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# A Robust Capon Beamforming Approach for Sparse Array Based on Importance Resampling Compressive Covariance Sensing

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**ABSTRACT** Reconstructing the interference-plus-noise covariance matrix instead of searching for the optimal diagonal loading factor for the sample covariance matrix is a good method for calculating the adaptive beamforming coefficients. However, when the directions-of-arrival (DOAs) and the number of the interferences are unknown and the steering vector has an error, the reconstructed interference-plus-noise covariance matrix might not be accurate, which degrades the performance of adaptive beamforming. Here, we propose a robust Capon beamforming approach, which is suited to the sparse array with the array steering error and the unknown interference DOAs. In particular, by drawing a modified optimization problem and the mean shift model of the interference covariance matrix, we propose the robust beamforming with the importance resampling based compressive covariance sensing, which is shown to outperform the classical beamforming method based on reconstructing the interference-plus-noise covariance matrix. The key to our approach is the new solution of the reconstructing method and the important functions. The excellent performance of the proposed approach for interference suppression is demonstrated via a number of numerical examples.

**INDEX TERMS** Adaptive beamforming, compressive covariance sensing, sparse antenna array, importance resampling.

## I. INTRODUCTION

Adaptive beamforming can form the narrow receiving beam pointing to the desired signal and nulls pointing to the interferences, which is widely used in radar, sonar, wireless communications, and other areas [1]–[3]. There are some ideal adaptive beamforming approaches which assume that the desired signal components are not present in the training data [1]. In practice, the assumption of signal-free training data may not be valid, thus the exact interference-plus-noise covariance matrix is usually unavailable. Moreover, various errors and nonideal factors exist in the practical antenna arrays, such as source wavefront distortions, look direction errors, imperfect array calibration, and distorted antenna shape [2]–[15]. Therefore, sufficient robustness of the adaptive beamformers is required. Because of their good resolution and interference rejection capability, the robust Capon beamforming methods based on the diagonal loading and subspace-based adaptive beamforming methods are widely used [3]–[5]. In the presence of an arbitrary unknown signal steering vector mismatch, worst-case optimization based beamformers in the form of convex second-order cone program [6] can be used.

In recent years, many new approaches to robust adaptive beamforming have still been proposed [7]–[15]. A novel method named spatial power spectrum sampling is proposed to reconstruct the interference-plus-noise covariance matrix in [7]. In [8], a robust diagonal loading algorithm against steering vector mismatch is proposed, which estimates beam-to-reference ratio firstly and then uses it as

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a weighting factor for variable diagonal loading. In [9], an outlier-resistant beamformer design criterion based on minimizing the expectation of the modulus of the array output with a regularization term being added for sidelobe suppression is proposed. In [10], a robust linear programming beamformer is proposed, which minimizes the norm of the output to exploit the non-Gaussianity and to achieve robustness against steering vector mismatch. To obtain a robust beamformer, the symmetric structure of the array weights is used to transform the non-convex constraint into a convex one without any relaxation or approximation [11]. In [12], the Capon beamformer and reconstructed interference-plus-noise covariance matrix constructed by the mutual coupling matrix are used for robust beamforming in the case of the mutual coupling effects. In order to address the issue of robust beamforming with the precise main beam control in the presence of arbitrary steering vector uncertainties, in [13], the lower and upper norm bounds of the beamformer weight vector are derived. The semidefinite relaxation technique is then employed as an approximate solver, ending up with iterative, grid search, and linearization solutions. For cost-effective low-rank techniques of designing robust adaptive beamforming algorithms, [14] proposes an algorithm based on the exploitation of the cross-correlation between the array observation data and output of the beamformer. In [15], attempts have been made to reconstruct the interference-plus-noise covariance matrix instead of searching for the optimal diagonal loading factor for the sample covariance matrix. To solve the steering vector error dominated by large DOA mismatch, a robust Capon beamforming approach is proposed in [16] and its essence is to decompose the actual steering vector into two components by oblique projection onto a subspace and then estimate the actual steering vector in two steps. A robust adaptive beamforming technique based on a modification of the robust Capon beamforming approach with approximate orthogonal projection onto the signal-plus-interference subspace is introduced in [17]. In [18], a middle subarray interference-plus-noise covariance matrix reconstruction approach is proposed to mitigate the mutual coupling problem in robust adaptive beamforming. Based on interference-plus-noise covariance matrix reconstruction with interference steering vector and power estimation, a robust adaptive beamforming algorithm is proposed in [19]. In [20], a robust adaptive beamforming using k-means clustering is proposed, which is a solution to high complexity of the reconstruction-based algorithm. An iterative adaptive approach based on angular sector reconstruction algorithm to reconstruct the interference-plus-noise covariance matrix is proposed in [21]. In [22], a new approach to MVDR beamforming which is suited to high-dimensional settings is proposed, and its key is the design of an optimized inverse covariance estimator, which applies eigenvalue clipping and shrinkage functions that are tailored to the MVDR application. Two new reconstruction-based robust adaptive beamformers by using the accurate iterative adaptive approach spectrum to combat the covariance matrix uncertainties and the steering vector mismatches are proposed in [23]. In [24], an enhanced eigenspace-based beamformer derived using the minimum sensitivity criterion is proposed with significantly improved robustness against steering vector errors. In [25], the fractional lower order covariance based minimum variance distortionless response beamformer is introduced, which has great white noise array gain. In [26], to solve inaccuracies in DOA, adding appropriate constraints in the determination of beamforming weights is used to improve the robustness of MVDR beamformers and optimizing array configurations is used to enhance system robustness. In [27], a deconvolution algorithm used in image deblurring to the conventional beam power of a uniform line array (spaced at half-wavelength) is applied to avoid the instability problems of common deconvolution methods. In [28], a novel robust beamforming method is devised to receive multipath signals effectively, which constructs a transformation matrix derived through high-order angle constraint to suppress the interferences with the directions of arrival of interference signals. In [29], based on analyzing the effect of nonuniform array configurations on adaptive beamforming for enhanced signal-to-interference-plus-noise ratio, three sparse array design methods are proposed. In [30], a novel coprime array adaptive beamforming algorithm is proposed, where both robustness and efficiency are well balanced.

Compressive covariance sensing is a novel approach to reconstruct the covariance matrix from the compressed signal, which can be applied to the numerous applications such as power-spectrum estimation, incoherent imaging, direction-of-arrival estimation, frequency estimation, and wideband spectrum sensing [31]–[33]. For the case of the reconstruction of the interference-plus-noise covariance matrix, compressive covariance sensing can be seen as the extensions of the reconstruction approaches. Moreover, a sparse antenna array can be expressed as a compressed antenna array, which is coincident with the application condition of compressive covariance sensing.

In practical applications, reconstructing the interference-plus-noise covariance matrix is a good method for calculating the beamformers. However, as the estimates of the directions-of-arrival (DOAs) and the number of the interferences might not be accurate enough, the reconstructed interference-plus-noise covariance matrix is different from the actual interference-plus-noise covariance matrix, which degrades the performance of the method. Thus, a more accurate interference-plus-noise covariance matrix is needed for better beamforming performance. In this paper, we show how to establish our robust Capon beamformer based on the compressive covariance sensing. The main contributions are summarized as follows. Firstly, different with the Stochastic Differential Equation (SDE) based group targets model described in [34], we provide a compound model of the interferences based on SDE, which is related to the time-varying interference angle positions and the interaction of the eigen-subspace corresponding to each interference during the DOA estimation process respectively. And the compound model is the basis of the probability distribution of the DOAs of the interferences. Secondly, based on three different cases of the DOA resolution and estimation of the interferences, the mean shift model of the interference covariance matrix is proposed to reconstruct the interference covariance matrix. These two models provide more accurate interference covariance matrix in the presence of unknown array steering error and time-varying interferences, which can be also interpreted as the probability distribution of the covariance matrix for the importance resampling. Finally, the importance resampling is used to reconstruct the interference covariance matrix with the proper weights which are related to the importance functions. Furthermore, three different importance functions are provided and their computational complexity is analyzed. Compared with the traditional beamformers, the robust Capon beamforming approach using the importance resampling based compressive covariance sensing has better output SINR of interference suppression.

This paper is organized as follows. In Section II, the adaptive beamforming optimization problem and the signal model with coherent sources, time-varying DOAs of interferences, sparse array and array steering error are formulated. In Section III, the compound model of the interferences based on SDE is established and the mean shift model for the reconstruction of the interference covariance matrix is introduced. Meanwhile, an importance resampling process and the selection of the importance function are discussed. Numerical examples for different scenarios are shown in Section IV and conclusions are drawn in Section V.

## II. PROBLEM FORMULATION

### A. BASIC DATA MODEL

We discuss the problem of robust digital beamforming for coherent signals and coherent interferences received by a uniform linear array (ULA) with $N'$ antennas and the inter-element spacing is $d$. At time $t$, there are $K$ narrow-band coherent signal sources and $P$ narrow-band coherent interferences which are expressed as $s_1(t), s_2(t), \ldots, s_K(t)$ and $\bar{j}_1(t)$, $\bar{j}_2(t), \ldots, \bar{j}_P(t)$ where the signals and interferences are phase-delayed amplitude-weighted replicas of one of them [35]. All of them have the identical center wavelength $\lambda$. The signals arrive from the DOAs $\theta_1, \theta_2, \ldots, \theta_K$, and the spatial nonstationary interferences arrive from the directions $\varphi_1(t)$, $\varphi_2(t), \ldots, \varphi_P(t)$, which are time-varying and can be written as

$$\varphi_p(t) = \varphi_p^0(t) + \Delta\varphi(t) \cdot rand(t) \tag{1}$$

where $\varphi_p^0(t)$ denotes the center of the $p$-th interference direction ($p = 1, 2, \ldots, P$), $\Delta\varphi(t) > 0$ corresponds to the variation range, $rand(t)$ represents a random number that varies uniformly between $[-1, 1]$.

