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Integrated Optimization of Train Stop Planning and Scheduling on Metro Lines With Express/Local Mode

ZHUJUN LI[1], BAOHUA MAO[1], YUN BAI[1], AND YAO CHEN[1,2]

[1]Key Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Ministry of Transport, Beijing Jiaotong University, Beijing 100044, China

[2]Department of Industrial Systems Engineering and Management, National University of Singapore, Singapore 119077, Singapore

Corresponding author: Baohua Mao (bhmao@bjtu.edu.cn)

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ABSTRACT Train stopping patterns and schedules play critical roles in the service design of metro lines with express/local mode. Previous studies on express/local mode generally handle the design of train stopping patterns and schedules independently, which cannot ensure the overall optimality of service provision. In this paper, a mixed integer nonlinear programming model is developed to collaboratively adjust train stopping patterns and schedules in order to minimize passenger travel time in express/local mode. With two types of train services provided, overtaking is allowed and a diverse set of passenger route choices is also incorporated into the proposed model. Linearization techniques are applied to transform this model from nonlinear to linear, which enables the proposed model to be handled by commercial linear solvers. To enhance computing efficiency in large-scale problems, a guided branch-and-cut algorithm is designed. The numerical examples on a test line and a real-world metro line are implemented to demonstrate the effectiveness of the proposed model and approach. The proposed approach is compared to existing approaches to identify the benefits of integrating train stop planning and scheduling decisions. Based on the computational time and the objective value, the guided branch-and-cut algorithm outperforms the direct use of the CPLEX solver.

INDEX TERMS Public transportation, train stop planning, train scheduling, express/local mode, integrated optimization, mixed integer linear programming.

I. INTRODUCTION

In the context of global trends towards urbanization and the ensuing expansion of urban areas, the spatial distance between the city center and suburbs grows gradually. With the swelling range of urban mobility, the travel distance of urban residents is also continuously increasing in recent decades. For example, an average daily journey has reached 19 km in Beijing in 2015 and takes travelers 52 minutes on average. Urban residents look forward to improvements to the efficiency of public transit service in response to the progressively growing urban scale. As the backbone of public transit networks, metros play important roles in the connectivity of urban areas. Metros are particularly important to the long-distance travel of urban residents, owing to its large capacity, high efficiency, and sufficient punctuality. However, all-stop schemes are increasingly insufficient to provide an attractive and competitive transit service with the extension of urban areas and metro lines. To further improve service efficiency, the express/local mode (Vuchic [1]) is commonly implemented by selecting suitable stopping service for trains. This service mode has been widely applied in many metropolises, including London, Tokyo, Paris, and New York.

In the express/local mode, two types of train service are provided to satisfy passenger demand in metro lines. The express trains, which only serve important stations, contribute to reducing the travel time for the passengers with relatively long travel distances. Local trains stop at all stations along the metro line to provide regular service, which enables passengers to arrive their destinations without transfer. To allow express and regular service to coexist, more than one track

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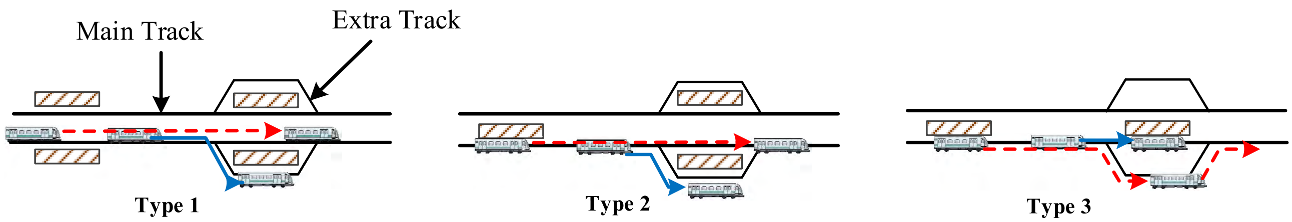

FIGURE 1. **Three exemplary layouts for tracks and platforms with the corresponding operation of express/local service.**

or takeover facilities are required. There are multiple layouts of track and platform designs for express/local operations. As shown in Fig. 1, two main tracks with some four-track stations are most extensively adopted in metro lines. With extra tracks at certain stations, local trains are able to stop at one track to make passengers get aboard or off; express trains are allowed to pass local trains through another track. Without such extra tracks, express trains are not permitted to overtake. The locations of the extra tracks are indispensable consideration to produce feasible service plans. This type of operations is possible only with precise scheduling and reliable operation [1].

The train service design with express/local mode is to determine a stopping pattern and the schedules. The general objective is to minimize passenger travel time. Two key challenges can be expected when devising a service plan. First, the allowance of overtaking calls for a methodical collaboration between stopping patterns and schedules. The locations of extra tracks are fundamentally linked to the train service design. Feasible locations for overtaking need to be considered when determining the stopping patterns. Train schedules must be adjusted to facilitate overtaking. Train stopping patterns and schedules should be designed collaboratively to allow overtaking where extra tracks are furnished, and to avoid conflicts between two consecutive trains. Efficient service with minimal passenger travel time is possible only by good coordination of train stop planning and scheduling considered in a unified objective. Second, passenger travel time is difficult to formulate, as passengers could have multiple route choices, given that two types of train service are provided. Travel time differs due to the multiple route choices, which in turn depends on the designed stopping plans and schedules. The analysis of passenger routes choices is indispensable to enhance service efficiency of express/local mode.

The train planning process is classically comprised of five sequential processes (Bussieck *et al.* [2]): demand analysis, line planning (stopping patterns, train consists, and frequency), train scheduling, train circulation planning, and crew scheduling. These five processes are generally handled individually due to the complexity of integrated problems. However, this separation may lead to poor coordination between train stop planning and scheduling. A train stopping pattern may not be implemented due to the failure to find a feasible train schedule. The need to iteratively adjust the train stopping patterns and schedules is highly possible to be required for express/local mode, especially with limited overtaking facilities. In addition, a sequential process would not result, in general, in an overall optimal solution. This process could lead to inefficient train service. Hence, an integrated optimization approach of train stop planning and scheduling for the metro lines with express/local mode is proposed in this paper. The main contributions of this paper are as follows:

1) This study takes a step forward in the collaborative optimization of train stopping patterns and schedules. This integrated approach considers the overtaking behavior between express and local trains, and multiple route choices introduced by two types of train service.
2) A mixed-integer nonlinear programming model is formulated to minimize passenger travel time. Linearization techniques are applied to transform this model from nonlinear to linear, which enables the proposed model to be handled by commercial linear solvers.
3) A guided branch-and-cut algorithm is developed to improve computing efficiency for large-scale problems. The proposed algorithm extends the applicability of the proposed strategy in practice.
4) Experiments on a test line and a real-world metro line are conducted to indicate the ability to produce efficient solutions using the proposed method. The benefits of the integrated strategy and the developed algorithm are demonstrated through a comparison to traditional approaches and alternative approaches in real-world case studies.

The rest of this paper is organized as follows. Section 2 presents a review of relevant literature. In Section 3, the mathematical model is formulated to obtain a train stopping pattern and schedules for both express and local trains. In Section 4, the nonlinear objective function and corresponding constraints are transformed into linearized expression. A guided branch-and-cut algorithm is presented to find efficient solutions to large-scale problems. In Section 5 and Section 6, the effectiveness of the proposed method is demonstrated by applying it to a test line and a real-world metro line. Finally, conclusions and future work are given in Section 7.

