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Enhanced NMF Separation of Mixed Signals in Strong Noise Environment

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ABSTRACT Separation of mixed signals from a noisy environment without prior conditions is one of the difficulties in blind signal separation. To solve the problem of poor separation effect of mixed signals in a strong noise environment, we propose an enhanced non-negative matrix factorization method in this paper. By extending the Kullback–Leibler divergence form, this method adopts a new target signal and noise estimation algorithm to overcome the shortcomings of existing methods in noise estimation. Furthermore, combining with the least squares algorithm, the computational complexity is effectively reduced, and the computational efficiency of the algorithm is improved while the source signals are well estimated. The theoretical analysis and simulation results show that the proposed algorithm is better than the existing algorithms in terms of the source signal separation from mixed signals with noise, especially when the signal and noise energy are equivalent and the mixed signals are completely obliterated in the noise, the proposed algorithm has more obvious advantages than the existing algorithms, while the operation efficiency has been improved.

INDEX TERMS Signal separation, non-negative matrix factorization, Kullback-Leibler divergence, least squares.

I. INTRODUCTION

In the field of signal processing, the problems of multiple signals mixing are often encountered. In these problems, the source signals before being mixed are unknown and the mixing process is also unknown. How to recover the source signals from the mixed signals is one of the research hotspots and emphases in signal processing [1]. Blind signal separation (BSS) extracts source signals from observed mixed signals, which is widely used in digital communication [2], [3], speech signal processing [4], [5], medical diagnosis [6], [7] and image processing [8].

Nowadays, the methods of independent component analysis (ICA) [9], sparse component analysis (SCA) [10] and non-negative matrix factorization (NMF) [11] are usually used for blind separation of mixed signals. The core idea of independent component analysis is to minimize the statistical relationship between each signal source [12]. FastICA algorithm is a typical ICA optimization algorithm [13]. However, the ICA is not suitable for underdetermined systems and requires only one Gaussian signal in the source signals [14]. Sparse component analysis can separate underdetermined systems, but the source signals are required to be sparse and not suitable for all mixed cases [15].

From a mathematical point of view, when the observed mixed signal matrix is positive, the NMF method makes it possible to solve the problem of signal separation. In fact, time-domain signals, such as speech signals and noise, which may not necessarily satisfy the non-negative conditions, can be transformed to non-negative signals by signal transformation (such as FFT).

In 1999, D. Lee and H. Sueng proposed the NMF [16], which attracted great attention in the academic circles. NMF takes the advantages of simple calculation, fast factorization and obvious physical properties of the results. Various signal separation algorithms based on NMF have been studied and proposed. The basic NMF algorithm includes NMF algorithm

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based on Euclidean distance and NMF algorithm based on Kullback-Leibler (KL).

NMF does not require statistical independence of the source, nor does it have non-Gaussian restrictions, and is also applicable to non-sparse cases. NMF is used for target signal separation, which can effectively separate several linear mixed signals, especially in the field of image signal separation [17]–[21]. For image processing, Yong Peng *et al.* successively put forward the graph regularized discriminative NMF (GDNMF) and Flexible NMF model with adaptively learned graph regularization (FNMFG) with guaranteed convergence and relatively low complexity [22]–[24]. In the speech signals separation, the transformed non-negative signals have also been applied to some extent.

In speech signal processing, the mixed speech signal will inevitably be affected by noise from all aspects, which makes it difficult to recognize the separated speech signal. To solve the problem of NMF separation in noisy environment, K. Kwon proposed a speech enhancement technique which combines statistical model with NMF to update the speech and noise online [25]. M. Sun decomposed the input noise amplitude spectrum into low-order noise parts and sparse speech-like parts to achieve noise and speech estimation [26]. S. Wood and others proposed an unsupervised dictionary learning algorithm based on NMF combined with generalized cross-correlation (GCC) spatial positioning blind source separation algorithm GCC-NMF [27]. A. Vaghmare characterized the noisy signal as NMF with sparse constraints to eliminate noise [28]. The above algorithms add a lot of constraints to NMF algorithm, which improves the noise reduction effect and reduces the separation efficiency.

In this paper, an enhanced NMF algorithm is proposed by extending the NMF model directly with another idea. Based on the extended KL divergence, the algorithm can effectively separate the source signals and interference noise, and its computational complexity is equivalent to that of the basic NMF algorithm without noise. To further reduce the computational complexity, a new algorithm combining extended KL divergence with least squares is proposed.

The rest of this paper is organized as follows: Section 2 summarizes the NMF separation model of mixed signals; Section 3 proposes an enhanced NMF algorithm based on extended KL divergence, which establishes and solves the unconstrained optimization cost function; Section 4 proposes a new method combining extended KL divergence with least squares; Section 5 selects mixed speech signals for implementation to verify the separation effect of the algorithm.

In this paper, a large number of operations for matrix row, column and element are involved. For a matrix $\mathbf{A}$, $(\mathbf{A})_{ij}$ is the element of $i-$th row and $j-$th column for the matrix $\mathbf{A}$; $(\mathbf{A})_{.j}$ is the $j-$th column vector of the matrix $\mathbf{A}$; $(\mathbf{A})_{i.}$ is the $i-$th row vector of the matrix $\mathbf{A}$.

II. NMF SEPARATION OF MIXED SIGNALS

The $n-$ dimension sources are mixed to produce the $m-$ dimension mixed observations. It may be assumed that the signal mixing mode is linear mixing. For convolutional mixing, it can be transformed into linear mixing by signal transformation. Suppose the unknown source signals matrix is $\mathbf{H} \in R^{n\times L}$; the observation data matrix is $\mathbf{V} \in R^{m\times L}$; the mixed matrix is $\mathbf{W} \in R^{m\times n}$. L is the length of the signal data, $L \gg m, n$. The signal mixing model can be obtained as shown in (1).

$$\mathbf{V} = \mathbf{WH} \tag{1}$$

The hybrid model is consistent with NMF: a non-negative matrix is decomposed into the product of two non-negative matrices, that is, for an arbitrary non-negative matrix $\mathbf{V}$, the NMF algorithm can decompose it into the product of a non-negative matrix $\mathbf{W}$ and a non-negative matrix $\mathbf{H}$. Therefore, the extraction of the target signal from the mixed signal can be realized by NMF.

