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Advanced Combination Localization Algorithm Based on Trilateration for Dynamic Cluster Network

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ABSTRACT Dynamic cluster network localization requires taking the following factors into consideration: 1) the localization time of network nodes; 2) the communication load among all cluster nodes, and; 3) the presence of the abnormal distance measurement data. However, most of the existing network localization algorithms only focus on localization accuracy. To solve the localization problem of the dynamic cluster network nodes, an improved combined trilateral localization algorithm is proposed. This proposed algorithm not only inherits the advantages of trilateration but also addresses the error propagation and accumulation problem with the strategies of anchor nodes selection in combined trilateral localization. Furthermore, by filtering the candidate positions, this algorithm can deal with some abnormal distance measurement data existing in the localization process. Finally, the extensive simulation of the algorithm is performed considering the actual unmanned aerial vehicle (UAV) cluster network, and the experimental results demonstrate the proposed algorithm can achieve high localization accuracy and have robustness as well as good adaptability to the dynamic network.

INDEX TERMS Error propagation and accumulation, combination localization algorithm based on trilateration, data anomalies, fault-tolerant, localization time.

I. INTRODUCTION

With the development of electronic components and radio communication technologies, applications in dynamic cluster network have attracted more and more researchers' attentions, such as UAV swarms, mobile robot swarms and mobile sensor network and so on. As a new information acquisition platform, dynamic network can monitor and collect various kinds of information of monitored objects at certain areas in real time, expand the capability of people to interact with the real world remotely, and accomplish some dangerous tasks which human beings cannot do it. Localization research is the technical support of network application because the foundation of network application is the location information.

In general, localization techniques for network nodes are divided into two categories: range-free and range-based. The range-free localization algorithm is quite simple and corresponding hardware requirement is low. It is generally used in rough localization of network nodes. For example, the DV-hop algorithm [1] uses a distance vector routing to calculate the network average hop distance, and then estimate the node location of the network. The Monte Carlo localization algorithm [2], [3] estimates the location of the mobile node in the next time with probabilistic statistics method.

To achieve high localization accuracy, it is necessary to obtain precise node spacing in most cases. Some range-based localization algorithm such as: multidimensional scaling(MDS) [4]–[6] through the distance matrix between the nodes of the whole network to reconstructed the relative position between nodes, but its drawback is high computational complexity; Aiming for the problem of error propagation and accumulation in trilateral [7], [8] and multilateral [9] localization algorithm, some improved algorithms are proposed. Such as confidence-based iterative trilateral

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localization algorithm [10] improves the localization accuracy by quantifying the geometric distribution of the unknown node's neighbor anchor nodes. However, some of the prior probabilities required to calculate the quality of the anchor nodes distribution are often difficult to obtain. N-times trilateral centroid with weight algorithm [11] can locate blind nodes by the combination of all neighbor anchor nodes. When the number of neighbor anchor nodes is large, the algorithm does not consider its computational complexity. Robust localization algorithm [12], [13] could reduce the computational complexity by locally positioning network nodes with pairwise combination anchor nodes, but unfortunately the localization error is simultaneously increased. The previous study [14] proposes a method to improve the positioning accuracy by pre-optimizing the layout of anchor nodes, but the deployment of anchor nodes only applied to indoor or small area scenarios where nodes can be manually deployed. The iterative multilateral positioning algorithm [15] based on energy constraint takes into account the balance between node energy and positioning accuracy, but due to the different physical properties of network nodes, its energy model is difficult to generalize to other network positioning.

Most of the methods mentioned above are concerned with static networks or single network topology, and less researches are attention to the localization of dynamic network nodes. Kumar et al. proposed a velocity-assisted multidimensional scaling localization algorithm [16] to locate the mobile network, through the measurement of relative velocity. Using the Doppler shift measurement technique to measure the relative velocity between nodes, and then applying it to the MDS stress function to locate the mobile network, but only can be used in that condition which nodes have small moving speed and localization time has not be limited. Besides, the problem of high computational complexity of MDS has still not been solved. The extended kalman filter can be used for the positioning of dynamic systems such as mobile robots [17] and underwater vehicles [18], but the premise is that the dynamic models of the mobile network is known. Qu et al. proposed a fault-tolerant multi-UAV positioning system [19], the azimuth and inclination angle information between the UAVs is used for positioning, which requires additional angle measuring device. Hlavacs [20] proposed a method of using signal propagation time to estimate distance between the nodes which complete the relative localization of target clusters through cooperation between dynamic network nodes to improve the localization precision of this method through continuous communication between the cluster nodes. However they didn't consider communication load of the network.

In our studies, the feature of dynamic network nodes positioning is fully considered. Based on the trilateration, an improved combined trilateral localization algorithm is proposed. The algorithm can achieve good results in localization accuracy, fault tolerance and overall performance. The specific contributions are as follows:

The advanced combined trilateral localization algorithm based on the trilateration technology framework, keeps its simple and efficient advantages, and it only needs to communicate with its one-hop neighbor nodes in the localization process. The communication cost of this technology is relatively low. The premise is that this algorithm can be applied to dynamic network nodes.

This algorithm solves the error accumulation problem of trilateration applied to the actual large-scale network through multiple combinations trilateral localization. At the same time, the anchor nodes selection strategy can reduce computational cost of the algorithm by controlling the number of neighbor anchor nodes participating in the combination, and then ensuring the real-time requirements of dynamic nodes for locations information.

Filtering candidate point sets eliminates the position with large errors, further improves the positioning accuracy, and enables the algorithm to process a certain percentage of abnormal distance measurement information.