II. RELATED WORKS

Train stopping patterns and schedules are of great importance to determine a detailed operation plan for metro lines with express/local mode. Because of the complexity of the integrated problem, train stopping patterns and schedules are generally optimized sequentially in previous studies.

TABLE 1. Comparison with work related to the integrated optimization of train stopping and scheduling.

Paper	Overtake	Goal	Route choices	Algorithm
Wang et al. [16]	No	Min Travel time Min Energy	None	SQP
Yang et al. [17]	Yes	Min Dwell time delay	None	CPLEX
Qi et al. [18]	Yes	Min On-board time	None	CPLEX
Jiang et al. [19]	Yes	Min Train shift Max Profit	None	Heuristic algorithm
Jamili et al. [20]	No	Min Travel time	None	Heuristic algorithm
Lin et al. [21]	No	Max Profit	None	Genetic algorithm
Yue et al. [22]	Yes	Max Profit	None	Heuristic algorithm
Gao et al. [23]	No	Min Travel time	None	Iterative algorithm
Altazin et al. [24]	No	Min Waiting time Min Delay	None	NM
This paper	Yes	Min Travel time	Yes	Guided branch-and-cut algorithm

The train stop planning problem aims to determine the set of stations that each train will serve along the given train path, such that passenger demand constraints and train capacity constraints are satisfied. There has been extensive research on mathematical optimization methods for train stop planning problem for a single line, e.g. [3]–[9]. With the prescribed stop schemes, the scheduling problem consists of determining the arrival and departure times for each train at the corresponding stations, e.g. [10]–[15]. This decomposition helps to transform a complicated problem into two sub-tasks of manageable size.

Although the benefit is quite obvious, researchers and engineers have gradually realized that the sequential process fails to guarantee overall optimal service, or even a feasible operation plan in one iteration. For example, a train stop scheme needs to be redefined when headway constraints are unable to be resolved in scheduling. With the increasing efficiency in currently available hardware and methods, researchers have shifted their focus to integrated optimization.

Train stop planning and scheduling have been handled in an integrated model in many works, as their decision variables are closely related. Table 1 presents the relating integrated researches in previous literature. Wang *et al.* [16] proposed a nonlinear optimization model to optimize train scheduling with skip-stop strategy to minimize passenger travel time in urban rail transit. Yang *et al.* [17] proposed the first mixed-integer linear programming model to optimize the stop plans and train schedule collaboratively on high-speed railway lines with the aim to minimize dwell times at intermediate stations and delay in departure times at the origin station in comparison to those favored by operators. Continuous variables were used to express the departure and arrival times at the stations, and binary variables were used to represent the train sequence and the choice of whether to stop at a station. The optimization software GAMS with the CPLEX solver was used to code this large-scale optimization model and then generate approximate optimal solutions. Based on this paper, Qi *et al.* [18] extended the optimization model by considering trains with different pre-specified operation zones instead of having each train operate from the originating station to the terminating station. To schedule more passenger trains in a highly-congested line, Jiang *et al.* [19] proposed an integer linear programming model to optimize train stopping patterns and schedules with the goal of maximizing the total profit. Jamili and Aghaee [20] considered a case in which passenger demand fluctuated within a certain range and further proposed a linear model to produce a robust stop plan and timetable. To investigate algorithm efficiency, Lin and Ku [21] proposed two genetic algorithms, namely binary-coded genetic algorithm (BGA) and integer-coded genetic algorithm (IGA), to optimize train stopping patterns and scheduling with the goal of maximizing the profit of a rail company. A column-generation-based heuristic algorithm was designed in Yue *et al.* [22] to search for the optimal solution of train stop scheme and schedule in a large-scale case with the aim of maximizing total profit. In addition, integrated optimization strategies have also been used to address rescheduling problems. A linear optimization model to reschedule trains with the skip-stop strategy is proposed in Gao *et al.* [23] to reduce the number of stranded passengers more quickly. Altazin *et al.* [24] developed a real-time rescheduling model with the skip-stop strategy to minimize both delay propagation and the waiting time of passengers.

The research listed above provides a strong reference for the integrated optimization methods for the express/local mode. Nevertheless, some limitations could be expected if these approaches are applied directly. First, overtaking was not allowed in a part of the listed research, such as [16], [20], [21], [23], and [24], which limited the efficiency of express service for passengers with long traveling distance. Among those that did allow overtaking, the integrated optimization models focus on railway systems instead of metros, which aimed to maximize operating profits rather than minimize passenger travel time. To our knowledge, Qi *et al.* [18] is the only paper that both allows overtaking and aims to minimize travel time on railway corridors. However, their objective functions only considered on-board time, and neglected passenger waiting time and transfer time. This omission can lead to inefficient solutions for express/local mode because passenger waiting time is crucial to the service efficiency of metros. Passenger transfer time also carries great importance to the total passenger travel time due to multiple route choices in express/local mode.

Most existing studies on the express/local service mode handled the train stopping patterns [25]–[28] and schedules [29], [30] individually. To the authors' knowledge, there is only one paper, Shi *et al.* [31] which optimized stopping patterns and schedules simultaneously for the express/local mode, which is shown in Table 2. Shi *et al.* [31] presented an integrated optimization model with the aim to minimize

TABLE 2. Comparison with work related to integrated optimization on express/local mode.

Paper	Goal (Min)	Routes choices	Constraints	Algorithm
Shi et al. [28]	Running time Num. of trains	None	Headway	CPLEX
This paper	Travel time	Yes	Headway Overtaking Locations	Guided branch-and-cut algorithm

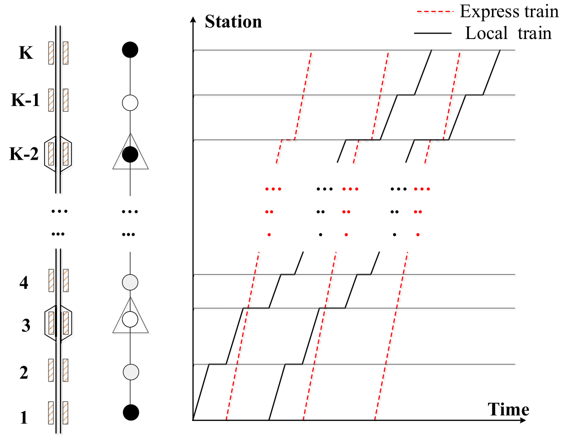

FIGURE 2. A layout of a metro line and an exemplary train service plan with express/local operation.

train operation time and the number of trains operated, which mainly focused on the benefits to operators. Nevertheless, with increasingly more importance placed on the quality of public transit service, passenger benefits should be given adequate attention. Specifically, one of the initial motivations to adopt express/local mode is to decrease passenger travel time. It is therefore of great importance to design express/local mode from the perspective of passenger travel efficiency.

III. MATHEMATICAL FORMULATION

The mathematical model is presented in this section to collaboratively optimize the train stopping patterns and schedules. The problem description and assumptions are demonstrated. The symbol notations are then introduced. Subsequently, the integrated optimization model of train stop planning and scheduling are proposed.

