum_{i=1}^{n} \mu_i |P^* P_i|$

$$\min(\mu_1 \sqrt{(x^* - x_1)^2 + (y^* - y_1)^2} + \ldots + \mu_n \sqrt{(x^* - x_n)^2 + (y^* - y_n)^2}) \quad (3)$$

Then P^* can be referred to as the optimal aggregation point, as shown in Figure 1.

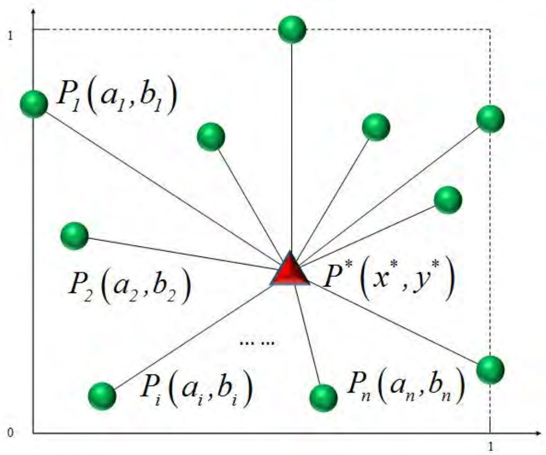

FIGURE 1. **A schematic diagram of aggregation point P^*.**

Comparing the plane point set (2) with definition 1, we can provide the following theorem:

Theorem 1: After mapping the judgment matrices (1) of P experts to the set of two-dimensional points (2), the point P^* is the optimal aggregation point of P experts. The mapping relationship satisfies the following condition:

$$A^{(i)} \subseteq R^2, \quad i = 1, 2, \ldots, p; p \geq 2 \quad (4)$$

It can be observed that the coordinate value of the optimal aggregation point P^* is the value of the reciprocal element corresponding to the optimal aggregation matrix A^*. The point P^* represents the decision-making willingness of the group and it is also the optimal aggregation point of the decision makers. However, with the increase of the points on the plane, it is difficult to solve this problem. Solving the optimal aggregation point becomes an NP-hard problem. This paper uses the optimal aggregation model and the efficient aggregation algorithm to solve this problem.

C. PLANT GROWTH SIMULATION ALGORITHM

The PGSA is an intelligent optimization algorithm based on the heuristic principle of plant phototropism, which was proposed in 2005 by Li *et al.* [30]. Durmus *et al.* [31] founded that PGSA compared with modern heuristic algorithms has the following two major advantages. First, PGSA determines the direction through the morphological concentration on the branches, and its operation speed is fast. Second, PGSA avoids the construction of new computational target functions with low dependency on parameter settings. In 2014, Li *et al.* [32] attempted to combine PGSA with group decision making to solve the optimal aggregation point problem. Since the establishment of PGSA, many scholars at home and abroad have paid great attention to it and applied it to their respective research fields [33]–[39].Compared with other intelligent algorithms such as MTACO, BA, and GA, PGSA has achieved better results [40]–[43].

The principle of PGSA is based on the plant phototropism theory as the heuristic criterion. It takes the solution space of the optimization problem as the growth environment of plants, takes the optimal solution as the light source, simulates the phototropic mechanism of real plants, and establishes the deduction mode of the rapid growth of branches and leaves under the environment of different light intensities.

The core of PGSA is established on the basis of plant system deduction based on the growth rules and probabilistic growth model based in the plant phototropism theory. The optimization model formed by the combination of the above two methods is to realize the process of artificial plants from the initial state to the complete form final state (no new branch growth) in the solution space of optimization problem. The diagram of the optimal path is shown in Figure 2.

In the probabilistic model of PGSA, the entire growth space of plants is considered as a viable domain. Let M be the length of the branch. There are T growing points named $S_M = (S_{M1}, S_{M2}, \cdots S_{MT})$ on the trunk, and the morphological concentration of each growing point is $P_M = (P_{M1}, P_{M2}, \cdots P_{MT})$.

Let $m (m < M)$ be the unit length of the branch trunk, on which there are r growing points named $s_m = (s_{m1}, s_{m2}, \cdots s_{mr})$. The morphological concentration of each growing point is $p_m = (p_{m1}, p_{m2}, \cdots p_{mr})$. The morphological

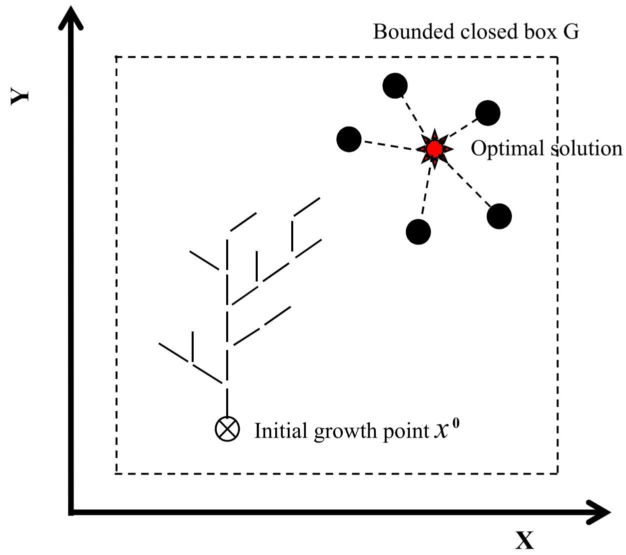

FIGURE 2. Optimal path diagram of PGSA.

concentrations of these growing points on the branches are calculated as follows:

$$p_{Mi} = \frac{f(x_0) - f(S_{Mi})}{\sum_{i=1}^{T} (f(x_0) - f(S_{Mi})) + \sum_{j=1}^{r} \left(f(x_0) - f\left(S_{mj}\right)\right)} \quad (5)$$

$$p_{mj} = \frac{f(x_0) - f(S_{mi})}{\sum_{i=1}^{T} (f(x_0) - f(S_{Mi})) + \sum_{j=1}^{r} \left(f(x_0) - f\left(S_{mj}\right)\right)} \quad (6)$$

where x_0 is the initial growing point, and $f(x)$ is the backlight function of these growing points. As the illumination intensity increases, the function value will decrease. We can derive $\sum_{i=1}^{K} p_{Mi} + \sum_{j=1}^{q} p_{mj} = 1$. The phototropism mechanism of plants can then be established. The algorithm is described as assuming that there are $K + 1$ growing points on the branches, and the corresponding morphological concentration is $p_1, p_2, \ldots, p_{K+1}$. According to the randomness of the algorithm, a value in the interval of [0, 1] is randomly generated, which will be used as the new growing points next time. The values of T and r will change with the growth of plants, and the new growing points can be added to the set of original growing points until no new branches are generated.

