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Second-Order Reputation Promotes Cooperation in the Spatial Prisoner's Dilemma Game

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ABSTRACT Reputation can significantly improve the level of cooperation in human societies. In recent years, most research efforts considered binary image scores or first-order evaluation models, and second-order criteria were considered only in well-mixed populations. In this paper, we therefore study the impact of four typical second-order reputation evaluation models in the spatial prisoner's dilemma game. Second-order reputation evaluation entails that an individual's image score is updated not only in accordance with his own strategy, but also in accordance with the reputation of the neighbors. We introduce a value for the reputation step length such that individuals can only maximize their reputation if they cooperate with the surrounding high-reputation individuals, and then thus become influential individuals in the population. By means of systematic Monte Carlo simulations, we show that all four rules promote cooperation beyond spatial reciprocity in the considered prisoner's dilemma game, and we also show that the longer the reputation step length the higher the level of cooperation. These results shed light on how reputation in structured populations affects cooperative behavior, and they might have important implications for human group dynamics and for cooperation in human societies in general.

INDEX TERMS Evolutionary game theory, cooperation dynamics, prisoner's dilemma game, reputation mechanism, second order evaluation.

I. INTRODUCTION

In the contemporary society, the human life would be unimaginable without the high level of cooperation that our species have arrived at. Thus, how to understand the persistence and emergence of cooperation within the population is a long-standing puzzle, which has become one of 25 crucial scientific problems to be resolved in the 21st century [1], [2]. Among them, the evolutionary game theory [3]–[5] provides a powerful framework to illustrate the dilemma of cooperation, and many classical game models have been proposed to analyze the individual strategy choice when confronting the conflict, such as the prisoner's dilemma game, snowdrift game and public goods game. In particular, Nowak [6] summarized 5 key mechanisms to support the evolution of cooperation including the kin selection [7], direct and indirect reciprocity [8], [9], group selection [10]–[12] and spatial or network reciprocity [13]–[16]. In recent years many studies have been devoted to exploring which mechanisms improve the level of cooperation in a population, and several comprehensive reviews have been written that cover recent progress in the field of evolutionary game theory and cooperation [17]–[23].

As mentioned above, one important mechanism to favor the collective cooperation is indirect reciprocity [24], [25]. Since indirect reciprocity does not require repeated interaction among players, cooperative individuals can accumulate a good prestige through helping others so that they can get the help from others in the future, which substantially explains the cooperative behavior among strange individuals. At this scenario, the potential cooperation between agents is mainly based on their reputation. Therefore, the reputation mechanism plays an important role to support the evolution of cooperation through the indirect reciprocity, and has become

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an active topic in the field of evolutionary game theory in the recent years.

However, the first step towards building a reliable reputation system is to decide how to evaluate the goodness of an individual strategy or action. Thus, it is crucial to construct a feasible criterion to judge whether one specific strategy or action is good or not. At present, most works adopted the classical 'first-order evaluation' rule, in which a donor will be evaluated to be good if he cooperates or to be bad if he defects regardless of the reputation of his recipient. To name a few examples, Fu et al. [26] investigated the effect of reputation on the individual partner-switching process, and they found that the cooperation prevails if players are able to alter their behavioral strategies and their social interaction partnerships according to the reputation situation. Wang et al. [27] also found that the existence of inferring reputation can promote the evolution of cooperation in spatial social dilemma games.

Nevertheless, the aforementioned scoring mechanism cannot lead to the evolution of cooperation through indirect reciprocity under some circumstances with errors or random noises, but the cooperation through indirect reciprocity tends to be evolutionarily stable if players utilize the standing criterion [28]. In the standing scheme, the defection against a bad recipient does not destroy the donor's reputation. Several other works [29]–[32] have also demonstrated that the standing strategy, based on the standing criterion, overcomes the issue of unjustified defection and maintains the cooperation. In particular, Ohtsuki and Iwasa [33], [34] checked all the possible methods to assign the reputation for a player if his action, his current reputation and the opponent's reputation are provided, and they found after exhaustive searches that eight reputation dynamics, named the 'leading eight' strategies, can maintain the evolutionarily stable strategy with a high level of cooperation. Recently, Santos et al. [35] further examined the complexity of higher-order reputation system, and showed that simple moral principles can elicit cooperation even in complex environments. Yet, all these previous works started from the well-mixed population, and little works are devoted to the second-order reputation evaluation on the spatial lattices or complex networks, and here we try to fill in this gap to investigate the second-order evaluation in the spatial prisoner's dilemma on the lattices. The current results indicate that the introduction of the second-order reputation mechanism will further foster the evolution of cooperation.

