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A New Failure Mode and Effects Analysis Method Based on Dempster–Shafer Theory by Integrating Evidential Network

HONGFEI WANG, XINYANG DENG, ZHUO ZHANG, AND WEN JIANG
School of Electronics and Information, Northwestern Polytechnical University, Xi'an 710072, China

Corresponding authors: Xinyang Deng (xinyang.deng@nwpu.edu.cn) and Wen Jiang (jiangwen@nwpu.edu.cn)

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ABSTRACT Failure mode and effects analysis (FMEA) is one of the most effective pre-accident prevention methods. Risk priority number (RPN) approach is a traditional method in the FMEA for risk evaluation. However, there are some shortcomings in the traditional RPN method. In this paper, we propose an FMEA approach based on Dempster–Shafer theory (DST) in an uncertainty evaluation environment. An evidential network (EN) method is proposed to establish a new model for risk evaluation in the FMEA, and we propose a novel approach to determine the conditional belief mass table (CBMT) of the non-root node. In addition, subjective weight and objective weight are integrated to determine the weights of risk factors, which can fully reflect the importance of risk factors. A numerical case is provided to illustrate the practical application of the proposed method, and the results show that this method is reasonable and effective.

INDEX TERMS Failure mode and effects analysis, risk priority number, Dempster-Shafer theory, evidential network.

I. INTRODUCTION

Failure mode and effects analysis (FMEA) is one of the most effective pre-accident prevention methods. Risk evaluation is a crucial step in FMEA to identify high-risk failure modes so as to prevent unexpected failure scenarios [1]. In the risk evaluation of FMEA, risk priority number (RPN) approach is a classical method for the purpose of ranking the failure modes. RPN is an aggregated index which is obtained by multiplying risk factors occurrence (O), severity (S), and detection (D) of a failure mode [2], [3]. Although the RPN method is useful in some real applications, it still has several shortcomings: the three risk factors (O,S,D) are assumed to have the same importance, which may lead to inaccurate results when the FMEA is applied in a practical application [4]–[6]; the three risk factors may produce an identical RPN value, whereas the risk implication may be totally different [4]–[6]; it is difficult to give the crisp evaluations of the risk factors for experts [7], [8]; the most controversial shortcoming of RPN method is the failure to address uncertainties in risk evaluation [9]–[11].

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In the risk evaluation of FMEA, there are various uncertainties in FMEA team members' assessments due to the different expertise and backgrounds, such as imprecision, ambiguity, and incompleteness. Various theories have been proposed to deal with all kinds of uncertainties, such as Dempster-Shafer theory (DST) [12]–[14], fuzzy set theory [15], [16], evidential reasoning [17], belief entropy [18]–[20], belief function [21]–[23], Z-number [24], D-number [25], [26], and so on [27]–[30]. Among these methods, DST [31], [32] is effective in dealing with indeterminate information, which is widely used in uncertain information processing [33] and risk analysis [34]. Many methods based on DST have been proposed to improve the effectiveness of the traditional FMEA [35]–[39]. In [36], the mean value of RPN (MVRPN) was proposed to evaluate the risk in FMEA, where the Dempster's combination rule in DST was modified to fuse the experts' uncertain assessments. In [37], the authors improved the MVRPN method by adding a new process of generating basic probability assignment (BPA), which can deal with the conflict information effectively. Liu et al. [39] proposed an improved FMEA method in which the weights of experts are determined by the evidence distance and belief entropy. The above literature reviews mainly focus on the uncertainty contained in the experts' assessments, the uncertainty in experts' knowledge about the relationships between input factors for risk evaluation in FMEA, however, is not well addressed. Thus, a new FMEA method based on DST by integrating evidential network (EN) [40] is proposed to simultaneously represent the uncertainty in the experts' assessments and in experts' knowledge about the relationships between input factors.

In this FMEA method, DST is applied to represent both the probabilistic uncertainty and epistemic uncertainty in the experts' assessments, and EN provides the framework for representing experts' knowledge about the relationships between input factors. In this paper, we use an improved EN which avoids the combination explosion of the number of states when implemented in large real-world applications, and a novel approach to determine the conditional belief mass table (CBMT) of the non-root node is proposed. Besides, the proposed FMEA method uses linguistic terms in the uncertainty evaluation structure, which overcomes the limitation of crisp assessments in the traditional RPN approach, the results of the assessments can provide valuable information for the risk evaluation process. In addition, linguistic terms allow experts to give more moderate judgments according to their knowledge bases, which makes the evaluation process easier to implement. To fully reflect both experts' knowledge and intrinsic information of the evaluation data, both subjective and objective weighting methods are utilized to determine the weights of risk factors.

By summarizing this work, there are three main contributions. At first, in the risk evaluation of FMEA, multi-experts assessment is generally used when evaluating failure modes with respect to risk factors. In previous studies, multi-criteria decision-making (MCDM) methods are often employed to integrate the assessments of experts. In this paper, we aggregate the evaluations of multiple experts from the perspective of network reasoning under uncertain environment. As an extension of Bayesian network (BN), EN provides an effective framework for representing experts' knowledge about the relationships between input factors. In addition, an improved EN is employed to integrate the assessments of multiple experts, which avoids the combination explosion of the number of states when implemented in large real-world applications. At second, the knowledge is represented by a conditional probability table (CPT) in BN and a CBMT in EN. However, how to determine the CBMT in EN is still an open issue. To handle this issue, we propose a new method based on experts' assessments to determine the CBMT. In this method, we can get a probability distribution from the results of experts' assessments, then the CBMT can be determined by the Shannon entropy. At third, both subjective weight and objective weight are considered to reflect the relative importance of risk factors, within the objective weight is determined by calculating the entropy of data for quantification of uncertain degree coming from the experts and evaluation process.

The rest of the paper is organized as follows. Section II presents the literature related to FMEA improvements. Section III gives a brief introduction about Dempster-Shafer theory, pignistic probability transformation, some uncertainty measures, evidential network approach, and fuzzy set theory. The proposed FMEA model is presented in Section IV. Section V gives a numerical case to demonstrate the effectiveness of the proposed model. Finally, a brief conclusion is made in Section VI.

II. LITERATURE REVIEW

FMEA is an effective tool to identify and eliminate potential failure modes in the pre-prevention phase. However, the conventional FMEA method has many shortcomings which affect its effectiveness [4]. To overcome the drawbacks, various risk priority methods have been proposed [41], [42]. For example, Liu et al. [43] developed an effective FMEA approach by using interval-valued intuitionistic fuzzy sets (IVIFSs) and the multi-attributive border approximation area comparison (MABAC) method. Moreover, the authors developed an optimal model based on the maximum cross-entropy to determine the weight vectors of risk factors. In their another study [44], an advanced integrated FMEA model based on cloud model theory and hierarchical TOPSIS was proposed, where the cloud model theory can effectively address the uncertainty of experts' linguistic expressions and the hierarchical TOPSIS was applied to rank the risk of failure modes for the advantages in dealing with complex risk analysis problems. Song *et al.* [45] developed a risk evaluation model using the TOPSIS method under fuzzy environment to prioritize failure modes. Li and Chen [46] proposed an evidential FMEA method by integrating fuzzy belief structure and grey relational projection method(GRPM). In this method, GRPM was applied to address assessments by fusing different evaluation criteria to deal with the deficiencies in conventional FMEA. Song *et al.* [47] proposed a risk priority model based on rough set theory and TOPSIS for obtaining a more rational rank of failure modes. This method took the advantages of rough set theory to handling the vagueness and uncertainty in FMEA.

