IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received April 29, 2019, accepted June 2, 2019, date of publication June 13, 2019, date of current version September 4, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2922748

Adaptive Finite-Time Control of Nonlinear Quantized Systems With Actuator Dead-Zone

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ABSTRACT This paper investigates a finite-time control problem of nonlinear quantized systems with actuator dead-zone in a non-strict feedback form. By combining a simplified dead-zone model and the sector-bound characteristic of a hysteretic quantizer, the control difficulties caused by the coexistence of unknown actuator dead-zone and control signal quantization effect are overcome. By applying the approximation ability of neural network systems, an novel neural adaptive controller is constructed, which can compensate the unknown control gain. The designed neural controller can ensure the transient performance of nonlinear quantized systems with actuator dead-zone in finite-time. Based on the Bhat and Bernstein theorem, the finite-time stability of system is proved. Finally, a numerical example is given to verify the validity of the proposed approach.

INDEX TERMS Adaptive neural control, backstepping technique, unknown dead-zone, nonlinear quantized systems, finite-time stability.

I. INTRODUCTION

Over the past few decades, the adaptive fuzzy or neural control based on approximation has received extensive attention. In [1]–[15], fuzzy logical or neural network systems were used to model the unknown nonlinear functions, and the adaptive controllers were designed by combing adaptive technique with backstepping. Although some achievements have been made in [1]–[15], signal quantization was neglected. Therefore, the problem of quantized control for linear and non-linear systems has also caught the attention of many scholars. The stability of a category of linear quantized systems was investigated in [16], [17]. In addition, the quantized control of a kind of uncertain nonlinear systems was studied in [18]- [22]. Compared with the schemes in [16], [17], the system models in [18]–[22] do not need to be completely known by the designer. By applying the backstepping technique, [23]–[27] proposed some input quantization approaches to compensate the unknown control gain. Unlike some existing control strategies for input quantized nonlinear systems, the proposed control schemes in [23]–[27] do not require the global Lipschitz condition, and the quantization parameters can be unknown to the designer. It is noteworthy that the adaptive fuzzy control in [23] and [25] requires that the systems have strict-feedback or pure-feedback forms, respectively. In theory, when the system does not meet such structural conditions, the system can not be controlled by the controller by employing the adaptive fuzzy method. Furthermore, [28] discussed the adaptive tracking control issue for a more general category of non-strict feedback systems with quantized input. Although some achievements have been made, the above literatures [16]–[28] on quantized work do not consider the problem of actuator dead-zone.

It is well known that the actuator dead-zone can degrade the systems performance and even lead to the system instability. In order to deal with the dead-zone nonlinearity, a variety of approaches have been proposed in [29]–[32] to design the controllers. Among them, there are two main methods are usually employed. One method, as shown in [33]–[35], is to compensate the impact by creating a slick inverse of dead-zone. The another method, as described in [36]–[39], is facilitate the control design by establishing a simplified dead-zone model. The above achievements are based on the

The associate editor coordinating the review of this article and approving it for publication was Ruilong Deng.

implicit assumption that the control commands transmitted between the physical actuator and the controller does not exhibit quantization phenomena. The work [40] investigates the issue of stochastic nonlinear quantized systems with actuator dead-zone, the closed-loop system was verified to be bounded in probability by using the Lyapunov synthesis. However, the aforementioned control researches in [16]–[40] focus on the problem of the infinite time stability. In theory, the control schemes in [16]–[40] can not ensure the transient response of the system in finite-time.

As a class of time-optimal control, the finite-time control has been caught more and more attention by scholars in recent years. The key characteristic of the finite-time control is to make the system reach equilibrium in finite-time and keep equilibrium thereafter. Reference [41] constructed a sliding controller by establishing a terminal mode sliding manifold. In order to solve the flutter phenomena caused by discontinuous controller, the Lyapunov theory of finite-time stability has been built for the first time in [42], [43]. On the base of the Lyapunov stability theory in [42], [43], the finite-time stability issues of the systems were discussed in [44]–[48]. Reference [49] discussed the problem of adaptive-robust stabilization of the Furuta's pendulum around unstable equilibrium, [50] designed a robust control for stabilization of a kind of nonlinear perturbed system with matched and unmatched disturbances, the controllers in [49] and [50] guarantee the ultimate uniform bound stabilization or controllers based on attractive ellipsoid method. In order to ensure the finite-time stability of nonlinear systems, some control programs were designed in [51]–[53] for a kind of nonlinear systems with hysteretic characteristics. By employing more appropriate Lyapunov-Krasovskii functional, the finite-time stability of time-varying delay systems was discussed, see [54], [55]. Furthermore, [56]–[58] presented some finite-time control strategies for a category of nonlinear systems with actuation failures. In addition, for a kind of nonlinear systems with dead-zone, several finite-time tracking control programs were investigated in [59]–[65]. In [66], a new adaptive finite-time output-feedback control method was proposed for a kind of nonlinear quantized systems with unmeasurable states. However, the scheme in [66] does not take into account the influence of the actuator dead-zone on the control performance. To the best of our knowledge, up to now, the finite-time quantized control problem of a category of non-strict feedback nonlinear systems with actuator dead-zone has not been studied, although it has a great potential in networked control systems. For the above-mentioned discussions, this manuscript is devoted to solving the finite-time control problem of non-strict feedback nonlinear quantized systems with actuator dead-zone. The contributions of this paper are highlighted as follows.

(1) Compared with the researches of nonlinear quantized systems with actuator dead-zone, a finite-time control strategy is developed. The proposed control scheme ensures the transient performance of quantized system with actuator dead-zone in finite-time, and a valid finite-time solution is obtained for nonlinear quantized systems with actuator dead-zone in this article.

(2) According to the structural properties of radial basis function (RBF) neural networks (NNs), the structure of the plants in this manuscript is in non-strict feedback form. Hence, the control scheme is more challenging and the control system is more common.

The rest of this manuscript is arranged as follows. The second part provides necessary preparation and problem statement. The third part addresses a tracking problem of non-strict feedback nonlinear quantized systems with actuator dead-zone, and the finite-time convergence is proved. The fourth part verifies the effectiveness of the controller through an example. The fifth part gives the conclusion and points out the future research direction.

II. PREREQUISITES AND PROBLEM FORMULATION

A. PREREQUISITES

Lemma 1 (see [63]): Considering the nonlinear system $\rho = f(\rho, \varsigma)$, for smooth positive definite function $V(\rho) \in C^1$, if there exist scalars $c > 0$, $d > 0$, and $0 < \sigma < 1$ satisfying that

$$\dot{V}(\rho) \leq -cV^{\sigma}(\rho) + d, \quad t \geq 0. \tag{1}$$

then, the nonlinear system $\dot{\rho} = f(\rho, \varsigma)$ is SGPFS.

Remark 1: Similar to the finite-time investigates in [50] and [67], Lemma 1 gives an important criterion of SGPFS, which will be employed in the subsequent finite-time stability analysis.

Lemma 2 (see [56]): For $\iota_k \in R,\ k = 1, \ldots, n,\ 0 < p \leq 1$, one has:

$$\Big(\sum_{k=1}^{n} |\iota_k|\Big)^p \leq \sum_{k=1}^{n} |\iota_k|^p \leq n^{1-p}\Big(\sum_{k=1}^{n} |\iota_k|\Big)^p. \tag{2}$$

Lemma 3 (see [68]): For $\forall \xi \in R$ and $\forall \epsilon > 0$, the following relationship can be obtained:

$$0 \leq |\xi| - \xi \tanh\left(\frac{\xi}{\epsilon}\right) \leq \kappa \epsilon. \tag{3}$$

where κ is a constant that meets $\kappa = e^{-(\kappa+1)}$, i.e., $\kappa = 0.2785$.

Lemma 4 (see [23]): For $\dot{\hat{\theta}}(t) = -\gamma\hat{\theta}(t) + \kappa\nu(t)$, if $\nu(t) > 0$ and $\hat{\theta}(t_0) \geq 0$ are satisfied, we have $\theta(t) \geq 0$ for $\forall t \geq t_0$, where $\gamma > 0$ and $\kappa > 0$ denote the design parameters.

Lemma 5 (see [69]): For any real variables υ and ω, the following inequality holds:

$$|\upsilon|^{\tau}|\omega|^{\gamma} \leq \frac{\tau}{\tau+\gamma}\lambda|\upsilon|^{\tau+\gamma} + \frac{\gamma}{\tau+\gamma}\lambda^{\frac{-\tau}{\gamma}}|\omega|^{\tau+\gamma}. \tag{4}$$

where $\tau > 0$, $\gamma > 0$ and $\lambda > 0$ represent the design parameters.

Remark 2: Same as the finite-time studies in [1], [2], [66], Lemma 2 and Lemma 5 will be used widely to cope with the inequality (61) and (63).