NMF algorithm is a new multivariate statistical analysis method with simple principle and concise algorithm, easy to understand and execute. In order to get the final desired factorization matrix, it is necessary to optimize the objective function and establish the update rules of the factor matrix in the iterative process. Only in this way can the final goal of matrix factorization be achieved.

Signal separation boils down to some parameters or direct signal estimation under given conditions. These parameters or signal estimates minimize or maximize a given objective function, or cost function, which is called optimization. The optimization under given certain conditions is called constrained optimization. That is to say, solving parameters or estimating signals can be transformed into establishing suitable target functions.

In order to complete the factorization of non-negative matrices, we must find an objective function to measure the degree of similarity between the matrices before and after factorization. In the process of solving the problem, all matrices must be non-negative. The commonly used objective functions are the objective function under maximum likelihood estimation, Euclidean distance and KL divergence, as shown from (2) to (4) respectively.

$$F = \sum_{i=1}^{m}\sum_{j=1}^{n}\left((\mathbf{V})_{ij}\log_2(\mathbf{WH})_{ij} - (\mathbf{WH})_{ij}\right) \tag{2}$$

$$E = \frac{1}{2}\|\mathbf{V}-\mathbf{WH}\|^2 = \frac{1}{2}\sum_{i,j}\left((\mathbf{V})_{ij} - (\mathbf{WH})_{ij}\right)^2 \tag{3}$$

$$D = \sum_{i,j}\left((\mathbf{V})_{ij}\ln\frac{(\mathbf{V})_{ij}}{(\mathbf{WH})_{ij}} - (\mathbf{V})_{ij} + (\mathbf{WH})_{ij}\right)^2 \tag{4}$$

Because KL divergence is more sensitive than that of Euclidean distance and more suitable for speech estimation under low energy observation, this paper uses the objective function under KL divergence. Then the NMF problem is transformed into an optimization problem, as shown in (5).

$$[\mathbf{W},\mathbf{H}] = \arg\min \mathrm{D}\left(\mathbf{V}||\mathbf{WH}\right) \quad s.t.\ (\mathbf{W})_{ij} \geq 0, \quad (\mathbf{H})_{ij} \geq 0 \tag{5}$$

Iterated by the gradient descent method, the incremental iteration rules of $\mathbf{H}$ and $\mathbf{W}$ can be written as follows.

$$(\mathbf{H})_{au} \leftarrow (\mathbf{H})_{au} + \eta_{au}\left[\sum_i (\mathbf{W})_{ia}\frac{(\mathbf{V})_{iu}}{(\mathbf{WH})_{iu}} - \sum_i (\mathbf{W})_{ia}\right] \tag{6}$$

$$(\mathbf{W})_{ia} \leftarrow (\mathbf{W})_{ia} + \eta_{ia}\left[\sum_u (\mathbf{H})_{au}\frac{(\mathbf{V})_{iu}}{(\mathbf{WH})_{iu}} - \sum_u (\mathbf{H})_{au}\right] \tag{7}$$

Unconstrained optimization problems cannot guarantee that the results are non-negative by gradient descent. Then gradient descent method can be turned into a multiplication algorithm. When $\eta_{au} = \frac{(\mathbf{H})_{au}}{\sum_i (\mathbf{W})_{ia}}$, $\eta_{ia} = \frac{(\mathbf{W})_{ia}}{\sum_i (\mathbf{H})_{au}}$, iteration rules of $\mathbf{H}$ and $\mathbf{W}$ are transformed into:

$$(\mathbf{H})_{kj} \leftarrow (\mathbf{H})_{kj}\frac{\sum_i (\mathbf{W})_{ik}(\mathbf{V})_{ij}/(\mathbf{WH})_{ij}}{\sum_i (\mathbf{W})_{ik}} \tag{8}$$

$$(\mathbf{W})_{ik} \leftarrow (\mathbf{W})_{ik}\frac{\sum_j (\mathbf{H})_{kj}(\mathbf{V})_{ij}/(\mathbf{WH})_{ij}}{\sum_j (\mathbf{H})_{kj}} \tag{9}$$

The iteration rules of (8) and (9) are consistent with (6) and (7).

The above analysis is the usual NMF algorithm. The algorithm does not consider the influence of noise or take noise as one of the signals. The NMF under noise is studied below.

III. NMF UNDER NOISE

A. NMF SEPARATION WITH NOISY SIGNAL

When the mixed system is noisy, the signal mixing model under the noise condition is shown in the following expressions.

$$\mathbf{V} = \mathbf{WH} + \mathbf{N} \tag{10}$$

where $\mathbf{N}$ represents additive noise, $\mathbf{V}, \mathbf{N} \in R^{m\times L}$, $\mathbf{W} \in R^{m\times n}$, $\mathbf{H} \in R^{n\times L}$. Correspondingly, the expanded KL divergence is:

$$D = \sum_{i,j}\left((\mathbf{V})_{ij}\ln\frac{(\mathbf{V})_{ij}}{(\mathbf{WH})_{ij}+(\mathbf{N})_{ij}} - (\mathbf{V})_{ij} + (\mathbf{WH})_{ij} + (\mathbf{N})_{ij}\right) \tag{11}$$

Under the condition of non-negative constraint, $(\mathbf{W})_{ij} \geq 0$, $(\mathbf{H})_{ij} \geq 0$, $(\mathbf{N})_{ij} \geq 0$. For coefficient regularization, $\sum_i (\mathbf{W})_{ij} = 1$. Therefore:

$$\begin{aligned}\sum_{i,j}\left(\sum_r (\mathbf{W})_{ir}(\mathbf{H})_{rj} + (\mathbf{N})_{ij}\right) &= \sum_{r,j}(\mathbf{H})_{rj}\sum_i (\mathbf{W})_{ir} + \sum_{i,j}(\mathbf{N})_{ij} \\ &= \sum_{r,j}(\mathbf{H})_{rj} + \sum_{i,j}(\mathbf{N})_{ij} \\ &= \sum_{i,j}(\mathbf{V})_{ij}\end{aligned} \tag{12}$$

