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Game Theoretic Strategies Design for Monostatic Radar and Jammer Based on Mutual Information

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ABSTRACT In electronic warfare, the conflict relationship between the radar and the jammer can be modeled using game theory. In this paper, the strategies design problem for the monostatic radar and the jammer is investigated within the framework of Stackelberg game and egalitarian game. The radar waveform and the jammer power spectrum density are regarded as their strategies respectively and mutual information criterion is used to formulate the utility function. In the Stackelberg game, Stackelberg Equilibrium (SE) strategies of the radar and the jammer are derived based on a two-stage optimization method. In the egalitarian game, the existence condition of Nash equilibrium (NE) is investigated and the corresponding NE strategies are also given. If the existence condition is not satisfied, it is pointed out that the SE strategies are still acceptable as safe strategies from the perspective of game theory. The simulation results are presented and the performances of the SE strategies are compared with other strategies.

INDEX TERMS Game theory, anti-jamming and jamming, strategy design, mutual information.

I. INTRODUCTION

Radar electronic countermeasure (ECM) and electronic counter-countermeasures (ECCM) have been a challenging area for a number of years. The ECM systems aim at preventing the enemy's radar from working correctly, while the objective of ECCM systems is to protect the radar from being jammed [1]–[3]. Traditional ECM and ECCM techniques are designed to operate against dumb opponents, where the opponent's countermeasures are not taken into consideration. For example, the spot noise is a kind of active ECM technique which is designed to jam the radar with fixed carrier frequency [4]. However, if the radar takes countermeasures such as frequency agility, the spot noise can not work well. As a result, traditional ECM and ECCM techniques will be confronted with a bigger challenge than before on account of that the radar and the jammer are both becoming smarter.

Game theory is a kind of mathematical tool that focuses on modeling the conflict and cooperative relationship between rational and intelligent decision-makers [5]. It is appropriate to apply game theory to analyze the strategies of the smart radar and the jammer. By doing that, the opponent's countermeasures can be taken into consideration when designing the ECCM or the ECM techniques [6]–[8]. In [6], the radar and the jammer are modeled as informed players in a non-cooperative two-person zero-sum (TPZS) game. The effects of jamming on target detection performance of the radar with a constant false alarm rate processor are investigated, and several important conclusions are given. The power game between a smart statistic multiple-input multiple-output (MIMO) radar and a smart jammer is well studied based on mutual information (MI) in [7]. The strategies of the radar and the jammer are studied from the perspective of hierarchical and symmetric games, which mainly differ from whether the decision-making process is symmetric. In [8], the work in [7] is extended by taking the effects of clutter into consideration and a novel two-step water-filling algorithm is proposed.

Motivated by the work in [7] and [8], we investigate the game between the smart monostatic radar and the smart jammer in this paper. In the game, the strategy of the monostatic radar is the transmit waveform. Transmit waveform design is of vital importance because many critical radar performance metrics, such as the signal-to-noise ratio (SNR), the range resolution, and so on, have a close relationship with the radar waveform [9]. It should be emphasized that the strategy of the radar is a spectrally designed waveform and the algorithm

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proposed in [10] can be used to acquire constant modulus waveform. As for the jammer, power spectrum density (PSD) is regarded as its strategy.

With regard to the radar waveform optimization, MI criterion is one of the most commonly used criteria, which is used to formulate the utility function in the game mentioned above. In fact, MI has been used as a criterion in the radar waveform design problem for many years. Bell points out that target classification ability or average measurement error will benefit from the improvement of the MI between the received signal and the target impulse response. Based on that, a waveform design method termed water-filling is proposed in [11]. Matched waveforms design based on MI criterion is investigated in [12] and the relationship between MI and SNR is analyzed. On the foundation of the MI criterion, two MIMO radar waveform design methods are presented in [13] and [14]. Note that the countermeasures of the jammer are not taken into consideration among these MI-based waveform design methods. The performance of these methods may degrade heavily since the jammer will take countermeasures in practice.

The main contributions of this paper are summarized as follows.

(1)The TPZS game theory model is used to characterize the relationship between the monostatic radar and the jammer based on the MI utility function. Two kinds of game theory models, including egalitarian game and Stackelberg game, are taken into consideration.

(2)In the Stackelberg game, where the radar and the jammer are asymmetric, the Stackelberg equilibrium (SE) strategies of the radar and the jammer are derived based on the two-step optimization method.

(3)In the egalitarian game, where the radar and the jammer are symmetric, the existence condition of the NE is derived and the NE strategies are also given. In addition, if the NE does not exist, it is pointed out that the SE strategies are still acceptable from a game theory perspective.

The remainder of the paper is organized as follows. In section 2, the signal model is first presented. On this basis, the formulation of the MI utility function is then introduced briefly. In addition, the basic concepts of the egalitarian game and Stackelberg game are given. In section 3, the SE strategies are derived in detail when the radar and the jammer are in the Stackelberg game. In section 4, the existence condition of the NE is investigated and the NE strategies are also given. Simulation results are shown in section 5 and then conclusions are drawn in section 6.

II. PROBLEM FORMULATION

A. SIGNAL MODEL

The signal model, as depicted in Fig.1, is considered in this paper. $x(t)$ is a finite-energy deterministic waveform with energy E_x and duration T. It is transmitted by the monostatic radar and scattered by the extended target whose random impulse response is $\mathbf{h}(t)$. The resulting signal $\mathbf{z}(t)$ is corrupted by the zero-mean addictive Gaussian noise process $\mathbf{n}(t)$ with PSD $S_{nn}(f)$ and the signal-independent jamming signal $\mathbf{j}(t)$ released by the jammer. The total signals are received by the radar and filtered by an ideal band-pass filter whose impulse response is $b(t)$.