B. PROBLEM DESCRIPTION

Think about the non-strict feedback nonlinear system as follows:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x), & 1 \le i \le n-1, \\ \dot{x}_n = \Gamma(Q(u)) + f_n(x), \\ y = x_1. \end{cases} \tag{5}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ represents the state vector, $y \in \mathbb{R}$ represents the system output, $u \in \mathbb{R}$ represents the control signal. $f_i(.) : \mathbb{R}^n \to \mathbb{R}$ represents an unknown smooth nonlinear function. $\Gamma(Q(u))$ denotes the system input influenced by quantization and actuator dead-zone, which can be designed as follows:

$$\Gamma(Q(u)) = \begin{cases} k_r(Q(u) - b_r), & Q(u) \ge b_r, \\ 0, & b_l < Q(u) < b_r, \\ k_l(Q(u) - b_l), & Q(u) \le b_l. \end{cases} \tag{6}$$

where b_l and b_r stand for the breakpoints of the actuator nonlinearity. The parameter k_r represents the right slope of the actuator dead-zone and the parameter k_l denotes the left slope of the actuator dead-zone. The quantizer $Q(u(t))$ is the hysteretic, which can be expressed as follows:

$$Q(u) = \begin{cases} u_i sgn(u), & \frac{u_i}{1+\delta} < |u| \le u_i, \dot{u} < 0, or \\ & u_i < |u| \le \frac{u_i}{1-\delta}, \dot{u} > 0. \\ u_i(1+\delta)sgn(u), & u_i < |u| \le \frac{u_i}{1-\delta}, \dot{u} < 0, \\ & \frac{u_i}{1-\delta} < |u| \le \frac{u_i(1+\delta)}{1-\delta}, \dot{u} > 0. \\ 0, & 0 \le |u| < \frac{u_{min}}{1+\delta}, \dot{u} < 0, or \\ & \frac{u_{min}}{1+\delta} \le |u| \le u_{min}, \dot{u} > 0. \\ Q(u(t^-)), & othercases. \end{cases} \tag{7}$$

where $sgn(.)$ is a signum function, which is defined as follows:

$$sgn(u) = \frac{u}{|u|} = \begin{cases} 1, & u > 0 \\ 0, & u = 0 \\ -1, & u < 0 \end{cases} \tag{8}$$

$u_i = \rho^{1-i} u_{min} (i = 1, 2, \ldots)$, $\rho \in (0, 1)$ represents a measure of quantization density, the parameters $u_{min} > 0$, and $\delta = \frac{1-\rho}{1+\rho}$. Therefore, $Q(u) \in U = \{0, \pm u_i, \pm u_i(1+\delta), i = 1, 2, \ldots\}$. In the hysteresis quantizer (7), u_{min} determines the range of dead-zone for $Q(u)$.

Remark 3: What needs to be pointed out is, compared with the existing quantized researches, this paper considers the unknown actuator dead-zone of the non-strict feedback nonlinear system. And this study will provide a stability analysis scheme for the actuator nonlinearity based on quantitative control system.

Assumption 1: There exist a smooth monotonously increasing function β_i: $R^+ \longrightarrow R^+, i = 1, 2, \ldots, n$, under the initial value of $\beta_i(0) = 0$, we have:

$$|f_i(x)| \le \beta_i(\|x\|).$$

What needs to be pointed out is that β_i is a monotone increasing function, which satisfies $\beta_i(0) = 0$, that means $\beta_i(\sum_{k=1}^n a_k) \le \sum_{k=1}^n \beta_i(n a_k)$, where $a_k > 0$. There exists a smooth function $h_i(s)$ such that $\beta_i(s) = s h_i(s)$. which results in

$$\beta_i(\sum_{k=1}^n a_k) \le \sum_{k=1}^n n a_k h_i(n a_k) \tag{9}$$

This inequality will be widely applied in the following processes.

Assumption 2: There are design parameters $\varpi > 0$ and $\bar{\varpi} > 0$, such that:

$$0 < \varpi \le k_l \le \bar{\varpi}, \quad 0 < \varpi \le k_r \le \bar{\varpi}.$$

The existence of quantization and dead-zone increases the difficulty of controller design, it is necessary to introduce the following Lemma.

Lemma 6 (see [40]): $\Gamma(Q(u))$ can be broken down as follows:

$$\Gamma(Q(u)) = \bar{H}(u)u + \bar{G}(t). \tag{10}$$

where

$$\begin{aligned} \varpi(1-\delta) \le \bar{H}(u) \le \bar{\varpi}(1+\delta), \\ |\bar{G}(t)| \le \bar{\varpi} \max\{|b_r|, |b_l|\} + \bar{\varpi} u_{\min}. \end{aligned} \tag{11}$$

C. NEURAL NETWORK SYSTEMS

Since system (5) contains the unknown function $f_i(.) : \mathbb{R}^n \to \mathbb{R}$, therefore, radial basis function (RBF) neural networks (NNs) will be adopted to approximate $f_i(.)$. The RBF NNs is described in the following form:

$$f_{nn}(X) = \Phi^T \xi(X).$$

where $X \in \Omega_x \subset \mathbb{R}^q$ represents the input vector, $\Phi = [\phi_1, \ldots, \phi_l]^T \in \mathbb{R}^l$ denotes the weight vector of RBF NNs, $l(> 1)$ denotes the number of nodes, $\xi(X) = [\xi_1(X), \ldots, \xi_l(X)]^T \in \mathbb{R}^l$ represents the basis function vector, and $\xi_i(X)$ as shown below:

$$\xi_i(X) = \exp[-\frac{(X - v_i)^T (X - v_i)}{\zeta^2}], \quad 1 \le i \le l.$$

where $v_i = [v_{i1}, \ldots, v_{iq}]^T$ is the center of the receiving field, and $\zeta > 0$ denotes the width of the Gaussian function.

$f(X)$ is a continuous function define on a compact set Ω_x. For $\forall \varepsilon > 0$, there exists an RBF NNs $\Phi^{*T}\xi(X)$, makes the following formula holds:

$$f(X) = \Phi^{*T}\xi(X) + \delta(X), \quad \forall X \in \Omega_x. \tag{12}$$

where $\delta(X)$ is the approximation error, Φ^* represents the ideal weight vector, and the inequality $|\delta(X)| < \varepsilon$ holds. when the number of nodes l is large enough, one has:

$$\Phi^* = \arg\min_{\Phi \in \mathbb{R}^l} \{ \sup_{X \in \Omega_x} |f(X) - \Phi^T \xi(X)| \}.$$

Lemma 7 (see [10]): Let $\xi(X) = [\xi_1(X), \ldots, \xi_l(X)]^T$ denotes the basis function vector of an RBF NNs, and $X =$

$[x_1, \ldots, x_n]^T$ denotes the input vector. For $\forall m \le n$, let $X_m = [x_1, \ldots, x_m]$, we have:

$$\|\xi(X)\|^2 \le \|\xi(X_m)\|^2.$$

Remark 4: The above lemma provides a simple but useful structural property of RBF NNs, which is helpful to handle the whole state variable in the process of backstepping design.

III. CONTROLLER DESIGN PROCESS

In this part, an adaptive controller for the system (5) will be designed. In order to begin the backstepping design procedure, $\Phi_i^*(X_i)$ will be employed to approximate the unknown nonlinear function, define $\theta_i = \|\Phi_i^*\|^2, i = 1, 2, \cdots, n$, where Φ_i^* represents the ideal weight vector of RBF NNs. Define $\hat{\theta}_i$ as the estimate of θ_i, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ as an estimation error. Then, define the error variables as follows:

$$\begin{aligned} z_1 &= x_1 - y_d, \\ z_i &= x_i - \alpha_{i-1}, \quad i = 2, \cdots, n. \end{aligned} \tag{13}$$

where α_i represents the virtual control function, it can be expressed as follows:

$$\alpha_i = -c_i z_i^{2\sigma-1} - \frac{1}{2a_i^2}\hat{\theta}_i z_i \xi_i^T(Z_i)\xi_i(Z_i), \tag{14}$$

where $\xi_i(Z_i)$ $\Big(Z_i = [\bar{x}_i^T, \bar{\hat{\theta}}_{i-1}^T, \bar{y}_d^{(i)T}]^T$ with $\bar{\hat{\theta}}_{i-1} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{i-1}]^T$, $\bar{y}_d^{(i)} = [y_d, y_d^{(1)}, \ldots, y_d^{(i)}]^T$, $i = 1, 2, \ldots, n.\Big)$ denotes the basis function vector of RBF NNs. $c_i > 0$, $a_i > 0$, and $\sigma = \frac{2l-1}{2l+1}(l > 2, l \in n)$ represent the design parameters.

The controller can be expressed as follows:

$$u = -\frac{c_n}{1-\delta} z_n^{2\sigma-1} - \frac{z_n \hat{\theta}_n \xi_n^T(Z_n)\xi_n(Z_n)}{2a_n^2(1-\delta)}, \tag{15}$$

where $c_n > 0$ and $a_n > 0$ are two parameters.

The adaptive law of $\hat{\theta}$ is chosen as:

$$\dot{\hat{\theta}}_i = \frac{q_i}{2a_i^2} z_i^2 \xi_i^T \xi_i - \gamma_i \hat{\theta}_i, \ \hat{\theta}_i(0) \ge 0, \quad i = 1, 2, \ldots, n. \tag{16}$$

where $q_i > 0$ and $\gamma_i > 0$ denote the design parameters.

Assumption 3: For the desired trajectory y_d and its i-th derivative $y_d^{(i)}$, $i = 1, 2, \ldots, n$, one has:

$$|y_d^{(i)}| \le d, \quad i = 0, 1, 2, \ldots, n.$$

where $d \ge 0$ denotes a constant.

Lemma 8: For $z_i = x_i - \alpha_{i-1}$, $i = 1, 2, \ldots, n$, one can get the relation as follows:

$$\|x\| \le \sum_{i=1}^{n} |z_i| \omega_i(z_i, \hat{\theta}_i) + d \tag{17}$$

where $\omega_j(z_i, \hat{\theta}_i) = 1 + c_i z_i^{2\sigma-2} + \frac{1}{2a_i^2}\hat{\theta}_i \xi_i^T(Z_i)\xi_i(Z_i)$, $\omega_n(z_n, \hat{\theta}_n) = 1$, $i = 1, 2, \ldots, n-1$.

