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Integral Sliding Mode Strategy for Robust Consensus of Networked Higher Order Uncertain Non Linear Systems

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ABSTRACT This paper focuses on an integral sliding mode technique-based consensus control protocol design for networked high order uncertain nonlinear systems. The nonlinear agents (nodes), which comprises of a leader and followers are networked via a fixed topology with a directed graph. A consensus among the leader and followers is achieved by first defining consensus error dynamics and then an integral manifold based distributed control protocols are designed. These distributed protocols steer the respective consensus error dynamics to equilibrium even in the presence of uncertainties. The robustness is achieved from the very start of the process by enforcing sliding mode at the initial time instant. The sliding mode enforcement and the closed loop stability analysis are presented in the form of a theorem. The theoretical results are verified via the simulation results of a numerical example.

INDEX TERMS Networked control systems, nonlinear systems, robustness, sliding mode control, uncertain systems.

I. INTRODUCTION

The applications of nonlinear control strategies, in the current technological era, are very wide and far fascinating. Apart from single system's control, its applications can be found in networked nonlinear systems [1]. To report a few, very fascinating applications of networked systems, can be found in formation control [2]–[4], rendezvous control of non-holonomic agents [5], and [6], smart grids applications [7], [8] and sensor networks [8]. Among such applications, the cooperative control protocol design is focused by big number of researchers. In case of networked systems, the nodes (often called agents) are either forced to follow a specific pattern of motion trajectories or the agents are supposed to follow a leader which is termed as consensus with theleader motion. In the leader follower system, the leader remains quite independent of the behaviour of the followers where as the followers are highly influenced by the leader dynamics. In practice, very rare and minimum information of the leader are available to the followers which is most possibly be the position and velocity information. It is also noted that the available information may quite possibly be available to a small portion of the network. Therefore, the investigation of control strategies for the consensus and formation of the networked systems, with limited information, become more significant.

In the networked systems, the followers must be capable enough to share the available information with the agents which are not in direct contact with the leader. Based on the available information, error dynamics can be defined which are either termed as consensus error dynamics or synchronization error dynamics. These kind of problems are also referred, in the literature, as cooperative regulation problem (consensus) and cooperative tracking problems (synchronization) [9]–[11]. In consensus, the distributed control protocols are designed to drive all the error dynamics of the networked agents to a consensus equilibrium which mainly depend on the initial conditions (see for instance; [11] and [12]).

The leader followers scenarios, for electro-mechanical systems, are very much focused in the existing literature (see for instance; [13]–[16]. In [13] Neural Networks (NNs) based uncertain dynamics estimations were performed for a network of first order Multi-Input Multi-Output (MIMO) uncertain systems. However, it suffered form unavoidable ultimate boundedness. In [14] Terminal Sliding Mode Control (TSMC) strategy based distributed control was employed to second order linear networked uncertain systems. However, the singularity existence in the manifold reduced its significance. Das and Lewis [15] and [16] developed adaptive

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distributed laws for the networks of single and double integrator uncertain systems. However, the requirement of knowing the non zero Eigen value of the Laplacian matrix limits its applications. In [17] a network of uncertain second order MIMO systems, with an undirected graph and fixed topology, was studied and Chebyshev Neural Networks (CNNs) based distributed TSMC was proposed to compensate the uncertain dynamics and bounded external disturbances. Recently, a network of second order linear uncertain system was considered in [18] and second order sliding mode based distributed laws were developed which resulted in finite time error convergence. The results were acceptable, however, this strategy was limited only to linear systems with matched uncertainties. In addition, this strategy showed sensitivity to disturbances in the reaching phase (a fundamental phase of conventional sliding mode control). A network of second order linear systems, in the presence of matched and mismatched uncertainties, was considered in [19]. An Integral Sliding Mode Control (ISMC) based protocols were designed which invoked an extended observers and NNs for the estimation and compensation of the mismatched uncertainties. A network of second order systems was targeted in [20] via a synthetic approach of backstepping control and Fourier Series (FS). The developed results are far interesting and worthy. However, this technique lacks in robustness against uncertainties. Moreover, some interesting results via the synergy of ISMC and FS could be extracted which will be more appealing and practical. An ISMC based distributed control protocols, subject to directed graph and fixed topology, were developed for networked uncertain nonlinear systems in the presence of matched uncertainties [21]. This strategy eliminate reaching phase which, consequently, resulted in enhanced robustness. However, it was employable only to electro-mechanical systems.

