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Cyclic Hybrid Double-Channel Quantum Communication via Bell-State and GHZ-State in Noisy Environments

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ABSTRACT In this paper, a scheme for cyclic hybrid double-channel quantum communication is proposed by using the product state of three Bell states and three Greenberger-Horne-Zeilinger (GHZ) states as the quantum channel. It shows that Alice teleports a single-qubit state to Bob and prepares a single-qubit state for Charlie, Bob teleports a single-qubit state to Charlie and prepares a single-qubit state for Alice, while Charlie teleports a single-qubit state to Alice and prepares a single-qubit state for Bob. The quantum channel is constructed by using Hadamard (H) and Controlled-NOT ($CNOT$) operations. Participants reconstruct the desired states by performing Bell-state measurements, single-qubit measurements, and unitary transformations. Compared with existing schemes, this new scheme improves the efficiency and capacity of quantum communication because it constructs a cyclic and bidirectional quantum communication and simultaneously supports two communication protocols, quantum teleportation and remote state preparation. Only single-qubit measurements, two-qubit measurements, and basic unitary transformations are utilized in the scheme, so our operation complexity is lower than others. Thus, the scheme is likely to be implemented through physical experiments in the future. Besides this, we discuss the impact of noisy environments (amplitude-damping, phase-damping noise, bit-flip noise, and phase-flip noise) in the scheme and calculate the fidelities of the output states. It is demonstrated that the fidelities only depend on the coefficients of the initial state and the decoherence rate.

INDEX TERMS Amplitude-damping noise, bell-state measurement, phase-damping noise, quantum teleportation, remote state preparation, single-qubit measurement.

I. INTRODUCTION

As a unique quantum phenomenon, quantum entanglement plays a crucial role in quantum communication because it could transmit information between places far apart without a channel connection [1]. Two typical applications of quantum entanglement are quantum teleportation (QT) and remote state preparation (RSP).

QT is a technique for transmitting the information of an unknown quantum state from a sender to a spatially remote receiver by using a pre-shared quantum entangled state as the quantum channel and some classical information.

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The first QT scheme was proposed in 1993 by Bennett et al. [2] who found that a single-qubit state could be transmitted from a sender (Alice) to a receiver (Bob) by using an Einstein-Podolsky-Rosen (EPR) as the quantum channel. The scheme required two bits of classical information to complete the transmission; this scheme was experimentally demonstrated in 1998 [3]. Since then, the technique has attracted a great deal of attention from researchers, and various related schemes have been presented [4]–[7]. To improve the capacity and security of quantum communication, many variants of QT schemes were put forward, such as controlled QT (CQT) [8], [9], bidirectional QT (BQT) [10]–[13], and controlled BQT (CBQT) [14], [15].

In 2019, Zhou et al. [16], [17] proposed two schemes. One realized bidirectional transmitting of three-qubit state and single-qubit by using a six-qubit cluster state [16]. The other presented a cyclic and bidirectional information transmission among three users [17].

As a variant of QT, the RSP scheme was first introduced in 2000 by Lo et al. [18]. Different from QT, the sender knows the complete information of the quantum state in RSP, but the quantum state is not required for measurement [19]. Thus, the cost of classical communication is lower. For this reason, considerable effort has been devoted to the study of this technique from researchers, producing a number of schemes [20]–[27] and many variants of RSP schemes, such as controlled RSP (CRSP) [28]–[30], controlled bidirectional RSP (CBRSP) [31]–[33], and joint RSP (JRSP) [34], [35]. However, these schemes only focus on one-way directional or bidirectional quantum state preparation. In 2018, Zhang et al. [36] and Sang et al. [37] presented two schemes for cyclic RSP by respectively using three multi-qubit GHZ-type states and a ten-qubit entangled state as the quantum channel. In 2018, Fang et al. [38] and Wu et al. [39] each presented one scheme for bidirectional and hybrid quantum communication, which allowed the sender and receiver to exchange their states with each other, whether they knew the state or not. Different from QT or RSP schemes, these two schemes are hybrid protocols for quantum communication, and the channels are multipurpose. However, they are only available for two-party communications.

Based on the schemes in [15], [17], [36]–[39], we propose a scheme for hybrid double-channel quantum communication for three parties by using the product state of three Bell-state and three GHZ-state as the quantum channel. The three participants, Alice, Bob, and Charlie, are all senders and receivers, and there are two communication channels, QT and RSP, between each pair of them. In QT, Alice could teleport a single-qubit state to Bob, Bob could teleport a single-qubit state to Charlie, and Charlie could teleport a single-qubit state to Alice. In RSP, Alice could prepare a single-qubit state for Charlie, Charlie also could prepare a single-qubit state for Bob, and Bob also could prepare a single-qubit state for Alice. Compared with QT or RSP schemes, the scheme is more efficient because it constructs a cyclic hybrid double-channel quantum communication protocol by simultaneously supporting QT and RSP. Also, the scheme is likely to be implemented through physical experiments because only single-qubit measurements, two-qubit measurements, and basic unitary transformations are employed. By increasing the Bell-state and GHZ-state in the quantum channel, the scheme could become a new N-party scheme (N>3). In addition, we discuss the scheme in four noisy environments including amplitude-damping, phase-damping noise, bit-flip noise, and phase-flip noise. The fidelities of the output states are calculated, and it is demonstrated that the fidelities only depend on the coefficients of the initial state and the decoherence rate.

The rest paper is organized as follows. In Section II, we present the scheme in detail, including the construction of the quantum channel, quantum measurement operations, and unitary transformation operations. In Section III, the scheme is discussed in four noisy environments (amplitude-damping, phase-damping noise, bit-flip noise, and phase-flip noise). In Section IV, we analyze the efficiency of the scheme and make a comparison with some existing schemes. Finally, some conclusions and further work are given in Section V.

II. SCHEME FOR CYCLIC HYBRID DOUBLE-CHANNEL QUANTUM COMMUNICATION

In this section, we present the scheme for cyclic hybrid double-channel quantum communication. In this scheme, three participants, Alice, Bob, and Charlie, are assumed to be located at three places. Alice has an unknown single-qubit state $|\phi\rangle_a$ and a known single-qubit state $|\xi_1\rangle$ as follows:

$$|\phi\rangle_a = \alpha_0|0\rangle + \alpha_1|1\rangle \tag{1}$$
$$|\xi_1\rangle = a_0 e^{i\theta_0}|0\rangle + a_1 e^{i\theta_1}|1\rangle \tag{2}$$

where α_0, α_1, a_0, a_1, θ_0, and θ_1 ($\theta_0, \theta_1 \in [0, 2\pi)$) are real numbers with the normalization conditions $|\alpha_0|^2 + |\alpha_1|^2 = 1$ and $|a_0|^2 + |a_1|^2 = 1$. Alice wants to teleport $|\phi\rangle_a$ to Bob and prepare $|\xi_1\rangle$ for Charlie.

Simultaneously, Bob possesses two arbitrary single-qubit states $|\phi\rangle_b$ and $|\xi_2\rangle$, where $|\phi\rangle_b$is an unknown state and $|\xi_2\rangle$is a known state. They are presented as follows:

$$|\phi\rangle_b = \beta_0|0\rangle + \beta_1|1\rangle \tag{3}$$
$$|\xi_2\rangle = b_0 e^{i\varepsilon_0}|0\rangle + b_1 e^{i\varepsilon_1}|1\rangle \tag{4}$$

where β_0, β_1, b_0, b_1, ε_0, and $\varepsilon_1(\varepsilon_0, \varepsilon_1 \in [0, 2\pi)$)are real numbers, satisfying the normalization conditions $|\beta_0|^2 + |\beta_1|^2 = 1$ and $|b_0|^2 + |b_1|^2 = 1$. Bob would like to send $|\phi\rangle_b$ to Charlie and help Alice prepare$|\xi_2\rangle$.

Furthermore, Charlie also owns the following two single-qubit states:

$$|\phi\rangle_c = \lambda_0|0\rangle + \lambda_1|1\rangle \tag{5}$$
$$|\xi_3\rangle = c_0 e^{i\gamma_0}|0\rangle + c_1 e^{i\gamma_1}|1\rangle \tag{6}$$

where $|\phi\rangle_c$ not known by Charlie is to be transmitted to Alice, and $|\xi_3\rangle$ known by Charlie is to be prepared for Bob. In (5), λ_0 and λ_1 are real numbers with the normalization condition$|\lambda_0|^2 + |\lambda_1|^2 = 1$. In (6), c_0, c_1, γ_0, and $\gamma_1(\gamma_0\gamma_1 \in [0, 2\pi)$) are real numbers, where c_0 and c_1 satisfy the normalization condition $|c_0|^2 + |c_1|^2 = 1$.

To realize the cyclic hybrid double-channel quantum communication, first, three participants need to pre-share a quantum channel, which is the product state of three Bell-state and three GHZ-state as shown:

$$\begin{aligned}|\psi\rangle = \frac{1}{8}&(|00\rangle+|11\rangle)_{1,2}\otimes(|00\rangle+|11\rangle)_{3,4}\otimes(|00\rangle+|11\rangle)_{5,6}\\&\otimes(|000\rangle+|111\rangle)_{7,89}\otimes(|000\rangle+|111\rangle)_{10,11,12}\\&\otimes(|000\rangle+|111\rangle)_{13,14,15}.\end{aligned} \tag{7}$$

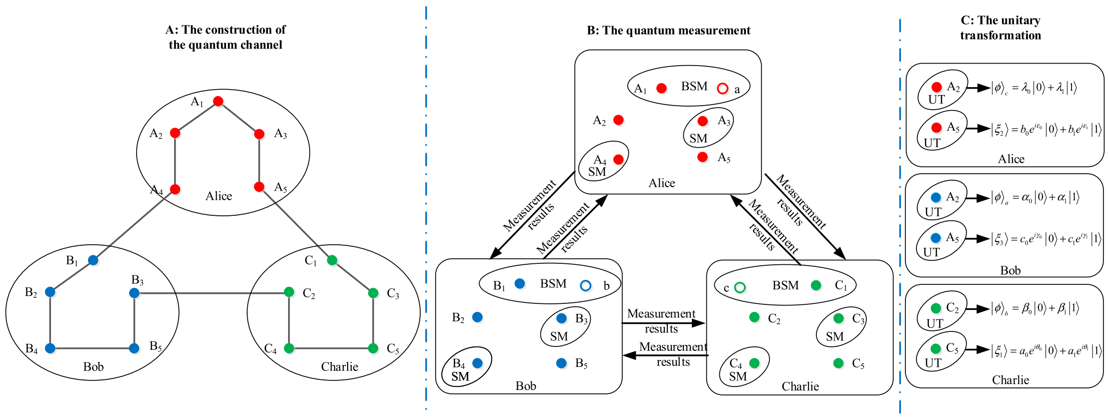

FIGURE 1. **The schematic of the scheme, where BSM denotes Bell-state measurement, SM denotes single-qubit measurement, and UT denotes unitary transformation.**