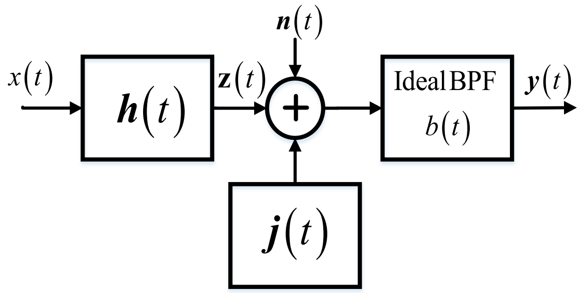

FIGURE 1. Block diagram of signal model.

$\mathbf{h}(t)$ is a finite-energy finite-duration random process and it can be defined by multiplying a stationary Gaussian random process $\mathbf{g}(t)$ with a rectangular window function $a(t)$ with duration T_h [12], [15]. Note that $\mathbf{h}(t)$ is not a true stationary Gaussian random process and has limited energy. As a result, its PSD is not available to describe the scattering characteristic of the extended target.

However, an alternative approach can be used to do that. If $\mathbf{H}(f)$ is denoted as the Fourier transform counterpart of $\mathbf{h}(t)$, then an energy spectral density (ESD) can be defined as follows [11], [12], [15]

$$\xi_H(f) = E[|\mathbf{H}(f)|^2], \tag{1}$$

where $E[\bullet]$ represents the expectation operation. The mean of $\mathbf{H}(f)$ is denoted as $\mu_H(f) = E[H(f)]$, then the energy spectral variance (ESV) is given below

$$\sigma_H^2(f) = E[|\mathbf{H}(f) - \mu_H(f)|^2]. \tag{2}$$

Generally, $\mu_H(f)$ is assumed to be zero and then the ESV and ESD functions are equivalent [11]. Note that ESV is different from PSD and it expresses the average energy of a finite-energy, zero-mean random process.

B. UTILITY FUNCTION

A barrage jammer, which is a common type of active suppression jammer [4], is taken into consideration in this paper. It is assumed that $\mathbf{j}(t)$ is a Gaussian random process whose PSD is $P_j(f)$ and is independent of the transmit waveform.

The observed signal $\mathbf{y}(t)$ can be expressed by (3). Let T_b be the duration of the ideal band pass filter then the duration of the convolution output $\mathbf{y}(t)$ is $\tilde{T} = T + T_b + T_h$.

$$\mathbf{y}(t) = b(t) * [x(t) * \mathbf{h}(t) + \mathbf{n}(t) + \mathbf{j}(t)] \tag{3}$$

The MI between the received signal $\mathbf{y}(t)$ and the target random impulse response $\mathbf{h}(t)$ given a deterministic transmit waveform $x(t)$ is chosen as the utility function of the game and it can be approximated by (4) [12]

$$I(\mathbf{h}(t); \mathbf{y}(t) | x(t)) = \tilde{T} \int_{\mathcal{W}} \ln \left\{ 1 + \frac{|X(f)|^2 \sigma_H^2(f)}{\tilde{T}\left[S_{nn}(f) + P_j(f)\right]} \right\} df, \quad (4)$$

where $\mathcal{W} = [f_0 - \frac{W}{2}, f_0 + \frac{W}{2}]$ is the frequency interval to which the energy of the radar waveform is confined and $|X(f)|^2$ is the magnitude-squared spectrum of $x(t)$. With the assumption of $T \gg T_h$ and $T \gg T_b$, $\tilde{T}$ approximately equals to T [11], [12].

Strictly speaking, $x(t)$ is not limited in the frequency domain but an approximate bandwidth W can be acquired by confining the majority of the signal energy to that frequency interval. The derivation of (4) is described in detail in [12]. The main difference between the utility function and the approximate MI in [12] is that a signal-independent jamming signal is incorporated in (4).

The classification performance and the measurement error are closely related to the MI between the parameter X which is measured and the measurement Y. According to the two propositions in [16], it can be concluded that the greater the MI between X and Y becomes, the better performance we can obtain.

Just like the measurement problem mentioned above, the performance of the radar systems is also closely related to the MI between the received signal and the target random impulse response. As a result, larger $I(\mathbf{h}(t); \mathbf{y}(t)|x(t))$ will lead to a better performance in both classification and measurement of the radar systems.

C. GAME THEORY BACKGROUND

In game theory, zero-sum game is a kind of competitive game and the sum of the outcomes of all the players is equal to zero [17]. That means the players are in a strict competitive situation. If there are only two persons in the zero-sum game, then the game is usually called TPZS game.

In a TPZS game, let $\mathbf{a}$ be the minimizer, $\mathbf{b}$ be the maximizer and $f(\mathbf{a}, \mathbf{b})$ be the utility function of the game. Here the TPZS game can be divided into egalitarian game and Stackelberg game according to whether the domination of the players exists [18].

In the egalitarian game, the two players are symmetric, which means they have no prior information about the strategies of their opponent. Therefore, the two players arrive at their strategies independently and NE is usually used to solve the egalitarian game. At the NE point, no player can benefit from the unilateral change of his strategy. The pure-strategy NE $(\mathbf{a}^*, \mathbf{b}^*)$ can be defined as follows [18]

$$f(\mathbf{a}, \mathbf{b}^*) \geq f(\mathbf{a}^*, \mathbf{b}^*) \geq f(\mathbf{a}^*, \mathbf{b}). \quad (5)$$

In the Stackelberg game, just opposite to the egalitarian game, the two players are asymmetric. That is to say one player will dominate the decision process. The player mentioned above is called the leader who knows his strategy will be intercepted by his opponent. The other player will take actions rationally to the leader's strategy and is called the follower [18]. With conservativeness and rationality assumptions, the leader will adopt a safe strategy which can avoid the worst case and then the game will lead to a SE. When the minimizer $\mathbf{a}$ is the leader, the SE strategies of $\mathbf{a}$ and $\mathbf{b}$ can be calculated by solving (6). When the maximizer $\mathbf{b}$ is the leader, the only difference is that the "minmax" is replaced by the "maxmin" and the SE strategies can be obtained by solving (7) [18].

$$\min_{\mathbf{a}\in\mathcal{A}} \max_{\mathbf{b}\in\mathcal{B}} f(\mathbf{a}, \mathbf{b}) \quad (6)$$

$$\max_{\mathbf{b}\in\mathcal{B}} \min_{\mathbf{a}\in\mathcal{A}} f(\mathbf{a}, \mathbf{b}) \quad (7)$$

In this paper, the jammer is the minimizer $\mathbf{a}$ and the radar is the maximizer $\mathbf{b}$. The mutual information in (4) is the utility function $f(\mathbf{a}, \mathbf{b})$. In the following two sections, the strategies of the radar and the jammer are analyzed when they are involved in the Stackelberg game and the egalitarian game. In the Stackelberg game, it should be emphasized that the SE strategies of the radar and the jammer exist for sure because every two-person finite game admits a Stackelberg strategy for the leader [18] and the equilibrium strategy of the follower is just any best response to the leader's SE strategy.