So the optimization model can be described as:

$$\begin{aligned}[\mathbf{W},\mathbf{H},\mathbf{N}] = \arg\min D\left(\mathbf{V}||\mathbf{WH}+\mathbf{N}\right) \\ s.t.\ \sum_i (\mathbf{W})_{ij} = 1, \\ (\mathbf{W})_{ij} \geq 0, \\ (\mathbf{H})_{ij} \geq 0, \\ (\mathbf{N})_{ij} \geq 0, \\ \sum_{r,j}(\mathbf{H})_{rj} + \sum_{i,j}(\mathbf{N})_{ij} = \sum_{i,j}(\mathbf{V})_{ij}\end{aligned} \tag{13}$$

The multiplication algorithm is used to iterate as follows.

$$(\mathbf{H})_{kj} \leftarrow (\mathbf{H})_{kj}\frac{\sum_i (\mathbf{W})_{ik}(\mathbf{V})_{ij}/\left((\mathbf{WH})_{ij}+(\mathbf{N})_{ij}\right)}{\sum_i (\mathbf{W})_{ik}} \tag{14}$$

$$(\mathbf{W})_{ik} \leftarrow (\mathbf{W})_{ik}\frac{\sum_j (\mathbf{H})_{kj}(\mathbf{V})_{ij}/\left((\mathbf{WH})_{ij}+(\mathbf{N})_{ij}\right)}{\sum_j (\mathbf{H})_{kj}} \tag{15}$$

$$(\mathbf{N})_{ij} \leftarrow (\mathbf{N})_{ij}\frac{(\mathbf{V})_{ij}}{(\mathbf{WH})_{ij}+(\mathbf{N})_{ij}} \tag{16}$$

Considering the normalization of coefficients, the iterative process can be described as the following matrix form:

$$\mathbf{W} \leftarrow \mathbf{W}\odot\left(\left(\mathbf{V}\bigcirc(\mathbf{WH}+\mathbf{N})\right)\mathbf{H}^{\mathrm{T}}\right) \tag{17}$$

$$\mathbf{H} \leftarrow \mathbf{H}\odot\left(\mathbf{W}^{\mathrm{T}}\left(\mathbf{V}\bigcirc(\mathbf{WH}+\mathbf{N})\right)\right) \tag{18}$$

$$\mathbf{N} \leftarrow \mathbf{N}\odot\left(\mathbf{V}\bigcirc(\mathbf{WH}+\mathbf{N})\right) \tag{19}$$

where $\odot$ is the Hadamard product of the matrices, with $\bigcirc$ representing the corresponding elements of the matrices.

B. ALGORITHM CONVERGENCE ANALYSIS

Define$\{\mathbf{W}^{(t)}, \mathbf{H}^{(t)}, \mathbf{N}^{(t)}\}$ as the result of the $t-$th iteration, then $\mathbf{V}^{(t)} = \mathbf{W}^{(t)}\mathbf{H}^{(t)} + \mathbf{N}^{(t)}$. The divergence deviation $D\left(\mathbf{V}^{(t)}\right) = \sum_{i,j}\left((\mathbf{V})_{ij}\ln\frac{(\mathbf{V})_{ij}}{(\mathbf{V})_{ij}^{(t)}} - (\mathbf{V})_{ij} + (\mathbf{V})_{ij}^{(t)}\right)$ is non-negative, if and only if $\mathbf{V}^{(t)} = \mathbf{V}$ then $D = 0$. It is only necessary to prove that the monotonicity of $D^{(t)}$ does not increase during each iteration.

In order to prove the convergence of the algorithm, an auxiliary function is introduced.

Definition 1: $G\left(\mathbf{V},\mathbf{V}^{(t)}\right)$ is the auxiliary function of $D\left(\mathbf{V}\right)$, if $G\left(\mathbf{V},\mathbf{V}^{(t)}\right) \geq D\left(\mathbf{V}\right)$ and $G\left(\mathbf{V},\mathbf{V}\right) = D\left(\mathbf{V}\right)$.

Lemma 1: Under the iterative rule $\mathbf{V}^{(t+1)} = \arg\min G\left(\mathbf{V},\mathbf{V}^{(t)}\right)$, the divergence function D is not increasing.

Proof: Only if $\mathbf{V}^{(t)}$ is local minima of $G\left(\mathbf{V},\mathbf{V}^{(t)}\right)$, $D\left(\mathbf{V}^{(t+1)}\right) = D\left(\mathbf{V}^{(t)}\right)$. It is proved that only local minima are real. If the derivative of D exists and is continuous within a minimal field of $\mathbf{V}^{(t)}$, $\nabla D\left(\mathbf{V}^{(t)}\right) = 0$.Therefore, each iteration by $\mathbf{V}^{(t+1)} = \arg\min G\left(\mathbf{V}, \mathbf{V}^{(t)}\right)$ will eventually converge to a local minimum $\mathbf{V}_{\min} = \arg\min D(\mathbf{V})$. As is shown in the following expression: $D(\mathbf{V}_{\min}) \leq \cdots \leq D\left(\mathbf{V}^{(t+1)}\right) \leq D\left(\mathbf{V}^{(t)}\right) \leq \cdots \leq D\left(\mathbf{V}^{(2)}\right) \leq D\left(\mathbf{V}^{(1)}\right) \leq D\left(\mathbf{V}^{(0)}\right)$.