The structure of this paper is organized as following: in section II, we will introduce the basic concepts of localization and the theory of the trilateral localization algorithm. Section III details problems of the dynamic cluster network localization and corresponding solutions of the combination trilateral localization algorithm. The application and superiority of the algorithm are verified through the simulation experiment of the engineering parameters based on actual unmanned aerial vehicle cluster in section IV. Finally, it concluded our studies and future works prospect in section V.

II. RELATED WORK

A. BASIC CONCEPTS

Network node localization goal is to use the existing conditions such as nodes connectivity, distance between adjacent nodes and so on, to obtain nodes relative or absolute location information. Some commonly used localization basic concepts are introduced.

1) ANCHOR NODE

some nodes in the network known positions are often called anchor node or beacon node, their positions can be obtained through GPS device or manual deployment. While nodes that need to be localized in the network are called unknown nodes or blind nodes. If there is no anchor nodes, the absolute location of nodes cannot be obtained, only relative location within the network cluster can be got.

2) NEIGHBOR NODE

wireless network node exists the concept of communication range, a node within an other node's communication radius is called that node's neighbor node.

3) LOCALIZATION ERROR

The distance between the located node's position and the actual position of the node is usually used as the localization

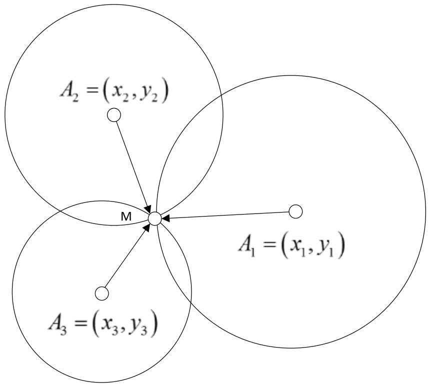

FIGURE 1. Schematic diagram of trilateration.

error, its calculate formula as following:

$$Error = \frac{\sum_{i=1}^{n} \sqrt{(x_i - \tilde{x}_i)^2 + (y_i - \tilde{y}_i)^2}}{n} \tag{1}$$

where, n indicates the number of nodes needs to be localized, (x_i, y_i) represents the real coordinates of the node, and $(\tilde{x}_i, \tilde{y}_i)$ represent the localization coordinates.

Localization error is the most important criteria to evaluate localization algorithm. Additionally, the size of the network, computational complexity and so on, must to be consider, which will be specific analyzed in the subsequent algorithm design and simulation experiments part will specific analysis.

This paper assumes that the number of nodes with unknown position is n, and the number of anchor nodes with known position is m. d_{ij} represent the measurement distance between node i and node j.

B. TRILATERATION

Trilateral localization algorithm is a most common range-based method while has simple and efficient features, and while the distance between nodes measurement error is smaller, we can get a higher localization accuracy. its theory as following:

If there is an unknown node and the distance between three non-collinear anchor nodes in two-dimensional plane, the coordinates of the unknown node can be determined. Now assuming that the coordinates of the three anchor nodes are: $A_1 = (x_1, y_1)$, $A_2 = (x_2, y_2)$, and $A_3 = (x_3, y_3)$ respectively, and coordinates of the blind node M is (x, y), and the distance of the node to each anchor node are:d_1, d_2, and d_3 respectively. The schematic diagram is shown in figure 1 below.

According to two-dimensional space distance calculation formula, a nonlinear system can be obtained as following:

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 = d_1^2 \\ (x - x_2)^2 + (y - y_2)^2 = d_2^2 \\ (x - x_3)^2 + (y - y_3)^2 = d_3^2 \end{cases} \tag{2}$$

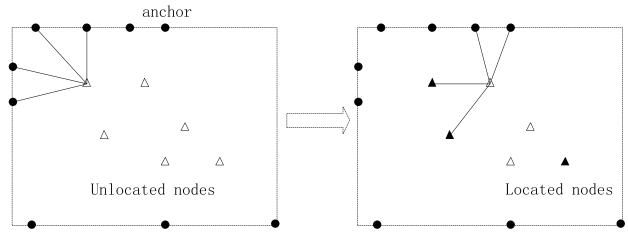

FIGURE 2. Cluster trilateration localization sketch map.

Solving this system of equations yields the coordinates of node M.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2(x_1 - x_3) & 2(y_1 - y_3) \\ 2(x_2 - x_3) & 2(y_2 - y_3) \end{bmatrix}^{-1} \times \begin{bmatrix} x_1^2 - x_3^2 + y_1^2 - y_3^2 + d_3^2 - d_1^2 \\ x_2^2 - x_3^2 + y_2^2 - y_3^2 + d_3^2 - d_2^2 \end{bmatrix} \tag{3}$$

C. ERROR PROPAGATION AND ACCUMULATION PROBLEM

Due to the mobility of nodes, dynamic cluster network leads to the network topologies changeable and has a wide range of coverage. Therefore, it belongs to large-scale multi-hop network. The diagram of the trilateration algorithm used to localize that network nodes is show in figure 2:

As is shown in figure 2, anchor nodes require additional equipment to obtain location information, which caused additional deployment cost. Hence, for most practical applications, the number of anchor nodes is relatively small. Meanwhile, because of the diversity of network application, anchor nodes are often deployed at the boundary of the coverage area of network. Therefore, some blind nodes cannot directly connect to the anchor nodes, which cannot obtain sufficient information for localization. As a result, iterative processing is required, treating nodes that has been located as anchor nodes to help other nodes localization.

Specifically, the anchor node broadcasts its location information to its one-hop neighbor. Subsequently, the unknown node receives the location from anchor node and measures the distance between itself and it. Once an unknown node obtains enough (Two-dimensional plane at least three non-collinear) positions of the anchor node and the corresponding distance information, a trilateration is used to locate the node coordinate. After that, upgrade the node that has been located to an anchor node, and repeat this process until all the nodes in the cluster are located.

