Received May 14, 2019, accepted May 28, 2019, date of publication June 7, 2019, date of current version August 13, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2921720

Finite-Time Adaptive Fuzzy Consensus Stabilization for Unknown Nonlinear Leaderless Multi-Agent Systems With Unknown Output Dead-Zone

HAINAN YANG, YINGZHAN LIAN, XINYU ZHANG, QIUSHI YE, AND XIANNAN CAO
School of Automation, Guangdong University of Technology, Guangzhou 510006, China

Corresponding author: Yingzhan Lian (13925163688@139.com)

This work was supported in part by the Science and Technology Planning Project of Guangdong Province under Project 2016B090912004, in part by the Guangzhou Municipal Science and Technology under Project 201802010045, and in part by the Nansha Science and Technology under Project 2017CX011.

ABSTRACT In this paper, the finite-time adaptive consensus stabilization is investigated for unknown nonlinear leaderless multi-agent systems with unknown output dead-zone. Different from the previous results on the multi-agent systems with unknown dead-zone, the dead-zone nonlinear is researched in the output channel. By using the recursive backstepping design method and the universal approximation ability of fuzzy logic systems, a local controller for each follower is constructed. Meanwhile, a Lyapunov-based logic switching rule is applied to handle the unknown control gain problems aroused by the output dead-zone nonlinearities. It is, thus, shown that the established adaptive control protocol can assure the finite-time state synchronization of each node. Besides, the state synchronization error between any adjacent followers also converges to a small region of zero when time tends to T_0. Finally, the two simulations are conducted to further verify our theoretical results.

INDEX TERMS Multi-agent systems, finite-time stability, synchronization tracking, adaptive backstepping, unknown output dead-zone.

I. INTRODUCTION

During the past several decades, adaptive finite-time stabilization [1]–[4] has received great attentions of the control community, since the system states can achieve a faster convergence rate in real-world applications. However, the weakness of the above result were based on the assumption that the uncertain nonlinearities are either to be linearly parameterized or to satisfy Lipschitz continuous, which limited the development of network control system. Backstepping technique [5] is a powerful tool. At the same time, due to the approximation property of fuzzy logic systems (FLS) and neural networks (NNs), the FLS/NNs-based backstepping is able to deal with the adaptive control problem when the structural information of the nonlinear plant functions is uncertain. Following this approach, very recently, many interesting progresses have been devoted to adaptive finite-time stabilization for a class of uncertain nonlinear systems, e.g., see [6]–[11] and references therein.

On the other hand, the distributed consensus, as a fundamental problem of multi-agent system control, has attracted tremendous attentions in recent years. This is due to the fact that consensus problem has a widespread applications in formation control (unmanned air vehicle, robotic teams) [12]–[15], sensor networks [16], containment control [17] and so forth. The so-called consensus problem demands all the agents in one group can reach an agreement. Following this principle, the progress of consensus control witnesses numerous remarkable results reported [18]–[22]. In addition, the consensus problem is widely used on the topic of stochastic multi-agent systems [23]–[26]. In [24], [25] Ma and Wang *et al.* proposed an excellent consensus algorithm and comprehensive review for a class of stochastic multi-agent systems. Furthermore, the communication protocols play an important role in the study of closed-loop systems [22], [27]–[30]. Wan and Wang *et al.* investigated the state estimation problem of genetic regulatory networks (GRNs) under the Round-Robin (RR) protocol and stochastic communication protocols (SCPs) in [27]–[29], respectively. The research results in [27]–[29]

The associate editor coordinating the review of this manuscript and approving it for publication was Lifeng Ma.

cannot extended to the consensus tracking problem. It is worth mention that all the aforementioned results are limited since they have not considered the influence of dead-zone.

Dead-zone is one of the most important non-smooth nonlinearities for its widespread existence in practical devices, e.g., hydraulic and electronic servo values [31], biomedical systems [32]. It is a common source of destabilizing the closed-loop system and limiting the control performance. In the past years, many research efforts have been devoted to the adaptive compensation of dead-zone [33]–[36]. To name just a few, in [33], by constructing the dead-zone inverse, the effects of the dead-zone are canceled; by decomposing the dead-zone into a combination of linear terms and disturbance-like terms [34]. More recently, as the rapidly development of network control systems, some promising progresses have been reported on the adaptive control for multi-agent systems with dead-zone [37], [38]. In [37], [38], the consensus tracking problem for uncertain nonlinear high-order multi-agent systems with unknown dead-zone input was developed. However, all these results are input dead-zone not the output dead-zone. It is worth to mention that both input dead-zone and output dead-zone may arouse the unknown control gain problem. The adaptive compensation problem of output dead-zone is more difficult that the one of input dead-zone, since the unknown control gain aroused by output dead-zone is time-varying.

In general, the Nussbaum-type function method is a promising tool to handle control directions uncertainties. By employing this design tool originally proposed in [39], the unknown time-varying control gain has been first addressed. Recently, plentiful results have been obtained on the topic [36], [40]–[44]. For example, in order to address the non-identical partially unknown multiple control gains of multi-agent systems, Chen *et al.* [45] proposed a novel family of Nussbaum approach. For multiple input multiple output (MIMO), using Nussbaum gain, Zhao *et al.* [46] realized the tracking regulation of n-dimensional nonlinear systems with unknown control gain. Unfortunately, it is well known that Nussbaum approach only can guarantee that the control error asymptotically reach a region around the origin, which cannot solved the finite-time stabilization for multi-agent systems with unknown output dead-zone. Therefore, there are few results reported on finite-time adaptive consensus stabilization for multi-agent systems with output dead-zone.

Motivated by the mentioned observations above, this study investigates the finite-time adaptive coordination stabilization problem for unknown nonlinear leaderless multi-agent systems, where the unknown dead zone nonlinear exists in the output channel. Compared with the existing consensus works [37], [38], [47]–[49], the main contributions of this paper are listed below.

1) Different from the previous studies on the consensus problems, they [47]–[49] have been investigated the finite-time adaptive control for multi-agent systems, while ignored the effects of the dead-zone. However, in [37], [38], thinking that the dead-zone nonlinear is an important nonlinearity, but never considered the convergence rate of system states in a finite time. Therefore, this study considers the finite-time stability and dead-zone nonlinear in the output mechanism of multi-agent systems simultaneously.
2) Note that the unknown control gain of each agent aroused by the output dead-zone nonlinearities is time-varying. However, the Nussbaum approach cannot addressed the finite-time stabilization for multi-agent systems with output dead-zone because of it asymptotically convergent property. It makes the control design and stability analysis become more complicated. In order to overcome this obstacle, a Lyapunov-based logic switching rule [4], [50], [51] is employed in the finite-time adaptive consensus control design process.

The outline of this paper is as follows. In Section II, the problem statement and the related preliminaries are given. In Section III, the finite-time adaptive fuzzy consensus stabilization control protocol is designed and the finite-time state synchronization stability of closed-loop system is also proofed. And then, a numerical simulation examples are provided to further verify our theoretical results in Section IV. Finally, we summarize the conclusions and our future research direction in Section V.

Notations: The following notations are used throughout this paper. R^n denotes the real n-dimensional space. $R^+_{even} \triangleq \{p \in R : p > 0$ and p denotes a positive even integer$\}$; $R^-_{odd} \triangleq \{q \in R : q < 0$ and q denotes a negative odd integer$\}$; and C^i denotes a set of functions whose ith derivatives are continuous and differentiable.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. PROBLEM FORMULATION

In this paper, the leaderless multi-agent systems contain $k(k \geq 2)$ followers (denoted by 1 to k), where the unknown dead-zone nonlinear exists in the output channel. The model of ith$(i = 1, \ldots, k)$ follower is described as:

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} + f_{i,1}(\overline{x}_{i,1}) \\ \dot{x}_{i,2} &= x_{i,3} + f_{i,2}(\overline{x}_{i,2}) \\ &\vdots \\ \dot{x}_{i,n} &= u_i + f_{i,n}(x_{i,n}) \\ y_i &= D_i(x_{i,1}) \end{aligned} \tag{1}$$

where $\overline{x}_{i,k} = [x_{i,1}, x_{i,2}, \ldots, x_{i,k}] \in \mathbb{R}^k$ denotes the measurable state vectors of the ith follower and $x_{i,n} = [x_{i,1}, x_{i,2}, \ldots, x_{i,n}] \in \mathbb{R}^n$. $f_{i,k}(\overline{x}_{i,k})(k = 1, 2, \ldots, n)$ is an unknown nonlinear function. $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the control input and the output of each subsystem, respectively. The dead-zone $D_i(x_{i,1}) \in \mathbb{R}$ is considered in the output channel of multi-agent systems, which can be formulated by the following form:

$$D_i(x_{i,1}) = \begin{cases} m_{r_{i,k}}(x_{i,1} - v_{r_{i,k}}), & x_{i,1} \geq v_{r_{i,k}} \\ 0, & v_{l_{i,k}} \leq x_{i,1} \leq v_{r_{i,k}} \\ m_{l_{i,k}}(x_{i,1} - v_{l_{i,k}}), & x_{i,1} \leq v_{l_{i,k}} \end{cases} \tag{2}$$

where the unknown parameters $v_{r_{i,k}} > 0$ and $v_{l_{i,k}}$ represent the breakpoints of the output nonlinearity, $m_{r_{i,k}}$ and $m_{l_{i,k}}$ are the slope of the output dead zone.

Remark 1: Compared with the synchronization problem of multi-agent systems with dead-zone in [37], [38], the dead-zone nonlinear in these results is input dead-zone nonlinear (see Fig. 1(a)) not the output dead-zone nonlinear (see Fig. 1(b)). It is worth mentioning that the output dead-zone nonlinear is investigated in the unknown nonlinear multi-agent systems, i.e., the dead-zone $D_i(x_{i,1})$ is considered in the output channel.

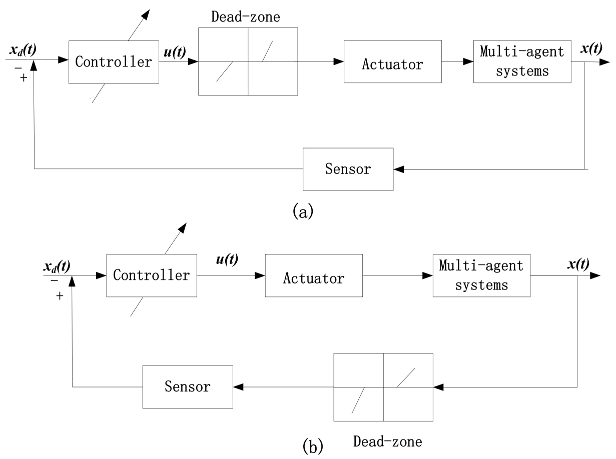


FIGURE 1. The plant with dead-zone.

Remark 2: Due to the effects of manufacturing, the dead-zone parameters may be different among all sensors. That means some signs of the output dead-zone directions are positive, but rest of signs are negative. Therefore, the output dead-zones may arouse the unknown non-identical control gain problems [45]. By the uncertainties of control directions, they will bring great difficulties for the finite-time control protocol design.