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Lemma 2:

$$
\begin{aligned}
&G\left(\mathbf{h}, \mathbf{h}^{(t)}\right) \\
&= \sum_i \left(\boldsymbol{v}_{(i)} \log_2 \boldsymbol{v}_{(i)} - \boldsymbol{v}_{(i)}\right) \\
&\quad + \sum_{ik}\left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)} + \mathbf{N}_{(ik)}\right) - \sum_i \boldsymbol{v}_{(i)} \frac{\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}}{\sum_k \left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}\right)} \\
&\cdot \left(\log_2\left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)} + \mathbf{N}_{(ik)}\right) - \log_2 \frac{\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}}{\sum_k \left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}\right)}\right)
\end{aligned}
$$

is the auxiliary function of

$$
\begin{aligned}
D(\mathbf{h}) = \sum_i \Bigg(&\boldsymbol{v}_{(i)} \log_2 \frac{\boldsymbol{v}_{(i)}}{\sum_k \left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)} + \mathbf{N}_{(ik)}\right)} - \boldsymbol{v}_{(i)} \\
&+ \sum_k \left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)} + \mathbf{N}_{(ik)}\right)\Bigg).
\end{aligned}
$$

Proof: Obviously, $G(\mathbf{h}, \mathbf{h}) = F(\mathbf{h})$, so only $G\left(\mathbf{h}, \mathbf{h}^{(t)}\right) \geq D(\mathbf{h})$ needs to be proved. From the property of logarithmic function, $-\log_2 \sum_k \left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)} + \mathbf{N}_{(ik)}\right) \leq -\sum_k \mathbf{a}_{(k)} \log_2 \frac{\mathbf{W}_{(ik)}\mathbf{h}_{(k)} + \mathbf{N}_{(ik)}}{\mathbf{a}_{(k)}}$, while $\sum_k \mathbf{a}_{(k)} = 1$, Set $\mathbf{a}_{(k)} = \frac{\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}}{\sum_a \left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}\right)}$, then

$$
\begin{aligned}
&-\log_2 \sum_a \left(\mathbf{W}_{(ia)}\mathbf{h}_{(k)} + \mathbf{N}_{(ik)}\right) \leq -\sum_k \frac{\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}}{\sum_k \left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}\right)} \\
&\cdot \left(\log_2\left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)} + \mathbf{N}_{(ik)}\right) - \log_2 \frac{\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}}{\sum_a \left(\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + !\mathbf{N}_{(ik)}\right)}\right).
\end{aligned}
$$

Therefore $G\left(\mathbf{h}, \mathbf{h}^{(t)}\right) \geq D(\mathbf{h})$.

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Theorem 1: Under the iterative rule $(\mathbf{H})_{kj} \leftarrow (\mathbf{H})_{kj} \frac{\sum_i (\mathbf{W})_{ik}(\mathbf{V})_{ij}/(\mathbf{WH+N})_{ij}}{\sum_i (\mathbf{W})_{ik}}$, $(\mathbf{W})_{ik} \leftarrow (\mathbf{W})_{ik} \frac{\sum_j (\mathbf{H})_{kj}(\mathbf{V})_{ij}/(\mathbf{WH+N})_{ij}}{\sum_j (\mathbf{H})_{kj}}$ and $(\mathbf{N})_{ij} \leftarrow (\mathbf{N})_{ij} \frac{(\mathbf{V})_{ij}}{(\mathbf{WH+N})_{ij}}$, The divergence $\mathrm{D}(\mathbf{V}||\mathbf{WH}+\mathbf{N})$ does not increase monotonously, and the algorithm converges when $\mathbf{H}$ and $\mathbf{W}$ are locally optimal.

Proof: For $G\left(\mathbf{h}, \mathbf{h}^{(t)}\right)$, the minimum value can be obtained by gradient method:

$$
\begin{aligned}
\frac{\partial G\left(\mathbf{h}, \mathbf{h}^{(t)}\right)}{\partial \mathbf{h}_{(k)}} &= -\sum_i \left(\boldsymbol{v}_{(i)} \frac{\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)}}{\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}}\right) + \sum_i \mathbf{W}_{(ik)}\mathbf{h}_{(k)} \\
&= 0.
\end{aligned}
$$

Therefore, the corresponding iteration rule is: $\mathbf{h}_{(k)}^{(t+1)} \leftarrow \frac{\mathbf{h}_{(k)}^{(t)}}{\sum_i \mathbf{W}_{(ik)}} \sum_i \frac{\boldsymbol{v}_{(i)}\mathbf{W}_{(ik)}}{\mathbf{W}_{(ik)}\mathbf{h}_{(k)}^{(t)} + \mathbf{N}_{(ik)}}$.

Since $\mathbf{H}$ and $\mathbf{W}$ are symmetrical, the same method can be used to prove that the iterative rules for $\mathbf{W}$ are also valid.

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IV. NNMF COMBINED WITH LEAST SQUARES

Equations (17) to (19) contain a large number of matrix multiplication and division operations with low efficiency, which can be combined with the least squares (LS) algorithm to optimize to reduce the matrix division. $\mathbf{V} = \mathbf{WH} + \mathbf{N}$, $\mathbf{V}, \mathbf{N} \in R^{m\times L}$, $\mathbf{W} \in R^{m\times n}$, $\mathbf{H} \in R^{n\times L}$, among them $m \geq n$, $L \gg m, n$. For $m \geq n$, the definition of $\mathbf{W}^{\dagger 1}$ is the left pseudo inverse of $\mathbf{W}$, if $\mathbf{W}^{\dagger 1}\mathbf{W} = \mathbf{I}$. For $n \ll L$, the definition of $\mathbf{H}^{\dagger 2}$ is the right pseudo inverse of $\mathbf{H}$, if $\mathbf{HH}^{\dagger 2} = \mathbf{I}$. Therefore $\mathbf{H}^{\dagger 2} = \mathbf{H}^{\mathrm{T}}\left(\mathbf{HH}^{\mathrm{T}}\right)^{-1}$.