III. KEY METHODS

A. THE AGGREGATION IDEA

In this research, the judgment matrix of each decision maker can be mapped to two-dimensional plane coordinates. Thus, the points on the plane coordinates are made to correspond one-to-one with the decision information of each decision maker. For the group decision problem of e decision maker judgment matrices with n attributes, it can be considered as a set of points consisting of plane points. We can use $A^{(i)} \subseteq R^2, i = 1, 2, \ldots, p; p \geq 2$ to express the mapping relationship between decision maker judgment matrices and plane points.

The key step of decision making is to aggregate all decision maker preference matrices into a single aggregation matrix. The decision maker preference is projected onto the two-dimensional coordinate, and the decision maker preference aggregation matrix is constructed by calculating the optimal aggregation point. The aggregation idea of this paper is summarized as follows. Firstly, all expert preference matrices are projected onto the two-dimensional coordinates. Secondly, we can express the difference between preferences of the two decision makers by the Euclidean distance between two points. Thirdly, the closer the distance is, the nearer the optimal aggregation point is. Conversely, the farther it is, the greater the differences are between the experts' opinions.

B. CORE PROCESS FOR SEARCHING OPTIMAL AGGREGATION POINT BASED ON PGSA

The similar growing structures are defined in accordance with the four directions of east, west, north, and south, and they continuously produce new branches. The new branch is rotated at a 90° angle, that is, $\alpha = 90°$. The length of branches can set to be $l/1000$(l is the length of the bounded closed box). Assuming that there are n known vector groups $x = (x_1, x_2, \cdots x_n) \in X$, where X is the bounded closed box in R^n. To obtain the optimal aggregation point x, the core process of PGSA is as follows:

Step 1: Set the initial growing point $x_0 \in X$ and the step length λ (it can be set to be $l/1000$). Set $X_{\min} = x^0$, $F_{\min} = f(x^0)$ where $f(x^0)$ is the backlight function of x_0.

Step 2: Take x^0 as the thought center. Through this point, we can make a line segment parallel to the x-axis and y-axis, extending $a_1 \leq x_1^0 \leq b_1, a_2 \leq x_2^0 \leq b_2, \ldots a_t \leq x_t^0 \leq b_t$ as the new branches. Search for the germination point $S_{i_1, j_1}^0 (1 \leq i_1 \leq t, 1 \leq j_1 \leq k_1)$ from the generated branches in λ, where S_{i_1, j_1}^0 denotes the coordinate of the j_1 germination point on the i_1 branches.

Step 3: Compare the values of $f(S_{i_1, j_1}^0)$ with $F_{\min}$.

If $f(S_{i_1, j_1}^0) < F_{\min}$, then $X_{\min} = S_{g_1, j_1}^0$, $F_{\min} = f(S_{i_1, j_1}^0)$. Otherwise, keep $X_{\min}$ and $F_{\min}$ unchanged.

Step 4: If $F_{\min} \leq f(S_{i_1, j_1}^0)$, then the growth hormone concentration at this growing point is $P_{S_{i_1, j_1}^0} = 0$; otherwise, use Eq. (7) to calculate $P_{S_{i_1, j_1}^0}$:

$$P_{S_{i_1, j_1}^0} = \frac{f(x^0) - f(S_{i_1, j_1}^0)}{\sum_{i_1=1}^{t} \sum_{j_1=1}^{k_1} [f(x^0) - f(S_{i_1, j_1}^0)]} \quad (7)$$

Step 5: The growth hormone concentration of all growing points is established as an interval [0, 1]. Suppose that δ_0 is a random number in this interval, then

$$\sum_{i_1=1}^{r_1} \sum_{j_1=1}^{t_1-1} P_{S_{i_1, j_1}^0} < \delta_0 \leq \sum_{i_1=1}^{r_1} \sum_{j_1=1}^{t_1} P_{S_{i_1, j_1}^0} \quad (8)$$

Select $S^0_{r_1,t_1}$ as the new growing point, and $x^1 = S^0_{r_1,t_1} X_{\min} = x^1, F_{\min} = f(x^1)$.

Step 6: Take x^1 as the thought center, through this point we can make a line segment parallel to the x-axis and y-axis, extending $a_1 \le x_1^1 \le b_1, a_2 \le x_2^1 \le b_2, \ldots a_t \le x_t^1 \le b_t$ as the new branches. Search for the germination point $S^1_{i_1,j_1}(1 \le i_2 \le t, 1 \le j_2 \le k_1)$ from the generated branches in λ.

Step 7: Compare the values of $f(S^1_{i_2,j_2})$ with $F_{\min}$. If $f(S^1_{i_2,j_2}) < F_{\min}$, then $X_{\min} = S^1_{i_2,j_2}, F_{\min} = f(S^1_{i_2,j_2})$. Otherwise, keep $X_{\min}$ and $F_{\min}$ unchanged.

Step 8: Calculate $P_{S^0_{i_1,j_1}}$ and $P_{S^1_{i_2,j_2}}$. If $f(x^0) \le f(S^1_{i_1,j_1})$, then $P_{S^0_{i_1,j_1}} = 0$; otherwise, use Eq. (9) to calculate $P_{S^0_{i_1,j_1}}$:

$$P_{S^0_{i_1,j_1}} = \frac{f(x^0) - f(S^0_{i_1,j_1})}{\sum_{i_1=1}^{t}\sum_{j_1=1}^{k_1}[f(x^0) - f(S^0_{i_1,j_1})] + \sum_{i_1=1}^{t}\sum_{j_1=1}^{k_2}[f(x^0) - f(S^1_{i_2,j_2})]} \tag{9}$$