Note that the analyses of the egalitarian game are based on the results obtained in the Stackelberg game so the Stackelberg game based strategies design is first presented in section 3.

III. STACKELBERG GAME BASED STRATEGIES DESIGN

In electronic warfare, the roles of the radar and the jammer are asymmetric in most cases, which means the radar or the jammer knows its strategies will be intercepted by its opponent. For example, cognitive radar has the ability of acquiring the type and even the detailed parameters of the jammer. So in this section, the radar and the jammer are modeled within the framework of Stackelberg game as introduced above.

A. RADAR AS THE LEADER

For the sake of analysis, it is assumed that the jammer has enough capability of intercepting the radar's waveform and the radar is aware of that. In the Stackelberg game, the radar is the leader who wants to maximize the MI; the jammer, just the opposite, is the follower who will take countermeasures to the radar's strategy to minimize the MI. The mathematical expression can be formulated by (8) [18]

$$\max_{|X(f)|^2} \min_{P_j(f)} \tilde{T} \int_{\mathcal{W}} \ln \left\{ 1 + \frac{|X(f)|^2 \sigma_H^2(f)}{\left[S_{nn}(f) + P_j(f)\right]\tilde{T}} \right\} df$$
$$s.t. \int_{\mathcal{W}} P_j(f) df = P, \int_{\mathcal{W}} |X(f)|^2 df = E_x, \quad (8)$$

where P is the transmit power constraint for the jammer.

As described above, the SE strategies of the radar and the jammer when the radar is the leader are the solutions to the "maxmin" problem in (8). A two-stage optimization method is used to solve the problem and the detailed solving process is described as below.

Stage 1: Fix the radar waveform $|X(f)|^2$ to reduce the "maxmin" problem to a minimization problem as shown in (9).

$$\min_{P_j(f)} \tilde{T} \int_{\mathcal{W}} \ln\left\{1+\frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f)+P_j(f)\right]\tilde{T}}\right\} df$$
$$s.t. \int_{\mathcal{W}} P_j(f)df = P \tag{9}$$

Check the second order condition of the integrand in (9) and it can be found that it is strictly convex with respect to $P_j(f)$ [19]. As a result, the optimal jammer PSD with respect to the given radar waveform exists, which can be solved by the Lagrange multiplier method [20].

Based on the Lagrange multiplier method, the objective function shown in (10) can be obtained

$$\Phi\left(P_j(f),\lambda_1\right) = \tilde{T} \int_{\mathcal{W}} \ln\left\{1+\frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f)+P_j(f)\right]\tilde{T}}\right\} df + \lambda_1\left(P-\int_{\mathcal{W}} P_j(f)df\right), \tag{10}$$

where λ_1 is the Lagrange multiplier and it is determined by $\int_{\mathcal{W}} P_j(f)df = P$.

Minimization of (10) with respect to $P_j(f)$ is equivalent to minimizing the following equation.

$$\phi\left(P_j(f),\lambda_1\right)=\tilde{T}\ln\left\{1+\frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f)+P_j(f)\right]\tilde{T}}\right\}-\lambda_1 P_j(f) \tag{11}$$

Take the derivative of (11) with respect to $P_j(f)$ and set it to zero. After that, the optimal solution to (9) can be obtained and it is expressed as follows.

$$P_j(f) = \max\left\{0, \sqrt{\left[\frac{\sigma_H^2(f)|X(f)|^2}{2\tilde{T}}\right]^2+\frac{\sigma_H^2(f)|X(f)|^2}{\lambda_1}} - \frac{\sigma_H^2(f)|X(f)|^2}{2\tilde{T}} - S_{nn}(f)\right\} \tag{12}$$

Stage 2: Substitute the result in (12) into the cost function in (8) and then a maximization problem as shown below needs to be solved

$$\max_{|X(f)|^2} \tilde{T} \int_{\mathcal{W}} \ln\left\{1+\frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f)+P_j(f)\right]\tilde{T}}\right\} df$$
$$s.t. \begin{cases} P_j(f) = \max\left\{0, \sqrt{A(f)^2+B(f)}-A(f) - S_{nn}(f)\right\} \\ \int_{\mathcal{W}} |X(f)|^2 df = E_x \end{cases}, \tag{13}$$

where $A(f) = \frac{\sigma_H^2(f)|X(f)|^2}{2\tilde{T}}$ and $B(f) = \frac{\sigma_H^2(f)|X(f)|^2}{\lambda_1}$ are used for notational brevity.

After the substitution of $P_j(f)$ into the cost function in (13), it can be found that the result is a strict concave function with respect to $|X(f)|^2$. Therefore, Lagrange multiplier method is still applied and the following objective function can be acquired

$$K\left(|X(f)|^2,\lambda_2\right) = \tilde{T} \int_{\mathcal{W}} \ln\left\{1+\frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f)+P_j(f)\right]\tilde{T}}\right\} df + \lambda_2\left[E_x - \int_{\mathcal{W}} |X(f)|^2 df\right] \tag{14}$$

where λ_2 is Lagrange multiplier which can be calculated by $\int_{\mathcal{W}} |X(f)|^2 df = E_x$.