In the context of applications, a class of non-holonomic mobile robots was considered and adaptive formation control laws were developed in [22]. The aforementioned strategies were mainly focused on the networks of second order linear and nonlinear agents with bounded uncertainties and disturbances. The cooperative control protocols of networked higher order uncertain systems, in Brunovsky form [23], were an extension of [15], and [16]. A strategy of NNs was used to estimate the drift terms and uncertain input channels to compensate the uncertainties. However, this method resulted in the asymptotic convergence of the error dynamics to a bounded vicinity of the origin. In [24] higher order MIMO networked systems were studied for the synchronization purpose. These MIMO agents were operated under unknown disturbances and NNs based non-singular controls were developed. It was found that by adjusting the control parameter, the bounds of the tracking errors may not easily be reduced. The control parameters must be chosen carefully to ensure asymptotic convergence. The strategy of [23] was extended in [25] while using neuro-adaptive sliding mode strategy. Unfortunately, it resulted in some shortcoming like: the boundedness of the estimated weights of the NNs can not be guaranteed with the given tuning laws always. In addition, it is some what difficult to guarantee the boundedness of the control inputs in remark 1. A second order sliding mode strategy for the consensus of networked higher order nonlinear system was proposed in [26]. The results are good, however, the devised distributed laws were designed subject to bounded disturbances affected by the states and inputs, which theoretically became questionable.

In this article, the ISMC control protocol design [21] is extended to a class of networked higher order uncertain nonlinear systems. It inherits the features like sliding mode enforcement from the very start which, consequently, develops robustness against (results in invariance to) parametric variations and disturbances from the very start. This strategy also provides the ease of finite time stabilization of the consensus error dynamics by choosing the first component to be finite time stabilizing law. One may also suppress the adverse effects of mismatched data loss by designing a time domain H_∞ control component. Thus, this strategy is more significant in application to networked systems as compared to the conventional SMC techniques as well as TSMC techniques which very often resulted in high frequency vibrations and singular sliding surfaces, respectively. In the main work, consensus (synchronization) errors based dynamics are developed which operates under matched disturbances and states dependent uncertainties. The uncertainties are rejected by the discontinuous control components of the Integral Sliding Mode Control Protocol (ISMCP) (which is designed to enforce sliding mode along an integral manifold) and the uncertainties free linear system is driven by the Linear Quadratic Regulator (LQR). The (closed loop) stability of the overall consensus error dynamics is rigorously presented in a theorem and are also verified via the simulation results of a network of one leader and four followers. The remaining paper is managed as follows.

The problem formulation along with some relevant preliminaries is presented in Section II. The ISMC based distributed control protocols design and the closed loop-stability is presented in Section III. An illustrative example is given in Section IV to verify the theoretical results. Section V is based on the discussion of the simulation results. The paper is concluded in Section VI.

II. PROBLEM FORMULATION

A. DEFINITIONS

A network of $n+1$ agents is considered in this work. The leader's dynamics have a subscript 0 which makes its dynamic equations different from the dynamics of the n followers. The leader and followers share information through a directed graph. The governing dynamics of an i^{th} follower are represented via the following state space equations

$$\begin{aligned} \dot{x}_{ij} &= x_{ij}+1 \quad i=1,2,\ldots,n \quad j=1,2,\ldots,n-1 \\ \dot{x}_{in} &= f_i(x_i)+g_i(x_i)u_i+\Delta_i(x_i,t) \end{aligned} \tag{1}$$

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T \in \Re^n$ is a measurable states vector of the i^{th} follower, $\Delta_i(x_i, t)$ represents the uncertainty which affects the follower i and u_i is the applied control protocol. Furthermore, it is important to note that $f_i(x)$, $g_i(x)$ are sufficiently smooth vector fields.

Now referring to (1), it is suitable to consider some realistic assumptions.

Assumption 1: To ensure the controllability of each agent, let's assume that $g_i(x_i) \neq 0$ for $x_i \in \Re^n$.

Assumption 2: The uncertainties terms are considered bounded i.e.,

$$||\Delta_i(x_i, t)|| \leq C_i, \quad i = 1, 2, \ldots, n \tag{2}$$

where C_i are positive constants and $||.||$ refers to the Euclidean norm.
The leader, which is supposed to be followed, is represented by the following equations

$$\begin{aligned} \dot{x}_{0r} &= x_{0r+1} \quad for\ r = 1, 2, \ldots, n-1 \\ \dot{x}_{0n} &= f_0(t, x_0) \end{aligned} \tag{3}$$

where $x_0 = [x_{01}, x_{02}, \ldots, x_{0n}]^T \in \Re^n$ is the states vector of the leader system, $f_0(t, x_0)$ is continuous and bounded in t and x_0 with $f_0(t, 0) = 0$ [25]. In the literature, a graph is expressed by $\mathcal{G} = \{\mathcal{V}, \ \mathcal{E}\}$ with a non empty set of leader and followers ($n + 1$ nodes) $\mathcal{V} = \{\mathcal{V}_0, \mathcal{V}_1, \ldots, \mathcal{V}_n\}$ and a non empty set of edges/arcs $\mathcal{E}$. It should be noted that the considered graph is directed which means that the node i can share information with node j but the reverse may not necessarily be true. However, in an undirected graph the communication in both way remains true and possible. A weighted adjacency matrix related to a fixed topology can be expressed mathematically as follows

$$A = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ a_{10} & a_{11} & \ldots & a_{1n} \\ a_{20} & a_{21} & \ldots & a_{2n} \\ \vdots & \ldots & \ddots & \vdots \\ a_{n0} & a_{n1} & \ldots & a_{nn} \end{bmatrix}$$