We assume that qubits 1, 6, 7, 8, and 15 belong to Alice, qubits 2, 3, 12, 13, and 14 belong to Bob, and qubits 4, 5, 9, 10, and 11 belong to Charlie. The quantum channel can be rewritten as follows:

$$|\psi\rangle = \frac{1}{8}(|00\rangle+|11\rangle)_{A_1B_2}\otimes(|00\rangle+|11\rangle)_{B_1C_2}\otimes(|00\rangle+|11\rangle)_{C_1A_2} \otimes(|000\rangle+|111\rangle)_{A_3A_4C_5}\otimes(|000\rangle+|111\rangle)_{C_3C_4B_5} \otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}. \quad (8)$$

Thus, the quantum state of the whole system can be expressed as follows:

$$|\omega\rangle_{18} = |\phi_a\rangle\otimes|\phi_b\rangle\otimes|\phi_c\rangle\otimes|\psi\rangle = \frac{1}{8}(\alpha_0|0\rangle+\alpha_1|1\rangle)_a\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_b\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_c \otimes(|00\rangle+|11\rangle)_{A_1B_2}\otimes(|00\rangle+|11\rangle)_{B_1C_2}\otimes(|00\rangle+|11\rangle)_{C_1A_2} \otimes(|000\rangle+|111\rangle)_{A_3A_4C_5}\otimes(|000\rangle+|111\rangle)_{C_3C_4B_5} \otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}. \quad (9)$$

Next, three participants carry out Bell-state measurements and single-qubit measurements on their own particles, and broadcast the measurement results to others. Finally, they recover the desired states by implementing appropriate unitary transformations. This scheme is schematically illustrated in Fig. 1. The scheme can be divided into three stages including the construction of the quantum channel, the quantum measurement and the unitary transformation.

A. THE CONSTRUCTION OF THE QUANTUM CHANNEL

In the scheme, the product state of three Bell-state and three GHZ-state is used as the quantum channel, which can be constructed by Hadamard (*H*) and Controlled-NOT (*CNOT*) operations. The *H* operation can change a single state into a superposed state as follows:

$$|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle), \quad |1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle). \quad (10)$$

The *CNOT* is a two-qubit gate: one is the control qubit and the other is the target qubit. If the control qubit is set to 0, the target qubit would be left alone. If the control qubit is set to 1, the target qubit would be reversed. The function of the *CNOT* operation can be illustrated by Equation (11):

$$|00\rangle\to|00\rangle,\ |01\rangle\to|01\rangle,\ |10\rangle\to|11\rangle,\ |11\rangle\to|10\rangle. \quad (11)$$

The construction process of the quantum channel is detailed as follows:

Step 1: The product state of fifteen-qubit states initialized to $|0\rangle$ is used as the input state $|\psi_0\rangle$in the quantum circuit, which can be expressed as:

$$|\psi_0\rangle = |0\rangle_1\otimes|0\rangle_2\cdots\otimes|0\rangle_{15}. \quad (12)$$

Step 2: We perform some *H* operations on qubits 1, 3, 5, 7, 10, and 13, and then the quantum state $|\psi_0\rangle$ is converted to $|\psi_1\rangle$ as follows:

$$|\psi_1\rangle = \frac{1}{8}(|00\rangle+|10\rangle)_{1,2}\otimes(|00\rangle+|10\rangle)_{3,4}\otimes(|00\rangle+|10\rangle)_{5,6} \otimes(|000\rangle+|100\rangle)_{7,89}\otimes(|000\rangle+|100\rangle)_{10,11,12} \otimes(|000\rangle+|100\rangle)_{13,14,15}. \quad (13)$$

Step 3: Several *CNOT* operations are performed on the following qubit pairs (1, 2), (3, 4), (5, 6), (7, 8), (7, 9), (10, 11), (10, 12), (13, 14), and (13, 15), where qubits 1, 3, 5, 7, 10, and 13 are used as control qubits, and qubits 2, 4, 6, 8, 9, 11, 12, 14, and 15 are target qubits. Afterwards, the quantum channel is reconstructed as follows:

$$|\psi\rangle = \frac{1}{8}(|00\rangle+|11\rangle)_{1,2}\otimes(|00\rangle+|11\rangle)_{3,4}\otimes(|00\rangle+|11\rangle)_{5,6} \otimes(|000\rangle+|111\rangle)_{7,89}\otimes(|000\rangle+|111\rangle)_{10,11,12} \otimes(|000\rangle+|111\rangle)_{13,14,15}. \quad (14)$$

The process of the construction of the quantum channel is illustrated by Fig. 2.

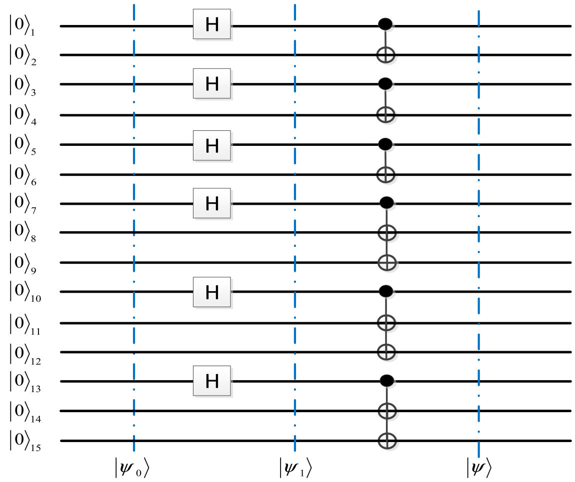

FIGURE 2. **The construction of the quantum channel.**

B. THE QUANTUM MEASUREMENTS

From the schematic given in Fig. 1, there are nine measurement operations in the scheme, namely, the Bell-state measurements and six single-qubit measurements.

Op. 1: Alice performs a Bell-state measurement on her qubits a and A_1 based on the following basis:

$$\begin{aligned} \left|\phi^+\right\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ \left|\phi^-\right\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\ \left|\psi^+\right\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ \left|\psi^-\right\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \end{aligned} \tag{15}$$

The measurement result has four possibilities, $\left|\phi^+\right\rangle_{aA_1}$, $\left|\phi^-\right\rangle_{aA_1}$, $\left|\psi^+\right\rangle_{aA_1}$, and $\left|\psi^-\right\rangle_{aA_1}$. If the measurement operator is $\left|\phi^+\right\rangle_{aA_1}\left\langle\phi^+\right|_{aA_1}$, then the measurement result is $\left|\phi^+\right\rangle_{aA_1}$, and the remaining qubits might collapse into a new state as follows:

$$\begin{aligned} &|\omega\rangle_{16} \\ &= \frac{1}{8\sqrt{2}}(\langle 00|+\langle 11|)_{aA_1} \otimes (\alpha_0|0\rangle+\alpha_1|1\rangle)_a \otimes (|00\rangle+|11\rangle)_{A_1B_2} \\ &\quad \otimes (\beta_0|0\rangle+\beta_1|1\rangle)_b \otimes (\lambda_0|0\rangle+\lambda_1|1\rangle)_c \otimes (|00\rangle+|11\rangle)_{B_1C_2} \\ &\quad \otimes (|00\rangle+|11\rangle)_{C_1A_2} \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5} \\ &\quad \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5} \\ &= \frac{1}{8\sqrt{2}}(\langle 00|+\langle 11|)_{aA_1} \otimes (\alpha_0|00\rangle|0\rangle+\alpha_0|01\rangle|1\rangle \\ &\quad +\alpha_1|10\rangle|0\rangle+\alpha_1|11\rangle|1\rangle)_{aA_1B_2} \otimes (\beta_0|0\rangle+\beta_1|1\rangle)_b \\ &\quad \otimes (\lambda_0|0\rangle+\lambda_1|1\rangle)_c \otimes (|00\rangle+|11\rangle)_{B_1C_2} \otimes (|00\rangle+|11\rangle)_{C_1A_2} \\ &\quad \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5} \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \\ &\quad \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5} = \frac{1}{8\sqrt{2}}(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2} \\ &\quad \otimes (\beta_0|0\rangle+\beta_1|1\rangle)_b \otimes (\lambda_0|0\rangle+\lambda_1|1\rangle)_c \otimes (|00\rangle+|11\rangle)_{B_1C_2} \\ &\quad \otimes (|00\rangle+|11\rangle)_{C_1A_2} \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5} \\ &\quad \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5}. \end{aligned} \tag{16}$$

Op. 2: Bob carries out a Bell-state measurement on his qubits b and B_1 based on the same basis. The outcome also has four possibilities, $\left|\phi^+\right\rangle_{bB_1}$, $\left|\phi^-\right\rangle_{bB_1}$, $\left|\psi^+\right\rangle_{bB_1}$, and $\left|\psi^-\right\rangle_{bB_1}$. If the result is $\left|\phi^+\right\rangle_{bB_1}$, then the rest qubits would collapse into the following state:

$$\begin{aligned} |\omega\rangle_{14} &= \frac{1}{16}((\langle 00|+\langle 11|)_{bB_1} \otimes (\beta_0|0\rangle+\beta_1|1\rangle)_b \otimes (|00\rangle \\ &\quad +|11\rangle)_{B_1,C_2} \otimes (\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2} \otimes (\lambda_0|0\rangle+\lambda_1|1\rangle)_c \\ &\quad \otimes (|00\rangle+|11\rangle)_{C_1A_2} \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5} \\ &\quad \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5} \\ &= \frac{1}{16}((\langle 00|+\langle 11|)_{bB_1} \otimes (\beta_0|00\rangle|0\rangle+\beta_0|01\rangle|1\rangle \\ &\quad +\beta_1|10\rangle|0\rangle+\beta_1|11\rangle|1\rangle)_{bB_1C_2} \otimes (\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2} \\ &\quad \otimes (\lambda_0|0\rangle+\lambda_1|1\rangle)_c \otimes (|00\rangle+|11\rangle)_{C_1A_2} \\ &\quad \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5} \\ &\quad \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5} \\ &= \frac{1}{16}(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2} \otimes (\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2} \\ &\quad \otimes (\lambda_0|0\rangle+\lambda_1|1\rangle)_c \otimes (|00\rangle+|11\rangle)_{C_1A_2} \\ &\quad \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5} \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \\ &\quad \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5}. \end{aligned} \tag{17}$$