A. PROBLEM DESCRIPTION

In this paper, a single-direction metro line with K stations is considered for the express/local service design, where stations are numbered as 1 to K, as shown in Fig. 2. All train services start from station 1 and end at station K. The local services stop at all stations, while express services only stop at certain stations. Stations at which both express and local trains stop are defined as major stations, like station 1, station (K − 1), and station K in Fig 2. The stations at which only local trains stop are defined as minor stations. Overtaking is allowed to provide efficient express/local services, where the express trains overtake the local service at stations with extra

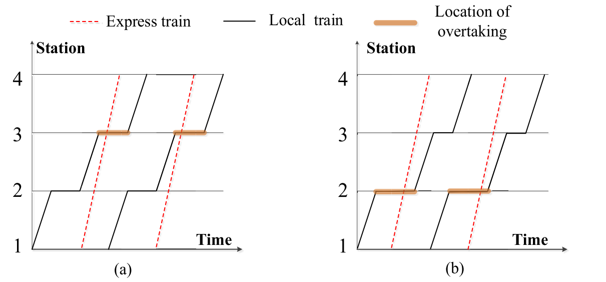

FIGURE 3. Typical examples of different trains schedules with overtaking at (a) station 3 and (b) station 2 under the same stopping plans.

tracks, like station 3 and station (K − 2) in Fig 2. Express trains are allowed to stop or pass directly when they overtake local trains.

This paper investigates the optimal design of express/local service on a metro line. We design the train stopping patterns and schedules with the objective of minimizing the passenger travel time. The stopping pattern is formed by determining the major stations where express trains stop. The train schedules are determined based on departure headways between express and local trains at the first station and dwell times at subsequent stations. The locations where overtaking occurs are also scheduled, which largely depends on stopping patterns, and could be mildly adjusted by modifying dwell times at stations. Fig. 3 illustrates how train dwell times can affect the location of overtaking, even with an identical stopping pattern. A metro line with four stations and three links is considered in Fig. 3. With the same stopping pattern, the overtaking location could be altered from station 3 to station 2 if the dwell time at station 2 is extended. The collaborative determination of train stopping patterns, schedules, and overtaking locations enables an efficient and precise service plan with express/local mode.

B. ASSUMPTIONS

In order to facilitate the model formulation, the following assumptions are made.

Assumption 1: During the studied time horizon, passenger origin-destination demand is known in advance and the arrival time of passengers at each station is uniformly distributed.

Assumption 2: All trains travel at the same speed between two consecutive stations. But, express trains and local trains could have different dwell times at each station.

Assumption 3: To maintain the regularity of timetables for passengers, the headway between two successive local trains are identical. Two consecutive express trains also maintain a uniform headway. As analysis methods under different ratios are similar, the ratio of express to local trains is assumed to be 1:1 in this paper, which means express and local trains depart alternatively. In reality, such regular schedules are extensively implemented in public transit service, as it is quite easy for passengers to follow. Numerous metro lines have adopted regular service schedules, including the Taoyuan airport MRT in Taiwan, and the Bangkok Airport Link in Thailand.

TABLE 3. Notation and description.

Notation	Description
Indices and parameters	
K	Number of stations
k,m,i,j	Index of station
L, X	Index of train service type, local and express
h_0	Departure interval between two successive express trains or local trains at the first station
t_k	Link running time of trains between station k and station k+1.
q	Acceleration and deceleration time of a train when it stops at a station.
h_d	Minimum departure interval between two successive trains at the first station
h_e	Minimum headway between two successive trains
h_w	Minimum time interval between a train's departure and the next train's arrival at a station
d_{min}	Minimum dwell time at a station
d_{max}^L	Maximum dwell time for a local train at a station
d_{max}^X	Maximum dwell time for an express train at a station
φ_{ij}	Number of passengers from station i to station j during every period h_0
M	A large positive number
Decision variables	
x_k	A binary variable. If express trains stop at station k, $x_k = 1$, otherwise $x_k = 0$.
o_k	A binary variable. If express trains overtake local trains at station k, $o_k = 1$, otherwise $o_k = 0$.
h_{LX}	Departure time interval between a local train and the subsequent express train at the first station
d_k^L	Dwell time of a local train in station k
d_k^X	Dwell time of an express train in station k
Auxiliary variables	
T_{ij}	Total travel time for passengers who depart from station i to station j
A_k^L	Arrival time at station k of the first local train
D_k^L	Departure time at station k of the first local train
A_k^X	Arrival time at station k of the first express train
D_k^X	Departure time at station k of the first express train
r_{ij}^1	The first major station between station i and station j
r_{ij}^2	The last major station between station i and station j
ω_{ij}	Percentage of passengers who would like to take express trains, even if they need to transfer or wait longer.

C. NOTATION

Before the model formulation, the necessary parameters are defined as follows.

D. CONSTRAINTS

In this section, system constraints related to train stop planning and scheduling are formulated to ensure that the resulting service plans are feasible. These constraints include link running time constraints, station dwell time constraints, and headway constraints.

1) DWELL WELL TIME AND DEPARTURE INTERVAL

Dwell times at station should not be less than the minimum dwell time d_{min}, which guarantees adequate time for passengers to board or alight. Similarly, it should not be more than the maximum dwell time d_{max} with considering the passenger service quality, i.e.,

$$d_{min} \le d_k^L \le d_{max}^L, \quad k = 1, 2, \ldots, \mathrm{K} \tag{1}$$

$$d_{min} \le d_k^X \le d_{max}^X, \quad k \in \{m | x_m = 1,\ m = 1, 2, \ldots, \mathrm{K}\} \tag{2}$$

2) ARRIVAL AND DEPARTURE TIME

The first local train departs from the first station at the beginning of the study period, i.e.,

$$A_1^L = D_1^L = 0 \tag{3}$$

$$A_1^X = D_1^X = h_{LX} \tag{4}$$

Given dwell time $d_k^l$and d_k^x, departure time interval h_{lx}, stopping patterns of express trains x_k, link running time t_k, and acceleration and deceleration time q, the arrival and departure times for the first local and the first express train should satisfy the following equations.

$$D_k^L = \sum_{m=1}^{k-1} t_m + \sum_{m=2}^{k} \left(q + d_m^L\right), \quad k = 2, 3, \ldots, \mathrm{K} \tag{5}$$

$$A_k^L = \sum_{m=1}^{k-1} t_m + \sum_{m=2}^{k} \left(q + d_m^L\right) - d_k^L, \quad k = 2, 3, \ldots, \mathrm{K} \tag{6}$$

$$D_k^X = h_{LX} + \sum_{m=1}^{k-1} t_m + \sum_{m=2}^{k} x_m \cdot \left(q + d_m^X\right), \quad k = 2, 3, \ldots, \mathrm{K} \tag{7}$$

$$A_k^X = h_{LX} + \sum_{i=1}^{k-1} t_m + \sum_{i=2}^{k} x_m \cdot \left(q + d_m^X\right) - d_k^X, \quad k = 2, 3, \ldots, \mathrm{K} \tag{8}$$

3) HEADWAY CONSTRAINTS

Headway constraints are formulated to keep the safety distance between two consecutive trains in metro lines. At the originating station, the departure interval between two successive trains should not be less than the minimum time interval h_d, i.e.,

$$h_{LX} \ge h_d \tag{9}$$

$$h_0 - h_{LX} \ge h_d \tag{10}$$

At other stations, the positional relationship of express and local trains at other stations can be categorized into four cases considering whether express trains stop or overtake local trains.