The other relevant notions contain the sub-graph which is expressed as $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$. The followers topology is expressed as follows

$$\bar{A} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$$

In addition, let $\bar{d}_i = \sum_{j=1}^{n} a_{ij}$, $\bar{D} = diag[\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_n]$ and the Laplacian is defined to be $\bar{L} = \bar{D} - \bar{A} \in \Re^{n\times n}$ for the followers topology. Furthermore, $a_{ij} = 1$ if $(\mathcal{V}_j, \mathcal{V}_i) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. The leader and followers connectivity is expressed in the form of a matrix $\bar{B} = diag[b_1, b_2, \ldots, b_n]$ with $b_i = 0$ if an agent have no connection with the leader and $b_i = 1$ in case of suitable and proper connection. Regarding the leader, the following assumptions are defined.

Assumption 3: The leader driving force $f_0(t, x_0)$ is bounded and its boundedness is enough information for the distributed control protocols of the followers.

Assumption 4: The position and velocity measurements of the leader are available to the connected followers.

The main control objective is that the followers must have a consensus with the leader's states i.e., the followers should track the leader in the presence of uncertainties. This task can be achieved by defining the following consensus errors equations

$$e_{ik} = \sum_{j=1}^{n} a_{ij}(x_{ik} - x_{jk}) + b_i(x_{ik} - x_{0k}), \quad k = 1, 2, \ldots, n \tag{4}$$

By following (4) along the leader and followers dynamics (1) and (3), the error dynamics (for an agent i) can be obtained as follows

$$\begin{aligned} \dot{e}_{i1} &= e_{i2} \\ \dot{e}_{i2} &= e_{i3} \\ &\vdots \\ \dot{e}_{in} &= \left(\sum_{j=1, j\neq i}^{n} a_{ij} + b_i\right)\Big(f_i(x) + g_i(x)u_i\Big) \\ &\quad - \sum_{j=1, j\neq i}^{n} a_{ij}\Big(f_j(x)+g_j(x)u_j\Big) - b_i f_0(x, t) + h_i(x, t) \end{aligned} \tag{5}$$

where $h_i(x, t) = \left(\sum_{j=1, j\neq i}^{n} a_{ij} + b_i\right)\Delta_i(x, t) - \sum_{j=1, j\neq i}^{n} a_{ij}\Delta_j(x, t)$ is a lumped uncertain term which depends only the states of the networked agents. This lumped uncertain term $h_i(x, t)$ is bounded by the virtues of Assumption 2. Regarding the consensus error dynamics (5), it is suitable to re-assume the following.

Assumption 5: The error dynamics in (5) are assumed fully controllable i.e.,

$$\Big(\sum_{j=1, j\neq i}^{n} a_{ij} + b_i\Big) g_i(x) \neq 0, \quad for\ j = 1, 2, \ldots, n.$$

Now the problem (5) is the reformulated problem. The regulation of the states of (5) to origin leads to the consensus among the leader and the connected followers. Hence a regulation problem becomes the main task. Now, the consensus protocol design can be presented in the next section.

Remark 1: The errors dynamics in (5) can be expressed in a compact form as follows

$$\begin{aligned} \dot{Y}_1 &= Y_2 \\ \dot{Y}_2 &= Y_3 \\ &\vdots \\ \dot{Y}_n &= \big(\bar{L} + \bar{B}\big)\big(f(x) + g(x)u + \Delta(x, t) - \bar{1}f_0(t, x)\big) \end{aligned} \tag{6}$$

where

$$Y_1 = [e_{11}, e_{21}, e_{31}, \ldots, e_{n1}],$$

$$\begin{aligned} Y_2 &= [e_{12}, e_{22}, e_{32}, \ldots, e_{n2}], \\ &\vdots \\ Y_n &= [e_{1n}, e_{2n}, e_{3n}, \ldots, e_{nn}], \\ f(x) &= [f_1(x_1), f_2(x_2), \ldots, f_n(x_n)]^T, \\ g(x) &= diag[g_1(x_1), g_2(x_2), \ldots, g_n(x_n)], \\ u &= [u_1, u_2, \ldots, u_n]^T, \end{aligned}$$

and

$$\Delta(t, x) = [\Delta_1(t, x_1), \Delta_2(t, x_2), \ldots, \Delta_n(t, x_n)].$$

This kind of compact presentation remains helpful when one deals with the overall stability analysis.