If $f(x^0) \le f(S^1_{i_2,j_2})$, then $P_{S^1_{i_2,j_2}} = 0$; otherwise, use Eq. (10) to calculate $P_{S^1_{i_2,j_2}}$:

$$P_{S^1_{i_2,j_2}} = \frac{f(x^0) - f(S^1_{i_2,j_2})}{\sum_{i_1=1}^{t}\sum_{j_1=1}^{k_1}[f(x^0) - f(S^0_{i_1,j_1})] + \sum_{i_1=1}^{t}\sum_{j_1=1}^{k_2}[f(x^0) - f(S^1_{i_2,j_2})]} \tag{10}$$

Step 9: The growth hormone concentration of the growing points is established as an interval [0, 1]. Suppose that δ_1 is a random number in this interval, if

$$\sum_{i_1=1}^{r_2}\sum_{j_1=1}^{t_2-1} P_{S^0_{i_1,j_1}} < \delta_1 \le \sum_{i_1=1}^{r_2}\sum_{j_1=1}^{t_2} P_{S^0_{i_1,j_1}} \tag{11}$$

Then select $S^0_{r_1,t_1}$ as the new growing point, set $x^2 = S^0_{r_1,t_1}$, $X_{\min} = x^2, F_{\min} = f(x^2)$. Otherwise, if

$$\sum_{i_1=1}^{t}\sum_{j_1=1}^{k_1} P_{S^0_{i_1,j_1}} + \sum_{i_1=1}^{r_2}\sum_{j_1=1}^{t_2-1} P_{S^1_{i_2,j_2}} < \delta_1 \le \sum_{i_1=1}^{t}\sum_{j_1=1}^{k_1} P_{S^0_{i_1,j_1}} + \sum_{i_1=1}^{r_2}\sum_{j_1=1}^{t_2} P_{S^1_{i_2,j_2}} \tag{12}$$

Then select $S^1_{r_2,t_2}$ as the new growing point, set $x^2 = S^1_{r_1,t_1}$, $X_{\min} = x^2, F_{\min} = f(x^2)$.

Step 10: Repeat steps 6 to 9 until the value of $F_{\min}$ remains unchanged. Then $x^* = X_{\min}$ is the global optimal solution and the process of iteration has stopped.

C. THE IMPROVEMENT OF PGSA

In the previous research of the PGSA method,the search step size always needed to be determined manually. However,for many problems in the actual research process, the space of feasible domain is quite different. When the space of the feasible domain is relatively small, the algorithm calculation process is simple, fast and the result is accurate. As the feasible domain space becomes large, the determination of the search step size will become difficult. Therefore, the Sierpinski carpet is introduced here to improve the PGSA (as shown in Figure 3).

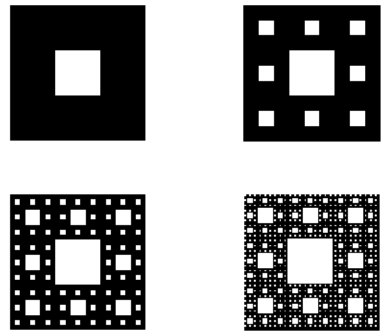

FIGURE 3. The Sierpinski carpet map.

The principle of the method is to remove the inner points by using the four vertices of the square as the initial growing points. The initial growing point is then established according to the fractal principle.

Key steps of the improved PGSA algorithm are as follows:

Step 1: Determine $e_t \in X$ is the initial growing point, X is the bounded closed box in R^n and these initial growing points e_t are randomly uniform points in the bounded closed box.

Step 2: Calculate the growth probability of each growing point (t is the number of growing points).

$$P_t = \frac{\sum_{i=1}^{n}(1/|e_t E_i|)}{\sum_{t=1}^{v}\sum_{i=1}^{n}(1/|e_t E_i|)}(t = 1, 2, \ldots v) \tag{13}$$

Step 3: Establish the probability of each growing point within the interval of 0-1 and select the growing points e_t of this iteration by random numbers.

Step 4: Determine step length λ (it is set to be $l/1000$ in this paper).The growing points e_t grow with the L-system of $\alpha = 90°$ and new growing points will replace e_t.

Step 5: If the new growth point is no longer generated and the preset number of iterations is reached, the calculation will stop. The optimal aggregation point is obtained. Otherwise, return to **Step 2**.

IV. NUMERICAL ANALYSIS

In the above sections, we have discussed the optimal aggregation model and the efficient method (PGSA) in detail. Now let us look at two numerical examples.

A. EXAMPLE 1

In this example, Xu [12] developed a theoretic basis for the application of WGMM. Let us assume the following four judgment matrices for a decision problem that are given by

TABLE 1. Expert preference coordinates and aggregation point coordinates.

	(a_{12}, a_{21})	(a_{13}, a_{31})	(a_{14}, a_{41})	(a_{23}, a_{32})	(a_{24}, a_{42})	(a_{34}, a_{43})
1	(4, 1/4)	(6, 1/6)	(7, 1/7)	(3, 1/3)	(4, 1/4)	(2, 1/2)
2	(5, 1/5)	(7, 1/7)	(9, 1/9)	(4, 1/4)	(6, 1/6)	(2, 1/2)
3	(3, 1/3)	(5, 1/5)	(8, 1/8)	(4, 1/4)	(5, 1/5)	(2, 1/2)
4	(4, 1/4)	(5, 1/5)	(6, 1/6)	(3, 1/3)	(3, 1/3)	(2, 1/2)
5	(3.936, 0.254)	(5.692, 0.176)	(7.417, 0.135)	(3.464, 0.289)	(4.356, 0.230)	(2, 1/2)
6	(4, 1/4)	(5, 1/5)	(8, 1/8)	(3, 1/3)	(5, 1/5)	(2, 1/2)

TABLE 2. Aggregation weight and consistency test.

	Aggregation weight	Consistency test
This paper	$w = (0.604, 0.236, 0.103, 0.057)^T$	$CR = 0.03888889 < 0.1$
Literature[12]	$w = (0.610, 0.237, 0.094, 0.059)^T$	$CR = 0.044 < 0.1$

four experts. The four judgment matrices are shown below:

$$A_1 = \begin{bmatrix} 1 & 4 & 6 & 7 \\ 1/4 & 1 & 3 & 4 \\ 1/6 & 1/3 & 1 & 2 \\ 1/7 & 1/4 & 1/2 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 5 & 7 & 9 \\ 1/5 & 1 & 4 & 6 \\ 1/7 & 1/4 & 1 & 2 \\ 1/9 & 1/6 & 1/2 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 3 & 5 & 8 \\ 1/3 & 1 & 4 & 5 \\ 1/5 & 1/4 & 1 & 2 \\ 1/8 & 1/5 & 1/2 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1 & 4 & 5 & 6 \\ 1/4 & 1 & 3 & 3 \\ 1/5 & 1/3 & 1 & 2 \\ 1/6 & 1/3 & 1/2 & 1 \end{bmatrix}$$

The points on the symmetrical position of the main diagonal of the judgment matrix are mapped to the two-dimensional coordinate points. Let us assume that the four experts have equal weights. We employ our method to calculate the coordinates of the two-dimensional decision preference points of each expert, that is, the aggregation point coordinates of the group decision, as shown in Table 1. No.1-4 represents the preference point coordinates of each of the four experts, respectively. No. 5 is the aggregation point coordinates of literature [12]; No. 6 is the aggregation coordinates of this paper.

