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Precoding Design for Two-Way MIMO Full-Duplex Amplify-and-Forward Relay Communication Systems

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ABSTRACT Full duplex (FD) multiple-input multiple-output (MIMO) relaying can significantly increase the spectral efficiency of cooperative communication systems. This paper examines the problem of linear source and relay precoder and destination combiner design for two-way MIMO FD amplify-and-forward (AF) relay communication systems. The effect of the residual loop interference (LI) due to imperfect LI cancellation is considered in the design. Two algorithms are proposed to minimize the mean squared error (MSE) of the received signals at the destinations. The first is a tri-step alternating iterative algorithm and the second is a bi-step iterative algorithm. The convergence and complexity of these algorithms are analyzed. Simulation results are presented, which show that the proposed two-way FD system provides approximately double the achievable rate of the corresponding half-duplex (HD) system when the residual LI is low. Furthermore, the bi-step algorithm shows comparable performance to the tri-step algorithm and has a lower computational complexity.

INDEX TERMS Amplify-and-forward (AF), full-duplex (FD), loop interference (LI), MIMO, precoding, sum achievable rate, two-way relay.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) relay communication systems have been extensively investigated in recent years because they can enhance capacity by increasing coverage and reliability [1]. In an amplify-and-forward (AF) relay system, the relay node amplifies the received signal and then forwards the amplified signal to the destination node. Since the relay only performs amplification, the complexity of this strategy is much lower than decode-and-forward (DF), which is a regenerative relaying scheme. In half-duplex (HD) relay systems [2]–[4], communications from the source to destination requires two time slots so the source node transmits only half of the time, which limits the efficiency.

In contrast to one-way relaying which needs four time slots to exchange information between a source and destination, two-way relaying only needs two time slots to complete a round of information exchange. Therefore, two-way relaying has a higher efficiency than one-way relaying. Physical-layer network coding (PNC) which exploit the self-information at the nodes has been used with two-way relaying [5]–[9]. There are two steps in HD two-way relaying communications. First, the nodes transmit their signals to the relay during the multiplexing access (MAC) phase. Then the relay broadcasts (BC) the received signal to the two nodes. Each node can cancel the interference they generate from the signal received from the relay to recover the signal transmitted by the other node.

In [5], a two-way relaying scheme which approaches the sum capacity of the MIMO cellular two-way relay channel was investigated. In order to achieve efficient interference-free decoding at the relay, a non-linear lattice-based precoding technique was used to compensate for the inter-stream interference. The sum capacity of the proposed system was asymptotically achieved in the high signal-to-noise ratio (SNR) region. The tradeoff between the capacity and diversity-multiplexing of the two-way relay channel was examined in [6]. An iterative algorithm was proposed to maximize the achievable rate with AF relaying subject to minimum signal-to-interference-plus-noise ratio (SINR) constraints. An energy efficient two-way AF relay system with

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multiple antennas at both the sources and relay was presented in [7]. The transmit power was minimized while satisfying the quality of service (QoS) requirements of both sources. Transmit beamformers and receive combiners were designed with a zero forcing (ZF) based relay precoding matrix. In [8], it was shown that the optimal diversity-multiplexing gain tradeoff can be achieved using a compress-and-forward (CF) strategy in which the relay quantizes its received signal and transmits the corresponding codeword.

Multiple-input, multiple-output (MIMO) can be employed to improve the transmission reliability and enhance the channel capacity of a wireless communication system. Employing MIMO in a two-way relay system is an efficient way to increase the performance over single antenna systems. In order to fully realize the benefits of MIMO two-way relaying, precoding should be employed at both the source nodes and relay by making use of channel state information (CSI) [10]–[15]. In [10], a nonlinear precoder design was presented for a MIMO two-way relay system using minimum mean squared error (MMSE) decision feedback equalizers. The design first considers the nonlinear source precoding at the two sources with a fixed relay precoder, and then considers the joint precoder design to incorporate the relay precoder. In [11], a constrained optimization problem with respect to the relay precoder was formulated for the general case of multiple relays each with multiple antennas. Under the assumption that complete CSI is available at the relays, the problem was converted to a convex optimization problem with respect to only the non-zero entries of the relay precoder matrix, which leads to a closed-form relay precoding solution.

In [12], a low-complexity joint beamforming and power management scheme was proposed. The beamformer first aligns the channel matrices of the node pairs and then decomposes the aligned channel into parallel subchannels. It was shown that this scheme improves the sum capacity and can be used to lower the required transmit power. Two iterative algorithms were proposed in [14] for joint source and relay precoder design based on the MSE criterion in a MIMO two-way relay system. In this system, two multiple antenna source nodes exchange information with the help of a multiple antenna amplify-and-forward relay. In [15], the problem of precoder design to suppress co-channel interference in a multiuser two-way relay system was considered. The uplink performance including the overall MSE and sum rate was optimized while maintaining individual downlink SINR requirements.

While most of the results in the literature focus on half-duplex relay systems [8]–[15], the development of new signal processing techniques and antenna designs has made FD relaying in MIMO systems a reality [16], [17]. A full-duplex AF relay system under Nakagami-*m* fading was considered in [19] and closed-form expressions for the outage probability and ergodic capacity were derived. In [20], an interference suppression scheme was developed to mitigate the residual LI and interference in a multiuser FD relaying system. Rather than using HD two-way relaying as in [8]–[15], a FD two-way relay design was presented in [18]. It was shown that FD relaying can achieve almost double the capacity of HD relaying if there is no residual LI. In [21], distributed space-time coding was investigated for a two-way FD relaying network which allows relay communications in both directions simultaneously. The direct source to destination link was also considered. A two-way FD relaying system with residual LI was presented in [22]. Exact and approximate closed-form expressions were given for the outage probability with both perfect and imperfect channel state information (CSI). A joint precoder and combiner design that maximizes the end-to-end (e2e) performance was investigated in [23]. ZF LI suppression at the relay was considered and a closed-form solution was obtained. In [24], rate and outage probability tradeoffs were examined for full-duplex one-way and two-way relaying systems considering the residual LI. The joint design of relay and receiver beamforming was considered in [25] for a full-duplex two-way amplify-and-forward relay system with imperfect cancellation of the loopback self-interference by minimizing the mean square error under a relay transmit power constraint. In the above results, the residual LI was assumed to have a Gaussian distribution, but a Rician distribution was obtained in [29].

An algorithm was presented in [26] to maximize the e2e performance by jointly optimizing the beamforming matrix at an AF relay and the transmit power at the source. If multiple antennas are employed at both the source and destination, the channel sum rate increases linearly with the minimum number of antennas [14]. In contrast to [26] which employs only a single antenna at the source and destination, this paper considers a MIMO FD two-way relay system where the source, relay and destination have multiple antennas. Further, the AF protocol with physical layer network coding is employed. As this is a FD system, the residual loop interference at the relay is considered. Because signals are known at the source nodes, the LI cancellation at the source nodes is assumed to be better than at the relay. Thus, the focus here is on the residual LI at the relay node. The source precoders, relay precoder and destination combiners are optimized using the MSE criterion. As the original optimization problem is highly non-convex and a closed-form solution is intractable, it is translated into three subproblems which can be solved iteratively. It is shown that this algorithm converges to an local optimal solution. Since the computational complexity of the proposed tri-step iterative algorithm is high, a low complexity bi-step iterative approach is obtained. Results are presented which show that this bi-step iterative algorithm provides performance comparable to that with the tri-step iterative algorithm, so the complexity-performance tradeoff is favorable. The sum achievable rate improvement with FD relaying over HD relaying is illustrated, and the effects of the residual LI are examined.

The remainder of this paper is organized as follows. In Section II, the MIMO two-way full-duplex relay system model is introduced, and the problem formulation is

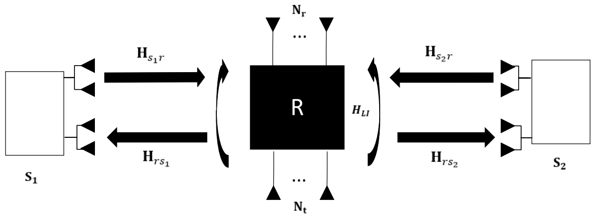

FIGURE 1. The MIMO two-way FD AF relay system model.

presented in Section III. Two iterative algorithms for solving the proposed optimization problem are developed in Section IV. The sum mean squared error (MSE) performance, sum achievable rate and complexity of the proposed algorithms are analyzed in Section V. Numerical results are presented to demonstrate the performance improvement with FD relaying and precoding. Finally, some conclusion are given in Section VI.

Notation: Throughout this paper, the following notation is used. Bold uppercase, bold lowercase and normal letters denote matrices, vectors and scalars, respectively. vec$(\cdot)$ denotes matrix vectorization and $\otimes$ denotes the matrix Kronecker product. $tr\{\cdot\}$ is the trace of a matrix and $\mathbf{I}_N$ is the $N \times N$ identity matrix. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the real and imaginary parts, respectively.

II. SYSTEM MODEL

We consider a three node, two-way MIMO full-duplex (FD) relay system. As shown in Fig. 1, two sources S_1 and S_2, each equipped with N_{s_1} transmit and N_{s_2} receive antennas, want to exchange messages via a relay R. The relay operates in full-duplex (FD) mode with physical layer network coding [26], and has N_r and N_t antennas to receive and transmit, respectively. The transmit and receive antennas are assumed to be identical at all nodes. The non-regenerative relay amplifies the received signals from both source nodes and then broadcasts the resulting signal to the destinations simultaneously. Therefore, communications between the two sources is accomplished in one time slot compared to a half-duplex (HD) system that requires two time slots. Note that in the two-way relay system, the source nodes are the destination nodes during the relay broadcast phase.