In this paper, firstly, we introduce in detail the prisoner's dilemma game model based on second-order reputation evaluation in Section 2. After that, in Section 3, we present the main results, as obtained by means of systematic Monte Carlo simulations. Finally, we conclude with a summary of the main conclusions, a discussion of potential implications, and we also point out directions for future research.

II. MODEL

In this section, we will firstly introduce our prisoner's dilemma game (*PDG*) model with second-order reputation evaluation. To be exact, our model is built on a regular lattice network with periodic boundaries. The size of the lattice is $L \times L$, each individual (i.e., player) occupies one node on the lattice and has $k = 4$ nearest neighbors. Within each time step, every individual will play the prisoner's dilemma game with his 4 nearest neighbors and collect the corresponding payoffs, respectively. In addition, on average, each player has one chance to update his current strategy through imitating the strategy of one randomly selected nearest neighbor. After enough iterations, the system arrives at the stationary state and we can count various quantities within the whole population.

Initially, each individual has the 50% likelihood to be designated as a cooperator (C) or a defector (D). In the prisoner's dilemma game, two players will make the decision at the same time. When two players both choose the same strategy, they will obtain the reward (R) for the cooperation or the punishment (P) of the defection; while they adopt different strategies, the cooperative player will get the sucker's payoff (S) but the defective one cannot resist the temptation (T) to defect. Also, the payoff ranking needs to satisfy the order: $T > R > P > S$, which leads to the scenario that the defection is the optimal action, that is, defection is the Nash equilibrium of prisoner's dilemma game model. However, like many previous works, we use the weak prisoner's dilemma game model, without lacking the generality, as follows,

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \end{array}$$

where T is set to be $b > 1.0$ and $R = 1$, but $P = S = 0$, that is, the ranking order of prisoner's dilemma game is not strictly obeyed. Nevertheless, this weak prisoner's dilemma game model can capture most features of the strict prisoner's dilemma game model and only one parameter b is considered here. Based on this matrix, each player (say, player i) can accumulate his game payoffs through playing the game with his every nearest neighbors, we denote it π_i. As each player will play game for 4 times with 4 different neighbors, after these 4 games, the total payoff of player i can be summed up as follows,

$$\Pi_i = \Sigma_j \pi_i^j \tag{1}$$

where j represents one of 4 nearest neighbors of player i.

After the game payoff calculation, player i will enter the phase of reputation update. Each individual will have an image score to depict his own reputation, namely, the reputation value R_i, which evolves $[1, 100]$ at the initial step. To some extent, the value of reputation of one player will characterize his influence in the population. In most of reputation-based game models, the reputation value will be increased only provided that one player adopts the cooperating action, or else his image score will decrease, that is, first-order reputation criterion. However, when we judge

TABLE 1. Four typical reputation assessment rules.

Update rules \ Strategy of i / Type of j	C/G	D/G	C/B	D/B
Stern Judging	$+p$	$-p$	- 1	+ 1
Simple Standing	$+p$	$-p$	+ 1	+ 1
Shunning	$+p$	$-p$	- 1	- 1
Image Scoring	$+p$	$-p$	+ 1	- 1

whether the action of one player is beneficial to the population, it would be fair if we not only consider the individual strategy to be cooperative (C) or defective (D), but also take the reputation of his opponent into account. Generally, we term this rule to be the second-order reputation evaluation. Starting from the second-order rule, we introduce a reputation threshold Z, which determines the goodness of individual reputation, into the model. When $R_i \geq Z$, the individual is called a high-reputation individual (denoted as G), otherwise it is a low-reputation individual (denoted as B). Assuming that the focal player i is playing the prisoner's dilemma game with one of his nearest neighbors (say, player j), we consider 4 assessment rules based on the strategy (C or D) of individual i and the reputation of player j (G or B) according to some previous works and named as "Simple Standing" [29], [30], "Stern Judging" [36], [37], "Shunning" [38] and "Image Scoring" [24]. We have shown them in Table 1. In Table 1, each row of the table represents a second-order evaluation criterion, and the four columns from left to right represent a combination of the strategy of the individual i and the type of his opponent j. For example, (C/G) means that the individual i chooses a cooperative strategy, while his opponent is a high-reputation individual, and so on. The values in the table indicate the change of individual i's reputation in the corresponding situation. In each of the second-order evaluation criteria, individual will always increases his reputation for p points if the situation is (C/G), and decreases if (D/G). We can adjust the size of p to reflect the extent to which high-reputation individuals influence his neighbors' reputation.