Our design objective is to develop a finite-time adaptive concensus control protocol u_i and adaption parameter law $\dot{\hat{\theta}}_i$ for multi-agent systems (1) to guarantee that each follower can faster reach the state synchronization when time tends to T_0, i.e., the following theoretical result holds:

$$\lim_{t\to T_0} |y_i(t)-y_j(t)| = \lim_{t\to T_0} |D_i(x_{i,1})-D_j(x_{j,1})| = 0, \quad \forall t \geq T_0 \tag{3}$$

where i, j represent the subscript of each follower, respectively.

The following assumptions are introduced for achieving our design objective.

Assumption 1: Although the output dead-zone is unknown and time-varying, there exists an unknown but bounded compact set Ω and scalar η such that

$$|\dot{D}_i| \leq \eta, \quad \eta \in \Omega \tag{4}$$

Assumption 2: If a constant satisfies $0.5 < \gamma < 1$ and $\gamma \in R^+_{odd}$, there exists unknown continuous function $\aleph_{i,k}(\bar{x}_{i,k}) \geq 0$ for $i \in M$, such that

$$|f_{i,k}(\bar{x}_{i,k})| \leq |\bar{x}_{i,k}|^{\frac{2\gamma-1}{\gamma}} \aleph_{i,k}(\bar{x}_{i,k}) \tag{5}$$

Remark 3: Assumption 2 is similar to a common Proposition for finite-time adaptive tracking control of nonlinear systems [3], [4], [47]. In practice, many systems satisfy Assumption 2 (e.g., unicycle-type mobile [52]). Therefore, Assumption 2 does not bring too much conservatism in the establishment of the main results.

Before drawing out our main results, the preliminaries about graph theory and some key lemmas and definitions are introduced.

B. GRAPH THEORY

In this paper, the communication topology graph of the k n-order agents is considered, which represented by $G = (V, E)$. $V = \{v_1, \ldots, v_k\}$ denotes a set of nodes or agents and $E \subseteq V \times V$ indicates an edge set. There exists a directed path from every node to others, we believed that the directed graphy is strongly connected. Moreover, $M = [a_{ij}] \in \mathbb{R}^{k\times k}$ is denoted as the connectivity matrix of topology graph, where $a_{ij} > 0$, if $k_i = \{v_j|(v_i, v_j)\} \in E$ and $a_{ij} = 0$, otherwise. It is assumed that each node has no self edge. The in-degree matrix is defined as $D = \mathrm{diag}(d_1, \ldots, d_k) \in \mathbb{R}^{k\times k}$, where $d_i = \sum(a_{ij})$. Then, the graphy Laplacian matrix L is obtained from $L = D - M$.

Assumption 3: In the leaderless multi-agent systems, it is assume that all followers can get information from each other. That means the communication graph topology is undirected.

Assumption 4: For multi-agent systems (1) with unknown output dead-zone. It is note that the graph G is connected, then we suppose that the packet dropouts [53]–[56] will never happen.

Remark 4: The communication protocol is an important issue for data transmissions in the communication channels, which can efficiently avoid the undesired data collision. More recently, several remarkable results have been reported on the adaptive state estimation control for GRNs under the RR protocol and SCPs [27]–[29]. It should be mentioned that these results are based on single system. These protocols will be infeasible when plant is the multi-agent system. To solve this problem, the graph topology [17], [20], [37] has been widely recognized as one of the most promising protocol for the consensus control of nonlinear multi-agent systems.

C. SOME KEY LEMMAS AND DEFINITIONS

Lemma 1 ([57]): For $z, y \in R$, if $0 < p = p_1/p_2 \leq 1$, where $p_1 > 0$ and $p_2 > 0$ are odd integers, the following inequality holds:

$$|z^p - y^p| \leq 2^{1-p}|z - y|^p. \tag{6}$$

Lemma 2 ([58]): If there exists two numbers $e \in (0, +\infty)$, $f \in (0, +\infty)$ and $g(x, y) > 0$ is a real-value function. Then

$$|x|^e|y|^f \leq \frac{e}{e+f}g(x,y)|x|^{e+f} + \frac{f}{e+f}g^{-e/f}(x,y)|y|^{e+f}. \quad (7)$$

Lemma 3 ([59]): For $s_l \in R$, $l \in M$, if $0 < \varpi \leq 1$, then

$$\Big(\sum_{l=1}^{\pi}|s_l|\Big)^{\varpi} \leq \sum_{l=1}^{\pi}|s_l|^{\varpi} \leq \pi^{1-\varpi}\Big(\sum_{l=1}^{\pi}|s_l|\Big)^{\varpi}. \quad (8)$$

Definition 1 ([60]): Given a plant $\dot{\chi} = f(\chi, t)$ and $f(0) = 0$. Assume that there are continuous differentiable function $V(x)$ and constants $c > 0$, $\alpha \in (0, 1)$ such that

$$\begin{aligned}&(1)V(x) > 0\\ &(2)dV(x) + c(V(x))^{\alpha} \leq 0\end{aligned} \quad (9)$$

then $V(x)$ reaches 0 when time tends to T_0. Meanwhile, the finite convergent time T_0 satisfies:

$$T_0 \leq \frac{V(x(0))^{1-\alpha}}{c(1-\alpha)}. \quad (10)$$

Hence, we called that the origin of plant $\dot{\chi} = f(\chi, t)$ is semi-global finite-time stable (SGFTS).

Lemma 4 (Universal Approximation Theorem [61]): For any continuous function $H(x)$ defined on a compact set Ω. If given a constant $\epsilon > 0$, there exists a FLS $\Theta^T S(x)$ such that

$$\sup_{x\in\Omega}|H(x) - \Theta^T S(x)| \leq \epsilon \quad (11)$$

where $\Theta = [\Theta_1, \Theta_2, \ldots, \Theta_L] \in R^L$ denotes the ideal weight vector. $S(x) = [S_1(x), S_2(x), \ldots, S_L(x)]/\sum_{i=1}^{L} S_i(x)$ represents the fuzzy basis function vector and L stands for the number of fuzzy rules with $L > 0$.

Lemma 5 ([3]): There exists real numbers $x_i, i = 1, 2, \ldots, n$ and $g \in (0, 1]$,

$$(|x_1| + \ldots + |x_n|)^g \leq |x_1|^g + \ldots + |x_n|^g \quad (12)$$

III. FINITE-TIME ADAPTIVE FUZZY CONSENSUS STABILIZATION CONTROL PROTOCOL DESIGN

In this subsection, by combining recursive backstepping design method and approximation ability of fuzzy logic systems (Part A) with Lyapunov-based logic switching rule (Part B), the finite-time adaptive consensus stabilization control protocol is established for unknown nonlinear leaderless multi-agent systems (1) with unknown output dead-zone. Then, with the developed control protocol, the output states $y_i (i = 1, 2, \ldots, k)$ of all the follower nodes can be faster reach the finite-time state synchronization. Meanwhile, the state synchronization error between any adjacent followers converges to a small region of zero when time tends to T_0.

For the convenience of design, a constant with the Euclidean norm is defined as:

$$\theta_i = \max\{\| \Theta_{i,m} \|^2; 0 \leq m \leq n\}, \quad (13)$$

where $\Theta_{i,m}$ are the FLS weight vectors. $\theta_i > 0$, since $\| \Theta_{i,m} \|$ is the Euclidean norm. $\hat{\theta}_i$ represents the estimated error of θ_i, where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

A. CONTROL PROTOCOL DESIGN

Step 1. Considered the following coordinate transformation:

$$s_{i,1} = y_i - y_j = D_i(x_{i,1}) - D_j(x_{j,1}), \quad i \in \Psi, j \in N_i \quad (14)$$

where $\Psi = \{1, 2, \ldots, k\}$, $s_{i,1}$ stands for the synchronization error of neighborhood node. Then, we have

$$\begin{aligned}\dot{s}_{i,1} &= \dot{D}_i(x_{i,1})[x_{i,2} + f_{i,1}(x_{i,1})] - \dot{D}_j(x_{j,1})[x_{j,2} + f_{j,1}(x_{j,1})]\\ &\leq \dot{D}_i(x_{i,1})[x_{i,2} + f_{i,1}(x_{i,1})]\end{aligned} \quad (15)$$

Denote $s = [s_{1,1}, \ldots, s_{k,1}]^T$. Construct a Lyapunov function as

$$\begin{aligned}V_1 &= \frac{1}{2}s^T Ls + \sum_{i=1}^{k}\frac{\tilde{\theta}_i^2}{2q_i}\\ &= \frac{1}{4}\sum_{i=1}^{k}\Big[\sum_{j\in N_i} a_{ij}(s_{i,1} - s_{j,1})^2\Big] + \sum_{i=1}^{k}\frac{\tilde{\theta}_i^2}{2q_i}\end{aligned} \quad (16)$$

Based on (15), a direct calculation gives

$$\begin{aligned}\frac{dV_1}{dt} &= s^T L\dot{s} - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i\\ &= \sum_{i=1}^{k}\Big[\sum_{j\in N_i} a_{ij}(s_{i,1} - s_{j,1})\Big]\\ &\quad \times \dot{D}_i(x_{i,2} + f_{i,1}(x_{i,1})) - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i\end{aligned} \quad (17)$$

Set

$$\upsilon_{i,1} = \sum_{j\in N_i} a_{ij}(s_{i,1} - s_{j,1}), \quad i \in \Psi \quad (18)$$

Then

$$\frac{dV_1}{dt} = \sum_{i=1}^{k}\upsilon_{i,1}\dot{D}_i(x_{i,2} + f_{i,1}(x_{i,1})) - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \quad (19)$$

A simple calculation gives

$$\begin{aligned}\frac{dV_1}{dt} &= \sum_{i=1}^{k}\upsilon_{i,1}\dot{D}_i(x_{i,2} + f_{i,1}(x_{i,1})) - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i\\ &= \sum_{i=1}^{k}\Big[\upsilon_{i,1}\dot{D}_i(x_{i,2} - x_{i,2}^*)\Big] + \sum_{i=1}^{k}\upsilon_{i,1}\dot{D}_i x_{i,2}^*\\ &\quad + \sum_{i=1}^{k}\upsilon_{i,1}\dot{D}_i f_{i,1}(x_{i,1}) - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i.\end{aligned} \quad (20)$$