Considering the ambiguity and uncertainty in FMEA team members' assessments, fuzzy set theory [15] provides a useful framework to represent the uncertainty in FMEA. Kumru *et al.* [5] used fuzzy logic approach to remove the deficiencies in traditional FMEA, and proposed a fuzzy-based FMEA to improve the purchasing process of a public hospital. An improved FMEA methodology, which integrated the intuitionistic fuzzy set (IFS) and the decision-making trial and evaluation laboratory (DEMATEL) approach to rank the risk of failure modes, was presented by Chang *et al.* [6]. Yang *et al.* [10] proposed a fuzzy rule-based Bayesian reasoning (FuRBaR) method for the risk prioritization of failure modes. The method had advantages in handling the drawbacks of conventional fuzzy logic approaches in FMEA.

Yeh and Chen [11] proposed an improved RPN computation based on fuzzy theory, which reduced the uncertainties related to experts' assessments to enhance the accuracy of the RPN values. Fuzzy set theory improves the accuracy of fault critical analysis, but it may not be able to effectively deal with uncertain information which is aleatory and epistemic in nature [46]. Other studies carried out to represent the ambiguity and epistemic uncertainty in risk assessment process by using evidential reasoning method [35]–[39], D numbers [48], and so on. For example, Bian *et al.* [48] used a risk priority model based on D numbers and technique for the order of preference by similarity to ideal solution (TOPSIS) to rank the risk of failure modes in FMEA, where D numbers can effectively represent the uncertain information. Certa *et al.* [38] proposed a DST-based FMEA method to deal with the epistemic uncertainty often affecting the experts' opinions on risk factors. In spite of DST provides an effective framework to represent the uncertainty in FMEA, the uncertainty in experts' knowledge about the relationships between input factors for risk evaluation in FMEA is not well addressed. As an extension of BN, EN provides the tool for representing experts' knowledge about the relationships between input factors. Therefore, in this paper, we propose a new FMEA method based on DST by integrating EN to simultaneously represent the uncertainty in the experts' assessments and in experts' knowledge about the relationships between input factors.

III. PRELIMINARIES

A. DEMPSTER-SHAFER THEORY

DST is introduced by Dempster [31] and developed by Shafer [32], which is widely used to deal with uncertain information [33], [34], [49].

1) BASIC DEFINITION

In DST, a finite set of N mutually exclusive and exhaustive elements is called the frame of discernment (FOD), symbolized as $\Theta = \{L_1, L_2, \ldots, L_i, \ldots, L_N\}$. The power set of Θ is denoted by 2^Θ which composed of 2^N elements:

$$2^\Theta = \{\emptyset, \{L_1\}, \ldots, \{L_N\}, \{L_1, L_2\}, \ldots, \{L_1, L_2, \ldots, L_i\}, \ldots, \Theta\}. \quad (1)$$

A BPA is a mapping from 2^Θ to [0,1], defined as $m : 2^\Theta \rightarrow [0, 1]$, which satisfies the following condition:

$$\sum_{A \in 2^\Theta} m(A) = 1 \quad and\ m(\emptyset) = 0. \quad (2)$$

In a mass function, $m(A)$ represents the supporting degree to focal element A. When $m(A) > 0$, A is called a focal element. Assume that there are two independent BPAs indicated by m_1 and m_2, they are combined with Dempster's combination rule as follows:

$$m(A) = \begin{cases} 0 & A = \emptyset \\ \dfrac{1}{1-K} \displaystyle\sum_{B \cap C = A} m_1(B)m_2(C) & A \neq \emptyset, \end{cases} \quad (3)$$

where

$$K = \sum_{B \cap C = \emptyset} m_1(B)m_2(C), \quad (4)$$

K is a normalization constant, reflecting the conflict between the two BPAs m_1 and m_2.

2) PIGNISTIC PROBABILITY TRANSFORMATION

The pignistic probability transformation (PPT) [50] assigns the probability of a multiple-element set to singleton sets using the principle of insufficient reason. That means, a belief interval is distributed into the crisp ones, which is called bet probability (BetP). Let m be a BPA on Θ, the resulting PPT for the singleton $x \in \Theta$ is given by:

$$BetP(\{x\}) = \sum_{x \in A \subseteq \Theta} \frac{m(A)}{|A|}, \quad (5)$$

where A is the focal element of m, and $|A|$ is the cardinality of A which denotes the number of elements in A.

3) UNCERTAINTY MEASURES

Entropy concept in information theory can be considered as a criterion for the degree of uncertainty represented by a discrete probability distribution. Shannon proposed a mathematical theory of communication which can evaluate the expected information content of a certain message, called Shannon entropy [51]. Shannon entropy is defined as follows:

$$H = -\sum_{i=1}^{N} p_i \log_b p_i, \quad (6)$$

where N is the number of basic states in a state space, p_i is the probability of state i appears satisfying $\sum_{i=1}^{N} p_i = 1$, b is the base of logarithm. In the DST framework, a generalized Shannon entropy called Deng entropy [52] is proposed to measure the uncertainty of belief functions. Deng entropy is defined as follows:

$$H_d = -\sum_{A \subseteq \Theta} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1}, \quad (7)$$

where m is a mass function defined on the FOD Θ, A is the focal element of m, and $|A|$ is the cardinality of A. Especially, if all the focal elements are singletons, Deng entropy degenerates into Shannon entropy.

B. EVIDENTIAL NETWORK APPROACH

The evidential network [40] extends the BN to the context of belief function theory to some extent. An evidential network is defined as a directed acyclic graph $G = ((N, A), M)$, where (N, A) stands for a graph consisting of nodes' set N and arcs'

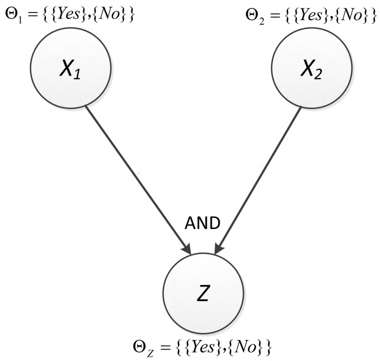

FIGURE 1. An example of evidential network.

set A, M represents a set of belief mass distributions associated with these nodes. If a node is a root node, which has no parents, its priori belief mass table is defined; otherwise, for a non-root node, which has some parent nodes, then its belief mass distribution is defined by a CBMT quantifying the relation between the node and its parents. Then, the mass function of the node concerned can be obtained by the inference algorithm. The following example is given to illustrate the reasoning process.

Example 1: Assume that there are three nodes: X_1, X_2, and Z, their FOD are Θ_1, Θ_2, Θ_Z respectively. X_1 and X_2 are root nodes, Z is the child node of X_1 and X_2, and they have the same FOD $\Theta_1 = \Theta_2 = \Theta_Z = \{\{Yes\}, \{No\}\}$. Then, a graph G consisting of three nodes can be constructed, as shown in Fig. 1. Since X_1 and X_2 are root nodes, their belief mass tables are defined prior as two mass functions m_{X_1} and m_{X_2}.

$$\begin{cases} m_{X_1}(\{Yes\}) = 0.3 \\ m_{X_1}(\{No\}) = 0.5 \\ m_{X_1}(\{Yes, No\}) = 0.2, \end{cases} \quad \begin{cases} m_{X_2}(\{Yes\}) = 0.2 \\ m_{X_2}(\{No\}) = 0.7 \\ m_{X_2}(\{Yes, No\}) = 0.1. \end{cases}$$