Since "cooperating with a high-reputation individual" (C/G) and "defecting with a high-reputation individual" (D/G) are two opposite behaviors, and it is obvious that cooperation is a kind behavior and defection can be regarded as a malicious one. Thus, we can control the degree of reward and punishment for these two kinds of behaviors by adjusting the value of p, what we called "reputation step length": The value of p determines the change in the reputation of individual i for his cooperative or defective behavior when faced with a high reputation opponent. The bigger the value of p, the more differentiated between the reward and punishment. Here, p is often set as an integer greater than 1. As mentioned above, each player needs to conduct 4 games with his nearest partner, and the reputation of each individual will be updated for 4 times within one time step.

After that, each individual will have a chance to update his current strategy but just once in the population on average, that is, asynchronous strategy transfer is implemented in the current model. Player i will randomly select one of his nearest neighbors (e.g., player j) to imitate this neighbor's strategy, meanwhile player j can also collect his total payoff with the same computing procedure as player i. Thus, we can perform the strategy imitation for player i with the following probability dictated by the Fermi-like rule,

$$Prob(s_i \leftarrow s_j) = \omega_j \frac{1}{1 + e^{\frac{-(\Pi_j - \Pi_i)}{K}}} \tag{2}$$

where K represents the noise factor, or we call its reverse $1/K$ as the strength of strategy selection, which reflects the uncertainty during the individual strategy adoption. $K \to 0$ means that the update of strategy is certain, and the individual is closer to the complete rationality at this time; otherwise, when $K \to \infty$ means that the individual is in a noisy environment and cannot make a rational decision; ω_j is a pre-factor that depends on the reputation of the individual j. When j is a high-reputation individual ($R_j \geq Z$), $\omega_j = 1$, and the individual has a higher influence so that it is easier for him to pass his own strategy on to the surrounding neighbors. When j is a low-reputation individual ($R_j < Z$), ω_j is set to be a value equals to ω which is often smaller than 1.0, that is, it is more difficult to adapt the strategy of neighbor j.

After the sub-steps mentioned above are all completed, a full Monte Carlo Simulation (MCS) step has been finished. In our simulation, Monte Carlo simulation was often performed up to $50,000$ steps. The average frequency of cooperators of the final $5,000$ steps was taken as the result at the stationary state (ρ_c). At the initial setup, the lattice size is set to be $L = 100$ and the reputation of each individual is taken as a random number between 1 and 100. In addition, we set $K = 0.1$ and $MCS = 50000$ if not clearly stated. In addition, larger lattice size (e.g., $L = 200$ or $L = 400$) are also tested and the qualitatively same results can be reproduced.

III. NUMERICAL SIMULATION RESULTS

At first, we plot the relationship between the temptation factor b and the frequency of cooperators ρ_c at the stationary state for different values of Z in Fig.1, where the X-axis represents the temptation factor b and the Y-axis denotes the frequency of cooperators ρ_c in the population. Here, we illustrate the Monte Carlo simulation results under all 4 second-order reputation evaluation rules, which include Stern Judging [panel (a)], Simple Standing [panel (b)], Shunning [panel (c)] and Image Scoring [panel (d)]. Inside each panel, the black line with squares represents the results of the traditional prisoner's dilemma game model, where no reputation model is introduced, and other colored lines represent the results under various reputation thresholds, in which Z ranges from 60 to 98. It can be clearly observed that under all 4 rules, the introduction of reputation can lead to the promotion of cooperation in the population when compared with the traditional prisoner's dilemma game. In general, the cooperation tends to be extinct for the tradition prisoner's dilemma