Since the system state function $f_{i,1}(x_{i,1})$ is unknown, it brings great difficulties for practical applications. According to the excellent approximation properties of FLS, then, $f_{i,1}(x_{i,1})$ can approximated by a FLS as

$$f_{i,1} = \Theta_{i,1}^T S_{i,1}(X_{i,1}) + \delta_{i,1}(X_{i,1}), \quad |\delta_{i,1}(X_{i,1})| \leq \varepsilon_{i,1} \quad (21)$$

where $X_{i,1}=x_{i,1}$, $\delta_{i,1}(X_{i,1})$ refers to the accuracy and $\varepsilon_{i,1}$ is a design parameter and $\varepsilon_{i,1}>0$. Using Young's inequality and the Euclidean norm of $\parallel \Theta_{i,m} \parallel^2$, if $a_{i,1}>0$, we have

$$\upsilon_{i,1}f_{i,1} \leq \frac{\theta_i}{2a_{i,1}^2}\upsilon_{i,1}^2 S_{i,1}^T S_{i,1} + \frac{a_{i,1}^2}{2} + \frac{\upsilon_{i,1}^2}{2} + \frac{\varepsilon_{i,1}^2}{2} \tag{22}$$

Using assumption 1, substituting (21) and (22) into (20) produces

$$\begin{aligned} \frac{dV_1}{dt} &= \sum_{i=1}^{k} \upsilon_{i,1}\dot{D}_i(x_{i,2}+f_{i,1}(x_{i,1})) - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \\ &\leq \sum_{i=1}^{k}\Big[\upsilon_{i,1}\dot{D}_i(x_{i,2}-x_{i,2}^*)\Big] + \sum_{i=1}^{k}\upsilon_{i,1}\dot{D}_i x_{i,2}^* \\ &\quad + \sum_{i=1}^{k}\eta(\frac{\theta_i}{2a_{i,1}^2}\upsilon_{i,1}^2 S_{i,1}^T S_{i,1} + \frac{\upsilon_{i,1}^2}{2}) \\ &\quad + k\sigma_{i,1} - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \end{aligned} \tag{23}$$

where $\sigma_{i,1}=\eta(\frac{a_{i,1}^2}{2}+\frac{\varepsilon_{i,1}^2}{2})$.

Select the virtual control protocol for the unknown time-varying dead-zone

$$\begin{aligned} x_{i,2}^* &= -\Gamma_i\upsilon_{i,1}^{r_2}n - \eta\frac{\hat{\theta}_i}{2a_{i,1}^2}\upsilon_{i,1}^2 S_{i,1}^T S_{i,1} - \eta\frac{\upsilon_{i,1}^2}{2} \\ &:= -\Gamma_i\upsilon_{i,1}^{r_2}\beta_1 - \eta\frac{\hat{\theta}_i}{2a_{i,1}^2}\upsilon_{i,1}^2 S_{i,1}^T S_{i,1} - \eta\frac{\upsilon_{i,1}^2}{2} \end{aligned} \tag{24}$$

where $\beta_1=n$, $r_i=1+(i-1)\tau$, $i=1,\ldots,n+1$, $\tau=-R_{even}^{+}/R_{odd}^{-}$. Define $\Gamma_i := \kappa_i\Upsilon(i)$, where the switching functions $\kappa_i\in\{-1,+1\}$ is related to the tunable parameter i and designed later, $\Upsilon(\cdot)\geq 0$ is a function that increases with the parameter i. For the purposes of convenience, we assumed that parameter i is fixed, so κ_i and $\Upsilon(i)$ are also fixed.

Based on virtual control protocol (24), the time derivative of V_1 is described as

$$\begin{aligned} \frac{dV_1}{dt} &\leq -n\sum_{i=1}^{k}\upsilon_{i,1}^{2+\tau} + \sum_{i=1}^{k}[\upsilon_{i,1}\dot{D}_i(x_{i,2}-x_{i,2}^*)] \\ &\quad - \sum_{i=1}^{k}(\dot{D}_i\Gamma_i-1)\upsilon_{i,1}^{1+r_2}\beta_1 \\ &\quad - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i - \eta\frac{q_i}{2a_{i,1}^2}\upsilon_{i,1}^2 S_{i,1}^T S_{i,1}) + k\sigma_{i,1} \end{aligned} \tag{25}$$

Step 2. Define $\upsilon_{i,2}=x_{i,2}^{1/r_2}-{x_{i,2}^*}^{1/r_2}$, $i\in\Psi$, and choose the following Lyapunov function:

$$V_2 = V_1 + \sum_{i=1}^{k}\sum_{z\in N_i} W_z, \qquad z=2,..,n \tag{26}$$

where

$$W_z = \int_{x_{i,z}^*}^{x_{i,z}} \left(e^{1/r_z} - {x_{i,z}^*}^{1/r_z}\right)^{2-r_z} de$$

From the Proposition 1 in [3], we have

$$\begin{aligned} \frac{dV_2}{dt} &\leq -n\sum_{i=1}^{k}\upsilon_{i,1}^{2+\tau} + \sum_{i=1}^{k}\Big[\upsilon_{i,1}\dot{D}_i(x_{i,2}-x_{i,2}^*)\Big] \\ &\quad - \sum_{i=1}^{k}(\dot{D}_i\Gamma_i-1)\upsilon_{i,1}^{1+r_2}\beta_1 - \sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i \\ &\quad - \eta\frac{q_i}{2a_{i,1}^2}\upsilon_{i,1}^2 S_{i,1}^T S_{i,1}) + k\sigma_{i,1} + \sum_{i=1}^{k}\upsilon_{i,2}^{2-r_2}\Big(x_{i,3} \\ &\quad + f_{i,2}(\overline{x}_{i,2})\Big) + (2-r_2)\sum_{i=1}^{k}\frac{\partial\Big[-{x_{i,2}^*}^{1/r_2}\Big]}{\partial x_{i,1}}\dot{x}_{i,1} \\ &\quad \times \int_{x_{i,2}^*}^{x_{i,2}}\left(e^{1/r_2}-{x_{i,2}^*}^{1/r_2}\right)^{1-r_2} de \end{aligned} \tag{27}$$

We will estimate each term of the above inequality (27). First, according to Lemmas 1 and 2, the following result holds:

$$\begin{aligned} |\upsilon_{i,1}\dot{D}_i(x_{i,2}-x_{i,2}^*)| &\leq |\upsilon_{i,1}|\cdot|\eta|\cdot|(x_{i,2}^{1/r_2})^{r_2} - ({x_{i,2}^*}^{1/r_2})^{r_2}| \\ &\leq 2^{1-r_2}|\eta|\cdot|\upsilon_{i,1}|\cdot|\upsilon_{i,2}|^{r_2} \\ &\leq \frac{1}{3}\upsilon_{i,1}^{2+\tau} + \varphi_1\upsilon_{i,2}^{2+\tau} \end{aligned} \tag{28}$$

where constant $\varphi_1>0$.

For the term of unknown function $f_{i,2}(\overline{x}_{i,2})$, from the Assumption 2, there exists C^1 functions $\gamma_{i,m}(\overline{x}_{i,m})\geq 0$, where $2\leq m\leq n$,

$$|f_{i,2}(\overline{x}_{i,2})| \leq |(\upsilon_{i,1}|^{r_2}+|\upsilon_{i,2}|^{r_2})\gamma_{i,2}(\overline{x}_{i,2}) \tag{29}$$

then we have

$$\begin{aligned} |\upsilon_{i,2}^{2-r_2}f_{i,2}| &\leq |\upsilon_{i,2}|^{2-r_2}\Big(\sum_{z=1}^{2}|\upsilon_{i,z}|^{1+2\tau}\Big)\gamma_{i,2}(\overline{x}_{i,2}) \\ &\leq \frac{1}{3}\upsilon_{i,1}^{1+r_2} + \upsilon_{i,2}^{1+r_2}\gamma_{i,2}(\overline{x}_{i,2}) \end{aligned} \tag{30}$$

Due to the complexity of virtual control $x_{i,2}^*$ in (24), that brings great difficulty for our design. In order to overcome this issue, we adopted a power integrator approach as following:

There exists some C^1 functions $E_{m,l}(\overline{x}_{i,m})$, where $2\leq m\leq n$, $1\leq l\leq m-1$, such that

$$\left|\frac{\partial\Big[-{x_{i,2}^*}^{1/r_2}\Big]}{\partial x_{i,1}}\dot{x}_{i,1}\right| \leq \Big(|\upsilon_{i,1}|^{r_2}+|\upsilon_{i,2}|^{r_2}\Big)E_{2,1}(\overline{x}_{i,2}). \tag{31}$$

Additionally, using Lemma 1

$$\begin{aligned} \left|\int_{x_{i,2}^*}^{x_{i,2}}\left(e^{1/r_2}-{x_{i,2}^*}^{1/r_2}\right)^{1-r_2} de\right| &\leq |\upsilon_{i,2}|^{1-r_2}|x_{i,2}-x_{i,2}^*| \\ &\leq 2^{1-r_2}|\upsilon_{i,2}|^{1-r_2}|\upsilon_{i,2}|^{r_2} \\ &\leq 2|\upsilon_{i,2}|. \end{aligned} \tag{32}$$

based on Lemma 2 and above inequality, we have

$$\left|(2-r_2)\frac{\partial\Big[-{x_{i,2}^*}^{1/r_2}\Big]}{\partial x_{i,1}}\dot{x}_{i,1}\int_{x_{i,2}^*}^{x_{i,2}}\left(e^{1/r_2}-{x_{i,2}^*}^{1/r_2}\right)^{1-r_2} de\right|$$

$$\leq 2(2-r_2)|\upsilon_{i,2}|\Big(\sum_{z=1}^{2}|\upsilon_{i,z}|^{r_2}\Big)E_{2,1}(\overline{x}_{i,2})$$
$$\leq \frac{1}{3}\upsilon_{i,1}^{1+r_2}+\upsilon_{i,2}^{1+r_2}h_{i,2}(\overline{x}_{i,2}) \tag{33}$$

Substituting (28), (30) and (33) into (27) results into

$$\begin{aligned}\frac{dV_2}{dt} \leq& -(n-1)\sum_{i=1}^{k}\upsilon_{i,1}^{2+\tau}-\sum_{i=1}^{k}(\dot{D}_i\Gamma_i-1)\upsilon_{i,1}^{1+r_2}\beta_1\\ &-\sum_{i=1}^{k}\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i-\eta\frac{q_i}{2a_{i,1}^2}\upsilon_{i,1}^2S_{i,1}^TS_{i,1})+k\sigma_{i,1}\\ &+\sum_{i=1}^{k}\upsilon_{i,2}^{2-r_2}x_{i,3}+\sum_{i=1}^{k}\upsilon_{i,2}^{2+\tau}\Big[\varphi_1+(\gamma_2(\overline{x}_{i,2})\\ &+h_{i,2}(\overline{x}_{i,2}))\Big].\end{aligned} \tag{34}$$