Let $\mathbf{s}_i[n] \in \mathbb{C}^{L\times 1}$ represent the $L \times 1$ signal vector transmitted at time n for source node i, $i = 1, 2$. Without loss of generality, we assume that $L \leq \min\{N_{s_i}, N_t, N_r\}$, $i = 1, 2$. In addition, it is assumed that $\mathbb{E}[\mathbf{s}_i[n]\mathbf{s}_i[n]^H] = \mathbf{I}_L$, where $(\cdot)^H$ represents conjugate transpose (Hermitian) and $\mathbb{E}$ denotes expectation. A linear precoding matrix $\mathbf{B}_i[n]$ is applied to the signal vector $\mathbf{s}_i[n]$ before transmission. The received signal at the relay can be expressed as

$$\mathbf{y}_R[n] = \mathbf{H}_{S_1R}[n]\mathbf{B}_1[n]\mathbf{s}_1[n] + \mathbf{H}_{S_2R}[n]\mathbf{B}_2[n]\mathbf{s}_2[n] + \mathbf{H}_{LI}[n]\mathbf{t}[n] + \mathbf{n}_R[n], \quad (1)$$

where $\mathbf{H}_{S_iR}[n] \in \mathbb{C}^{N_r\times N_{s_i}}$ is the ith source to relay channel matrix, $\mathbf{H}_{RS_i} \in \mathbb{C}^{N_{s_i}\times N_t}$ is the relay to ith destination channel matrix, $\mathbf{H}_{LI}[n] \in \mathbb{C}^{N_r\times N_t}$ is the loop interference (LI) channel matrix, and $\mathbf{n}_r[n] \in \mathbb{C}^{N_r\times 1}$ is an independent and identically distributed (i.i.d.) noise matrix. After employing a LI cancellation technique, (1) can be written as

$$\mathbf{y}_R[n] = \mathbf{H}_{S_1R}[n]\mathbf{B}_1[n]\mathbf{s}_1[n] + \mathbf{H}_{S_2R}[n]\mathbf{B}_2[n]\mathbf{s}_2[n] + \mathbf{H}_{LI}[n]\mathbf{t}[n] + \mathbf{T}[n] + \mathbf{n}_R[n], \quad (2)$$

where $\mathbf{t}[n]$ is the loop interference at time n, and $\mathbf{T}[n] = -\mathbf{H}_{LI}[n]\mathbf{t}[n]$ when perfect LI cancellation is applied. However, in an actual system $\mathbf{T}[n] = -\mathbf{H}_{LI}[n]\tilde{\mathbf{t}}[n]$ where $\tilde{\mathbf{t}}[n]$ is a noisy version of $\mathbf{t}[n]$ due to imperfect LI cancellation. As discussed in [28], $\mathbf{y}_R[n]$ can be rewritten as

$$\mathbf{y}_R[n] = \mathbf{H}_{S_1R}[n]\mathbf{B}_1[n]\mathbf{s}_1[n] + \mathbf{H}_{S_2R}[n]\mathbf{B}_2[n]\mathbf{s}_2[n] + \mathbf{H}_{LI}[n]\Delta\mathbf{t}[n] + \mathbf{n}_R[n], \quad (3)$$

where $\Delta\mathrm{t}[n] = \mathrm{t}[n] - \tilde{\mathrm{t}}[n]$ and $\mathbf{H}_{LI}[n]\Delta\mathrm{t}[n]$ is the residual LI after imperfect LI cancellation.

At time $n + 1$, the full-duplex relay applies a precoding matrix $\mathbf{F}[n + 1] \in \mathbb{C}^{N_t\times N_t}$ to the received signal and then broadcasts the result to the nodes. The received signal at node i can be expressed as

$$\begin{aligned}\mathbf{y}_i[n+1] &= \mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{y}_R[n] + \mathbf{n}_{D_i}[n+1]\\ &= \mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{H}_{S_iR}[n]\mathbf{B}_i[n]\mathbf{s}_i[n]\\ &\quad+\mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{H}_{S_{\bar{i}}R}[n]\mathbf{B}_{\bar{i}}[n]\mathbf{s}_{\bar{i}}[n]\\ &\quad+\mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{H}_{LI}[n]\Delta\mathbf{t}[n]\\ &\quad+\mathbf{H}_{RS_i}[n+1]\mathbf{F}[n+1]\mathbf{n}_R[n+1] + \mathbf{n}_{D_i}[n], \quad (4)\end{aligned}$$

where $\bar{i} = 2$ if $i = 1$ and $\bar{i} = 1$ if $i = 2$.

Similar to [15], we assume that the channel characteristics of each link change very slowly so they can be perfectly estimated using pilot symbols or training sequences. The channel $\mathbf{H}_{S_iR}$ at the relay can be estimated by S_i sending a training sequence. The LI channel $\mathbf{H}_{LI}$ can be estimated at the relay by sending an N_t-symbol pilot sequence. Although channel reciprocity does not hold exactly, as discussed in [2], [11], [14], [26] for the purposes of analysis it can be assumed to hold during the MAC and BC phases so that $\mathbf{H}_{S_iR} = \mathbf{H}^T_{RS_i}$. As the antennas are identical, the back propagated self-interference term $\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{s}_i$ from (4) can be canceled. Further, we assume that the channel variations during the precoder update interval are relatively small so the time index has no influence on the precoder design and can be omitted. Therefore, (4) can be expressed as

$$\mathbf{y}_i = \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{s}_{\bar{i}} + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_iR}\mathbf{H}_{LI}\Delta\mathbf{t} + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{n}_R + \mathbf{n}_{D_i}. \quad (5)$$

A combiner $\mathbf{W}_i \in \mathbb{C}^{N_{s_{\bar{i}}}\times L}$ is employed on the received signal at node i, so the estimated signal from node $\bar{i}$ can be written as

$$\hat{\mathbf{s}}_{\bar{i}} = \mathbf{W}_i^H\mathbf{y}_i. \quad (6)$$

Since i.i.d. noise with zero mean and unit variance is assumed, $\mathbb{E}[\mathbf{n}_R\mathbf{n}_R^H] = \sigma^2_{n,r}\mathbf{I}_{N_r}$ and $\mathbb{E}[\mathbf{n}_{D_i}\mathbf{n}_{D_i}^H] = \sigma^2_{n,d}\mathbf{I}_{N_{s_i}}$, where

$\sigma_{n,r}^2 = 1$ and $\sigma_{n,d}^2 = 1$ are the variances of $\mathbf{n}_R$ and $\mathbf{n}_{D_i}$, respectively. The problem now is how to design the linear precoders $\mathbf{B}_i$ and $\mathbf{F}$ and the linear combiners $\mathbf{W}_i$ to minimize the sum mean squared error (MSE) of the received signals at the destinations.

III. PROBLEM FORMULATION

In this section, we first formulate the joint source and relay precoding optimization problem to minimize the sum MSE in the MIMO two-way relay system. Considering the received signal (5) and the estimated signal after applying the linear combiner (6), the MSE at node i can be expressed as

$$\begin{aligned} J_i &= \mathbb{E}[(\hat{\mathbf{s}}_{\bar{i}} - \mathbf{s}_{\bar{i}})(\hat{\mathbf{s}}_{\bar{i}} - \mathbf{s}_{\bar{i}})^H] \\ &= tr\{(\mathbf{W}_i^H \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}} - \mathbf{I}_{N_{S_i}})(\mathbf{W}_i^H \mathbf{H}_{RS_i}\mathbf{F} \\ &\quad \times \mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}} - \mathbf{I}_{N_{S_i}})^H + \mathbf{W}_i^H \mathbf{C}_{n_i}\mathbf{W}_i\}, \end{aligned} \tag{7}$$

where $\mathbf{C}_{n_i} = \sigma_t^2 \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_iR}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{H}_{S_iR}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H \mathbf{H}_{RS_i}^H + \mathbf{I}_{N_{D_i}}$, and σ_t^2 is the variance of $\Delta\mathbf{t}$.

The problem is to find the matrices $\mathbf{F}, \mathbf{B}_i, \mathbf{W}_i$ such that the sum MSE at the two destinations is minimized. The optimization problem can be formulated as

$$\min_{\mathbf{B}_i,\mathbf{F},\mathbf{W}_i, i=1,2} \; J_1 + J_2 \tag{8a}$$

$$s.t.\; tr\left(\mathbf{F}\left(\sum_{i=1}^{2}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{S_iR}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}\right)\mathbf{F}^H\right) \le P_r \tag{8b}$$

$$tr\left(\mathbf{B}_i\mathbf{B}_i^H\right) \le P_i \tag{8c}$$

where $P_i > 0$ and $P_r > 0$ are the power constraints at source node i and the relay, respectively.

IV. THE PROPOSED ITERATIVE ALGORITHMS

The original optimization problem in (8) is highly non-convex and a closed-form solution is intractable. Thus, in this section two algorithms are proposed to solve this problem. One is a tri-step iterative algorithm and the other is a bi-step iterative approach with lower computational complexity.