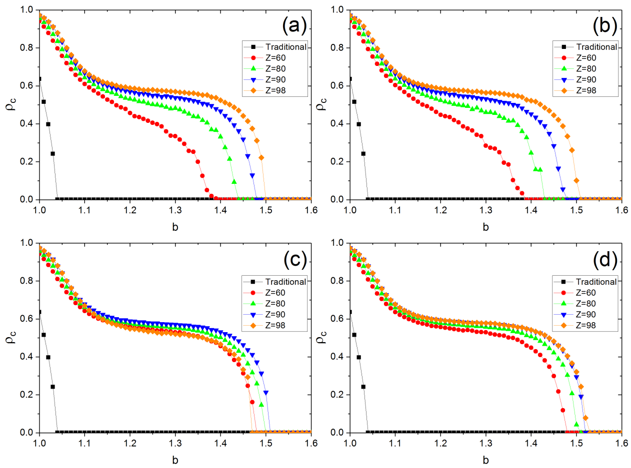

FIGURE 1. The relationship between the frequency of cooperators ρ_c at the stationary state and the temptation factor b under different reputation thresholds under four different evaluation rules. From panel (a) to (d), the second-order evaluations rules are set to be Stern Judging, Simple Standing, Shunning and Image Scoring, respectively. Pre-factor ω_j which indicates individuals' influence equals to 0.05. Other parameters are set to be: MCS = 50000, $p = 5$, $L = 100$, respectively.

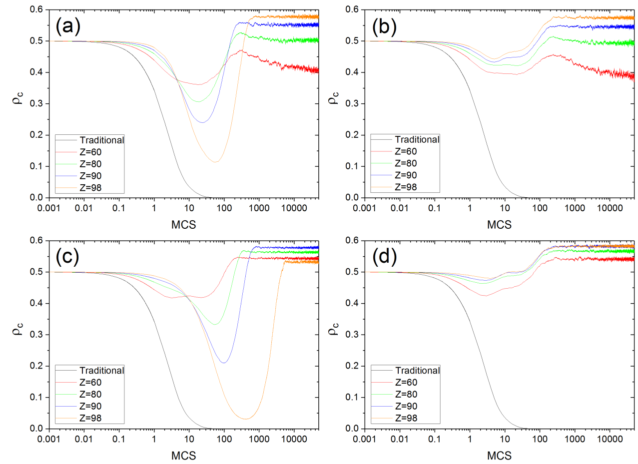

FIGURE 2. Frequency of cooperators $\rho_c(t)$ at each Monte Carlo Simulation step when the reputation threshold Z takes different values under four different evaluation rules. From panel (a) to (d), the second-order evaluations rules are set to be Stern Judging, Simple Standing, Shunning and Image Scoring, respectively. The temptation factor $b = 1.25$, MCS = 50000, $p = 5$, $\omega = 0.05$ and $L = 100$, respectively.

game model when b approaches around 1.04, but this value leading to the extinction of cooperation (b_c) has been greatly increased, which means that the introduction of cooperation has significantly fostered the cooperation. As an example, b_c arrives at 1.4 in "Stern Judging" and "Simple Standing" rules, and even becomes up to 1.45 for the "Shunning" and "Image Scoring" rules. In addition, we can also find that b_c gradually increases as a consequence of the augmentation of the threshold Z.

The above-mentioned results indicate that the introduction of the reputation threshold Z creates an obvious difference between individuals with higher reputation and lower reputation. Individuals need to cooperate with high-reputation individuals to gain more reputation values, thereby to increase their influence and the possibility to spread their strategies to neighbors in the future. In contrast, individuals who defect with high-reputation individuals will make their reputation be largely reduced, finally leading the reputation to be less than the threshold Z, which causes this individual's influence to become smaller, and thus it is more difficult to persuade their neighbors to imitate his own strategy. The reputation difference among individuals renders the cooperation strategy easier to spread among the population, and as Z increases, the impact becomes more apparent. However, for the rule of "Shunning" and "Image Scoring", it seems that they are not very sensitive to the change of Z. Since there is a commonnality for these two rules that the defection of low-reputation individuals only reduces the player's reputation for 1 point, it can be believed that whether punishes the defection of low-reputation individuals determines whether the cooperation rate is sensitive to Z as the defection parameter b varies.

Next, we illustrate the frequency of cooperators ρ_c at each time step for different thresholds Z in Fig.2, where 4 different second-order evaluation rules are adopted in panel (a) [Stern Judging], panel (b) [Simple Standing], panel (c) [Shunning] and panel (d) [Image Scoring]. Similarly, inside each panel, the black line represents the evolution of ρ_c under the traditional prisoner's dilemma game model with $b = 1.25$ and other lines denote the results with the reputation setup, in which the reputation threshold is set to be $Z = 60$, 70, 80 and 98, respectively. It is clearly shown that when no reputation is introduced (i.e., second-order evaluation is also absent at this scenario), $b = 1.25$ is not enough to support the existence of cooperative behavior and the frequency if cooperators is quickly decreased to zero in all 4 panels. However, when the reputation is introduced, the frequency of cooperators will still decline a little for the first several time steps due to the invasion of defectors. During this enduring periods, the difference between high-reputation individuals and low-reputation individuals is gradually widened, and individuals with higher reputation will spread their own strategies to the surrounding neighbors with the higher probability, while those with lower reputation will find it difficult to continually diffuse their own ones. As the high-reputation individuals continuously increase, they can organize into the clusters to foster the cooperative individuals, which will lead to the situation avoiding the extensive invasion of defectors and finally arrive at the equilibrium state between cooperators and defectors.