Let $f'_{i,2}(\overline{x}_{i,2})=\gamma_{i,2}(\overline{x}_{i,2})+h_{i,2}(\overline{x}_{i,2})$. Meanwhile, due to $2+\tau=1+r_2$, set $\upsilon_{i,2}^{2+\tau}\leq\upsilon_{i,2}$. Based on the FLS, the unknown function $f'_{i,2}(\overline{x}_{i,2})$ can be expressed as

$$f'_{i,2}=\Theta_{i,2}^TS_{i,2}(X_{i,2})+\delta_{i,2}(X_{i,2}),\quad |\delta_{i,2}(X_{i,2})|\leq\varepsilon_{i,2} \tag{35}$$

where $X_{i,2}=[\overline{x}_{i,2},\overline{v}_{i,2}]^T$, note that the definition of $\overline{v}_{i,2}$ is similar to $\overline{x}_{i,2}$. $\delta_{i,2}(X_{i,2})$ refers to the approximation error and $\varepsilon_{i,2}$ is a design parameter and $\varepsilon_{i,2}>0$. By Young's inequality, if given a $a_{i,2}>0$, we have

$$\upsilon_{i,2}f'_{i,2}\leq\eta(\frac{\theta_i}{2a_{i,2}^2}\upsilon_{i,2}^2S_{i,2}^TS_{i,2}+\frac{a_{i,2}^2}{2}+\frac{\upsilon_{i,2}^2}{2}+\frac{\varepsilon_{i,2}^2}{2}) \tag{36}$$

A virtual controller is selected as

$$\begin{aligned}x_{i,3}^* &= -\upsilon_{i,2}^{r_3}[n-1+\varphi_1]-\eta\frac{\hat{\theta}_i}{2a_{i,2}^2}\upsilon_{i,2}^2S_{i,2}^TS_{i,2}-\eta\frac{\upsilon_{i,2}^2}{2}\\ &:= -\upsilon_{i,2}^{r_3}\beta_2-\eta\frac{\hat{\theta}_i}{2a_{i,2}^2}\upsilon_{i,2}^2S_{i,2}^TS_{i,2}-\eta\frac{\upsilon_{i,2}^2}{2}\end{aligned} \tag{37}$$

which satisfies that

$$\begin{aligned}\frac{dV_2}{dt} \leq& -(n-1)\sum_{i=1}^{k}\Big(\upsilon_{i,1}^{2+\tau}+\upsilon_{i,2}^{2+\tau}\Big)\\ &-\sum_{i=1}^{k}(\dot{D}_i\Gamma_i-1)\upsilon_{i,1}^{1+r_2}\beta_1\\ &+\sum_{i=1}^{k}\upsilon_{i,2}^{2-r_2}[x_{i,3}-x_{i,3}^*]-\sum_{i=1}^{k}\Big(\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i\\ &-\sum_{z=1}^{2}\eta\frac{q_i}{2a_{i,z}^2}\upsilon_{i,z}^2S_{i,z}^TS_{i,z})\Big)+k\sigma_{i,2}\end{aligned} \tag{38}$$

where $\sigma_{i,2}=\sigma_{i,1}+\eta(\frac{a_{i,2}^2}{2}+\frac{\varepsilon_{i,2}^2}{2})$.

Inductive Step: For $i=1,\ldots,k$, according to the definition of virtual controller, such as $\upsilon_{i,2}=x_{i,2}^{1/r_2}+x_{i,2}^{*\,1/r_2}$. At step $m-1$, assume that there exists a set of positive constants $\beta_1,\ldots,\beta_{m-1}$, and a C^1 Lyapunov function

$$\begin{aligned}V_{m-1} =& V_1+\sum_{i=1}^{k}\int_{x_{i,2}^*}^{x_{i,2}}\Big(e^{1/r_2}-x_{i,2}^{*\,1/r_2}\Big)^{2-r_2}de\\ &+\cdots+\sum_{i=1}^{k}\int_{x_{i,m-1}^*}^{x_{i,m-1}}\Big(e^{1/r_{m-1}}\\ &-x_{i,m-1}^{*\,1/r_{m-1}}\Big)^{2-r_{m-1}}de\end{aligned} \tag{39}$$

such that

$$\begin{aligned}\frac{dV_{m-1}}{dt} =& -(n-m+2)\sum_{i=1}^{k}\Big(\upsilon_{i,1}^{2+\tau}+\upsilon_{i,2}^{2+\tau}+\ldots\\ &+\upsilon_{i,m-1}^{2+\tau}\Big)-\sum_{i=1}^{k}(\dot{D}_i\Gamma_i-1)\upsilon_{i,1}^{1+r_2}\beta_1\\ &+\sum_{i=1}^{k}\upsilon_{i,m-1}^{2-r_{m-1}}[x_{i,m}-x_{i,m}^*]+k\sigma_{i,m-1}\\ &-\sum_{i=1}^{k}\Big(\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i-\sum_{z=1}^{m-1}\eta\frac{q_i}{2a_{i,z}^2}\upsilon_{i,z}^2S_{i,z}^TS_{i,z})\Big)\end{aligned} \tag{40}$$

where $\sigma_{i,m-1}=\sigma_{i,m-2}+\eta(\frac{a_{i,m-1}^2}{2}+\frac{\varepsilon_{i,m-1}^2}{2})$.

Next, we will proven the above inequality (40) also holds at step m. Select Lyapunov function as

$$V_m=V_{m-1}+\sum_{i=1}^{k}\int_{x_{i,m}^*}^{x_{i,m}}\Big(e^{1/r_m}-x_{i,m}^{*\,1/r_m}\Big)^{2-r_m}de \tag{41}$$

By (41), along system (1), we get

$$\begin{aligned}\frac{dV_m}{dt} \leq& -(n-m+2)\sum_{i=1}^{k}\Big(\upsilon_{i,1}^{2+\tau}+\upsilon_{i,2}^{2+\tau}+\ldots\\ &+\upsilon_{i,m-1}^{2+\tau}\Big)+\sum_{i=1}^{k}\upsilon_{i,m-1}^{2-r_{m-1}}[x_{i,m}-x_{i,m}^*]\\ &-\sum_{i=1}^{k}\Big(\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i-\sum_{z=1}^{m-1}\eta\frac{q_i}{2a_{i,z}^2}\upsilon_{i,z}^2S_{i,z}^TS_{i,z})\Big)\\ &+(2-r_m)\sum_{i=1}^{k}\sum_{l=1}^{m-1}\frac{\partial[-x_{i,m}^{*1/r_m}]}{\partial x_{i,l}}\dot{x}_{i,l}\int_{x_{i,m}^*}^{x_{i,m}}\Big(e^{1/r_m}\\ &-x_{i,m}^{*\,1/r_m}\Big)^{2-r_m}de-\sum_{i=1}^{k}(\dot{D}_i\Gamma_i-1)\upsilon_{i,1}^{1+r_2}\beta_1\\ &+k\sigma_{i,m-1}\end{aligned} \tag{42}$$

Similar to (28), it can be found that

$$\begin{aligned}|\upsilon_{i,m-1}^{2-r_{m-1}}(x_{i,m}-x_{i,m}^*)| \leq& 2^{1-r_m}|\upsilon_{i,m-1}|^{2-r_{m-1}}|\upsilon_{i,m}|^{r_m}\\ \leq& \frac{1}{3}\upsilon_{i,m-1}^{2+\tau}+\varphi_m\upsilon_{i,m}^{2+\tau}\end{aligned} \tag{43}$$

where φ_m stands for a constant and $\varphi_m\geq0$.

The computation of term $(\partial[-x_{i,m}^{*1/r_m}]/\partial x_{i,l})\dot{x}_{i,l}$ in (42) is similar to (33), and given as follows:

$$\begin{aligned}&\Bigg|(2-r_m)\frac{\partial\left[-x_{i,m}^{*\ 1/r_m}\right]}{\partial x_{i,l}}\dot{x}_{i,l}\int_{x_{i,m}^*}^{x_{i,m}}\Big(e^{1/r_m}\\&\quad -x_{i,m}^{*\ 1/r_m}\Big)^{1-r_m}de\Bigg|\\&\leq 2(2-r_m)|\upsilon_{i,m}|\Big(\sum_{z=1}^{m}|\upsilon_{i,z}|^{r_m}\Big)E_{m,l}(\overline{x}_{i,m})\\&\leq \frac{1}{3}\sum_{z=1}^{m-1}\upsilon_{i,z}^{1+r_2}+\upsilon_{i,m}^{1+r_2}h_{i,m}(\overline{x}_{i,m})\end{aligned}\tag{44}$$

Similarly, substituting (43) and (44) into (42), one has

$$\begin{aligned}\frac{dV_m}{dt}&\leq -(n-m+2)\sum_{i=1}^{k}\Big(\upsilon_{i,1}^{2+\tau}+\upsilon_{i,2}^{2+\tau}+\ldots\\&\quad+\upsilon_{i,m-1}^{2+\tau}\Big)+\sum_{i=1}^{k}\upsilon_{i,m-1}^{2-r_{m-1}}[x_{i,m}-x_{i,m}^*]\\&\quad-\sum_{i=1}^{k}\Big(\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i-\sum_{z=1}^{m-1}\eta\frac{q_i}{2a_{i,z}^2}\upsilon_{i,z}^2S_{i,z}^TS_{i,z})\Big)\\&\quad+\sum_{i=1}^{k}\upsilon_{i,m}^{2-r_m}x_{i,m+1}+\sum_{i=1}^{k}\upsilon_{i,2}^{2+\tau}\Big[\varphi_1+\ldots+\varphi_m\\&\quad+(\gamma_{i,m}(\overline{x}_{i,m})+h_{i,m}(\overline{x}_{i,m}))\Big]+k\sigma_{i,m}.\end{aligned}\tag{45}$$

According to $f_{i,m}'(\overline{x}_{i,m})=\gamma_{i,m}(\overline{x}_{i,2})+h_{i,m}(\overline{x}_{i,m})$. Meanwhile, due to $2+\tau=1+r_2$, set $\upsilon_{i,m}^{2+\tau}\leq\upsilon_{i,m}$. Based on the FLS, the unknown function $f_{i,m}'(\overline{x}_{i,m})$ can be approximated by a FLS as

$$f_{i,m}'=\Theta_{i,m}^TS_{i,m}(X_{i,m})+\delta_{i,m}(X_{i,m}),\quad |\delta_{i,m}(X_{i,m})|\leq\varepsilon_{i,m}\tag{46}$$

where $X_{i,m}=[\overline{x}_{i,m},\overline{v}_{i,m}]^T$, $\delta_{i,m}(X_{i,m})$ refers to the approximation error and $\varepsilon_{i,m}$ is a design parameter and $\varepsilon_{i,m}>0$. By Young's inequality, if given a $a_{i,m}>0$, we have

$$\upsilon_{i,m}f_{i,m}'\leq\eta(\frac{\theta_i}{2a_{i,m}^2}\upsilon_{i,m}^2S_{i,m}^TS_{i,m}+\frac{a_{i,m}^2}{2}+\frac{\upsilon_{i,m}^2}{2}+\frac{\varepsilon_{i,m}^2}{2})\tag{47}$$

Clearly, chosen a virtual control protocol as

$$\begin{aligned}x_{i,m+1}^*&=-\upsilon_{i,m}^{r_{m+1}}[n-m+1+\varphi_m]\\&\quad-\eta\frac{\hat{\theta}_i}{2a_{i,m}^2}\upsilon_{i,m}^2S_{i,m}^TS_{i,m}-\eta\frac{\upsilon_{i,m}^2}{2}\\&:=-\upsilon_{i,m}^{r_{m+1}}\beta_m-\eta\frac{\hat{\theta}_i}{2a_{i,m}^2}\upsilon_{i,m}^2S_{i,m}^TS_{i,m}-\eta\frac{\upsilon_{i,m}^2}{2}\end{aligned}\tag{48}$$

then

$$\begin{aligned}\frac{dV_m}{dt}&\leq-(n-m+1)\sum_{i=1}^{k}\Big(\upsilon_{i,1}^{2+\tau}+\upsilon_{i,2}^{2+\tau}+\ldots\\&\quad+\upsilon_{i,m}^{2+\tau}\Big)-\sum_{i=1}^{k}(\dot{D}_i\Gamma_i-1)\upsilon_{i,1}^{1+r_2}\beta_1\\&\quad+\sum_{i=1}^{k}\upsilon_{i,m}^{2-r_m}[x_{i,m+1}-x_{i,m+1}^*]+k\sigma_{i,m}\\&\quad-\sum_{i=1}^{k}\Big(\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i-\sum_{z=1}^{m}\eta\frac{q_i}{2a_{i,z}^2}\upsilon_{i,z}^2S_{i,z}^TS_{i,z})\Big)\end{aligned}\tag{49}$$

where $\sigma_{i,m}=\sigma_{i,m-1}+\eta(\frac{a_{i,m}^2}{2}+\frac{\varepsilon_{i,m}^2}{2})$.