A. TRI-STEP ALGORITHM

A tri-step algorithm [2], [14] is presented here which is based on alternating optimization that updates one group of precoders at a time while fixing the others to solve the corresponding convex subproblems to obtain $\mathbf{B}_i$, $\mathbf{F}$ and $\mathbf{W}_i$. First, given $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{F}$, we find the optimal combining matrices $\mathbf{W}_1$ and $\mathbf{W}_2$. Since the power constraints in (8b) and (8c) are not related to the destination combiners $\mathbf{W}_1$ and $\mathbf{W}_2$, the optimization problem is unconstrained and so is given by

$$\min_{\mathbf{W}_i, i=1,2} J_{W_1} + J_{W_2} \tag{9}$$

where

$$\begin{aligned} J_{W_i} &= tr\{\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H\mathbf{W}_i \\ &\quad + \mathbf{I}_{N_{S_i}} - \mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}} - \mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H \\ &\quad \times \mathbf{H}_{RS_i}^H\mathbf{W}_i + \mathbf{W}_i^H\mathbf{C}_{n_i}\mathbf{W}_i\}. \end{aligned} \tag{10}$$

Differentiating J_{W_i} with respect to $\mathbf{W}_i$ and setting the result to zero, the optimal combining matrix can be expressed as

$$\begin{aligned} \mathbf{W}_i &= (\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H + \mathbf{C}_{n_i})^{-1} \\ &\quad \times \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}},\; i = 1, 2. \end{aligned} \tag{11}$$

This solution is also known as a Wiener filter [2].

Second, the optimal relay precoding matrix $\mathbf{F}$ is obtained by assuming $\mathbf{W}_i$ and $\mathbf{B}_i$, $i = 1, 2$, are fixed and solving the optimization problem

$$\begin{aligned} &\min_{\mathbf{F}} \; J_1 + J_2 \\ &s.t.\; tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) \le P_r \end{aligned} \tag{12}$$

where $\mathbf{K}_x = \left(\sum_{i=1}^{2}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{S_iR}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}\right)$, $i = 1, 2$. The MSE at S_i is

$$\begin{aligned} J_i &= tr(\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H \\ &\quad - \mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F} - \mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H \\ &\quad + \mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H + \mathbf{W}_i\mathbf{W}_i^H \\ &\quad + \sigma_t^2\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H + \mathbf{I}_L) \\ &= tr(\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{K}_{x_{\bar{i}}}\mathbf{F}^H - \mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}} \\ &\quad \times \mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F} - \mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H \\ &\quad + \sigma_{n_r}^2\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H + \mathbf{W}_i\mathbf{W}_i^H \\ &\quad + \sigma_t^2\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H + \mathbf{I}_L) \end{aligned} \tag{13}$$

where $\mathbf{K}_{x_{\bar{i}}} = \mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H$. As with similar problems [14], (13) is convex so the optimal relay precoder can be obtained by employing the KKT conditions. The Lagrangian function of (13) is

$$\mathcal{L} = J_1 + J_2 + \lambda(tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) - P_r), \tag{14}$$

where $\lambda \ge 0$ is the Lagrange multiplier. Differentiating $\mathcal{L}$ with respect to $\mathbf{F}$ with $\mathbf{B}_i$ and $\mathbf{W}_i$ fixed and equating the result to zero gives

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial\mathbf{F}} &= \mathbf{H}_{RS_1}^H\mathbf{W}_1\mathbf{W}_1^H\mathbf{H}_{RS_1}\mathbf{F}\mathbf{K}_{x_2} - \mathbf{H}_{RS_1}^H\mathbf{W}_1\mathbf{B}_2^H\mathbf{H}_{S_2R}^H \\ &\quad + \mathbf{H}_{RS_2}^H\mathbf{W}_2\mathbf{W}_2^H\mathbf{H}_{RS_2}\mathbf{F}\mathbf{K}_{x_1} - \mathbf{H}_{RS_2}^H\mathbf{W}_2\mathbf{B}_1^H\mathbf{H}_{S_1R}^H \\ &\quad + \lambda\mathbf{F}\mathbf{K}_x + \mathbf{H}_{RS_1}^H\mathbf{W}_1\mathbf{W}_1^H\mathbf{H}_{RS_1}\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ &\quad + \mathbf{H}_{RS_2}^H\mathbf{W}_2\mathbf{W}_2^H\mathbf{H}_{RS_2}\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ &= \mathbf{K}_{r_1}\mathbf{F}(\mathbf{K}_{x_2} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ &\quad + \mathbf{K}_{r_2}\mathbf{F}(\mathbf{K}_{x_1} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ &\quad - \mathbf{K}_r + \lambda\mathbf{F}\mathbf{K}_x = 0, \end{aligned} \tag{15}$$

where

$$\begin{aligned} \mathbf{K}_{r_1} &= \mathbf{H}_{RS_1}^H\mathbf{W}_1\mathbf{W}_1^H\mathbf{H}_{RS_1}, \\ \mathbf{K}_{r_2} &= \mathbf{H}_{RS_2}^H\mathbf{W}_2\mathbf{W}_2^H\mathbf{H}_{RS_2}, \\ \mathbf{K}_r &= \mathbf{H}_{RS_1}^H\mathbf{W}_1\mathbf{B}_2^H\mathbf{H}_{S_2R}^H + \mathbf{H}_{RS_2}^H\mathbf{W}_2\mathbf{B}_1^H\mathbf{H}_{S_1R}^H. \end{aligned}$$

From the properties of the vector operators, $\text{vec}(\mathbf{AXB})) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X})$ and $\text{vec}(\mathbf{A}+\mathbf{B}) = \text{vec}(\mathbf{A}) + \text{vec}(\mathbf{B})$, so

$$\text{vec}(\mathbf{K}_r) = ((\mathbf{K}_{x_2} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_1})\text{vec}(\mathbf{F}) + ((\mathbf{K}_{x_1} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_2})\text{vec}(\mathbf{F}). \quad (16)$$

The optimal solution is then

$$\mathbf{F} = \text{mat}\{[(\mathbf{K}_{x_2} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_1} + (\mathbf{K}_{x_1} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_2} + (\lambda\mathbf{K}_x^H)^T \otimes \mathbf{I}_{N_r}]^{-1}\text{vec}(\mathbf{K}_r)\}, \quad (17)$$

where $\text{mat}\{\cdot\}$ is the inverse operation of $\text{vec}(\cdot)$.

In the case $\lambda = 0$, we have

$$\mathbf{F} = \text{mat}\{[(\mathbf{K}_{x_2} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_1} + (\mathbf{K}_{x_1} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})^T \otimes \mathbf{K}_{r_2}]^{-1}\text{vec}(\mathbf{K}_r)\}, \quad (18)$$

and

$$\lambda(tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) - P_r) = 0. \quad (19)$$

If $\mathbf{F}$ in (18) satisfies the condition in (19), then (18) is the optimal relay precoder. Otherwise, let $\lambda > 0$ so that

$$tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) \le P_r. \quad (20)$$

Substituting (17) into (20) and solving the resulting nonlinear equation gives

$$tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) = P_r. \quad (21)$$

In this case, $\mathbf{F}$ decreases with λ because of the inverse in (17). The optimal λ that satisfies $tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) = P_r$ can then be readily obtained using numerical methods such as bisection search.

An upper bound on λ can be found by following an approach similar to that in [14]. Let $\mathbf{K}_r = \mathbf{E}_1 + \mathbf{E}_2$ where $\mathbf{E}_1 = \mathbf{K}_{r_1}\mathbf{F}(\mathbf{K}_{x_2} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) + \mathbf{K}_{r_2}\mathbf{F}(\mathbf{K}_{x_1} + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})$ and $\mathbf{E}_2 = \lambda\mathbf{F}\mathbf{K}_x = 0$. If $\mathbf{F}$ and λ are the optimal primal and dual solutions of (8), respectively, then

$$\mathbf{F} = \frac{1}{\lambda}\mathbf{E}_2\mathbf{K}_x^{-1}, \quad (22)$$

and

$$tr(\mathbf{F}\mathbf{K}_x\mathbf{F}^H) = tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{K}_x\mathbf{K}_x^{-1}\mathbf{E}_2^H) = tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_2^H) = P_r. \quad (23)$$

On the other hand, if $\lambda > 0$ we have

$$tr\left(\frac{1}{\lambda^2}\mathbf{K}_r\mathbf{K}_x^{-1}\mathbf{K}_r^H\right) = tr(\frac{1}{\lambda^2}(\mathbf{E}_1+\mathbf{E}_2)\mathbf{K}_x^{-1}(\mathbf{E}_1+\mathbf{E}_2)^H) = tr(\frac{1}{\lambda^2}\mathbf{E}_1\mathbf{K}_x^{-1}\mathbf{E}_1^H) + tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_2^H) + tr(\frac{1}{\lambda^2}\mathbf{E}_1\mathbf{K}_x^{-1}\mathbf{E}_2^H) + tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_1^H) \quad (24)$$

Applying the matrix property that if $\mathbf{Z_1} \ge 0$ and $\mathbf{Z_2} \ge 0$ then $tr(\mathbf{Z_1}\mathbf{Z_2}) \ge 0$ gives $tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_2^H) \ge 0$, so that

$$tr(\mathbf{E}_1\mathbf{F}^H) = tr(\mathbf{K}_{r_1}\mathbf{F}\mathbf{K}_{x_2}\mathbf{F}^H + \mathbf{K}_{r_2}\mathbf{F}\mathbf{K}_{x_1}\mathbf{F}^H) \ge 0, \quad (25)$$

$$tr(\mathbf{E}_1\mathbf{F}^H) = tr(\frac{1}{\lambda^2}\mathbf{E}_1\mathbf{K}_x^{-1}\mathbf{E}_2^H) \ge 0, \quad (26)$$

and

$$tr(\frac{1}{\lambda^2}\mathbf{E}_2\mathbf{K}_x^{-1}\mathbf{E}_1^H) \ge 0. \quad (27)$$

Since all the terms in (24) are greater than or equal zero, it can be concluded that

$$tr(\frac{1}{\lambda^2}\mathbf{K}_r\mathbf{K}_x^{-1}\mathbf{K}_r^H) \ge P_r, \quad (28)$$

so $\lambda \le \sqrt{\frac{\mathbf{K}_r\mathbf{K}_x^{-1}\mathbf{K}_r^H}{P_r}}$ which is an upper bound on λ.