Maximization of $K\left(|X(f)|^2,\lambda_2\right)$ with respect to $|X(f)|^2$ is equivalent to maximizing the following equation.

$$k\left(|X(f)|^2,\lambda_2\right) = \tilde{T}\ln\left\{1+\frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f)+P_j(f)\right]\tilde{T}}\right\} - \lambda_2|X(f)|^2 \tag{15}$$

Substitute $P_j(f)$ in (12) into (15) and the following expression can be obtained.

$$k\left(|X(f)|^2,\lambda_2\right) = \tilde{T}\ln\left\{1+\frac{1}{\sqrt{\frac{1}{4}+\frac{\tilde{T}^2}{\lambda_1\sigma_H^2(f)}\frac{1}{|X(f)|^2}}-\frac{1}{2}}\right\} - \lambda_2|X(f)|^2 \tag{16}$$

Taking the derivative of $k\left(|X(f)|^2,\lambda_2\right)$ with respect to $|X(f)|^2$ and setting it to zero yields the solution to (13), which is also the solution of the radar waveform $|X(f)|^2$ to the "maxmin" problem. The result is given by

$$|X(f)|^2 = \sqrt{\left(\frac{2\tilde{T}^2}{\lambda_1\sigma_H^2(f)}\right)^2+\left(\frac{\tilde{T}}{\lambda_2}\right)^2} - \frac{2\tilde{T}^2}{\lambda_1\sigma_H^2(f)}. \tag{17}$$

Substituting (17) into (12) and simplifying the result, the solution of the jammer PSD $P_j(f)$ to the "maxmin" problem is obtained as shown below.

$$P_j(f) = \max\left\{0, \frac{\tilde{T}}{\lambda_1}+\frac{\sigma_H^2(f)}{2\lambda_2} - \sqrt{\frac{\tilde{T}}{\lambda_1}^2+\left(\frac{\sigma_H^2(f)}{2\lambda_2}\right)^2} - S_{nn}(f)\right\} \tag{18}$$

After the stage 1 and the stage 2, the solutions of $|X(f)|^2$ and $P_j(f)$ to the "maxmin" problem in (8) are both acquired, which are also the SE strategies of the radar and the jammer.

Note that the strategies in (17) and (18) are meaningful in theory, however, it is not practical to implement them in real electronic warfare. The reason is that the Lagrange multipliers are needed but they are difficult to be calculated.

It is obvious that a Lagrange multiplier pair $\{\lambda_1, \lambda_2\}$ needs to satisfy both $\int_{\mathcal{W}} P_j(f)df = P$ and $\int_{\mathcal{W}} |X(f)|^2 df = E_x$ at the same time, which is a highly nonlinear system of equations and will result in a high computational complexity.

In order to simplify the calculation of the Lagrange multipliers, the first-order Taylor approximation to $P_j(f)$ is applied and the results are shown as follows.

$$P_j(f) \approx \max\left\{0, \frac{1}{2\lambda_2}\sigma_H^2(f) - S_{nn}(f)\right\} \tag{19}$$

The approximation in (19) is reasonable and efficient due to the following two reasons.

The first is that the complexity of solving the Lagrange multipliers λ_1 and λ_2 is greatly reduced by the simplification operation. By doing that, the influence of λ_1 is eliminated and only a one-dimensional search method is needed to obtain λ_2. Then λ_2 is substituted into (17) and the same method can be used to acquire λ_1.

The second is that the characteristic of the jammer PSD $P_j(f)$ remains the same after the simplification operation. Examine the SE strategies and it can be found that both the radar waveform $|X(f)|^2$ and jammer PSD $P_j(f)$ get larger as $\sigma_H^2(f)$ gets larger. That is to say, the radar and the jammer are willing to "pour" more energy to the frequency interval in which the target ESV is large. With respect to the approximate $P_j(f)$, it will also become larger with the increase of $\sigma_H^2(f)$. As a result, it can be concluded that the characteristic of the jammer PSD is not changed.

In addition, it should be highlighted that the solution pair $\{|X(f)|^2, P_j(f)\}$ to (8) may not be unique, meaning that the SE strategies of the radar and the jammer may also not be unique. This is due to the fact that it is difficult to determine whether the solutions to the nonlinear system of equations mentioned above are unique. However, the MI value will be the same even if there are multiple SEs.

B. JAMMER AS THE LEADER

Let the radar have enough capability of sensing the existence of the jamming signal and the jammer knows that. Therefore, the jammer is the leader and the radar is the follower. The jammer wants to minimize the MI while the radar is just the opposite. The mathematical expression of the above process is given by (20) [18]. Similar to the case in which the radar is the leader, the SE strategies of the radar and the jammer are the solutions to the "minmax" problem expressed in (20).

$$\begin{aligned}&\min_{P_j(f)}\max_{|X(f)|^2} \tilde{T}\int_{\mathcal{W}} \ln\left\{1 + \frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f) + P_j(f)\right]\tilde{T}}\right\}df\\ &s.t. \int_{\mathcal{W}} |X(f)|^2 df = E_x, \int_{\mathcal{W}} P_j(f)df = P\end{aligned} \tag{20}$$

The two-stage optimization method is still used to solve the problem. The detailed solution procedure is the same as section 3.A so it is ignored and only some important results are given below.

Stage 1: The "minmax" problem is reduced to a "max" problem by fixing the jammer PSD $P_j(f)$ and the maximization problem is given by

$$\begin{aligned}&\max_{|X(f)|^2} \tilde{T}\int_{\mathcal{W}} \ln\left\{1 + \frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f) + P_j(f)\right]\tilde{T}}\right\}df\\ &s.t. \int_{\mathcal{W}} |X(f)|^2 df = E_x.\end{aligned} \tag{21}$$

Examine the second-order condition and we can verify that the integrand of (21) is strictly concave with respect to $|X(f)|^2$. Applying Lagrange multiplier method, the optimal radar waveform can be acquired in (22)

$$|X(f)|^2 = \max\left(0, \frac{\tilde{T}}{\lambda_3} - \frac{\left[S_{nn}(f) + P_j(f)\right]\tilde{T}}{\sigma_H^2(f)}\right), \tag{22}$$

where λ_3 is determined by $\int_{\mathcal{W}} |X(f)|^2 df = E_x$.