Proof: Let $x = [x_1, \ldots, x_n]^T$, $z = [z_1, \ldots, z_n]^T$, and $\alpha = [\alpha_1, \ldots, \alpha_n]^T$. Then, one has

$$\begin{aligned} \|x\| &\le \sum_{i=1}^{n} |x_i| = \sum_{i=1}^{n} |z_i + \alpha_{i-1}| \\ &\le \sum_{i=1}^{n} |z_i| + \sum_{i=1}^{n-1} |\alpha_i| + y_d \qquad (18) \\ &\le \sum_{i=1}^{n} |z_i| + |y_d| + \sum_{i=1}^{n-1} |z_i| \Big(c_i z_i^{2\sigma-2} + \frac{1}{2a_i^2}\hat{\theta}_i \xi_i^T(Z_i)\xi_i(Z_i) \Big) \\ &\le \sum_{i=1}^{n} |z_i| \omega_i(z_i, \hat{\theta}_i) + d. \qquad (19) \end{aligned}$$

Step 1: For the non-strict feedback nonlinear system (5), the derivative of z_1 is

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_d = x_2 + f_1(x) - \dot{y}_d. \tag{20}$$

Think about Lyapunov function candidate as follows:

$$V_1 = \frac{z_1^2}{2} + \frac{\tilde{\theta}_1^2}{2q_1}, \tag{21}$$

where $q_1 > 0$ represents a design parameter.

Differentiating V_1 yields:

$$\dot{V}_1 = z_1(z_2 + \alpha_1 + f_1(x) - \dot{y}_d) - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{q_1}. \tag{22}$$

According to Young's inequality, Assumption 1, (9) and Lemma 8, the following inequality can be obtained:

$$\begin{aligned} z_1 f_1 &\le |z_1| \beta_1(\|x\|) \le |z_1| \beta_1 \Big(\sum_{l=1}^{n} |z_l| \omega_l(z_l, \hat{\theta}_l) + d \Big) \\ &\le \sum_{l=1}^{n} |z_1| \beta_1 \Big((n+1)|z_l| \omega_l(z_l, \hat{\theta}_l) \Big) \\ &\quad + |z_1| \beta_1 \Big((n+1)d \Big) \\ &\le \frac{n}{2} z_1^2 + \sum_{l=1}^{n} z_l^2 \bar{\beta}_1^2(z_l, \hat{\theta}_l) \\ &\quad + |z_1| \beta_1 \Big((n+1)d \Big), \end{aligned} \tag{23}$$

where

$$\bar{\beta}_1^2(z_l, \hat{\theta}_l) = \frac{1}{2}(n+1)^2 \omega_l^2(z_l, \hat{\theta}_l) h_1^2 \Big((n+1)|z_l| \omega_l(z_l, \hat{\theta}_l) \Big).$$

Then, the following inequality can be obtained by using Lemma 3 to the term $|z_1|\beta_1\Big((n+1)d\Big)$ in (23)

$$\begin{aligned} |z_1|\beta_1((n+1)d) &\le \tanh\Big(\frac{z_1\beta_1((n+1)d)}{\epsilon_1}\Big) \\ &\quad \times z_1 \beta_1\Big((n+1)d\Big) + \kappa\epsilon_1. \end{aligned} \tag{24}$$

where $\epsilon_1 > 0$ denotes a design parameter.

Applying Young's inequality, one has:

$$z_1 z_2 \le \frac{z_1^2}{2} + \frac{z_2^2}{2} \tag{25}$$

Substituting (14), (23)–(25) into (22), we have:

$$\dot{V}_1 \le -\frac{1}{2}z_1^2 - c_1 z_1^{2\sigma} - \frac{1}{2a_1^2}\hat{\theta}_1 z_1^2 \xi_1^T(Z_1)\xi_1(Z_1) + z_1\bar{f}_1 - \frac{\tilde{\theta}_1\dot{\hat{\theta}}_1}{q_1} + \sum_{l=2}^{n} z_l^2 \bar{\beta}_1^2(z_l,\hat{\theta}_l) - z_1^2\sum_{k=2}^{n-1}\sum_{j=1}^{k}\bar{\beta}_j^2(z_1,\hat{\theta}_1) + \frac{z_2^2}{2} + \kappa\epsilon_1, \quad (26)$$

where

$$\bar{f}_1 = \frac{(n+2)z_1}{2} + z_1\bar{\beta}_1^2(z_1,\hat{\theta}_1) - \dot{y}_d + \beta_1\Big((n+1)d\Big)\tanh\Big(\frac{z_1\beta_1((n+1)d)}{\epsilon_1}\Big) + z_1\sum_{k=2}^{n-1}\sum_{j=1}^{k}\bar{\beta}_j^2(z_1,\hat{\theta}_1).$$

It is clear that $\bar{f}_1$ is a function of $\bar{\beta}_j$. There exist an RBF NNs $\Phi_1^{*T}\xi_1(X_1)$, it can be used to approximate $\bar{f}_1$. Namely, for $\forall\varepsilon_1 > 0$,

$$\bar{f}_1 = \Phi_1^{*T}\xi_1(X_1) + \delta_1(X_1), \quad |\delta_1(X_1)| \le \varepsilon_1. \quad (27)$$

where $X_1 = (x_1,\ldots,x_n,y_d,\dot{y}_d)$.

By combining (27), Young's inequality and Lemma 7, the following inequality can be obtained:

$$\begin{aligned} z_1\bar{f}_1 &= z_1(\Phi_1^{*T}\xi_1(X_1) + \delta_1(X_1)) \\ &\le |z_1|(\|\Phi_1^*\|\|\xi_1(X_1)\| + \varepsilon_1) \\ &\le \frac{1}{2a_1^2}z_1^2\theta_1\xi_1^T(Z_1)\xi_1(Z_1) + \frac{1}{2}a_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2. \end{aligned} \quad (28)$$

where $\theta_1 = \|\Phi_1^*\|^2$, $Z_1 = (x_1,y_d,\dot{y}_d)$, and $a_1 > 0$ denotes a design parameter.

Substituting (16) and (28) into (26), the following inequality holds:

$$\dot{V}_1 \le -c_1 z_1^{2\sigma} + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + \frac{\gamma_1}{q_1}\tilde{\theta}_1\hat{\theta}_1 + \frac{z_2^2}{2} + \kappa\epsilon_1 + \sum_{l=2}^{n} z_l^2\bar{\beta}_1^2(z_l,\hat{\theta}_l) - z_1^2\sum_{k=2}^{n-1}\sum_{j=1}^{k}\bar{\beta}_j^2(z_1,\hat{\theta}_1). \quad (29)$$

Furthermore, according to $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ and Young's ineuqality, the following inequality can be obtained:

$$\begin{aligned} \frac{\gamma_1}{q_1}\tilde{\theta}_1\hat{\theta}_1 &= \frac{\gamma_1}{q_1}\tilde{\theta}_1(\theta_1 - \tilde{\theta}_1) \\ &= \frac{\gamma_1}{q_1}\tilde{\theta}_1\theta_1 - \frac{\gamma_1}{q_1}\tilde{\theta}_1^2 \\ &\le \frac{\gamma_1}{2q_1}\tilde{\theta}_1^2 + \frac{\gamma_1}{2q_1}\theta_1^2 - \frac{\gamma_1}{q_1}\tilde{\theta}_1^2 \\ &= -\frac{\gamma_1}{2q_1}\tilde{\theta}_1^2 + \frac{\gamma_1}{2q_1}\theta_1^2 . \end{aligned} \quad (30)$$

Then, the inequality (29) can be rewritten as follows:

$$\dot{V}_1 \le -c_1 z_1^{2\sigma} - \frac{\gamma_1}{2q_1}\tilde{\theta}_1^2 + \sum_{l=2}^{n} z_l^2\bar{\beta}_1^2(z_l,\hat{\theta}_l) - z_1^2\sum_{k=2}^{n-1}\sum_{j=1}^{k}\bar{\beta}_j^2(z_1,\hat{\theta}_1) + \frac{z_2^2}{2} + \lambda_1. \quad (31)$$

where

$$\lambda_1 = \kappa\epsilon_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + \frac{\gamma_1}{2q_1}\theta_1^2.$$

Step i ($2 \le i \le n-1$): Let us assume that

$$V_{i-1} = \sum_{j=1}^{i-1}(\frac{1}{2}z_j^2 + \frac{1}{2q_j}\tilde{\theta}_j^2),$$

satisfies

$$\dot{V}_{i-1} \le -\sum_{j=1}^{i-1}\Big(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2\Big) + \sum_{s=1}^{i-1}\sum_{k=1}^{s}\sum_{l=i}^{n} z_l^2\bar{\beta}_k^2(z_l,\hat{\theta}_l) - \sum_{s=1}^{i-1}\sum_{k=i}^{n-1}\sum_{j=1}^{k} z_s^2\bar{\beta}_j^2(z_s,\hat{\theta}_s) + \frac{z_i^2}{2} + \lambda_{i-1}, \quad (32)$$

where

$$\lambda_{i-1} = \sum_{j=1}^{i-1}(\kappa\epsilon_j + \frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2} + \frac{\gamma_j}{2q_j}\theta_j^2).$$

And then, let us consider a Lyapunov function candidate as follows:

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2q_i}\tilde{\theta}_i^2. \quad (33)$$

Differentiating z_i yields:

$$\dot{z}_i = \dot{x}_i - \dot{\alpha}_{i-1} = x_{i+1} + f_i(x) - \dot{\alpha}_{i-1}. \quad (34)$$

where

$$\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Big(x_{j+1} + f_j(x)\Big) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)}. \quad (35)$$

Differentiating V_i yields:

$$\dot{V}_i = \dot{V}_{i-1} + z_i(z_{i+1} + \alpha_i + f_i(x) - \dot{\alpha}_{i-1}) - \frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \quad (36)$$