From Table 1, we can obtain the aggregation matrix A^* of this paper and the aggregation matrix $\overline{A}$ of the literature [12]. The aggregation weight and consistency test are shown in Table 2.

$$A^* = \begin{bmatrix} 1 & 4 & 5 & 8 \\ 1/4 & 1 & 3 & 5 \\ 1/5 & 1/3 & 1 & 2 \\ 1/8 & 1/5 & 1/2 & 1 \end{bmatrix}$$

$$\overline{A} = \begin{bmatrix} 1 & 3.936 & 5.692 & 7.417 \\ 0.254 & 1 & 3.464 & 4.356 \\ 0.176 & 0.289 & 1 & 2 \\ 0.135 & 0.230 & 1/2 & 1 \end{bmatrix}$$

In order to intuitively represent the expert preference points and aggregation points, the aggregation process can be expressed in the two-dimensional coordinate system by the MATLAB simulation, as shown in **Fig. 4** to **Fig. 9**. The "hexagonal point" in the figure represents the aggregation point obtained in this paper, and the "square point" represents the aggregation point obtained in reference [12]. When the hexagonal point appears, this signifies that the aggregation points obtained in this paper coincide with the point obtained in literature [12].

Assuming that S_1 is the weighted sum of Euclidean distances from the aggregation point in literature [12] to each expert preference points, S_2 is the sum of weighted Euclidean