In the case where the array steering vector contains errors, we denote the error vector as $\Delta_x$, which is bounded by a known constant $\varepsilon > 0$, that is

$$\|\Delta_x\| \le \varepsilon \tag{2}$$

Then, the actual array steering vector can be expressed as $\mathbf{c}$, which is written as

$$\mathbf{c} = \mathbf{a} + \Delta_x \tag{3}$$

where $\mathbf{a}$ is the ideal array steering vector without error, which is denoted as

$$\mathbf{a} = [1, e^{j2\pi d \sin\theta/\lambda}, \cdots, e^{j2\pi(N'-1)d\sin\theta/\lambda}]^H \tag{4}$$

The observation errors induced by the error of the array steering vector at the $n$-th sensor is denoted as $\Delta_n^x$, which is also assumed Gaussian distribution with zero mean. Therefore, the observation model for the $n$-th sensor of the actual array is

$$\begin{aligned}\bar{x}_n(t) &= x_n(t) + \Delta_n^x \\ &= \sum_{k=1}^{K} s_k(t) e^{j2\pi(n-1)d\sin\theta_k/\lambda} \\ &\quad + \sum_{p=1}^{P} \bar{j}_p(t) e^{j2\pi(n-1)d\sin\varphi_p/\lambda} + \xi_n(t) + \Delta_n^x\end{aligned} \tag{5}$$

where $\xi_n(t)$ is the noise of the $n$-th antenna element, which is assumed Gaussian distribution with zero mean and variance $\sigma_\xi^2$, and $x_n(t)$ is the observation model for the $n$-th antenna element without the error of the array steering vector, which is written as

$$\begin{aligned}x_n(t) = &\sum_{k=1}^{K} s_k(t) e^{j2\pi(n-1)d\sin\theta_k/\lambda} \\ &+ \sum_{p=1}^{P} \bar{j}_p(t) e^{j2\pi(n-1)d\sin\varphi_p/\lambda} + \xi_n(t)\end{aligned} \tag{6}$$

The signals and the interferences are in the far field of the antenna array, which can be considered as the plane wave.

Equation (5) can be rewritten as

$$\bar{x}_n(t) = \bar{x}_n^0(t) + \xi_n(t) + \Delta_n^x \tag{7}$$

where $\bar{x}_n(t)$ is the observation model for the $n$-th antenna element without the noise and the error of the array steering vector, which is written as

$$\begin{aligned}\bar{x}_n^0(t) = &\sum_{k=1}^{K} s_k(t) e^{j2\pi(n-1)d\sin\theta_k/\lambda} \\ &+ \sum_{p=1}^{P} \bar{j}_p(t) e^{j2\pi(n-1)d\sin\varphi_p/\lambda}\end{aligned} \tag{8}$$

The output data of the array can be expressed as

$$\begin{aligned}\bar{\mathbf{x}}(t) &= [\bar{x}_1(t), \bar{x}_2(t), \cdots, \bar{x}_{N'}(t)]^T \\ &= \bar{\mathbf{x}}^0(t) + \Delta\bar{\mathbf{x}}(t)\end{aligned} \tag{9}$$

where

$$\bar{\mathbf{x}}^0(t) = [\bar{x}_1^0(t), \bar{x}_2^0(t), \cdots, \bar{x}_{N'}^0(t)]^T \tag{10}$$

$$\Delta\bar{\mathbf{x}}(t) = [\xi_1(t) + \Delta_1^x, \xi_2(t) + \Delta_2^x, \cdots, \xi_N(t) + \Delta_{N'}^x]^T \tag{11}$$

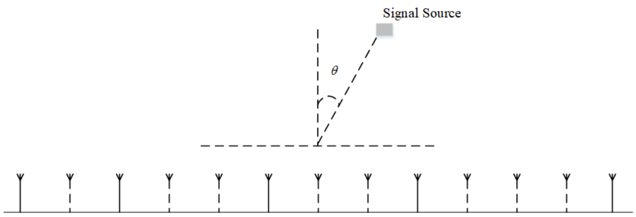


**FIGURE 1.** Illustration of a sparse antenna array.

Considering a sparse array [36], shown in Fig. 1 (the DOA of the signal source is $\theta$), we only reserve $N$ active antennas (shown in the solid line) which have indices $q_0, \ldots, q_{N-1}$. The data vector received by this subarray at time $t$, which can be seen as a compressed observation, is given by $\bar{\mathbf{y}}(t) = [\bar{\mathbf{x}}_{q_0}(t), \ldots, \bar{\mathbf{x}}_{q_{N-1}}(t)]^T$. Then the output signals can be expressed as

$$\begin{aligned}\bar{\mathbf{y}}(t) &= \mathbf{\Phi}\bar{\mathbf{x}}(t)\\ &= \Phi\bar{\mathbf{x}}^0(t) + \Phi\Delta\bar{\mathbf{x}}(t)\end{aligned} \tag{12}$$

The matrix $\Phi$ which is $N \times N'$ dimension, contains ones at the positions $(i, j)$ and zeros elsewhere, where $i = 1, 2, \ldots$, $N$, and $j = q_0, \ldots, q_{N-1}$ respectively. Therefore, the sparse matrix $\Phi$ has at most one nonzero element at each row or column, which can be randomly selected. When the signals and interferences are mixed together, the accurate interference covariance matrix cannot be always obtained from the receiving data directly. The following section considers how to reconstruct the interference covariance matrix.

### B. ADAPTIVE BEAMFORMING OPTIMIZATION PROBLEM

The DOAs of the interferences can be estimated using the MUSIC [37] algorithm based on the spatial smoothing technique [35]. As the interference DOAs are $\varphi_i$, where $i = 1, 2, \ldots, P$, the reconstructed interference covariance matrix can be expressed as

$$\begin{aligned}\mathbf{R}_c &= \sum_{i=1}^{P} \alpha_i \mathbf{\Phi}\mathbf{c}_i\mathbf{c}_i^H\mathbf{\Phi}^H\\ &= \sum_{i=1}^{P} \alpha_i \mathbf{\Phi}(\mathbf{a}_i + \mathbf{\Delta}_i)(\mathbf{a}_i + \mathbf{\Delta}_i)^H\mathbf{\Phi}^H\\ &= \sum_{i=1}^{P} \alpha_i \mathbf{\Phi}\mathbf{a}_i\mathbf{a}_i^H\mathbf{\Phi}^H + \Delta\mathbf{\Psi}_1 + \Delta\mathbf{\Psi}_2 + \mathbf{\Delta}\end{aligned} \tag{13}$$

where

$$\begin{aligned}\Delta\mathbf{\Psi}_1 &= \sum_{i=1}^{P} \alpha_i \mathbf{\Phi}\mathbf{a}_i\mathbf{\Delta}_i^H\mathbf{\Phi}^H\\ \Delta\mathbf{\Psi}_2 &= \sum_{i=1}^{P} \alpha_i \mathbf{\Phi}\mathbf{\Delta}_i\mathbf{a}_i^H\mathbf{\Phi}^H\\ \Delta &= \sum_{i=1}^{P} \alpha_i \mathbf{\Phi}\mathbf{\Delta}_i\mathbf{\Delta}_i^H\mathbf{\Phi}^H\end{aligned} \tag{14}$$

And $\mathbf{c}_i$ is the actual array steering vector corresponding to the middle of $i$-th interference, $\mathbf{\Delta}_i$ is its error of the array steering vector, $\mathbf{a}_i$ is the ideal array steering vector, $\alpha_i$ is the power corresponding to each sub-interval, where $i = 1, 2, \ldots, P$.

Let the actual value of the array response be more than one, that is

$$\left|\mathbf{w}^H\mathbf{c}\right| \geq 1 \tag{15}$$

Formula (15) can be rewritten as

$$\min\left|\mathbf{w}^H\mathbf{c}\right| \geq 1 \tag{16}$$

Based on (3), the constraint in (16) is equivalent to

$$\min\left|\mathbf{w}^H\mathbf{a} + \mathbf{w}^H\mathbf{\Delta}\right| \geq 1 \tag{17}$$

Furthermore, using the Cauchy Schwartz inequality and (2), we have

$$\left|\mathbf{w}^H\mathbf{a} + \mathbf{w}^H\mathbf{\Delta}\right| \geq \left|\mathbf{w}^H\mathbf{a}\right| - \left|\mathbf{w}^H\mathbf{\Delta}\right| \geq \left|\mathbf{w}^H\mathbf{a}\right| - \varepsilon\left\|\mathbf{w}\right\| \tag{18}$$

Therefore, we have

$$\min\left|\mathbf{w}^H\mathbf{a} + \mathbf{w}^H\mathbf{\Delta}\right| = \left|\mathbf{w}^H\mathbf{a}\right| - \varepsilon\left\|\mathbf{w}\right\| \tag{19}$$

where we assume that $\left|\mathbf{w}^H\mathbf{a}\right| > \varepsilon\left\|\mathbf{w}\right\|$.