The relation between Z and its parents X_1 and X_2 is quantified by a CBMT, as shown in Table 1 for example. Then the inference result regarding node Z can be obtained by the following formula:

$$m_Z(A) = \sum_{M_Z=\{x_1\wedge x_2\wedge\cdots\wedge x_{n-1}\rightarrow A\}} \prod_{i=1,\cdots,n-1} m_{X_i}(x_i) m_Z^{M_Z}(A), \quad with \quad A \subseteq \Theta_Z \quad and\ x_i \subseteq \Theta_i, \tag{8}$$

where $m_{X_i}(x_i)$ represents the mass value when node X_i is in the x_i state; M_Z represents that the state of the Z node is in A state when the other $n-1$ nodes take the corresponding state. $m_Z^{M_Z}(A)$ represents the mass value of node Z in the A state when the other nodes satisfy the corresponding conditions in the CBMT. The calculation process is as follows:

$$\begin{aligned} &m_Z(\{Yes\}) \\ &= \sum_{M_Z=\{x_1\wedge x_2\rightarrow\{Yes\}\}} \prod_{i=1,2} m_{X_i}(x_i) m_Z^{M_Z}(\{Yes\}) \\ &= 0.3\times(0.2\times 1+0.7\times 0.3+0.1\times 0.3)+0.5\times(0.2\times 0.3 \\ &\quad +0.1\times 0.1)+0.2\times(0.2\times 0.3+0.7\times 0.1+0.1\times 0.1) \\ &= 0.195, \end{aligned}$$

$$\begin{aligned} &m_Z(\{No\}) \\ &= \sum_{M_Z=\{x_1\wedge x_2\rightarrow\{No\}\}} \prod_{i=1,2} m_{X_i}(x_i) m_Z^{M_Z}(\{No\}) \\ &= 0.3\times(0.7\times 0.3+0.1\times 0.1)+0.5\times(0.2\times 0.2+0.7\times 1 \\ &\quad +0.1\times 0.3)+0.2\times(0.2\times 0.1+0.7\times 0.4+0.1\times 0.1) \\ &= 0.513, \end{aligned}$$

$$\begin{aligned} &m_Z(\{Yes, No\}) \\ &= \sum_{M_Z=\{x_1\wedge x_2\rightarrow\{Yes,No\}\}} \prod_{i=1,2} m_{X_i}(x_i) m_Z^{M_Z}(\{Yes, No\}) \\ &= 0.3\times(0.7\times 0.4+0.1\times 0.6)+0.5\times(0.2\times 0.5+0.1 \\ &\quad \times 0.6)+0.2\times(0.2\times 0.6+0.7\times 0.5+0.1\times 0.8) \\ &= 0.292. \end{aligned}$$

Compared to the CPT in BN, the CBMT could include not only singletons but also subsets of FODs as the basic reasoning components. Therefore, the evidential network has provided the ability to simultaneously handle random uncertainty and epistemic uncertainty. However, for a non-root node, the size of its CBMT increases exponentially with the rise of the cardinalities of its parents' FODs. For example, if node X_1 and node X_2 have 10 elements in the frame of discernment, then the CBMT of Z has to be a table with size of 1023×1023. It is a big challenge to generate such a huge CBMT. A novel evidential network approach presented by Deng and Jiang [53] overcomes the shortcoming in the original evidential network approach. Consider only the single subset state of the parent node when generating the CBMT in the novel evidential network approach. According to the novel evidential network approach, the CBMT given in Table 1 is changed to Table 2. Compared to the original evidential network, the size of node's CBMT is much smaller. And the novel evidential network approach also gets the ability of expressing imprecise knowledge, which has formed the major advantage of evidential networks [53].

C. FUZZY SET THEORY

Fuzzy set theory was first proposed by Zadeh [15] in 1965, which can deal with the uncertainty problems in real-life decision situations. A fuzzy set $\tilde{A}$ is defined on a universe X may be given as:

$$\tilde{A} = \left\{\left\langle x, \mu_{\tilde{A}}(x)\right\rangle | x \in X\right\}, \tag{9}$$

where $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ is the membership function A. The membership value $\mu_{\tilde{A}}(x)$ describes the degree of $x \in X$ belonging to $\tilde{A}$.

A fuzzy number $\tilde{A} = (a_l, a_c, a_u)$ is called to be a triangular fuzzy number if its membership function is given as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_l \\ \dfrac{x-a_l}{a_c-a_l} & a_l \le x \le a_c \\ \dfrac{a_u-x}{a_u-a_c} & a_c \le x \le a_u \\ 0 & x > a_u, \end{cases} \tag{10}$$

TABLE 1. **The conditional belief mass table for node *Z*.**

	X_1	{Yes}			{No}			{Yes,No}		
	X_2	{Yes}	{No}	{Yes,No}	{Yes}	{No}	{Yes,No}	{Yes}	{No}	{Yes,No}
Z	{Yes}	1	0.3	0.3	0.3	0	0.1	0.3	0.1	0.1
	{No}	0	0.3	0.1	0.2	1	0.3	0.1	0.4	0.1
	{Yes,No}	0	0.4	0.6	0.5	0	0.6	0.6	0.5	0.8

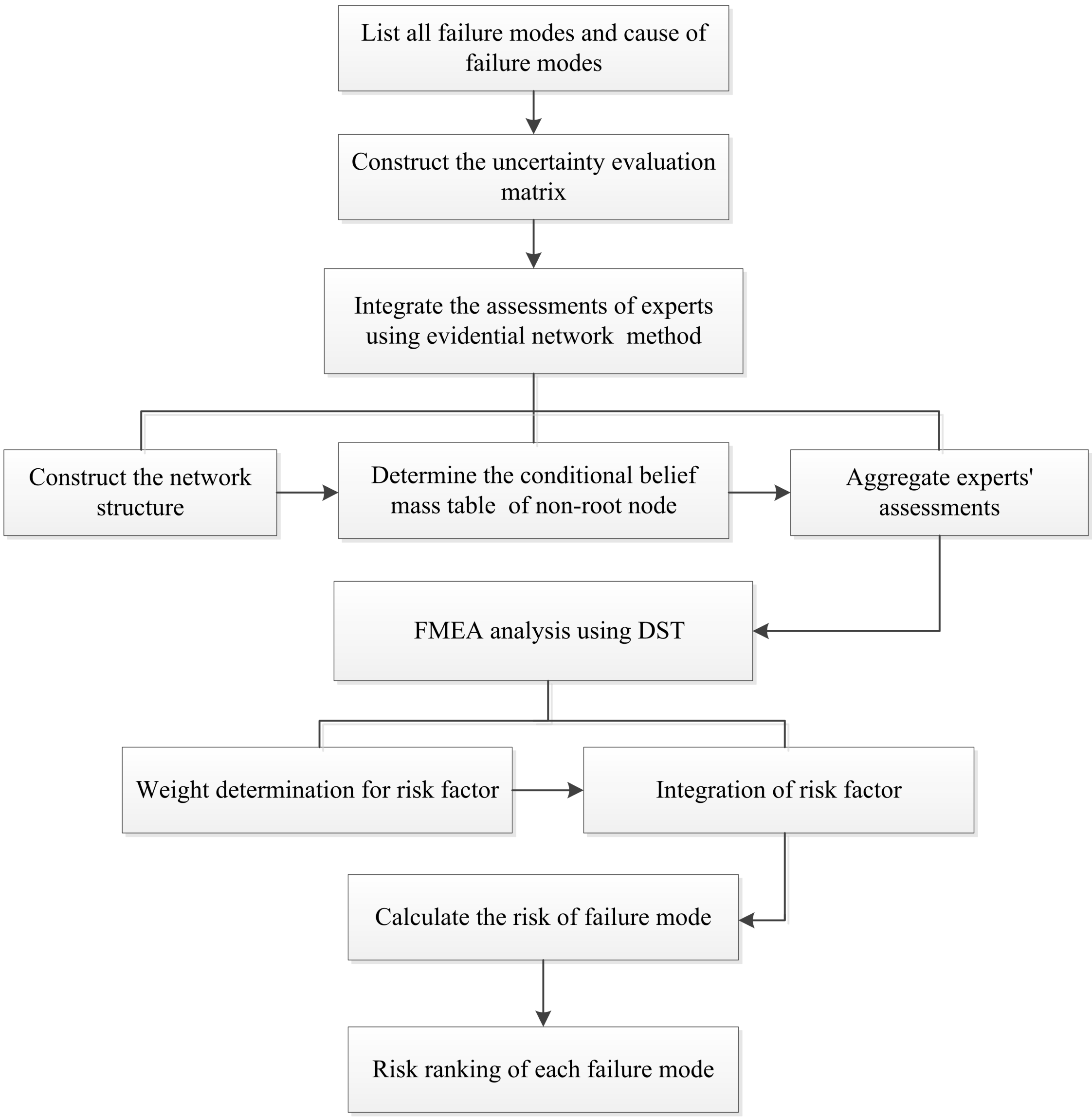

FIGURE 2. **The flowchart of the proposed FMEA method.**

where a_l and a_u are the lower and upper bounds of support of triangular fuzzy number $\tilde{A}$, respectively. And a_c is the core of triangular fuzzy number $\tilde{A}$. The triangular fuzzy number $\tilde{A}$ can be convert into a crisp number $\alpha_{\tilde{A}}$ by the following formula:

$$\alpha_{\tilde{A}} = \frac{\int x\mu_{\tilde{A}}(x)dx}{\int \mu_{\tilde{A}}(x)dx}. \tag{11}$$