Similarly, there exists constant gain β_n, when step m equal to step n, the following result holds:

$$\begin{aligned}\frac{dV_n}{dt}&\leq-\sum_{i=1}^{k}\Big(\upsilon_{i,1}^{2+\tau}+\ldots+\upsilon_{i,n}^{2+\tau}\Big)-\sum_{i=1}^{k}(\dot{D}_i\Gamma_i\\&\quad-1)\upsilon_{i,1}^{1+r_2}\beta_1+\sum_{i=1}^{k}\upsilon_{i,n}^{2-r_n}[x_{i,n+1}-x_{i,n+1}^*]\\&\quad-\sum_{i=1}^{k}\Big(\frac{1}{q_i}\tilde{\theta}_i(\dot{\hat{\theta}}_i-\sum_{z=1}^{n}\eta\frac{q_i}{2a_{i,z}^2}\upsilon_{i,z}^2S_{i,z}^TS_{i,z})\Big)+k\sigma_{i,n}\end{aligned}\tag{50}$$

where $\sigma_{i,n}=\sigma_{i,n-1}+\eta(\frac{a_{i,n}^2}{2}+\frac{\varepsilon_{i,n}^2}{2})$.

Select the virtual control protocol for multi-agent systems

$$x_{i,n+1}^*:=-\upsilon_{i,n}^{r_{n+1}}\beta_n-\eta\frac{\hat{\theta}_i}{2a_{i,n}^2}\upsilon_{i,n}^2S_{i,n}^TS_{i,n}-\eta\frac{\upsilon_{i,n}^2}{2}\tag{51}$$

Let

$$\begin{aligned}u_i&=x_{i,n+1}^*\\&:=-\upsilon_{i,n}^{r_{n+1}}\beta_n-\eta\frac{\hat{\theta}_i}{2a_{i,n}^2}\upsilon_{i,n}^2S_{i,n}^TS_{i,n}-\eta\frac{\upsilon_{i,n}^2}{2}\end{aligned}\tag{52}$$

Based on the above derivation, then the actual control protocol as follows:

$$\begin{aligned}u_i&=-\beta_n\upsilon_{i,n}^{r_{n+1}}\\&=-\beta_n\Big[x_{i,n}^{\frac{1}{r_n}}+\eta\frac{\hat{\theta}_i}{2a_{i,n-1}^2}\upsilon_{i,n-1}^2S_{i,n-1}^TS_{i,n-1}+\eta\frac{\upsilon_{i,n-1}^2}{2}\\&\quad+\beta_{n-1}^{\frac{1}{r_n}}\Big[x_{i,n-1}^{\frac{1}{r_{n-1}}}+\ldots+\Gamma_i^{\frac{1}{r_2}}\beta_1^{\frac{1}{r_2}}\Big[\sum_{j\in N_i}a_{ij}(D_i(x_{i,1})\\&\quad-D_j(x_{j,1}))\Big]\Big]\Big]^{r_n+\tau}-\eta\frac{\hat{\theta}_i}{2a_{i,n}^2}\upsilon_{i,n}^2S_{i,n}^TS_{i,n}-\eta\frac{\upsilon_{i,n}^2}{2}.\end{aligned}\tag{53}$$

Meanwhile, the adaption parameter law $\dot{\hat{\theta}}_i$ is selected as

$$\dot{\hat{\theta}}_i=\sum_{z=1}^{n}\eta\frac{q_i}{2a_{i,z}^2}\upsilon_{i,z}^2S_{i,z}^TS_{i,z}-z_{i,0}\hat{\theta}_i\tag{54}$$

where $z_{i,0}\geq 0$.

Substituting (51), (53) and (54) into (50), then

$$\frac{dV_n}{dt}\leq-\sum_{i=1}^{k}\Big(\upsilon_{i,1}^{2+\tau}+\upsilon_{i,2}^{2+\tau}+\ldots+\upsilon_{i,n}^{2+\tau}\Big)\tag{55}$$

where

$$V_n = V_1 + \sum_{i=1}^{k}\Bigg(\int_{x_{i,2}^*}^{x_{i,2}}\left(e^{1/r_2} - {x_{i,2}^*}^{1/r_2}\right)^{2-r_2} de + \cdots + \int_{x_{i,n}^*}^{x_{i,n}}\left(e^{1/r_n} - {x_{i,n}^*}^{1/r_n}\right)^{2-r_n} de\Bigg). \quad (56)$$

Next, we will analyze the ultimate boundedness of Lyapunov function candidate V_n. Based on $s = [s_{i,1}, \ldots, s_{k,1}]^T$, one has

$$\sum_{i=1}^{k}\upsilon_{i,1}^2 = (L^{1/2}s)^T L(L^{1/2})s \geq \hat{\varrho}_1 s^T Ls \quad (57)$$

where $\hat{\varrho}_1$ is a positive constant. Moreover, using Lemma 1, for $2 \leq l \leq n$, we can obtained

$$\left|\int_{x_{i,l}^*}^{x_{i,l}}\left(e^{1/r_l} - {x_{i,l}^*}^{1/r_l}\right)^{2-r_l} de\right| \leq |\upsilon_{i,l}|^{2-r_l}|x_{i,l} - x_{i,l}^*| \leq 2^{2-r_l}|\upsilon_{i,l}|^{2-r_l}|\upsilon_{i,l}|^{r_l} = 2^{2-r_l}\upsilon_{i,l}^2. \quad (58)$$

Substituting (57) and (58) into (56), the following relation holds:

$$V_n \leq \omega\sum_{i=1}^{k}(\upsilon_{i,1}^2 + \upsilon_{i,2}^2 + \ldots + \upsilon_{i,n}^2) \quad (59)$$

where ω is a positive constant. Combined (55) with the above analysis, it is easy to see that

$$\frac{dV_n}{dt} \leq -\omega^{-\frac{2+\tau}{2}} V_n^{\frac{2+\tau}{2}} - \sum_{i=1}^{k}(\dot{D}_i\Gamma_{i,1} - 1)\upsilon_{i,1}^{1+r_2}\beta_1 \quad (60)$$

B. LYAPUNOV-BASED LOGIC SWITCHING LAW

In order to overcome the unknown control gain arising from unknown output dead-zone and analyze the finite-time consensus stability of such systems (1), where the unknown dead-zone exists in the output channel of multi-agent. Based on the parameter i is adjustable, hence, we provided the following logic switching rule to overcome the obstacle aroused by output dead-zone.

$$s_0 = \begin{cases} i, & \text{if } V_1(x(t)) \leq \hbar(t, t_i, x_{i,m}(t_i), \pi),\ t \geq t_i \\ i+1, & \text{otherwise} \end{cases} \quad (61)$$

where π stands for a positive constant. $V_1(x(t))$ is derived from (7), meanwhile $\hbar(t, t_i, x_{i,m}(t_i), \pi) = [(V_1(x(t_i)) + \pi)^{1-\zeta} - 0.25(1-\zeta)(t - t_i)]^{[1/(1-\zeta)]}$ for $t_i \leq t \leq t_i + [4(V_1(x(t_i)) + \pi)^{1-\zeta}/(1-\zeta)]$, $\hbar(t, t_i, x_{i,m}(t_i), \pi) = 0$, otherwise. The set of the switching time is defined by:

$$t_{i+1} := \inf\{t | t > t_i, V_1(x(t)) > \hbar(t, t_i, x_{i,m}(t_i), \pi)\}. \quad (62)$$

However, for the output dead-zone of each subsystem, we can see that the signs of $\dot{D}_i$ have two cases, i.e., either positive (represented by $+1$) or negative (represented by -1). The multi-agent systems (1) have k subsystems. Therefore, the signs of the dead-zone $\dot{D}_i$ have 2^k possible cases which can be explicitly considered. Generally, represented by $\kappa^{(0)}, \kappa^{(1)}, \ldots, \kappa^{(2^k-1)}$. Finally, for $t \in [t_0, +\infty)$, $i = 0, 1, 2, \ldots$, the switching vector is established as

$$\kappa(t) = \begin{cases} \Gamma(i)\kappa^{(0)}, & \text{if } i = \vartheta \cdot 2^k \\ \Gamma(i)\kappa^{(1)}, & \text{if } i = \vartheta \cdot 2^k + 1 \\ \vdots & \vdots \\ \Gamma(i)\kappa^{(2^k-1)}, & \text{if } i = \vartheta \cdot 2^k + 2^k - 1 \end{cases} \quad (63)$$

where the number of period $\vartheta := (i/2^k)$, and $i = 0, 1, 2, \ldots$

According to the above backstepping design procedure, the main conclusions of this paper are summarized as follows:

Theorem 1 (Finite-Time Adaptive Fuzzy Consensus Stabilization Regulation Protocol): For unknown nonlinear leaderless multi-agent systems (1) with unknown output dead-zone, if the control protocol (53), parameter adaptive law (54), logic switching rule (61) and candidate switching vector (63) are selected, then there exists a time constant T_0, such that the control objective is achieved for state synchronization of each follower when time tends to T_0. Meanwhile, the state synchronization error of each follower converges to a small region of zero, i.e., $\lim_{t\to T_0}|y_i(t) - y_j(t)| = \lim_{t\to T_0}|D_i(x_{i,1}) - D_j(x_{j,1})| = 0, \forall t \geq T_0$. We also can called that the origin of multi-agent systems (1) is semi-global finite-time stable (SGFTS).