Thus, the optimization problem (17) can be transformed into the following minimization problem

$$\min(\left|\mathbf{w}^H\mathbf{a}\right| - \varepsilon\left\|\mathbf{w}\right\|) \geq 1 \tag{20}$$

In [6], it is proved that the constraint $\min(\left|\mathbf{w}^H\mathbf{a}\right| - \varepsilon\left\|\mathbf{w}\right\|)$ in (20) is equivalent to $\left|\mathbf{w}^H\mathbf{a}\right| = \varepsilon\left\|\mathbf{w}\right\| + 1$. Thus, (20) can be rewritten as

$$\left|\mathbf{w}^H\mathbf{a} - 1\right|^2 = \varepsilon^2\mathbf{w}^H\mathbf{w} \tag{21}$$

Then, according to the optimization problem in [6] and (21), the adaptive beamforming problem can be transformed into the following constrained minimization problem

$$\min_{\mathbf{w},\,\mathbf{R}_c} \mathbf{w}^H\mathbf{R}_c\mathbf{w} \text{ subject to } \begin{cases}\left|\mathbf{w}^H\mathbf{a} - 1\right|^2 = \varepsilon^2\mathbf{w}^H\mathbf{w}\\ \mathbf{R}_c = \sum_{i=1}^{P} \alpha_i\mathbf{\Phi}\mathbf{a}_i\mathbf{a}_i^H\mathbf{\Phi}^H\\ \qquad + \Delta\mathbf{\Psi}_1 + \Delta\mathbf{\Psi}_2 + \mathbf{\Delta}\end{cases} \tag{22}$$

When $\mathbf{R}_c$ is known, $\mathbf{w}$ in (22) can be solved with (68) in Appendix A. Therefore, the optimization problem in (22) is transformed as a covariance fitting problem, which needs to reconstruct the interference covariance matrix and the equation can be written as

$$\mathbf{R}_c = \mathbf{\Phi}_c\mathbf{\Phi}\mathbf{R}_{org} \tag{23}$$

where $\mathbf{R}_{org}$ is the actual interference covariance matrix of the original non-sparse array, $\mathbf{\Phi}_c$ is the corresponding transformation matrix.

From the form of the transformation formula (23), it is close to the formula of the compressive sensing problem. From [33], we know that this problem can be taken as the compressive covariance sensing problem, and we can recover the original $\mathbf{R}_{org}$ according to the constructed $\mathbf{R}_c$. However, the transformation matrix $\mathbf{\Phi}_c$ is also unknown, and the exact original covariance matrix $\mathbf{R}_{org}$ cannot be obtained directly by the compressive sensing algorithm. Therefore, the modification approach of the reconstructed interference covariance matrix is needed.

## III. NEW ROBUST CAPON BEAMFORMING BASED ON COMPRESSIVE COVARIANCE SENSING

### A. COMPOUND MODEL OF THE INTERFERENCES BASED ON SDE

In [34], a dynamical model based on SDE for target states conditional upon group structure is introduced. Based on the dynamical model, here we suppose that $\hat{\varphi}_{t,i}$ is the compound DOA, and $\hat{\varphi}^i_{t,i}$ is the time-varying DOA of the $i$-th interference at time $t$, and $\hat{\varphi}^s_{t,i}$ is the DOA estimate of the $i$-th interference at time $t$ by the eigen-subspace algorithm, we have

$$\hat{\varphi}_{t,i} = \frac{1}{2}\left(\hat{\varphi}^i_{t,i} + \hat{\varphi}^s_{t,i}\right) \tag{24}$$

*Definition 1:* The compound model of the interferences written as (24) is a dynamical model based on SDE for the DOA and the DOA estimates of interferences, which is divided into two parts. One part is the time-varying interference DOA, and the other part is the interaction of the eigen-subspace corresponding to each interference during the interference DOA estimation process.

According to the description of SDE [34], we have

$$d\dot{\hat{\varphi}}^i_{t,i} = -\gamma_1 \dot{\hat{\varphi}}^i_{t,i} dt + d\hat{\mathbf{W}}^i_{t,i} \tag{25}$$

$$d\dot{\hat{\varphi}}^s_{t,i} = \{-\alpha[\hat{\varphi}^s_{t,i} - f_2(\hat{\varphi}^s_{t,i})] - \gamma_2 \dot{\hat{\varphi}}^s_{t,i}]\}dt + d\hat{\mathbf{W}}^s_{t,i} \tag{26}$$

where $\hat{\mathbf{W}}^i_{t,i}$ is a Brownian motion which is related to the time-varying interference DOA, $f_2(\hat{\varphi}^s_{t,i})$ is the mean of all of the interference DOAs, $\hat{\mathbf{W}}^s_{t,i}$ is another Brownian motion which is related to the DOA estimation. $\gamma_1$, $\gamma_2$ and $\alpha$ are positive, which can be interpreted as the strength of the pull towards the mean of all of the interference DOAs. At time $t$, the "mean reversion" term $\gamma_1 \dot{\hat{\varphi}}^i_{t,i}$ and $\gamma_2 \dot{\hat{\varphi}}^s_{t,i}$ simply prevent the velocities drifting up to very large values.

Let $\hat{\boldsymbol{\varphi}}_t = \left[\hat{\varphi}_{t,1}, \dot{\hat{\varphi}}_{t,1}, \hat{\varphi}_{t,2}, \dot{\hat{\varphi}}_{t,2}, \ldots, \hat{\varphi}_{t,P}, \dot{\hat{\varphi}}_{t,P}\right]^T$, according to (24)

$$\hat{\boldsymbol{\varphi}}_t = \frac{1}{2}\mathbf{H}\boldsymbol{\varphi}_t \tag{27}$$

where

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots & 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 & \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 1 \end{bmatrix}_{2P\times 4P} \tag{28}$$

$$\boldsymbol{\varphi}_t = \begin{bmatrix} \hat{\boldsymbol{\varphi}}^i_t \\ \hat{\boldsymbol{\varphi}}^s_t \end{bmatrix} \tag{29}$$

$$\hat{\boldsymbol{\varphi}}^i_t = \left[\hat{\varphi}^i_{t1}, \dot{\hat{\varphi}}^i_{t1}, \hat{\varphi}^i_{t,2}, \dot{\hat{\varphi}}^i_{t,2}, \ldots, \hat{\varphi}^i_{t,P}, \dot{\hat{\varphi}}^i_{t,P}\right]^T \tag{30}$$

$$\hat{\boldsymbol{\varphi}}^s_t = \left[\hat{\varphi}^s_{t1}, \dot{\hat{\varphi}}^s_{t1}, \hat{\varphi}^s_{t,2}, \dot{\hat{\varphi}}^s_{t,2}, \ldots, \hat{\varphi}^s_{t,P}, \dot{\hat{\varphi}}^s_{t,P}\right]^T \tag{31}$$

According to [34], $\boldsymbol{\varphi}_t$ and its transition probability obey Gaussian distribution, and $\hat{\boldsymbol{\varphi}}^i_t$ and $\hat{\boldsymbol{\varphi}}^s_t$ are also Gaussian distribution with different variances. According to (27), the $\hat{\varphi}_{t,i}$ in $\hat{\boldsymbol{\varphi}}_t$ is a compound Gaussian model.

Suppose the $\hat{\varphi}_{t,i}$ in $\hat{\boldsymbol{\varphi}}_t$ obeys a compound Gaussian distribution with a mean of $\tilde{\boldsymbol{\theta}}_t$ and a variance of $\boldsymbol{\varepsilon}^2_t$, where $\boldsymbol{\varepsilon}^2_t$ is a diagonal matrix. Suppose $\tilde{\boldsymbol{\theta}}_t = \begin{bmatrix} \hat{\theta}^i_{t,1}, \dot{\hat{\theta}}^i_{t,1}, \hat{\theta}^i_{t,2}, \dot{\hat{\theta}}^i_{t,2}, \ldots, \hat{\theta}^i_{t,P}, \dot{\hat{\theta}}^i_{t,P}, \\ \hat{\theta}^s_{t,1}, \dot{\hat{\theta}}^s_{t,1}, \hat{\theta}^s_{t,2}, \dot{\hat{\theta}}^s_{t,2}, \ldots, \hat{\theta}^s_{t,P}, \dot{\hat{\theta}}^s_{t,P} \end{bmatrix}^T$, $\boldsymbol{\varepsilon}^2_t$ has a diagonal element of $\left(\hat{\varepsilon}^i_{t,1}\right)^2$, $\left(\dot{\hat{\varepsilon}}^i_{t,1}\right)^2$, $\left(\hat{\varepsilon}^i_{t,2}\right)^2$, $\left(\dot{\hat{\varepsilon}}^i_{t,2}\right)^2$, ..., $\left(\hat{\varepsilon}^i_{t,P}\right)^2$, $\left(\dot{\hat{\varepsilon}}^i_{t,P}\right)^2$, $\left(\hat{\varepsilon}^s_{t,1}\right)^2$, $\left(\dot{\hat{\varepsilon}}^s_{t,1}\right)^2$, $\left(\hat{\varepsilon}^s_{t,2}\right)^2$, $\left(\dot{\hat{\varepsilon}}^s_{t,2}\right)^2$, ..., $\left(\hat{\varepsilon}^s_{t,P}\right)^2$, $\left(\dot{\hat{\varepsilon}}^s_{t,P}\right)^2$, are the mean and variance of the time-varying DOA of the $i$-th interference at time $t$, $\hat{\theta}^s_{t,i}$ and $\left(\hat{\varepsilon}^s_{t,i}\right)^2$ are the mean and variance of the DOA estimate of the $i$-th interference at time $t$ by the eigen-subspace algorithm. Suppose $\hat{\theta}_{t,i}$ and $\left(\hat{\varepsilon}_{t,i}\right)^2$ are the mean and the variance of the compound angle position $\hat{\varphi}_{t,i}$, which are expressed as

$$\hat{\theta}_{t,i} = \frac{1}{2}\left(\hat{\theta}^i_{t,i} + \hat{\theta}^s_{t,i}\right) \tag{32}$$

$$\left(\hat{\varepsilon}_{t,i}\right)^2 = \frac{1}{2}\left[\left(\hat{\varepsilon}^i_{t,i}\right)^2 + \left(\hat{\varepsilon}^s_{t,i}\right)^2\right] \tag{33}$$