Stage 2: By substituting the optimal radar waveform into the cost function in (20), a minimization problem is obtained as shown below.

$$\begin{aligned}&\min_{P_j(f)} \tilde{T}\int_{\mathcal{W}} \ln\left\{1 + \frac{|X(f)|^2\sigma_H^2(f)}{\left[S_{nn}(f) + P_j(f)\right]\tilde{T}}\right\}df\\ &s.t.\begin{cases}|X(f)|^2 = \max\left(0, \dfrac{\tilde{T}}{\lambda_3} - \dfrac{\left[S_{nn}(f) + P_j(f)\right]\tilde{T}}{\sigma_H^2(f)}\right)\\ \int_{\mathcal{W}} P_j(f)df = P\end{cases}\end{aligned} \tag{23}$$

Solve (23) with Lagrange multiplier method and the result, which is also the solution of the jammer PSD $P_j(f)$ to the "minmax" problem, can be obtained as shown in (24)

$$P_j(f) = \max\left\{0, \frac{1}{\lambda_4} - S_{nn}(f)\right\}, \tag{24}$$

where λ_4 is determined by $\int_{\mathcal{W}} P_j(f)df = P$. Substitute the solution in (24) into the optimal radar waveform in (22) and then the solution of the radar waveform $|X(f)|$ to the "minmax" problem can be acquired in (25).

$$|X(f)|^2 = \max\left(0, \frac{\tilde{T}}{\lambda_3} - \frac{\max\left\{S_{nn}(f), \frac{1}{\lambda_4}\right\}\tilde{T}}{\sigma_H^2(f)}\right) \tag{25}$$

As for the Lagrange multipliers, λ_4 can be calculated using the one-dimensional search method. When λ_4 is obtained, it is substituted into (25) and the same method is applied to get λ_3.

Until now, the solutions to the "minmax" problem previously mentioned, which are also the SE strategies of the radar and the jammer, have been solved.

The SE strategies of the radar and the jammer are both water-filling solutions. Taking the SE strategy of the radar as an example, the explanation for that is given as follows. The constant term $\frac{\tilde{T}}{\lambda_3}$ in (25) provides the upper boundary of the solution and the term $\frac{\max\left\{S_{nn}(f), \frac{1}{\lambda_4}\right\}\tilde{T}}{\sigma_H^2(f)}$ gives the lower boundary of the solution. The energy of the radar waveform is filled in the area formed by the lower boundary and the upper boundary in the same way in which water distributes itself in a vessel. As a result, the SE strategy of the radar is a water-filling solution and so is the SE strategy of the jammer.

As for the radar, although its SE strategy is a water-filling solution, it is different from [11] because the countermeasures of the jammer are taken into consideration. As for the jammer, if $S_{nn}(f)$ is constant for all $f \in \mathcal{W}$, the jammer's PSD is also constant and the Lagrange multiplier λ_4 simply equals to $\frac{W}{P+WS_{nn}(f)}$. If $S_{nn}(f)$ is not constant, then the jammer's PSD is also not constant. However, the power of the jamming signal at the radar receiver is usually far larger than the noise power so the effect of $S_{nn}(f)$ can be negligible.

Different from the case in which the radar is the leader, the SE strategies of the radar and the jammer under this circumstance are unique. As we can see from (24), the SE strategy of the jammer is only decided by λ_4 and once it is obtained, the SE strategy of the radar can be obtained by λ_3. Therefore, the SE strategies here are unique.

IV. EGALITARIAN GAME BASED STRATEGIES DESIGN

In the previous section, the game in which the players are asymmetric is investigated. With the assumption of conservation and rationality, the players will not regret their strategies when the game is over [18]. But if they are involved in the egalitarian game, meaning that they arrive at their strategies independently, the SE strategies are not the best strategies. If the NE exists, the radar and the jammer prefer to the NE strategies rather than the SE strategies [5], [18].

In this section, the existence condition of the egalitarian game is investigated. If the NE condition is satisfied, the NE strategies of the radar and the jammer are given. However, if the NE condition is not satisfied, the possible safe strategies are also presented.

According to the proposition 5 in [7], the NE of a TPZS game on a continuous space is the saddle point of their utility function. That is to say $(\mathbf{a}^*, \mathbf{b}^*)$ must satisfy the following equation

$$\left(\mathbf{a}^*, \mathbf{b}^*\right) = arg\, \min_{\mathbf{a}\in\mathcal{A}} \max_{\mathbf{b}\in\mathcal{B}} f\,(\mathbf{a}, \mathbf{b}) = arg\, \max_{\mathbf{b}\in\mathcal{B}} \min_{\mathbf{a}\in\mathcal{A}} f\,(\mathbf{a}, \mathbf{b})\,. \tag{26}$$

Denote $\{X^r, P_j^r\}$ as the solution of (8) and $\{X^j, P_j^j\}$ as the solution of (20). $I(X, Y)$ is the value of the MI between the received signal and the target random impulse response if the radar adopts the strategy X and the jammer adopts the strategy Y. Here X and Y belong to the set $\{X^r, X^j\}$ and $\{P_j^r, P_j^j\}$, respectively.

Based on (26), if the NE exists, $I(X^r, P_j^r)$ must be equal to $I(X^j, P_j^j)$. Therefore, the NE existence condition is equivalent to the condition that $I(X^r, P_j^r)$ is equal to $I(X^j, P_j^j)$. According to whether the target ESV is constant in the given bandwidth, two kinds of situations are taken into consideration and we investigate whether the equality condition holds in each situation.

Situation 1: The target ESV is not a constant number. In this situation, strict inequality $I(X^r, P_j^r) < I(X^j, P_j^j)$ holds so the saddle point does not exist. That means the NE of the egalitarian game does not exist. Here the proof is given below.