Op. 3: Charlie carries out a Bell-state measurement on his qubits c and C_1based on the same bases, and the result can be $\left|\phi^+\right\rangle_{cC_1}$, $\left|\phi^-\right\rangle_{cC_1}$, $\left|\psi^+\right\rangle_{cC_1}$, or $\left|\psi^-\right\rangle_{cC_1}$. If $\left|\phi^+\right\rangle_{cC_1}$ is the result, then the corresponding collapsed states of other qubits would be as follows:

$$\begin{aligned} |\omega\rangle_{12} &= \frac{1}{16}((\langle 00|+\langle 11|)_{cC_1} \otimes (\lambda_0|0\rangle+\lambda_1|1\rangle)_c \\ &\quad \otimes (|00\rangle+|11\rangle)_{C_1A_2} \otimes (\beta_0|0\rangle+\beta_1|1\rangle)_{C_2} \\ &\quad \otimes (\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2} \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5} \\ &\quad \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5} \\ &= \frac{1}{16\sqrt{2}}((\langle 00|+\langle 11|)_{cC_1} \otimes (\lambda_0|00\rangle|0\rangle+\lambda_0|01\rangle|1\rangle) \\ &\quad +\lambda_1|10\rangle|0\rangle+\lambda_1|11\rangle|1\rangle)_{cC_1A_2} \otimes (\beta_0|0\rangle+\beta_1|1\rangle)_{C_2} \\ &\quad \otimes (\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2 \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5}} \\ &\quad \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5} \\ &= \frac{1}{16\sqrt{2}}(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2} \otimes (\beta_0|0\rangle+\beta_1|1\rangle)_{C_2} \\ &\quad \otimes (\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2} \\ &\quad \otimes (|000\rangle+|111\rangle)_{A_3A_4C_5} \otimes (|000\rangle+|111\rangle)_{C_3C_4B_5} \\ &\quad \otimes (|000\rangle+|111\rangle)_{B_3B_4A_5}. \end{aligned} \tag{18}$$

Op. 4, 5: Alice performs the first single-qubit measurement on his qubit A_3 based on the orthogonal basis:

$$\begin{aligned} |\eta_1\rangle &= a_0|0\rangle + a_1|1\rangle \\ |\eta_2\rangle &= a_1|0\rangle - a_0|1\rangle. \end{aligned} \tag{19}$$

The outcome is $|\eta_1\rangle_{A_3}$or $|\eta_2\rangle_{A_3}$with equal probability. If the outcome is $|\eta_1\rangle_{A_3}$, then the collapsed states of other qubits can be written as:

$$\begin{aligned}|\omega\rangle_{11} &= \frac{1}{16\sqrt{2}}(a_0\langle 0|+a_1\langle 1|)_{A_3}\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2}\\ &\quad\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2}\\ &\quad\otimes(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2}\otimes(|000\rangle+|111\rangle)_{A_3A_4C_5}\\ &\quad\otimes(|000\rangle+|111\rangle)_{C_3C_4B_5}\\ &\quad\otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}\\ &= \frac{1}{16\sqrt{2}}(a_0\langle 0|+a_1\langle 1|)_{A_3}\otimes(|0\rangle|00\rangle+|1\rangle|11\rangle)_{A_3A_4C_5}\\ &\quad\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2}\\ &\quad\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2}\otimes(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2}\\ &\quad\otimes(|000\rangle+|111\rangle)_{C_3C_4B_5}\\ &\quad\otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}\\ &= \frac{1}{16\sqrt{2}}\otimes(a_0|00\rangle+a_1|11\rangle)_{A_4C_5}\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2}\\ &\quad\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2}\\ &\quad\otimes(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2}\otimes(|000\rangle+|111\rangle)_{C_3C_4B_5}\\ &\quad\otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}\end{aligned}\tag{20}$$

Subsequently, Alice carries out the second single-qubit measurement on her qubit A_4, where the basis is related to the first measurement result. If it is $|\eta_1\rangle_{A_3}$, then the corresponding measurement basis can be constructed as follows:

$$\begin{aligned}\left|\rho_1^1\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\theta_0}|0\rangle+e^{-i\theta_1}|1\rangle\right)\\ \left|\rho_2^1\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\theta_0}|0\rangle-e^{-i\theta_1}|1\rangle\right).\end{aligned}\tag{21}$$

Otherwise, the measurement basics should be chosen as:

$$\begin{aligned}\left|\rho_1^2\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\theta_1}|0\rangle+e^{-i\theta_0}|1\rangle\right)\\ \left|\rho_2^2\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\theta_1}|0\rangle-e^{-i\theta_0}|1\rangle\right).\end{aligned}\tag{22}$$

After the two single-qubit measurements are performed, the remaining qubits are collapsed into a new state. If the two measurement results are $|\eta_1\rangle_{A_3}$ and $\left|\rho_1^1\right\rangle_{A_4}$, then the collapsed states would be:

$$\begin{aligned}|\omega\rangle_{10} &= \frac{1}{32}\left(e^{i\theta_0}\langle 0|+e^{i\theta_1}\langle 1|\right)\otimes(a_0|0\rangle|0\rangle+a_1|1\rangle|1\rangle)_{A_4C_5}\\ &\quad\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2}\\ &\quad\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2}\otimes(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2}\\ &\quad\otimes(|000\rangle+|111\rangle)_{C_3C_4B_5}\\ &\quad\otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}\\ &= \frac{1}{32}\left(a_0e^{i\theta_0}|0\rangle+a_1e^{i\theta_1}|1\rangle\right)_{C_5}\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2}\\ &\quad\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2}\\ &\quad\otimes(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2}\otimes(|000\rangle+|111\rangle)_{C_3C_4B_5}\\ &\quad\otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}.\end{aligned}\tag{23}$$

Op. 6, 7: At the same time, Charlie carries out two single-qubit measurements on his qubits C_3 and C_4 based on the appropriate measurement basis. The first measurement basis can be written as:

$$\begin{aligned}|\delta_1\rangle &= c_0|0\rangle+c_1|1\rangle\\ |\delta_2\rangle &= c_1|0\rangle-c_0|1\rangle.\end{aligned}\tag{24}$$

Also, the second measurement basis can be constructed according to the first measurement result. If the first basis is $|\delta_1\rangle_{C_3}$, then the measurement basis is given as:

$$\begin{aligned}\left|\mu_1^1\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\gamma_0}|0\rangle+e^{-i\gamma_1}|1\rangle\right)\\ \left|\mu_2^1\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\gamma_0}|0\rangle-e^{-i\gamma_1}|1\rangle\right).\end{aligned}\tag{25}$$

If the first basis is $|\delta_2\rangle_{C_3}$, then the measurement basis is constructed as:

$$\begin{aligned}\left|\mu_1^2\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\gamma_1}|0\rangle+e^{-i\gamma_0}|1\rangle\right)\\ \left|\mu_2^2\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\gamma_1}|0\rangle-e^{-i\gamma_0}|1\rangle\right).\end{aligned}\tag{26}$$

If the two measurement results are $|\delta_1\rangle_{C_3}$ and $\left|\mu_1^1\right\rangle_{C_4}$, then the rest states of the whole system can be collapsed into the following state:

$$\begin{aligned}|\omega\rangle_8 &= \frac{1}{32\sqrt{2}}\left(e^{i\gamma_0}\langle 0|+e^{i\gamma_1}\langle 1|\right)_{C_4}\otimes(c_0\langle 0|+c_1\langle 1|)_{C_3}\\ &\quad\otimes(|0\rangle|0\rangle|0\rangle+|1\rangle|1\rangle|1\rangle)_{C_3C_4B_5}\\ &\quad\otimes\left(a_0e^{i\theta_0}|0\rangle+a_1e^{i\theta_1}|1\rangle\right)_{C_5}\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2}\\ &\quad\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2}\\ &\quad\otimes(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2}\otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}\\ &= \frac{1}{32\sqrt{2}}\left(c_0e^{i\gamma_0}|0\rangle+c_1e^{i\gamma_1}|1\rangle\right)_{B_5}\\ &\quad\otimes\left(a_0e^{i\theta_0}|0\rangle+a_1e^{i\theta_1}|1\rangle\right)_{C_5}\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2}\\ &\quad\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2}\otimes(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2}\\ &\quad\otimes(|000\rangle+|111\rangle)_{B_3B_4A_5}.\end{aligned}\tag{27}$$

Op. 8, 9: Like Alice and Charlie, Bob also needs to implement two single-qubit measurements on his qubits B_3 and B_4. The first measurement basis can be given as:

$$\begin{aligned}|\tau_1\rangle &= b_0|0\rangle+b_1|1\rangle\\ |\tau_2\rangle &= b_1|0\rangle-b_0|1\rangle.\end{aligned}\tag{28}$$

If the measurement result is $|\tau_1\rangle_{B_3}$, then the second measurement basis can be expressed as:

$$\begin{aligned}\left|\chi_1^1\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\beta_0}|0\rangle+e^{-i\beta_1}|1\rangle\right)\\ \left|\chi_2^1\right\rangle &= \frac{1}{\sqrt{2}}\left(e^{-i\beta_0}|0\rangle-e^{-i\beta_1}|1\rangle\right).\end{aligned}\tag{29}$$

TABLE 1. **Partial possible measurement results AND THE corresponding unitary transformations.**