Third, the optimal source precoders $\mathbf{B}_i$, $i = 1, 2$, are derived using the previously obtained $\mathbf{F}$ and $\mathbf{W}_i$. From (8b), updating the source precoder can affect the power constraint at the relay. Thus the relay power constraint in (8b) should be included, so (8) is rewritten as

$$\min_{\mathbf{B}_i, i=1,2} J_{B_1} + J_{B_2}$$
$$s.t.\ tr(\mathbf{B}_i\mathbf{B}_i^H) \le P_i$$
$$tr(\sum_{i=1}^{2}\mathbf{H}_{S_iR}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{B}_i^H) \le \bar{P}_r \quad (29)$$

where $\bar{P}_r = P_r - tr(\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H)$. Now let

$$\mathbf{K}_{O_i} = \mathbf{H}_{S_iR}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_iR}, \quad (30)$$
$$J_{B_i} = tr\{\mathbf{K}_{S_i1}\mathbf{B}_i\mathbf{B}_i^H - 2\Re(\mathbf{K}_{S_i2}\mathbf{B}_{\bar{i}}) + \mathbf{K}_{S_i3}\}, \quad (31)$$
$$\mathbf{K}_{S_i1} = \mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H\mathbf{W}_i\mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}, \quad (32)$$
$$\mathbf{K}_{S_i2} = \mathbf{W}_i^H\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}, \quad (33)$$
$$\mathbf{K}_{S_i3} = \mathbf{W}_i^H\mathbf{C}_{n_i}\mathbf{W}_i + \mathbf{I}_{N_{S_i}}. \quad (34)$$

Applying the trace operator identity $tr\{\mathbf{ABCD}\} = (\text{vec}(\mathbf{D})^T)^T(\mathbf{C}^T \otimes \mathbf{A})\text{vec}(\mathbf{B})$ gives

$$J_{B_i} = \hat{\mathbf{b}}_{\bar{i}}^H\hat{\mathbf{O}}_i\hat{\mathbf{b}}_{\bar{i}} - 2\Re\{\hat{\mathbf{a}}_i^T\hat{\mathbf{b}}_{\bar{i}}\} + tr\{\mathbf{K}_{S_i3}\}, \quad i = 1, 2, \quad (35)$$

where $\hat{\mathbf{O}}_i = \mathbf{I}_N \otimes \mathbf{K}_{S_i1}$, $\hat{\mathbf{a}}_i = \text{vec}(\mathbf{K}_{S_i2}^T)$ and $\hat{\mathbf{b}}_i = \text{vec}(\mathbf{B}_i)$. Because $\hat{\mathbf{O}}_i$ is positive semidefinite (PSD), (34) can be transformed into

$$J_{B_i} = \| \hat{\mathbf{O}}_i^{\frac{1}{2}}\hat{\mathbf{b}}_{\bar{i}} \|_2^2 - 2\Re\{\hat{\mathbf{a}}_i^T\hat{\mathbf{b}}_{\bar{i}}\} + tr\{\mathbf{K}_{S_i3}\}, \quad i = 1, 2. \quad (36)$$

To eliminate the $\Re\{\cdot\}$ operation, let $\mathbf{b}_i = [\Re\{\hat{\mathbf{b}}_i^T\}, \Im\{\hat{\mathbf{b}}_i^T\}]^T$, $i = 1, 2$. This gives

$$J_{B_i} = \mathbf{b}_{\bar{i}}^T\mathbf{O}_i\mathbf{b}_{\bar{i}} - 2\mathbf{a}_i^T\mathbf{b}_{\bar{i}} + tr\{\mathbf{K}_{S_i3}\}, \quad i = 1, 2. \quad (37)$$

where $\mathbf{O}_i = \tilde{\mathbf{O}}_i^T\tilde{\mathbf{O}}_i$ with $\tilde{\mathbf{O}}_i = \begin{bmatrix} \Re\{\hat{\mathbf{O}}_i^{\frac{1}{2}}\} & -\Im\{\hat{\mathbf{O}}_i^{\frac{1}{2}}\} \\ \Im\{\hat{\mathbf{O}}_i^{\frac{1}{2}}\} & \Re\{\hat{\mathbf{O}}_i^{\frac{1}{2}}\} \end{bmatrix}$, and $\mathbf{a}_i = \left[\Re\{\hat{\mathbf{a}}_i^T\}\ -\Im\{\hat{\mathbf{a}}_i^T\}\right]^T$, $i = 1, 2$. In addition, for the power constraints in (29) we have $tr\{\mathbf{B}_i\mathbf{B}_i^H\} = \mathbf{b}_i^T\hat{\mathbf{E}}_i\mathbf{b}_i$ with

$\hat{\mathbf{E}}_i = \mathbf{I}_{2L^2\times 2L^2}$, $i = 1, 2$, and $tr\{\mathbf{K}_{o1}\mathbf{B}_1\mathbf{B}_1^H + \mathbf{K}_{o2}\mathbf{B}_2\mathbf{B}_2^H\} = \mathbf{b}_1^H\hat{\mathbf{E}}_3\mathbf{b}_1 + \mathbf{b}_2^H\hat{\mathbf{E}}_4\mathbf{b}_2$, where $\hat{\mathbf{E}}_3 = \tilde{\mathbf{E}}_3^T\tilde{\mathbf{E}}_3$ and $\hat{\mathbf{E}}_4 = \tilde{\mathbf{E}}_4^T\tilde{\mathbf{E}}_4$ are positive semidefinite matrices with

$$\tilde{\mathbf{E}}_3 = \begin{bmatrix} \Re\{(\mathbf{I}_N\otimes\mathbf{K}_{O_1})^{\frac{1}{2}}\} & -\Im\{(\mathbf{I}_N\otimes\mathbf{K}_{O_1})^{\frac{1}{2}}\} \\ \Im\{(\mathbf{I}_N\otimes\mathbf{K}_{O_1})^{\frac{1}{2}}\} & \Re\{(\mathbf{I}_N\otimes\mathbf{K}_{O_1})^{\frac{1}{2}}\} \end{bmatrix}, \quad (38)$$

and

$$\tilde{\mathbf{E}}_4 = \begin{bmatrix} \Re\{(\mathbf{I}_N\otimes\mathbf{K}_{O_2})^{\frac{1}{2}}\} & -\Im\{(\mathbf{I}_N\otimes\mathbf{K}_{O_2})^{\frac{1}{2}}\} \\ \Im\{(\mathbf{I}_N\otimes\mathbf{K}_{O_2})^{\frac{1}{2}}\} & \Re\{(\mathbf{I}_N\otimes\mathbf{K}_{O_2})^{\frac{1}{2}}\} \end{bmatrix}. \quad (39)$$

The resulting optimization problem has the form

$$\begin{aligned} &\min_{\mathbf{b}} \ \mathbf{b}^T\mathbf{O}\mathbf{b} - \mathbf{a}^T\mathbf{b} + tr\{\mathbf{K}_{S_13} + \mathbf{K}_{S_23}\} \\ &s.t.\ \mathbf{a}^T\mathbf{E}_1\mathbf{b} \le P_1,\ \mathbf{b}^T\mathbf{E}_2\mathbf{b} \le P_2 \\ &\quad\ \mathbf{a}^T\mathbf{E}_3\mathbf{b} \le \bar{P}_r \end{aligned} \quad (40)$$

where $\mathbf{O} = \begin{bmatrix} \mathbf{O}_2 & 0 \\ 0 & \mathbf{O}_1 \end{bmatrix}$, $\mathbf{a} = [2\mathbf{a}_2^T, 2\mathbf{a}_1^T]^T$, $\mathbf{b} = [\mathbf{b}_1^T, \mathbf{b}_2^T]^T$, $\mathbf{E}_1 = \begin{bmatrix} \hat{\mathbf{E}}_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{E}_2 = \begin{bmatrix} 0 & 0 \\ 0 & \hat{\mathbf{E}}_2 \end{bmatrix}$, and $\mathbf{E}_3 = \begin{bmatrix} \hat{\mathbf{E}}_3 & 0 \\ 0 & \hat{\mathbf{E}}_4 \end{bmatrix}$. Note that the term $tr\{\mathbf{K}_{S_13} + \mathbf{K}_{S_23}\}$ does not affect the optimization result and so can be ignored. Since $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ and $\mathbf{O}$ are positive semidefinite, the problem can be transformed into a convex QCQP problem and efficiently solved using CVX [27]. The algorithm to solve the optimization problem (8) is summarized in Algorithm 1.