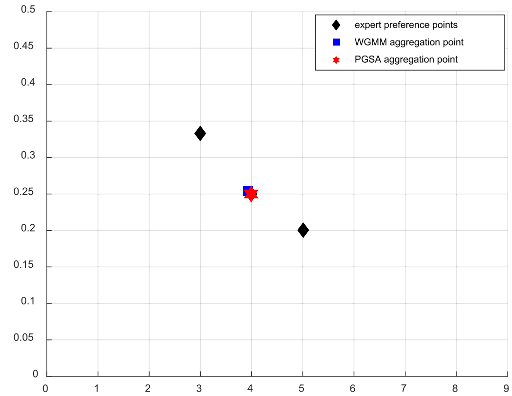

FIGURE 4. (a_{12}, a_{21}) **Expert preference points and aggregation points.**

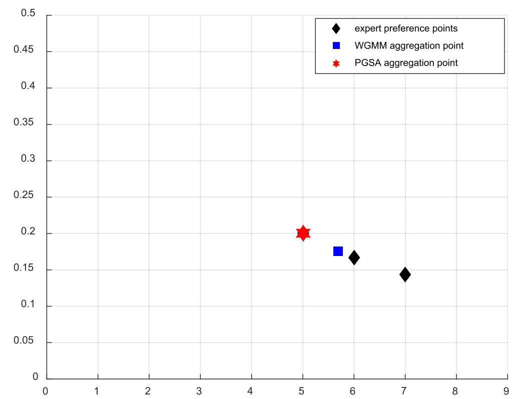

FIGURE 5. (a_{13}, a_{31}) **Expert preference points and aggregation points.**

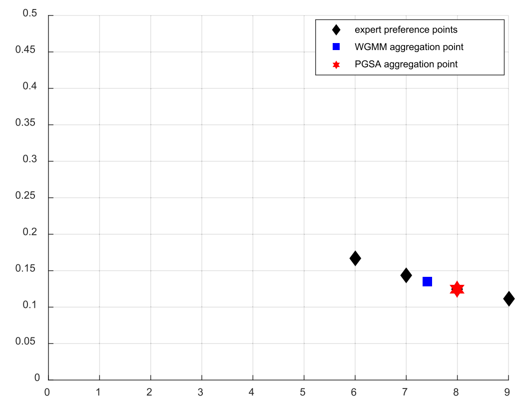

FIGURE 6. (a_{14}, a_{41}) **Expert preference points and aggregation points.**

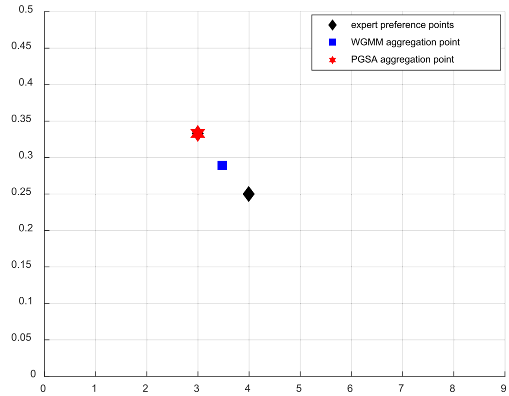

FIGURE 7. (a_{23}, a_{32}) **Expert preference points and aggregation points.**

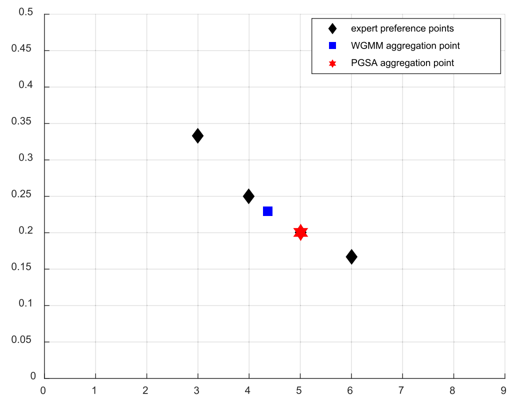

FIGURE 8. (a_{24}, a_{42}) **Expert preference points and aggregation points.**

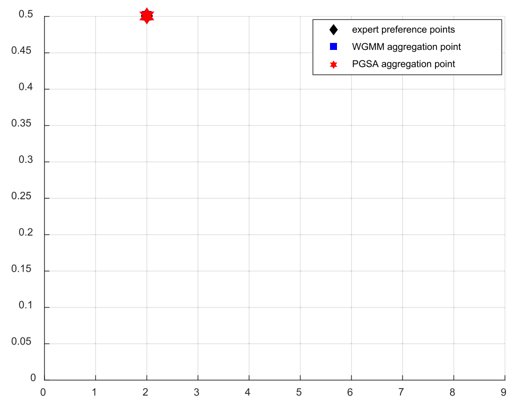

FIGURE 9. (a_{34}, a_{43}) **Expert preference points and aggregation points.**

distances from the aggregation point in this paper to each expert preference points, while ΔS is the difference between S_1 and S_2. The results obtained from the correlational analysis regarding the sum of Euclidean distances from the aggregation points to other given points can be compared in Table 3 and Table 4.

It is apparent from Table 3 and Table 4 that S_2 is less than S_1, except for the overlap between the aggregation points obtained in literature [12] and that obtained in this paper which is plotted in **Fig. 9**. This shows that the aggregation points obtained in this paper are better than the aggregation points obtained in [12].

Assuming that the weights of four experts are not equal, they are $\lambda_1 = 1/10$, $\lambda_2 = 2/10$, $\lambda_3 = 3/10$, $\lambda_4 = 4/10$. We employ our method to calculate the aggregation point coordinates of the group decision, as shown in Table 5. No.1-4 represents the preference point coordinates of each of the four experts, respectively. No. 5 is the aggregation point coordinates of the literature [12]. No. 6 is the aggregation coordinates of this paper.

TABLE 3. The comparison 1 between this paper and literature [12].

	The situation in Fig. 4	The situation in Fig. 5	The situation in Fig. 6
Literature [12]	S_1=2.132975	S_1=3.001393	S_1=4.000694
This paper	S_2=2.004715	S_2=3.001372	S_2=4.000690
Total path comparison	$S_2 < S_1$	$S_2 < S_1$	$S_2 < S_1$
Difference ΔS	$\Delta S = S_1 - S_2$=0.12826	$\Delta S = S_1 - S_2$=0.000021	$\Delta S = S_1 - S_2$=0.000004

TABLE 4. The comparison 2 between this paper and literature [12].

	The situation in Fig. 7	The situation in Fig. 8	The situation in Fig. 9
Literature [12]	S_1=2.007060	S_1=4.006411	S_1=0.000000
This paper	S_2=2.006932	S_2=4.006244	S_2=0.000000
Total path comparison	$S_2 < S_1$	$S_2 < S_1$	$S_2 = S_1$
Difference ΔS	$\Delta S = S_1 - S_2$=0.000128	$\Delta S = S_1 - S_2$=0.000167	$\Delta S = S_1 - S_2$=0.000000

TABLE 5. Expert preference coordinates and aggregation point coordinates.

	(a_{12}, a_{21})	(a_{13}, a_{31})	(a_{14}, a_{41})	(a_{23}, a_{32})	(a_{24}, a_{42})	(a_{34}, a_{43})
1	(4, 1/4)	(6, 1/6)	(7, 1/7)	(3, 1/3)	(4, 1/4)	(2, 1/2)
2	(5, 1/5)	(7, 1/7)	(9, 1/9)	(4, 1/4)	(6, 1/6)	(2, 1/2)
3	(3, 1/3)	(5, 1/5)	(8, 1/8)	(4, 1/4)	(5, 1/5)	(2, 1/2)
4	(4, 1/4)	(5, 1/5)	(6, 1/6)	(3, 1/3)	(3, 1/3)	(2, 1/2)
5	(3.837,0.261)	(5.446,0.184)	(7.204,0.139)	(3.464,0.287)	(4.134,0.242)	(2,1/2)
6	(4, 1/4)	(5, 1/5)	(7.185,0.142)	(3, 1/3)	(4.443,0.238)	(2,1/2)

From Table 5, we can obtain the aggregation matrix A^* of this paper and the aggregation matrix $\bar{A}$ of the literature [12]. The aggregation weight and consistency test are shown in Table 6.

$$A^* = \begin{bmatrix} 1 & 4 & 5 & 7.185 \\ 1/4 & 1 & 3 & 4.443 \\ 1/5 & 1/3 & 1 & 2 \\ 0.142 & 0.238 & 1/2 & 1 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1 & 3.837 & 5.446 & 7.204 \\ 0.261 & 1 & 3.464 & 4.134 \\ 0.184 & 0.287 & 1 & 2 \\ 0.139 & 0.242 & 1/2 & 1 \end{bmatrix}$$

As above, expert preference points and aggregation points are shown in **Fig. 10** to **Fig. 15.** The "hexagonal point" in the figure represents the aggregation point obtained in this paper, and the "square point" represents the aggregation

TABLE 6. Aggregation weight and consistency test.

	Aggregation weight	Consistency test
This paper	$w=(0.599,0.234,0.105,0.062)^{\mathrm{T}}$	$CR=0.04880639<0.1$
Literature[12]	$w=(0.601,0.239,0.098,0.062)^{\mathrm{T}}$	$CR=0.044<0.1$

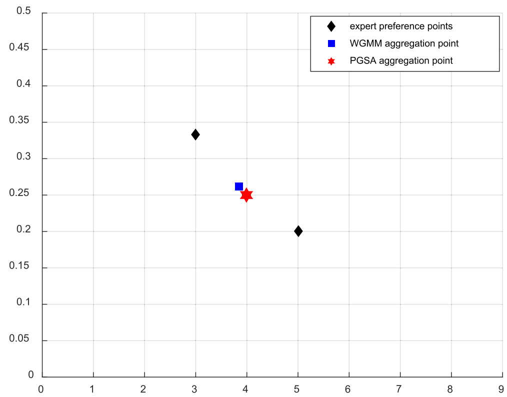

FIGURE 10. (a_{12}, a_{21}) Expert preference points and aggregation points.