Alice's BSM	Bob's BSM	Charlie's BSM	Alice's SPM1	Bob's SPM1	Charlie's SPM1	Alice's SPM2	Bob's SPM2	Charlie's SPM2	Unitary operation		
									Alice	Bob	Charlie
$\|\phi^+\rangle_{aA_1}$	$\|\phi^+\rangle_{bB_1}$	$\|\phi^+\rangle_{cC_1}$	$\|\eta_1\rangle_{A_3}$	$\|\tau_1\rangle_{B_3}$	$\|\delta_1\rangle_{C_3}$	$\|\rho_1^1\rangle_{A_4}$	$\|\chi_1^1\rangle_{B_4}$	$\|\mu_1^1\rangle_{C_4}$	$I_{A_2}\otimes I_{A_5}$	$I_{B_2}\otimes I_{B_5}$	$I_{C_2}\otimes I_{C_5}$
						$\|\rho_2^1\rangle_{A_4}$	$\|\chi_2^1\rangle_{B_4}$	$\|\mu_2^1\rangle_{C_4}$	$I_{A_2}\otimes\sigma^z_{A_5}$	$I_{B_2}\otimes\sigma^z_{B_5}$	$I_{C_2}\otimes\sigma^z_{C_5}$
			$\|\eta_2\rangle_{A_3}$	$\|\tau_2\rangle_{B_3}$	$\|\delta_2\rangle_{C_3}$	$\|\rho_1^2\rangle_{A_4}$	$\|\chi_1^2\rangle_{B_4}$	$\|\mu_2^1\rangle_{C_4}$	$I_{A_2}\otimes\sigma^x_{A_5}$	$I_{B_2}\otimes\sigma^x_{B_5}$	$I_{C_2}\otimes\sigma^x_{C_5}$
						$\|\rho_2^2\rangle_{A_4}$	$\|\chi_2^2\rangle_{B_4}$	$\|\mu_2^2\rangle_{C_4}$	$I_{A_2}\otimes\sigma^{xz}_{A_5}$	$I_{B_2}\otimes\sigma^{xz}_{B_5}$	$I_{C_2}\otimes\sigma^{xz}_{C_5}$
$\|\phi^-\rangle_{aA_1}$	$\|\phi^-\rangle_{bB_1}$	$\|\phi^-\rangle_{cC_1}$	$\|\eta_1\rangle_{A_3}$	$\|\tau_1\rangle_{B_3}$	$\|\delta_1\rangle_{C_3}$	$\|\rho_1^1\rangle_{A_4}$	$\|\chi_1^1\rangle_{B_4}$	$\|\mu_1^1\rangle_{C_4}$	$\sigma^z_{A_2}\otimes I_{A_5}$	$\sigma^z_{B_2}\otimes I_{B_5}$	$\sigma^z_{C_2}\otimes I_{C_5}$
						$\|\rho_2^1\rangle_{A_4}$	$\|\chi_2^1\rangle_{B_4}$	$\|\mu_2^1\rangle_{C_4}$	$\sigma^z_{A_2}\otimes\sigma^z_{A_5}$	$\sigma^z_{B_2}\otimes\sigma^z_{B_5}$	$\sigma^z_{C_2}\otimes\sigma^z_{C_5}$
			$\|\eta_2\rangle_{A_3}$	$\|\tau_2\rangle_{B_3}$	$\|\delta_2\rangle_{C_3}$	$\|\rho_1^2\rangle_{A_4}$	$\|\chi_1^2\rangle_{B_4}$	$\|\mu_2^1\rangle_{C_4}$	$\sigma^z_{A_2}\otimes\sigma^x_{A_5}$	$\sigma^z_{B_2}\otimes\sigma^x_{B_5}$	$\sigma^z_{C_2}\otimes\sigma^x_{C_5}$
						$\|\rho_2^2\rangle_{A_4}$	$\|\chi_2^2\rangle_{B_4}$	$\|\mu_2^2\rangle_{C_4}$	$\sigma^z_{A_2}\otimes\sigma^{xz}_{A_5}$	$\sigma^z_{B_2}\otimes\sigma^{xz}_{B_5}$	$\sigma^z_{C_2}\otimes\sigma^{xz}_{C_5}$
$\|\psi^+\rangle_{aA_1}$	$\|\psi^+\rangle_{bB_1}$	$\|\psi^+\rangle_{cC_1}$	$\|\eta_1\rangle_{A_3}$	$\|\tau_1\rangle_{B_3}$	$\|\delta_1\rangle_{C_3}$	$\|\rho_1^1\rangle_{A_4}$	$\|\chi_1^1\rangle_{B_4}$	$\|\mu_1^1\rangle_{C_4}$	$\sigma^x_{A_2}\otimes I_{A_5}$	$\sigma^x_{B_2}\otimes I_{B_5}$	$\sigma^x_{C_2}\otimes I_{C_5}$
						$\|\rho_2^1\rangle_{A_4}$	$\|\chi_2^1\rangle_{B_4}$	$\|\mu_2^1\rangle_{C_4}$	$\sigma^x_{A_2}\otimes\sigma^z_{A_5}$	$\sigma^x_{B_2}\otimes\sigma^z_{B_5}$	$\sigma^x_{C_2}\otimes\sigma^z_{C_5}$
			$\|\eta_2\rangle_{A_3}$	$\|\tau_2\rangle_{B_3}$	$\|\delta_2\rangle_{C_3}$	$\|\rho_1^2\rangle_{A_4}$	$\|\chi_1^2\rangle_{B_4}$	$\|\mu_2^1\rangle_{C_4}$	$\sigma^x_{A_2}\otimes\sigma^x_{A_5}$	$\sigma^x_{B_2}\otimes\sigma^x_{B_5}$	$\sigma^x_{C_2}\otimes\sigma^x_{C_5}$
						$\|\rho_2^2\rangle_{A_4}$	$\|\chi_2^2\rangle_{B_4}$	$\|\mu_2^2\rangle_{C_4}$	$\sigma^x_{A_2}\otimes\sigma^{xz}_{A_5}$	$\sigma^x_{B_2}\otimes\sigma^{xz}_{B_5}$	$\sigma^x_{C_2}\otimes\sigma^{xz}_{C_5}$
$\|\psi^-\rangle_{aA_1}$	$\|\psi^-\rangle_{bB_1}$	$\|\psi^-\rangle_{cC_1}$	$\|\eta_1\rangle_{A_3}$	$\|\tau_1\rangle_{B_3}$	$\|\delta_1\rangle_{C_3}$	$\|\rho_1^1\rangle_{A_4}$	$\|\chi_1^1\rangle_{B_4}$	$\|\mu_1^1\rangle_{C_4}$	$\sigma^{xz}_{A_2}\otimes I_{A_5}$	$\sigma^{xz}_{B_2}\otimes I_{B_5}$	$\sigma^{xz}_{C_2}\otimes I_{C_5}$
						$\|\rho_2^1\rangle_{A_4}$	$\|\chi_2^1\rangle_{B_4}$	$\|\mu_2^1\rangle_{C_4}$	$\sigma^{xz}_{A_2}\otimes\sigma^z_{A_5}$	$\sigma^{xz}_{B_2}\otimes\sigma^z_{B_5}$	$\sigma^{xz}_{C_2}\otimes\sigma^z_{C_5}$
			$\|\eta_2\rangle_{A_3}$	$\|\tau_2\rangle_{B_3}$	$\|\delta_2\rangle_{C_3}$	$\|\rho_1^2\rangle_{A_4}$	$\|\chi_1^2\rangle_{B_4}$	$\|\mu_2^1\rangle_{C_4}$	$\sigma^{xz}_{A_2}\otimes\sigma^x_{A_5}$	$\sigma^{xz}_{B_2}\otimes\sigma^x_{B_5}$	$\sigma^{xz}_{C_2}\otimes\sigma^x_{C_5}$
						$\|\rho_2^2\rangle_{A_4}$	$\|\chi_2^2\rangle_{B_4}$	$\|\mu_2^2\rangle_{C_4}$	$\sigma^{xz}_{A_2}\otimes\sigma^{xz}_{A_5}$	$\sigma^{xz}_{B_2}\otimes\sigma^{xz}_{B_5}$	$\sigma^{xz}_{C_2}\otimes\sigma^{xz}_{C_5}$

Otherwise, the measurement basis can be written as:

$$\left|\chi_1^2\right\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\beta_1}|0\rangle + e^{-i\beta_0}|1\rangle\right)$$
$$\left|\chi_2^2\right\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\beta_1}|0\rangle - e^{-i\beta_0}|1\rangle\right). \quad (30)$$

After the two single-qubit measurements are performed, the remaining states are collapsed into a new state. If two SM results are $|\tau_1\rangle_{B_3}$ and $\left|\chi_1^1\right\rangle_{B_4}$, then the collapsed state can be written as:

$$\begin{aligned}
|\omega\rangle_6 = & \frac{1}{64}(e^{i\beta_0}\langle 0| + e^{i\beta_1}\langle 1|)_{B_4} \otimes (b_0\langle 0| + b_1\langle 1|)_{B_3} \\
& \otimes (|0\rangle|0\rangle|0\rangle + |1\rangle|1\rangle|1\rangle)_{B_3B_4A_5} \\
& \otimes (c_0e^{i\gamma_0}|0\rangle + c_1e^{i\gamma_1}|1\rangle)_{B_5} \otimes (a_0e^{i\theta_0}|0\rangle + a_1e^{i\theta_1}|1\rangle)_{C_5} \\
& \otimes (\lambda_0|0\rangle + \lambda_1|1\rangle)_{A_2} \\
& \otimes (\beta_0|0\rangle + \beta_1|1\rangle)_{C_2} \otimes (\alpha_0|0\rangle + \alpha_1|1\rangle)_{B_2} \\
= & \frac{1}{64}(b_0e^{i\beta_0}|0\rangle + b_1e^{i\beta_1}|1\rangle)_{A_5} \\
& \otimes (c_0e^{i\gamma_0}|0\rangle + c_1e^{i\gamma_1}|1\rangle)_{B_5} \otimes (a_0e^{i\theta_0}|0\rangle + a_1e^{i\theta_1}|1\rangle)_{C_5} \\
& \otimes (\lambda_0|0\rangle + \lambda_1|1\rangle)_{A_2} \otimes (\beta_0|0\rangle + \beta_1|1\rangle)_{C_2} \\
& \otimes (\alpha_0|0\rangle + \alpha_1|1\rangle)_{B_2}. \quad (31)
\end{aligned}$$

Notably, for each participant, after completing the measurement operations, they need to announce their outcomes to the others via the classical channels. It is worth emphasizing that secret transmissions of the results is unnecessary as the results make no sense to any parties other than the receiver.