Algorithm 1 Tri-Step Iterative Algorithm to Design $\mathbf{B}_i$, $\mathbf{F}$ and $\mathbf{W}_i$

1: Initialize the algorithm with $\mathbf{B}_i^{(n)} = \sqrt{\frac{P_{si}}{L}}\mathbf{I}_L$, $\mathbf{F}^{(n)} = \sqrt{\frac{P_r}{tr(\sum_{i=1}^2 \mathbf{H}_{S_iR}\mathbf{B}_i^{(n)}(\mathbf{H}_{S_iR}\mathbf{B}_i^{(n)})^H + \mathbf{I}_{N_r})}}\mathbf{I}_{N_r}$, $i = 1, 2$, and set $n = 0$.
2: Update $\mathbf{W}_i^{(n)}$ using (11) with $\mathbf{F}^{(n)}$ and $\mathbf{B}_i^{(n)}$.
3: Update $\mathbf{F}^{(n+1)}$ using (17) and (28) with $\mathbf{W}_i^{(n)}$ and $\mathbf{B}_i^{(n)}$.
4: Update $\mathbf{B}_i^{(n+1)}$ by solving the problem (40)
5: using $\mathbf{W}_i^{(n)}$ and $\mathbf{F}^{(n+1)}$.
6: If
$$(\text{sum\_MSE}^{(n)} - \text{sum\_MSE}^{(n+1)})/\text{sum\_MSE}^{(n)} > \epsilon,$$
go to step 2.
7: End

B. BI-STEP ITERATIVE ALGORITHM

The tri-step iterative algorithm presented above provides good performance according to the results presented in Section V, but the computational complexity is high due to the number of iterations required for convergence. In this section, a bi-step iterative algorithm to obtain the source and relay precoding matrices is presented which has lower computational complexity than the tri-step algorithm. Applying the combiner (11) at the destinations, the MSE of the signal estimate at node i in (7) is a function of $\mathbf{B}_i$ and $\mathbf{F}$ given by

$$J_i = tr\{[\mathbf{I}_{N_{S_i}} + \mathbf{H}_i\mathbf{C}_{n_i}^{-1}\mathbf{H}_i^H]^{-1}\}, \quad (41)$$

where $\mathbf{H}_i = \mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}$. Thus, the joint source and relay precoder optimization problem for the proposed two-way full-duplex relaying system is

$$\min_{\mathbf{B}_i,\mathbf{F},i=1,2} \ J_1 + J_2 \quad (42a)$$
$$s.t.\ tr\left(\mathbf{F}\left(\sum_{i=1}^{2}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{S_iR}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_{D_i}}\right)\mathbf{F}^H\right) \le P_r \quad (42b)$$
$$tr(\mathbf{B}_i\mathbf{B}_i^H) \le P_i \quad (42c)$$

In this iterative algorithm, the source and relay precoders are found by solving two convex subproblems.

Assuming source matrices $\mathbf{B}_i$ satisfying (42c) are given, and eliminating the constraint in (42c), the relay matrix $\mathbf{F}$ is optimized by solving the following problem

$$\min_{\mathbf{F},i=1,2} \ J_1 + J_2 \quad (43a)$$
$$s.t.\ tr\left(\mathbf{F}\left(\sum_{i=1}^{2}\mathbf{H}_{S_iR}\mathbf{B}_i\mathbf{B}_i^H\mathbf{H}_{S_iR}^H + \sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_{D_i}}\right)\mathbf{F}^H\right) \le P_r \quad (43b)$$

It was proven in [13] that the optimal precoding in one-way relaying has parallel channels between the source and relay. Then, singular value decomposition (SVD) can be used between the relay and destination to match the eigenchannels in the two communication hops. Similar to the approach in [14], a heuristic channel parallelization method for bidirectional communications can be employed which uses generalized singular value decomposition (GSVD) for the MAC phase and SVD for the BC phase. Applying GSVD for the MAC phase channel pair $(\mathbf{H}_{S_1R})^H$, $(\mathbf{H}_{S_2R})^H$ gives

$$\mathbf{H}_{S_1R} = \mathbf{V}_h\Gamma_{h_1}\mathbf{U}_{h_1}^H, \quad (44)$$
$$\mathbf{H}_{S_2R} = \mathbf{V}_h\Gamma_{h_2}\mathbf{U}_{h_2}^H, \quad (45)$$

where $\mathbf{V}_h$ is a nonsingular $N_r \times N_r$ complex matrix, $\mathbf{U}_{h_1}^H$ and $\mathbf{U}_{h_2}^H$ are $L \times L$ unitary matrices, and $\Gamma_{h_1} = [\mathbf{0}_{(N_r-L)\times L}^T, \Lambda_{h_1}^T]^T$ and $\Gamma_{h_2} = [\Lambda_{h_2}^T, \mathbf{0}_{(N_r-L)\times L}^T]^T$ where Λ_{h_1} and Λ_{h_2} are $L \times L$ nonnegative diagonal matrices.

For the BC phase, since the superimposed signal is simultaneously transmitted from the relay to both nodes, a virtual point-to-point MIMO channel $\mathbf{H}_{rs} = [(\mathbf{H}_{RS_1})^T, (\mathbf{H}_{RS_2})^T]^T$ can be established. Employing SVD on $\mathbf{H}_{rs}$ gives

$$\mathbf{H}_{rs} = \mathbf{V}_g\Gamma_g\mathbf{U}_g^H, \quad (46)$$

where $\mathbf{V}_g$ and $\mathbf{U}_g$ are $2N_r \times 2N_r$ and $N_t \times N_t$ unitary matrices, respectively, $\Gamma_g = [\Lambda_g^T, \mathbf{0}_{(2N_r-N_t)\times N_t}^T]^T$, and Λ_g is an $N_t \times N_t$

nonnegative diagonal matrix. Employing SVD on $\mathbf{H}_{RS_1}$ and $\mathbf{H}_{RS_2}$ gives

$$\mathbf{H}_{RS_1} = \mathbf{V}_{g_1}\Gamma_g\mathbf{U}_g^H, \tag{47}$$
$$\mathbf{H}_{RS_2} = \mathbf{V}_{g_2}\Gamma_g\mathbf{U}_g^H, \tag{48}$$

where $\mathbf{V}_{g_1} = \mathbf{V}_g(1 : N_r, 1 : 2N_r)$ and $\mathbf{V}_{g_2} = \mathbf{V}_g(N_r + 1 : 2N_r, 1 : 2N_r)$. Note that $\mathbf{V}_{g_1}$ and $\mathbf{V}_{g_2}$ are not unitary matrices. Based on the solution of a similar problem in [14, (29)], the optimal relay precoding matrix obtained by solving problem (43) is

$$\mathbf{F} = \mathbf{U}_g\Lambda_F\mathbf{V}_h^{-1}, \tag{49}$$

and the ith source precoder is

$$\mathbf{B}_i = \mathbf{U}_{h_i}\Lambda_{B_i}\mathbf{V}_{B_i}, i = 1, 2. \tag{50}$$

Substituting (47) and (48) in (41) gives

$$J_i = tr\{[\mathbf{I}_L + (\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}})(\sigma_t^2\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H\mathbf{H}_{RS_i}^H + \mathbf{I}_{D_i})^{-1}(\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}})^H]^{-1}\}. \tag{51}$$

Due to the similarity between J_1 and J_2, we focus on the derivation of J_1 and the results for J_2 can be obtained using the same approach. Substituting (44)-(50) in (51) gives

$$\begin{aligned}J_1 &= tr\{[\mathbf{I}_{N_{s_1}} + (\mathbf{V}_{g_1}\Gamma_g\Lambda_F\Gamma_{h_2}\Lambda_{B_2}\mathbf{V}_{B_2}^H)^H \\ &\times(\mathbf{V}_{g_1}\Gamma_g\Lambda_F\mathbf{V}_h^{-1}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r}) \\ &\times\mathbf{V}_h^{-H}\Lambda_F^H\Gamma_g^H\mathbf{V}_{g_1}^H + \mathbf{I}_{N_{D_1}})^{-1} \\ &\times(\mathbf{V}_{g_1}\Gamma_g\Lambda_F\Gamma_{h_2}\Lambda_{B_2}\mathbf{V}_{B_2}^H)]^{-1}\}\end{aligned} \tag{52}$$

Denoting $\mathbf{B}_{h_{LI}} = \mathbf{V}_h^{-1}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{V}_h^{-H}$ and $\mathbf{B}_{g_i} = (\mathbf{V}_{g_i}\mathbf{V}_{g_i}^H)^{-1}$ gives

$$J_1 = tr\{[\mathbf{I}_L + (\Lambda_{B_2}\Gamma_{h_2}\Lambda_F\Gamma_g)(\Gamma_g\Lambda_F\mathbf{B}_{h_{LI}}\Lambda_F\Gamma_g + \mathbf{B}_{g_1})^{-1}(\Gamma_g\Lambda_F\Gamma_{h_2}\Lambda_{B_2})]^{-1}\} \tag{53}$$

It is obvious that the MSE covariance matrix in (53) is not diagonal since $\mathbf{B}_{h_{LI}}$ is a non-diagonal matrix. To solve this issue, let $\mathbf{C} = \Gamma_g\Lambda_F\mathbf{B}_{h_{LI}}\Lambda_F\Gamma_g + \mathbf{B}_{g_1}$, and $\mathbf{D} = \Lambda_{B_2}\Gamma_{h_2}\Lambda_F\Gamma_g$ so that

$$\begin{aligned}J_1 &= tr\{[\mathbf{I}_L + \mathbf{D}\mathbf{C}^{-1}\mathbf{D}]^{-1}\} \\ &= tr\{\mathbf{I}_L - (\mathbf{I}_L + \mathbf{D}^{-1}\mathbf{C}\mathbf{D}^{-1})^{-1}\},\end{aligned} \tag{54}$$

where the matrix inversion lemma $(\mathbf{I} + \mathbf{A}^{-1})^{-1} = \mathbf{I} - (\mathbf{I} + \mathbf{A})^{-1}$ has been applied. Since $tr\{\mathbf{A}^{-1}\} \geq \sum_i[\mathbf{A}(i, i)]^{-1}$ for any positive definite square matrix $\mathbf{A}$, we have