### B. RECONSTRUCTION OF THE INTERFERENCE COVARIANCE MATRIX

#### 1) THE CASE WHEN THE INTERFERENCES CAN BE RESOLVED AND THE ESTIMATES ARE UNBIASED

According to the compound Gaussian model of the interferences, the DOA of the $i$-th interference at time $t$ is located in $[\hat{\theta}_{t,i} - 3\hat{\varepsilon}_{t,i}, \hat{\theta}_{t,i} + 3\hat{\varepsilon}_{t,i}]$ with 0.997 probability [38]. In the case that the interferences can be resolved and the estimates are unbiased, the reconstructed covariance matrix can be expressed as

$$\mathbf{R}_{c,1} = \sum_{i=1}^{P} \int_{\hat{\theta}_{t,i}-3\hat{\varepsilon}_{t,i}}^{\hat{\theta}_{t,i}+3\hat{\varepsilon}_{t,i}} \alpha_i \mathbf{\Phi}\mathbf{R}_i(\theta)\mathbf{\Phi}^H d\theta + \Delta\mathbf{\Psi}_1 + \Delta\mathbf{\Psi}_2 + \mathbf{\Delta} \tag{34}$$

where $\mathbf{R}_i(\theta) = \mathbf{a}_i(\theta)\mathbf{a}^H_i(\theta)$ and $\hat{\varepsilon}_{t,i}$ represents the standard deviation, indicating the value of the interference angle changes within the interval at time $t$, which is derived in Appendix B.

#### 2) THE CASE WHEN THE INTERFERENCE CAN BE RESOLVED BUT THE ESTIMATES ARE BIASED

In the case that the interference can be resolved but the estimates are biased, the reconstructed covariance matrix can be expressed as

$$\mathbf{R}_{c,2} = \sum_{i=1}^{P} \int_{\hat{\theta}_{t,i}+\Delta\hat{\theta}_{t,i}-3\hat{\varepsilon}_{t,i}}^{\hat{\theta}_{t,i}+\Delta\hat{\theta}_{t,i}+3\hat{\varepsilon}_{t,i}} \alpha_i \mathbf{\Phi}\mathbf{R}_i(\theta)\mathbf{\Phi}^H d\theta + \Delta\mathbf{\Psi}_1 + \Delta\mathbf{\Psi}_2 + \mathbf{\Delta} \tag{35}$$

where $\Delta\hat{\theta}_{t,i}$ is the estimation bias of the $i$-th interference at time $t$.

#### 3) THE CASE WHEN THE INTERFERENCES CANNOT BE RESOLVED

If the interferences cannot be resolved, we suppose that the $p'$ intervals of the interference DOAs can be obtained, where $p' < P$. Suppose the possible $p'$ intervals of interference are $\left[\hat{\theta}_k - \frac{1}{2}\Delta\hat{\theta}_k, \hat{\theta}_k + \frac{1}{2}\Delta\hat{\theta}_k\right]$, where $k = 1, 2, \ldots, p'$, $\hat{\theta}_k$ is the center value of the $k$-th interval and $\Delta\hat{\theta}_k$ is the length of the $k$-th interval. The reconstructed covariance matrix can be expressed as

$$\mathbf{R}_{c,3} = \sum_{k=1}^{p'} \int_{\hat{\theta}_k-\frac{1}{2}\Delta\hat{\theta}_k}^{\hat{\theta}_k+\frac{1}{2}\Delta\hat{\theta}_k} \alpha_i \mathbf{\Phi}\mathbf{R}_i(\theta)\mathbf{\Phi}^H d\theta + \Delta\mathbf{\Psi}_1 + \Delta\mathbf{\Psi}_2 + \mathbf{\Delta} \tag{36}$$

#### 4) THE MEAN SHIFT MODEL OF INTERFERENCE COVARIANCE MATRIX

In order to find the most suitable models, based on the three models described above and the mean shift algorithm described in [39] and [40], we take $\mathbf{R}_{c,i}$ $(i = 1, 2, 3)$ as the samples in the $N \times N$ dimensional space, then we have the mean shift matrix $\mathbf{M}_{shift}$ as

$$\mathbf{M}_{shift} = \frac{\sum_{i=1}^{3} \Theta\left(w_i\right) w_i \mathbf{R}_{c,i}}{\sum_{i=1}^{3} \Theta\left(w_i\right) w_i} - \mathbf{R}_{org} = \mathbf{R}_{shift} - \mathbf{R}_{org} \tag{37}$$

where $\Theta(\cdot)$ is the kernel function, for example, it can be the sigmoid function. $\mathbf{R}_{org}$ is the actual interference covariance matrix of the compressed array. $w_i$ $(i = 1, 2, 3)$ are the weight coefficients, which are obtained by the importance function described below. And $\mathbf{R}_{shift}$ is the mean shift reconstruction of the interference covariance matrix, which is denoted as

$$\mathbf{R}_{shift} = \frac{\sum_{i=1}^{3} \Theta\left(w_i\right) w_i \mathbf{R}_{c,i}}{\sum_{i=1}^{3} \Theta\left(w_i\right) w_i} \tag{38}$$

### *C. ESTABLISHING AN IMPORTANCE RESAMPLING PROCESS*

In the following, the estimation process of $\mathbf{R}_{org}$ is regarded as a sequential signal processing, and the state-space and observation equations are established as

$$\begin{aligned} \mathbf{R}_{org}(t) &= f_t\left(\mathbf{R}_{org}(t-1), \mathbf{u}_t\right) \\ \mathbf{R}_c(t) &= g_t\left(\mathbf{R}_{org}(t), \mathbf{v}_t\right) \end{aligned} \tag{39}$$

where $f_t(\cdot)$ is a system transition function, $g_t(\cdot)$ is a measurement function, $\mathbf{u}_t$ and $\mathbf{v}_t$ are the noise vectors. $\mathbf{R}_{org}(t)$ is the actual interference covariance matrix at time $t$, $\mathbf{R}_c(t)$ is the reconstructed interference covariance matrix which is equivalent to a measurement data matrix. According to the compound model of the interferences based on SDE, (39) can be rewritten as

$$\begin{aligned} \mathbf{R}_{org}(t) &= \mathbf{R}_{org}(t-1) + \sum_{i=1}^{P} \Delta\mathbf{R}_i(\Delta\theta_{t-1}) \\ \mathbf{R}_c(t) &= \mathbf{R}_{org}(t) + \Delta\mathbf{R}_{shift}(t) \end{aligned} \tag{40}$$

where $\Delta\mathbf{R}_i(\Delta\theta_{t-1})$ is the difference of the actual interference covariance matrix induced by the DOA difference $\Delta\theta_t$ between time $t$-1 and time $t$. And $\Delta\mathbf{R}_{shift}(t)$ is the difference between the reconstructed interference covariance matrix and the actual interference covariance matrix at time $t$.

According to [41], the joint posteriori distribution of $\mathbf{R}_{org}(0), \mathbf{R}_{org}(1), \ldots, \mathbf{R}_{org}(t)$ in the case of the independent noise samples is approximated by the distribution of interest $\chi$, which is

$$\chi = \left\{\mathbf{R}_{org}(t)^{(m)}, w^{(m)}\right\}_{m=1}^{M} \tag{41}$$

where $\mathbf{R}_{org}(t)^{(m)}$ is the matrix formed by the elements of the particles, $w^{(m)}$ is the weight, and $M$ is the number of particles used in the approximation.

Thus, the actual interference covariance matrix $\mathbf{R}_{org}$ can be written as

$$\mathbf{R}_{org} = \sum_{m=1}^{M} w^{(m)} \mathbf{R}_{org}(t)^{(m)} = \mathbf{w}_M \cdot \tilde{\mathbf{R}}_{org}(t) \tag{42}$$

From (23) and (42), we have

$$\mathbf{R}_c = \mathbf{\Phi}_c \mathbf{\Phi} \mathbf{R}_{org} = \mathbf{w}_M \mathbf{\Phi}_c \mathbf{\Phi} \tilde{\mathbf{R}}_{org}(t) = \mathbf{w}_M \cdot \tilde{\mathbf{R}}_c(t) \tag{43}$$

where

$$\begin{aligned} \mathbf{w}_M &= \left[w^{(1)}, w^{(2)}, \ldots, w^{(M)}\right] \\ \tilde{\mathbf{R}}_{org}(t) &= \left[\mathbf{R}_{org}(t)^{(1)}, \mathbf{R}_{org}(t)^{(2)}, \ldots, \mathbf{R}_{org}(t)^{(M)}\right]^T \\ \tilde{\mathbf{R}}_c(t) &= \mathbf{\Phi}_c \mathbf{\Phi} \tilde{\mathbf{R}}_{org}(t) \\ \tilde{\mathbf{R}}_c(t) &= \left[\mathbf{R}_c(t)^{(1)}, \mathbf{R}_c(t)^{(2)}, \ldots, \mathbf{R}_c(t)^{(M)}\right]^T \\ \mathbf{R}_c(t)^{(m)} &= \mathbf{\Phi}_c \mathbf{\Phi} \mathbf{R}_{org}(t)^{(m)} \quad m = 1, 2, \cdots, M \end{aligned} \tag{44}$$

We denote the particle matrix of $\mathbf{R}_{shift}$ in (38) as $\mathbf{R}_{shift}(t)^{(m)}(m = 1, 2, \ldots, M)$, and let $\tilde{\mathbf{R}}_c(t) = \tilde{\mathbf{R}}_{shift}(t)$ and $\mathbf{R}_c(t)^{(m)} = \mathbf{R}_{shift}(t)^{(m)}$. From (43), we have

$$\mathbf{R}_c = \mathbf{w}_M \cdot \tilde{\mathbf{R}}_{shift}(t) = \sum_{m=1}^{M} w^{(m)} \mathbf{R}_{shift}(t)^{(m)} \tag{45}$$

In order to select the optimal weight combination, we establish the importance resampling based compressive covariance sensing method, as shown in Table 1.