As shown in Fig. 4, h_k^1 and h_k^2 represent the time interval between two successive trains' arrival and departure at station k, which should be not less than the minimum headway h_e. h_k^3 represents the time interval between a train's departure and the next train's arrival at a station, which is not less than the minimum headway h_w. These headway constraints are presented in cases as follows.

Case (i): Express trains stop and overtake local trains at station k.

Case (ii): Express trains do not stop but overtake local trains at station k.

For the case (i) and case (ii) in Fig. 4, the local train arrives at the station earlier but departs later than the express train. Due to the periodic nature in which express and local trains are scheduled, the safety interval at station k can be ensured by focusing on the first local train. Specifically, if the first

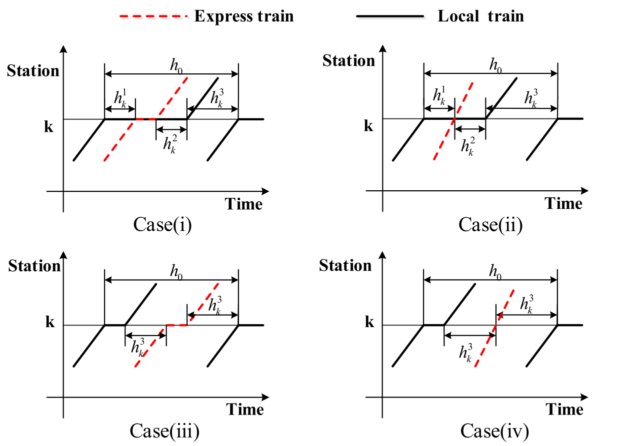

FIGURE 4. **An illustration for headway constraints of two successive trains.**

local train cannot collide with the express train, then there is no conflict at station k. Headway constraints for these two cases are as follows.

$$\left(A_k^X + h_0 \cdot \sum_{m=1}^{k-1} o_m\right) - A_k^L + M \cdot (1-o_i) \geq h_e, \quad k=2,3,\ldots,\mathrm{K} \tag{11}$$

$$D_k^L - \left(D_k^X + h_0 \cdot \sum_{m=1}^{k-1} o_m\right) + M \cdot (1-o_i) \geq h_e, \quad k=2,3,\ldots,\mathrm{K} \tag{12}$$

$$h_0 - d_k^L \geq h_w \tag{13}$$

Case (iii): Express trains stop but do not overtake local trains at station k.

Case (iv): Express trains do not stop at station k or overtake local trains either.

For case(iii) and case(iv) in Fig. 4, the local train arrives at the station and departs before the express train arrives, and vice versa. Similarly, the safety interval at station k can be ensured if there is no conflict between the first local train and the adjacent express trains. Headway constraints in these two cases are as follows.

$$\left(A_k^X + h_0 \cdot \sum_{m=1}^{k-1} o_m\right) - D_k^L + M \cdot o_i \geq h_w, \quad k=2,3,\ldots,\mathrm{K} \tag{14}$$

$$\left(D_k^X + h_0 \cdot \sum_{m=1}^{k-1} o_m\right) - A_k^L - M \cdot o_i \leq h_0 - h_w, \quad k=2,3,\ldots,\mathrm{K} \tag{15}$$

E. OBJECTIVE FUNCTION

The proposed model aims to reduce passenger travel time. In express/local mode, passengers may have different route choices, leading to different travel time. Based on the analysis of multiple route choices, the objective function is formulated.

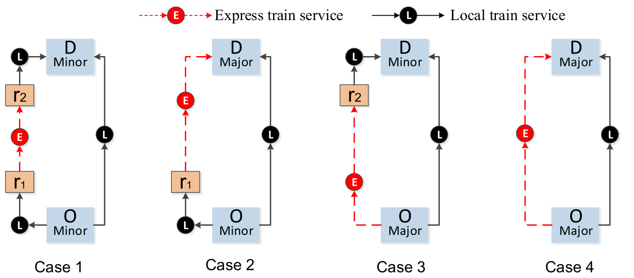

FIGURE 5. **Passenger route choices under four typical cases with express/local service.**

With two types of train service provided, passengers can be categorized into two groups. Some passengers, who are fully informed about train timetables, would like to take express trains to save travel time, even they need to transfer or wait longer. The rest are willing to take the first train subsequent to their arrival at the originating station, even if taking a local train would consume more time. Here ω_{ij} is introduced to describe the proportion of passengers who would like to seek the most efficient route to save their travel time. Passengers are assumed to transfer to express trains at the station r_{ij}^1, which is the closest major station from their originating station. Similarly, if passengers on express trains head to minor stations, they would transfer to local trains at the station r_{ij}^2, which is closest major station to their destination. Depending on their originating and terminating stations, passenger trips can be classified into four cases. Fig. 5 presents the passenger route choices under these four typical passenger trips. Passenger travel time is formulated as follows.

Case 1: From a minor station i to a minor station j

In case 1, ω_{ij} of passengers would like to transfer to express trains at station r_{ij}^1 and then transfer to local trains at station r_{ij}^2. The rest of them would like to take local trains to their destination directly. Passenger travel time is as follows

$$T_{ij}^{(1)} = \varphi_{ij} \cdot \Big(\frac{h_0}{2} + \omega_{ij} \cdot (A_j^L - D_i^L + h_0 + h_0 \cdot \sum_{m=1}^{r_{ij}^1 - 1} o_m - h_0 \cdot \sum_{m=1}^{r_{ij}^2} o_m) + (1-\omega_{ij}) \cdot (A_j^L - D_i^L)\Big) \tag{16}$$

$$r_{ij}^1 = min\left\{k \mid x_k = 1, k \epsilon Z^*, \ i < k \leq j\right\} \tag{17}$$

$$r_{ij}^2 = max\left\{k \mid x_k = 1, k \epsilon Z^*, \ i < k \leq j\right\} \tag{18}$$

Case 2: From a minor station i to a major station j

In case 2, ω_{ij} of passengers would like to transfer to express trains at station r_{ij}^1. The rest of them prefer taking local trains to their destination directly. Passenger travel time is as follows.

$$T_{ij}^{(2)} = \varphi_{ij} \cdot \Big(\frac{h_0}{2} + \omega_{ij} \cdot (A_j^X - D_i^L + h_0 \cdot \sum_{m=1}^{r_{ij}^1 - 1} o_m) + (1-\omega_{ij}) \cdot (A_j^L - D_i^L)\Big) \tag{19}$$

Case 3a: From a major station i to a major station j,and overtaking does not occur at station i.