III. CONTROL PROTOCOL DESIGN

In this section, the ISMCP design for the above mentioned system is presented in a comprehensive manner. Generally, the ISMC appears as an algebraic sum of two control components [26] i.e.,

$$u_i = u_{ai} + u_{bi} \tag{7}$$

where u_{ai} is the control input which regulates the system in sliding mode and u_{bi} is the control input which effectively diminishes the effects of matched disturbances and compensate the nonlinear terms. In the following study, the design of both the components will be outlined. To proceed to the design of u_{ai} the system defined in (5), without any disturbances, is expressed as follows

$$\begin{aligned} \dot{e}_{i1} &= e_{i2} \\ \dot{e}_{i2} &= e_{i3} \\ &\vdots \\ \dot{e}_{in} &= \psi_i(e_{i1}, e_{i2}, \ldots, e_{in}, u_j) + u_i \end{aligned} \tag{8}$$

where $\psi_i = \sum_{j=1,j\neq i}^{n}(a_{ij} + b_i)f_i(x_i) + (\sum_{j=1,j\neq i}^{n}(a_{ij} + b_i)$ $g_i(x) - 1)u_i - \sum_{j=1,j\neq i}^{n} a_{ij}\Big(f_j(x) + g_j(x)u_j\Big) - b_i f_0(x, t)$. For the sake of simplification the following assumption is made.

Assumption 6: It is assumed that in the very beginning the system (8) behaves linearly i.e., $\psi(Y_1, Y_2, \ldots, Y_n, u) = 0$.

Remark 2: The validity of this assumption depends on the fact that at time t_0 the sliding mode occurs. Therefore, the nonlinearities and disturbances are compensated exactly at the very start by u_{bi}. Consequently, the system operates under the action of the component u_{ai} in sliding mode from the initial time instant (for more details please follow the second half of the theorem 1 in the stability analysis).

Now, subject to Assumption 6 and Remark 2, the system (8) becomes

$$\begin{aligned} \dot{e}_{i1} &= e_{i2} \\ \dot{e}_{i2} &= e_{i3} \\ &\vdots \\ \dot{e}_{in} &= u_{ai} \end{aligned} \tag{9}$$

Here the main focus is the design of the control protocol component u_{ai}. For the sake of completion, the LQR based design is presented. The detailed expression of the control component u_{ai} appears as follows

$$u_{ai} = -k_{i1}e_{i1} - k_{i2}e_{i2} - \ldots - k_{in}e_{in} = -k_iE_i$$

where $k_i = [k_{i1}, k_{i2}, \ldots, k_{in}]$ is the gain's vector designed via the LQR procedure and $E_i = [e_{i1} \quad e_{i2} \ldots \quad e_{in}]^T$ represents a vector of the errors of an i^{th} agent. This part of the control protocol minimizes the Quadratic Cost Function

$$J_i = \frac{1}{2}\int_0^\infty \Big(E_iQ_iE_i^T + u_{ai}^T R_i u_{ai}\Big)\, dt$$

subject to the dynamics reported in (9). Note that, the designer parameters of the aforesaid system can be computed by solving the following algebraic Riccati equation

$$A_i^T P_i + P_iA_i - P_iB_iR_i^{-1}B_i^T P_i + Q_i = 0$$

and

$$k_i = R_i^T B_i P_i$$

where P_i and Q_i are symmetric and positive definite matrices and

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \ldots & \ldots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

This control component will steer the error dynamics to zero asymptotically after the enforcement of sliding mode.

Remark 3: The aforementioned control design of u_{ai} always confirms the asymptotic convergence of the system's states. This asymptotic convergence can be replaced with finite time convergence (to improve precision) if one design the said component via some finite time stabilizing laws e.g., [26], and [27].

Remark 4: If the agents are operating under uncertainties of both matched and mismatched kind, then the matched uncertainties can be nullified and the mismatched uncertainties can be suppressed to a tolerable band via the design of an H_∞ based ISMC controller [29], and [30]. On the other hand, both kind of uncertainties can also be handled by following the integral manifold design strategy [31]. For states dependent matched and mismatched uncertainties one may follow the strategy outlined in [32].

At this stage, the uncertainty rejecting and sliding mode enforcement control component will be presented.

For the i^{th} system, with dynamics in (5), the integral sliding manifold can be defined as follows

$$\sigma_i = \sum_{j=1}^{n} \lambda_{ij}e_{ij} + z_i \tag{10}$$

where λ_{ij} (with $\lambda_{in} = 1$) are the performance parameters which are chosen positive and z_i is an integral term which helps in the elimination of the reaching phase. By taking the time derivative of the integral manifold along the errors dynamics described in (8), one has

$$\dot{\sigma_i} = \sum_{j=1}^{n-1}\lambda_{ij}e_{ij+1} + \left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) f_i(x_i) + \left(\left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) g_i(x_i) - 1\right) u_{ai} + u_{ai} + \left(\left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) g_i(x_i)\right) u_{bi} - \sum_{j=1,j\neq i}^{n} a_{ij}\Big(f_j(x) + g_j(x)u_j\Big) - b_i f_0(x,t) + h_i(x,t) + \dot{z}_i \tag{11}$$

By choosing

$$\dot{z}_i = -\sum_{j=1}^{n}\lambda_{ij}e_{ij+1} - u_{ai} \tag{12}$$

Equation(11) becomes

$$\dot{\sigma_i} = \left(\left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) f_i(x_i) + \left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right)(g_i(x_i) - 1)u_{ai}\right) + \left(\left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) g_i(x_i)\right) u_{bi} - \sum_{j=1,j\neq i}^{n} a_{ij}\Big(f_j(x) + g_j(x)u_j\Big) - b_i f_0(x,t) + h_i(x,t)\Big) \tag{13}$$