The position of a previous located node is not exactly accurate because of the range measurement error. The inaccurate reference node will further increase the localization error of successor node, which is referred to error propagation and accumulation problem.

III. THE ADVANCED COMBINED TRILATERAL LOCALIZATION ALGORITHM

In view of key problems that need to be solved in the dynamic network localization, a corresponding solution is proposed by

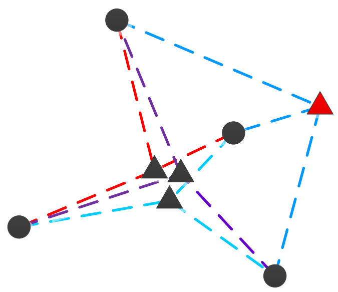

FIGURE 3. The diagram of combined trilateration localization.

the combined trilateral localization algorithm. Firstly, combined neighbor anchor nodes and multi-times trilateration localization to improve the accuracy of node position and to alleviate the problem of error propagation and accumulation. Then through the anchor node selection strategy, limiting the number of anchor nodes involved in the combination, which can greatly reduce the time complexity of the algorithm, in order to complete the localization in the time limit of dynamic network. Finally the filter operation of candidate points set can further improve the localization accuracy and ensure the fault tolerance of the combined trilateral localization algorithm.

A. COMBINED TRILATERAL LOCALIZATION ALGORITHM

In the positioning process, unknown nodes upgraded to anchor nodes after located, may led to more than one neighbor anchor nodes around a blind node location. The traditional trilateration only randomly select three anchor nodes in the neighbor to locate, in this case, the problem of high collinearity of the selected anchor nodes is severity, which results in a large error and influence of other nodes positioning.

The theory of combined trilateral localization is to combining the neighbor anchor nodes of unknown node in different combinations, and then to locate the unknown node with trilateration algorithm to obtain multi candidate localization results, finally enhance accuracy by this candidate sets. Its basic principle is shown in figure 3.

The black dots in the figure above represent the neighbor anchor node (initial or upgraded) around the unknown node. These anchor nodes are combined in proper order, a series of candidate positions are obtained with the trilateral localization algorithm, which are represented by triangle in the figure.

In summary we described the combined trilateral localization algorithm with pseudo code (algorithm I). Where neighbor_anchor_list represents the neighbor anchor node sets involved in localization, including location and corresponding distance information. This algorithm returns the candidate_list, then we can gain the location of the unknown node by calculating its mean value after filtering the list.

B. ANCHOR NODE SELECTION STRATEGY

Furthermore, assuming that the unknown node i has k neighbor anchor nodes, the value of $num = C_k^3$ will be large

Algorithm 1 Combined Trilateral Localization Algorithm

```
Function:combination_anchor
Input:neighbor_anchor_list(alias as node)
Output:candidate_list
Initialize:index = 0, count = size(node)
for i = 1 to count-2
    for j = i + 1 to count-1
        for k = j + 1 to count
            // use trilateral equation calculate unknown nodes coordinate
            candidate_list(index ++) =
                triposition(node(i), node(j), node(k));
        end
    end
end
```

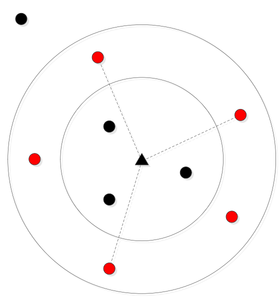

FIGURE 4. The diagram of selection anchor node.

if k is large. Therefore, corresponding candidate point sets will be huge caused by the execution time of localization algorithm increases significantly, which will cause greater time complexity for filter.

In order to solve the problem of time complexity caused by the combination of all neighbor anchor nodes, some nodes can be selected from neighbor anchor nodes to combine, which can guarantee the localization accuracy and meanwhile reduce time complexity of the algorithm. The selection of anchor nodes is shown in figure 4.

Assuming that the maximum number of neighbor anchor nodes involved in the localization is p (the selection of this value is related to the specific network scenario, the experiment part of this paper will test the optimal p value by specific experiments.). When the number of neighbor anchors of an unknown node is more than p, select proper nodes from the ring shown in figure 4. The outer boundary of the ring is the maximum communication range of the node(assumed the range is R), and the inner boundary(assumed the radius of the smaller circle is R1) removes the anchor nodes which are close to the unknown node, then only retains required anchor nodes in the end. The process of anchor node selection algorithm pseudo code as following:

Where neighbor_list represents all the neighbor anchor nodes of a node, and the returned neighbor_anchor_list is the selected anchor node set to participate in the localization,

Algorithm 2 Anchor Node Selection Algorithm

Function:select_anchor

```
Input:neighbor_list, p, R
Output:neighbor_anchor_list
Initialize:index = 0
if size(neighbor_list) <= p
        neighbor_anchor_list = neighbor_list;
else
        sort neighbor_list distance from big to small
        for node in neighbor_list
                if node.distance < R && node.distance > R1
&& size(neighbor_anchor_list) < p
                        neighbor_anchor_list(index ++) = node;
                end
            end
end
```

which is the necessary input for the combine trilateration algorithm. The above strategy chooses anchor nodes in the specific ring, the geometric distribution is better, and the collinearity will be low. From the trilateral equation, anchor nodes which are close to each other have great influence on the coefficient matrix of the trilateral equation, so we prior choose anchor nodes which are far away.

If the number of neighbor nodes of a node is less than 3, it can't be positioned by trilateral localization algorithm. At this time, the power of the node can be increased to make its communication range wider, or other localization constraints must be provided to it.