C. THE UNITARY TRANSFORMATIONS

After receiving all of the measurement results from the others, each participant reconstructs the desired quantum state by performing appropriate unitary transformations. To realize the end, several basic unitary transformations are employed, such as I, σ^x, and σ^z. Their matrix expressions and functions are presented as follows:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma^x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$I|0\rangle \rightarrow |0\rangle, \quad I|1\rangle \rightarrow |1\rangle$$
$$\sigma^x 0\rangle \rightarrow |1\rangle, \quad \sigma^x|1\rangle \rightarrow |0\rangle$$
$$\sigma^z 0\rangle \rightarrow |0\rangle, \quad \sigma^z|1\rangle \rightarrow -|1\rangle. \quad (32)$$

The quantum circuit of the scheme is presented in Fig. 3.

Fig. 3 shows nine quantum measurements that are performed in this scheme, which produce4^6 measurement results. For all outcomes, the appropriate unitary

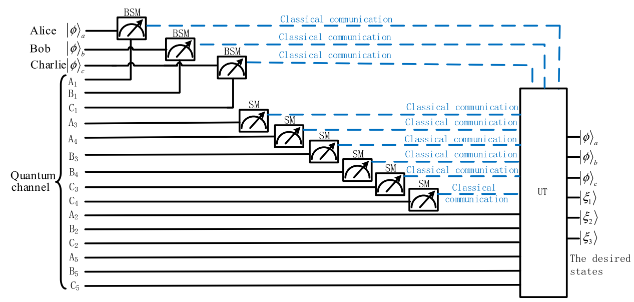

FIGURE 3. The quantum circuit of the scheme.

transformation could always be found to reconstruct the desired state. That is, the scheme could achieve a unit success possibility. However, the possible outcomes are too many to be presented entirely in this paper. Partial measurement results and the corresponding unitary transformations are presented in Table 1. For instance, if the nine measurement results are $|\phi^+\rangle_{a,A_1}$, $|\phi^+\rangle_{b,B_1}$, $|\phi^+\rangle_{c,C_1}$, $|\eta_1\rangle_{A_3}$, $|\rho_1^1\rangle_{A_4}$, $|\tau_1\rangle_{B_3}$, $|\chi_1^1\rangle_{B_4}$, $|\delta_1\rangle_{C_3}$, and $|\mu_1^1\rangle_{C_4}$, respectively. The unitary transformation $I_{B_2} \otimes I_{B_5}$ needs to be implemented by Alice to reconstruct the desired states $|\phi\rangle_c$ and $|\xi\rangle_b$, the unitary transformation $I_{B_2} \otimes I_{B_5}$ should be performed by Bob to recover the desired states $|\phi\rangle_a$ and $|\xi\rangle_c$, meanwhile, Charlie obtains the states $|\phi\rangle_b$ and $|\xi\rangle_a$ by completing the unitary transformation $I_{C_2} \otimes I_{c_5}$.

III. THE SCHEME IN NOISY ENVIRONMENTS

In the actual communication, it is impossible to have an ideal environment (noiseless environment). The communication channel is bound to be disturbed by noises. In this section, we discuss our scheme in four noisy environments, including amplitude-damping, phase-damping noise, bit-flip noise, and phase-flip noise. They can be expressed by Kraus operators [41] given in (33)- (36).

$$E_0^A = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-P_A} \end{bmatrix}, \quad E_1^A = \begin{bmatrix} 0 & \sqrt{P_A} \\ 0 & 0 \end{bmatrix} \tag{33}$$

$$E_0^P = \sqrt{1-P_P}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_1^P = \sqrt{P_P}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$E_2^P = \sqrt{P_P}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{34}$$

$$E_0^B = \sqrt{1-P_B}I = \sqrt{1-P_B}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$E_1^B = \sqrt{P_B}X = \sqrt{P_B}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{35}$$

$$E_0^W = \sqrt{1-P_W}I = \sqrt{1-P_W}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$E_1^W = \sqrt{P_W}Z = \sqrt{P_W}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{36}$$

where $P_A\,(P_A \in [0,1])$, $P_P\,(P_P \in [0,1])$ $P_B\,(P_B \in [0,1])$, and $P_W\,(P_W \in [0,1])$are the decoherence rates for amplitude-damping noise, phase-damping noise, bit-flip noise, and phase-flip noise, respectively. If the quantum channel pre-shared by participants is constructed by an institution named the qubits distribution center (QDC), the QDC distributes qubits A_1, A_2, A_3, A_4, and A_5 to Alice, qubits B_1, B_2, B_3, B_4, and B_5 to Bob, and qubits C_1, C_2, C_3, C_4, and C_5 to Charlie, after the preparation of the quantum channel. The quantum channel $|\psi\rangle$, considered in previous sections, is in a noiseless environment. The channel is a pure state, and its density matrix could be written as $\rho = |\psi\rangle\langle\psi|$. However, when it is affected by noisy environments, the pure state could be converted to a mixed state. The corresponding density matrix can be written as follows:

$$\begin{aligned}
&\Gamma^r(\rho) \\
&= \sum_m (E_m^r)_{A_1}(E_m^r)_{B_2}(E_m^r)_{B_1}(E_m^r)_{C_2}(E_m^r)_{C_1}(E_m^r)_{A_2} \\
&\quad\times (E_m^r)_{A_3}(E_m^r)_{A_4}(E_m^r)_{C_5}(E_m^r)_{B_3}(E_m^r)_{B_4}(E_m^r)_{A_5} \\
&\quad\times (E_m^r)_{C_3}(E_m^r)_{C_4}(E_m^r)_{B_5}\,\rho\,(E_m^r)^\dagger_{A_1}(E_m^r)^\dagger_{B_2}(E_m^r)^\dagger_{B_1} \\
&\quad\times (E_m^r)^\dagger_{C_2}(E_m^r)^\dagger_{C_1}(E_m^r)^\dagger_{A_2}(E_m^r)^\dagger_{A_3}(E_m^r)^\dagger_{A_4}(E_m^r)^\dagger_{C_5} \\
&\quad\times (E_m^r)^\dagger_{B_3}(E_m^r)^\dagger_{B_4}(E_m^r)^\dagger_{A_5}(E_m^r)^\dagger_{C_3}(E_m^r)^\dagger_{C_4}(E_m^r)^\dagger_{B_5}
\end{aligned} \tag{37}$$

where $r \in \{A, P, B, W\}$. If $r = A$, i.e., for amplitude-damping noise, then $m = 0, 1$; if $r = P$, i.e., for phase-damping noise, then $m = 0, 1, 2$; if $r = B$, i.e., for bit-flip noise, then $m = 0, 1$; and if $r = W$, i.e., for phase-flip noise, then $m = 0, 1$. Four concrete density matrices are shown as follows:

$$\begin{aligned}
&\Gamma^A(\rho) \\
&= \frac{1}{8}\{[|00\rangle + (1-P_A)|11\rangle]_{A_1B_2} \\
&\qquad \times [|00\rangle + (1-P_A)|11\rangle]_{B_1C_2} \\
&\quad \times [|00\rangle + (1-P_A)|11\rangle]_{C_1A_2} \\
&\quad \times \left[|000\rangle + \sqrt{(1-P_A)^3}|111\rangle\right]_{A_3A_4C_5} \\
&\quad \times \left[|000\rangle + \sqrt{(1-P_A)^3}|111\rangle\right]_{B_3B_4A_5} \\
&\quad \times \left[|000\rangle + \sqrt{(1-P_A)^3}|111\rangle\right]_{C_3C_4B_5} \\
&\quad \times [\langle 00| + (1-P_A)\langle 11|]_{A_1B_2} \times [\langle 00| + (1-P_A)\langle 11|]_{B_1C_2} \\
&\quad \times [\langle 00| + (1-P_A)\langle 11|]_{C_1,A_2} \\
&\quad \times \left[\langle 000| + \sqrt{(1-P_A)^3}\langle 111|\right]_{A_3A_4C_5} \\
&\quad \times \left[\langle 000| + \sqrt{(1-P_A)^3}\langle 111|\right]_{B_3B_4A_5} \\
&\quad \times \left[\langle 000| + \sqrt{(1-P_A)^3}\langle 111|\right]_{C_3C_4B_5} \\
&\quad + P_A^{15}\,(|000000000000000\rangle \\
&\quad \langle 000000000000000|)_{A_1B_2B_1C_2C_1A_2A_3A_4C_5B_3B_4A_5C_3C_4B_5}\}
\end{aligned} \tag{38}$$

$$\begin{aligned}&\Gamma^{P}(\rho)\\&=\frac{1}{8}\Big\{\left(1-P_{P}\right)^{15}[|00\rangle+|11\rangle]_{A_1B_2}\times[|00\rangle+|11\rangle]_{B_1C_2}\\&\quad\times[|00\rangle+|11\rangle]_{C_1A_2}\times[|000\rangle+|111\rangle]_{A_3A_4C_5}\\&\quad\times[|000\rangle+|111\rangle]_{B_3B_4A_5}\times[|000\rangle+|111\rangle]_{C_3C_4B_5}\\&\quad\times[\langle 00|+\langle 11|]_{A_1B_2}\times[\langle 00|+\langle 11|]_{B_1C_2}\\&\quad\times[\langle 00|+\langle 11|]_{C_1A_2}\times[\langle 000|+\langle 111|]_{A_3A_4C_5}\\&\quad\times[\langle 000|+\langle 111|]_{B_3B_4A_5}\times[\langle 000|+\langle 111|]_{C_3C_4B_5}\\&\quad+P_{P}^{15}(|000000000000000\rangle\\&\quad\langle 000000000000000|)_{A_1B_2B_1C_2C_1A_2A_3A_4C_5B_3B_4A_5C_3C_4B_5}\\&\quad+P_{P}^{15}(|111111111111111\rangle\\&\quad\langle 111111111111111|)_{A_1B_2B_1C_2C_1A_2A_3A_4C_5B_3B_4A_5C_3C_4B_5}\Big\}\end{aligned}\tag{39}$$