Remark 5: The conventional consensus methods about unknown control directions are based on Nussbaum-type function. By combining backstepping and Nussbaum-type gains, [45], [62] successfully addressed the adaptive consensus problem for multi-agent systems with unknown control directions. It is well known that the Nussbaum-type gains are conservative. Therefore, these results will be infeasible in the study of finite-time adaptive consensus stabilization.

Remark 6: Different from Nussbaum-type approaches, Lyapunov-based logic switching approaches have the advantage to reduce the conservativeness to analyze and synthesize closed-loop systems under the unknown control direction. More recently, several similar results have been reported on the finite-time adaptive control for nonlinear plants with unknown control directions [4], [50], [51]. In contrast with these results, the main advantages of our scheme can be listed as: i) we have fully considered the effects of the dead-zone in the output channel; and ii) we do not need to assume the structural parameters are nonlinearly parameterized, and hence, less restriction on the systems is required.

IV. SIMULATION RESULTS

In this section, two simulation examples are conducted to further verify our theoretical results.

Example 1: Consider a five-agent systems with unknown output dead-zones, the dynamic of each follower is modeled

TABLE 1. Parameters of dead-zone in Example 1.

	$m_{l_{i,k}}$	$m_{r_{i,k}}$	$v_{l_{i,k}}$	$v_{r_{i,k}}$
i=1	1	1.5	0.25	0.4
i=2	1.25	2	0.15	0.3
i=3	1.35	1.78	0.3	0.25
i=4	2	1	0.1	0.2
i=5	1	1	0	0.1

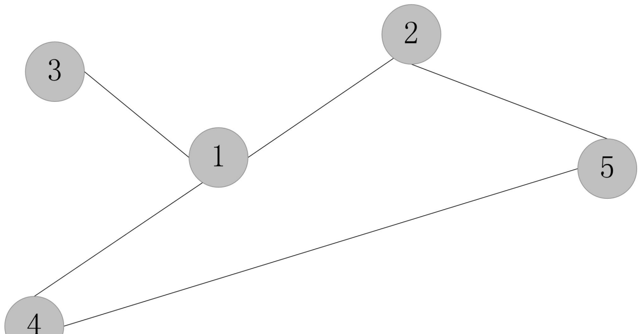


FIGURE 2. Information exchange topology of Example 1.

as follows:

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2} + f_{i,1}(\overline{x}_{i,1}),\\ \dot{x}_{i,2} &= x_{i,3} + f_{i,2}(\overline{x}_{i,2}),\\ \dot{x}_{i,3} &= u_i + f_{i,3}(x_{i,3}),\\ y_i &= D_i(x_{i,1}).\end{aligned} \tag{64}$$

where $f_{i,1}(\overline{x}_{i,1}) = \cos(x_{i,1}) + x_{i,1}/(1 + x_{i,1}^2)$, $f_{i,2}(\overline{x}_{i,2}) = (1/x_{i,1})\sin(x_{i,2})$ and $f_{i,3}(x_{i,3}) = x_{i,1}^2 x_{i,2} x_{i,3}$ are unknown nonlinear functions. The parameters of output dead-zones are listed in Table 1.

In order to verify the validity of Theorem 1, five followers are initialized as: $x_1(0) = [2, 0, -2]^T$, $x_2(0) = [-2.6, 0, 1.1]^T$, $x_3(0) = [5, -1, -1]^T$, $x_4(0) = [-4, 0, 1]^T$, $x_5(0) = [1, 1, -2]^T$. The adjacency weighted of communication graph topology (see Fig. 2) are chosen to be $d_1 = [0.01, 0.1, 0.09, 0.04, 0.05]^T$. The control design parameter are set as $a_{1,1} = a_{1,2} = 2$, $a_{1,3} = 3$, $a_{2,1} = 4$, $a_{2,2} = a_{2,3} = 2$, $a_{3,1} = 2$, $a_{3,2} = a_{3,3} = 4$, $a_{4,1} = 2$, $a_{4,2} = 3$, $a_{4,3} = 4$, $a_{5,1} = 2$, $a_{5,2} = 3$, $a_{5,3} = 1$, $q_i = 200$, $z_{i,0} = 10$, $\hat{\theta}_i = [0.2, 0.2, 0.2, 0.2, 0.2]^T$, $i = 1, \ldots, 5$, $\eta = 2$. Meanwhile, the gains of control protocol u_i are selected as $\beta_1 = 3$, $\beta_2 = 2$, $\beta_3 = 4$ and $\tau = -10/11$.

The simulation results are shown in Figs. 3-8. Based on Fig. 3, it is clear that although the output dead-zone nonlinearities exist in the multi-agent systems, the proposed control protocol can guarantee that the finite-time state synchronization of closed-loop system. Besides, from Fig. 7, it is found that the synchronization performance between any adjacent followers is also achieved when time tends to T_0. Therefore, the effectiveness our theoretical results is verified.

Example 2: In the second example, a simulation is conducted for the practical multi robotic manipulator systems

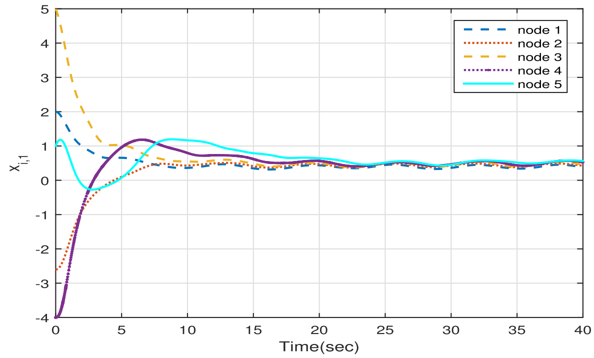


FIGURE 3. State $x_{i,1}$ of Example 1.

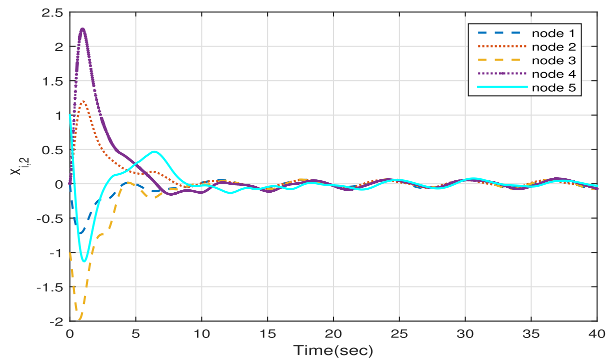


FIGURE 4. States $x_{i,2}$ of Example 1.

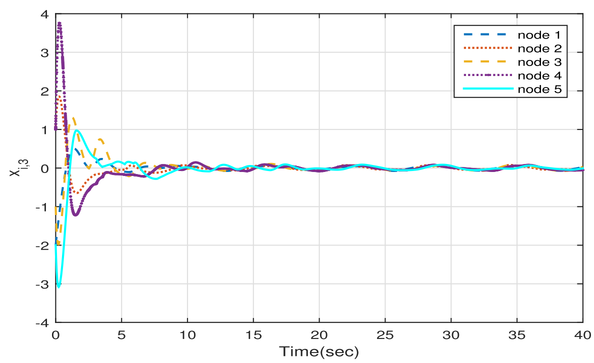


FIGURE 5. States $x_{i,3}$ of Example 1.

with unknown output dead-zone. The dynamic of such systems is described as follows:

$$\begin{aligned}M_i\ddot{q}_i + B_i\dot{q}_i + N_i\sin(q_i) &= U_i\\ y_i &= D_i(q_i)\end{aligned} \tag{65}$$

where q denotes the angle, while $\dot{q}$, and $\ddot{q}$ represent the velocity and acceleration of the robotic manipulator, respectively. U_i is the input electromechanical torque. $M_i = 1$ N·m denotes the moment of inertia, $B_i = 1$ N(ms)/rad denotes the viscous friction coefficient, $N_i = 10$ represents a positive constant related to the product of mass and gravity acceleration. We believe that there exists output dead-zone in the output channel of the robotic manipulator. For the

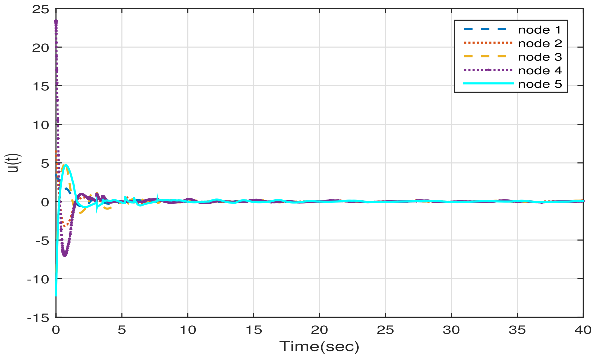


FIGURE 6. The control protocol u_i of Example 1.

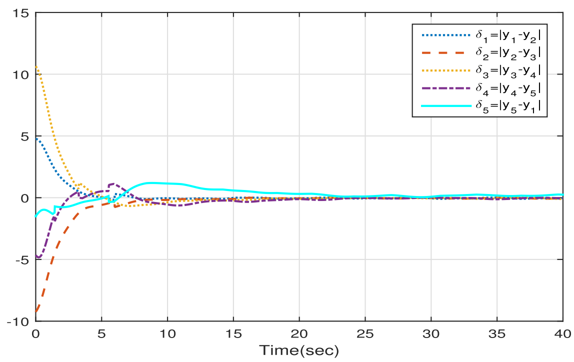


FIGURE 7. The synchronization error δ_i of Example 1.

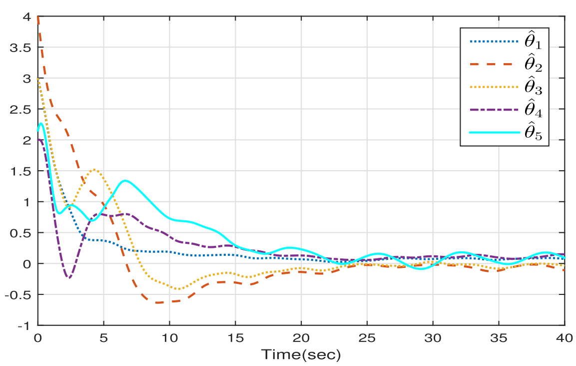


FIGURE 8. The parameter adaptive law $\hat{\theta}_i$ of Example 1.

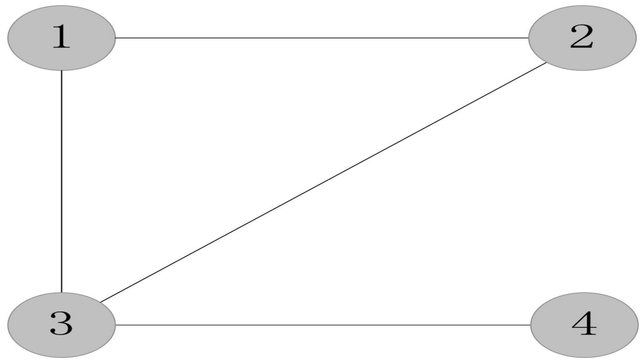


FIGURE 9. Information exchange topology of Example 2.