According to Assumption 1, Lemma 5, (9) and Lemma 8, the following inequality can be obtained:

$$\begin{aligned} z_i f_i &\le |z_i|\beta_i(\|x\|) \le |z_i|\beta_i\Big(\sum_{l=1}^{n}|z_l|\omega_l(z_l,\hat{\theta}_l) + d\Big) \\ &\le \sum_{l=1}^{n}|z_i|\beta_i\Big((n+1)|z_l|\omega_l(z_l,\hat{\theta}_l)\Big) \end{aligned}$$

$$+|z_i|\beta_i\Big((n+1)d\Big) \le \frac{n}{2}z_i^2+\sum_{l=1}^{n}z_l^2\bar{\beta}_i^2(z_l,\hat{\theta}_l)+|z_i|\beta_i\Big((n+1)d\Big), \tag{37}$$

$$\begin{aligned} z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j &\le |z_i|\sum_{j=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_j}|\beta_j\Big(\sum_{l=1}^{n}|z_l|\omega_l(z_l,\hat{\theta}_l)+d\Big)\\ &\le |z_i|\sum_{j=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_j}|\beta_j\Big((n+1)d\Big)\\ &\quad+|z_i|\sum_{j=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_j}|\\ &\quad\times\sum_{l=1}^{n}\beta_j\Big((n+1)|z_l|\omega_l(z_l,\hat{\theta}_l)\Big)\\ &\le |z_i|\sum_{j=1}^{i-1}|\frac{\partial\alpha_{i-1}}{\partial x_j}|\beta_j\Big((n+1)d\Big)\\ &\quad+\frac{n}{2}z_i^2\sum_{j=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_j})^2\\ &\quad+\sum_{j=1}^{i-1}\sum_{l=1}^{n}z_l^2\bar{\beta}_j^2(z_l,\hat{\theta}_l). \end{aligned} \tag{38}$$

where

$$\bar{\beta}_j^2(z_l,\hat{\theta}_l)=\frac{1}{2}(n+1)^2\omega_l^2(z_l,\hat{\theta}_l)h_j^2\Big((n+1)|z_l|\omega_l(z_l,\hat{\theta}_l)\Big).$$

Let

$$\chi_i=\beta_i((n+1)d)+\sum_{j=1}^{i-1}\Big|\frac{\partial\alpha_{i-1}}{\partial x_j}\Big|\beta_j\Big((n+1)d\Big),$$

and using Lemma 3, one can get the relationship as follows:

$$|z_i|\chi_i\le\kappa\epsilon_i+z_i\chi_i\tanh\Big(\frac{z_i\chi_i}{\epsilon_i}\Big). \tag{39}$$

where $\epsilon_i>0$ denotes a design parameter.

Meanwhile, applying Young's inequality, the following inequality can be obtained:

$$z_iz_{i+1}\le\frac{z_i^2}{2}+\frac{z_{i+1}^2}{2}. \tag{40}$$

Substituting (14), (32), (35) and (37)–(40) into (36), the following inequality can be obtained:

$$\begin{aligned} \dot{V}_i &\le -\sum_{j=1}^{i-1}\Big(c_jz_j^{2\sigma}+\frac{\gamma_j}{2q_j}\tilde{\theta}_j^2\Big)+\lambda_{i-1}\\ &\quad+\frac{1}{2}z_{i+1}^2-c_iz_i^{2\sigma}-\frac{1}{2}z_i^2+z_i\bar{f}_i\\ &\quad-\frac{\hat{\theta}_i}{2a_i^2}z_i^2\xi_i^T(Z_i)\xi_i(Z_i)-\frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i\\ &\quad+\kappa\epsilon_i+\sum_{s=1}^{i}\sum_{k=1}^{s}\sum_{l=i+1}^{n}z_l^2\bar{\beta}_k^2(z_l,\hat{\theta}_l)\\ &\quad-\sum_{s=1}^{i}\sum_{k=i+1}^{n-1}\sum_{j=1}^{k}z_s^2\bar{\beta}_j^2(z_s,\hat{\theta}_s), \end{aligned} \tag{41}$$

where

$$\begin{aligned} \bar{f}_i &= -\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1}-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j\\ &\quad+\frac{n}{2}z_i\sum_{j=1}^{i-1}\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2+\frac{n+3}{2}z_i\\ &\quad+\chi_i\tanh\Big(\frac{z_i\chi_i}{\epsilon_i}\Big)+z_i\sum_{s=1}^{n-1}\sum_{j=1}^{s}\bar{\beta}_j^2(z_i,\hat{\theta}_i)\\ &\quad-\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)}. \end{aligned}$$

Similarly, there exist an RBF NNs $\Phi_i^{*T}\xi_i(X_i)$, it can be used to approximate $\bar{f}_i$. Namely, for $\forall\varepsilon_i>0$, one can get the relation as follows:

$$\bar{f}_i=\Phi_i^{*T}\xi_i(X_i)+\delta_i(X_i),\quad |\delta_i(X_i)|\le\varepsilon_i. \tag{42}$$

where $X_i=(x_1,\ldots,x_n,\hat{\theta}_1,\ldots,\hat{\theta}_{i-1},y_d,\dot{y}_d,\ldots,y_d^{(i)})$.

By combining Young's inequality and Lemma 7, which is similar to the derivation in (28), the following inequality can be obtained:

$$\begin{aligned} z_i\bar{f}_i &= z_i(\Phi_i^{*T}\xi_i(X_i)+\delta_i(X_i))\\ &\le |z_i|(\|\Phi_i^*\|\|\xi_i(X_i)\|+\varepsilon_i)\\ &\le \frac{1}{2a_i^2}z_i^2\theta_i\xi_i^T(Z_i)\xi_i(Z_i)\\ &\quad+\frac{1}{2}a_i^2+\frac{1}{2}z_i^2+\frac{1}{2}\varepsilon_i^2. \end{aligned} \tag{43}$$

where $Z_i=(x_1,x_2,\ldots,x_i,\hat{\theta}_1,\hat{\theta}_2,\ldots,\hat{\theta}_{i-1},y_d,\dot{y}_d,\ldots,y_d^{(i)})$, $\theta_i=\|\Phi_i^*\|^2$ and $a_i>0$ denotes the design parameter.

Substituting (16) and (42)–(45) into (41), one has:

$$\begin{aligned} \dot{V}_i &\le -\sum_{j=1}^{i-1}\Big(c_jz_j^{2\sigma}+\frac{\gamma_j}{2q_j}\tilde{\theta}_j^2\Big)+\lambda_{i-1}+\frac{1}{2}z_{i+1}^2-c_iz_i^{2\sigma}\\ &\quad+\frac{1}{2a_i^2}z_i^2\theta_i\xi_i^T(Z_i)\xi_i(Z_i)+\frac{1}{2}a_i^2+\frac{1}{2}\varepsilon_i^2+\kappa\epsilon_i\\ &\quad-\frac{\hat{\theta}_i}{2a_i^2}z_i^2\xi_i^T(Z_i)\xi_i(Z_i)-\frac{1}{q_i}\tilde{\theta}_i(\frac{q_i}{2a_i^2}z_i^2\xi_i^T\xi_i-\gamma_i\hat{\theta}_i)\\ &\quad+\sum_{s=1}^{i}\sum_{k=1}^{s}\sum_{l=i+1}^{n}z_l^2\bar{\beta}_k^2(z_l,\hat{\theta}_l)\\ &\quad-\sum_{s=1}^{i}\sum_{k=i+1}^{n-1}\sum_{j=1}^{k}z_s^2\bar{\beta}_j^2(z_s,\hat{\theta}_s)\\ &= -\sum_{j=1}^{i-1}\Big(c_jz_j^{2\sigma}+\frac{\gamma_j}{2q_j}\tilde{\theta}_j^2\Big)+\lambda_{i-1}+\frac{1}{2}z_{i+1}^2-c_iz_i^{2\sigma}\\ &\quad+\frac{1}{2}a_i^2+\frac{1}{2}\varepsilon_i^2+\kappa\epsilon_i+\frac{\gamma_i}{q_i}\tilde{\theta}_i\hat{\theta}_i \end{aligned}$$

$$+\sum_{s=1}^{i}\sum_{k=1}^{s}\sum_{l=i+1}^{n} z_l^2\bar{\beta}_k^2(z_l,\hat{\theta}_l)$$
$$-\sum_{s=1}^{i}\sum_{k=i+1}^{n-1}\sum_{j=1}^{k} z_s^2\bar{\beta}_j^2(z_s,\hat{\theta}_s). \quad (44)$$

Furthermore, by employing Young's inequality and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, the following inequality can be obtained:

$$\begin{aligned}\frac{\gamma_i}{q_i}\tilde{\theta}_i\hat{\theta}_i &= \frac{\gamma_i}{q_i}\tilde{\theta}_i(\theta_i-\tilde{\theta}_i)\\ &= \frac{\gamma_i}{q_i}\tilde{\theta}_i\theta_i - \frac{\gamma_i}{q_i}\tilde{\theta}_i^2\\ &\le \frac{\gamma_i}{2q_i}\tilde{\theta}_i^2 + \frac{\gamma_i}{2q_i}\theta_i^2 - \frac{\gamma_i}{q_i}\tilde{\theta}_i^2\\ &= -\frac{\gamma_i}{2q_i}\tilde{\theta}_i^2 + \frac{\gamma_i}{2q_i}\theta_i^2. \end{aligned} \quad (45)$$