C. CANDIDATE POINT SET FILTER METHOD

In figure 3, the red triangle can be seen as the result of large localization error as the distance between it and other triangles are relatively large. The main reason may be that the geometric distribution of the three anchor nodes in this condition is poor, or a large error of distance between nodes, even the network node failure etc. Therefore, it is necessary to remove these abnormal locations from the candidate points set, and obtain the final localization result from the mean value of the remaining candidate points.

It is assumed that the candidate points set obtained by combined trilateration is:

$$candidate\_list = \left\{\left(x^j, y^j\right)\right\}_{j=1,2,\cdots,num} \tag{4}$$

where, $num = C_p^3$ represents the number of candidate positions. In theory, the candidate points set is basically concentrated together, but when there is a small amount of abnormal data or some combinations of large localization error, the corresponding candidate points will be far away from most of the others. So it can be assumed that the points service in the set obeys the normal distribution.

$$\sigma^x = \left[\frac{1}{num-1}\sum_{j=1}^{num}\left(x^j - \mu^x\right)^2\right]^{\frac{1}{2}} \tag{5}$$

$$\sigma^y = \left[\frac{1}{num-1}\sum_{j=1}^{num}\left(y^j - \mu^y\right)^2\right]^{\frac{1}{2}} \tag{6}$$

where, $\mu^x = \frac{1}{num}\sum_{j=1}^{num} x^j$, $\mu^y = \frac{1}{num}\sum_{j=1}^{num} y^j$ represent the mean value of coordinates. The equation of the normal distribution is as following:

$$p(x, y) = \frac{1}{2\pi\sigma^x\sigma^y} e^{-\frac{1}{2}\left[\left(\frac{x-\mu^x}{\sigma^x}\right)^2 + \left(\frac{y-\mu^y}{\sigma^y}\right)^2\right]} \tag{7}$$

Define $\eta = \sqrt{\left(\frac{x-\mu^x}{\sigma^x}\right)^2 + \left(\frac{y-\mu^y}{\sigma^y}\right)^2}$. Calculating $\eta\left(x^j, y^j\right)$ for each point in the set $S = \left\{\left(x^j, y^j\right)\right\}_{j=1,2,\cdots,num}$. Besides, deleting the candidate list's point which value is greater than a certain threshold γ (In the experiment of this paper, γ was set as 0.5 and 1 respectively, when there was no abnormal range measurement and there was existing abnormal range measurement.), finally the position of blind node can be gained by solving the mean value of remaining points.

D. SUMMARY OF COMBINED TRILATERAL ALGORITHM

A series of candidate points are obtained by selecting a specific number of neighbor anchor nodes and combining these anchor nodes multi-times trilateral positioning. The abnormal locations in candidate points set are filtered and the mean value is calculated as the final localization result. By combing a few anchor nodes, the time complexity of the combination algorithm can be significantly reduced. Furthermore the selection strategy of anchor nodes and the filter operation of candidate points set will solve the problem of instability results of trilateral algorithm, thus stabilize the localization error at a lower level. The problem of error propagation and accumulation can be alleviated, and the result of abnormal locations and large error can be eliminated, which makes the algorithm accurate and robust. In summary, the pseudo code of the combined trilateral localization algorithm is as follows:

E. ALGORITHM RATIONALITY ANALYSIS

The trilateration equation can be written as a matrix form as follows:

$$AX = b \tag{8}$$

where,

$$A = \begin{bmatrix} 2(x_1 - x_3) & 2(y_1 - y_3) \\ 2(x_2 - x_3) & 2(y_2 - y_3) \end{bmatrix},$$

$$b = \begin{bmatrix} x_1^2 - x_3^2 + y_1^2 - y_3^2 + d_3^2 - d_1^2 \\ x_2^2 - x_3^2 + y_2^2 - y_3^2 + d_3^2 - d_2^2 \end{bmatrix}.$$

Then gives the positioning coordinates $X = A^{-1}b$, its scalar form is exactly (3).

Algorithm 3 An Improved Combined Trilateral Localization Algorithm

Algorithm: combination_ trilateration

Input:anchor node list p R
Output:unknown node's coordinates
// anchor node broadcast coordinate
step1: configuration neighbor_list
step2: if size(neighbor_list) < 3
can't localization;
step3: neighbor_anchor_list = select_ anchor(neighbor_list p R);
step4: candidate_list = combination_anchor(neighbor_anchor_list);
step5: filter abnormal data from candidate_list and calculate the mean as (x, y);

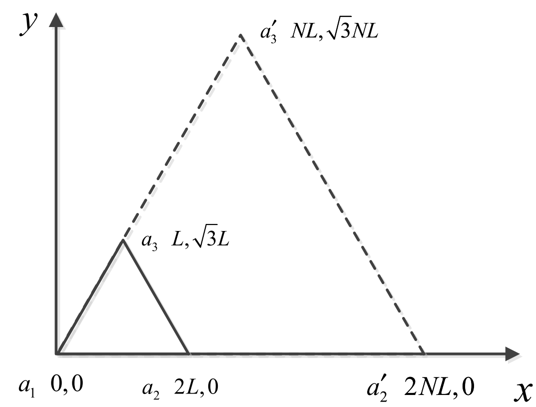

FIGURE 5. Different trilateral positioning cases under the same condition number.