As Z increases, the frequency of cooperators is also increased gradually except "Shunning" rule. Moreover, we find that in the rule of "Stern Judging" and "Shunning", the greater the value of the reputation threshold Z, the more severe the decline of cooperation rate at the beginning of evolution. What these rules in common is that they both reduce 1 point of reputation for the action of "cooperating with low-reputation individuals". Thus, we explain this

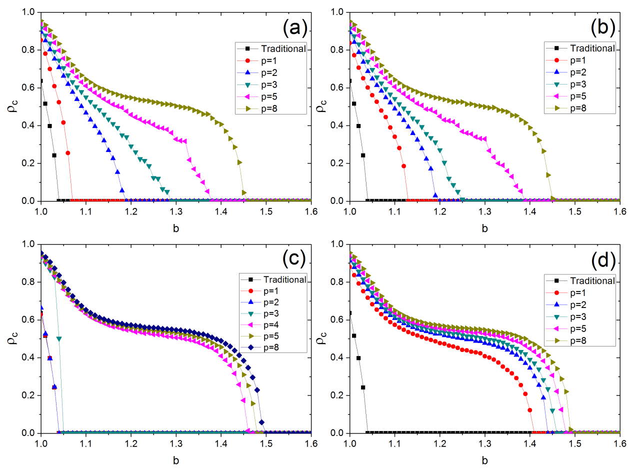

FIGURE 3. **The relationship between the frequency of cooperators ρ_c at the stationary state and the temptation factor *b* under different reputation step length *p* under four different evaluation rules. From panel (a) to (d), the second-order evaluations rules are set to be Stern Judging, Simple Standing, Shunning and Image Scoring, respectively. Other parameters: $Z = 60$, $\omega = 0.05$, MCS = 50000 and $L = 100$, respectively.**

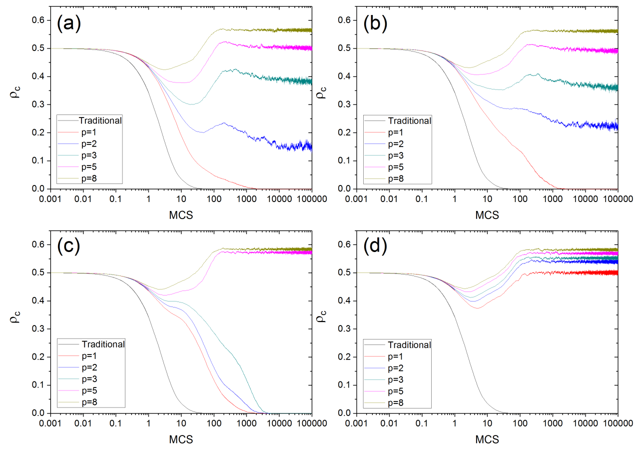

FIGURE 4. **Frequency of cooperators $\rho_c(t)$ at each Monte Carlo Simulation step when the reputation step length *p* takes different values under four different evaluation rules. From panel (a) to (d), the second-order evaluations rules are set to be Stern Judging, Simple Standing, Shunning and Image Scoring, respectively. Other parameters are set to be: $Z = 60$, $b = 1.17$, MCS = 100000, $\omega = 0.05$ and $L = 100$, respectively.**

behavior as follows: The existence of this rule makes the cooperators who are surrounded by low-reputation defectors at the beginning stage will continue to lose their reputation or image scores; The higher the threshold Z, the more easily these cooperators change into low-reputation individuals, and then their cooperation strategies are difficult to pass on to the neighboring players. Thus, the defective strategy continuously invades the population and then the frequency of cooperators will drop drastically at the beginning of evolution.