Now, by comparing the following reachability law [33] with (13)

$$\dot{\sigma_i} = -k_i(x,t)sign(\sigma_i)$$

the expression for the control component u_{b_i}, for the i^{th} agent, can be obtained as follows

$$u_{bi} = -\left(\Big(\sum_{j=1,j\neq i}^{n} a_{ij} + b_i)g_i(x_i)\right)^{-1} \times \left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) f_i(x_i) + \left(\left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) g_i(x_i) - 1\right) u_{ai} - \sum_{j=1,j\neq i}^{n} a_{ij}(f_j(x)+g_j(x)u_j) - b_i f_0(x,t) + k_i(x,t)sign(\sigma_i)\Big) \tag{14}$$

Now, both the components of the control protocol (7) are suitably presented for an i^{th} agent. The finite time enforcement of sliding mode, in closed loop, will be presented in the following study.

IV. STABILITY ANALYSIS

The sliding mode enforcement stability for an i^{th} networked agent, in closed loop, can be proved by stating the following theorem.

Theorem 1: Consider that Assumptions 1, 2, 5, and 6 are satisfied. The sliding mode can be enforced in finite time by the control component (14) along an integral sliding surface (10) if the gains of the control component u_{bi} are chosen according to (15).

$$k_i(x,t) \geq \eta_i + |h_i(x,t)| \tag{15}$$

In addition, the dynamics of the overall system, in sliding mode, are governed by the control component u_{ai}.

Proof: This theorem can be proved by considering a Lyapunov candidate function as follows

$$v_i = \frac{1}{2}\sigma_i^2 \tag{16}$$

Computing the time derivative of (16) along (11) and then utilizing the expression of $\dot{z}_i$ from (12), one has

$$\dot{v_i} = \sigma_i\left(\left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) f_i(x_i) + \left(\left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) g_i(x_i) - 1\right) u_{ai} + \left(\left(\sum_{j=1,j\neq i}^{n} a_{ij} + bi\right) g_i(x_i)\right) u_{bi} - \sum_{j=1,j\neq i}^{n} a_{ij}\Big(f_j(x)+g_j(x)u_j\Big) - b_i f_0(x,t) + h_i(x,t)\right) \tag{17}$$

Substituting the values of u_{b_i} from (15) (subject to Assumption 5 and 6), one gets

$$\dot{v_i} = \sigma_i\dot{\sigma_i} = \sigma_i\,(h_i(x,t) - k_i(x,t)sign(\sigma_i))$$

Now, making use of Assumption 2 and the inherent boundedness property of $h_i(x,t)$, one may have

$$\dot{v_i} \leq |\sigma_i||h_i(x,t)| - k_i(x,t)|\sigma_i|$$

or

$$\dot{v_i} \leq -|\sigma_i|(k_i(x,t) - |h_i(x,t)|) \tag{18}$$

Since the objective is to confirm sliding mode enforcement which can be ensured by proving (18) negative definite. Therefore, if one chooses $k_i(x,t) \geq \eta_i + |h_i(x,t)|$ then it will remain negative definite. Consequently, one can write the above inequality as follows

$$\dot{v_i} \leq -\eta_i|\sigma_i|$$
$$\dot{v_i} \leq -\sqrt{2}\eta_i v^{1/2}$$

where η_i is a small positive real constant to maintain the negative definiteness of (18). This differential inequality confirms that $\sigma_i \rightarrow 0$ in finite time $t_{\sigma_i} \leq \sqrt{2}\eta_i^{-1}\sqrt{v_i(\sigma_i(0))}$(see for details [34]). Since, the integral manifold (10) is designed in such a way that $\sigma_i(0) = 0$ holds. Hence, the sliding mode enforcement time is almost 0, i.e., sliding mode occurs from the time $t_0 = 0$.

The second part of the theorem can be proved by considering the (13). Posing $\sigma_i = 0$ and then calculating the

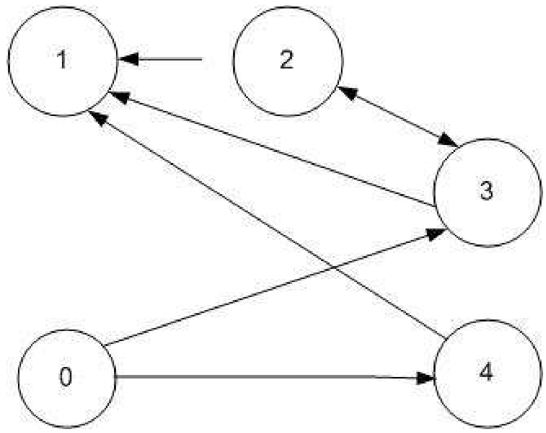

FIGURE 1. Topology of the network of four followers and a leader.

expression of the equivalent control protocol [33], one may get

$$u_{eq_i} = -\left(\left(\sum_{j=1,j\neq i}^{n} a_{ij}+bi\right)g_i(x_i)\right)^{-1} \times \left(\left(\sum_{j=1,j\neq i}^{n} a_{ij}+bi\right)f_i(x_i)+\left(\left(\sum_{j=1,j\neq i}^{n} a_{ij}+bi\right)(g_i(x_i)-1)u_{ai}\right) - \sum_{j=1,j\neq i}^{n} a_{ij}\Big(f_j(x)+g_j(x)u_j\Big)-b_if_0(x,t)+h_i(x,t)\right) \tag{19}$$

Using the expression (19) in (5), one gets (9). This confirms that the dynamics of an i^{th} agent are governed by the control component u_{a_i} in sliding mode. This proves the theorem.