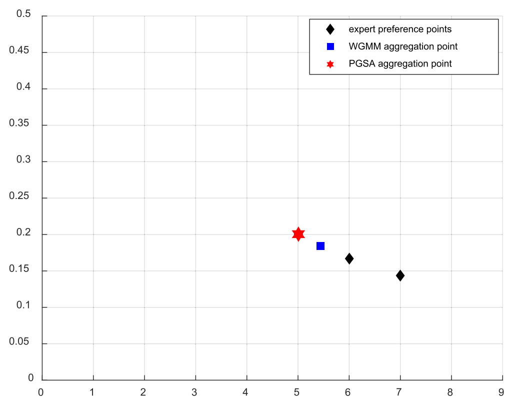

FIGURE 11. (a_{13}, a_{31}) Expert preference points and aggregation points.

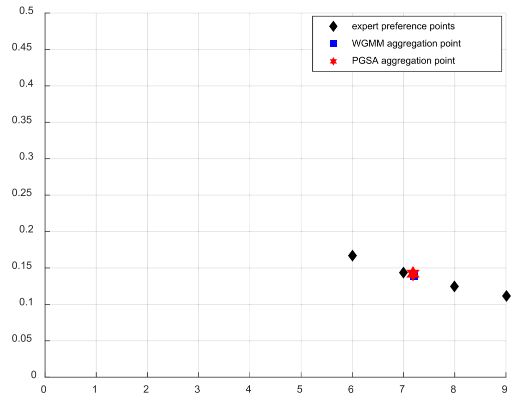

FIGURE 12. (a_{14}, a_{41}) Expert preference points and aggregation points.

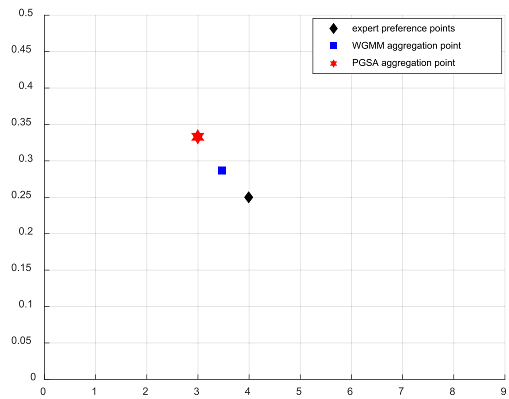

FIGURE 13. (a_{23}, a_{32}) Expert preference points and aggregation points.

point obtained in reference [12]. When the hexagonal point appears, this signifies that the aggregation points obtained in this paper coincide with the point obtained in literature [12].

Assuming that S_3 is the sum of weighted Euclidean distances from the aggregation point in literature [12] to each expert preference points, S_4 is the sum of weighted Euclidean distances from the aggregation point in this paper to each expert preference points, while ΔS is the difference between S_3 and S_4. The results obtained from the correlational analysis regarding the sum of Euclidean distances from the aggregation points to other given points can be compared in Table 7 and Table 8.

It is apparent from Table 7 and Table 8 that S_4 is less than S_3, except for the overlap between the aggregation points obtained in literature [12] and that obtained in this paper which is plotted in **Fig. 15**. This shows that the aggregation point obtained in this paper is better than the aggregation point obtained in [12].

B. EXAMPLE 2

This section refers to the numerical analysis given by Zhou and Kocaoglu [21] under the framework of AHP. He proposed the MDM method to support the group decision-making process which can help decision makers reach consensus. Suppose that there are four evaluators A, B, C, D, they have equal weights. The four comparison judgment matrices are as follows:

$$A=\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 2 & 1 & 2/3 & 1/2 \\ 3 & 3/2 & 1 & 3/4 \\ 4 & 2 & 4/3 & 1 \end{bmatrix}$$

TABLE 7. The comparison 1 between this paper and literature [12].

	The situation in Fig. 10	The situation in Fig. 11	The situation in Fig. 12
Literature [12]	S_3=0.566641	S_3=0.678737	S_3=1.100211
This paper	S_4=0.501290	S_4=0.500219	S_4=1.100208
Total path comparison	$S_4 < S_3$	$S_4 < S_3$	$S_4 < S_3$
Difference ΔS	$\Delta S = S_3 - S_4$=0.065351	$\Delta S = S_3 - S_4$=0.178518	$\Delta S = S_3 - S_4$=0.000002

TABLE 8. The comparison 2 between this paper and literature [12].

	The situation in Fig. 13	The situation in Fig. 14	The situation in Fig. 15
Literature [12]	S_3=0.501792	S_3=1.102102	S_3=0.000000
This paper	S_4=0.501733	S_4=1.101990	S_4=0.000000
Total path comparison	$S_4 < S_3$	$S_4 < S_3$	$S_4 = S_3$
Difference ΔS	$\Delta S = S_3 - S_4$=0.000059	$\Delta S = S_3 - S_4$=0.000112	$\Delta S = S_3 - S_4$=0.000000

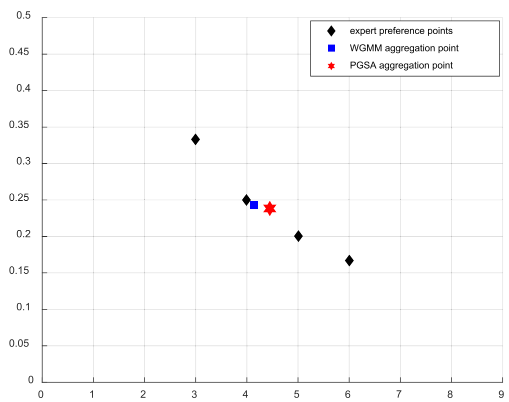

FIGURE 14. (a_{24}, a_{42}) Expert preference points and aggregation points.