$$\begin{aligned}J_1 &\leq \mathbf{I}_L - \sum_{i=1}^{L}[(\mathbf{I}_L + \mathbf{D}^{-1}\mathbf{C}\mathbf{D}^{-1})(i, i)] \\ &= tr\{\mathbf{I}_L - (\mathbf{I}_L + \mathbf{D}^{-1}\Lambda_c\mathbf{D}^{-1})^{-1}\} \\ &= tr\{[\mathbf{I}_L + \mathbf{D}\Lambda_c^{-1}\mathbf{D}]^{-1}\},\end{aligned} \tag{55}$$

so

$$J_1 \leq J_1^u = tr\{[\mathbf{I}_L + (\Gamma_g\Lambda_F\Gamma_{h_2}\Lambda_{B_2})(\Gamma_g\Lambda_F\Lambda_{B_{h_{LI}}}\Lambda_F\Gamma_g + \Lambda_{B_{g_1}})^{-1}(\Gamma_g\Lambda_F\Gamma_{h_2}\Lambda_{B_2})]^{-1}\}, \tag{56}$$

where $\Lambda_{B_{h_{LI}}}$ and $\Lambda_{B_{g_1}}$ are diagonal matrices containing the diagonal entries of $\mathbf{B}_{h_{LI}}$ and $\mathbf{B}_{g_1}$, respectively. Now the upper bound in (56) has a diagonal structure, so the precoders can be obtained by minimizing this bound. Assuming $\mathbf{P}_k = \Lambda_k^2$ for $k \in \{h_1, h_2, F, g, B_2, B_2\}$, the upper bound in (56) can be reformulated as

$$J_1^u = \sum_{n=1}^{L}\left(1 + \frac{p_g^n p_{h_2}^n p_F^n p_{B_2}^n}{\lambda_{B_{g_1}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n}\right)^{-1}, \tag{57}$$

where the p_k^n are the diagonal entries of $\mathbf{P}_k$ and λ_k^n, $k \in \{B_{g_1}, B_{g_2}, B_1, B_2, B_{h_{LI}}\}$ are the diagonal entries of Λ_k. The precoder design can then be simplified to the following optimization problem

$$\min_{p_F} \; J_1^u + J_2^u \tag{58a}$$
$$s.t. \; \sum_{n=1}^{L} p_F^n(p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) \leq P_r \tag{58b}$$

This problem is convex and thus can be solved using the KKT conditions. The Lagrangian function of (58) is

$$\begin{aligned}\mathcal{L} = \sum_{n=1}^{L}&\left[\frac{\lambda_{B_{g_1}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n}{\lambda_{B_{g_1}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n + p_g^n p_{h_2}^n p_F^n p_{B_2}^n}\right. \\ &+ \left.\frac{\lambda_{B_{g_2}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n}{\lambda_{B_{g_2}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n + p_g^n p_{h_1}^n p_F^n p_{B_1}^n}\right] \\ &+ \mu\left[\sum_{n=1}^{L} p_F^n(p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) - P_r\right],\end{aligned} \tag{59}$$

where $\mu \geq 0$ is the Lagrange multiplier. Taking the derivative with respect to p_F^n gives

$$\frac{\partial\mathcal{L}}{\partial p_F^n} = \sum_{i=1}^{2}\frac{-(\lambda_{B_{g_i}}^n p_{h_{\bar{i}}}^n p_g^n p_{B_{\bar{i}}}^n)}{(\lambda_{B_{g_i}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n + p_g^n p_{h_{\bar{i}}}^n p_F^n p_{B_{\bar{i}}}^n)^2} + \mu(p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) = 0 \tag{60}$$

and the complementarity condition can be expressed as

$$\mu\left[\sum_{n=1}^{L} p_F^n(p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) - P_r\right] = 0. \tag{61}$$

From (60) and (61) we have

$$p_F^n = \max[0, \mathrm{Root}(f^n)], \forall n, \tag{62}$$

where $\mathrm{Root}(f^n)$ denotes the maximum real root of

$$\begin{aligned}f^n &= \mu(p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) \\ &= \sum_{i=1}^{2}\frac{\lambda_{B_{g_i}}^n p_{h_{\bar{i}}}^n p_g^n p_{B_{\bar{i}}}^n}{(\lambda_{B_{g_i}}^n + p_g^n p_F^n \lambda_{B_{h_{LI}}}^n + p_g^n p_{h_{\bar{i}}}^n p_F^n p_{B_{\bar{i}}}^n)^2},\end{aligned} \tag{63}$$

and μ should be chosen to satisfy

$$\sum_{n=1}^{L} p_F^n(p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) = P_r. \tag{64}$$

This can be efficiently solved numerically as follows.

To find the optimal relay precoder $\mathbf{F}$ in (49), Λ_F must be found. The diagonal entries of Λ_F are given by p_F^n from (62), and (63) can be expressed as

$$\begin{aligned} f^n &= \mu(p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n) \\ &- \frac{\lambda_{B_{g_1}}^n p_{h_2}^n p_g^n p_{B_2}^n}{[\lambda_{B_{g_1}}^n + p_F^n(p_g^n \lambda_{B_{h_{LI}}}^n + p_{h_2}^n p_g^n p_{B_2}^n)]^2} \\ &- \frac{\lambda_{B_{g_2}}^n p_{h_1}^n p_g^n p_{B_1}^n}{[\lambda_{B_{g_2}}^n + p_F^n(p_g^n \lambda_{B_{h_{LI}}}^n + p_{h_1}^n p_g^n p_{B_1}^n)]^2} = 0. \quad (65)\end{aligned}$$

This can be rewritten as

$$\begin{aligned} f^n &= (p_F^n)^4[T^n(S_1^n)^2(S_2^n)^2)] + (p_F^n)^3[T^n(2R_2^n)(S_1^n)^2 S_2^n \\ &+ (2R_1^n)(S_2^n)^2 S_1^n] + (p_F^n)^2[T^n((R_1^n)^2(S_2^n)^2 \\ &+ (S_1^n)^2(R_2^n)^2 + 4R_1^n R_2^n S_1^n S_2^n) - Q_1^n(S_2^n)^2 \\ &- Q_2^n(S_1^n)^2] + (p_F^n)[T^n(2(R_1^n)^2 R_2^n s_2^n \\ &+ 2R_1^n(R_2^n)^2 s_1^n) - (2Q_1^n R_2^n s_2^n + 2Q_2^n R_1^n s_1^n)] \\ &+ T^n(R_1^n)^2(R_2^n)^2 - Q_1^n(R_2^n)^2 - Q_2^n(R_1^n)^2 = 0, \quad (66)\end{aligned}$$

where

$$\begin{aligned} T^n &= \mu(p_{h_1}^n p_{B_1}^n + p_{B_2}^n p_{h_2}^n + \lambda_{B_{h_{LI}}}^n), \\ Q_1^n &= \lambda_{B_{g_1}}^n p_{h_2}^n p_g^n p_{B_2}^n, \\ Q_2^n &= \lambda_{B_{g_2}}^n p_{h_1}^n p_g^n p_{B_1}^n, \\ R_1^n &= \lambda_{B_{g_1}}^n, \\ R_2^n &= \lambda_{B_{g_2}}^n, \\ S_1^n &= p_g^n \lambda_{B_{h_{LI}}}^n + p_{h_2}^n p_g^n p_{B_2}^n, \\ S_2^n &= p_g^n \lambda_{B_{h_{LI}}}^n + p_{h_1}^n p_g^n p_{B_1}^n. \end{aligned}$$

p_F^n is the maximum real root of the polynomial in (66). The precoder $\mathbf{F}$ in (49) is then obtained using Λ_F.

The next task is to obtain $\mathbf{B}_i$ using the optimal relay precoder $\mathbf{F}$ from (49). Using the identity

$$tr\{[\mathbf{I}_m+\mathbf{A}_{m\times n}\mathbf{B}_{n\times m}]^{-1}\}=tr\{[\mathbf{I}_n+\mathbf{B}_{n\times m}\mathbf{A}_{m\times n}]^{-1}\} + m - n,$$

the objective function (41) can be expressed as

$$\begin{aligned} J_i &= tr\{[\mathbf{I}_L + \mathbf{H}_i^H \mathbf{C}_{n_i}^{-1}\mathbf{H}_i]^{-1}\} + N_t - L \\ &= tr\{[\mathbf{I}_L + \mathbf{C}_{n_i}^{-\frac{1}{2}}\mathbf{H}_i\mathbf{H}_i^H\mathbf{C}_{n_i}^{-\frac{1}{2}}]^{-1}\} + N_t - L \\ &= tr\{[\mathbf{I}_L + \mathbf{C}_{n_i}^{-\frac{1}{2}}\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}\mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H\mathbf{H}_{S_{\bar{i}}R}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H\mathbf{C}_{n_i}^{-\frac{1}{2}}]^{-1}\} \\ &= tr\{[\mathbf{I}_L + \mathbf{F}_i\mathbf{Q}_{\bar{i}}\mathbf{F}_i^H]^{-1}\}, \quad (67)\end{aligned}$$

where

$$\mathbf{C}_{n_i} = \sigma_t^2\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{LI}\mathbf{H}_{LI}^H\mathbf{F}^H\mathbf{H}_{RS_i}^H + \mathbf{H}_{RS_i}\mathbf{F}\mathbf{F}^H\mathbf{H}_{RS_i}^H + \mathbf{I}_{N_{D_i}},$$

$\mathbf{Q}_i = \mathbf{B}_{\bar{i}}\mathbf{B}_{\bar{i}}^H$, and $\mathbf{F}_i = \mathbf{C}_{n_i}^{-\frac{1}{2}}\mathbf{H}_{RS_i}\mathbf{F}\mathbf{H}_{S_{\bar{i}}R}$. Using the above results, the original optimization problem can be transformed into

$$\begin{aligned} &\min_{\mathbf{Q}_i} \; tr\{[\mathbf{I}_L+\mathbf{F}_1\mathbf{Q}_2\mathbf{F}_1^H]^{-1}\}+tr\{[\mathbf{I}_L+\mathbf{F}_2\mathbf{Q}_1\mathbf{F}_2^H]^{-1}\} \quad (68a)\\ &s.t. \; tr\{(\mathbf{Q}_1(\mathbf{H}_{S_1R}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_1R}) \\ &\quad + (\mathbf{Q}_2(\mathbf{H}_{S_1R}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_1R})\} \le \bar{P}_r \quad (68b)\\ &\quad tr\{\mathbf{B}_i\mathbf{B}_i^H\} \le P_i \quad (68c)\end{aligned}$$

where $\bar{P}_r = P_r - tr(\mathbf{F}(\sigma_t^2\mathbf{H}_{LI}\mathbf{H}_{LI}^H + \mathbf{I}_{N_r})\mathbf{F}^H)$.