Remark 5: In practice, all the physical systems are available only with outputs. Therefore, the output feedback control strategy will be needed which will utilize the higher derivatives of the outputs. Thus, these derivatives are estimated via either higher order sliding mode differentiator [35] or high gain observers [36].

V. ILLUSTRATIVE EXAMPLE

In this section, the distributed ISMC protocols are designed for the network of third order leader and followers. The related network topology is shown in FIG.1. Note that all the followers are operated under the effect of matched uncertainties. In the forthcoming study, the descriptions of these systems are given.

A. SYSTEMS DESCRIPTION

The dynamics of the leader and followers are expressed via the following mathematical equations

$$\begin{aligned}
\dot{x}_{01} &= x_{02}\\
\dot{x}_{02} &= x_{03}\\
\dot{x}_{03} &= -x_{02}-2x_{03}+1+3sin(2t)+2cos(2t),\\
\dot{x}_{13} &= x_{12}\sin(x_{11})+\cos^2(x_{13})+(0.1+x_{12}^2)u_1+\xi_1,\\
\dot{x}_{23} &= -x_{21}x_{22}+0.01x_{21}\\
&\quad -0.01x_{21}^2+(1+\sin^2(x_{21}))u_2+\xi_2,\\
\dot{x}_{33} &= x_{32}+\sin(x_{33})+(1+\cos^2(x_{32}))u_3+\xi_3
\end{aligned}$$

and

$$\begin{aligned}
\dot{x}_{43} = {} & -3(x_{41}+x_{42}-1)^2(x_{41}+x_{42}+x_{43}-1)-x_{42}-x_{43}\\
& +0.5\sin(2t)+\cos(2t)+(1+0.5x_{42}^2)u_4+\xi_4.
\end{aligned}$$

Note that, the terms $\xi_i = 0.5sin(3t)$, $i = 1, 2, 3, 4$ represents the matched uncertain terms which affects the input channels. The graph in this study is directed in nature. The adjacency, the Laplacian and the interconnection matrices are expressed as follows

$$A=\begin{bmatrix} 0&0&0&0&0\\ 0&0&1&1&1\\ 0&0&0&1&0\\ 1&0&1&0&0\\ 1&0&0&0&0 \end{bmatrix} \quad \bar{L}=\begin{bmatrix} 3&-1&-1&-1\\ 0&1&-1&0\\ 0&-1&1&0\\ 0&0&0&0 \end{bmatrix}$$

and

$$\bar{B} = diag[0, 0, 1, 1].$$

All the agents are controllable and the main task is the establishment of consensus with the leader's trajectories. This job is done via the following control protocols which are explicitly designed in the previous section.

B. CONTROLLER DESIGN

The proposed design strategy can be employed by defining the consensus errors as follows

$$\begin{aligned}
e_{i1} &= \sum_{j=1}^{4} a_{ij}(x_{i1}-x_{j1})+b_i(x_{i1}-x_{01})\\
e_{i2} &= \sum_{j=1}^{4} a_{ij}(x_{i2}-x_{j2})+b_i(x_{i2}-x_{02})\\
&\vdots\\
e_{i4} &= \sum_{j=1}^{4} a_{ij}(x_{i4}-x_{j4})+b_i(x_{i4}-x_{04})
\end{aligned}$$

The integral manifold for an i^{th} agent is defined as follows

$$\sigma_i = \sum_{j=1}^{3}\lambda_{ij}e_{ij}+z_i$$

where λ_{ij} with $\lambda_{i3} = 1$ are the performance parameters of the control components which establish sliding modes from the initial time instant. The final expressions of the controllers, used in these simulations, are given below.

$$u_i = u_{ai} - \left(\Big(\sum_{j=1,j\neq i}^{4} a_{ij}+b_i\Big)g_i(x_i)\right)^{-1} \times \left(\Big(\sum_{j=1,j\neq i}^{4} a_{ij}+b_i\Big)f_i(x_i)+\left(\left(\sum_{j=1,j\neq i}^{4} a_{ij}+b_i\right)g_i(x_i)-1\right)u_{ai}-\sum_{j=1,j\neq i}^{4} a_{ij}(f_j(x)+g_j(x)u_j)-b_if_0(x,t)+k_i(x,t)sign(\sigma_i)\right) \tag{20}$$

TABLE 1. Parameters of the controllers used in this experiment.