Now used Δb to represent the ranging error, thus:

$$A\left(X + \Delta X\right) = b + \Delta b \tag{9}$$

Then positioning error:

$$\Delta X = A^{-1}\Delta b \tag{10}$$

Take the paradigm on the both sides of (8) and (10):

$$\|b\| = \|AX\| \le \|A\| \, \|X\| \tag{11}$$

$$\|\Delta X\| = \left\|A^{-1}\Delta b\right\| \le \left\|A^{-1}\right\| \|\Delta b\| \tag{12}$$

Union (11) and (12) we can obtainedčž

$$\frac{\|\Delta X\|}{\|X\|} \le \left\|A^{-1}\right\| \|A\| \frac{\|\Delta b\|}{\|b\|} \tag{13}$$

The coefficient matrix A is completely determined by the distribution of anchor nodes. If the position of anchor nodes also has errors, then there are:

$$\left(A + \Delta A\right)\left(X + \Delta X\right) = b + \Delta b' \tag{14}$$

Union (8) and (14) we can obtained:

$$\Delta X = A^{-1}\Delta b' - A^{-1}\Delta AX - A^{-1}\Delta A\Delta X \tag{15}$$

Take the paradigm on the both sides of (15):

$$\begin{aligned}\|\Delta X\| &= \left\|A^{-1}\Delta b' - A^{-1}\Delta AX - A^{-1}\Delta A\Delta X\right\| \\ &\le \left\|A^{-1}\right\| \left\|\Delta b'\right\| + \left\|A^{-1}\right\| \|\Delta A\| \, \|X\| \\ &\quad + \left\|A^{-1}\right\| \|\Delta A\| \, \|\Delta X\|\end{aligned} \tag{16}$$

Then simplified (16) we can get:

$$\begin{aligned}&\left(1 - \left\|A^{-1}\right\| \|\Delta A\|\right) \frac{\|\Delta X\|}{\|X\|} \\ &\le \frac{\left\|A^{-1}\right\| \left\|\Delta b'\right\|}{\|X\|} + \left\|A^{-1}\right\| \|\Delta A\| \\ &= \frac{\|A\| \left\|A^{-1}\right\| \left\|\Delta b'\right\|}{\|b\|} + \left\|A^{-1}\right\| \|\Delta A\|\end{aligned} \tag{17}$$

Derived from (17), it can be obtained that:

$$\frac{\|\Delta X\|}{\|X\|} \le \frac{\|A\| \cdot \left\|A^{-1}\right\|}{1 - \|A\| \cdot \left\|A^{-1}\right\| \frac{\|\Delta A\|}{\|A\|}} \left(\frac{\left\|\Delta b'\right\|}{\|b\|} + \frac{\|\Delta A\|}{\|A\|}\right) \tag{18}$$

where, $\frac{\|\Delta b'\|}{\|b\|}$ is the ranging error and $\frac{\|\Delta A\|}{\|A\|}$ is the anchor node position error, so the relative error $\frac{\|\Delta X\|}{\|X\|}$ is affected by the condition number of coefficient matrix [21]. In fact equation (18) is a theorem when $\left\|A^{-1}\right\| \|\Delta A\| < 1$.

We call the equation with a large condition number $cond\left(A\right) = \left\|A^{-1}\right\| \|A\|$ is "ill-condition equation". Small changes in coefficients of the ill-condition equation will also lead to large changes in the solution, as shown in the following example:

$$\begin{pmatrix} 2 & 6 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix} \tag{19}$$

The exact solution of equation (19) is $x = (1, 1)^T$, if there is some interference in the parameters of the equation at this time, for example:

$$\begin{pmatrix} 2 & 6 \\ 2 & 5.99999 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 8.00002 \end{pmatrix} \tag{20}$$

Then the solution shows that $x = (10, -2)^T$, although the parameter of the equation has only a little change, the solution of the equation is quite different. The coefficient matrix of equation (19) is irreversible, so the condition number is: ∞.

In trilateral positioning equation, the interference of equation parameters is due to the error of anchor nodes and the error of range measurement. The condition number of the equation mainly depends on the distribution of the anchor nodes which used to locate unknown nodes, and we can be found that the condition number is inversely proportional to the size of the smallest inner angle of the triangle with three anchor nodes, that is, the larger the smallest inner angle is, the smaller condition number is. When the equilateral triangle is formed, the minimum condition number is 5.1963.

In addition, as shown in figure 5 below, (a_1, a_2, a_3) and $\left(a_1, a_2', a_3'\right)$ are anchor nodes for positioning an unknown node, the condition number of these two triangles formed equation is the same.

because of

$$A^{-1} = \frac{1}{2(x_1 - x_3)(y_2 - y_3) - 2(x_2 - x_3)(y_1 - y_3)} \cdot \begin{bmatrix} y_2 - y_3 & -(y_1 - y_3) \\ -(x_2 - x_3) & x_1 - x_3 \end{bmatrix} \tag{21}$$

Therefore, it can be seen that when the three coordinates of the triangle are expanded by N times, we can get:

$$A'^{-1} = \frac{1}{N} A^{-1} \tag{22}$$

That is to say, anchor nodes with the same condition number have different influences on positioning results. In triangulation, the longer the shortest side of the triangle formed by three anchor nodes, the smaller the element value in A^{-1}. The error $A^{-1}(\Delta b)$ is smaller.

In summary, select the condition number of equation coefficient matrix smaller, that is the anchor node with better geometric distribution to locate unknown node, can avoid large positioning error due to poor anchor nodes distribution. Therefore, combined trilateral positioning algorithm is proposed, and through filter operation of candidate points set to reserve some better results. Furthermore, in order to further improve the localization accuracy and realize the fault-tolerance ability of the combined trilateral localization algorithm, and reduce the computational time complexity of large amount combinations for combined all anchor nodes, the corresponding anchor node selection strategy is proposed based on the coefficient matrix condition number theory and its inverse matrix computing method.