IV. THE PROPOSED FMEA MODEL

This new FMEA method, which is based on DST and EN approach, aims to find options with high risk. In the proposed FMEA method, the uncertainty evaluation matrix structure is applied to provide flexibilities in the assessments delivered by experts, which allows the experts to give moderate judgments according to their knowledge bases. Besides,

TABLE 2. **The new conditional belief mass table for node *Z*.**

	X_1	{Yes}		{No}	
	X_2	{Yes}	{No}	{Yes}	{No}
Z	{Yes}	1	0.3	0.3	0
	{No}	0	0.3	0.2	1
	{Yes,No}	0	0.4	0.5	0

TABLE 3. Interpretations of the linguistic terms [4].

Linguistic term	Severity	Occurrence	Detectability
(Very good)	A failure that has no effect on the system performance, the operator probably will not notice	It would be very unlikely for these failures to be observed even once	Failure remains undetected, such a defect would almost certainly be detected during inspection or test
Good	A failure that would cause slight annoyance to the operator but that would cause no deterioration to the system	Likely to occur once, but unlikely to occur more frequently	Defect remains undetected until inspection or test is carried out
Moderate	A failure that would cause a high degree of operator dissatisfaction or that causes noticeable but slight deterioration in system performance	Likely to occur more than once	Defect remains undetected until system performance is affected
Poor	A failure that causes significant deterioration in system performance and/or leads to minor injuries	Near certain to occur at least once	Defect remains undetected until system performance is severely reduced
(Very poor)	A failure that would seriously affect the ability to complete the task or cause damage, serious injury or death	Near certain to occur several times	Defect remains undetected until the system performance degrades to the extent that the task will not be completed

this method does not use the traditional RPN in the risk priority evaluation but uses the language terms gathered from experts to reflect the assessment of risk level. The flowchart in Fig. 2 shows the proposed approach for the FMEA process.

Assume there is a multiple criteria decision making problem with M failure modes FM_i toward the three risk factors RF_j (S, O, D). Moreover, judgments are presented by K experts. The proposed method can be summarized as the following phases:

Phase.1: List all failure modes and causes of failure modes in the system through historical data and expert knowledge.

Phase.2: Construct the uncertainty evaluation matrix. The severity of the associated effects (S), probability of occurrence (O), and detection to each failure mode (D) are considered as risk factors in the evaluation matrix. In addition, the relative importance among S, O, and D is taken into consideration by determining the weights of risk factors. The uncertainty evaluation matrix based on the judgment of the Kth expert is defined as follows:

$$M^K = \begin{array}{c} \\ FM_1 \\ \vdots \\ FM_i \\ \vdots \\ FM_M \end{array} \begin{array}{c} \begin{array}{ccc} S & O & D \end{array} \\ \begin{bmatrix} S_{11}^k & S_{12}^k & S_{13}^k \\ \vdots & \vdots & \vdots \\ S_{i1}^k & S_{i2}^k & S_{i3}^k \\ \vdots & \vdots & \vdots \\ S_{M1}^k & S_{M2}^k & S_{M3}^k \end{bmatrix} \end{array}, \tag{12}$$

where S_{ij}^k is an uncertainty evaluation structure addressed by the Kth expert for each failure mode versus each risk factor, which can be given by the following distribution mathematically:

$$S_{ij}^k = \{(L_n, \beta_n^k), n = 1, \cdots, N\}, \tag{13}$$

where L_n is the nth evaluation grade, β_n^k is a degree of belief. A distribution S_{ij}^k is completed if $\sum_{n=1}^{N} \beta_n^k = 1$. S_{ij}^k means that the Kth expert E_k believes that the FM_i with respect to RF_j has the nth evaluation grade L_n with belief degree β_n^k. For example, there is an expert who addresses complete assessment toward the risk factor S of failure mode FM_1 elaborately as follows:

$$S_{11} = \{(G, 0.8), (M, 0.1), (P, 0.1)\}.$$

To simplify the procedure, 3 Linguistic terms **Good(G)**, **Moderate(M)**, and **Poor(P)** are chosen to represent 3 diverse classes. The interpretations of these linguistic terms are given in Table 3. The numbers are the degrees of belief toward 3 distinct standards, which means the expert is 80% sure that the assigned amount of failure mode FM_1 is good, 10% is moderate, and 10% is poor with respect to the first risk factor S. In addition, the BPA generated from the uncertainty evaluation structure can be listed as below:

$$\begin{cases} m_{E_1}(\{G\}) = 0.8 \\ m_{E_1}(\{M\}) = 0.1 \\ m_{E_1}(\{P\}) = 0.1 \end{cases}$$

Phase.3: Integrate the assessments of experts using EN method.

In this phase, EN method aims to aggregate the assessments of k experts on failure mode FM_i with respect to risk factor RF_j.

Step 1: Construct the network structure

Assume that all experts have the same credibility. The evidential network structure graph can be constructed, as shown in Fig. 3. In the figure, all nodes have the same FOD $\Theta = \{L_1, L_2, \ldots, L_n, \ldots, L_N\}$, where $L_n(n : 1, 2, \ldots, N)$ is the nth evaluation grade. E_k is the Kth expert who addresses the assessment on failure mode FM_i with respect to risk factor RF_j, which is a root node in the evidential network structure, then its priori belief mass table is defined. E is the aggregated judgment on failure mode FM_i with respect to risk factor RF_j, which is a non-root node, then its belief mass distribution is defined by a CBMT.

Step 2: Determine the CBMT of non-root node E.

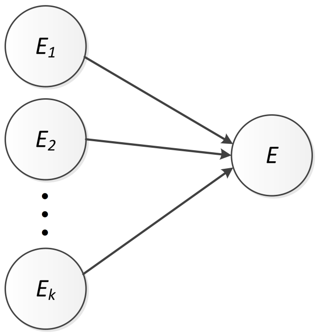

FIGURE 3. The evidential network structure graph.

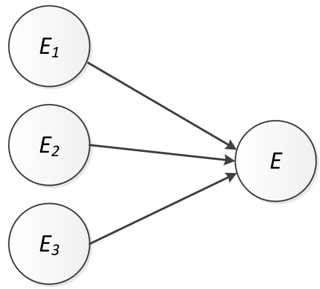

FIGURE 4. Example of constructing a graph to represent the relationship among experts.