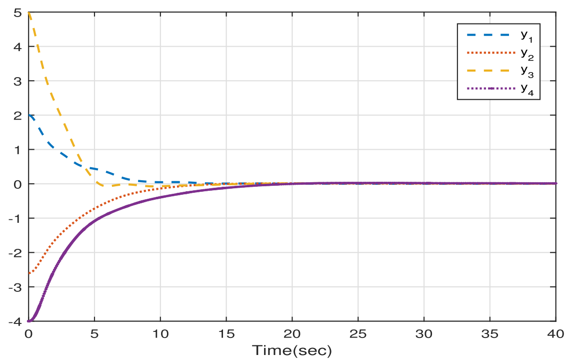


FIGURE 10. The output trajectories of Example 2.

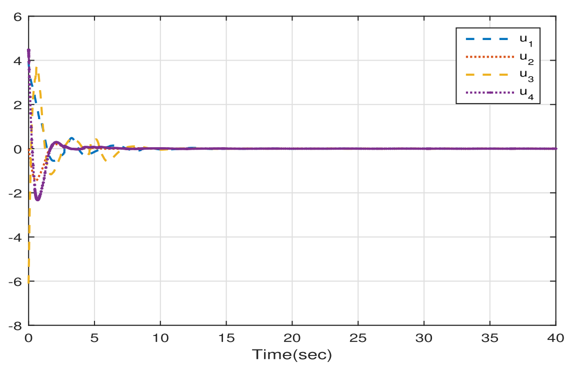


FIGURE 11. The control protocol u_i of Example 2.

convenience of analysis, by setting $x_{i,1} = q_i$ and $x_{i,2} = \dot{q}_i$. Then, the dynamics of multi robotic manipulators can be rewritten as follows:

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} \\ \dot{x}_{i,2} &= \frac{U_i}{M_i} - \frac{B_i x_{i,2}}{M_i} - \frac{N_i \sin(x_{i,1})}{M_i} \\ y_i &= D_i(x_{i,1}) \end{aligned} \tag{66}$$

Reuse the fuzzy membership functions established in Example 1. A finite-time adaptive consensus control strategy in Theorem 1 is applied to Example 2.

The output dead-zones are similar to Example 1 but $i = \{1, 2, 3, 4\}$. Initialize the state variables as $x_1(0) = [1.8, 0.1]^T, x_2(0) = [-2.6, 0.2]^T, x_3(0) = [4.5, -1]^T, x_4(0) = [-4.3, 0]^T$. The adjacency weighted of communication graph topology (see Fig. 9) are chosen to be $d_2 = [0.1, 0.04, 0.09, 0.04]^T$. The design parameters of the plant are set to $a_{1,1} = a_{1,2} = 3, a_{2,1} = 4, a_{2,2} = 2, a_{3,1} = 1, a_{3,2} = 3, a_{4,1} = 2, a_{4,2} = 4, q_i = 200, z_{i,0} = 8, \hat{\theta}_i = [0.25, 0.25, 0.25, 0.25]^T, i = 1, \ldots, 4, \eta = 1$. Meanwhile, the gains of control protocol u_i are selected as $\beta_1 = 2.3$, $\beta_2 = 1.2, \beta_3 = 3.2$ and $\tau = -10/11$.

Fig. 10-Fig. 13 show the simulation results. The trajectories of the ith subsystem output $y_i = D_i(x_{i,1})$ is given in Fig. 10. We can see that state synchronization of each robotic manipulator subsystem is achieved under the effect of output dead-zone. In addition, from the Fig. 12, it is

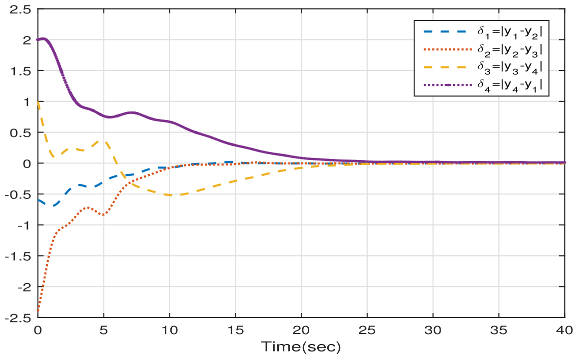


FIGURE 12. The synchronization error δ_i of Example 2.

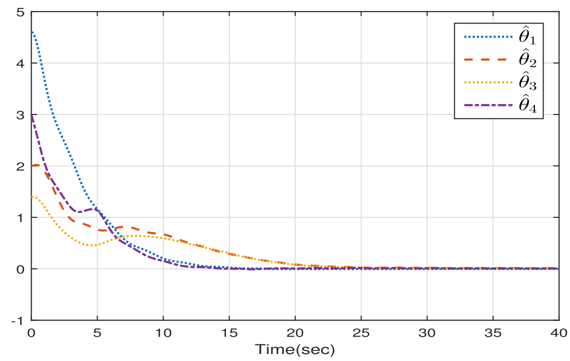


FIGURE 13. The parameter adaptive law $\hat{\theta}_i$ of Example 2.

quite obvious that the synchronization errors converge to a region around the origin when time tends to T_0. Therefore, the effectiveness our theoretical results is verified.

V. CONCLUSION

We have proposed a finite-time adaptive fuzzy concensus stabilization scheme for unknown nonlinear leaderless multi-agent systems with unknown output dead-zone in the output channel. By employing a Lyapunov-based logic switching rule and using the recursive backstepping design method and the universal approximation ability of fuzzy logic systems, the problem of unknown virtual control coefficient is handled and the finite-time adaptive concensus stabilization is also resolved, respectively. It is proved that the finite-time adaptive fuzzy concensus stabilization scheme can guarantee that state synchronization of each follower when time tends to T_0. Meanwhile, the synchronization errors between any adjacent followers also converge to a region around the zero in a finite time. Finally, simulation are conducted to further verify our theoretical results.

How to cope with the finite-time adaptive concensus stabilization for unknown nonlinear multi-agent systems with unknown unmodeled dynamics [63] is an open problem, which will be our future work.

REFERENCES

[1] S. P. Bhat and D. S. Bernstein, "Continuous finite-time stabilization of the translational and rotational double integrators," *IEEE Trans. Autom. Control*, vol. 43, no. 5, pp. 678–682, May 1998.

[2] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, Jan. 2000.

[3] X. Huang, W. Lin, and B. Yang, "Global finite-time stabilization of a class of uncertain nonlinear systems," *Automatica*, vol. 41, no. 5, pp. 881–888, May 2005.

[4] J. Wu, J. Li, G. Zong, and W. Chen, "Global finite-time adaptive stabilization of nonlinearly parametrized systems with multiple unknown control directions," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 7, pp. 1405–1414, Jul. 2017.

[5] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York, NY, USA: Wiley, 1995.

[6] Y. Yang, C. Hua, and X. Guan, "Adaptive fuzzy finite-time coordination control for networked nonlinear bilateral teleoperation system," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 3, pp. 631–641, Jun. 2014.

[7] F. Wang and X. Zhang, "Adaptive finite time control of nonlinear systems under time-varying actuator failures," *IEEE Trans. Syst., Man, Cybern. Syst.*, to be published.

[8] B. Chen, F. Wang, X. P. Liu, and C. Lin, "Finite-time adaptive fuzzy tracking control design for nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1207–1216, Jun. 2018.

[9] J. Na, S. Wang, Y.-J. Liu, Y. Huang, and X. Ren, "Finite-time convergence adaptive neural network control for nonlinear servo systems," *IEEE Trans. Cybern.*, to be published.

[10] C. Yang, Y. Jiang, J. Na, Z. Li, L. Cheng, and C.-Y. Su, "Finite-time convergence adaptive fuzzy control for dual-arm robot with unknown kinematics and dynamics," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 3, pp. 574–588, Aug. 2019.

[11] J. Xia, J. Zhang, W. Sun, B. Zhang, and Z. Wang, "Finite-time adaptive fuzzy control for nonlinear systems with full state constraints," *IEEE Trans. Syst., Man, Cybern. Syst.*, to be published.

[12] Z.-G. Wu, Y. Xu, Y.-J. Pan, P. Shi, and Q. Wang, "Event-triggered pinning control for consensus of multiagent systems with quantized information," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 11, pp. 1929–1938, Nov. 2018.

[13] C. Chen, Z. Liu, Y. Zhang, C. L. P. Chen, and S. Xie, "Saturated nussbaum function based approach for robotic systems with unknown actuator dynamics," *IEEE Trans. Cybern.*, vol. 46, no. 10, pp. 2311–2322, Oct. 2016.

[14] R. W. Beard, J. Lawton, and F. Y. Hadaegh, "A coordination architecture for spacecraft formation control," *IEEE Trans. Control Syst. Technol.*, vol. 9, no. 6, pp. 777–790, Nov. 2001.

[15] A. Zhang, D. Zhou, M. Yang, and P. Yang, "Finite-time formation control for unmanned aerial vehicle swarm system with time-delay and input saturation," *IEEE Access*, vol. 7, pp. 5853–5864, 2018.

[16] S. Kar and J. M. F. Moura, "Distributed consensus algorithms in sensor networks with imperfect communication: Link failures and channel noise," *IEEE Trans. Signal Process.*, vol. 57, no. 1, pp. 355–369, Jan. 2009.

[17] S. J. Yoo, "Distributed adaptive containment control of uncertain nonlinear multi-agent systems in strict-feedback form," *Automatica*, vol. 49, no. 7, pp. 2145–2153, 2013.

[18] G. Chen and F. L. Lewis, "Distributed adaptive tracking control for synchronization of unknown networked lagrangian systems," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 41, no. 3, pp. 805–816, Dec. 2011.

[19] H. Ma, Z. Wang, D. Wang, D. Liu, P. Yan, and Q. Wei, "Neural-network-based distributed adaptive robust control for a class of nonlinear multiagent systems with time delays and external noises," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 46, no. 6, pp. 750–758, Jun. 2016.

[20] Z. Peng, D. Wang, H. Zhang, and G. Sun, "Distributed neural network control for adaptive synchronization of uncertain dynamical multiagent systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 8, pp. 1508–1519, Aug. 2014.

[21] S. J. Yoo, "Distributed consensus tracking for multiple uncertain nonlinear strict-feedback systems under a directed graph," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 4, pp. 666–672, Apr. 2013.

[22] S. J. Yoo, "Connectivity-preserving consensus tracking of uncertain nonlinear strict-feedback multiagent systems: An error transformation approach," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 9, pp. 4542–4548, Sep. 2018.

[23] X. Wu, Y. Tang, J. Cao, and W. Zhang, "Distributed consensus of stochastic delayed multi-agent systems under asynchronous switching," *IEEE Trans. Cybern.*, vol. 46, no. 8, pp. 1817–1827, Aug. 2016.