Then, the inequality (44) can be rewritten as:

$$\begin{aligned}\dot{V}_i &\le -\sum_{j=1}^{i-1}\Big(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2\Big) + \lambda_{i-1} + \frac{1}{2}z_{i+1}^2 - c_i z_i^{2\sigma}\\ &\quad + \frac{1}{2}a_i^2 + \frac{1}{2}\varepsilon_i^2 + \kappa\epsilon_i - \frac{\gamma_i}{2q_i}\tilde{\theta}_i^2 + \frac{\gamma_i}{2q_i}\theta_i^2\\ &\quad + \sum_{s=1}^{i}\sum_{k=1}^{s}\sum_{l=i+1}^{n} z_l^2\bar{\beta}_k^2(z_l,\hat{\theta}_l)\\ &\quad - \sum_{s=1}^{i}\sum_{k=i+1}^{n-1}\sum_{j=1}^{k} z_s^2\bar{\beta}_j^2(z_s,\hat{\theta}_s)\\ &\le -\sum_{j=1}^{i}\Big(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2\Big) + \frac{z_{i+1}^2}{2} + \lambda_i\\ &\quad + \sum_{s=1}^{i}\sum_{k=1}^{s}\sum_{l=i+1}^{n} z_l^2\bar{\beta}_k^2(z_l,\hat{\theta}_l)\\ &\quad - \sum_{s=1}^{i}\sum_{k=i+1}^{n-1}\sum_{j=1}^{k} z_s^2\bar{\beta}_j^2(z_s,\hat{\theta}_s), \end{aligned} \quad (46)$$

where

$$\begin{aligned}\lambda_i &= \lambda_{i-1} + \kappa\epsilon_i + \frac{a_i^2}{2} + \frac{\varepsilon_i^2}{2} + \frac{\gamma_i}{2q_i}\theta_i^2\\ &= \sum_{j=1}^{i}\Big(\kappa\epsilon_j + \frac{a_j^2}{2} + \frac{\varepsilon_j^2}{2} + \frac{\gamma_j}{2q_j}\theta_j^2\Big).\end{aligned}$$

Step n. According to (5), (10), (13), we have:

$$\dot{z}_n = \bar{H}(u)u + \bar{G}(t) + f_n(x) - \dot{\alpha}_{n-1}, \quad (47)$$

where

$$\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\Big(x_{j+1} + f_j(x)\Big) + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j + \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)}. \quad (48)$$

Choose the Lyapunov function candidate as follows:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2q_n}\tilde{\theta}_n^2, \quad (49)$$

According to (46), the following inequality can be obtained:

$$\begin{aligned}\dot{V}_n &\le -\sum_{j=1}^{n-1}\Big(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2\Big) + \frac{z_n^2}{2} + \lambda_{n-1} - \frac{1}{q_n}\tilde{\theta}_n\dot{\hat{\theta}}_n\\ &\quad + \sum_{s=1}^{n-1}\sum_{k=1}^{s} z_n^2\bar{\beta}_k^2(z_n,\hat{\theta}_n) - \sum_{s=1}^{n-1}\sum_{k=1}^{n} z_s^2\bar{\beta}_k^2(z_s,\hat{\theta}_s)\\ &\quad + z_n(\bar{H}(u)u + \bar{G}(t) + f_n(x) - \dot{\alpha}_{n-1}). \end{aligned} \quad (50)$$

Applying Young's inequality and (11), one has:

$$z_n\bar{G}(t) \le \frac{1}{2}z_n^2 + \frac{1}{2}u_{\min}^2. \quad (51)$$

Substituting (51) into (50), we have:

$$\begin{aligned}\dot{V}_n &\le -\sum_{j=1}^{n-1}(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2) + \lambda_{n-1} + z_n\bar{f}_n\\ &\quad - \frac{1}{2}z_n^2 + z_n\bar{H}(u)u + \frac{1}{2}u_{\min}^2 - \frac{1}{q_n}\tilde{\theta}_n\dot{\hat{\theta}}_n, \end{aligned} \quad (52)$$

where

$$\begin{aligned}\bar{f}_n &= f_n(x) - \dot{\alpha}_{n-1} + z_n\sum_{s=1}^{n-1}\sum_{k=1}^{s}\bar{\beta}_k^2(z_n,\hat{\theta}_n)\\ &\quad - \sum_{s=1}^{n-1}\sum_{k=1}^{n} z_s^2\bar{\beta}_k^2(z_s,\hat{\theta}_s) + \frac{3}{2}z_n. \end{aligned} \quad (53)$$

Similarly, there exist an RBF NNs $\Phi_n^{*T}\xi_n(X_n)$, it can be used to approximate $\bar{f}_n$. Namely, for $\forall\varepsilon_n > 0$, one can get the relation as follows:

$$\bar{f}_n = \Phi_n^{*T}\xi_n(X_n) + \delta_n(X_n), \quad |\delta_n(X_n)| \le \varepsilon_n. \quad (54)$$

where $X_n = (x_1,\ldots,x_n,\hat{\theta}_1,\ldots,\hat{\theta}_{n-1},y_d,\dot{y}_d,\ldots,y_d^{(n)})$.

By combining Young's inequality and Lemma 7, which is similar to the derivation in (28) and (43), the following inequality can be obtained:

$$\begin{aligned}z_n\bar{f}_n &= z_n(\Phi_n^{*T}\xi_n(X_n) + \delta_n(X_n))\\ &\le |z_n|(\|\Phi_n^*\|\|\xi_n(X_n)\| + \varepsilon_n)\\ &\le \frac{1}{2a_n^2}z_n^2\theta_n\xi_n^T(Z_n)\xi_n(Z_n)\\ &\quad + \frac{1}{2}a_n^2 + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_n^2. \end{aligned} \quad (55)$$

where $Z_n = (x_1,\ldots,x_n,\hat{\theta}_1,\ldots,\hat{\theta}_{n-1},y_d,\dot{y}_d,\ldots,y_d^{(n)})$, $\theta_n = \|\Phi_n^*\|^2$ and $a_n > 0$ denotes a design parameter.

Applying Lemma 4, (11) and (15), one has:

$$z_n\bar{H}(u)u \le -c_n z_n^{2\sigma} - \frac{1}{2a_n^2}z_n^2\hat{\theta}_n\xi_n^T\xi_n. \quad (56)$$

Combing (16), (45), (55) and (56) to (52), we have:

$$\begin{aligned}\dot{V}_n \leq& -\sum_{j=1}^{n-1}(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2) + \lambda_{n-1} \\ &+ \frac{1}{2a_n^2} z_n^2 \theta_n \xi_n^T(Z_n)\xi_n(Z_n) \\ &+ \frac{1}{2}a_n^2 + \frac{1}{2}\varepsilon_n^2 - c_n z_n^{2\sigma} - \frac{1}{2a_n^2} z_n^2 \hat{\theta}_n \xi_n^T \xi_n \\ &+ \frac{1}{2}u_{\min}^2 - \frac{1}{q_n}\tilde{\theta}_n(\frac{q_n}{2a_n^2} z_n^2 \xi_n^T \xi_n - \gamma_i \hat{\theta}_i) \\ =& -\sum_{j=1}^{n-1}(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2) + \lambda_{n-1} + \frac{1}{2}a_n^2 \\ &+ \frac{1}{2}\varepsilon_n^2 - c_n z_n^{2\sigma} + \frac{1}{2}u_{\min}^2 + \frac{\gamma_n}{q_n}\tilde{\theta}_n\hat{\theta}_n, \end{aligned} \tag{57}$$

Furthermore, by employing Young's inequality and $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$, the following inequality can be obtained:

$$\frac{\gamma_n}{q_n}\tilde{\theta}_n\hat{\theta}_n \leq -\frac{\gamma_n}{2q_n}\tilde{\theta}_n^2 + \frac{\gamma_n}{2q_n}\theta_n^2. \tag{58}$$

Then, the inequality (57) can be rewritten as:

$$\begin{aligned}\dot{V}_n \leq& -\sum_{j=1}^{n-1}(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2) + \lambda_{n-1} + \frac{1}{2}a_n^2 \\ &+ \frac{1}{2}\varepsilon_n^2 - c_n z_n^{2\sigma} + \frac{1}{2}u_{\min}^2 - \frac{\gamma_n}{2q_n}\tilde{\theta}_n^2 + \frac{\gamma_n}{2q_n}\theta_n^2, \\ =& -\sum_{j=1}^{n}\left(c_j z_j^{2\sigma} + \frac{\gamma_j}{2q_j}\tilde{\theta}_j^2\right) + \lambda_n \end{aligned} \tag{59}$$

where

$$\lambda_n = \lambda_{n-1} + \frac{1}{2}a_n^2 + \frac{1}{2}\varepsilon_n^2 + \frac{1}{2}u_{\min}^2 + \frac{\gamma_n}{2q_n}\theta_n^2 \tag{60}$$

Theorem 1: Under Assumptions 1-3 and bounded initial conditions, the non-strict feedback nonlinear quantized system (5) preceded by input defined in (6) is considered. If the controller (15) and parameter adaptive law (16) are chosen, then there exists two constant o and ϑ, such that $|z_i| \leq o$ and $|y - y_d| \leq o$, where $o = \sqrt{2}\Big[\frac{d}{(1-\varrho)c}\Big]^{1/2\sigma}$, ϑ is defined in (66) and $c = \min\{2^\sigma c_j, \gamma_j, j = 1, 2, \ldots, n\}$, $d = \lambda_n + (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}} \sum_{i=1}^{n} \gamma_i$.

proof: Applying lemma 5, let $\omega = 1, \tau = \sigma, v = \frac{1}{2q_i}\tilde{\theta}_i^2, \gamma = 1-\sigma, \lambda = \frac{1}{\sigma}$, the following relationship can be obtained:

$$(\frac{1}{2q_i}\tilde{\theta}_i^2)^\sigma \leq (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}} + \frac{1}{2q_i}\tilde{\theta}_i^2 \tag{61}$$

Combining (59) and (61), one has:

$$\begin{aligned}\dot{V}_n \leq& -\sum_{j=1}^{n} c_j z_j^{2\sigma} - \sum_{j=1}^{n}\gamma_j\Big(\frac{1}{2q_j}\tilde{\theta}_j^2\Big)^\sigma + d \\ \leq& -c\sum_{j=1}^{n}(\frac{1}{2}z_j^2)^\sigma - c\sum_{j=1}^{n}\Big(\frac{1}{2q_j}\tilde{\theta}_j^2\Big)^\sigma + d \end{aligned} \tag{62}$$

where $c = \min\{2^\sigma c_j, \gamma_j, j = 1, 2, \ldots, n\}$, $d = \sum_{i=1}^{n}\frac{\gamma_i}{2q_i}\theta_i^2 + \sum_{i=1}^{n}\frac{1}{2}(a_i^2+\varepsilon_i^2)+\sum_{i=1}^{n-1}\kappa\epsilon_i+\frac{1}{2}u_{\min}^2+(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}}\sum_{i=1}^{n}\gamma_i$.