We now introduce positive semidefinite (PSD) matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ that satisfy

$$\begin{aligned} [\mathbf{I}_L + \mathbf{F}_1\mathbf{Q}_2\mathbf{F}_1^H]^{-1} &\le \mathbf{X}_1, \quad (69a)\\ [\mathbf{I}_L + \mathbf{F}_2\mathbf{Q}_1\mathbf{F}_2^H]^{-1} &\le \mathbf{X}_2, \quad (69b)\end{aligned}$$

and using the Schur complement gives

$$\begin{aligned} \begin{bmatrix} \mathbf{X}_1 & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L + \mathbf{F}_1\mathbf{Q}_2\mathbf{F}_1^H \end{bmatrix} &\succeq 0, \quad (70a)\\ \begin{bmatrix} \mathbf{X}_2 & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L + \mathbf{F}_2\mathbf{Q}_1\mathbf{F}_2^H \end{bmatrix} &\succeq 0, \quad (70b)\end{aligned}$$

where $\mathbf{X} \succeq 0$ means that $\mathbf{X}$ is PSD. Since the sum of two PSD matrices with the same dimensions is still PSD, let $\mathbf{X} = \mathbf{X}_1 + \mathbf{X}_2$ so that problem (68) can be converted to the following PSD programming optimization problem

$$\begin{aligned} &\min_{\mathbf{Q}_i,\mathbf{X}} \; tr\{\mathbf{X}\} \quad (71a)\\ &s.t. \begin{bmatrix} \mathbf{X}_1 & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L + \mathbf{F}_1\mathbf{Q}_2\mathbf{F}_1^H \end{bmatrix} \succeq 0 \quad (71b)\\ &\quad \begin{bmatrix} \mathbf{X}_2 & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L + \mathbf{F}_2\mathbf{Q}_1\mathbf{F}_2^H \end{bmatrix} \succeq 0 \quad (71c)\\ &\quad tr\{(\mathbf{Q}_1(\mathbf{H}_{S_1R}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_1R}) \\ &\quad\quad + (\mathbf{Q}_2(\mathbf{H}_{S_2R}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{S_2R})\} \le \bar{P}_r \quad (71d)\\ &\quad tr\{\mathbf{Q}_i\} \le P_i \quad (71e)\\ &\quad \mathbf{Q}_i \ge 0, i = 1, 2 \quad (71f)\end{aligned}$$

The CVX software package [27] can be used to solve problem (71). Then the source and relay precoding optimization problem given in (42) can be solved using the iterative algorithm given in Algorithm 2.

Algorithm 2 Bi-Step Iterative Algorithm to Design $\mathbf{B}_i$ and $\mathbf{F}$

Initialize the algorithm with $\mathbf{B}_i^{(n)} = \sqrt{\frac{P_{si}}{L}}\mathbf{I}_L$, $i = 1, 2$, and set $n = 0$.
$\mathbf{F}^{(n)}$ is first solved with (49) and (62) using $\mathbf{B}_i^{(n)}$.
Update the subproblem (71) using $\mathbf{F}^{(n)}$ to obtain $\mathbf{B}_i^{(n+1)}$.
If $(\text{sum\_MSE}^{(n)} - \text{sum\_MSE}^{(n+1)})/\text{sum\_MSE}^{(n)} > \epsilon$, go to step 2.
End

C. CONVERGENCE ANALYSIS AND COMPLEXITY COMPARISON

1) CONVERGENCE ANALYSIS

The tri-step algorithm can be shown to converge as follows. It is obvious that the subproblems are convex. It then follows

TABLE 1. Average number of iterations and average CPU time required for convergence.

SNR_{rd_i} (dB)	0	5	10	15	20	25	30
Tri-Step Algorithm	4	8	8	12	16	21	24
Bi-Step Algorithm	4	8	8	8	11	11	12
CPU time (Tri-Step Algorithm)	0.0067	0.0053	0.0105	0.0329	0.0610	0.0605	0.0650
CPU time (Bi-Step Algorithm)	0.0062	0.0050	0.0081	0.0115	0.3140	0.0365	0.0383

that each update of $\mathbf{B}_i$, $\mathbf{F}$ and $\mathbf{W}_i$ will decrease or at least not increase the value of the objective function, and thus the iterative algorithm converges to at least a local optimum solution. Similarity, the two subproblems in the bi-step algorithm are convex, so each update of $\mathbf{B}_i$ and $\mathbf{F}$ will decrease or at least not increase the value of the objective function, and thus the bi-step iterative algorithm also converges to at least a local optimum solution.

2) COMPLEXITY COMPARISON

The number of iterations required for convergence for the two algorithms is given in Table 1 for the same tolerance $\epsilon = 0.001$. The parameters used are $N_{s_1} = N_{s_2} = N_t = N_r = 2$ and $\text{SNR}_{s-r} = 30$ dB with SNR_{r-d} set to 0, 5, 10, 15, 20, 25 and 30 dB. The residual loop interference level is set to 10 dB and the number of trials is 1000. These results show that the proposed tri-step algorithm requires more iterations when the SNR is high. When the SNR is 10 dB or less, the algorithms require a similar number of iterations, but the results for this region are not important as the performance is poor. The average CPU time for the two iterative algorithms is also given in Table 1. The simulations were conducted on a Lenovo Thinkpad T470 laptop with an Intel core i7-7500U 2.70 GHz processor. These results show that the bi-step algorithm takes less time than the tri-step algorithm. Thus in terms of the tradeoff between complexity and performance, the bi-step algorithm is a better solution.

V. SIMULATION RESULTS

In this section, the performance of the proposed optimization algorithms is studied through numerical simulation. Flat-fading MIMO channels are considered so the entries of $\mathbf{H}_{S_iR}$ and $\mathbf{H}_{RS_i}$ are i.i.d. complex Gaussian random variables with zero mean and unit variance, As in the literature, the entries of $\mathbf{H}_{LI}$ are i.i.d. complex Gaussian random variables with zero mean and variance σ_{LI}^2.

The received SINR at node S_1 is

$$\Theta_1 = \frac{\frac{P_{s_1}}{N_{t_1}} \left\| \mathbf{H}_{RS_1}\mathbf{F}\mathbf{H}_{S_2R} \right\|^2}{\sigma_t^2 \left\| \mathbf{H}_{RS_1}\mathbf{F}\mathbf{H}_{LI} \right\|^2 + \left\| \mathbf{H}_{RS_1}\mathbf{F} \right\|^2 + \mathbf{I}_{N_{D_1}}}, \quad (72)$$

and the received SINR at node S_2 is

$$\Theta_2 = \frac{\frac{P_{s_2}}{N_{t_2}} \left\| \mathbf{H}_{RS_2}\mathbf{F}\mathbf{H}_{S_1R} \right\|^2}{\sigma_t^2 \left\| \mathbf{H}_{RS_2}\mathbf{F}\mathbf{H}_{LI} \right\|^2 + \left\| \mathbf{H}_{RS_2}\mathbf{F} \right\|^2 + \mathbf{I}_{N_{D_2}}}. \quad (73)$$

The achievable rates are given by $R_1 = \log_2 \det[\mathbf{I}_{N_r} + \Theta_1]$ and $R_2 = \log_2 \det[\mathbf{I}_{N_r} + \Theta_2]$, respectively, where $\det(\mathbf{A})$ denotes the determinant of $\mathbf{A}$. Therefore, the sum achievable rate of the proposed two-way FD relay system can be written as $R_{sum} = R_1 + R_2$.

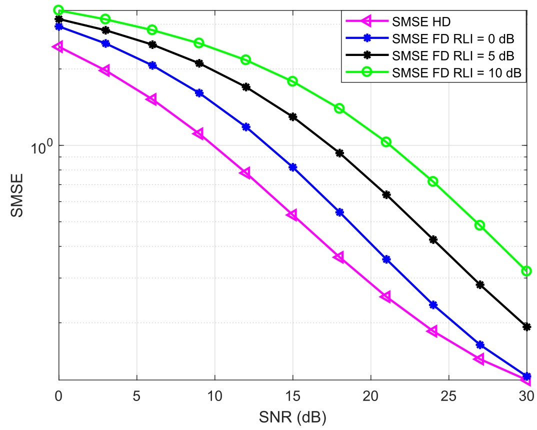

FIGURE 2. Tri-step algorithm sum MSE versus SNR_{s_i-r} with $\text{SNR}_{r-d_i} = 30$ dB.