**TABLE 1. The importance resampling based compressive covariance sensing method.**

| |
|---|
| Step 1: Initialization $M$, $\mathbf{R}_{shift}(t)^{(1)}$, $\mathbf{R}_{shift}(t)^{(2)}$, ..., $\mathbf{R}_{shift}(t)^{(M)}$, $w_c^{(1)}, w_c^{(2)}, \ldots, w_c^{(M)}$.<br>Step 2: Let $\Psi_1 = w_c^{(1)}\mathbf{R}_{shift}(t)^{(1)}$, and calculate the value of the importance function.<br>Step 3:<br>For m=2,3, ...,M<br>$\boldsymbol{\Psi}_m = \boldsymbol{\Psi}_{m-1} + w_c^{(m)}\mathbf{R}_{shift}(t)^{(m)}$<br>Use $\boldsymbol{\Psi}_m$ to calculate the value of the importance function, and if the result is greater than the previous value, we have $\boldsymbol{\Psi}_m = \boldsymbol{\Psi}_m$. Otherwise, $\boldsymbol{\Psi}_m = \boldsymbol{\Psi}_{m-1}$.<br>End for<br>Step 4: Let $\mathbf{R}_c = \boldsymbol{\Psi}_m$, and output $\mathbf{R}_c$. |

### D. THREE IMPORTANCE FUNCTIONS

#### 1) MAXIMIZATION OF THE BEAM-SPACE MUSIC SPECTRUM

The maximization of the beam-space MUSIC [42] spectrum is used as an importance function. Assuming that the number of the beamformers is $D$, the signals received by the antenna array are transformed by these $D$ beamformers. The role of the beamformers is to point the bearings to the sector of interest and its output can be expressed as

$$\mathbf{z}_j(t) = \sum_{i=1}^{N} t_{ji}^H \bar{\mathbf{y}}_i(t) = \mathbf{t}_j^H \bar{\mathbf{y}}(t), \quad j = 1, 2, \ldots, D \tag{46}$$

where $\mathbf{z}_j(t)$ is the $j$-th signal in the beam-space, $\bar{\mathbf{y}}_i(t)$ is the signal received by the $i$-th antenna element, which is the element of the signals vector $\bar{\mathbf{y}}(t)$ of the array space, $t_{ji}$ is the weight coefficient of $j$-th beamformer to $i$-th antenna element, $\mathbf{t}_j$ is the weight coefficient vector. The beam-space signals can be written as

$$\mathbf{z}(t) = \mathbf{T}^H \bar{\mathbf{y}}(t) \tag{47}$$

where $\mathbf{T} = [\mathbf{t}_1, \mathbf{t}_2, \ldots, \mathbf{t}_D]$ is the beamforming matrix which consists of the weight coefficients of these $D$ beamformers. The covariance matrix of $\mathbf{z}(t)$ is

$$\mathbf{R}_0 = E\{\mathbf{z}(t)\mathbf{z}^H(t)\} \tag{48}$$

The beam-space signal subspace $\mathbf{U}_{BS}$ and the beam-space noise subspace $\mathbf{U}_{BN}$ can be obtained by the eigen-decomposition of the matrix $\mathbf{R}_0$, then the beam-space MUSIC spectrum can be written as

$$p(\theta) = \frac{1}{(\mathbf{T}^H \boldsymbol{\Phi}\boldsymbol{a}(\theta))^H \mathbf{U}_{BN}\mathbf{U}_{BN}^H \mathbf{T}^H \boldsymbol{\Phi}\boldsymbol{a}(\theta)} \tag{49}$$

Assuming that the maximum amplitude of the beam-space MUSIC spectrum is $\max(p(\theta))$, the mean amplitude of the beam-space MUSIC spectrum is $\text{mean}(p(\theta))$, denote the difference between these two amplitudes as

$$\Delta p = \max(p(\theta)) - \text{mean}(p(\theta)) \tag{50}$$

Corresponding to the $m$-th particle matrix, $\Delta p$ can be obtained, which can be recorded as $\Delta p^{(m)}$. Thus, when the beam spatial MUSIC algorithm spectrum is maximized as an objective function, the weight $w_1^{(m)}$ is

$$w_1^{(m)} = \Delta p^{(m)} \tag{51}$$

#### 2) MAXIMUM LIKELIHOOD FUNCTION

According to the reference [43], the maximum likelihood estimation for the signal DOA $\theta$ is

$$\max_{\theta} \text{Tr}\left[\mathbf{P}_{\mathbf{T}^H\boldsymbol{\Phi}\boldsymbol{a}(\theta)}\mathbf{R}_0^{(m)}\right] \tag{52}$$

where $\text{Tr}(\cdot)$ is the trace of a matrix, and

$$\mathbf{P}_{\mathbf{T}^H\boldsymbol{\Phi}\boldsymbol{a}(\theta)} = \mathbf{T}^H\boldsymbol{\Phi}\boldsymbol{a}(\theta)\Big[(\boldsymbol{\Phi}\boldsymbol{a}(\theta))^H\mathbf{T}\mathbf{T}^H\boldsymbol{\Phi}\boldsymbol{a}(\theta)\Big]^{-1} \cdot (\boldsymbol{\Phi}\boldsymbol{a}(\theta))^H\mathbf{T} \tag{53}$$

which is a projection operator projected onto the space formed by the column vector of $\mathbf{T}^H\boldsymbol{\Phi}\boldsymbol{a}(\theta)$, and $\mathbf{R}_0^{(m)}$ is the beam-space covariance matrix of the $m$-th iterative. With the eigen-decomposition on $\mathbf{R}_0^{(m)}$, we have

$$\mathbf{R}_0^{(m)} = \sum_{i=1}^{D} \lambda_i^{(m)}\mathbf{e}_i^{(m)}\left(\mathbf{e}_i^{(m)}\right)^H \tag{54}$$

where $\lambda_i^{(m)}$ is the eigenvalue of matrix $\mathbf{R}_0^{(m)}$ and $\mathbf{e}_i^{(m)}$ is the eigenvector corresponding to $\lambda_i^{(m)}$. Thus, the maximum likelihood estimation is

$$\max_{\theta} \sum_{i=1}^{D} \lambda_i^{(m)} \left\|\mathbf{P}_{\mathbf{T}^H\boldsymbol{\Phi}\boldsymbol{a}(\theta)}\mathbf{e}_i^{(m)}\right\|^2 \tag{55}$$

According to (54), the maximum of maximum likelihood function which is also the value of the importance resampling function can be obtained. Thus, the weight $w_2^{(m)}$ is

$$w_2^{(m)} = \max_{\theta} \sum_{i=1}^{D} \lambda_i^{(m)} \left\|\mathbf{P}_{\mathbf{T}^H\boldsymbol{\Phi}\boldsymbol{a}(\theta)}\mathbf{e}_i^{(m)}\right\|^2 \tag{56}$$

#### 3) MATCHED FILTER MAGNITUDE AFTER THE ADAPTIVE BEAMFORMING

We consider the maximal value of the matched filter magnitude after the adaptive beamforming as an importance function, which is denoted as

$$\max \left\|\mathbf{t}_m^H\bar{\mathbf{y}}(t)\right\|^2 \quad m = 1, 2, \ldots, D \tag{57}$$

In (57), the weight vector $\mathbf{t}_m$ of the matched filter which points to the direction of the signal. With the constant beamforming matrix, the importance function in (57) is equivalent to

$$\max \left\|\mathbf{T}^H\bar{\mathbf{y}}(t)\right\|^2 \tag{58}$$

The problem can be furtherly rewritten as

$$\max \text{Tr}\left[\mathbf{T}^H\bar{\mathbf{y}}(t)\bar{\mathbf{y}}^H(t)\mathbf{T}\right] = \sum_{i=1}^{D}\lambda_i^2 \tag{59}$$

After the adaptive beamforming processing, the residual of the interferences can be written as

$$\zeta_c = \mathbf{T}^H \mathbf{R}_{org} \mathbf{T} \tag{60}$$

And the noise and the error components can be denoted as

$$\mathbf{\Delta}_{n\_e} = \mathbf{T}^H \mathbf{\Phi} \Delta\bar{\mathbf{x}}(t) \Delta\bar{\mathbf{x}}^H(t) \mathbf{\Phi}^H \mathbf{T} \tag{61}$$

With (60) and (61), we have the pre-whitening matrix, which is denoted as

$$\bar{\mathbf{W}}^{-1/2} = (\mathbf{T}^H \mathbf{R}_{org} \mathbf{T} + \mathbf{T}^H \mathbf{\Phi} \Delta\bar{\mathbf{x}}(t) \Delta\bar{\mathbf{x}}^H(t) \mathbf{\Phi}^H \mathbf{T})^{-1/2} \tag{62}$$