How to determine the CBMT of the non-root node in EN method is still an open issue. To handle this issue, we propose a new method to determine the CBMT of the non-root node E with Shannon entropy. As mentioned in section III-B, it is difficult to construct CBMT in the traditional way. Therefore, drawing on the method proposed by Deng [53], only consider the single subset state of the parent nodes when generating the CMBT. The process can be described as follows:

Assume $p(L_n)$ represents the frequency of experts' assessments as L_n on failure mode FM_i with respect to risk factor RF_j, then we can get a probability distribution P defined on Θ. The entropy of the probability distribution to the assessment can be calculated, which is taken as the mass value of the multi-element subset. Remove the mass value of the multi-element subset, the rest is assigned to the single subset elements according to the corresponding proportions, which is taken as the mass value of the single subset. The CBMT of the non-root node can be expressed as follows mathematically:

$$\begin{cases} m_P(\{L_n|L_n \in \Theta, p(L_n) \neq 0\}) = -\sum_{n=1}^{N} p(L_n)\log_k p(L_n), \\ m_P(L_n) = (1 - m_P(\{L_n|L_n \in \Theta, p(L_n) \neq 0\}))p(L_n), \\ L_n \in \Theta, \end{cases} \tag{14}$$

where N is the number of evaluation grades, k is the number of experts. It is worth noting that when all the experts' assessments are different from each other, then $p(L_n) = \frac{1}{k}$, $m_P(\{L_n|L_n \in \Theta, p(L_n) \neq 0\}) = m_P(\Theta)$ takes the maximum value 1, it means that we don't know how to distribute the reliability; when all the experts' assessments are identical to each other, for example, the assessments of all experts are L_1, then $p(L_1) = 1$, $m_P(L_1)$ takes the maximum value 1, it means assigning all the reliability to L_1.

Step 3: Aggregate experts' assessments

When determining the CBMT of non-root node E, the mass function can be calculated by (8), which is the aggregated judgment of k experts on failure mode FM_i with respect to risk factor RF_j.

The following example is given to illustrate the process.

Example 2: Assume there are three experts who give the assessments toward risk factor S of failure mode FM_1 elaborately as follows:

$$E_1(0.8, 0.1, 0.1),$$
$$E_2(0.7, 0.0, 0.3),$$
$$E_3(0.8, 0.2, 0.0).$$

There are three evaluation grades $\{L_1, L_2, L_3\} = \{G, M, P\}$. The evidential network structure graph can be constructed, as shown in Fig. 4. When the assessment given by the three experts is $\{E_1 = G, E_2 = G, E_3 = M\}$, we can get a probability distribution $P_1 : p(G) = {}^2\!/_3, p(M) = {}^1\!/_3, p(P) = 0$. Then the conditional mass function of the non-root node can be calculated as follows:

$$\begin{cases} m_{P_1}(\{G, M\}) = -({}^2\!/_3 \times \log_3 {}^2\!/_3 + {}^1\!/_3 \times \log_3 {}^1\!/_3) \\ = 0.579 \\ m_{P_1}(G) = (1 - 0.579) \times {}^2\!/_3 = 0.281 \\ m_{P_1}(M) = (1 - 0.579) \times {}^1\!/_3 = 0.14. \end{cases}$$

There are three experts and each one give three assessments to each failure mode versus each risk factor. The CBMT of E can be obtained by all combinations of the assessment given by the three experts, the result is shown in Table 4. Then the mass function of E can be calculated by (8), the result is as follows:

$$\begin{cases} m_E(\{G\}) = 0.57 \\ m_E(\{M\}) = 0.03 \\ m_E(\{P\}) = 0.044 \\ m_E(\{G, M\}) = 0.105 \\ m_E(\{G, P\}) = 0.158 \\ m_E(\{M, P\}) = 0.007 \\ m_E(\{G, M, P\}) = 0.086 \end{cases}$$

Phase.4: FMEA analysis using DST

It should be point that when evidence highly conflicts with each other, Dempster's combination rule is not efficient [54]. Many methods are proposed to deal with this problem [55]–[57]. One of the efficient methods is to modify the original data, it means Dempster's combination rule itself is not wrong. The typical method is an averaged algorithm based on the consideration of the ensemble potential by Murphy method [58]. However, Murphy method is only an average of evidence, it does not consider the relations between

TABLE 4. The conditional belief mass table for node *E*.

E_1	E_2	E_3	*E*						
			G	M	P	GM	GP	MP	GMP
G	G	G	1	0	0	0	0	0	0
G	G	M	0.281	0.14	0	0.579	0	0	0
G	G	P	0.281	0	0.14	0	0.579	0	0
G	M	G	0.281	0.14	0	0.579	0	0	0
G	M	M	0.14	0.281	0	0.579	0	0	0
G	M	P	0	0	0	0	0	0	1
G	P	G	0.281	0	0.14	0	0.579	0	0
G	P	M	0	0	0	0	0	0	1
G	P	P	0.14	0	0.281	0	0.579	0	0
M	G	G	0.281	0.14	0	0.579	0	0	0
M	G	M	0.14	0.281	0	0.579	0	0	0
M	G	P	0	0	0	0	0	0	1
M	M	G	0.14	0.281	0	0.579	0	0	0
M	M	M	0	1	0	0	0	0	0
M	M	P	0	0.281	0.14	0	0	0.579	0
M	P	G	0	0	0	0	0	0	1
M	P	M	0	0.281	0.14	0	0	0.579	0
M	P	P	0	0.14	0.281	0	0	0.579	0
P	G	G	0.281	0	0.14	0	0.579	0	0
P	G	M	0	0	0	0	0	0	1
P	G	P	0.14	0	0.281	0	0.579	0	0
P	M	G	0	0	0	0	0	0	1
P	M	M	0	0.281	0.14	0	0	0.579	0
P	M	P	0	0.14	0.281	0	0	0.579	0
P	P	G	0.14	0	0.281	0	0.579	0	0
P	P	M	0	0.14	0.281	0	0	0.579	0
P	P	P	0	0	1	0	0	0	0

the evidence. In this FMEA model, we use a weighted averaging combination method based on Deng entropy, which can consider the relative importance of risk factors. To fully reflect both experts' knowledge and intrinsic information, both subjective and objective weighting methods are utilized to determine the weight of risk factor RF_j.

Step 1: Weight determination for risk factor RF_j

According to expert knowledge, determine the subjective weight (w_{sj}) of risk factor RF_j. When determining the objective weight (w_{oj}) which is ascertained with intrinsic information, Deng entropy is well suited for measuring the relative contrast intensities of BPAs. Deng entropy is an efficient tool to measure uncertain information of evidence. The bigger the entropy, the more the elasticity of evidence, which should be given more attention. Therefore, the objective weight based on Deng entropy can be obtained as follows:

$$w_{oj} = \frac{H_{dj}}{\sum_{j=1}^{n} H_{dj}}, \tag{15}$$

with

$$H_{dj} = \sum_{i=1}^{m} H_{dij}, \tag{16}$$

where H_{dij} represents Deng entropy of each BPA on failure mode FM_i with respect to risk factor RF_j, it can be calculated by (7); m is the number of failure modes; n is the number of risk factors.

Then combine subjective weights from experts and objective weights from intrinsic information together to determine the comprehensive weights of risk factors. The comprehensive weight (w_j) can be obtained by multiplicative combination weighting method, the formula is as follows:

$$w_j = \frac{w_{sj} \times w_{oj}}{\sum_{j=1}^{n} w_{sj} \times w_{oj}}. \tag{17}$$

Step 2: Integration of risk factors

The DST is aimed to aggregate the risk factors of failure mode FM_i. Therefore, after determining the weights of the risk factors, we can get the weighted average of the evidence $\tilde{m}$:

$$\tilde{m}(L) = \sum_{j=1}^{n} w_j \times m_{ij}(L), \quad L \subseteq \Theta, \tag{18}$$

where $m_{ij}(L)$ is the evidence of risk factor RF_j toward failure mode FM_i, n is the number of risk factors. Finally, fusing the weighted average evidence $n - 1$ times by Dempster combination rule, we can get the aggregated evidence of failure mode FM_i:

$$m_{FM_i} = \underbrace{\tilde{m} \oplus \tilde{m} \oplus \ldots \oplus \tilde{m}}_{n-1 \; times}, \tag{19}$$

where m_{FM_i} represents the combined result of all the risk factors toward failure mode FM_i.