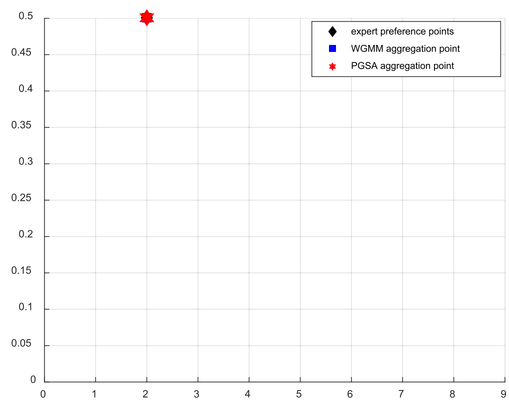

FIGURE 15. (a_{34}, a_{43}) Expert preference points and aggregation points.

$$B = \begin{bmatrix} 1 & 2 & 2/3 & 1/2 \\ 1/2 & 1 & 1/3 & 1/4 \\ 3/2 & 3 & 1 & 3/4 \\ 2 & 4 & 4/3 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1/3 & 1/2 & 1/4 \\ 3 & 1 & 3/2 & 3/4 \\ 2 & 2/3 & 1 & 1/2 \\ 4 & 4/3 & 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 4/3 & 2 & 4 \\ 3/4 & 1 & 3/2 & 3 \\ 1/2 & 2/3 & 1 & 2 \\ 1/4 & 1/3 & 1/2 & 1 \end{bmatrix}$$

The points on the symmetrical position of the main diagonal of the judgment matrix are mapped to the coordinate points on the plane. We employ our method to calculate the aggregation point coordinates of the group decision, as shown in Table 9. No.1-4 represents the preference point coordinates

TABLE 9. Expert preference coordinates and aggregation point coordinates.

	(a_{12},a_{21})	(a_{13},a_{31})	(a_{14},a_{41})	(a_{23},a_{32})	(a_{24},a_{42})	(a_{34},a_{43})
1	(1/2,2)	(1/3,3)	(1/4,4)	(2/3,3/2)	(1/2,2)	(3/4,4/3)
2	(2,1/2)	(2/3,3/2)	(1/2,2)	(1/3,3)	(1/4,4)	(3/4,4/3)
3	(1/3,3)	(1/2,2)	(1/4,4)	(3/2,2/3)	(3/4,4/3)	(1/2,2)
4	(4/3,3/4)	(2,1/2)	(4,1/4)	(3/2,2/3)	(3,1/3)	(2,1/2)
5	(0.5,2)	(0.5,2)	(0.375,2.667)	(1,1)	(0.75,1.333)	(0.75,1.333)
6	(0.670,1.887)	(0.5,2)	(0.25,4)	(1.5,0.667)	(0.643,1.905)	(0.75,1.333)

TABLE 10. Aggregation weight and consistency test.

	Aggregation weight	Consistency test
This paper	$w=(0.120,0.259,0.223,0.398)^T$	$CR=0.0685519<0.1$
Literature[21]	$w=(0.130,0.261,0.261,0.348)^T$	$CR=\ 0.<0.1$

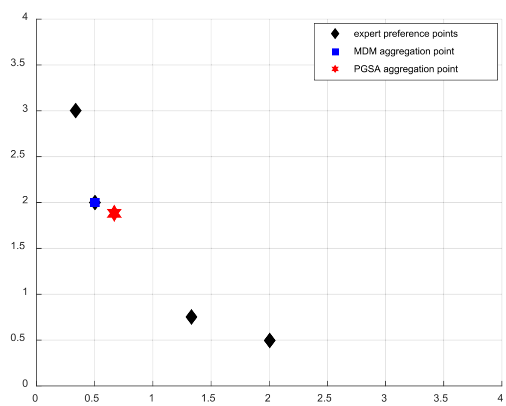

FIGURE 16. (a_{12}, a_{21}) **Expert preference points and aggregation points.**

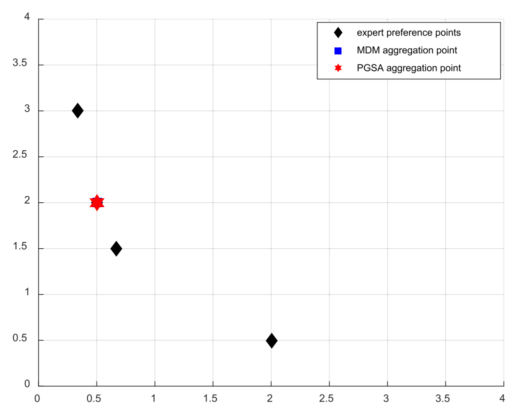

FIGURE 17. (a_{13}, a_{31}) **Expert preference points and aggregation points.**

of each of the four experts, respectively. No. 5 is the aggregation point coordinates of the literature [21]. No. 6 is the aggregation coordinates of this paper.

From Table 9, we can obtain the aggregation matrix A^* of this paper and the aggregation matrix $\bar{A}$ of the literature [21]. The aggregation weight and consistency test are as shown in Table 10.

$$A^* = \begin{bmatrix} 1 & 0.670 & 0.5 & 0.25 \\ 1.887 & 1 & 1.5 & 0.643 \\ 2 & 0.667 & 1 & 0.75 \\ 4 & 1.905 & 1.333 & 1 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.375 \\ 2 & 1 & 1 & 0.75 \\ 2 & 1 & 1 & 0.75 \\ 2.667 & 1.333 & 1.333 & 1 \end{bmatrix}$$

As above, expert preference points and aggregation points are shown in **Fig. 16** to **Fig. 21.**The "hexagonal point" in the figure represents the aggregation point obtained in this paper, and the "square point" represents the aggregation point obtained in reference [21]. When the hexagonal point appears, this signifies that the aggregation points obtained in this paper coincide with the point obtained in literature [21].

TABLE 11. The comparison 1 between this paper and literature [21].

	The situation in Fig. 16	The situation in Fig. 17	The situation in Fig. 18
Literature [21]	D_1=4.637427	D_1=3.662160	D_1=7.713201
This paper	D_2=4.604917	D_2=3.662160	D_2=7.318865
Total path comparison	$D_2 < D_1$	$D_2 = D_1$	$D_2 < D_1$
Difference ΔD	$\Delta D = D_1 - D_2$ =0.03251	$\Delta D = D_1 - D_2$ =0.000000	$\Delta D = D_1 - D_2$ =0.394336

TABLE 12. The comparison 2 between this paper and literature [21].