Furthermore, according to Lemma 2, we have:

$$\begin{aligned}\dot{V} = \dot{V}_n \leq& -c\Big(\sum_{j=1}^{n}\Big(\frac{z_j^2}{2} + \frac{1}{2q_j}\tilde{\theta}_j^2\Big)\Big)^\sigma + d \\ \leq& -cV^\sigma + d. \end{aligned} \tag{63}$$

Define

$$\Omega_z = \Big\{(z,\theta)\Big|V(t) < \Big[\frac{d}{(1-\varrho)c}\Big]^{1/\sigma}, \frac{1}{2} < \varrho < 1\Big\}.$$

Based on Bhat and Bernstein, if $\forall(z,\theta)\overline{\in}\Omega_z$ for $\forall t \in [0, t_\delta]$. We have $V \geq \Big[\frac{d}{(1-\varrho)c}\Big]^{1/\sigma} \geq \Big(\frac{d}{\varrho c}\Big)^{\frac{1}{\sigma}}$, namely, $d \leq \varrho c V^\sigma$ for $\forall t \in [0, t_\delta]$. Hence, according to (63), for $\forall t \in [0, t_\delta]$, the following relationship can be obtained:

$$\dot{V} \leq -c(1-\varrho)V^\sigma. \tag{64}$$

combining $V \geq \Big(\frac{d}{\varrho c}\Big)^{\frac{1}{\sigma}}$ and (64), we have:

$$t_\delta \leq \frac{[V(t_0)]^{1-\sigma}}{c(1-\varrho)(1-\sigma)}. \tag{65}$$

Let

$$\vartheta = \frac{[V(t_0)]^{1-\sigma}}{c(1-\varrho)(1-\sigma)}, \tag{66}$$

then

$$V(t) \leq \Big[\frac{d}{(1-\varrho)c}\Big]^{1/\sigma}, \quad \forall t \geq \vartheta. \tag{67}$$

In view of the definition of $V(t)$, we come to the conclusion that for $o = \sqrt{2}\Big[\frac{d}{(1-\varrho)c}\Big]^{1/2\sigma}$, we have $|z_i| \leq o$ and $|y - y_d| \leq o$. The proof is thus completed.

Remark 5: The research status of adaptive quantization control for non-strict feedback nonlinear systems can only ensure the stability of infinite-time. It should be mentioned that, its stability in finite-time can be proved by Therefrom 1, i.e., the tracking performance of the non-strick feedback nonlinear quantized system can be achieved when $t \geq \vartheta$.

IV. SIMULATION EXAMPLE

Example 1: Think about the non-strict feedback nonlinear system as follows:

$$\begin{aligned}\dot{x}_1 &= x_2 + (1-\sin^2 x_1)x_2, \\ \dot{x}_2 &= \Gamma(Q(u)) - 3.5x_2 + x_1^2 x_2^2, \\ y &= x_1, \end{aligned} \tag{68}$$

where $\Gamma(Q(u))$ represents the input of the system, and $Q(u)$ is defined in (7).

In order to examine the validity of Theorem 1, let $y_d = \sin(0.5t) + 0.5\sin(1.5t)$. The corresponding parameters are set as $\delta = 0.5$, $u_{min} = 0.2$, $k_l = k_r = 1.2$, $b_l = b_r = 0.6$. Consider the following control law and adaption law:

$$\alpha_1 = -c_1 z_1^{2\sigma-1} - \frac{1}{2a_1^2}\hat{\theta}_1 z_1 \xi_1^T(Z_1)\xi_1(Z_1).$$

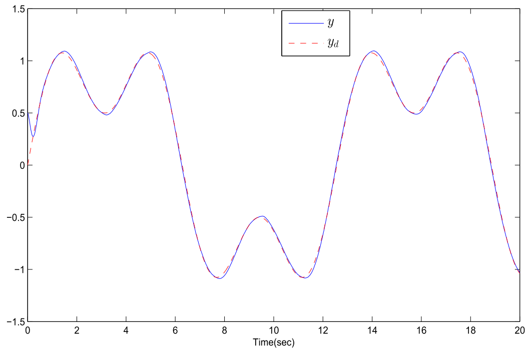

FIGURE 1. y and y_d of Example 1.

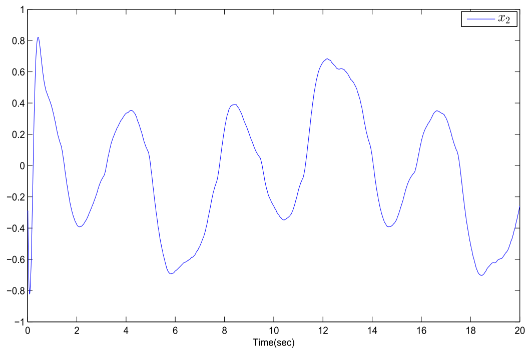

FIGURE 2. State variable x_2 of Example 1.

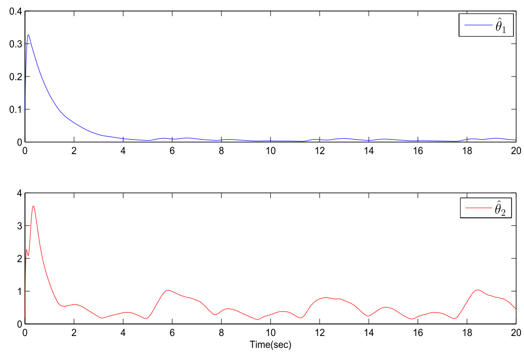

FIGURE 3. $\hat{\theta}_1$ and $\hat{\theta}_2$ of Example 1.

$$u = -\frac{c_2}{1-\delta} z_2^{2\sigma-1} - \frac{z_2\hat{\theta}_2\xi_2^T(Z_2)\xi_2(Z_2)}{2a_2^2(1-\delta)}.$$

$$\dot{\hat{\theta}}_i = \frac{q_i}{2a_i^2} z_i^2 \xi_i^T \xi_i - \gamma_i\hat{\theta}_i, \quad \hat{\theta}_i(0) \geq 0, \ i = 1, 2.$$

where $z_1 = x_1 - y_d, \ z_2 = x_2 - \alpha_1$.

In order to construct the basis vector function $\xi_i(X_i)$, for each input variable, the center of the receptive field is taken as $v = [-1.5, -1, -0.5, 0, 0.5, 1, 1.5]^T$ and the width of Gaussian function is $\zeta = \sqrt{2}$. Select the design parameters as follows: $c_1 = 8$, $c_2 = 6$, $a_1 = 0.8$, $a_2 = 1$, $q_1 = 20$, $q_2 = 25$, $\gamma_1 = 1$, $\gamma_2 = 2$. Select the initial conditions as $[x_1(0), x_2(0)]^T = [0.5, -0.3]^T$ and $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.1, 0.2]^T$. Fig.1-Fig.4 display the corresponding simulation results.

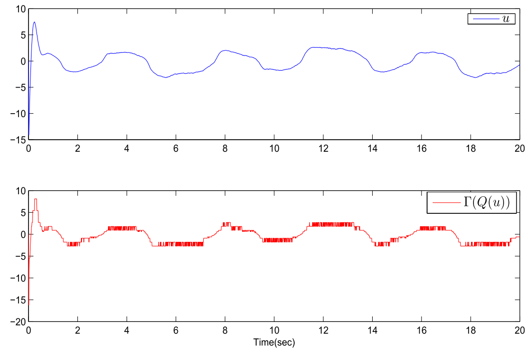

FIGURE 4. u and $\Gamma(Q(u))$ of Example 1.

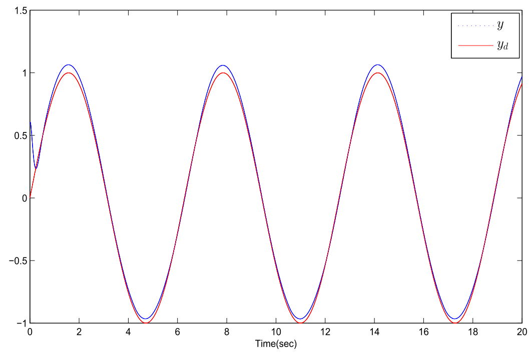

FIGURE 5. y and y_d of Example 2.

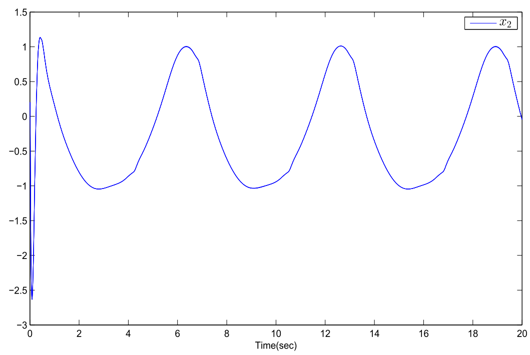

FIGURE 6. State variable x_2 of Example 2.

Example 2: To demonstrate the utility of the proposed scheme, consider a non-strict feedback nonlinear quantized system as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 + \frac{x_1^2}{1+x_1^2+x_2^2}, \\ \dot{x}_2 &= \Gamma(Q(u)) - x_2 + sin(x_1)x_2^2, \\ y &= x_1, \end{aligned} \tag{69}$$

where $\Gamma(Q(u))$ represents the input of the system defined in (6), and $Q(u)$ is defined in (7).

To examine the effectiveness of Theorem 1, the reference signal is set as $y_d = \sin t$. The corresponding parameters are set as $\delta = 0.4$, $u_{min} = 0.2$, $k_l = k_r = 1.2$, $b_l = b_r = 0.6$.