In order to explore the role of reputation evaluation, we will focus on the impact of reputation step length p on the evolution of cooperation under the second-order reputation evaluation model in Fig.3, where result under 4 different rules are plotted in 4 panels. As mentioned earlier, the greater the value of p, the more distinct the reward and punishment of reputation, and thus the reputation value also changes more intensely. We show the frequency of cooperators as a function of b for different p-values for a fixed reputation threshold $Z = 60$ in Fig.3, where it can be observed that the frequency of cooperators can be largely enhanced under the same parameter b as the p increases. For instance, for the "Simple standing" rule in panel (b) and $b = 1.1$, when we change the value of p from 1, 2, 3, 5 to 8, the frequency of cooperators ρ_c is equal to 0.34, 0.48, 0.54, 0.61 and 0.64, respectively. We can interpret this phenomenon as follows: when we increase the value of p, the reputation difference between individuals will become more intense, and cooperating with high-reputation individuals around them will add their reputation much more so that their reputation values can be rapidly increased and even exceed the threshold Z, which is initially set and kept unchanged during the evolution of cooperation; Henceforth, the cooperation strategy of high reputation individuals can be quickly transferred to their surrounding ones. Due to the introduction of p, interacting with high-reputation individuals becomes much more advantageous for the whole population. Thus, cooperating with these kinds of individuals can quickly increase one individual's reputation, otherwise defecting with such individuals will accelerate the loss of his reputation.

In all 4 evaluation rules, the frequency of cooperators at the stationary state ρ_c increases as p augments, which can be clearly seen from 4 panels in Fig.3. In particular, in the rule of "Shunning", the cooperation frequency is significantly improved when p is increased from 3 to 4. Since 4 is the number of nearest neighbors around the focal player, and we can illustrate the current phenomenon in the following way: In the rule of "Shunning", an individual wants to increase his reputation only by cooperating with a high-reputation individual. When $p = 4$, if there is a high-reputation individual in the surrounding neighbors, as long as the central individual adopts a cooperative strategy, his reputation value will always increase (cooperating with high-reputation individual will increase $p = 4$ points of reputation, and cooperating with other 3 low-reputation individuals will reduce a total of 3 points of reputation, which actually adds 1 point for his reputation), which will further help the cooperators at the boundary of the cooperative clusters. Through encouraging them to continuously increase their reputation, these boundary cooperators will become a high-reputation individuals so that the cooperation strategy can be continuously expanded, which leads to the emergence of cooperation within the whole population.

After that, we further provide the evolution of frequency of cooperators at each time step $[\rho_c(t)]$ for different p values. As shown in Fig.4, we plot $[\rho_c(t)]$ for 4 assessment rules illustrated in four panels under different values of p values, where the reputation threshold in all 4 rules is fixed to be $Z = 60$. It can be clearly shown that as the value of

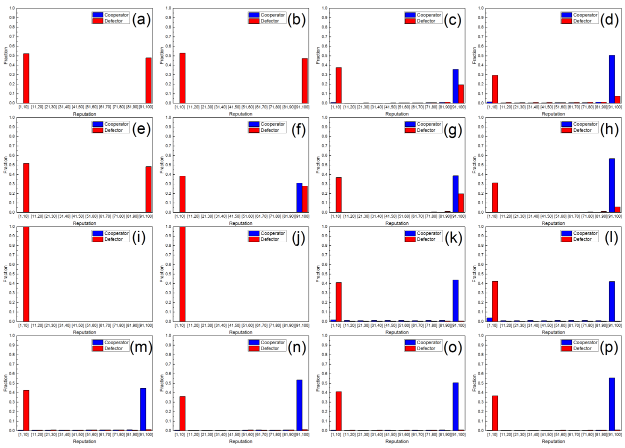

FIGURE 5. Histograms of the proportion of individuals within different reputation intervals under different values of *Z* and *p* under four different evaluation rules. Four rows from top to bottom represents "Stern Judging", "Simple Standing", "Shunning" and "Image Scoring", respectively. The blue columns represent cooperators, and the red columns represent defectors. The X-axis indicates the reputation intervals, and the Y-axis is the proportion of cooperators or defectors. The results in the figure were taken at MCS step 49000. The simulation parameters are set to be: $L = 100$, $\omega = 0.05$, $r = 1.25$, and MCS = 50000.