	The situation in Fig. 19	The situation in Fig. 20	The situation in Fig. 21
Literature [21]	D_1=3.910961	D_1=5.888189	D_1=2.215107
This paper	D_2=3.787257	D_2=5.717763	D_2=2.215107
Total path comparison	$D_2 < D_1$	$D_2 < D_1$	$D_2 = D_1$
Difference ΔD	$\Delta D = D_1 - D_2$ =0.123704	$\Delta D = D_1 - D_2$ =0.170426	$\Delta D = D_1 - D_2$ =0.000000

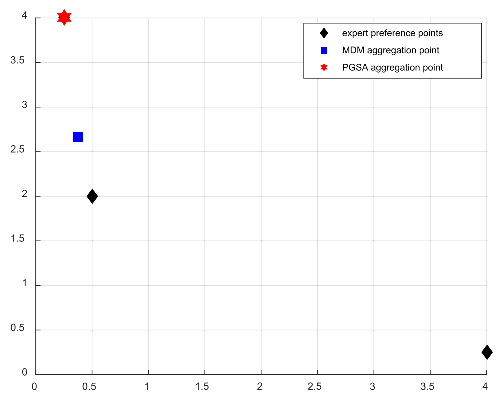

FIGURE 18. (a_{14}, a_{41}) **Expert preference points and aggregation points.**

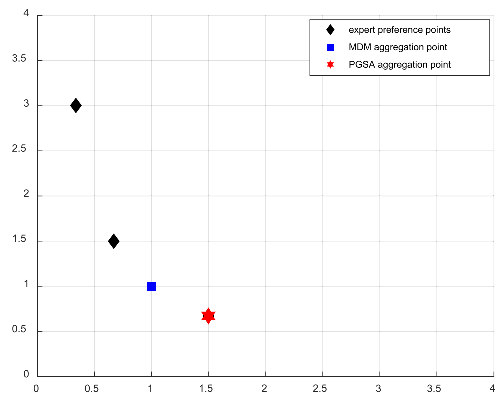

FIGURE 19. (a_{23}, a_{32}) **Expert preference points and aggregation points.**

Assuming that D_1 is the sum of weighted Euclidean distances from the aggregation point in literature [21] to each expert preference points, D_2 is the sum of weighted Euclidean distances from the aggregation point in this paper to each expert preference points, while ΔD is the difference between D_1 and D_2. The results obtained from the correlational analysis regarding the sum of Euclidean distances from the aggregation points to other given points can be compared in Table 11 and Table 12.

It is apparent from Table 11 and Table 12 that D_2 is less than D_1, except for the overlap between the aggregation points obtained in literature [21] and that obtained in this paper

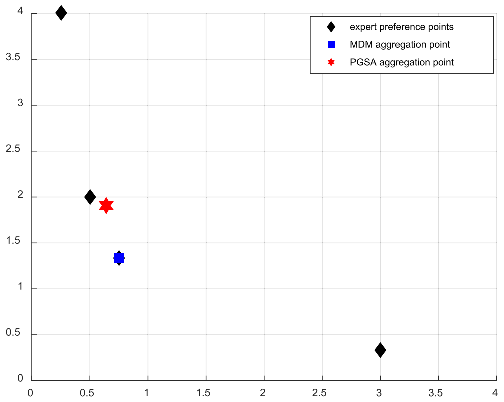

FIGURE 20. **(a_{24}, a_{42}) Expert preference points and aggregation points.**

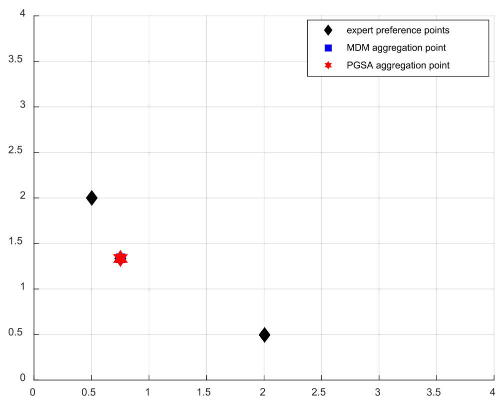

FIGURE 21. **(a_{34}, a_{43}) Expert preference points and aggregation points.**

which is plotted in **Fig. 17** and **Fig. 21**. This shows that the aggregation point obtained in this paper is better than the aggregation point obtained in [21].

V. CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

In this paper, a new aggregation method of judgment matrices is proposed, which is an optimal aggregation method in theory. The optimal aggregation points can be obtained by the optimal aggregation model and the efficient algorithm (we adopt the improved PGSA method in this paper).

The decision maker preference is projected onto the two-dimensional coordinates, and the aggregation matrix of decision maker preference is constructed by calculating the optimal aggregation points. Compared with Xu' method [12] and Zhou' method [21],the sum of Euclidean distances from the aggregation points to other given points in this paper is minimal. In addition, the method proposed in this paper can provide a new method to aggregate all decision maker preference matrices into a single aggregation matrix in AHP. The aggregation matrix is made up of the optimal aggregation points which are obtained by the proposed method in this paper. It is obtained by the iterative algorithm rather than calculated by the aggregation operator. The global optimal solutions are obtained by the PGSA since every iteration scans the points in the whole growth space. Therefore, it is reasonable that we establish the aggregation matrix by the proposed method in this paper.

It is apparent from this study that the method in this paper can solve the multi-points aggregation problems in the two-dimensional coordinates system. The validity and rationality of this method is also verified by the analysis and comparison of examples, which provides a new idea to solve the MAGDM problems.

However, there are some limitations of this method. When the expert preference points are increasing (more than fifty points), we cannot effectively calculate the distance by the proposed method.

In future research, we will focus on the following aspects. Firstly, we proposed a new aggregation method of judgment matrices in AHP. But in the real world, the MAGDM problems are very complicated. We will attempt to map the expert preference information into the spatial multi-dimensional coordinate points. Secondly, when the expert preference points are increasing (more than fifty points), we cannot effectively calculate the distance by the proposed method. We will try to adopt clustering algorithms such as spectral clustering, kernel clustering and quantum clustering to cluster expert preferences. In this way, the number of expert preference points can be reduced. Thirdly, we will develop a new approach for large-scale MAGDM problems with linguistic information combining the proposed method in this paper and the linguistic computational model [44].

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WEI LIU received the M.S. degree in management science and engineering from the Henan University of Technology, Zhengzhou, China, in 2014. She is currently pursuing the Ph.D. degree with the School of Internet of Things Engineering, Jiangnan University, Wuxi, China. Her research interests include group decision making and data mining.

LEI LI received the Ph.D. degree in management science from Tianjin University, Tianjin, China, in 2004. He held Postdoctoral position of mobile station of civil engineering with the Harbin Institute of Technology, Harbin, China, in 2005. He is currently a Professor and the Ph.D. Supervisor with the School of Internet of Things Engineering, Jiangnan University, Wuxi, China. His research interests include optimization theory and application, group decision making, and artificial intelligence.