Phase.5: Calculate the risk of each failure mode

In this FMEA model, the evaluation grade L_n is represented with triangular fuzzy number $\tilde{u}(L_n) = (u_n^l, u_n^c, u_n^u)$, which can be converted into a crisp number $\alpha(L_n)$ by (11). Through the PPT method, we can transform m_{FM_i} to a probability distribution P_{FM_i} by (5). Then risk of failure mode FM_i can be calculated as follows:

$$R_{FM_i} = \sum_{n=1}^{N} \alpha(L_n) \times P_{FM_i}(L_n), \tag{20}$$

where N is the number of evaluation grades. Note that R_{FM_i} is a positive indicator, while risk is a negative concept; therefore, the smaller the value of R_{FM_i} is, the higher the risk, which should be paid more attention to identify the related failure modes.

V. NUMERICAL CASE STUDY

In this section, a case study of a steel factory (steel factory of guilan) is provided to illustrate the effectiveness of the proposed FMEA method. In this case, ten failure modes for sheet steel production process are evaluated. The severity of the associated effects (S), probability of occurrence (O), and detection to each failure mode (D) are considered as risk factors, as shown in Fig. 5. The weights of risk factors (w_S, w_O, and w_D) are set as: 0.2, 0.3, and 0.1. According to the earlier study [59], the judgments in the decision matrix are taken by three experts with the same importance. The assessments are evaluated by a set of standards with three evaluation grades, and each evaluation grade is represented with a triangular fuzzy number. The following steps are given to illustrate the process of the proposed method.

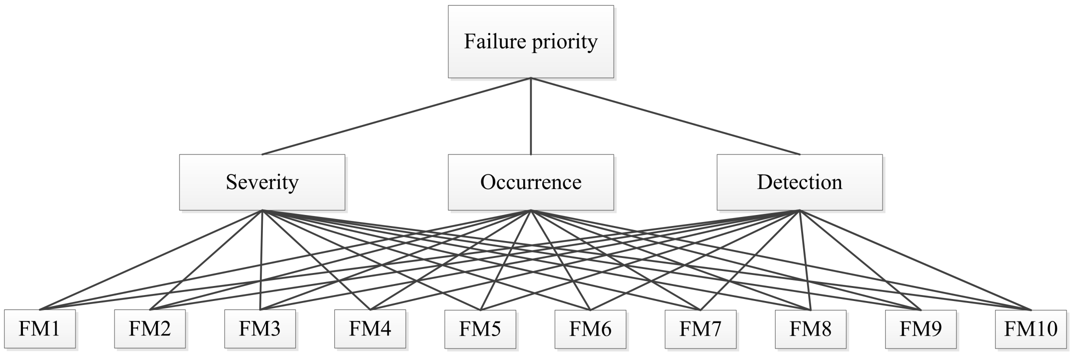

FIGURE 5. The evidential network structure graph [59].

TABLE 5. The FMEA of the sheet steel production process in Guilan steel factory [59].

No.	Failure mode (FM)	Cause of failure (CF)
1	Non-acceptable formation	Non-conductive scrap
2	Nipple thread pitted	Proper coverage not obtained
3	Arc formation loss	Proper gripping loss
4	Burn-out electrode	Cooler not working properly
5	Breaking of house of pipe	Wearing of pipe due to use
6	Problem in movement of arm	Severe leakage
7	Refractory damage	Due to slag
8	Formation of steam	Roof leak
9	Refractory line damage	By hot gas
10	Movement of roof stop	Jam of plunger in un loader valve

Step 1: List the failure modes and causes of failure throughout the system versus three risk factors, as shown in Table 5.

Step 2: Construct the uncertainty evaluation matrix based on the expert opinions. In the decision matrix, there are ten failure modes $FM_1, FM_2, \ldots, FM_{10}$, three risk factors S, O, D, and three experts E_1, E_2, E_3. Each assessment in the matrix is expressed as an uncertainty evaluation structure with three evaluation grades $\{L_1, L_2, L_3\} = \{G, M, P\}$. The uncertainty evaluation matrix is presented in Table 6.

Step 3: The assessments from three different experts in the decision matrix are aggregated by EN method. The CBMT of the non-root node can be obtained by (14), the mass function can be calculated by (8), which is the aggregated judgment of three experts on failure mode FM_i with respect to risk factor RF_j. The result is presented in Table 7.

Step 4: When using DST method to aggregate three risk factors, both subjective and objective weighting methods are utilized to determine the weight of risk factor RF_j. The subjective weights of the risk factors (S, O, D) are set as: 0.2, 0.3, and 0.1. Then calculate the objective weights by (15) and (16), we can have $w_{oS} = 0.338$, $w_{oO} = 0.314$, $w_{oD} = 0.348$. The comprehensive weights can be obtained by (17), which are $w_S = 0.344$, $w_O = 0.479$, $w_D = 0.177$. The risk factors are aggregated by using the improved D-S evidence theory in (18) and (19). The result is presented in Table 8.

Step 5: In this FMEA model, the evaluation grade L_n is represented with triangular fuzzy number $\tilde{u}(L_n) = (u_n^l, u_n^c, u_n^u)$.

TABLE 6. Uncertainty evaluation structure judgment of the sheet steel production process [59].

No.	Experts	Severity (S)	Occurrence (O)	Detectability (D)
FM_1	E_1	(0.8, 0.1, 0.1)	(0.1, 0.2, 0.7)	(0.2, 0.5, 0.3)
	E_2	(0.7, 0.0, 0.3)	(0.0, 0.4, 0.6)	(0.3, 0.4, 0.3)
	E_3	(0.8, 0.2, 0.0)	(0.1, 0.4, 0.5)	(0.2, 0.5, 0.3)
FM_2	E_1	(0.7, 0.1, 0.2)	(0.1, 0.2, 0.7)	(0.8, 0.1, 0.1)
	E_2	(0.7, 0.0, 0.3)	(0.0, 0.4, 0.6)	(0.7, 0.0, 0.3)
	E_3	(0.6, 0.4, 0.0)	(0.1, 0.4, 0.5)	(0.8, 0.2, 0.0)
FM_3	E_1	(0.8, 0.1, 0.1)	(0.0, 0.1, 0.9)	(0.2, 0.5, 0.3)
	E_2	(0.9, 0.0, 0.1)	(0.0, 0.2, 0.8)	(0.3, 0.4, 0.3)
	E_3	(0.7, 0.3, 0.0)	(0.1, 0.0, 0.9)	(0.2, 0.5, 0.3)
FM_4	E_1	(0.4, 0.4, 0.2)	(0.0, 0.1, 0.9)	(0.1, 0.2, 0.7)
	E_2	(0.3, 0.5, 0.2)	(0.0, 0.2, 0.8)	(0.0, 0.4, 0.6)
	E_3	(0.4, 0.4, 0.2)	(0.1, 0.0, 0.9)	(0.1, 0.4, 0.5)
FM_5	E_1	(0.4, 0.4, 0.2)	(0.2, 0.4, 0.4)	(0.7, 0.0, 0.3)
	E_2	(0.3, 0.5, 0.2)	(0.2, 0.4, 0.4)	(0.8, 0.2, 0.0)
	E_3	(0.4, 0.4, 0.2)	(0.1, 0.5, 0.4)	(0.6, 0.3, 0.1)
FM_6	E_1	(0.4, 0.4, 0.2)	(0.2, 0.4, 0.4)	(0.7, 0.0, 0.3)
	E_2	(0.3, 0.5, 0.2)	(0.2, 0.4, 0.4)	(0.8, 0.2, 0.0)
	E_3	(0.4, 0.4, 0.2)	(0.1, 0.5, 0.4)	(0.6, 0.3, 0.1)
FM_7	E_1	(0.4, 0.4, 0.2)	(0.2, 0.4, 0.4)	(0.1, 0.2, 0.7)
	E_2	(0.5, 0.5, 0.0)	(0.2, 0.4, 0.4)	(0.0, 0.4, 0.6)
	E_3	(0.6, 0.4, 0.0)	(0.1, 0.5, 0.4)	(0.1, 0.4, 0.5)
FM_8	E_1	(0.8, 0.1, 0.1)	(0.0, 0.1, 0.9)	(0.2, 0.5, 0.3)
	E_2	(0.9, 0.0, 0.1)	(0.0, 0.2, 0.8)	(0.3, 0.4, 0.3)
	E_3	(0.7, 0.3, 0.0)	(0.1, 0.0, 0.9)	(0.2, 0.5, 0.3)
FM_9	E_1	(0.4, 0.4, 0.2)	(0.2, 0.4, 0.4)	(0.7, 0.0, 0.3)
	E_2	(0.3, 0.5, 0.2)	(0.2, 0.4, 0.4)	(0.8, 0.2, 0.0)
	E_3	(0.4, 0.4, 0.2)	(0.1, 0.5, 0.4)	(0.6, 0.3, 0.1)
FM_{10}	E_1	(0.7, 0.0, 0.3)	(0.2, 0.4, 0.4)	(0.7, 0.3, 0.0)
	E_2	(0.8, 0.2, 0.0)	(0.4, 0.0, 0.6)	(0.3, 0.4, 0.3)
	E_3	(0.6, 0.3, 0.1)	(0.4, 0.0, 0.6)	(0.2, 0.5, 0.3)