$$\begin{aligned}&\Gamma^{B}(\rho)\\&=\frac{1}{8}\Big\{\left(1-P_{B}\right)^{15}[|00\rangle+|11\rangle]_{A_1B_2}\times[|00\rangle+|11\rangle]_{B_1C_2}\\&\quad\times[|00\rangle+|11\rangle]_{C_1A_2}\times[|000\rangle+|111\rangle]_{A_3A_4C_5}\\&\quad\times[|000\rangle+|111\rangle]_{B_3B_4A_5}\times[|000\rangle+|111\rangle]_{C_3C_4B_5}\\&\quad\times[\langle 00|+\langle 11|]_{A_1B_2}\times[\langle 00|+\langle 11|]_{B_1C_2}\\&\quad\times[\langle 00|+\langle 11|]_{C_1A_2}\times[\langle 000|+\langle 111|]_{A_3A_4C_5}\\&\quad\times[\langle 000|+\langle 111|]_{B_3B_4A_5}\times[\langle 000|+\langle 111|]_{C_3C_4B_5}\\&\quad+P_{B}^{15}[|11\rangle+|00\rangle]_{A_1B_2}\times[|11\rangle+|00\rangle]_{B_1C_2}\\&\quad\times[|11\rangle+|00\rangle]_{C_1A_2}\times[|111\rangle+|000\rangle]_{A_3A_4C_5}\\&\quad\times[|111\rangle+|000\rangle]_{B_3B_4A_5}\times[|111\rangle+|000\rangle]_{C_3C_4B_5}\\&\quad\times[\langle 11|+\langle 00|]_{A_1B_2}\times[\langle 11|+\langle 00|]_{B_1C_2}\\&\quad\times[\langle 11|+\langle 00|]_{C_1A_2}\times[\langle 111|+\langle 000|]_{A_3A_4C_5}\\&\quad\times[\langle 111|+\langle 000|]_{B_3B_4A_5}\times[\langle 111|+\langle 000|]_{C_3C_4B_5}\Big\}\end{aligned}\tag{40}$$

$$\begin{aligned}&\Gamma^{W}(\rho)\\&=\frac{1}{8}\Big\{\left(1-P_{W}\right)^{15}[|00\rangle+|11\rangle]_{A_1B_2}\times[|00\rangle+|11\rangle]_{B_1C_2}\\&\quad\times[|00\rangle+|11\rangle]_{C_1A_2}\times[|000\rangle+|111\rangle]_{A_3A_4C_5}\\&\quad\times[|000\rangle+|111\rangle]_{B_3B_4A_5}\times[|000\rangle+|111\rangle]_{C_3C_4B_5}\\&\quad\times[\langle 00|+\langle 11|]_{A_1B_2}\times[\langle 00|+\langle 11|]_{B_1C_2}\\&\quad\times[\langle 00|+\langle 11|]_{C_1A_2}\times[\langle 000|+\langle 111|]_{A_3A_4C_5}\\&\quad\times[\langle 000|+\langle 111|]_{B_3B_4A_5}\times[\langle 000|+\langle 111|]_{C_3C_4B_5}\\&\quad+P_{W}^{15}[|00\rangle+|11\rangle]_{A_1B_2}\times[|00\rangle+|11\rangle]_{B_1C_2}\\&\quad\times[|00\rangle+|11\rangle]_{C_1A_2}\times[|000\rangle-|111\rangle]_{A_3A_4C_5}\\&\quad\times[|000\rangle-|111\rangle]_{B_3B_4A_5}\times[|000\rangle-|111\rangle]_{C_3C_4B_5}\\&\quad\times[\langle 00|+\langle 11|]_{A_1B_2}\times[\langle 00|+\langle 11|]_{B_1C_2}\\&\quad\times[\langle 00|+\langle 11|]_{C_1A_2}\times[\langle 000|-\langle 111|]_{A_3A_4C_5}\\&\quad\times[\langle 000|-\langle 111|]_{B_3B_4A_5}\times[\langle 000|-\langle 111|]_{C_3C_4B_5}\Big\}\end{aligned}\tag{41}$$

For reconstructing the desired state, three participants have to perform three Bell-state measurements, and six single-qubit measurements based on appropriate measurement bases. The desired states were then reconstructed via unitary transformations. If the nine measurement bases are $\left(|\phi^+\rangle\langle\phi^+|\right)_{aA_1}$, $\left(|\phi^+\rangle\langle\phi^+|\right)_{bB_1}$, $\left(|\phi^+\rangle\langle\phi^+|\right)_{cC_1}$, $(|\eta_1\rangle\langle\eta_1|)_{A_3}$, $\left(|\rho_1^1\rangle\langle\rho_1^1|\right)_{A_4}$, $(|\tau_1\rangle\langle\tau_1|)_{B_3}$, $\left(|\chi_1^1\rangle\langle\chi_1^1|\right)_{B_4}$, $(|\delta_1\rangle\langle\delta_1|)_{C_3}$, and $\left(|\mu_1^1\rangle\langle\mu_1^1|\right)_{C_4}$, then the corresponding unitary transformations would be $I_{A_2}\otimes I_{A_5}\otimes I_{B_2}\otimes I_{B_5}\otimes I_{C_2}\otimes I_{C_5}$ to recover the desired states. If the scheme is implemented in amplitude-damping, phase-damping, bit-flip, and phase-flip noisy environments, then the output states can be obtained as $\left(\rho_{out}^{A}\right)_{A_2A_5B_2B_5C_2C_5}$, $\left(\rho_{out}^{P}\right)_{A_2A_5B_2B_5C_2C_5}$, $\left(\rho_{out}^{B}\right)_{A_2A_5B_2B_5C_2C_5}$, and $\left(\rho_{out}^{W}\right)_{A_2A_5B_2B_5C_2C_5}$ after performing these operations.

$$\begin{aligned}&\left(\rho_{out}^{A}\right)_{A_2A_5B_2B_5C_2C_5}\\&=\{[\alpha_0|0\rangle+(1-P_A)\alpha_1|1\rangle]_{B_2}\times[\beta_0|0\rangle+(1-P_A)\beta_1|1\rangle]_{C_2}\\&\quad\times[\lambda_0|0\rangle+(1-P_A)\,\lambda_1|1\rangle]_{A_2}\\&\quad\times\left[a_0e^{i\theta_0}|0\rangle+\sqrt{(1-P_A)^3}a_1e^{i\theta_1}|1\rangle\right]_{C_5}\\&\quad\times\left[b_0e^{i\varepsilon_0}|0\rangle+\sqrt{(1-P_A)^3}b_1e^{i\varepsilon_1}|1\rangle\right]_{A_5}\\&\quad\times\left[c_0e^{i\gamma_0}|0\rangle+\sqrt{(1-P_A)^3}c_1e^{i\gamma_1}|1\rangle\right]_{B_5}\\&\quad\times[\alpha_0\langle 0|+(1-P_A)\alpha_1\langle 1|]_{B_2}\times[\beta_0\langle 0|+(1-P_A)\beta_1\langle 1|]_{C_2}\\&\quad\times[\lambda_0\langle 0|+(1-P_A)\,\lambda_1\langle 1|]_{A_2}\\&\quad\times\left[a_0e^{i\theta_0}\langle 0|+\sqrt{(1-P_A)^3}a_1e^{i\theta_1}\langle 1|\right]_{C_5}\\&\quad\times\left[b_0e^{i\varepsilon_0}\langle 0|+\sqrt{(1-P_A)^3}b_1e^{i\varepsilon_1}\langle 1|\right]_{A_5}\\&\quad\times\left[c_0e^{i\gamma_0}\langle 0|+\sqrt{(1-P_A)^3}c_1e^{i\gamma_1}\langle 1|\right]_{B_5}\\&\quad+P_{A}^{15}\alpha_0^2\beta_0^2\lambda_0^2a_0^2b_0^2c_0^2\,(|000000\rangle\langle 000000|)_{B_2C_2A_2C_5A_5B_5}\Big\}\end{aligned}\tag{42}$$

$$\begin{aligned}&\left(\rho_{out}^{P}\right)_{A_2A_5B_2B_5C_2C_5}\\&=\Big\{\left(1-P_{P}\right)^{15}[\alpha_0|0\rangle+\alpha_1|1\rangle]_{B_2}\times[\beta_0|0\rangle+\beta_1|1\rangle]_{C_2}\\&\quad\times[\lambda_0|0\rangle+\lambda_1|1\rangle]_{A_2}\times\left[a_0e^{i\theta_0}|0\rangle+a_1e^{i\theta_1}|1\rangle\right]_{C_5}\\&\quad\times\left[b_0e^{i\varepsilon_0}|0\rangle+b_1e^{i\varepsilon_1}|1\rangle\right]_{A_5}\\&\quad\times\left[c_0e^{i\gamma_0}|0\rangle+c_1e^{i\gamma_1}|1\rangle\right]_{B_5}\times[\alpha_0\langle 0|+\alpha_1\langle 1|]_{B_2}\\&\quad\times[\beta_0\langle 0|+\beta_1\langle 1|]_{C_2}\\&\quad\times[\lambda_0\langle 0|+\lambda_1\langle 1|]_{A_2}\times\left[a_0e^{-i\theta_0}\langle 0|+a_1e^{-i\theta_1}\langle 1|\right]_{C_5}\\&\quad\times\left[b_0e^{-i\varepsilon_0}\langle 0|+b_1e^{-i\varepsilon_1}\langle 1|\right]_{A_5}\\&\quad\times\left[c_0e^{-i\gamma_0}\langle 0|+c_1e^{-i\gamma_1}\langle 1|\right]_{B_5}\\&\quad+P_{P}^{15}\alpha_0^2\beta_0^2\lambda_0^2a_0^2b_0^2c_0^2\,(|000000\rangle\langle 000000|)_{B_2C_2A_2C_5A_5B_5}\\&\quad+P_{P}^{15}\alpha_1^2\beta_1^2\lambda_1^2a_1^2b_1^2c_1^2\,(|111111\rangle\langle 111111|)_{B_2C_2A_2C_5A_5B_5}\Big\}\end{aligned}\tag{43}$$