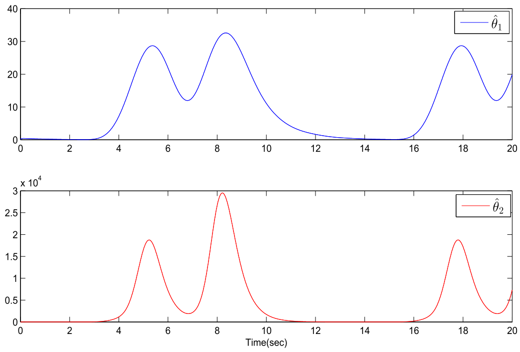

FIGURE 7. $\hat{\theta}_1$ **and** $\hat{\theta}_2$ **of Example 2.**

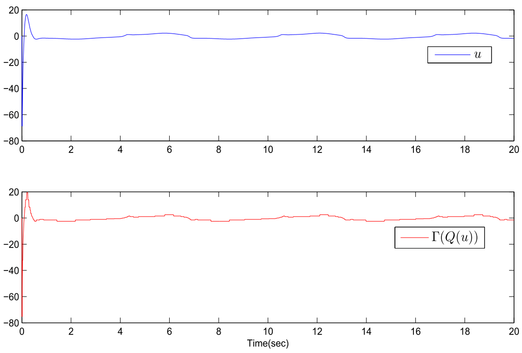

FIGURE 8. u **and** $\Gamma(Q(u))$ **of Example 2.**

Think about the following control law and adaption law:

$$\alpha_1 = -c_1 z_1^{2\sigma-1} - \frac{1}{2a_1^2}\hat{\theta}_1 z_1 \xi_1^T(Z_1)\xi_1(Z_1).$$

$$u = -\frac{c_2}{1-\delta} z_2^{2\sigma-1} - \frac{z_2\hat{\theta}_2\xi_2^T(Z_2)\xi_2(Z_2)}{2a_2^2(1-\delta)}.$$

$$\dot{\hat{\theta}}_i = \frac{q_i}{2a_i^2} z_i^2 \xi_i^T \xi_i - \gamma_i \hat{\theta}_i, \quad \hat{\theta}_i(0) \geq 0, \quad i = 1, 2.$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$.

To construct the basis vector function $\xi_i(X_i)$, for each input variable, the center of the receptive field is taken as $v = [-1.5, -1, -0.5, 0, 0.5, 1, 1.5]^T$ and the width of Gaussian function is $\zeta = \sqrt{2}$. Select the design parameters as follows: $c_1 = 10$, $c_2 = 12$, $a_1 = 1$, $a_2 = 1$, $q_1 = 20$, $q_2 = 25$, $\gamma_1 = 1$, $\gamma_2 = 2$. The initial conditions are chosen as $[x_1(0), x_2(0)]^T = [0.6, 0.2]^T$ and $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.2, 0.3]^T$. The corresponding simulation results are display by Fig.5-Fig.8.

V. CONCLUSION

In this article, a finite-time control design method is addressed for a category of non-strict feedback nonlinear quantized systems with actuator dead-zone. By using the relationship between the system input and the control signal, the problem of nonlinear quantized control is transformed into a conventional control problem of a nonlinear system with bounded perturbation and unknown control gain. By employing the structural properties of RBF NNs, a backstepping design method is extended from strict-feedback systems to a category of more common nonlinear systems. By applying the adaptive neural control based on approximation, an neural adaptive controller is constructed, which can ensure that the system output converges into a small enough neighborhood of the reference signal in finite-time, and all the signals of the closed-loop system remain bounded. According to the Bhat and Bernstein theorem, the finite-time stability of the nonlinear quantized system is proved. However, how to realize adaptive finite-time control of a class of stochastic nonlinear quantized system with actuator dead-zone is still a challenging problem, which may be considered in our future research topic.

REFERENCES

[1] B. Chen, F. Wang, X. P. Liu, and C. Lin, "Finite-time adaptive fuzzy tracking control design for nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1207–1216, Jun. 2018.

[2] B. Chen, X. P. Liu, S. S. Ge, and C. Lin, "Adaptive fuzzy control of a class of nonlinear systems by fuzzy approximation approach," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 6, pp. 1012–1021, Dec. 2012.

[3] F. Wang, B. Chen, Y. Sun, and C. Lin, "Finite time control of switched stochastic nonlinear systems," *Fuzzy Sets Syst.*, vol. 365, pp. 140–152, Jun. 2018.

[4] J. Gu, Y. Zhang, and H. Dong, "Dynamic behaviors of interaction solutions of (3+1)-dimensional Shallow Water wave equation," *Comput. Math. Appl.*, vol. 76, no. 6, pp. 1408–1419, 2018.

[5] S. Tong, K. Sun, and S. Sui, "Observer-based adaptive fuzzy decentralized optimal control design for strict-feedback nonlinear large-scale systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 569–584, Apr. 2018.

[6] B. Chen, X. Liu, and C. Lin, "Observer and adaptive fuzzy control design for nonlinear strict-feedback systems with unknown virtual control coefficients," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1732–1743, Jun. 2018.

[7] C. Hua, K. Li, and X. Guan, "Event-based dynamic output feedback adaptive fuzzy control for stochastic nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 3004–3015, Oct. 2018.

[8] X. Meng, F. Li, and S. Gao, "Global analysis and numerical simulations of a novel stochastic eco-epidemiological model with time delay," *Appl. Math. Comput.*, vol. 339, pp. 701–726, Dec. 2018.

[9] Y. Li, K. Li, and S. Tong, "Finite-time adaptive fuzzy output feedback dynamic surface control for MIMO nonstrict feedback systems," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 1, pp. 96–110, Jan. 2019.

[10] Y. Sun, B. Chen, C. Lin, and H. Wang, "Finite-time adaptive control for a class of nonlinear systems with nonstrict feedback structure," *IEEE Trans. Cybern.*, vol. 48, no. 10, pp. 2774–2782, Oct. 2018.

[11] F. Liu, "Boundedness and continuity of maximal operators associated to polynomial compound curves on Triebel-Lizorkin spaces," *Math. Inequalities Appl.*, vol. 21, no. 1, pp. 25–44, 2019.

[12] F. Wang, Z. Liu, Y. Zhang, and C. L. P. Chen, "Adaptive fuzzy control for a class of stochastic pure-feedback nonlinear systems with unknown hysteresis," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 1, pp. 140–152, Jan. 2016.

[13] W. Lv and F. Wang, "Adaptive tracking control for a class of uncertain nonlinear systems with infinite number of actuator failures using neural networks," in *Advances in Difference Equations*, 2017. doi: 10.1186/s13662-017-1426-5.

[14] B. Chen, H. Zhang, X. Liu, and C. Lin, "Neural observer and adaptive neural control design for a class of nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 9, pp. 4261–4271, Sep. 2017.

[15] F. Wang, B. Chen, C. Lin, and X. Li, "Distributed adaptive neural control for stochastic nonlinear multiagent systems," *IEEE Trans. Cybern.*, vol. 47, no. 7, pp. 1795–1803, Jul. 2017.

[16] S. Tatikonda and S. Mitter, "Control under communication constraints," *IEEE Trans. Autom. Control*, vol. 49, no. 7, pp. 1056–1068, Jul. 2004.

[17] F. Liu, Z. Fu, and S. T. Jhang, "Boundedness and continuity of Marcinkiewicz integrals associated to homogeneous mappings on Triebel–Lizorkin spaces," *Frontiers Math. China*, vol. 14, no. 1, pp. 95–122, 2019.