With the pre-whitening processing, we have

$$\max \operatorname{Tr}\left[\bar{\mathbf{W}}^{-1/2} \mathbf{T}^H \bar{\mathbf{y}}(t) \bar{\mathbf{y}}^H(t) \mathbf{T} \bar{\mathbf{W}}^{-1/2}\right] = \max \sum_{i=1}^{D} \bar{\lambda}_i^2 \tag{63}$$

where $\bar{\lambda}_i$ $(i = 1, 2, \ldots, D)$ are the eigenvalues of the matrix after the pre-whitening processing, which are arranged by descending order. In the case of single signal, we have $\bar{\lambda}_i = \sigma^2$, where $2 \le i \le D$. And the problem in (59) can be rewritten as

$$\begin{aligned} &\max \operatorname{Tr}\left[\bar{\mathbf{W}}^{-1/2} \mathbf{T}^H \bar{\mathbf{y}}(t) \bar{\mathbf{y}}^H(t) \mathbf{T} \bar{\mathbf{W}}^{-1/2}\right] \\ &\quad = \max \sum_{i=1}^{D} \bar{\lambda}_i^2 \\ &\quad = \max \left[\bar{\lambda}_1^2 + (D-1)\sigma^2\right] \\ &\quad \to \max \bar{\lambda}_1^2 \end{aligned} \tag{64}$$

Formulas (58) and (64) are basically equivalent, but due to the computational complexity, (58) is chosen as the objective function. Thus, when using the matched filter magnitude after the adaptive beamforming as the objective function, the weight $w_3^{(m)}$ is

$$w_3^{(m)} = \max \left\| \mathbf{T}^H \bar{\mathbf{y}}(t) \right\|^2 \tag{65}$$

#### 4) COMPUTATIONAL COMPLEXITY OF THE IMPORTANCE FUNCTIONS

Suppose the number of the elements of the sparse array is $N$, which is generated from a uniform linear array with $N'$ elements, the number of expected signals is $K$, and the number of interferences is $P$. The number of the beamformers is $D$, and the number of the angle searching grids is $Q$. Because the calculation speed of multiplication is much slower than the addition, only the multiplication calculations of the importance functions are counted here.

(1) According to [44], when the beam-space MUSIC spectrum is maximized as an importance function, the computational complexity of the eigen-decomposition of the beam-pace covariance matrix is $O(D^3)$, and the computational complexity of $p(\theta)$ in (49) is $O(DN'(N-1) + (D+1)(D-K))$. Therefore, the computational complexity of this importance function is $O(QDN'(N-1)+(D+1)(D-K)+D^3)$.

(2) When using the maximum likelihood estimation as the importance function, the computational complexity of the eigen-decomposition of the covariance matrix after beamforming is $O(D^3)$, and the computational complexity of $\mathbf{P}_{\mathbf{T}^H \mathbf{\Phi} \boldsymbol{a}(\theta)}$ is $O(DN'(N-1) + D + D^2)$. The computational complexity of the maximum likelihood estimation in (84) is $O(D^3 + D)$, so the computational complexity of this importance function is $O(2D^3 + DN'(N-1) + D^2 + 2D)$.

(3) When using the matched filter magnitude after the adaptive beamforming as the importance function, the computational complexity is $O(D(N+1))$.

### E. IMPLEMENT OF THE PROPOSED APPROACH

The proposed robust Capon beamforming approach using the importance resampling based compressive covariance sensing is summarized in Table 2.

**TABLE 2. The proposed robust capon beamforming approach.**

| |
|---|
| Step 1:<br>- Use (34), (35), (36), (38) to obtain $M$ covariance matrices $\mathbf{R}_{shift}(t)^{(1)}, \mathbf{R}_{shift}(t)^{(2)}, \ldots, \mathbf{R}_{shift}(t)^{(M)}$.<br>Step 2:<br>For m=1,2, …,M<br>- Calculate the weights of $w_c^{(m)}$ based on the importance functions.<br>- When using the maximization of the beam-space MUSIC spectrum as the importance function, calculate ${w_c}^{(m)}$ in (51).<br>- When using the maximum likelihood estimation as the importance function, calculate ${w_c}^{(m)}$ in (56).<br>- When using the matched filter magnitude after the adaptive beamforming as the importance function, calculate ${w_c}^{(m)}$ in (65).<br>End for.<br>Step 3:<br>- Use the method in Table 1 to calculate the final estimated interference covariance matrix $\mathbf{\Psi}_c$. Thus, $\mathbf{R}_\tau$ is obtained and the beamformer $\mathbf{w}$ is calculated with (68) in Appendix A. |

*Remark 1:* Generally, for the fixed DOA difference between multiple sources, the DOA estimation of independent sources is much easier and has higher accuracy than the DOA estimation of coherent sources. Therefore, although the proposed approach in this paper is based on the coherent interferences, it is also suitable for the independent interferences.

## IV. SIMULATIONS

In order to verify the validity of the proposed method, we introduce the input signal to noise ratio (SNR) and the output signal to interference plus noise ratio (SINR) [6], which can be written as

$$\mathrm{SNR} = \frac{\sigma_s^2}{\sigma_0^2}, \quad \mathrm{SINR} = \frac{\mathbf{w}^H \mathbf{R}_S \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}} \tag{66}$$

where $\sigma_s^2$ is the signal power, $\sigma_0^2$ is the noise power, $\mathbf{R}_S$ is the signal covariance matrix, $\mathbf{R}_{i+n}$ is the covariance matrix of the interference plus noise. The kernel function in (37) is set as the sigmoid function, which is written as

$$\Theta(w_i) = \frac{\rho}{1+e^{-\varsigma(w_i-\vartheta)}} \quad w_i \in \left[w_{\min}, w_{\max}\right] \tag{67}$$

where $\rho$ is the ratio of $w_i$ and $\rho = 2$ is selected in this simulation. $\vartheta = w_{\max}$ and $\varsigma = 10/\left(w_{\max} - w_{\min}\right)$, where $w_{\max}$ and $w_{\min}$ are the minimum and maximum values of $w_i$ in (37), $i = 1, 2, 3$ respectively.

Consider the receiving antenna array of a radar system, which is a sparse antenna array with 16 identical antennas. This sparse antenna array is generated from a uniform linear array with 32 elements, which can be randomly selected. The ratio of the distance between the neighboring elements $d$ to the wavelength is 0.5. And we use a $16 \times 32$ dimensional sparse matrix $\mathbf{\Phi}$, which has at most one nonzero element at each row or column. In the following simulations, we set $\mathbf{\Phi}(1,1) = 1$, $\mathbf{\Phi}(2,2) = 1$, $\mathbf{\Phi}(3,4) = 1$, $\mathbf{\Phi}(4,7) = 1$, $\mathbf{\Phi}(5,8) = 1$, $\mathbf{\Phi}(6,11) = 1$, $\mathbf{\Phi}(7,14) = 1$, $\mathbf{\Phi}(8,15) = 1$, $\mathbf{\Phi}(9,17) = 1$, $\mathbf{\Phi}(10,20) = 1$, $\mathbf{\Phi}(11,23) = 1$, $\mathbf{\Phi}(12,24) = 1$, $\mathbf{\Phi}(13,26) = 1$, $\mathbf{\Phi}(14,29) = 1$, $\mathbf{\Phi}(15,30) = 1$, $\mathbf{\Phi}(16,32) = 1$, and other elements are zero.

The received signals and the interferences are coherent, and the error of the array steering vector is considered in our interference covariance reconstruction equation. According to (3), the error of the array steering vector has the same dimensionality as the array steering vector, the elements of which are modeled as Gaussian random number with zero mean and variance 0.2 in our simulation. $M$ is 5, which is the number of particles for importance resampling. The number of the data samples is 64 and the noise is the additive white Gaussian noise. The variation range of the interference DOA $\Delta\varphi(t)$ is 1°. In the following simulations, 10 repetitions are carried on for each different SNR to calculate the output SINR.

Four approaches are considered in the following simulations, which are the standard Capon beamformer algorithm [3] (which is referred to as SCB), the Capon beamformer with diagonal loading (which is referred to as DL-CB), the Capon beamformer with diagonal loading and of which covariance matrix is reconstructed (which is referred to as DL-RE-CB), the proposed robust Capon beamformer based on the compressive covariance sensing without the importance resampling (which is referred to as CCS-RCB), and the proposed robust Capon beamformer based on the compressive covariance sensing with the importance resampling using the maximization of the beam-space MUSIC spectrum, the maximum likelihood estimation and the matched filter magnitude after the adaptive beamforming as the importance function respectively (which are referred to as BS-IR-CCS, ML-IR-CCS and MF-IR-CCS respectively).

In the first simulation, we consider one target signal with the DOA 0° and two interferences of which the azimuths are 20° and 40° respectively. The interference is time-varying according to (1), and the time-varying numerical intervals can be obtained by the estimation dynamically and can be expressed as $[20° - \Delta\varphi(t), 20° + \Delta\varphi(t)]$ and $[40° - \Delta\varphi(t), 40° + \Delta\varphi(t)]$ respectively. The beampatterns of the first simulation are shown in Fig. 2 in the case of SNR=10 dB. It is shown that the approach proposed in this paper has a quite deeper interferences nulling than the other beamformers.