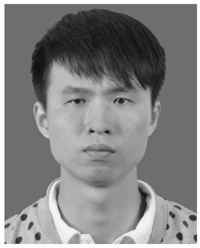
[24] L. Ma, Z. Wang, and H.-K. Lam, "Mean-square H_∞ consensus control for a class of nonlinear time-varying stochastic multiagent systems: The finite-horizon case," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 7, pp. 1050–1060, Mar. 2017.
[25] L. Ma, Z. Wang, Q.-L. Han, and Y. Liu, "Consensus control of stochastic multi-agent systems: A survey," *Sci. China Inf. Sci.*, vol. 60, no. 12, p. 120201, Dec. 2017.
[26] S. Li, M. J. Er, and J. Zhang, "Distributed adaptive fuzzy control for output consensus of heterogeneous stochastic nonlinear multiagent systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1138–1152, Jun. 2018.
[27] X. Wan, Z. Wang, M. Wu, and X. Liu, "H_∞ state estimation for discrete-time nonlinear singularly perturbed complex networks under the round-robin protocol," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published.
[28] X. Wan, Z. Wang, M. Wu, and X. Liu, "State estimation for discrete time-delayed genetic regulatory networks with stochastic noises under the round-robin protocols," *IEEE Trans. Nanobiosci.*, vol. 17, no. 2, pp. 145–154, Apr. 2018.
[29] X. Wan, Z. Wang, Q.-L. Han, and M. Wu, "Finite-time H_∞ state estimation for discrete time-delayed genetic regulatory networks under stochastic communication protocols," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 65, no. 10, pp. 3481–3491, Oct. 2018.
[30] L. Zou, Z. Wang, H. Gao, and F. E. Alsaadi, "Finite-horizon H_∞ consensus control of time-varying multiagent systems with stochastic communication protocol," *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 1830–1840, Aug. 2017.
[31] J. G. Truxal, *Control Engineers' Handbook*. New York, NY, USA: McGraw-Hill, 1958.
[32] C. A. Sacher and G. Inbar, "Tracking of the muscle recruitment characteristic during adaptive control of the electrically stimulated knee," in *Proc. 12th Annu. Int. Conf. IEEE Eng. Med. Biol. Soc.*, Nov. 1990, pp. 2315–2317.
[33] G. Tao and P. V. Kokotovic, "Adaptive control of plants with unknown dead-zones," *IEEE Trans. Autom. Control*, vol. 39, no. 1, pp. 59–68, Jan. 1994.
[34] X.-S. Wang, C.-Y. Su, and H. Hong, "Robust adaptive control of a class of nonlinear systems with unknown dead-zone," *Automatica*, vol. 40, no. 3, pp. 407–413, 2004.
[35] F. Wang, Z. Liu, and G. Lai, "Fuzzy adaptive control of nonlinear uncertain plants with unknown dead zone output," *Fuzzy Sets Syst.*, vol. 263, pp. 27–48, Mar. 2015.
[36] Z. Liu, F. Wang, Y. Zhang, X. Chen, and C. L. P. Chen, "Adaptive tracking control for a class of nonlinear systems with a fuzzy dead-zone input," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 1, pp. 193–204, Feb. 2015.
[37] F. Wang, Z. Liu, Y. Zhang, and B. Chen, "Distributed adaptive coordination control for uncertain nonlinear multi-agent systems with dead-zone input," *J. Franklin Inst.*, vol. 353, no. 10, pp. 2270–2289, 2016.
[38] C. Hua, L. Zhang, and X. Guan, "Distributed adaptive neural network output tracking of leader-following high-order stochastic nonlinear multiagent systems with unknown dead-zone input," *IEEE Trans. Cybern.*, vol. 47, no. 1, pp. 177–185, Jan. 2017.
[39] R. D. Nussbaum, "Some remarks on a conjecture in parameter adaptive control," *Syst. Control Lett.*, vol. 3, no. 5, pp. 243–246, 1983.
[40] Y. Wen and X. Ren, "Neural networks-based adaptive control for nonlinear time-varying delays systems with unknown control direction," *IEEE Trans. Neural Netw.*, vol. 22, no. 10, pp. 1599–1612, Oct. 2011.
[41] S. Tong, C. Liu, Y. Li, and H. Zhang, "Adaptive fuzzy decentralized control for large-scale nonlinear systems with time-varying delays and unknown high-frequency gain sign," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 41, no. 2, pp. 474–485, Apr. 2011.
[42] J. Yu, P. Shi, C. Lin, and H. Yu, "Adaptive neural command filtering control for nonlinear mimo systems with saturation input and unknown control direction," *IEEE Trans. Cybern.*, to be published.
[43] Y. H. Choi and S. J. Yoo, "Minimal-approximation-based distributed consensus tracking of a class of uncertain nonlinear multiagent systems with unknown control directions," *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 1994–2007, Aug. 2017.
[44] W. Wang, D. Wang, Z. Peng, and T. Li, "Prescribed performance consensus of uncertain nonlinear strict-feedback systems with unknown control directions," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 46, no. 9, pp. 1279–1286, Sep. 2016.
[45] C. Chen, C. Wen, Z. Liu, K. Xie, Y. Zhang, and C. P. Chen, "Adaptive consensus of nonlinear multi-agent systems with non-identical partially unknown control directions and bounded modelling errors," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4654–4659, Sep. 2017.
[46] K. Zhao, Y. Song, J. Qian, and J. Fu, "Zero-error tracking control with pre-assignable convergence mode for nonlinear systems under nonvanishing uncertainties and unknown control direction," *Syst. Control Lett.*, vol. 115, pp. 34–40, May 2018.
[47] H. Du, G. Wen, G. Chen, J. Cao, and F. Alsaadi, "A distributed finite-time consensus algorithm for higher-order leaderless and leader-following multiagent systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 7, pp. 1625–1634, Jul. 2017.
[48] M. Cai, Z. Xiang, and J. Guo, "Adaptive finite-time consensus protocols for multi-agent systems by using neural networks," *IET Control Theory Appl.*, vol. 10, no. 4, pp. 371–380, 2016.
[49] Z. Li and H. Ji, "Finite-time consensus and tracking control of a class of nonlinear multiagent systems," *IEEE Trans. Autom. Control*, vol. 63, no. 12, pp. 4413–4420, Dec. 2018.
[50] J. Wu, W. S. Chen, and J. Li, "Global finite-time adaptive stabilization for nonlinear systems with multiple unknown control directions," *Automatica*, vol. 69, no. 1, pp. 298–307, 2016.
[51] W. Chen, C. Wen, and J. Wu, "Global exponential/finite-time stability of nonlinear adaptive switching systems with applications in controlling systems with unknown control direction," *IEEE Trans. Autom. Control*, vol. 63, no. 8, pp. 2738–2744, Aug. 2018.
[52] K. D. Do and J. Pan, "Nonlinear formation control of unicycle-type mobile robots," *Robot. Auton. Syst.*, vol. 55, no. 3, pp. 191–204, 2007.
[53] X. Zhao, X. Zheng, C. Ma, and R. Li, "Distributed consensus of multiple Euler–Lagrange systems networked by sampled-data information with transmission delays and data packet dropouts," *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 3, pp. 1440–1450, Jul. 2017.
[54] Y.-J. Pan, H. Werner, Z. Huang, and M. Bartels, "Distributed cooperative control of leader–follower multi-agent systems under packet dropouts for quadcopters," *Syst. Control Lett.*, vol. 106, pp. 47–57, Aug. 2017.
[55] Z. Liu, W. Yan, H. Li, and M. Small, "Cooperative output regulation problem of multi-agent systems with stochastic packet dropout and time-varying communication delay," *J. Franklin Inst.*, vol. 355, no. 17, pp. 8664–8682, 2018.
[56] X. Sui, Y. Yang, X. Xu, S. Zhang, and L. Zhang, "The sampled-data consensus of multi-agent systems with probabilistic time-varying delays and packet losses," *Phys. A, Stat. Mech. Appl.*, vol. 492, pp. 1625–1641, Feb. 2018.
[57] C. Qian and W. Lin, "A continuous feedback approach to global strong stabilization of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 46, no. 7, pp. 1061–1079, Jul. 2001.
[58] C. Qian and W. Lin, "Non-Lipschitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization," *Syst. Control Lett.*, vol. 42, no. 3, pp. 185–200, 2001.
[59] G. H. Hardy, J. E. Littlewood, and G. Pólya, *Inequalities*. Cambridge, U.K.: Cambridge Univ. Press, 1952, p. 89.
[60] Z. Zhu, Y. Xia, and M. Fu, "Attitude stabilization of rigid spacecraft with finite-time convergence," *Int. J. Robust Nonlinear Control*, vol. 21, no. 6, pp. 686–702, Apr. 2011.
[61] L.-X. Wang and J. M. Mendel, "Fuzzy basis functions, universal approximation, and orthogonal least-squares learning," *IEEE Trans. Neural Netw.*, vol. 3, no. 5, pp. 807–814, Sep. 1992.
[62] W. Chen, X. Li, W. Ren, and C. Wen, "Adaptive consensus of multi-agent systems with unknown identical control directions based on a novel Nussbaum-type function," *IEEE Trans. Autom. Control*, vol. 59, no. 7, pp. 1887–1892, Jul. 2014.
[63] Q. Zhou, H. Li, C. Wu, L. Wang, and C. K. Ahn, "Adaptive fuzzy control of nonlinear systems with unmodeled dynamics and input saturation using small-gain approach," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 8, pp. 1979–1989, Aug. 2017.

HAINAN YANG received the B.S. degree from the School of Chemistry and Chemical Engineering, Guizhou University, Guiyang, China, in 2017. He is currently pursuing the M.S. degree with the School of Automation, Guangdong University of Technology, Guangzhou, China. His current research interests include multi-agent systems, neural networks control, adaptive fuzzy control, and switched systems.

YINGZHAN LIAN received the B.S. and M.S. degrees from the School of Automation, Central South University, Changsha, China, in 1985 and 1995, respectively. He is currently an Associate Professor with the Guangdong University of Technology. His current research interests include multi-agent systems, network control systems, and adaptive fuzzy control.

QIUSHI YE received the B.S. degree from the School of Mechanical Engineering, Lanzhou Jiaotong University, Lanzhou, China, in 2017. He is currently pursuing the M.S. degree with the School of Automation, Guangdong University of Technology, Guangzhou, China. His current research interests include multi-agent systems and adaptive fuzzy control.

XINYU ZHANG is currently pursuing the B. Eng. degree with the School of Automation, Guangdong University of Technology, Guangzhou, China, in 2019. Her current research interests include adaptive control theories, big data analytics and mining, machine learning and complex system modeling, and intelligent control based on data and knowledge.



XIANNAN CAO received the B.S. degree from the School of Electrical Engineering, University of South China, Hengyang, China, in 2018. He is currently pursuing the M.S. degree with the School of Automation, Guangdong University of Technology, Guangzhou, China. His current research interests include multi-agent systems, neural networks control, and adaptive fuzzy control.