The inequality (27) is first proven.

$$I(X^r, P_j^r) < I(X^r, P_j^j) \tag{27}$$

Given the radar waveform X^r, P_j^r is the optimal solution to the minimization problem as described in the stage 1 of section 3.A, which results in $I(X^r, P_j^r) \leq I(X^r, P_j^j)$. In section 3.A, it is pointed out that the integrand in (9) is strictly convex with respect to $P_j(f)$. That means if the strategies of the jammer P_j^r and P_j^j are not equal, $I(X^r, P_j^r) < I(X^r, P_j^j)$ will hold. In situation 1, the target ESV is not constant so the strategies of the jammer P_j^r and P_j^j are not equal obviously. As a result, strict inequality $I(X^r, P_j^r) < I(X^r, P_j^j)$ holds.

Then the following strict inequality is proven.

$$I(X^r, P_j^j) < I(X^j, P_j^j) \tag{28}$$

Given the jammer PSD P_j^j, X^j is the optimal solution to the maximization problem as described in the stage 1 of section 3.B, which leads to $I(X^r, P_j^j) \leq I(X^j, P_j^j)$. As mentioned above, the integrand in (21) is strictly concave with respect to $|X(f)|^2$. With the target ESV not constant in the given bandwidth, the radar strategies X^r and X^j are also not equal. As a consequence, $I(X^r, P_j^j) < I(X^j, P_j^j)$ holds.

Therefore, it can be concluded that strict inequality $I(X^r, P_j^r) < I(X^r, P_j^j) < I(X^j, P_j^j)$ holds. The saddle point, which requires the value of "minmax" to equal to the value of "maxmin", does not exist.

Situation 2: The target ESV is a constant number. In this situation, $I(X^r, P_j^r) = I(X^j, P_j^j)$ holds and the NE exists. The proof is given below.

From the above analyses in situation 1 we can know that the equality of P_j^r and P_j^j leads to $I(X^r, P_j^r) = I(X^r, P_j^j)$. In situation 2, the strategies of the jammer P_j^r and P_j^j are both constant in the given bandwidth because of the constant target ESV. If the given jammer transmit power is the same, P_j^r and P_j^j are equal, leading to $I(X^r, P_j^r) = I(X^r, P_j^j)$. Similarly, with the target ESV a constant number and the same radar waveform energy, X^r and X^j are also the same and then $I(X^r, P_j^j) = I(X^j, P_j^j)$ holds. Therefore, it can be concluded that $I(X^r, P_j^r) = I(X^j, P_j^j)$ holds and the NE exists.

On the basis of the analyses in situation 1 and situation 2, the following conclusion can be drawn.

Conclusion:

(1) When the target ESV is constant, the NE exists and the NE strategies of the radar and the jammer are just distributing the energy or power evenly in the given bandwidth.

(2) When the target ESV is not constant, the NE does not exist and no NE strategies are available.

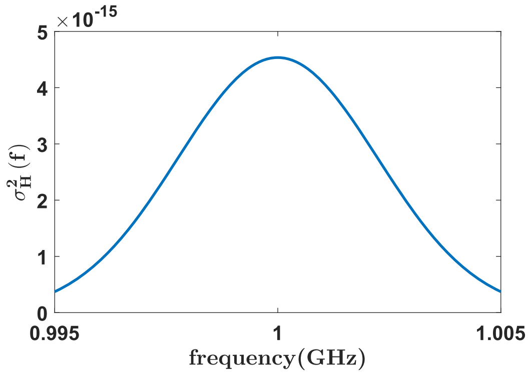

FIGURE 2. Target ESV.

Although the NE does not exist when the target ESV is not constant, the SE strategies are still acceptable for both the radar and the jammer no matter who the leader is. From the perspective of game theory, the explanation is given below.

Actually, $I(X^j, P_j^j)$ is the loss-ceiling [18] of the jammer. It means that $I(X^j, P_j^j)$ is the upper bound of the MI no matter what strategies the radar adopts with the jammer strategy fixed by P_j^j. $I(X^r, P_j^r)$ is the gain-floor [18] of the radar. That is to say that this strategy provides a lower bound of the MI whatever strategies the jammer takes if the radar keeps X^r as its strategy. In a word, SE strategies guarantee the radar and the jammer safe strategies under the conservativeness and rational assumption. So in this view, the SE strategies are still acceptable.

V. NUMERICAL RESULTS

In this section, simulation results are provided and the basic simulation parameters and assumptions are given below. It is assumed that there is a monostatic radar equipped with an antenna whose gain G is $30dB$. The carrier frequency of the radar is $f_0 = 1GHz$ and the bandwidth of the transmit waveform is $W = 10MHz$. Hence, the radar frequency interval is $\mathcal{W} = [f_0 - \frac{W}{2}, f_0 + \frac{W}{2}] = [0.995GHz, 1.005GHz]$. The duration of the radar waveform is $T = 10ms$ and its energy can be expressed by $E_x = P_x T$, where P_x is the average transmit power of the radar. The target, which is $R = 10km$ away from the radar and in the main lobe of the radar, is equipped with a self-screening jammer whose antenna gain is 10 dB.

The ESV, as shown in Fig.2, is given by

$$\sigma_H^2(f) = \beta \exp\left\{-\alpha(f - f_0)^2\right\}, \tag{29}$$

where α and β are constants that describe the characteristics of the target ESV. α is related to the radar bandwidth and it describes how fast $\sigma_H^2(f)$ decreases as $|f - f_0|$ increases. β characterizes the magnitude of the target ESV and it represents how much the ratio, the received power P_R to the transmit power P_T of the radar, changes when there is a $1m^2$ change in the radar cross section (RCS) [11]. β can be defined