For the dimension of $\mathbf{W}$, $m \geq n$, and if $\|\mathbf{V} - \mathbf{WH}\|^2$ is the smallest, then the equation $\mathbf{V} = \mathbf{WH}$ has a unique solution, which is the least squares problem. The renewal formula of the optimal solution of $\mathbf{W}$ and $\mathbf{H}$ is:

$$
\mathbf{W} \leftarrow \mathbf{VH}^{\dagger 2} = \mathbf{VH}^{\mathrm{T}}\left(\mathbf{HH}^{\mathrm{T}}\right)^{-1} \tag{20}
$$

$$
\mathbf{H} \leftarrow \mathbf{W}^{\dagger 1}\mathbf{V} = \left(\mathbf{W}^{\mathrm{T}}\mathbf{W}\right)^{-1}\mathbf{W}^{\mathrm{T}}\mathbf{V} \tag{21}
$$

NMF algorithm based on the LS is directly applied to signal separation to obtain the LS-based NMF signal separation algorithm, abbreviated as LS-NMF. The algorithm needs $\left(2mn + m^2 + n^2\right)L$ multiplication and two matrix inverse operations every update.

Correspondingly, for the noisy signal model $\mathbf{V} = \mathbf{WH} + \mathbf{N}$, the renewal formula of the optimal solution of $\mathbf{H}$ and $\mathbf{W}$ is:

$$
\mathbf{W} \leftarrow (\mathbf{V} - \mathbf{N})\mathbf{H}^{\dagger 2} = (\mathbf{V} - \mathbf{N})\mathbf{H}^{\mathrm{T}}\left(\mathbf{HH}^{\mathrm{T}}\right)^{-1} \tag{22}
$$

$$
\mathbf{H} \leftarrow \mathbf{W}^{\dagger 1}(\mathbf{V} - \mathbf{N}) = \left(\mathbf{W}^{\mathrm{T}}\mathbf{W}\right)^{-1}\mathbf{W}^{\mathrm{T}}(\mathbf{V} - \mathbf{N}) \tag{23}
$$

Combining (19), we can get the hybrid algorithm as follows.

$$
\begin{cases}
\mathbf{W} \leftarrow (\mathbf{V} - \mathbf{N})\mathbf{H}^{\mathrm{T}}\left(\mathbf{HH}^{\mathrm{T}}\right)^{-1} \\
\mathbf{H} \leftarrow \left(\mathbf{W}^{\mathrm{T}}\mathbf{W}\right)^{-1}\mathbf{W}^{\mathrm{T}}(\mathbf{V} - \mathbf{N}) \\
\mathbf{N} \leftarrow \mathbf{N} \odot (\mathbf{V} \bigcirc (\mathbf{WH} + \mathbf{N}))
\end{cases} \tag{24}
$$

$\mathbf{HH}^{\mathrm{T}}$ and $\mathbf{W}^{\mathrm{T}}\mathbf{W}$ are $n \times n$ dimensional matrices, whose inverse computation is far less than that of $m \times L$ dimension operation $\mathbf{V} \bigcirc (\mathbf{WH} + \mathbf{N})$.