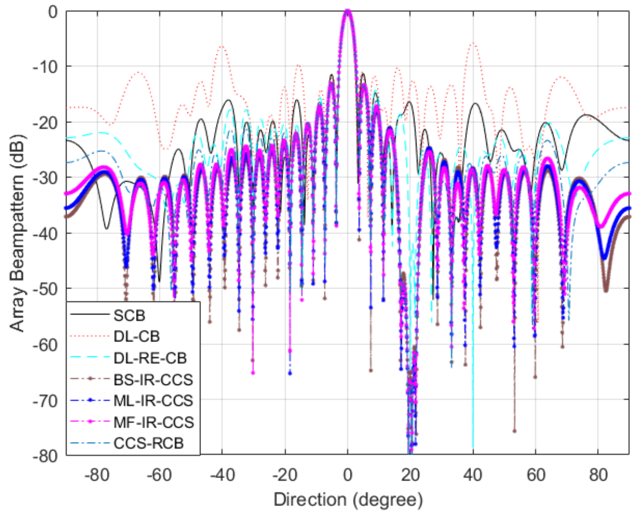


**FIGURE 2. Beampatterns of the first simulation (SNR=10 dB).**

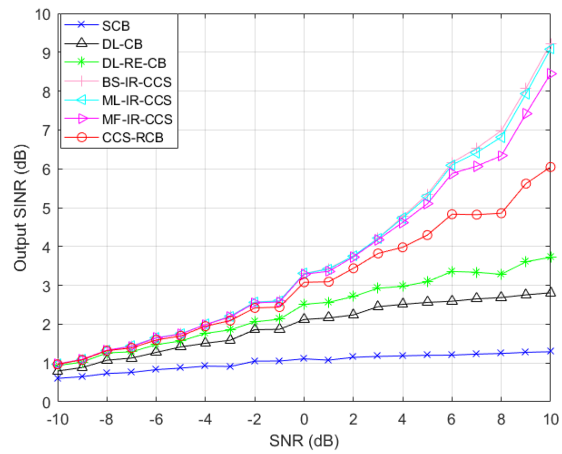


**FIGURE 3. Output SINR versus SNR of the first simulation.**

Fig. 3 gives the output SINR under the case of SNR ranging from -10 dB to 10 dB. We can see that in the case of low SNR, the CCS-RCB, BS-IR-CCS, ML-IR-CCS, MF-IR-CCS have higher output SINR than other three beamformers. In the case of high SNR, BS-IR-CCS, ML-IR-CCS and MF-IR-CCS have higher output SINR than other four beamformers. Although BS-IR-CCS has the highest output SINR, the curves of BS-IR-CCS, ML-IR-CCS and MF-IR-CCS have little difference.

In the second simulation, we consider one target signal with the DOA 0° and three interferences of which the azimuths are 20°, 21° and 22° respectively. The interference is time-varying according to (1), and the time-varying numerical intervals are expressed as $[20° - \Delta\varphi(t), 20° + \Delta\varphi(t)]$, $[21° - \Delta\varphi(t), 21° + \Delta\varphi(t)]$, $[22° - \Delta\varphi(t), 22° + \Delta\varphi(t)]$ respectively. The beampatterns of the second simulation are shown in Fig. 4 in the case of SNR=10 dB. It is shown that compared with the other three approaches, the proposed approaches have quite deeper interferences nulling. Fig. 5 is

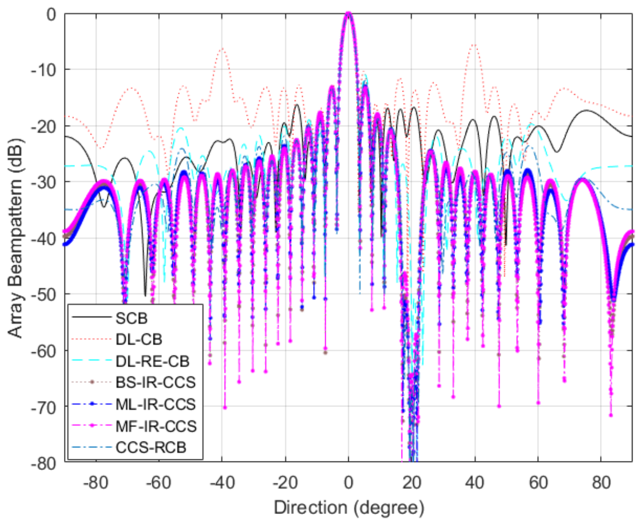


**FIGURE 4. Beampatterns of the first simulation (SNR=10 dB).**

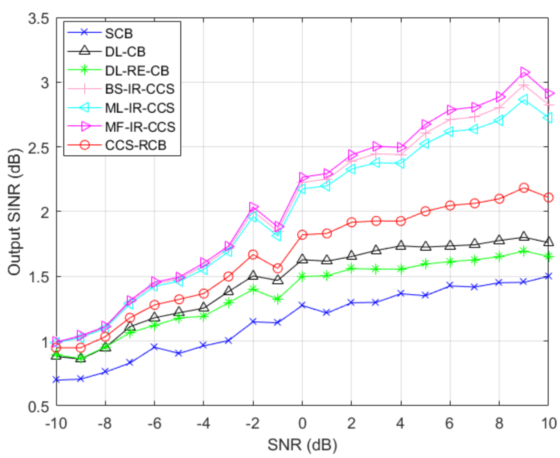


**FIGURE 5. Output SINR versus SNR of the first simulation.**

the output SINR of the second simulation under the case of SNR ranging from -10 dB to 10 dB. We can see that BS-IR-CCS, ML-IR-CCS and MF-IR-CCS have higher output SINR than other four beamformers. Although MF-IR-CCS has the highest output SINR, the curves of BS-IR-CCS, ML-IR-CCS and MF-IR-CCS have little difference.

In the third simulation, we consider one target signal with the DOA 0° and three interferences of which the azimuths are $-18°$, $-12°$ and $-10°$ respectively. The time-varying numerical interval of the interferences can be expressed as $[-18° - \Delta\varphi(t), -18° + \Delta\varphi(t)]$ and $[-12° - \Delta\varphi(t), -12° + \Delta\varphi(t)]$, $[-10° - \Delta\varphi(t), -10° + \Delta\varphi(t)]$ respectively. The beampatterns of the third simulation are shown in Fig. 6 in the case of SNR=10 dB. Also, the proposed approaches have quite deeper interferences nulling. Fig. 7 is the output SINR of the third simulation. Although BS-IR-CCS has the highest output SINR, the curves of BS-IR-CCS, ML-IR-CCS and MF-IR-CCS have little difference.

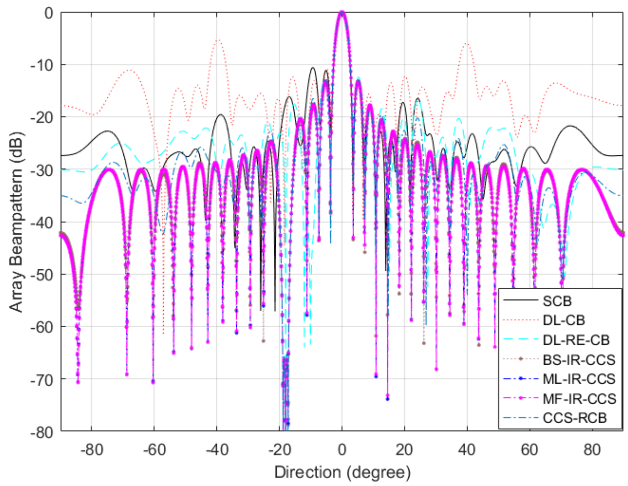


**FIGURE 6. Beampatterns of the first simulation (SNR=10 dB).**

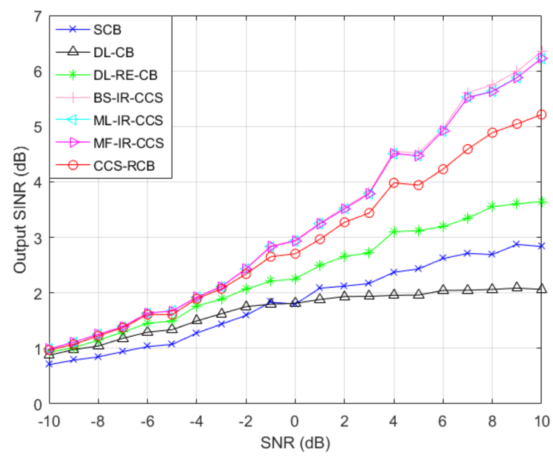


**FIGURE 7. Output SINR versus SNR of the first simulation.**

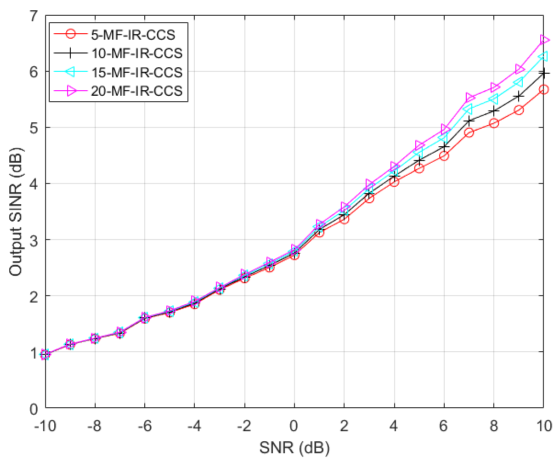


**FIGURE 8. Output SINR versus SNR of the fourth simulation.**

*Remark 2:* Although the output SINR curves of BS-IR-CCS, ML-IR-CCS and MF-IR-CCS have little difference in the three simulations, MF-IR-CCS is the most preferable beamformer because of its computational efficiency.