F. ALGORITHM TIME COMPLEXITY ANALYSIS

In a dynamic network, the position of mobile nodes has time validity, so the time complexity of the localization algorithm and the communication cost of the dynamic cluster can't be so large.

From (3), we can see that the time complexity of the trilateration is $O(1)$, and the main time spend of the combined trilateration algorithm is to combine the neighbor anchor nodes and filter the candidate points set. Assuming that there are k neighbor nodes around a blind node, the combined trilateration calculate times is: $\frac{1}{6}\left(k^3 - 3k^2 + 2k\right)$, when k is large, the time complexity of the algorithm is close to $O\left(k^3\right)$. When a smaller threshold value p is confirmed, the time complexity is much smaller than $O\left(k^3\right)$. For example, when $p < 9$, the time complexity will be less than $O\left(k^2\right)$. The above section III.B focuses on controlling the p-value in a smaller range under an expected positioning accuracy. And the filter operation only needs to be judged once for each point in the candidate points set.

On the other hand, the trilateration algorithm is a distributed algorithm. Localization between nodes does not interfere with each other and they are performed at the same time. And only needs to maintain one-hop neighbor node's connection, therefore, the cost of communication is low. The time complexity to localization the whole network only needs to consider the localization rounds of the algorithm, assuming that the value is q. Since the whole network is connected, the value of q will not be much large, following simulation experiments also illustrate this. Therefore, values of p and q are controlled in a smaller range, the localization time complexity of our algorithm for the whole network will be very low.

IV. SIMULATION EXPERIMENT

The MATLAB simulation tool is used to simulate the combined trilateration localization algorithm in this paper. The configuration of the computer is as follows, System: windows 7; processor: Intel(R) Core(TM) i5-4570 CPU 3.20GHZ; memory: 4.0GB.

In order to verify performances of the algorithm of this paper, we must consider in two aspects. The first one is to choose important parameters of the algorithm which can determine best performance of our algorithm. Then it needs to be compared with other methods under same conditions to verify the superiority of our algorithm. Furthermore, as the above mentioned our algorithm has the fault-tolerant ability, it is necessary to design corresponding experiments for relevant verification. The results and theoretical analysis of each experiment are important basis for our algorithm to practical applications.

Our experiment is carried out based on the actual UAV cluster parameters, and in this paper we assumed that the altitude of unmanned aerial vehicles has small difference, i.e. take into account the localization in the two-dimensional plane. In the area of 1000*1000 meters, 100 UAVs are randomly deployed, which initial speed is a random value in 0-400m/s, the acceleration varies randomly from 0 to 100 m/s2, and the direction changes randomly within 0-360 degrees. At the boundary of the region, an anchor node is placed every 50 meters, so the unknown node's number n = 100, the anchor node's number m = 80.

In the simulation experiment, a certain distance error is added to the real distance between two UAVs as the actual measurement distance. Assuming that the communication radius of the UAV and the measurement distance of the UWB range technology [23], [24] are all 200 meters, and the range error obeys uniform distribution in [10, 30]cm.

The experiment result is mainly evaluated by the nodes localization error. And in order to ensure reliability of the simulation experiment, the result of each experiment is the average value of 30 times simulations.

A. EXPERIMENT OF DIFFERENT PARAMETERS

First of all, the value of parameter p is simulated, which is the maximum number of neighbor anchor nodes retained for combined localization. Taking different values for p, the error bar diagram of network localization error is obtained as shown in figure 6.

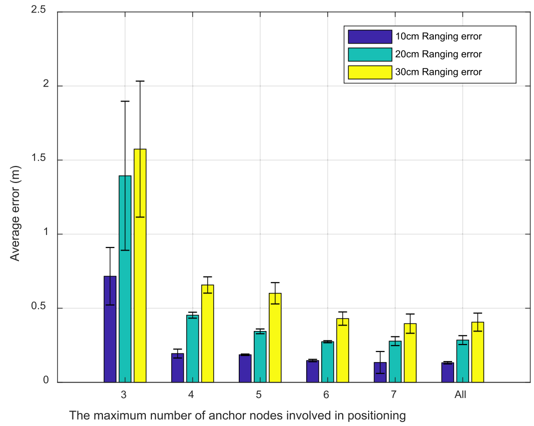

FIGURE 6. Different P value localization results under 100 nodes.

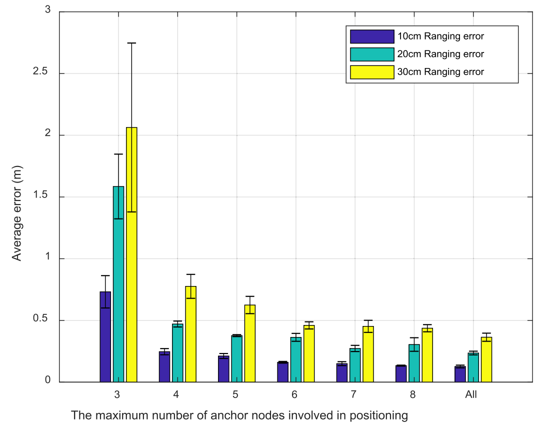

FIGURE 7. Different P value localization results under 200 nodes.

TABLE 1. The running time of the algorithm with different number of combined anchor nodes.

Anchor nodes	3	4	5	6	7	8	all
Algorithm time (s)	0.318	0.354	0.422	0.529	0.663	0.806	2.389

In figure 6 combining three anchor nodes is the original trilateral localization method, only select three anchor nodes from the unknown node's neighbors simply and randomly. Because of random selection the collinearity of that three anchor nodes may be high, which leads to greater localization error. Besides the combination of all neighbor anchor nodes neglect the whole number of neighbor anchor nodes, that is no matter how many neighbor anchor nodes are, they are both combined in proper order.