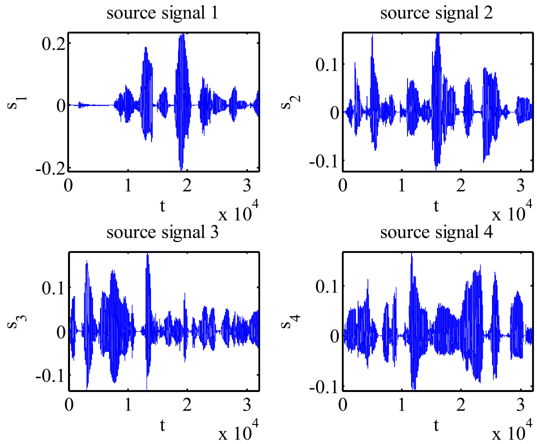

FIGURE 1. **Original signals.**

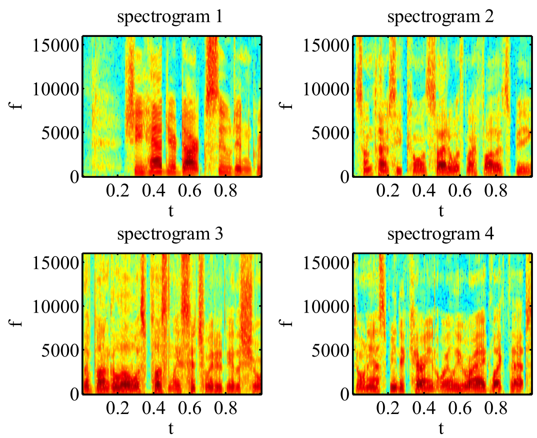

FIGURE 2. **Spectrograms of original signals.**

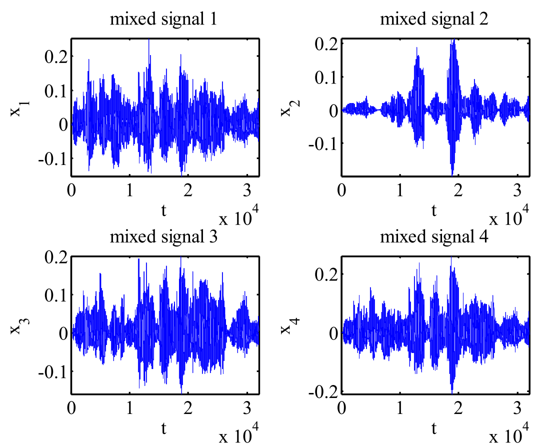

FIGURE 3. **Mixed signals.**

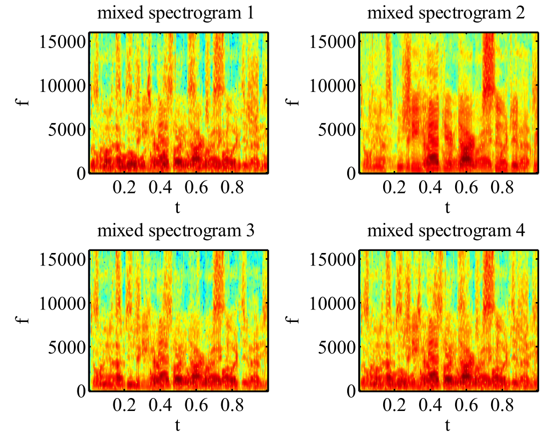

FIGURE 4. **Spectrograms of mixed signals.**

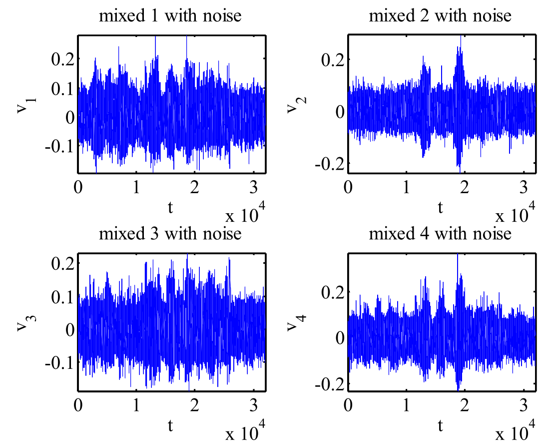

FIGURE 5. **Mixed signal when the SNR is 0 dB.**

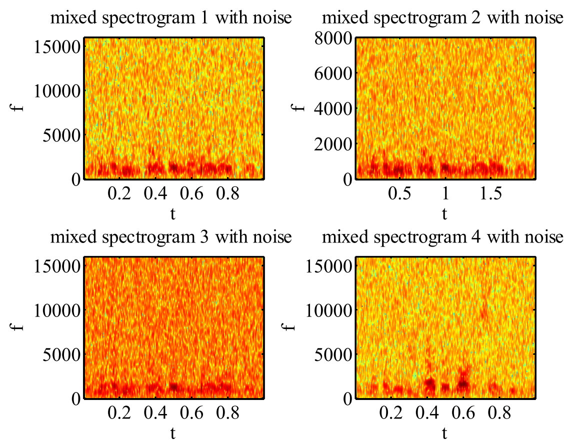

FIGURE 6. **Spectrograms of mixed signal when the SNR is 0 dB.**

V. SIMULATION AND PERFORMANCE ANALYSIS

A. EXPERIMENTAL SIGNAL

The experimental signals are taken from the TIMIT standard speech library. The four typical original signals and their spectrograms are shown in Fig. 1 and Fig. 2.

A mixing matrix is randomly generated to mix the source signals. The mixed signals and their spectrograms are shown in Fig. 3 and Fig. 4.

Each mixed signal contains multiple signals, so multiple voices overlap and mutually interfere. In this case, the usual ICA or NMF algorithm can be used to effectively separate the signals. But in the noise environment, the separation effect will drop sharply. The mixed signals and their spectrograms are shown in Fig. 5 and Fig. 6 when the signal-to-noise ratio is 0dB.

Fig. 5 and Fig. 6 show that the signal is almost completely annihilated when the SNR is 0 dB, and the existence of speech

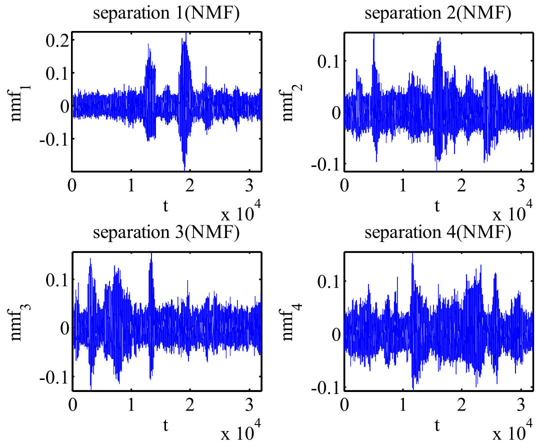

FIGURE 7. Separation by NMF method.

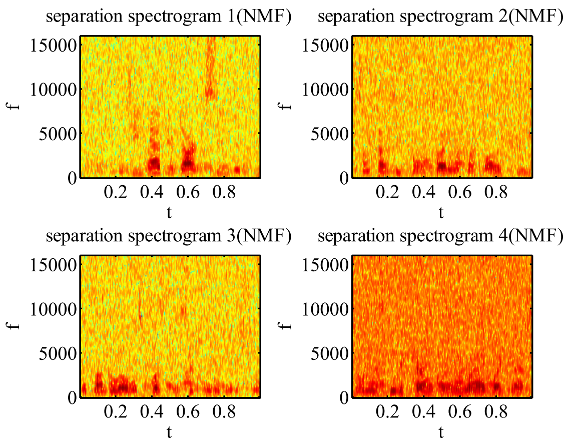

FIGURE 8. Spectrograms of separation by NMF method.

can hardly be recognized. In this case, ICA or basic NMF method is used to separate the signals, but the separation effect is poor. The separation effect is directly shown by NMF method, as shown in Fig. 7 and Fig. 8.

Fig. 7 and Fig. 8 show that the signal can be separated to a certain extent by using NMF directly, but the separation is not thorough, and the influence of noise is very large, so the separated speech is difficult to be identified.

B. EXPERIMENTAL RESULTS AND PERFORMANCE ANALYSIS

The unsupervised noise removal technique based on constrained NMF (UNMF) proposed in literature 28 is used to separate the source signals and identify the speech content. Noise can also be removed to some certain extent, but in the case of low signal-to-noise ratio, the effect of noise cancellation is limited, as shown in Fig. 9 and Fig. 10.

Using the proposed NNMF algorithm, the effect of signal separation and noise cancellation is better than that of UNMF algorithm, as shown in Fig. 11 and Fig. 12.

The proposed LS-NNMF algorithm can also eliminate noise more effectively, while the effect of signal separation

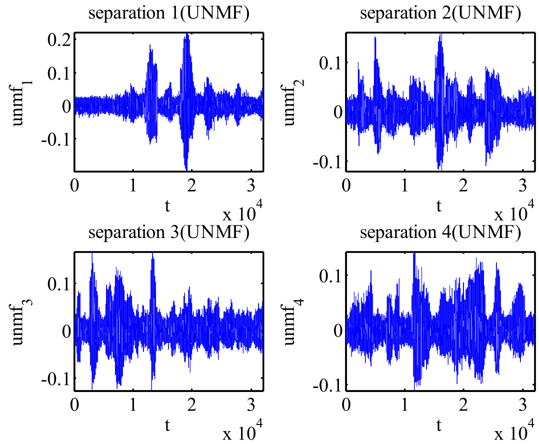

FIGURE 9. Separation by UNMF method.