In the fourth simulation, we consider the role of the number of particles for importance resampling $M$ on the output SINR of the MF-IR-CCS. The simulation conditions are the same as the first simulation. We consider the different selections of the number of particles for importance resampling $M$ which are 5, 10, 15, 20 respectively (referred as 5-MF-IR-CCS, 10-MF-IR-CCS, 15-MF-IR-CCS, 20-MF-IR-CCS). Fig. 8 is the output SINR of the fourth simulation. It is shown that the number of particles for importance resampling $M$ is more and the output SINR is larger.

## V. CONCLUSION

We have shown how to obtain the robust Capon beamformer based on the compressive covariance sensing with the importance resampling method. Three importance functions are selected to generate the weights of the resampling particles, which are the maximization of the beam-space MUSIC spectrum, the maximum likelihood function and the matched filter magnitude after the adaptive beamforming. The proposed robust Capon beamformer based on the compressive covariance sensing with the importance resampling is much less sensitive to coherent sources, time-varying DOA and steering vector mismatches. They have much better interference rejection capability than the robust beamformer based on the standard interference covariance reconstruction. Since the computation of the MF-IR-CCS is much more efficient, MF-IR-CCS is the most preferable beamformer in engineering application. Moreover, the output SINR is proportional to the number of particles for importance resampling. The excellent performance of our proposed methods for interference rejection has been demonstrated via a number of numerical examples.

## APPENDIX A

When $P$ and $\alpha_i$ are known, $\mathbf{w}$ in (22) can be solved by minimizing function [45] $H(\mathbf{w},\lambda) = \mathbf{w}^2\mathbf{R}_c\mathbf{w} + \lambda(\varepsilon^2\mathbf{w}^H\mathbf{w} - \mathbf{w}^H\mathbf{a}\mathbf{a}^H\mathbf{w} + \mathbf{w}^H\mathbf{a} + \mathbf{a}^H\mathbf{w} - 1)$, where $\lambda$ is the Lagrange multiplier. Take the gradient for $\mathbf{w}$ of $H(\mathbf{w},\lambda)$ and let it equal to zero, we get $\mathbf{w} = -\lambda(\mathbf{R}_c + \lambda\varepsilon^2\mathbf{I} - \lambda\mathbf{a}\mathbf{a}^H)^{-1}\mathbf{a}$. Using the matrix inversion rule, we have

$$\begin{aligned}\mathbf{w} &= \frac{\lambda}{\lambda\mathbf{a}^H(\mathbf{R}_c+\lambda\varepsilon^2\mathbf{I})^{-1}\mathbf{a}-1}(\mathbf{R}_c+\lambda\varepsilon^2\mathbf{I})^{-1}\mathbf{a}\\ &= \frac{\lambda}{\lambda\mathbf{a}^H\left(\sum_{i=1}^{J}\alpha_i\mathbf{\Phi}\mathbf{a}_i\mathbf{a}_i^H\mathbf{\Phi}^H+\lambda\varepsilon^2\mathbf{I}\right)^{-1}\mathbf{a}-1}\\ &\quad\cdot\left(\sum_{i=1}^{J}\alpha_i\mathbf{\Phi}\mathbf{a}_i\mathbf{a}_i^H\mathbf{\Phi}^H+\lambda\varepsilon^2\mathbf{I}\right)^{-1}\mathbf{a}\end{aligned} \tag{68}$$

where $\lambda$ can be obtained by solving the root of the scalar equation $f(\lambda) = \lambda^2\sum_{i=1}^{n}\frac{\bar{\mathbf{c}}_i^2\gamma_i}{(1+\lambda\gamma_i)^2} - 2\lambda\sum_{i=1}^{n}\frac{\bar{\mathbf{c}}_i^2}{(1+\lambda\gamma_i)} - 1$, where $\boldsymbol{\gamma} \in \mathbf{R}^n$ are the diagonal elements of $\Gamma$, $\gamma_i(i=1,2,\ldots,n)$ is the element of $\boldsymbol{\gamma}$. The values of $\boldsymbol{\gamma}$ are known as the generalized eigenvalues of $\mathbf{Q}$ and $\mathbf{R}_c$ are the roots of the equation $\det(\mathbf{Q}-\boldsymbol{\gamma}\mathbf{R}_c)=0$, where $\mathbf{Q}=\varepsilon^2\mathbf{I}-\mathbf{a}\mathbf{a}^T$, $\bar{\mathbf{c}}=\mathbf{V}^T\mathbf{R}_c^{-\frac{1}{2}}\mathbf{a}$, $\bar{\mathbf{c}}_i$ is the element of $\bar{\mathbf{c}}$. $\mathbf{V}^T$ and $\Gamma$ satisfy the eigenvalue decomposition $\mathbf{V}\Gamma\mathbf{V}^T = \mathbf{R}_c^{-\frac{1}{2}}\mathbf{Q}(\mathbf{R}_c^{-\frac{1}{2}})^T$, $\mathbf{I}$ is an identity matrix of which dimension is identical to $\mathbf{a}\mathbf{a}^T$. The roots $\lambda$ should satisfy $\lambda > \hat{\lambda}$, where $\hat{\lambda} = \frac{-1-|\bar{\mathbf{c}}_j|(\gamma_j+\bar{\mathbf{c}}_j^2)^{-\frac{1}{2}}}{\gamma_j}$, $\bar{\mathbf{c}}_j = \mathbf{v}_j^T\mathbf{R}_c^{-\frac{1}{2}}\mathbf{a}$, $\mathbf{v}_j$ is the eigenvector associated with the negative eigenvalue $\gamma_j$.

## APPENDIX B

The DOAs of the interferences are estimated by using the MUSIC [37] algorithm based on the spatial smoothing technique [35]. From [46], [47], using the MUSIC algorithm combined with the spatial smoothing technique, if there are $N_k$ subarrays, and each subarray is of size $L = N' - N_k + 1$, we know that the asymptotic variance of the error in the direction estimate of the $k$-th interference is given by

$$\begin{aligned}E[\delta\theta_k^2] = \frac{2}{d_\theta(\theta_k)^2CL^2}\{&\sum_{m,n=1}^{L}\underline{\boldsymbol{\beta}}^H\mathbf{R}(m,n)\underline{\boldsymbol{\beta}}\underline{\boldsymbol{\alpha}}^H\mathbf{N}(m,n)\underline{\boldsymbol{\alpha}}\\ &+Re[\sum_{m,n=1,m\neq n}^{L}\underline{\boldsymbol{\beta}}^H\mathbf{N}(m,n)\underline{\boldsymbol{\alpha}}\underline{\boldsymbol{\beta}}^H\mathbf{N}(m,n)\underline{\boldsymbol{\alpha}}]\}\end{aligned} \tag{69}$$

where $\delta\theta_k$ denotes the error in the estimate of the DOA of the $k$-th interference, $C$ is the number of snapshots. And $Re[\cdot]$ denotes the real part of a complex value. The scalar value $d_\theta(\theta_k)$ is given by the matrix product $\underline{\mathbf{d}}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\underline{\mathbf{d}}(\theta)$, where $\underline{\mathbf{d}}(\theta)$ is the Brandwood vector derivative of $\mathbf{a}(\theta)$. The matrix $\mathbf{R}(m,n)$ is given by $\mathbf{F}(m)\mathbf{R}\mathbf{F}(n)^T$, where $\mathbf{R}$ is the covariance matrix of array data before the spatial smoothing processing and the matrix $\mathbf{F}(k) \in C^{L\times N'}$ is defined by

$$\mathbf{F}(k)_{ij} = \begin{cases}1 & \text{if } j = i+k-1 \text{ and } 1 \le i \le L\\ 0 & \text{otherwise}\end{cases} \tag{70}$$

$\underline{\boldsymbol{\alpha}}$ and $\underline{\boldsymbol{\beta}}$ are defined as

$$\underline{\boldsymbol{\alpha}} = \mathbf{E}_n\mathbf{E}_n^H\underline{\mathbf{d}}_S(\theta_k) \quad \text{and } \underline{\boldsymbol{\beta}} = \mathbf{U}_S\Lambda_S^{-1}\mathbf{U}_S^H\underline{\mathbf{a}}_S(\theta_k) \tag{71}$$

where $\mathbf{E}_n \in \mathbf{C}^{N'\times(N'-P)}$ is the matrix of column noise eigenvectors. The matrix $\mathbf{U}_S \in \mathbf{C}^{L\times P}$ contains the $P$ interference eigenvectors of the smoothed covariance matrix $\mathbf{R}_S$, $\Lambda_S \in \mathbf{R}^{P\times P}$ is is a diagonal matrix containing the $P$ smoothed interferences eigenvalues. The matrix $\mathbf{N}(m,n)$ is defined in a similar way to $\mathbf{R}(m,n)$ with $\mathbf{N}(m,n) = \sigma^2\mathbf{F}(m)\mathbf{F}(n)^T$, where $\sigma^2$ is the variance of the Gaussian white noise. The vector $\underline{\mathbf{a}}_S(\theta_k)$ is the steering vector for the smoothed array and $\underline{\mathbf{d}}_S(\theta_k)$ is its derivative.

As the angle estimation errors obey a Gaussian distribution, we know the $k$-th angle estimation error $\Delta\theta_k \sim N(0, E[\delta\theta_k^2])$, $k = 1, 2, \ldots, K$.

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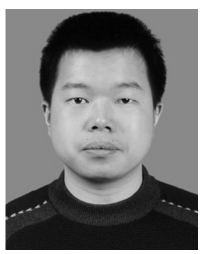

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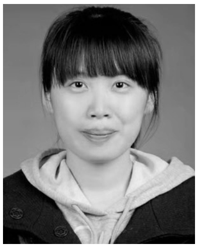

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