$$\begin{aligned}
&\left(\rho_{out}^{B}\right)_{A_2A_5B_2B_5C_2C_5}\\
&= \Big\{\left(1-P_{B}\right)^{15}[\alpha_0|0\rangle+\alpha_1|1\rangle]_{B_2}\times[\beta_0|0\rangle+\beta_1|1\rangle]_{C_2}\\
&\quad\times[\lambda_0|0\rangle+\lambda_1|1\rangle]_{A_2}\times\left[a_0e^{i\theta_0}|0\rangle+a_1e^{i\theta_1}|1\rangle\right]_{C_5}\\
&\quad\times\left[b_0e^{i\varepsilon_0}|0\rangle+b_1e^{i\varepsilon_1}|1\rangle\right]_{A_5}\\
&\quad\times\left[c_0e^{i\gamma_0}|0\rangle+c_1e^{i\gamma_1}|1\rangle\right]_{B_5}\times[\alpha_0\langle 0|+\alpha_1\langle 1|]_{B_2}\\
&\quad\times[\beta_0\langle 0|+\beta_1\langle 1|]_{C_2}\\
&\quad\times[\lambda_0\langle 0|+\lambda_1\langle 1|]_{A_2}\times\left[a_0e^{-i\theta_0}\langle 0|+a_1e^{-i\theta_1}\langle 1|\right]_{C_5}\\
&\quad\times\left[b_0e^{-i\varepsilon_0}\langle 0|+b_1e^{-i\varepsilon_1}\langle 1|\right]_{A_5}\\
&\quad\times\left[c_0e^{-i\gamma_0}\langle 0|+c_1e^{-i\gamma_1}\langle 1|\right]_{B_5}+P_{P}^{15}[\alpha_0|0\rangle+\alpha_1|1\rangle]_{B_2}\\
&\quad\times[\beta_0|0\rangle+\beta_1|1\rangle]_{C_2}\\
&\quad\times[\lambda_0|0\rangle+\lambda_1|1\rangle]_{A_2}\times\left[a_0e^{i\theta_0}|0\rangle+a_1e^{i\theta_1}|1\rangle\right]_{C_5}\\
&\quad\times\left[b_0e^{i\varepsilon_0}|0\rangle+b_1e^{i\varepsilon_1}|1\rangle\right]_{A_5}\times\left[c_0e^{i\gamma_0}|0\rangle+c_1e^{i\gamma_1}|1\rangle\right]_{B_5}\\
&\quad\times[\alpha_0\langle 0|+\alpha_1\langle 1|]_{B_2}\times[\beta_0\langle 0|+\beta_1\langle 1|]_{C_2}\\
&\quad\times[\lambda_0\langle 0|+\lambda_1\langle 1|]_{A_2}\times\left[a_0e^{-i\theta_0}\langle 0|+a_1e^{-i\theta_1}\langle 1|\right]_{C_5}\\
&\quad\times\left[b_0e^{-i\varepsilon_0}\langle 0|+b_1e^{-i\varepsilon_1}\langle 1|\right]_{A_5}\\
&\quad\times\left[c_0e^{-i\gamma_0}\langle 0|+c_1e^{-i\gamma_1}\langle 1|\right]_{B_5}\Big\} \qquad (44)
\end{aligned}$$

$$\begin{aligned}
&\left(\rho_{out}^{W}\right)_{A_2A_5B_2B_5C_2C_5}\\
&= \Big\{\left(1-P_{W}\right)^{15}[\alpha_0|0\rangle+\alpha_1|1\rangle]_{B_2}\times[\beta_0|0\rangle+\beta_1|1\rangle]_{C_2}\\
&\quad\times[\lambda_0|0\rangle+\lambda_1|1\rangle]_{A_2}\times\left[a_0e^{i\theta_0}|0\rangle+a_1e^{i\theta_1}|1\rangle\right]_{C_5}\\
&\quad\times\left[b_0e^{i\varepsilon_0}|0\rangle+b_1e^{i\varepsilon_1}|1\rangle\right]_{A_5}\\
&\quad\times\left[c_0e^{i\gamma_0}|0\rangle+c_1e^{i\gamma_1}|1\rangle\right]_{B_5}\times[\alpha_0\langle 0|+\alpha_1\langle 1|]_{B_2}\\
&\quad\times[\beta_0\langle 0|+\beta_1\langle 1|]_{C_2}\times[\lambda_0\langle 0|+\lambda_1\langle 1|]_{A_2}\\
&\quad\times\left[a_0e^{-i\theta_0}\langle 0|+a_1e^{-i\theta_1}\langle 1|\right]_{C_5}\\
&\quad\times\left[b_0e^{-i\varepsilon_0}\langle 0|+b_1e^{-i\varepsilon_1}\langle 1|\right]_{A_5}\\
&\quad\times\left[c_0e^{-i\gamma_0}\langle 0|+c_1e^{-i\gamma_1}\langle 1|\right]_{B_5}+P_{W}^{15}[\alpha_0|0\rangle+\alpha_1|1\rangle]_{B_2}\\
&\quad\times[\beta_0|0\rangle+\beta_1|1\rangle]_{C_2}\times[\lambda_0|0\rangle+\lambda_1|1\rangle]_{A_2}\\
&\quad\times\left[a_0e^{i\theta_0}|0\rangle-a_1e^{i\theta_1}|1\rangle\right]_{C_5}\times\left[b_0e^{i\varepsilon_0}|0\rangle-b_1e^{i\varepsilon_1}|1\rangle\right]_{A_5}\\
&\quad\times\left[c_0e^{i\gamma_0}|0\rangle-c_1e^{i\gamma_1}|1\rangle\right]_{B_5}\times[\alpha_0\langle 0|+\alpha_1\langle 1|]_{B_2}\\
&\quad\times[\beta_0\langle 0|+\beta_1\langle 1|]_{C_2}\times[\lambda_0\langle 0|+\lambda_1\langle 1|]_{A_2}\\
&\quad\times\left[a_0e^{-i\theta_0}\langle 0|-a_1e^{-i\theta_1}\langle 1|\right]_{C_5}\\
&\quad\times\left[b_0e^{-i\varepsilon_0}\langle 0|-b_1e^{-i\varepsilon_1}\langle 1|\right]_{A_5}\\
&\quad\times\left[c_0e^{-i\gamma_0}\langle 0|-c_1e^{-i\gamma_1}\langle 1|\right]_{B_5}\Big\} \qquad (45)
\end{aligned}$$

As shown in Section II, in an ideal environment, the desired state can be reconstructed determinately as:

$$\begin{aligned}
|\omega\rangle &= \left(b_0e^{i\beta_0}|0\rangle+b_1e^{i\beta_1}|1\rangle\right)_{A_5}\otimes\left(c_0e^{i\gamma_0}|0\rangle+c_1e^{i\gamma_1}|1\rangle\right)_{B_5}\\
&\quad\otimes\left(a_0e^{i\theta_0}|0\rangle+a_1e^{i\theta_1}|1\rangle\right)_{C_5}\otimes(\lambda_0|0\rangle+\lambda_1|1\rangle)_{A_2}\\
&\quad\otimes(\beta_0|0\rangle+\beta_1|1\rangle)_{C_2}\otimes(\alpha_0|0\rangle+\alpha_1|1\rangle)_{B_2}. \qquad (46)
\end{aligned}$$

However, in noisy environments, the desired state may fail to be recovered. The fidelity of the output state can be calculated as:

$$\begin{aligned}
F^A &= \langle\omega|\rho_{out}^{A}|\omega\rangle\\
&= \Big\{\Big\{\left[\alpha_0^2+(1-P_A)^2\alpha_1^2\right]\times\left[\beta_0^2+(1-P_A)^2\beta_1^2\right]\\
&\quad\times\left[\lambda_0^2+(1-P_A)^2\lambda_1^2\right]\times\left[a_0^2+\sqrt{(1-P_A)^3}a_1^2|1\rangle\right]\\
&\quad\times\left[b_0^2+\sqrt{(1-P_A)^3}b_1^2|1\rangle\right]\times\Big[c_0^2\\
&\quad+\sqrt{(1-P_A)^3}c_1^2|1\rangle\Big]\Big\}^2\\
&\quad+P_A^{15}\alpha_0^4\beta_0^4\lambda_0^4a_0^4b_0^4c_0^4\Big\} \qquad (47)
\end{aligned}$$

$$\begin{aligned}
F^P &= \langle\omega|\rho_{out}^{P}|\omega\rangle = \Big\{\left(1-P_{P}\right)^{15}+P_A^{15}\alpha_0^4\beta_0^4\lambda_0^4a_0^4b_0^4c_0^4\\
&\quad+P_A^{15}\alpha_1^4\beta_1^4\lambda_1^4a_1^4b_1^4c_1^4\Big\} \qquad (48)
\end{aligned}$$

$$F^B = \langle\omega|\rho_{out}^{B}|\omega\rangle = \left\{\left(1-P_{B}\right)^{15}+P_{B}^{15}\right\} \qquad (49)$$

$$\begin{aligned}
F^W &= \langle\omega|\rho_{out}^{W}|\omega\rangle = \Big\{\left(1-P_W\right)^{15}+P_W^{15}\left(a_0^2-a_1^2\right)^2\\
&\quad\times\left(b_0^2-b_1^2\right)^2\left(c_0^2-c_1^2\right)^2\Big\} \qquad (50)
\end{aligned}$$

From the above calculation results, it can be found that in the amplitude-damping, phase-damping, and phase-flip noisy environments, the fidelity of the output state depends on the coefficients of the initial state and the decoherence rate. However, in the bit-flip noisy environment, the fidelity of the output state depends only on the decoherence rate. In order to research the trend of the fidelity of the output state, with the change of the decoherence rate in four types of noisy environment, a specific case is given in Fig. 4. We assume that $\alpha_0 = \beta_0 = \lambda_0 = a_0 = b_0 = c_0 = \alpha_1 = \beta_1 = \lambda_1 = a_1 = b_1 = c_1 = 1\big/\sqrt{2}$ and $P_A = P_P = P_B = P_W = P$.

Fig. 4 demonstrates that in the amplitude-damping, phase-damping, and phase-flip noisy environments, the fidelity of the output state decreases as the decoherence rate increases. The fidelity (F^A, F^P, F^W) of the output state reaches 1 when $P = 0$ and reaches 0 when $P = 1$. Furthermore, in the bit-flip noisy environment, the fidelity (F^B) of the output state decreases initially and then increases as the decoherence rate increases. F^B reaches 1 when $P = 0$ or $P = 1$ and reaches 0 when $P = 0.4$.