In case 3a, two types of train service are provided in the originating station. ω_{ij} of passengers would like to take the express train only. The rest of them will take the first following train after their arrival, no matter which type of train service it provides. The rest of passengers are assumed that half take the express train and another half take the local train. This assumption is applicable since the arrival time is uniformly distributed, and the difference in the time interval is accepted [27]. Passenger travel time is as follows.

$$T_{ij}^{(3a)} = \varphi_{ij} \cdot (\omega_{ij} \cdot (\frac{h_0}{2} + A_j^X - D_i^X) + (1-\omega_{ij}) \cdot (\frac{h_0}{4} + \frac{1}{2} \cdot \left(A_j^X - D_i^X\right) + \frac{1}{2} \cdot \left(A_j^L - D_i^L\right)) \quad (20)$$

Case 3b: From a major station i to a major station j, and overtaking occurs at station i.

In case 3b, ω_{ij} of passengers would like to take the express train only. The rest of them are assumed to take the local train since the local train always arrives earlier and departs later than the express train at station i. Passenger travel time is as follows.

$$T_{ij}^{(3b)} = \varphi_{ij} \cdot (\omega_{ij} \cdot (\frac{h_0}{2} + A_j^X - D_i^X) + (1 - \omega_{ij}) \cdot (\frac{h_0}{2} + A_j^L - D_i^L) \quad (21)$$

Case 4a: From a major station i to a minor station j,and overtaking does not occur at station i.

Similar to case 3a, passenger travel time in case 4a is as follows.

$$T_{ij}^{(4a)} = \varphi_{ij} \cdot (\omega_{ij} \cdot (\frac{h_0}{2} + A_j^L - D_i^X + h_0 - h_0 \cdot \sum_{m=1}^{r_{ij}^2} o_m) + (1-\omega_{ij}) \cdot (\frac{h_0}{4} + \frac{1}{2} \cdot (A_j^L - D_i^X + h_0 - h_0 \cdot \sum_{m=1}^{r_{ij}^2} o_m) + \frac{1}{2} \cdot (A_j^L - D_i^L)) \quad (22)$$

Case 4b: Passengers from a major station i to a minor station j, and overtaking occurs at station i.

Similar to case 3b, passenger travel time in case 4b is as follows.

$$T_{ij}^{(4b)} = \varphi_{ij} \cdot (\omega_{ij} \cdot (\frac{h_0}{2} + A_j^L - D_i^X + h_0 - h_0 \cdot \sum_{m=1}^{r_{ij}^2} o_m) + (1 - \omega_{ij}) \cdot (\frac{h_0}{2} + A_j^L - D_i^L) \quad (23)$$

In reality, passengers will not transfer from local to express trains, if the transfer cannot reduce their travel time. Passengers would not benefit from transfers when overtaking does not occur between their origin and destination stations. Therefore, if taking the local train requires less time, all passengers would like to take the local trains to the destination without transfer. In this case, passenger travel time is as follows.

$$T_{ij}^{*} = \varphi_{ij} \cdot (\frac{h_0}{2} + A_j^L - D_i^L) \quad (24)$$

Similarly, passengers would not benefit from waiting longer to take the next express train when overtaking does not occur between their origin and destination stations. All passengers would like to take the first coming train no matter which service it provides. In this case, passenger travel time is as follows.

$$T_{ij}^{**} = \varphi_{ij} \cdot (\frac{h_0}{4} + \frac{1}{2} \cdot \left(A_j^X - D_i^X\right) + \frac{1}{2} \cdot \left(A_j^L - D_i^L\right)) \quad (25)$$

The objective function is defined as follows.

$$\begin{aligned} Min \sum\nolimits_{i=1}^{K-1} \sum\nolimits_{j=1}^{K} & ((1 - x_i) \cdot (1 - x_j) \cdot min\{T_{ij}^{(1)}, T_{ij}^{*}\} \\ & + (1 - x_i) \cdot x_j \cdot min\{T_{ij}^{(2)}, T_{ij}^{*}\} \\ & + x_i \cdot x_j \cdot (1 - o_i) \cdot min \left\{T_{ij}^{(3a)}, T_{ij}^{**}\right\} \\ & + x_i \cdot x_j \cdot o_i \cdot min \left\{T_{ij}^{(3b)}, T_{ij}^{**}\right\} \\ & + x_i \cdot (1 - x_j) \cdot (1 - o_i) \cdot min\{T_{ij}^{(4a)}, T_{ij}^{*}\} \\ & + x_i \cdot (1 - x_j) \cdot o_i \cdot minT_{ij}^{(4b)}, T_{ij}^{*}) \end{aligned} \quad (26)$$

In the function (26), passenger travel time is equal to the minimum of two cases. One is that all passenger take local trains directly; the other is that passengers have multiple route choices. Note that some constraints including multiplying variables and the minimum functions, the presented model is a nonlinear model.

IV. SOLUTION ALGORITHM

In this section, some linear techniques are presented to transform the proposed nonlinear model into a linear one. Then a guided branch-and-cut method is presented to solve large-scale cases.

A. LINEARIZATION

In order to find an exact solution, we transform the model above into a linear programming model. The three linearization methods applied in this process are as follows.

1) PERCENTAGE ω_{ij}

ω_{ij} is introduced to describe the proportion of passengers who would like to take express trains to save travel time, even they would transfer or wait longer. Gao *et al.* [30] pointed out that the percent of these passengers could be defined as

$$\omega_{ij} = (j - i)/K \quad (27)$$

In this way, the percent ω_{ij} is expressed as a linear function of the locations of origin station i and destination station j. The linear formulation indicates that the longer distance passengers plan to travel, the more of them would like to take express trains, even if they need to transfer.

2) DEPARTURE AND ARRIVAL TIME OF EXPRESS TRAINS

In Constraint (7) and (8), the departure and arrival times of the first express trains, which includes the product of stopping decision variables and dwell times, are expressed as quadratic constraints. We introduce another five linear constraints to substitute Constraint (2), (7) and (8) as follows.

$$D_k^X = h_{LX} + \sum_{m=1}^{k-1} t_m + \sum_{m=2}^{k} (x_m \cdot q + d_m^X), \quad k = 2, 3, \ldots, \mathrm{K} \tag{28}$$

$$A_k^X = h_{LX} + \sum_{m=1}^{k-1} t_m + \sum_{m=2}^{k} (x_m \cdot q + d_m^X) - d_k^X, \quad k = 2, 3, \ldots, \mathrm{K} \tag{29}$$

$$0 \le d_k^X \le d_{max}^X \tag{30}$$

$$d_k^X - M \cdot x_k \le d_{max}^X \tag{31}$$

$$d_k^X + M \cdot (1 - x_k) \ge d_{min}^X \tag{32}$$

Through these constraints, dwell time for express trains at a station is equal to 0 if express trains do not stop, and dwell time is no less than d_{min}^X if express trains stop at the station

3) MINIMUM AND MAXIMUM FUNCTIONS

In the objective function (26), minimum and maximum functions are applied. Since they are easy to be transferred into linear expressions with 0-1 variables and a large positive number M, we use the target function for case1 as an example.

$$T_{ij}^{(1+*)} = min\{T_{ij}^{(1)}, T_{ij}^{*}\} \tag{33}$$

$$T_{ij}^{(1+*)} \ge T_{ij}^{(1)} \tag{34}$$

$$T_{ij}^{(1+*)} \ge T_{ij}^{*} \tag{35}$$

$$T_{ij}^{(1+*)} \le T_{ij}^{(1)} + M \cdot \beta_{ij}^{(1)} \tag{36}$$

$$T_{ij}^{(1+*)} \le T_{ij}^{(1)} + M \cdot (1 - \beta_{ij}^{(1)}) \tag{37}$$

$$\beta_{ij}^{(1)} \in \{0, 1\} \tag{38}$$

Combined with the methods mentioned above, our model can be transformed into a mixed-integer linear programming model. The optimal solution of the linear model can be easily obtained by some commercial software, like CPLEX, GUROBI, and so on.