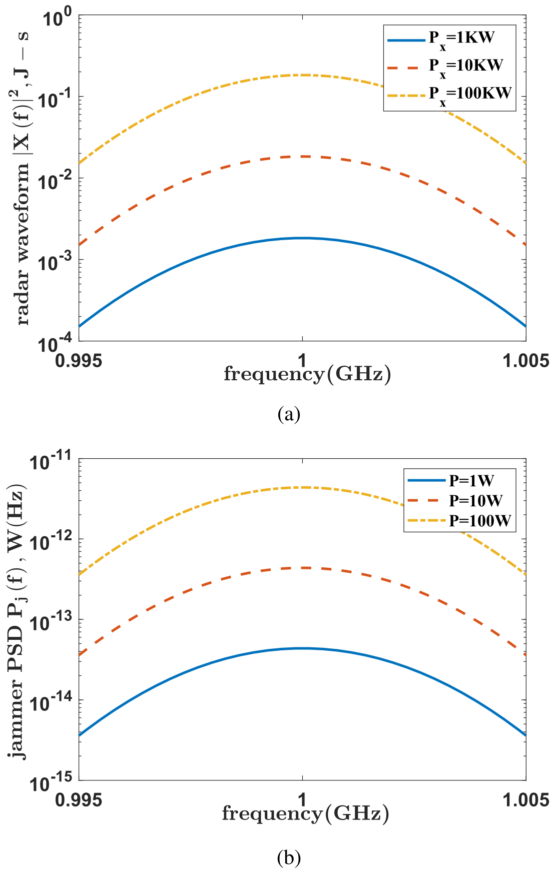

FIGURE 3. SE strategies of the radar and the jammer when the radar is leader. (a) Radar waveform $|X(f)|^2$ for radar average transmit power $P_x = \{1KW, 10KW, 100KW\}$ and jammer transmit power $P = 10W$. (b) Jammer PSD $P_j(f)$ for radar average transmit power $P_x = 10KW$ and jammer transmit power $P = \{1W, 10W, 100W\}$.

as follows

$$\beta = \frac{\Delta P_R}{P_T} = \frac{A_e^2 \Delta\sigma}{4\pi\lambda^2 R^4}, \quad Ae = \frac{G\lambda^2}{4\pi}, \tag{30}$$

where λ is the wavelength of the radar, R is the distance between the radar and the target, $\Delta\sigma = 1m^2$ represents a $1m^2$ change in the RCS and A_e is the effective aperture. When the bandwidth is $10MHz$, α is set to $10^{-13}s^2$ according to [11]. By (30), β can be obtained and it is equal to 4.5354×10^{-14}. Here the receiver noise is addictive white Gaussian noise and the PSD of the noise S_{nn} equals to kT_s, where k is the Boltzmann's constant and $T_s = 300K$ is the effective noise temperature.

A. STACKELBERG GAME BASED STRATEGIES DESIGN

1) RADAR AS THE LEADER

In this subsection, the SE strategies are shown in Fig.3 when the radar is the leader.

The radar's strategies are given in Fig.3(a). Here the jammer transmit power P is fixed and the radar average transmit power P_x is changed. The jammer's strategies are given in Fig.3(b) with the radar average transmit power P_x fixed and the jammer transmit power P changed.

It can be seen from Fig.3(a) and Fig.3(b) that the amplitude of the radar waveform and the jammer PSD will increase with the increase of the radar average transmit power P_x and jammer transmit power P, respectively. Compared with the target ESV in Fig.2, it is obvious to find that both the radar and the jammer will allocate more energy in the frequencies with larger target ESV.

2) JAMMER AS THE LEADER

In this subsection, the SE strategies of the radar and the jammer are illustrated in Fig.4 when the jammer is the leader.

When the jammer is the leader, the radar SE strategy is a water-filling solution and more energy will be allocated to the frequencies with larger target ESV as show in Fig.4(a) and Fig.4(b). With the increase of the jammer transmit power, the radar tends to allocate its energy in a narrower frequency interval as shown in Fig.4(a). With the increase of the radar average transmit power, just the opposite, the radar is willing to allocate more energy in a wider frequency as shown in Fig.4(b).

The jammer SE strategy is also a water-filling solution and the PSD of the jammer is constant in the radar bandwidth as depicted in Fig.4(c).

B. EGALITARIAN GAME BASED STRATEGIES DESIGN

1) SITUATION 1

Fig.5(a) and Fig.5(b) show the MI for different radar average transmit power and jammer transmit power when the radar and the jammer are the leader, respectively. In section 4, it is proven that the NE does not exist in situation 1, meaning that I_{minmax} is always greater than I_{maxmin} as shown in Fig.5(c).

As discussed before, SE strategies provide the gain-floor for the radar and the loss-ceiling for the jammer. In order to give a further explanation, the following experiments are conducted. Two other strategies except the SE strategies are used as a comparison. One is the uniform strategy which distributes the energy or power uniformly in the radar frequency bandwidth. The other one is the random strategy which distributes the energy or power randomly in the radar frequency bandwidth.

(1) When the radar is the leader, it is assumed that the radar always adopts the SE strategy and the jammer will take the SE strategy, uniform strategy and random strategy respectively. As shown in Fig.6(a), the SE strategy indeed provides a lower bound, which is called gain-floor in game theory.

(2) When the jammer is the leader, it is assumed that the jammer always adopts the SE strategy and the radar will take the SE strategy, uniform strategy and random strategy respectively. An upper bound, called loss-ceiling in game theory, is guaranteed by the SE strategy as shown in Fig.6(b).

In fact, any strategies can be used and the conclusion remains unchanged. Here the two strategies are just taken as an example.