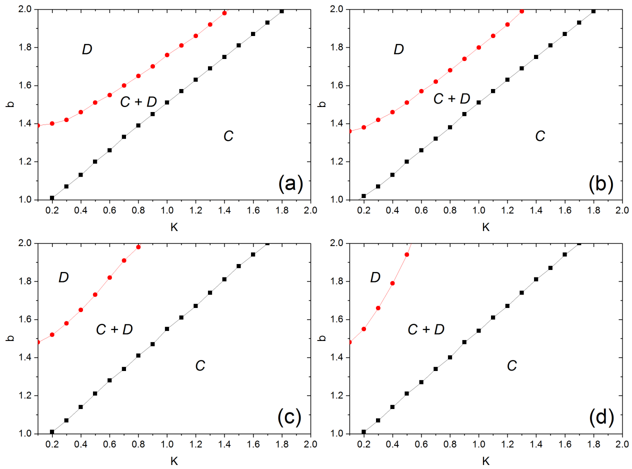

FIGURE 6. The phase transition process of cooperation and defection under four different evaluation rules. The X-axis indicates *K*, which is called the noise factor, and the Y-axis indicates the temptation factor *b*. From panel (a) to (d), the second-order evaluations rules are set to be Stern Judging, Simple Standing, Shunning and Image Scoring, respectively. The black line represents the critical value from the full cooperation to mixed state, and the red line represents this value from mixed state to full defection. The reputation threshold *Z* is set to be 60, while other parameters are fixed to be $\omega = 0.05$, MCS = 50000, $p = 5$ and $L = 100$.

p increases, the frequency of cooperators at the stationary state (ρ_c) will eventually arrive at a higher value. As an example, in the rule of "Image Scoring", ρ_c will steadily increase when p is varied from 1 to 8. Even if the evolution of collective cooperation tends to be extinct, the time taken under the case with a higher p value is longer than that in the traditional prisoner's dilemma game, that is, the cooperation strategy will last longer after the p value is introduced. For instance, $p = 1$ leads to the full defection after around 1000 steps under the rule of "Stern Judging", while the cooperation tends to be extinct after 30 steps in the traditional prisoner's dilemma game; and similar cases can be found in the case of $p = 1, 2, 3$ when "Shunning" rule is adopted in the reputation evaluation.

In Fig.5, we further plot the fraction of individuals with reputation falling within different reputation intervals so as to scrutinize the distribution of the strategies in the population. We provide the results regarding the distribution of reputation under different thresholds Z and reputation step length p. From top to bottom, each row of panels represent the distribution of strategists within different intervals under 4 evaluation rules such as Stern Judging [from panel (a) to (d)], Simple Standing [from panel (e) to (h)], Shunning [from panel (i) to (l)] and Image Scoring [from panel (m) to (p)], respectively. The values of reputation in all panels are taken at MCS step 49000. We divided 10 different intervals of reputation, which are equally drawn from 1 to 100, and count the proportion of individuals within each reputation interval in the total population. It is clearly indicated that the individual's reputation is mainly concentrated on two different intervals no matter what the rule is, which are often locating in the minimum and the maximum one as shown in Fig.5. We can observe that for all 4 rules, increasing p or Z can promote the emergence of high-reputation cooperators and elevate the frequency of cooperators at the stationary state. Furthermore, we find that a considerable number of high-reputation defectors appear in the rule of "Stern Judging" and "Simple Standing", as shown in panel (a) to (h). The possible reason can be depicted as follows: both these rules stipulate that "defecting low-reputation individuals" will just increase 1 point reputation value of the focal player, and then defectors surrounded by low-reputation individuals will utilize this rule to improve his reputation; Finally, these defectors may even become high-reputation ones. However, as p increases, defectors who rely on this approach to increase their reputation become less and less. On the one hand, it is more convenient and efficient for any player to improve their reputation by cooperating with high-reputation individuals; on the other hand, the cluster size of cooperators tends to become larger and larger so that the survival space of defectors will be further squeezed. On the contrary, there exist some low-reputation cooperators in the rule of "Shunning" as result of the sharp decline of cooperation frequency at the initial stage under this rule as shown in Fig.2, that is, cooperators surrounded by low-reputation defectors will continue to lose their reputation although some of them still maintain a cooperative strategy, but at this time they have become low-reputation cooperators, which is also confirmed in previous works [39], [40].