B. GUIDED BRANCH-AND-CUT ALGORITHM

Linearization is a commonly used strategy to make the functions and constraints of an optimization model easier to process. Many optimization solvers have developed advanced approaches to solve a linear model of manageable size. However, the low efficiency of these solvers is inevitable when dealing with large-scale problems, since there could be a huge number of variables and constraints. For example, when the proposed model contains 18 stations, the number of variables and constraints could exceed 10,000 and 17,000, respectively. Applying an optimization solver directly is not efficient enough, especially given the large proportion of binary variables in the proposed formulation.

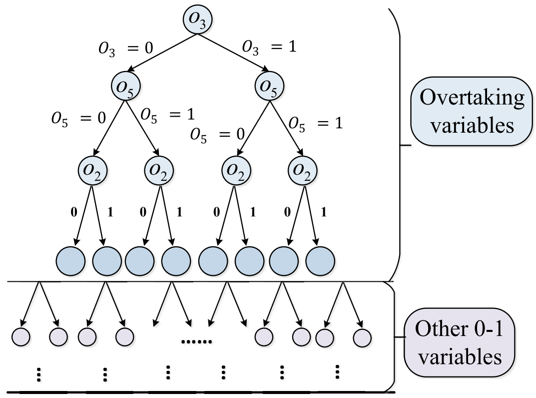

FIGURE 6. A guided branch-and-cut algorithm with a special ordered set.

As reported in Linderoth and Savelsbergh [32], introducing problem-specific branching rules is an effective way to reduce the size of the branching tree, and thus speed up the solution process. A guided branch-and-cut algorithm is proposed to solve the problem. In general, the implemented branching rule when considering binary problems is to branch on a single variable, thus creating two new subproblems. The order of branching variables is determined according to the solution to the LP relaxation problem, such as choosing the variable taking the variable $\tilde{x}_j$ for which is closest to 0.5, e.g., [33] and [34]. In the proposed linear model, overtaking variables and train stopping variables are mutually dependent. Overtaking variables are also primarily linked to variables pertaining to train scheduling and passenger routes choices. Considering the bridge role served by the overtaking variables, prioritized branching is recommended. A special ordered set approach is applied in the algorithm, as shown in Fig. 6. Overtaking variables are first branched in a specific order. The following is the iterative procedure implemented to solve the problems, as summarized in the following.

Guided approaches:

Step 1 (Initialization): Initialize overtaking variables as continuous variables $0 \le o_k \le 1$and a node set $\pi = \{o_k \mid k = 1, 2, \ldots, K\}$. Initialize a binary tree by selecting overtaking binary variables to develop new branches. The overtaking variables are selected according to passenger demand at station k during the research time period, where stations with less demand are selected first.

Step 2 (Node Selection): Search the binary tree in a depth-first or width-first way. Choose note $m$fromset M to be the next searching note as the order set.

Step 3 (Solving): Solve the linear relaxation problems using CPLEX solver within the time limits of 1,800 seconds. Let $\tilde{T}_m$ be the target value of the LR in node m. Record the optimized integer solution as $\bar{o}_m$ and target value as $\bar{T}_m$ respectively. Let the relative gap produced by CPLEX be ρ_m, which is defined as $\rho_m = (\bar{T}_m - \tilde{T}_m) \big/ \tilde{T}_m$.

Step 4 (Pruning): If ρ_m <0.01%, then cut the residual branches and notes below the note m. If $\bar{T}_m \le n, n \in \pi$, cut the residual branches and notes below the note n. Remove note m from set π.

TABLE 4. Passenger flow within fixed headway h_0.

	St.1	St.2	St.3	St.4	St.5
St.1	-	50	50	50	1000
St.2		-	50	50	50
St.3			-	50	50
St.4				-	50
St.5					-

* St. is short for Station

TABLE 5. Dwell time for local and express trains.

Station	Type	h_{LX}	2	3	4
Dwell time (s)	Express	120	0	0	0
	Local		105	30	30

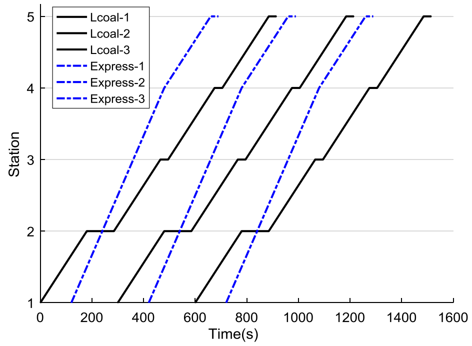

FIGURE 7. Time-distance diagram with express/local mode.

Step 5 (Output): If $\pi = \{\emptyset\}$, then output the optimized solution $\bar{o}_m$ with the minimum value of $\bar{T}_m$. If not, go back to Step 2.

V. DISCUSSION ON A TEST METRO LINE

A hypothetical metro line including 5 stations and each station is equipped with extra tracks is employed to illustrates how the proposed model works. Using CPLEX, the model is solved in 2 seconds (relative gap < 0.01) on a personal computer with 2.38GHz CPU and 4GB of RAM.

Table 4 presents the passenger demand φ_{ij} within h_0. Since we are only concerned with the upstream direction, the OD matrix is an upper triangular one. The link running time is equal to 120 seconds. The values of other parameters are $h_0 =$ 300s, $h_w = 45$s, $h_e = 45$s, q $=$ 60s, $d_{min} = 30$s, $d^L_{max} =$ 150s, $d^X_{max} = 90$s, M $= 1 \times 10^6$.

A. OPTIMIZED RESULT

The model involves 569 variables, 410 of which are binary. The passenger travel time of the optimized service plan is 939,000 seconds. Table 5 shows dwell time at each station. Fig. 7 shows the corresponding time-distance diagram.

In Fig. 7, express trains only stop at station 1 and station 5. Express trains overtake local trains at station 2 but do not stop. In Table 5, dwell time of local trains at station 2 is 105 seconds to enable overtaking at this station; dwell times at other stations are 30 seconds, which are equal to the minimum dwell time.

TABLE 6. Comparison between standard mode and express/local mode.

Result			Standard mode	Express/local
Stopping patterns	Express service		all	1-5
Overtaking stations			None	2
Passenger travel time (s)	Total		1,073,250	939,000 (-12.51%)
	St. 1 – St. 5*		885,000	709,500 (-19.83%)
		Waiting time	75,000	135,000
		On-board time	810,000	162,000
	Other OD pairs		188,250	229,500 (+21.91%)
		Waiting time	33,750	67,500
		On-board time	154,500	162,000

* St. is short for Station.