Controller Parameters					
λ_{11}	λ_{12}	λ_{21}	λ_{22}	λ_{31}	λ_{32}
5	2.2	15.2	5.2	25.2	8.2
λ_{41}	λ_{42}	k_{11}	k_{12}	k_{13}	k_{21}
35.2	10.2	8000	4200	4000	7980
k_{22}	k_{23}	k_{31}	k_{32}	k_{33}	k_{41}
5500	4500	7000	6000	5600	8000
k_{42}	k_{43}	K_1	K_2	K_3	K_4
7000	6500	301.1	310.1	500.1	400.1

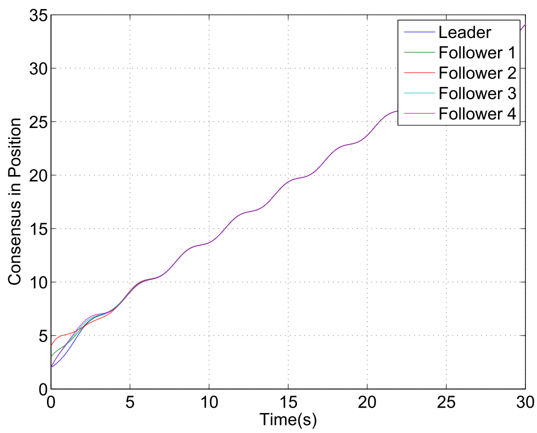

FIGURE 2. Position Tracking of the leader by four followers.

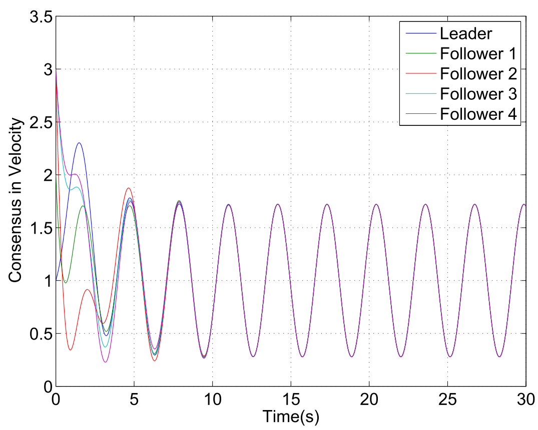

FIGURE 3. Velocity Tracking of the leader by four followers.

where $k_i = [k_{i1}\ k_{i2}\ k_{i3}]$ and $E_i = [e_{i1}\ e_{i2}\ e_{i3}]^T$. The gains of the control protocols, used in this simulation, are reported in TABLE 1.

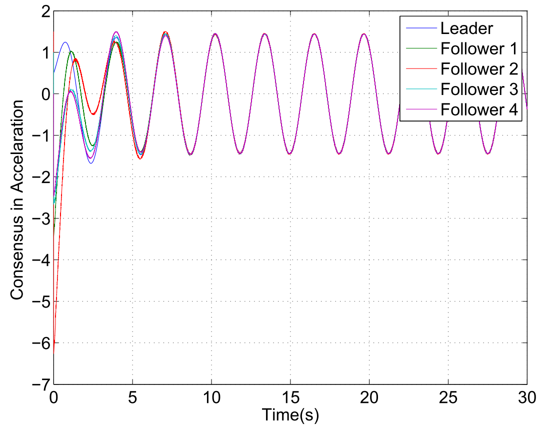

FIGURE 4. Acceleration Tracking of the leader by four followers.

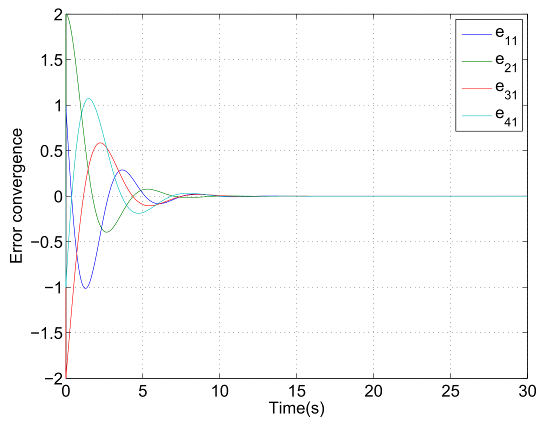

FIGURE 5. Position error convergence of four followers.

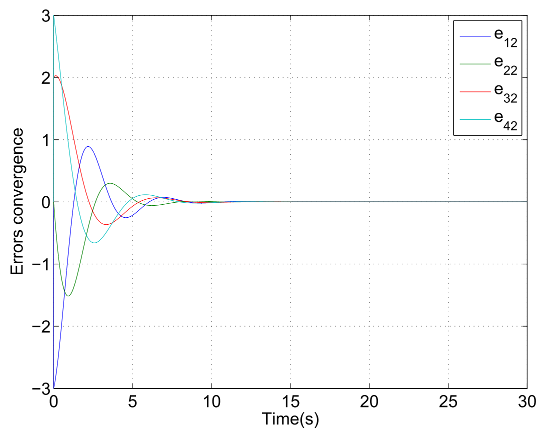

FIGURE 6. Velocity error convergence of four followers.