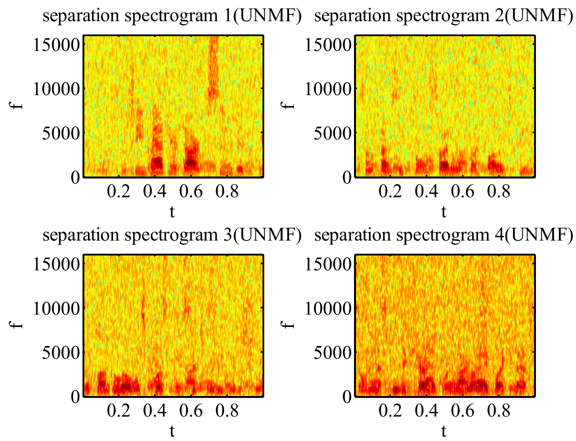

FIGURE 10. Spectrograms of separation by UNMF method.

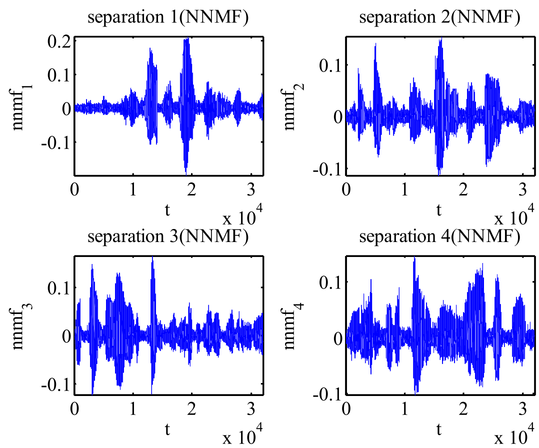

FIGURE 11. Separation by NNMF method.

is similar to that of the NNMF algorithm, as shown in Fig. 13 and Fig. 14.

C. RESULTS AND DISCUSSION

This section discusses the performance evaluation of the proposed methods. We use the typical speech signal of TIMIT

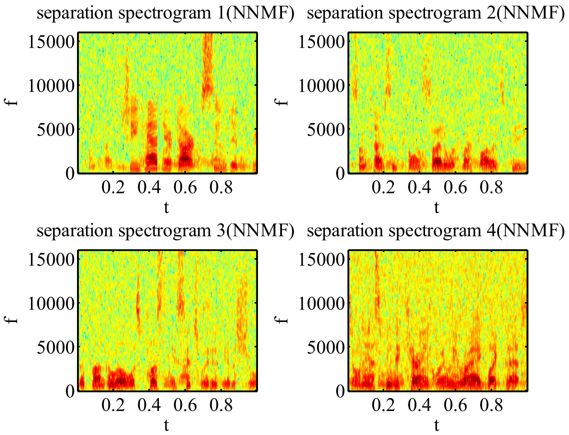

FIGURE 12. Spectrograms of separation by NNMF method.

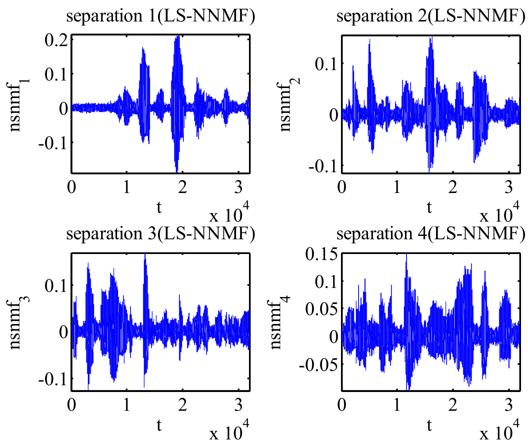

FIGURE 13. Separation by LS-NNMF method.

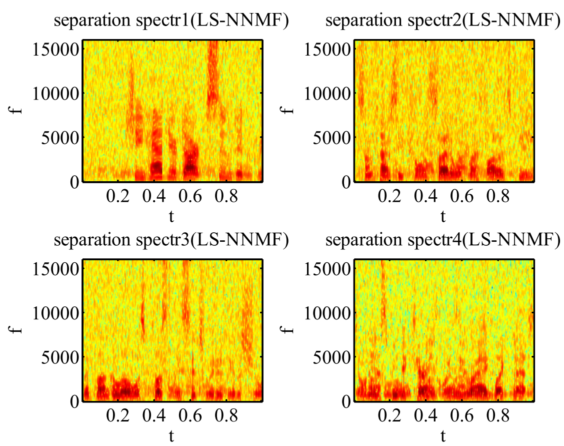

FIGURE 14. Spectrograms of separation by LS-NNMF method.

standard speech library with noises of different intensity. Algorithms of NMF, UNMF, NNMF and LS-NNMF were used to separate the signals from each other, and the time domain signals and spectrograms were displayed and compared with each other when SNR was zero. For the separation of mixed signals under different SNR, we employ three

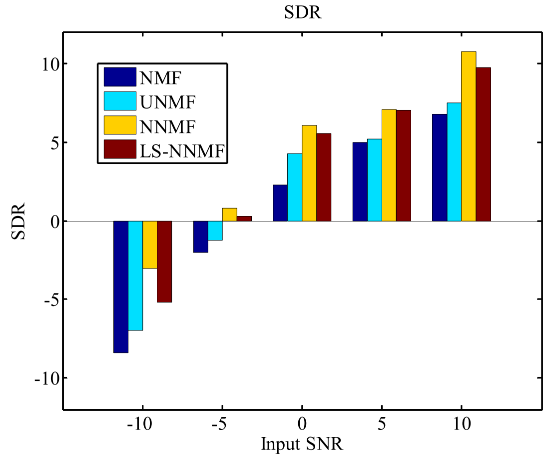

FIGURE 15. The SDR of separate signals from different algorithms.