In order to verify the influence of different network scale on parameters and final localization results, we take n = 200 and do same experiments as above. The experimental results obtained are shown in figure 7. Compared with 100 nodes, the node density of 200 node network increases. When n is equal to 100, the whole network localization rounds q is between 4 to 8 by statistics, and the maximum number of neighbor anchor nodes k is between 10 to 21; while when n is equal to 200, q is between 3 to 5 and k is between 16 to 26.

Under the current computer configuration, we calculated the running time of the algorithm in the network with 200 nodes with different maximum number of combined anchor nodes. and the results are shown in table 1 below.

As show in TABLE 1 we can found that with the increase of the number of anchor nodes, the computation time of the algorithm increases rapidly. For the mobile nodes, there will be a large location offset, and the cost of computation will also increase. And the denser of network nodes is, set the smaller of the p value is, will reduces the time complexity of the algorithm more significantly.

Combined with figures 6 and 7, it can be concluded that whether the number of UAVs in the network is 100 or 200, the localization error decreases with the increasing number of combined anchor nodes. At the same time, it is found that when p takes a relatively small value, it also obtains almost close localization results as combined all the neighbor anchor nodes, and will reduce computation significantly. For example, when $n = 100$, make $p = 7$; $n = 200$, make $p = 8$, both the time complexity and localization accuracy satisfied the application requirements.

In different network scenarios, the appropriate p value can be selected to balance requirements of localization accuracy and time complexity.

TABLE 2. Comparison the results of different localization algorithms.

Average localization error (m)		Different range error		
		10cm	20cm	30cm
Different localization algorithms	combined trilateral localization algorithm	0.134	0.278	0.446
	The original trilateral algorithm	0.732	1.394	1.585
	multilateral localization algorithms	0.131	0.290	0.439
	MDS localization algorithm	5.897	6.011	6.122

B. ALGORITHM CONTRAST EXPERIMENT

In order to further verify the superiority of our algorithm, the combined trilateration localization algorithm is compared with the original trilateration algorithm, Least Squares algorithms[22](multilateral localization), and MDS localization algorithm. The parameter of the network are the same as above, $n = 100$, the combined trilateration localization algorithm $p = 7$, and the MDS algorithm needs to construct the distance matrix of the whole network, we use Dijkstra shortest route algorithm to calculate the distance between multihop nodes. Localization error as the evaluation indicator is to estimate different algorithm, results is shown in Table 2 as following:

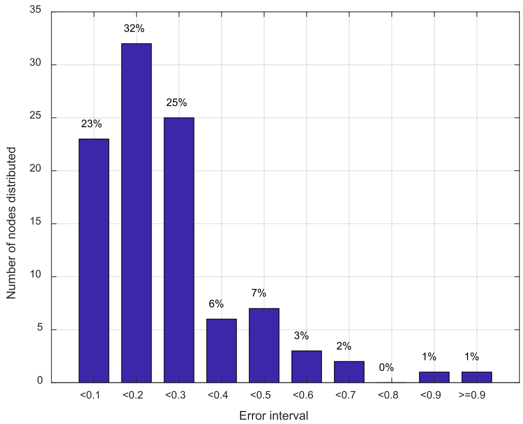

FIGURE 8. Node distribution in different error zones.

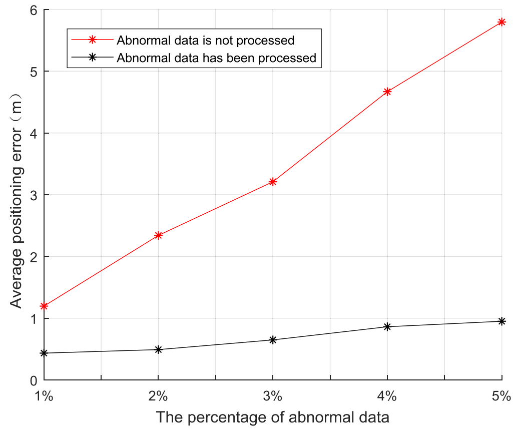

FIGURE 9. Localization results under abnormal data.

Firstly, it can be seen that among the above four positioning algorithms, the result of combined trilateral positioning and multilateral positioning is best. This is because of the more anchor nodes involved in positioning, the more constraints are given to the blind nodes, so the higher positioning accuracy of the results. However as the number of anchor nodes reaches to a certain extent, the accuracy will not increase any more, instead it needs to maintain more neighbor anchor nodes.

The information of multiple anchor nodes in multilateral positioning actually provides an average optimal positioning result, the more anchor nodes involved in positioning, the better positioning accuracy is. However, when the number of anchor nodes reaches a certain value, the improvement of the algorithm is very limited. When the range error is large or there are some abnormal measurement data, the disadvantage of the average strategy of least squares is reflected (experimental results under abnormal data are shown in Section IV.C). Therefore, the performance of the combined trilateral positioning algorithm is best.

The original trilateral positioning has a large error, because the random selection has the high collinearity of anchor nodes. The algorithm in this paper generates candidate point sets, which can ensure the localization error always be in a lower level. The error of the MDS localization algorithm is largest, because the shortest route is used to obtain the distance between multi-hop nodes, which caused the larger error between the obtained distance and the actual distance. Meanwhile the time complexity of MDS algorithm is $O\left(n^3\right)$, so it is difficult to apply it in the localization of fast moving dynamic cluster network nodes.

In order to grasp the positioning situation of the whole network, preset parameter the range error is 20 cm, we statistics the error distribution of all nodes under the combined trilateral localization algorithm, the result is shown in figure 8 below.