Finally, we depict the phase transition process under different K to deeply understand the evolution of cooperation in Fig.6. Four panels depict the phase diagrams for 4 different rules, which include Stern Judging, Simple standing, Shunning and Image Scoring and so on. The current simulations clearly indicate that the main difference between these 4 rules is distinguished by the coexistence region of cooperators and defectors. From ''Stern Judging'' to ''Image Scoring'', the space of $C + D$ becomes larger and larger, and the area occupied by pure defectors is also squeeze. That is, the rule of ''Image Scoring'' can best suppress the expansion of defectors since ''Image Scoring'' will reduce individual's reputation for all defection strategies and will increase individual's reputation for all cooperation strategies. At the same time, in the case of fixed b, an appropriate increase in K can suppress the spread of defection strategy and promote the formation of cooperation. For example, in the rule of ''Image Scoring'', when $b = 1.4$ is fixed, $K = 0.4$ is located in the mixed state of cooperation and defection; while K is increased to 1.0, the entire group arrives at a cooperative state. Additionally, when we keep K constant, increasing the value of b will illicit the individuals to adopt the defection strategy for the larger game payoff, which is also consistent with the results observed in Fig. 1.

IV. DISCUSSION

In summary, we explore the role of second-order reputation evaluation in the spatial prisoner's dilemma game in detail. In our model, each individual i has an image score R_i to describe his reputation, and this score directly determines an individual influence and the ability to spread his strategy to his neighbors. Meanwhile, we introduce a reputation threshold (Z) to determine whether an individual owns a high reputation. If $R_i \geq Z$, player i has a good reputation and he will have a strong transmission ability, in which we set his influencing factor $\omega_i = 1$; Otherwise ($R_i < Z$), $\omega < 1$ and then his transmission ability is weaker and he has little influence. In addition, an individual will update his reputation after he interacts with one of his neighbors so that his reputation will update for 4 times within each time step. During the reputation evaluation, we adopt 4 typical second-order rules to update the value of R_i as shown in Table 1, in which not only the individual i's strategy is considered, but also the reputation of his neighbor j is taken into account. R_i can only be maximized when i chooses to cooperate with a high-reputation neighbor, and his reputation will be deducted to the greatest extent when he defects. In the other two cases (cooperating with low-reputation individuals or defecting low-reputation individuals), the judgment of the two cases will be different according to the specific rule.

Through extensive numerical simulations, it can be clearly found that the introduction of reputation could significantly improves the frequency of cooperators within the population. Except the rule of ''Shunning'', the bigger the reputation threshold Z, the higher the stationary frequency of cooperators. Furthermore, the frequency of cooperators at the steady state under all four reputation assessment rules can be increased as the value of reputation step length p. The simulation results indicate that the introduction of Z and p can promote the establishment of reputation mechanism and further enhance the level of cooperation in the entire population.

To some extent, our model has a practical significance under the realistic societies. Considering there are two companies working together, one of which is a large or state-owned enterprise (e.g., the high-reputation individuals in the model) and the other one is a small or private company (e.g., the low-reputation individuals in the model). When the small company does not fulfil the contract, his reputation will be lost a large quantity as he has interacted with a large enterprise (a good reputation); However, if the large enterprise fails to fulfil this contract, according to the four different evaluation rules, the public opinion in the society will be different: some will think that the large enterprise should not destroy the cooperative relationship (Shunning and Image Scoring in the model), while others may think that the small company have a poor reputation and defected is also reasonable (Stern Judging and Simple Standing).

In future works, the second-order reputation evaluation models mentioned here can be further improved to the higher-order evaluation so that we can consider the more realistic scenarios. As an example, we perform the reputation evaluation in accordance with the actor's strategy & reputation and his opponent's reputation, we can formulate a third-order model for the prisoner's dilemma game. However, the number of third-order reputation models is $((2^2)^2)^2 = 256$, which is the most complicated evaluation model, and it is also the one with the most variety and the most diverse research direction. In addition to improving the order of evaluation rules, there are also several other directions for the second-order evaluation model to be further studied. For instances, the topology for the agents to play the game in this model is just a regular lattice with periodic boundaries, which are far from the ones in real society, and we can consider the more real network topologies, such as small world, scale free, and even the dynamic networks and so on. Moreover, individuals may have other strategies besides cooperation and defection, for example they may not participate in the game, or engage in punishment or rewarding, or competitiveness [41]. The approach can also be considered in conjunction with heterogeneous updating mechanisms [42].

The current model provides useful insights into understanding the emergence of cooperation and collective dynamics [43], and we hope that our results can also improve our understanding of cooperation in groups, in particular where large-scale cooperation is especially important. Examples include the vaccination of epidemic diseases [44]–[47], the responsible use of antibiotics [48], synchronization of neurons [49], [50], or mitigating adverse effects of overexploitation and climate change [20].

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