Suppose the utilities of evaluation grades are

$$\tilde{u}(L_1) = \tilde{u}(G) = (0.5, 0.7, 0.9)$$
$$\tilde{u}(L_2) = \tilde{u}(M) = (0.3, 0.5, 0.7)$$
$$\tilde{u}(L_3) = \tilde{u}(P) = (0.1, 0.3, 0.5)$$

The triangular fuzzy number can be converted into a crisp number $\alpha(L_n)$ by (11), we have $\alpha(G) = 0.7$, $\alpha(M) = 0.5$, $\alpha(P) = 0.3$. Through the PPT method, we can transform the BPAs of aggregated risk factors to probability distribution by (5). Then the risk of failure mode R_{FM_i} can be calculated by (20). The result is presented in Table 9. From the result, it is found that the failure mode FM_4 has the highest risk and the failure mode FM_{10} has the lowest risk. To note that, all experts have the same assessments on failure modes FM_3

TABLE 7. The assessments after aggregating the experts' opinions by using EN.

No.	$m_S(.)^*$	$m_O(.)^*$	$m_D(.)^*$
FM_1	(0.57,0.03,0.04,0.11,0.16,0.01,0.09)	(0.02,0.15,0.36,0.02,0.05,0.33,0.08)	(0.08,0.23,0.11,0.13,0.06,0.19,0.2)
FM_2	(0.43,0.05,0.05,0.15,0.14,0.02,0.16)	(0.02,0.15,0.36,0.02,0.05,0.33,0.08)	(0.57,0.03,0.04,0.11,0.16,0.01,0.09)
FM_3	(0.62,0.05,0.02,0.18,0.07,0,0.06)	(0.01,0.04,0.74,0,0.04,0.15,0.03)	(0.08,0.23,0.11,0.13,0.06,0.19,0.2)
FM_4	(0.16,0.2,0.06,0.22,0.07,0.1,0.19)	(0.01,0.04,0.74,0,0.04,0.15,0.03)	(0.02,0.15,0.36,0.02,0.05,0.33,0.08)
FM_5	(0.16,0.2,0.06,0.22,0.07,0.1,0.19)	(0.05,0.2,0.18,0.08,0.06,0.25,0.18)	(0.47,0.05,0.04,0.17,0.13,0.01,0.12)
FM_6	(0.16,0.2,0.06,0.22,0.07,0.1,0.19)	(0.05,0.2,0.18,0.08,0.06,0.25,0.18)	(0.47,0.05,0.04,0.17,0.13,0.01,0.12)
FM_7	(0.27,0.21,0.01,0.35,0.03,0.02,0.1)	(0.05,0.2,0.18,0.08,0.06,0.25,0.18)	(0.02,0.15,0.36,0.02,0.05,0.33,0.08)
FM_8	(0.62,0.05,0.02,0.18,0.07,0,0.06)	(0.01,0.04,0.74,0,0.04,0.15,0.03)	(0.08,0.23,0.11,0.13,0.06,0.19,0.2)
FM_9	(0.16,0.2,0.06,0.22,0.07,0.1,0.19)	(0.05,0.2,0.18,0.08,0.06,0.25,0.18)	(0.47,0.05,0.04,0.17,0.13,0.01,0.12)
FM_{10}	(0.47,0.05,0.04,0.17,0.13,0.01,0.12)	(0.13,0.03,0.28,0.04,0.25,0.08,0.19)	(0.16,0.17,0.05,0.22,0.1,0.06,0.23)

* $m_S(.) = m_S(\{G\},\{M\},\{P\},\{G,M\},\{G,P\},\{M,P\},\{G,M,P\})$
$m_O(.) = m_O(\{G\},\{M\},\{P\},\{G,M\},\{G,P\},\{M,P\},\{G,M,P\})$
$m_D(.) = m_D(\{G\},\{M\},\{P\},\{G,M\},\{G,P\},\{M,P\},\{G,M,P\})$

TABLE 8. The BPAs of aggregated risk factors.

No.	$m(\{G\},\{M\},\{P\},\{G,M\},\{G,P\},\{M,P\},\{G,M,P\})$
FM_1	(0.246, 0.214, 0.448, 0.01, 0.015, 0.064, 0.003)
FM_2	(0.364, 0.157, 0.394, 0.014, 0.019, 0.049, 0.003)
FM_3	(0.239, 0.084, 0.647, 0.01, 0.005, 0.014, 0.001)
FM_4	(0.032, 0.137, 0.785, 0.009, 0.005, 0.031, 0.002)
FM_5	(0.254, 0.415, 0.189, 0.052, 0.021, 0.059, 0.01)
FM_6	(0.254, 0.415, 0.189, 0.052, 0.021, 0.059, 0.01)
FM_7	(0.14, 0.517, 0.222, 0.046, 0.008, 0.062, 0.005)
FM_8	(0.239, 0.084, 0.647, 0.01, 0.005, 0.014, 0.001)
FM_9	(0.254, 0.415, 0.189, 0.052, 0.021, 0.059, 0.01)
FM_{10}	(0.569, 0.067, 0.236, 0.035, 0.071, 0.013, 0.01)

TABLE 9. The result of R_{FM_i} and risk priority ranking.

Failure modes	$BetP(\{G\},\{M\},\{P\})$	R_{FM_i}	Rank
FM_1	(0.26,0.252,0.488)	0.454	4
FM_2	(0.381,0.189,0.429)	0.49	6
FM_3	(0.247,0.096,0.657)	0.418	2
FM_4	(0.039,0.158,0.803)	0.347	1
FM_5	(0.294,0.474,0.233)	0.512	7
FM_6	(0.294,0.474,0.233)	0.512	7
FM_7	(0.169,0.572,0.259)	0.482	5
FM_8	(0.247,0.096,0.657)	0.418	2
FM_9	(0.294,0.474,0.233)	0.512	7
FM_{10}	(0.625,0.094,0.281)	0.569	10

and FM_8, so they hold the same risk values, which means the same attention should be paid toward them. The case of failure modes FM_5, FM_6, and FM_9 is the same as that of FM_3 and FM_8. The risk ranking of all failure modes from high to low is $FM_4 > FM_3 = FM_8 > FM_1 > FM_7 > FM_2 > FM_5 = FM_6 = FM_9 > FM_{10}$.