As show in figure 8, the localization error of 80% nodes is less than 0.3, which is close to the average localization error of the network. In addition, only 7% nodes localization error is larger than 0.5, and only 1 % nodes localization error is larger than 0.9. This stability of localization results demonstrates the feasibility of combined trilateral localization algorithm. No outliers are generated, which directly proves the efficiency of the anchor nodes selection strategy and candidate points set filter method.

C. ABNORMAL DATA EXPERIMENT

Due to the mobility of the node, dynamic network is easily disturbed by the environment, which leads to the anomaly of the distance measurement. The combined trilateration localization algorithm has ability to deal with abnormal data through the selection of anchor nodes and the filter of the candidate points set.

Assuming normal conditions that the range error between nodes is 20cm, and the abnormal range measurement error is 10 m. As shown in figure 9, it is the localization result of different proportions of abnormal range in the network.

The percentage of abnormal data refers to the corresponding proportion of outliers in distance measure between nodes of the whole network. The result of unhandled anomaly localization means the selection of anchor nodes is random, and the candidate set is not filtered, and the mean value is obtained directly. The localization result of dealing with the anomaly is the algorithm in this paper.

It can be seen from the experimental results above, the location error will increase sharply when there is a certain proportion of range anomalies. In this paper, our method eliminates the abnormal measure data through the selection of anchor nodes and the filter of the candidate points set strategy, and ensures a relatively low localization error under a small abnormal proportion.

In order to further confirm the fault tolerance ability of the proposed algorithm, under the same experimental parameters, we designed experiment compare the combined trilateral positioning with the multilateral positioning method and two robust positioning algorithms [12], [13] ([12]: Bilateration, [13]: Novel Robust Trilateration Method) when there is presence of outlier ranging data. The positioning result is shown in Figure 10 below:

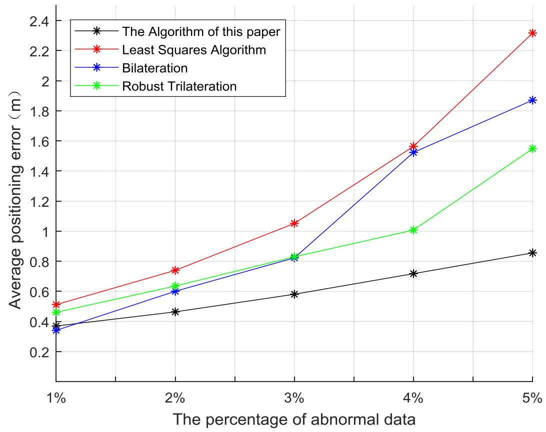

FIGURE 10. Comparison with others positioning algorithm under abnormal data.

As shown in figure 10, it can be found that among the four methods, our proposed positioning algorithm has the best fault-tolerant ability, the reason is the selection strategy of anchor nodes and the filtering operation of candidate points set can eliminate abnormal data, and thus ensuring the positioning accuracy of the algorithm. However, the positioning error of the multilateral positioning algorithm increases sharply in presence of abnormal data, this is because multilateral positioning uses the least squares strategy, and the balanced approach causes the normal positioning information to be affected. The method in literature [12] use two anchor nodes to determine two candidate position, and then filter the candidate positions to get the final positioning result. The localization method locating the position through two anchor nodes is increases the location error, which makes it difficult to select the threshold in the filtering stage. In addition, the way through the candidate locations to weighted anchor nodes to determine abnormal data need large communication overhead. The fault-tolerant ability of literature [13] is slightly better than that of literature [12]. The method is combines all neighbor anchor nodes and directly average the candidate points set to get the positioning result, so when the proportion of abnormal data is slightly larger, that algorithm no longer has ability to deal with, furthermore its computational complexity of combined all neighbor anchor nodes is large.

Therefore, the experimental results show that the improved combined trilateral localization algorithm has high positioning accuracy and fault-tolerant ability. Combined with the analysis of the time complexity of the algorithm mentioned above, it can be concluded that combined trilateral localization algorithm has better comprehensive performance.

V. CONCLUSION

On the basis of the trilateral localization algorithm, advanced combination localization algorithm based on trilateration is proposed. Firstly, by combining neighbor anchor nodes and multiple times trilateration localization, it is possible to prevent the result position which with large or unacceptable errors, due to the higher collinearity of anchor nodes. Candidate selection based on optimal positions can effectively reduce nodes localization error and mitigate the problem of error propagation and accumulation. Then choosing a smaller number of anchor nodes, can greatly reduce the time complexity of the algorithm, especially in the dense network. Finally, the robustness of the algorithm is guaranteed by anchor nodes choice and candidate points set filter operation, it allows a small percentage of abnormal range data appear in the network. The simulation experiments section, from the algorithm's own parameters confirmation, comparison with other methods, and the abnormal data tested aspects.

The advanced combined trilateral localization algorithm is an efficient, high-accuracy distributed algorithm, whose nodes only need to communicate with their one-hop neighbor nodes. It is appropriate for specific area dynamic cluster network, some satellite restrictions, adverse wireless environment dynamic network nodes localization and so on. On the basis of this article, the main consideration in the follow-up works is to extend the algorithm to three-dimensional space. In addition to except process the range measurement abnormal, it is also necessary to take into account the problem of network abnormality in various situations such as node failure and malicious attack.

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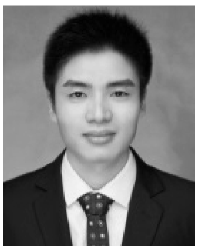

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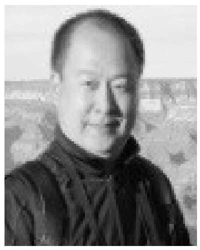

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