C. SIMULATION RESULTS

The proposed control protocol (as reported in (18)) are employed to a network of five agents which include one leader and four followers. The control protocol's robustness and performances are examined against the matched disturbances $0.5sin(3t)$ and states dependent uncertainties. The leader's position trajectory is closely followed tracked by the four followers as depicted in FIG. 2. It is evident that in a very short time of almost 3 seconds consensus among the leader and followers is established. Similarly, the consensus in velocities and accelerations of the networked agents are shown in FIG. 3, and FIG. 4, respectively. The convergence of the consensus errors (in positions, velocities and accelerations) are shown in FIG. 5, FIG. 6, and FIG. 7, respectively. The integral manifolds are shown in FIG. 8. The manifolds convergence show that the integral sliding modes are enforced with alleviated chattering of magnitude 0.1. In addition, the sliding surfaces stay at zero from time t_0 which confirmed the theoretical claim made in the control

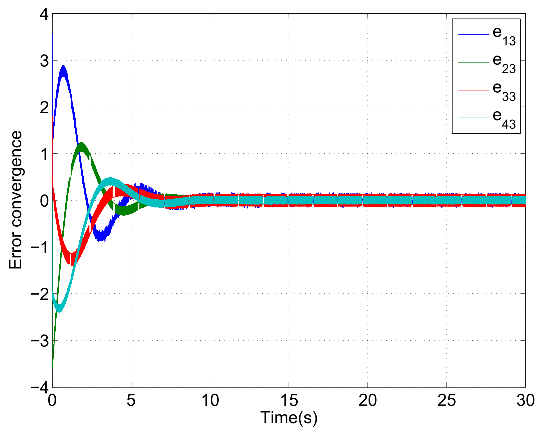

FIGURE 7. Acceleration error convergence of four followers.

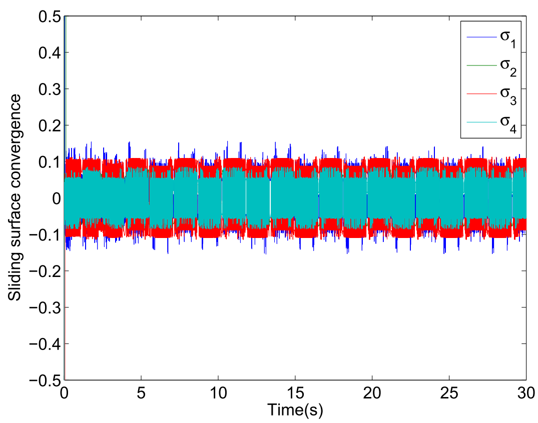

FIGURE 8. Integral manifolds of the four followers.

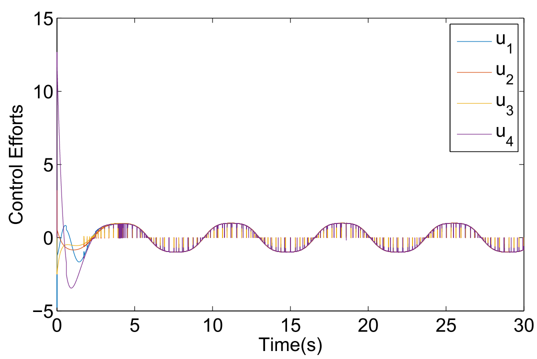

FIGURE 9. Control efforts for the consensus establishment.

design section. This also confirms very robust nature of the proposed ISMCP from the very start of the process. The applied controlled efforts for the consensus establishment among the positions, velocities and accelerations are displayed in FIG. 9. It is evident that the control efforts evolve with suppressed chattering. Having looked at these Figures, one may easily note that the sliding modes are enforced from the very start which ensures invariance to matched disturbances and states depending uncertainties from the very start of the process. In addition, the control inputs exhibits with suppressed chattering and the consensus errors are driven to zero by the linear control components (u_{ai}) asymptotically. On the other hand, the disturbances are rejected via the discontinuous control component u_{bi}. The systems evolve with far appealing benefits like suppressed chattering and enhanced robustness as compared to the conventional sliding mode technique.

VI. CONCLUSION

In this paper, the integral sliding mode control protocol design technique is generalized for a class of networked higher order nonlinear systems. The nonlinear dynamics of the agents are assumed to be in controllable canonical form whereas the leader is assumed to be driven by a nonlinear bounded smooth function. Distributed control protocols, for the consensus among the leader and followers, are designed by first defining proper consensus errors dynamics and then integral manifolds are designed. Discontinuous control laws are designed which ensure the sliding mode enforcement (along the integral manifolds) from the start and the consensus among all the agents is established asymptotically by the control component. The overall closed loop analysis is presented in the form of a comprehensive theorem and the claims (made in theoretical results) are verified by the simulation results for a network of one leader and four followers which communicated with a fixed topology and a directed graph.

In future, these networked agents will be studied with varying topologies, communication delays (in term of time) and data loss which may give birth to mismatch uncertainties.

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