The performance of the proposed precoding algorithms for a two-way MIMO full-duplex relaying system is examined in terms of the sum mean squared error (MSE) and the sum achievable rate. The results are compared with those of the corresponding half-duplex (HD) relay system. Note that the precoding algorithms for the HD system are the same as for the FD system except that the residual LI term is zero. Further, the achievable rate for the HD system is reduced by half because two time slots are required for information exchange between the two nodes. The signal-to-noise ratios (SNRs) of the source-to-relay and relay-to-destination channels are $\text{SNR}_{s_i-r} = \frac{P_{s_i}}{N_t}$ and $\text{SNR}_{r-d_i} = \frac{P_r}{N_r}$, respectively. For simplicity, it is assumed that perfect channel state information (CSI) is available for all channels. Further, $N_{s_1} = N_{s_2} = N_t = N_r = L = 2$ is assumed in all simulations. The extension to the case with more than 2 antennas is straightforward. All the results are averaged over 1000 trials with independent channel realizations. As discussed in [17], the residual LI can vary from 0 dB to 15 dB larger than the channel noise. Therefore, the residual LI levels considered here are 0 dB, 5 dB and 10 dB. The convergence tolerance for the tri-step iterative algorithm is set to $\epsilon = 10^{-6}$ and the maximum number of iterations is 30.

Fig. 2 presents the sum MSE of the proposed tri-step iterative method versus SNR_{s-r} with $\text{SNR}_{r-d} = 30$ dB. It is clear that the FD system has a higher sum MSE than the HD system due to the existence of residual LI. Further, the sum MSE increases as the residual LI level increases.

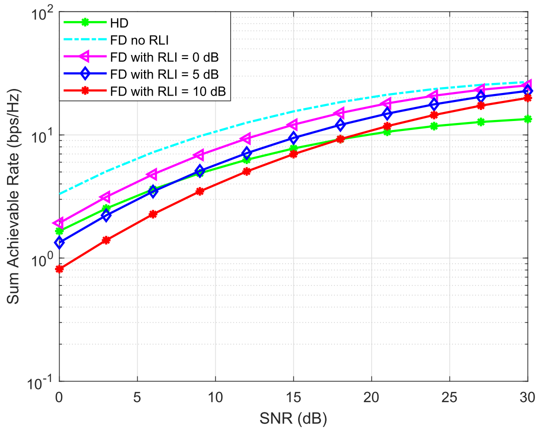

FIGURE 3. Tri-step algorithm sum achievable rate versus SNR_{s_i-r} with $\text{SNR}_{r-d_i} = 30$ dB.

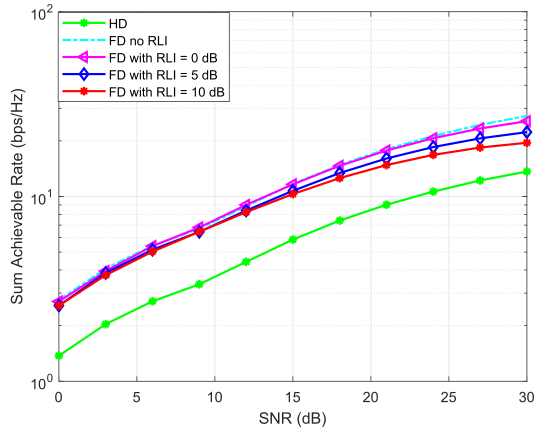

FIGURE 5. Tri-step algorithm sum achievable rate versus SNR_{r-d_i} with $\text{SNR}_{s_i-r} = 30$ dB.

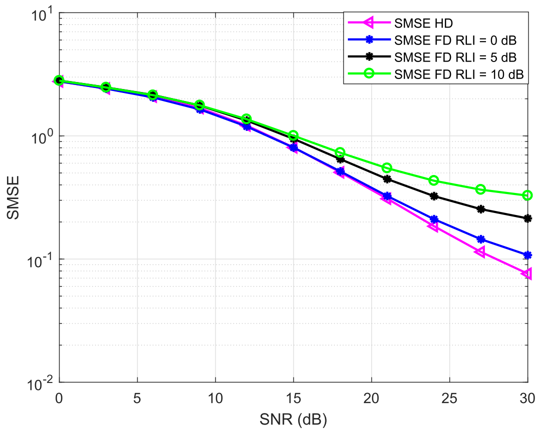

FIGURE 4. Tri-step algorithm sum MSE versus SNR_{r-d_i} with $\text{SNR}_{s_i-r} = 30$ dB.

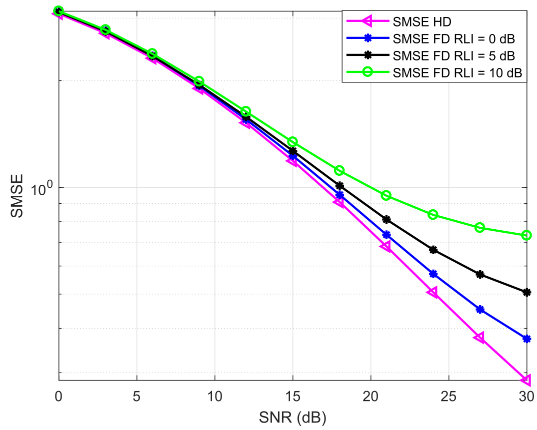

FIGURE 6. Bi-step algorithm sum MSE versus SNR_{r-d_i} with $\text{SNR}_{s_i-r} = 30$ dB.

Fig. 3 presents the achievable rate of the HD and FD systems. The FD system sum achievable rate is twice that of the HD system when the LI is canceled completely. The FD system outperforms the HD system when $\text{SNR}_{s-r} \geq$ 17 dB for all levels of residual LI. Further, when the residual LI level is greater than 5 dB, the HD system outperforms the FD system only when $\text{SNR}_{s-r} < 10$ dB. The HD system outperform the FD system when the residual LI is great than 10 dB and $\text{SNR}_{s-r} < 17$ dB.

Figs. 4 and 5 present the sum MSE and sum achievable rate with a fixed SNR of 30 dB between the source and relay and an SNR between the relay and destination from 0 dB to 30 dB. The sum MSE in Fig. 4 is better than that in Fig. 2 in the low SNR_{r-d} region because a higher transmit power at the relay results in greater residual LI. Fig. 5 shows that the sum achievable rate of the FD system is always higher than that of the HD system for the residual LI levels considered.

Figs. 6 and 7 present the sum MSE and achievable rate for the proposed bi-step algorithm with a fixed SNR of 30 dB between the source and relay and an SNR between the relay and destination from 0 dB to 30 dB. In Fig. 6, the HD system has a higher sum MSE than the FD system for all residual LI levels. The sum MSE of the FD system is degraded as the residual LI level increases. Fig. 7 shows that the sum achievable rate of the FD system is greater than that of the HD system for all values of residual LI.

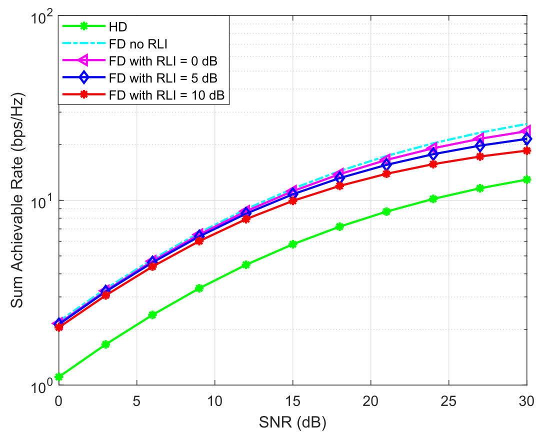

FIGURE 7. Bi-step algorithm sum achievable rate versus SNR_{r-d_i} with $\text{SNR}_{s_i-r} = 30$ dB.

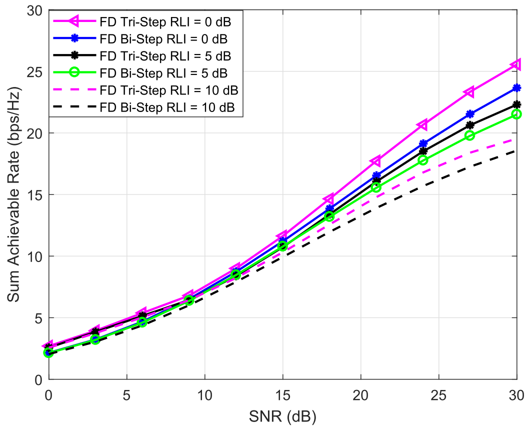

FIGURE 8. Sum achievable rate for the tri-step and bi-step algorithms.

Fig. 8 presents the sum achievable rate for the proposed tri-step and bi-step iterative algorithms. This shows that the rates of the two algorithms are comparable. This performance-complexity tradeoff is an important consideration in the design of practical MIMO FD relay systems.

VI. CONCLUSION

In this paper, locally optimal source and relay precoding and destination combiner design was considered for MIMO two-way full-duplex (FD) relay communication systems. To make the optimization problem tractable, two efficient MSE based algorithms were developed to obtain the source and relay precoding and destination combining matrices. The tri-step iterative algorithm provides optimal solutions to the three corresponding subproblems, while the bi-step iterative algorithm provides optimal solutions to the two corresponding subproblems. The convergence of the algorithms was examined, and the effect of the residual loop interference at the relay on the sum achievable rate was evaluated. Simulation results were presented which demonstrate that both algorithms outperform the corresponding HD relay system in terms of sum achievable rate and sum MSE.

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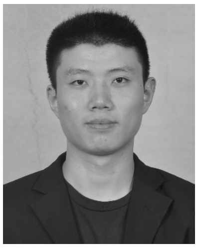

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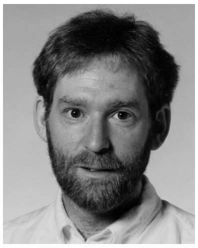

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