The above-mentioned case was also studied by Vahdani *et al.* [59] and Li and Chen [46]. In literature [46], Li et al. dealted with the risk evaluation in FMEA with GRPM method. Vahdani et al. proposed an improved FMEA method based on fuzzy belief technique for order of preference by similarity to ideal solution (FB-TOPSIS) in [59]. Besides, In [36], [37], the authors developed the improved FMEA models in DST framework. Therefore, we use the same case to implement the MVRPN [36] and improved MVRPN [37] to illustrate the validation of this proposed method. The comparison results are given in Table 10.

From the results shown in Table 10, it can be found that all the methods identify that the failure mode FM_4 has the highest risk. It should be noted that the methods in [36], [37] have the same risk priority ranking. This is because that the improved MVRPN in [37] mainly deal with the case where different and precise values of the evaluation in [36], while this case does not appear in the assessment of the sheet steel production process. In addition, if we divide all the failure modes into two groups in the risk ranking by using FB-TOPSIS, MVRPN, and improved MVRPN, the first group which has higher risk is composed by failure modes FM_4, FM_7, FM_3, FM_8, FM_1, the second group having lower risk includes FM_2, FM_5, FM_6, FM_9, FM_{10}. By using the proposed method, we can also obtain the same classification that FM_4, FM_3, FM_8, FM_1, FM_7 are in the first group with higher risk and FM_2, FM_5, FM_6, FM_9, FM_{10} are in the second group with lower risk. It can be discovered that in the second group, the risk priority ranking by the proposed method is the same as that by MVRPN and improved MVRPN. A close look at the first group, the FB-TOPSIS, MVRPN, and improved MVRPN give a higher risk to FM_7 in comparison with FM_1, while FM_7 is ranked behind FM_1 in the risk priority ranking by the proposed method. The main reason may be that FB-TOPSIS, MVRPN, and improved MVRPN do not consider the objective weights of risk factors which are obtained by the intrinsic information. Besides, according to Table 6, it can be found that FM_1 has a higher level of the probability of occurrence in comparison with FM_7, and the occurrence has the greatest weight. Therefore, FM_1 should be given a higher priority than FM_7. A close look at FM_{10}, the ranking is in front of FM_2 in FB-TOPSIS, while FM_{10} ranks behind FM_2 in the proposed method. However, FM_{10} has the lowest level of the probability of occurrence according to Table 6, and GRPM, MVRPN, and improved MVRPN give the least priority to FM_{10}. Thus, it is reasonable to rank FM_{10} behind FM_2. Comparing the risk priority ranking of failure modes by using the proposed method with GRPM, it can be found that the ranking results are basically the same. The difference is that FM_7 ranks in front of failure mode FM_2 in the proposed method, while the ranking of FM_7 is behind FM_2 in GRPM. In addition, the ranking of failure

TABLE 10. Risk ranking of failure modes by using different FMEA methods.

FMEA methods	Rank									
	1	2	3	4	5	6	7	8	9	10
FB-TOPSIS [59]	FM_4	FM_3	FM_8	FM_7	FM_1	FM_{10}	FM_5	FM_6	FM_9	FM_2
GRPM [46]	FM_4	FM_3	FM_8	FM_1	FM_2	FM_5	FM_6	FM_9	FM_7	FM_{10}
MVRPN [36]	FM_4	FM_7	FM_3	FM_8	FM_1	FM_2	FM_5	FM_6	FM_9	FM_{10}
improved MVRPN [37]	FM_4	FM_7	FM_3	FM_8	FM_1	FM_2	FM_5	FM_6	FM_9	FM_{10}
Proposed method	FM_4	FM_3	FM_8	FM_1	FM_7	FM_2	FM_5	FM_6	FM_9	FM_{10}

mode FM_7 is in front of FM_2 in FB-TOPSIS, MVRPN, and improved MVRPN. Therefore, it is reasonable to rank FM_7 in front of FM_2. Through the above analysis and comparison, it shows that the proposed method is effective for risk evaluation in FMEA.

In addition, in our previous studies on risk evaluation in FMEA [60], [61], the focuses are mainly on the uncertainty in the experts' assessments and evaluation process, ignoring the uncertainty in experts' knowledge about the relationships between input factors for risk evaluation in FMEA. Thus, we proposed a new FMEA method based on DST by integrating EN to simultaneously represent the uncertainty in the experts' assessments and in experts' knowledge about the relationships between input factors. Besides, both subjective weight and objective weight are considered to reflect the relative importance of risk factors and avoid failure modes from being underestimated or overestimated, which is different from our previous work. Furthermore, the assessments of multiple experts are integrated from the perspective of network reasoning, which provides a new solution for the risk evaluation in FMEA.

VI. CONCLUSIONS

In this paper, a new FMEA method based on DST by integrating EN is proposed to improve risk evaluation process. In this proposed method, DST is applied to represent both the probabilistic uncertainty and epistemic uncertainty in the experts' assessments, and EN provides the framework for representing experts' knowledge about the relationships between input factors. Besides, an improved EN is employed to integrate the assessments of multiple experts, which avoids the combination explosion of the number of states when implemented in large real-world applications, and we propose a novel approach to determine the CBMT of the non-root node. Additionally, the proposed FMEA method uses linguistic terms in the uncertainty evaluation structure, which overcomes the limitation of crisp assessments in the traditional RPN approach and allows experts to give more moderate judgments in the evaluation process. To the end, both subjective weight and objective weight are utilized to determine the weights of risk factors, which can fully reflect the importance of risk factors and avoid failure modes from being underestimated or overestimated. Finally, a case study of a steel factory is provided to illustrate the effectiveness of the proposed FMEA method. By comparing with other FMEA methods, the results show that the proposed method is effective for risk evaluation in FMEA.

This study provides a new solution for the risk evaluation in FMEA from the perspective of network reasoning, which can be easily applied in industries fields. In future research, the relative importance of each expert can be studied in FMEA analysis. Besides, how to construct the CBMT of the non-root node when its parents' FODs are different needs further study. Finally, we will explore other technologies such as MCDM methods to improve the risk evaluation in FMEA.

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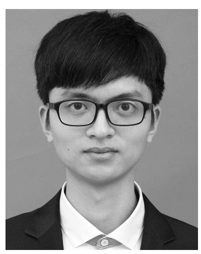

HONGFEI WANG received the bachelor's and master's degrees from Northeast Agricultural University, Harbin, China, in 2016 and 2018, respectively. He is currently pursuing the Ph.D. degree in electronic science and technology with Northwestern Polytechnical University. His main research interests include uncertain information processing and fault diagnosis.

XINYANG DENG received the bachelor's, master's, and Ph.D. degrees from Southwest University, Chongqing, China, in 2010, 2013, and 2016, respectively, and the joint Ph.D. degree Vanderbilt University, in 2016. He was supported by the China Scholarship Council. Since 2017, he has been an Associate Professor with the School of Electronics and Information, Northwestern Polytechnical University. His main research interests include multi-source information fusion, Dempster–Shafer evidence theory, uncertain information modeling and processing, and evolutionary game theory. He is a Reviewer for several SCI journals.

ZHUO ZHANG received the bachelor's and master's degrees from Northeast Agricultural University, Harbin, China, in 2016 and 2018, respectively. She is currently pursuing the Ph.D. degree in electronic science and technology with Northwestern Polytechnical University. Her main research interests include uncertain information processing and deep learning.

WEN JIANG received the bachelor's and master's degrees from Information Engineering University, Zhengzhou, China, in 1994 and 1997, respectively, and the Ph.D. degree from Northwestern Polytechnical University, Xi'an, China, in 2009, where she is currently a Professor and a Doctoral Advisor with the School of Electronics and Information. From 2012 to 2013, she was a Visiting Scholar with the University of Miami. She is a Commentator of the National Natural Science Foundation of China. Her main research interests include information fusion and intelligent information processing. She is a Reviewer of several SCI and EI journals. She is a member of the Information Integration Branch, China Aviation Society.