[18] J. Liu and N. Elia, "Quantized feedback stabilization of non-linear affine systems," *Int. J. Control*, vol. 77, no. 3, pp. 239–249, Feb. 2004.
[19] W. Ren and J. Xiong, "Quantized feedback stabilization of nonlinear systems with external disturbance," *IEEE Trans. Autom. Control*, vol. 63, no. 9, pp. 3167–3172, Sep. 2018.
[20] Z. Shi, J. Wang, Q. Li, and H. Cheng, "Control optimization and homoclinic bifurcation of a prey–predator model with ratio-dependent," in *Advances in Difference Equations*, 2019. doi: 10.1186/s13662-018-1933-z.
[21] W. KhanLin, Y. Lin, S. U. Khan, and N. Ullah, "Quantized adaptive decentralized control for interconnected nonlinear systems with actuator faults," *Appl. Math. Comput.*, vol. 320, pp. 175–189, Mar. 2018.
[22] G. Lai, Z. Liu, Y. Zhang, C. L. P. Chen, and S. Xie, "Asymmetric actuator backlash compensation in quantized adaptive control of uncertain networked nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 2, pp. 294–307, Feb. 2017.
[23] Z. Liu, F. Wang, Y. Zhang, and C. L. P. Chen, "Fuzzy adaptive quantized control for a class of stochastic nonlinear uncertain systems," *IEEE Trans. Cybern.*, vol. 46, no. 2, pp. 524–534, Feb. 2016.
[24] B. Chen, X. Lin, and W. Zhang, "On the system entropy and energy dissipativity of stochastic systems and their application in biological systems," *Complexity*, 2018. doi: 10.1155/2018/1628472.
[25] J. Wu, Z.-G. Wu, J. Li, G. Wang, H. Zhao, and W. Chen, "Practical adaptive fuzzy control of nonlinear pure-feedback systems with quantized nonlinearity input," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 49, no. 3, pp. 638–648, Mar. 2019.
[26] J. Wang, H. Zhu, C. L. Zhang, Z. Chen, Y. Huang, W. Chen, X. Huang, and F. Wang, "Adaptive hyperbolic tangent sliding-mode control for building structural vibration systems for uncertain earthquakes," *IEEE Access*, vol. 6, pp. 74728–74736, 2018.
[27] F. Wang, Z. Liu, Y. Zhang, and C. L. P. Chen, "Adaptive quantized controller design via backstepping and stochastic small-gain approach," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 2, pp. 330–343, Apr. 2016.
[28] Z. Yu, Y. Dong, S. Li, and F. Li, "Adaptive quantized tracking control for uncertain switched nonstrict-feedback nonlinear systems with discrete and distributed time-varying delays," *Int. J. Robust Nonlinear Control*, vol. 28, no. 5, pp. 1145–1164, Mar. 2017.
[29] H. Wang *et al.*, "Observer-based adaptive fuzzy tracking control for a class of MIMO nonlinear systems with unknown dead zones and time-varying delays," *Int. J. Syst. Sci.*, vol. 50, no. 3, pp. 546–562, 2019.
[30] Y. Li, H. Cheng, J. Wang, and Y. Wang, "Dynamic analysis of unilateral diffusion Gompertz model with impulsive control strategy," in *Advances in Difference Equations*, 2018. doi: 10.1186/s13662-018-1484-3.
[31] Y. Li and G. Yang, "Adaptive fuzzy decentralized control for a class of large-scale nonlinear systems with actuator faults and unknown dead zones," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 5, pp. 729–740, 2017.
[32] W. Yang and S. C. Tong, "Robust stabilization of switched fuzzy systems with actuator dead zone," *Neurocomputing*, vol. 173, pp. 1028–1033, Jan. 2016.
[33] Z. Liu, G. Lai, Y. Zhang, and C. L. P. Chen, "Adaptive fuzzy tracking control of nonlinear time-delay systems with dead-zone output mechanism based on a novel smooth model," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 6, pp. 1998–2011, Dec. 2015.
[34] J. Wang, H. Cheng, H. Liu, and Y. Wang, "Periodic solution and control optimization of a prey-predator model with two types of harvesting," in *Advances in Difference Equations*, 2018. doi: 10.1186/s13662-018-1499-9.
[35] S. Tong and Y. Li, "Adaptive fuzzy decentralized output feedback control for nonlinear large-scale systems with unknown dead-zone inputs," *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 5, pp. 913–925, Oct. 2013.
[36] C. Li, J. Yi, H. Wang, G. Zhang, and J. Li, "Interval data driven construction of shadowed sets with application to linguistic word modelling," *Inf. Sci.*, to be published. doi: 10.1016/j.ins.2018.11.018.
[37] C. Tian, C. Li, G. Zhang, and Y. Lv, "Data driven parallel prediction of building energy consumption using generative adversarial nets," *Energy Buildings*, vol. 186, pp. 230–243, Mar. 2019.
[38] X. Wang, Z. Wang, and H. Shen, "Dynamical analysis of a discrete-time SIS epidemic model on complex networks," *Appl. Math. Lett.*, vol. 94, pp. 292–299, Aug. 2019.
[39] B. Chen, X. Liu, K. Liu, and C. Lin, "Fuzzy approximation-based adaptive control of nonlinear delayed systems with unknown dead zone," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 2, pp. 237–248, Apr. 2014.
[40] F. Wang, Z. Liu, Y. Zhang, and C. L. P. Chen, "Adaptive quantized fuzzy control of stochastic nonlinear systems with actuator dead-zone," *Inf. Sci.*, vols. 370–371, pp. 385–401, Nov. 2016.
[41] C. P. Tan, X. Yu, and Z. Man, "Terminal sliding mode observers for a class of nonlinear systems," *Automatica*, vol. 46, no. 8, pp. 1401–1404, 2010.
[42] S. P. Bhat and D. S. Bernstein, "Continuous finite-time stabilization of the translational and rotational double integrators," *IEEE Trans. Autom. Control*, vol. 43, no. 5, pp. 678–682, May 1998.
[43] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, 2000.
[44] G. Chen and Y. Yang, "Finite-time stability of switched nonlinear time-varying systems via indefinite Lyapunov functions," *Int. J. Robust Nonlinear Control*, vol. 28, no. 5, pp. 1901–1912, 2018.
[45] F. Zhu, X. Meng, and T. Zhang, "Optimal harvesting of a competitive n-species stochastic model with delayed diffusions," *Math. Biosci. Eng.*, vol. 16, no. 3, pp. 1554–1574, 2019.
[46] J. Zhang, J. Xia, W. Sun, G. Zhuang, and Z. Wang, "Finite-time tracking control for stochastic nonlinear systems with full state constraints," *Appl. Math. Comput.*, vol. 338, pp. 207–220, Dec. 2018.
[47] W. S. Lv, F. Wang, and Y. Li, "Adaptive finite-time tracking control for nonlinear systems with unmodeled dynamics using neural networks," in *Advances in Difference Equations*, 2018. doi: 10.1186/s13662-018-1615-x.
[48] R. Yang and L. Sun, "Finite-time robust control of a class of nonlinear time-delay systems via Lyapunov functional method," *J. Franklin Inst.*, vol. 356, no. 3, pp. 1155–1176, 2019. doi: 10.1016/j.jfranklin.2018.08.029.
[49] P. Ordaz and A. Poznyak, "Adaptive-robust stabilization of the Furuta's pendulum via attractive ellipsoid method," *J. Dyn. Syst. Meas. Control*, vol. 138, no. 2, 2016, Art. no. 021005.
[50] P. Ordaz, M. Ordaz, C. Cuvas, and O. Santos, "Reduction of matched and unmatched uncertainties for a class of nonlinear perturbed systems via robust control," *Int. J. Robust Nonlinear Control*, vol. 29, no. 8, pp. 2510–2524, 2019.
[51] W. Lv, F. Wang, and Y. Li, "Finite-time adaptive fuzzy output-feedback control of MIMO nonlinear systems with hysteresis," *Neurocomputing*, vol. 296, pp. 74–81, Jun. 2018.
[52] W. Lv and F. Wang, "Finite-time adaptive fuzzy tracking control for a class of nonlinear systems with unknown hysteresis," *Int. J. Fuzzy Syst.*, vol. 20, no. 3, pp. 782–790, 2018.
[53] W. Liu, D. W. C. Ho, S. Xu, and B. Zhang, "Adaptive finite-time stabilization of a class of quantized nonlinearly parameterized systems," *Int. J. Robust Nonlinear Control*, vol. 27, no. 18, pp. 4554–4573, 2017.
[54] L. Zhang, W. Qi, Y. Kao, X. Gao, and L. Zhao, "New results on finite-time stabilization for stochastic systems with time-varying delay," *Int. J. Control, Automat. Syst.*, vol. 16, no. 2, pp. 649–658, 2018.
[55] X. Yang and X. Li, "Finite-time stability of linear non-autonomous systems with time-varying delays," in *Advances in Difference Equations*, 2018. doi: 10.1186/s13662-018-1557-3.
[56] F. Wang and X. Zhang, "Adaptive finite time control of nonlinear systems under time-varying actuator failures," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 9, pp. 1845–1852, Sep. 2019. doi: 10.1109/TSMC.2018.2868329.
[57] F. Wang, Z. Liu, Y. Zhang, and C. L. P. Chen, "Adaptive finite-time control of stochastic nonlinear systems with actuator failures," *Fuzzy Sets Syst.*, to be published. doi: 10.1016/j.fss.2018.12.005.
[58] F. Wang, Z. Liu, X. Li, Y. Zhang, and C. L. P. Chen, "Observer-based finite time control of nonlinear systems with actuator failures," *Inf. Sci.*, vol. 500, pp. 1–14, Oct. 2019.
[59] W. Lv, F. Wang, and L. Zhang, "Adaptive fuzzy finite-time control for uncertain nonlinear systems with dead-zone input," *Int. J. Control Autom. Syst.*, vol. 16, no. 5, pp. 2549–2558, 2018.
[60] J. Liu, C. Wang, X. Li, and X. Cai, "Adaptive finite-time practical consensus protocols for second-order multiagent systems with nonsymmetric input dead zone and uncertain dynamics," *J. Franklin Inst.*, vol. 356, no. 6, pp. 3217–3244, 2019.
[61] Q. Chen, X. Ren, J. Na, and D. Zheng, "Adaptive robust finite-time neural control of uncertain PMSM servo system with nonlinear dead zone," *Neural Comput. Appl.*, vol. 28, no. 12, pp. 3725–3736, 2016.
[62] Y. Li, Y. Li, Y. Liu, and H. Cheng, "Stability analysis and control optimization of a prey-predator model with linear feedback control," *Discrete Dyn. Nature Soc.*, 2018.

[63] H. Li, S. Zhao, W. He, and R. Lu, "Adaptive finite-time tracking control of full state constrained nonlinear systems with dead-zone," *Automatica*, vol. 100, pp. 99–107, Feb. 2019.
[64] F. Gao, Y. Wu, and Y. Liu, "Finite-time stabilization for a class of switched stochastic nonlinear systems with dead-zone input nonlinearities," *Int. J. Robust Nonlinear Control*, vol. 28, no. 9, pp. 3239–3257, 2018.
[65] Z. Zhang, G. Duan, and M. Hou, "Global finite time stabilization of pure-feedback systems with input dead-zone nonlinearity," *J. Franklin Inst.*, vol. 354, no. 10, pp. 4073–4101, 2017.
[66] F. Wang, B. Chen, C. Lin, J. Zhang, and X. Meng, "Adaptive neural network finite-time output feedback control of quantized nonlinear systems," *IEEE Trans. Cybern.*, vol. 48, no. 6, pp. 1839–1848, Jun. 2018.
[67] P. Ordaz, "Nonlinear robust output stabilization for mechanical systems based on luenberger-like controller/observer," *J. Dyn. Syst. Meas. Control*, vol. 139, no. 8, 2017.
[68] M. M. Ploycarpou and P. A. Ioannou, "A robust adaptive nonlinear control design," *Automatica*, vol. 32, no. 3, pp. 423–427, 1996.
[69] C. Qian and W. Lin, "Non-Lipschitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization," *Syst. Control Lett.*, vol. 42, no. 3, pp. 185–200, 2001.

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