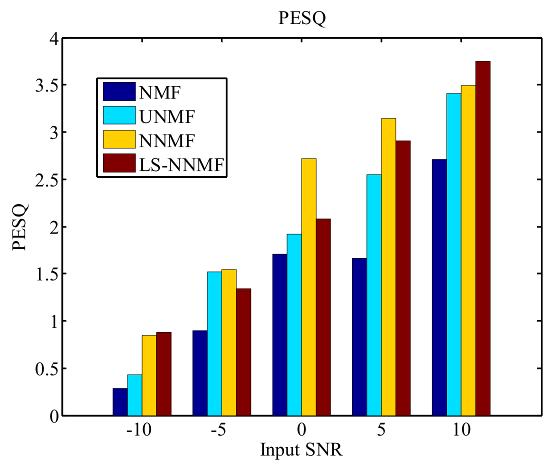

FIGURE 16. The PESQ of separate signals from different algorithms.

evaluation indicators– signal-to-interference ratio (SDR), speech quality perception assessment (PESQ) and short-term target intelligibility (STOI) to compare the separation effect. Among them, SDR is used to measure the effect of interference noise suppression, the greater the SDR is, the better the suppression effect will be; the PESQ is used to measure the distorted speech signal, the greater the PESQ is, the better the speech fidelity will be; the STOI is used to measure the intelligibility of speech, the greater the STOI, the better the intelligibility will be. The specific performance indicators are shown from Fig. 15 to Fig. 17.

The simulation results show that the separation performance of four methods at different input signal-to-noise ratios. UNMF algorithm, NNMF algorithm and LS-NNMF algorithm are significantly superior to the general NMF algorithm in three indicators. The proposed NNMF algorithm and LS-NNMF algorithm are superior to UNMF algorithm in terms of SDR, PESQ and STOI. Especially under the condition of low SNR, the performance of the proposed NNMF algorithm and LS-NNMF algorithm is improved significantly.

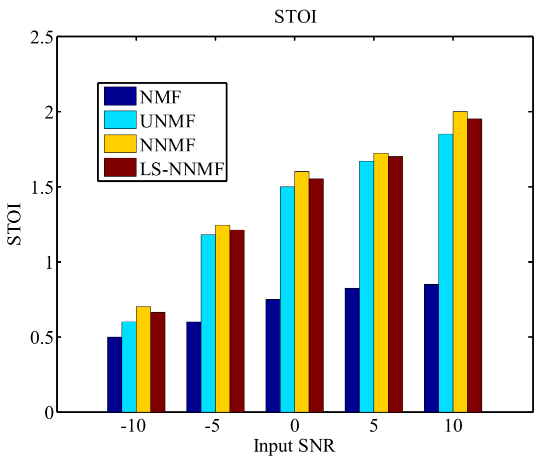

FIGURE 17. The STOI of separate signals from different algorithms.

TABLE 1. Comparison of computational complexity.

Algorithm	Times of multiplication
NNMF	$\left(6mn+m^2+n^2\right)L$
LS-NNMF	$\left(2mn+m^2+n^2\right)L$

The computational complexity of NNMF algorithm and LS-NNMF algorithm is compared as follows.

Since the iterative mode of **N** in both methods is the same, only the computational amount of **W** and **H** needs to be verified. For Eq. 17, $\left(3mn+n^2\right)L$ times of multiplication and division are required; While for Eq. 18, $\left(3mn+m^2\right)L$ times of multiplication and division are required. In other words, in each iteration of NNMF algorithm, the sum of **W** and **H** needs to be multiplied and divided $\left(6mn+m^2+n^2\right)L$ times.

For equation 22, $\left(mn+n^2\right)L$ times multiplication and 1 matrix inversion are required. For equation 23, $\left(mn+m^2\right)L$ times multiplication and 1 matrix inversion are required. In other words, in each iteration of LS-NNMF algorithm, the sum of **W** and **H** requires $\left(2mn+m^2+n^2\right)L$ times of multiplication and 2 times of small matrix inversion. Since L is the length of the signal data, $L \gg m, n$, the inverse operation time of small matrix can be ignored. The comparison of computation amount between the two algorithms is shown in Table 1.

When NNMF algorithm is adopted, each iteration will be carried out $4mnL$ times multiplication and division operation more than LS-NNMF algorithm. In practical application scenarios, m and n are usually not far apart. When $m = n$, NNMF algorithm requires $8n^2L$ times of multiplication and division. In comparison, LS-NNMF algorithm only requires $4n^2L$ times of multiplication and division, and the operation complexity is only half of NNMF algorithm. Adding equation 19, which is used in both iterations, the efficiency of LS-NNMF algorithm is still higher than that of NNMF algorithm.

VI. ABBREVIATIONS

BSS: Blind signal separation;
ICA: Independent component analysis;
SCA: Sparse component analysis;
NMF: non-negative matrix factorization;
FFT: Fast Fourier transform;
KL: Kullback-Leibler;
LS: least squares;
UNMF: unsupervised noise removal NMF;
NNMF: Enhanced NMF separation in strong noise environment;
LS-NNMF: LS-based NNMF
SNR: signal-to-noise ratio;
SDR: Signal to Interference Ratio;
PESQ: Perception Evaluation of Speech Quality;
STOI: Short-term Object Intelligibility

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The authors declare that all relevant raw data are taken from the TIMIT speech library. Any scientist wishing to use them can get them easily. They declare that we have no conflicts of interest, and we confirm that the funding did not lead to any conflict of interests regarding the publication of this manuscript. They have read and approved the final manuscript. They would like to express their sincere gratitude to the editor and anonymous reviewers for their constructive comments which have helped to improve the quality of this paper.

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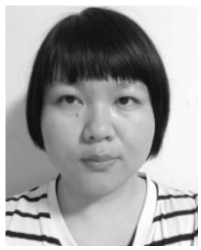

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