B. COMPARISON TO STANDARD MODE

The comparison between standard mode, in which each train stops at each station for 30 seconds, and the optimized plan with express/local mode is shown in Table 6. Table 6 reveals that the proposed strategy decreases passenger travel time by 12.51%, from 1,073,250 s to 939,000 s. Specifically, total travel time from station 1 to station 5 decreases by 19.83%, which is mainly due to passenger on-board time sharply decreasing from 810,000 s to 162,000 s. Meanwhile, travel time increases by 21.91% for other OD stations pairs, primarily due to waiting time increasing from 33,750 s to 67,500 s. The increase in waiting time is reasonable since service frequency at those stations skipped by express trains is reduced. This result shows that the proposed strategy is able to reduce passenger travel time with express/local mode. This mode is typically applicable for metro lines with high long-distance travel demand.

VI. A PRACTICAL APPLICATION OF THE WORK

To further test the computational performance, the following discussion intends to apply the proposed model to design the train stopping patterns and schedules for a real-world metro line in a city, which locates in the southern part of China. Our workstation is a personal computer with Intel(R) Xeon(R) E5-2620 v3 @ 2.40 GHz CPU and 32.00 GB RAM, using the Microsoft Windows XP(64 bit) OS.

The metro line connects the eastern suburban, urban center and western suburban, as shown in Fig.7. The stations are numbered from 1 to 18 as the upstream direction. Table 7 shows the running time between two successive stations. Station 2 and Station 18 connects with a railway station and a bus terminal respectively, which attracts many passengers get aboard or off at these two stations. Station 13 locates in the city center. Station 6, Station 11 and Station 15 are equipped with overtaking facilities, which are marked with "△" in Fig. 8.

A. OPTIMIZED RESULT

The model involves about 10,930 variables, of which 9,432 are binary. The passenger travel time with express/local

TABLE 7. Link running time.

Link	1	2	3	4	5	6	7	8	9
Time(s)	120	180	120	180	120	180	60	120	120
Link	10	11	12	13	14	15	16	17	
Time(s)	120	120	120	120	120	120	120	180	

FIGURE 8. Illustration for stations and line for metro line.

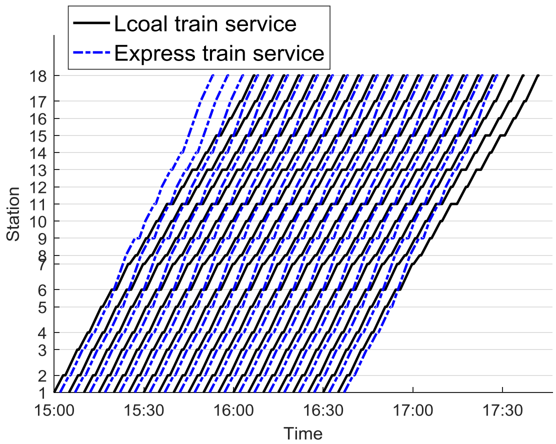

FIGURE 9. Time-distance diagram for express/local mode in the real-world line.

mode is 8,696,782 seconds. The time-distance diagram for optimized the train stopping patterns and schedules through is shown in Fig. 9.

Compared with standard mode, total travel time in the optimized strategy is reduced by 11.31%, from 9,805,620 s to 8,696,782 s. As shown in Fig 9, express trains only provide service to 9 stations, which is half of the total number of stations in this metro line. During the journey, express trains overtake local trains three times, at stations 6, 11 and 15. Dwell time for local trains at these stations are 150 s, 90 s, and 90 s, respectively, to enables the overtaking. Dwell time of express trains are 45 s at station 2 and 60 s at station 13, which facilitates the headway constraints between two successive trains.

B. COMPARISON TO RELATED WORK

In this section, the comparison of the performance of the proposed strategy in this paper and other strategies is presented. Table 8 shows the comparison result. Existing approaches

TABLE 8. Comparison between optimized plans produced by different strategies.

Result	Traditional Strategy	Alternative Strategy	This Paper
Target function	Travel time	Running time	Travel time
Decision Variable	Stopping	Stopping Scheduling	Stopping Scheduling
Stopping pattern	1-2-3-4-5-6-7-8-9-10-11-13-14-15-16-17-18	1-3-4-9-10-13-18	1-2-4-5-7-8-9-13-18
Overtaking stations	None	6,11,15	6,11,15
Travel time (s)	9,655,155 (-1.53%)	9,174,263 (-6.44%)	8,696,782 (-11.31%)

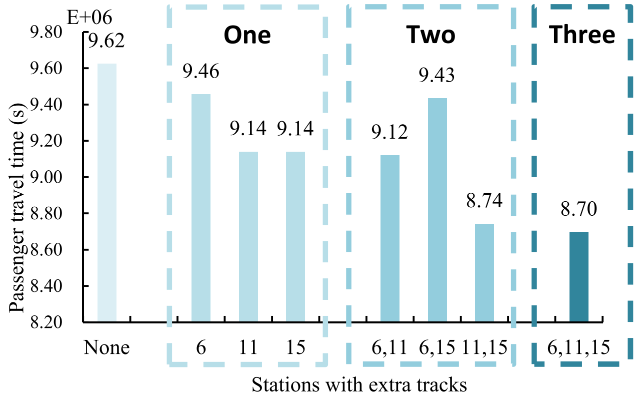

FIGURE 10. Sensitivity analysis of the number of stations with extra tracks.

for express/local mode are concluded into two strategies, e.g. the traditional strategy and alternative strategy. The traditional strategy refers to the two-stage optimization approach, in which the train stopping patterns and schedules are designed separately. Train stopping patterns are optimized to reduce passenger travel time with a sketch of schedule, for example, dwell time is assumed as 60 seconds when trains stop and 0 otherwise, such as Luo *et al.* [26] and Sun *et al.* [27]. Train schedules are designed to reduce passenger travel time with detailed analysis of diverse route choices, which is formulated based on Gao *et al.* [30]. The alternative strategy optimizes train stopping patterns and schedules simultaneously but simplifies the target function as minimizing train running time instead of passenger travel time, including Shi *et al.* [31]. The performance of these two types of model is compared with the approach presented in this paper.

In Table 8, Passenger travel time in the optimized plans by traditional strategy and alternative strategy decrease by 1.53% and 6.44% compared to standard mode, respectively, which are less than 11.31% by the model in this paper. The optimized plan by traditional strategy has skipped one station and decreases passengers travel time mildly. The improvement obtained by using the traditional strategy is limited as the stopping pattern and schedules are not optimized collaboratively. The alternative strategy decreases the passenger

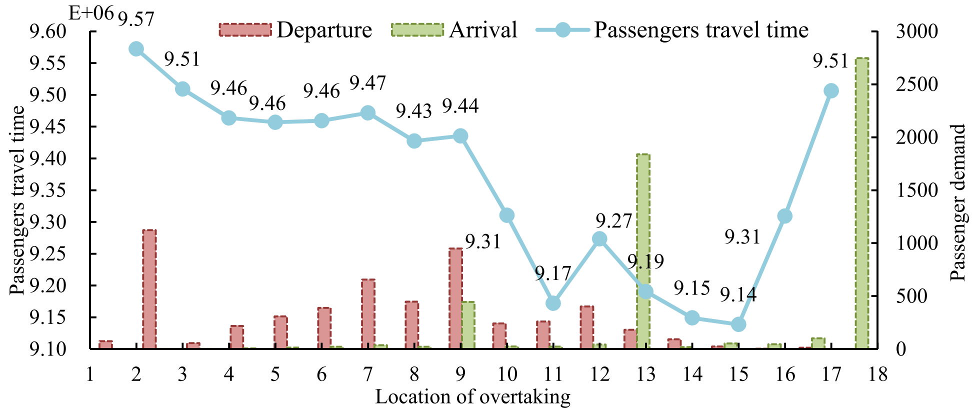

FIGURE 11. Sensitivity analysis on the locations of overtaking stations.