Fig. 5 shows the variations of the fidelities with different coefficients of the initiated state and decoherence rate. From Fig. 5 (a)-(d), it can be clearly observed that in

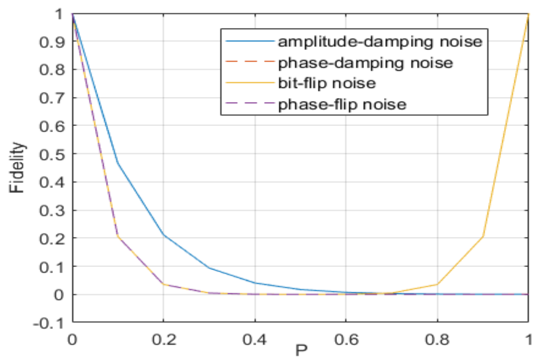

FIGURE 4. The trend of the fidelity of the output state with the change of the decoherence rate in four types of noisy environments.

the amplitude-damping, phase-damping and phase-flip noisy environments, the fidelity (F^A, F^P, F^W) of the output state decreases as the decoherence rate (P_A,P_P,P_W) increases. While in the bit-flip noisy environment, the fidelity (F^B) of the output state decreases initially and then increases as the decoherence rate (P_B) increases. Fig. 5 (e) and (f) illustrate that in the amplitude-damping, phase-damping noisy environments, the fidelity of the output state slowly increases with the coefficients of the initiated state.

IV. ANALYSIS AND COMPARISON

The most notable innovation of our scheme is that it constructs a cyclic hybrid double-channel quantum communication of QT and RSP for three parties, where QT is used to transmit unknown quantum states, and RSP is used to prepare known quantum states. That is, the qubit could be sent through our scheme whether it is known or not.

In this scheme, without the controller and any auxiliary qubit, the product state of three Bell-state and three GHZ-state is used as the quantum channel. The scheme's efficiency can be calculated with the following manner proposed by Banerjee and Pathak [40].

$$\kappa = \frac{q_s}{q_u + b_t}, \tag{51}$$

where q_s denotes the number of states to be transmitted, q_u indicates the quantum resource consumption, and b_t means the classical resource consumption. Based on the equation, the efficiency of the scheme is:

$$\kappa = \frac{6}{15 + 12} \approx 22\%. \tag{52}$$

Table 2 presents a comparison with other schemes [15], [17], [36]–[39] in terms of six aspects, including the type of scheme (TOS), the number of qubits transmitted (NQT), the quantum resource consumption (QRC), the classical resource consumption (CRC), the auxiliary operations (AO), and the efficiency (κ).

From TABLE 2, the following conclusions can be drawn:

1) The efficiency of the proposed scheme is slightly better than those of the schemes in [15], [36]–[39], but it is no better than the scheme in [17]. However, our scheme has a larger transmission capacity than the scheme in [17] because the arbitrary state could carry more information via the amplitude and phase than the special- type state that transmitted in the scheme in [17].

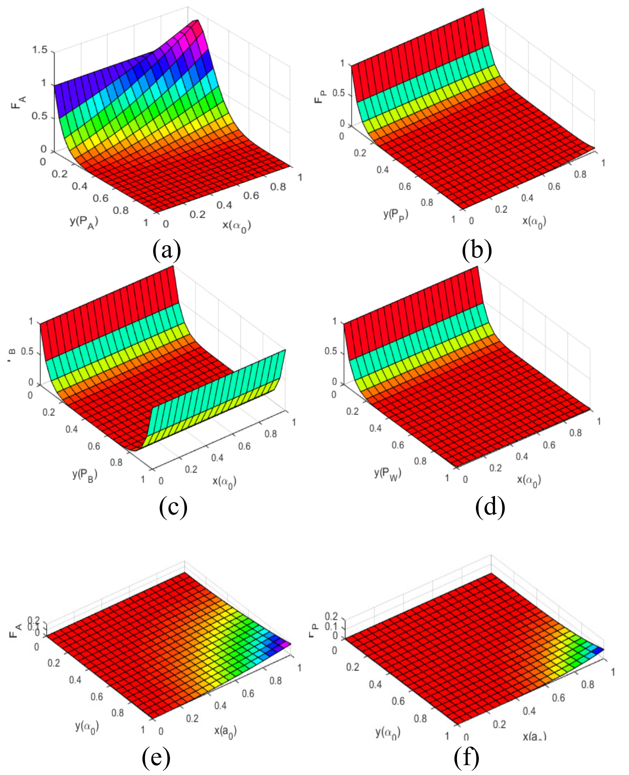

FIGURE 5. The variations of the fidelities with different coefficients of the initiated state and decoherence rate: (a) for amplitude-damping noise with $x = \alpha_0$,$\beta_0 = \lambda_0 = a_0 = b_0 = c_0 = 1/2$, $\beta_1 = \lambda_1 = a_1 = b_1 = c_1 = \sqrt{3}/2$; (b) for phase-damping noise with $x = \alpha_0$,$\beta_0 = \lambda_0 = a_0 = b_0 = c_0 = \sqrt{3}/2$, $\beta_1 = \lambda_1 = a_1 = b_1 = c_1 = \sqrt{1}/2$; (c) for bit-flip noise with $x = \alpha_0$,$\beta_0 = \lambda_0 = a_0 = b_0 = c_0 = 1/2$, $\beta_1 = \lambda_1 = a_1 = b_1 = c_1 = \sqrt{3}/2$; (d) for phase-flip noise with $x = \alpha_0$,$\beta_0 = \lambda_0 = a_0 = b_0 = c_0 = 1/2$, $\beta_1 = \lambda_1 = a_1 = b_1 = c_1 = \sqrt{3}/2$; (e) for amplitude-damping noise with $x = a_0$, $y = \alpha_0$, $P = 1$, $\beta_0 = \lambda_0 = b_0 = c_0 = 1/2$, $\beta_1 = \lambda_1 = b_1 = c_1 = \sqrt{3}/2$; (f) for phase-damping noise with $x = a_0$, $y = \alpha_0$, $P = 1$, $\beta_0 = \lambda_0 = b_0 = c_0 = \sqrt{3}/2$, $\beta_1 = \lambda_1 = b_1 = c_1 = \sqrt{1}/2$.

2) In comparison with the single QT schemes in [15], [17] and the single RSP schemes in [36], [37], the proposed scheme and schemes in [38], [39] are more efficient because they are hybrid protocols for quantum communication, and the channels are multi-purpose. Furthermore, when compared with the schemes of two-party communications in [38], [39], the proposed scheme is superior because it is both cyclic and bidirectional between three parties.

3) Only single-qubit measurements, Bell-state measurements, and basic unitary transformations are used in the proposed scheme. Hence, compared with other schemes, its computational complexity is lower, and it is more likely to be implemented through physical experiments in the future.

4) Another remarkable advantage of the scheme is that the impact of four noisy environments (amplitude-damping,

TABLE 2. Comparison with other schemes, where BSM is short for Bell-state measurement, SM for single-qubit measurement, and TM for two-qubit measurement.

Scheme	TOS	NQT	QRC	CRC	AO	κ
Ref. [15]	QT	1 arbitrary single-qubit, 1 arbitrary two-qubit	7	7	3 BSM, 1 SM	21%
Ref. [17]	QT	3 two-qubit entangled state	12	12	6 BSM	25%
Ref. [36]	RSP	3 arbitrary single-qubit	9	6	6 SM	20%
Ref. [37]	RSP	3 arbitrary single-qubit	10	6	6 SM	19%
Ref. [38]	QT& RSP	1 arbitrary single-qubit, 1 arbitrary two-qubit	7	7	1BSM, 2 TM, 1 SM	21%
Ref. [39]	QT& RSP	2 arbitrary single-qubit	7	6	1BSM, 4 SM	15%
Ours	QT& RSP	6 arbitrary single-qubit	15	12	3 BSM, 6 SM	22%

phase-damping noise, bit-flip noise, and phase-flip noise) is considered. Therefore, the scheme lays a foundation for the practical application in the future.

5) Besides, based on the proposed scheme, a new scheme for N-party ($N>3$) is devised by increasing the Bell-state and GHZ-state in the quantum channel. The quantum channel is given as follows:

$$\begin{aligned}|\psi\rangle = \left(\frac{1}{2}\right)^n & (|00\rangle + |11\rangle)_{1,2} \\ & \otimes (|00\rangle + |11\rangle)_{3,4} \otimes \cdots \otimes (|00\rangle + |11\rangle)_{2N-1,2N} \\ & \otimes (|000\rangle + |111\rangle)_{2N+1,2N+2,2N+3} \\ & \otimes (|000\rangle + |111\rangle)_{2N+4,2N+5,2N+6} \\ & \otimes \cdots \otimes (|000\rangle + |111\rangle)_{5N-2,5N-1,5N}. \end{aligned} \tag{53}$$

V. CONCLUSIONS AND FURTHER WORK

In this paper, a cyclic hybrid double-channel quantum communication scheme among three participants has been proposed for the first time. In this scheme, each participant can transmit an unknown single-qubit state and a known single-qubit state to the other two via QT and RSP, respectively. First, the product state of three Bell-state and three GHZ-state was constructed by H and $CNOT$ operations and used as the quantum channel. Second, three participants carry out Bell-state measurements and single-qubits measurements on their qubits, and inform the measurement results to each other via classical channels. Finally, the desired quantum states were reconstructed by using appropriate unitary transformations on the corresponding qubits with a success probability of 100%. Compared with the existing schemes, our scheme is more efficient because it can construct a cyclic and bidirectional quantum communication and simultaneously supports two communication protocols, QT and RSP. Furthermore, as the only single-qubit measurements, two-qubit measurements, and basic unitary transformations are employed in this scheme, the complexity of this scheme could be lower than the other existing schemes. Thus, the proposed scheme could be more likely to be implemented through physical experiments in the future. In addition, we assessed our scheme performance in four noisy environments (amplitude-damping, phase-damping noise, bit-flip noise, and phase-flip noise) and presented the fidelities of the output states. The results showed that the fidelity only depended on the coefficients of the initial state and the decoherence rate. However, the effective solutions of this scheme in noisy environments were not investigated, which is a part of our further work.

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