TABLE 9. Comparison to optimized plans by applying CPLEX directly.

Result	CPLEX	Guided branch-and-cut algorithm
Stopping patterns	1-2-4-5-6-7-8-9-13-18	1-2-4-5-7-8-9-13-18
Overtaking stations	11,15	6,11,15
Travel time (s)	8,742,939	8,696,782
Relative gap	19.53%	<0.01%
Calculation time (s)	86,400	35,813

travel time slightly, but it is less than the reduction brought about by the proposed strategy. This mainly comes from the simplified target function of the alternative strategy, which omits the analysis on multiple route choices. Although the simplified calculation of target function might increase the solving efficiency of the model, it will sacrifice the service competence of the optimized plan.

C. COMPARISON OF ALGORITHMS

In this section, the proposed algorithm is compared to using the CPLEX solver directly. The CPLEX solver is a well-recognized tool to solve mix-integer programming model. The CPLEX solver is applied to solve the linearized model, and the time limit is set to 24 hours. If the solver cannot found a solution with a relative gap of less than 0.01% within 24 hours, the optimal solution that it has obtained will be given. The relative gap is a measure of the degree of solution convergence.

As shown in Table 9, applying CPLEX to solve the model directly obtains an optimized solution whose relative gap is about 19.53% within 24 hours. In contrast, the guided branch-and-cut algorithm produces an optimal plan with a relative gap of less than 0.01%. The calculation time is less than 10 hours. This result indicates the advantage of the proposed algorithm to find an optimal solution than using CPLEX directly, especially when dealing with large-scale problems.

D. ANALYSIS OF LOCATIONS OF OVERTAKING

In this set of experiment, the influence of overtaking locations on passenger travel time is analyzed. The number of stations with extra tracks and their locations are analyzed in Fig. 10 and Fig. 11.

Different locations of extra tracks in the real-world metro line are discussed in Fig. 10. The number of stations with extra tracks can be divided into four cases: none, one, two and three. When the number of overtaking stations increases, passenger travel time generally decreases but not strictly. For example, passenger travel time with overtaking station 6 and 15 is even more than the case with only one overtaking station in station 11 or station 15. This result demonstrates that more overtaking stations are not sure to bring higher service efficiency for the express/local mode.

Fig. 11 presents the relationship between the locations of stations with extra tracks and passenger travel time. Each station is assumed to be equipped with extra tracks, except for the station 1 and station 18. In Fig. 11, passenger travel time generally has a similar fluctuation pattern and trend with passenger volume at stations. Passenger travel time decreases first and then increases significantly as the passenger volume decreases and then escalates. Therefore, it is recommended to equip the overtaking facilities at some stations with relatively low passenger demand. Locating the extra tracks in the middle of the metro line, or slightly close to the terminal station might be conducive to an efficient service plan.

VII. CONCLUSION

This paper proposes a strategy to optimize train stopping patterns and schedules simultaneously in metro lines with express/local mode. To minimize passenger travel time, a mixed integer linear programming model is formulated to produce an efficient stopping pattern and schedules. In this model, headway constraints are considered both in stop planning and train scheduling. Overtaking is allowed at certain stations, making the model applicable when the locations of extra tracks are predetermined. Multiple route choices under different stopping patterns and schedules are considered to calculate the passenger travel time. A guided branch-and-cut algorithm is designed to deal with large-scale problems.

The effectiveness of the proposed approach is demonstrated through a test and a real-world metro line. The test case shows that the express/local mode decreases passenger travel time in comparison with standard mode, in which all trains stop at each station. Passenger on-board time decreases noticeably although waiting time increases marginally. Furthermore, the proposed approach is compared to existing approaches to identify the benefits of integrating train stop planning and scheduling decisions. In the real-world case, passenger travel time decreases by 11.31% than standard mode. This reduction is more than the 1.53% reduction produced by traditional two-stage strategy, in which the train stopping patterns and schedules are designed separately. This reduction is also more than the 6.44% reduction produced by the alternative strategy used in existing works.

A comparison to using the CPLEX solver directly is conducted to demonstrate the advantages of the guided branch-and-cut algorithm with large-scale problems. Applying CPLEX to solve the model directly obtains an optimized solution whose relative gap is about 19.53% within 24 hours. In contrast, the guided branch-and-cut algorithm produces an optimal plan with a relative gap of less than 0.01%. Finally, the sensitivity of different locations of extra tracks is also analyzed. It is recommended to equip the overtaking facilities at some stations with relatively low passenger demand.

Our future work will focus on the following aspects. (1) A two-direction metro line with express/local mode can be considered, in which train overtaking and passenger route choices will be more complicated; (2) Improvement on the proposed solution approaches for the management of larger case studies can be investigated, since the efficiency of the proposed algorithm is merely adequate. (3) Stochastic passenger demand can be considered to enhance the reliability of the express/local service pattern. A more general optimization model on train service design will be researched in future works.

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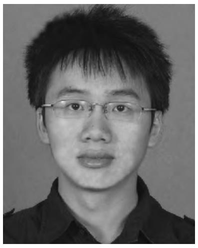
ZHUJUN LI received the bachelor's degree from Central South University, in 2014. She is currently pursuing the Ph.D. degree with the School of Traffic and Transportation, Beijing Jiaotong University. She has since enrolled in Beijing Jiaotong University in a combined master's-Ph.D. program. She is currently a visiting Ph.D. student with the University of Toronto. Her research interests include the modeling and simulation of public transportation systems, scheduling of public transit service, and train control strategies.

BAOHUA MAO received the B.Eng. degree in transportation management from Central South University, Changsha, China, in 1983, and the Ph.D. degree in engineering science from Beijing Jiaotong University, Beijing, China, in 1993. Since 1998, he has been a Professor with Beijing Jiaotong University, where he is also the Director of the Integrated Transport Research Center. His research interests include transport policies, transport management, operational optimization, and railway simulation. He serves as the Chief Editor for the *Journal of Transportation Systems Engineering and Information Technology* and a reviewer for many SCI/SSCI journals.



YUN BAI received the B.Eng. degree in mechanical and electrical engineering from Central South University, Changsha, China, in 2005, and the Ph.D.degree in transport planning and management from Beijing Jiaotong University, Beijing, China, in 2010, where he is an Associate Professor of transport engineering and planning. His research interests include railway simulation, train operation, traffic control in rail networks, and energy conservation in transportation.

YAO CHEN was born in Fuzhou, Jiangxi, China, in 1993. He received the B.S. and Ph.D. degrees from Beijing Jiaotong University, in 2013 and 2018, respectively. He is currently a Postdoctoral Research Fellow with the National University of Singapore. His research interests include operation research on public transit and car-sharing systems.