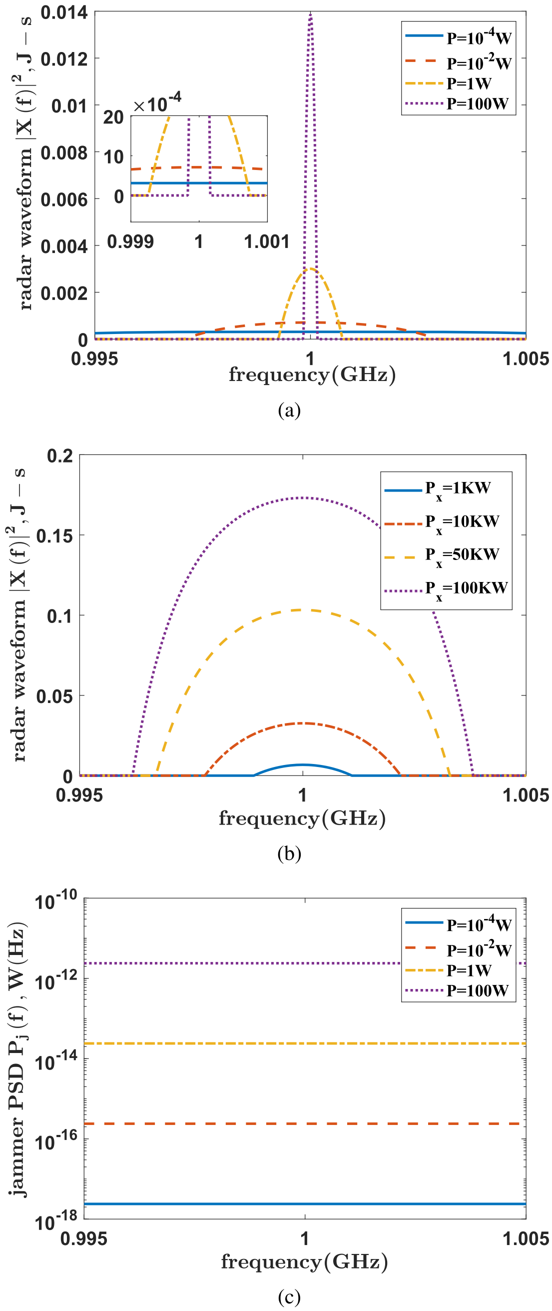

FIGURE 4. SE strategies for the radar and the jammer when the jammer is leader. (a) Radar waveform $|X(f)|^2$ for radar average transmit power $P_x = 10KW$ and jammer transmit power $P = \{10^{-4}W, 10^{-2}W, 1W, 100W\}$. (b) Radar waveform $|X(f)|^2$ for radar average transmit power $P_x = \{1KW, 10KW, 50KW, 100KW\}$ and jammer transmit power $P = 1W$. (c) Jammer PSD $P_j(f)$ for radar average transmit power $P_x = 10KW$ and jammer transmit power $P = \{10^{-4}W, 10^{-2}W, 1W, 100W\}$.

2) SITUATION 2

Here the constant target ESV, which can be expressed as $\sigma_H(f) = C, f \in [f_0 - \frac{W}{2}, f_0 + \frac{W}{2}]$, is used in the experiment. The same as Fig.5(a) and Fig.5(b), $I(X^r, P_j^r)$ and $I(X^j, P_j^j)$ for different radar average transmit power and jammer transmit power are presented in Fig.7(a) and Fig.7(b) respectively.

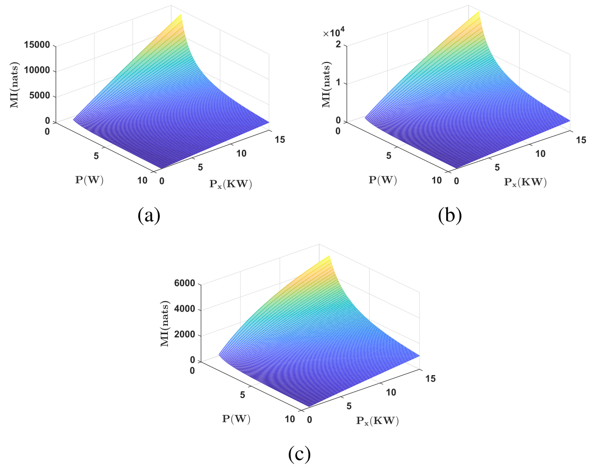

FIGURE 5. MI and their difference with respect to different P_x and P in situation 1. (a) I_{maxmin}. (b) I_{minmax}. (c) $I_{minmax} - I_{maxmin}$.

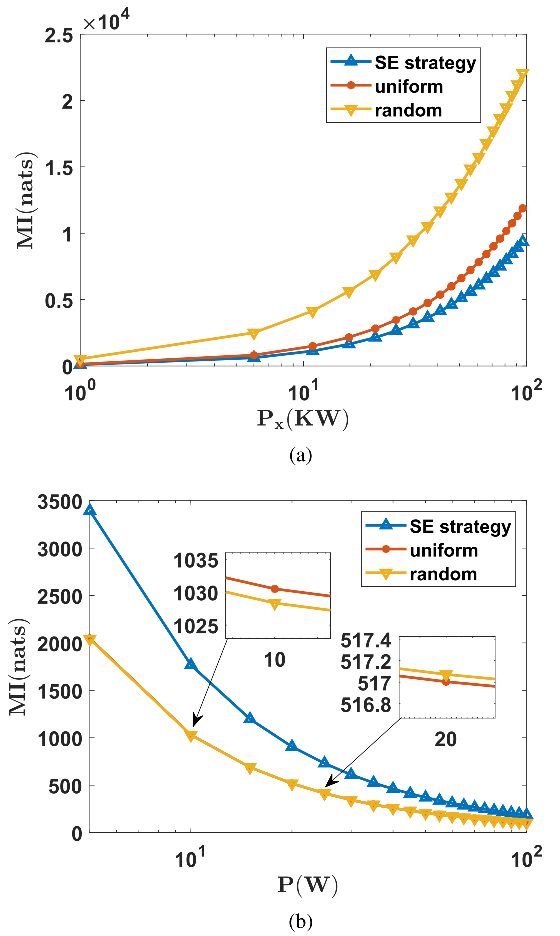

FIGURE 6. Gain-floor and loss-ceiling for the radar and the jammer respectively. (a) Gain-floor for radar. (b) Loss-ceiling for jammer.

The result of $I(X^j, P_j^j) - I(X^r, P_j^r)$ is shown in Fig.7(c). Obviously, the difference between $I(X^r, P_j^r)$ and $I(X^j, P_j^j)$ is zero and the NE exists.

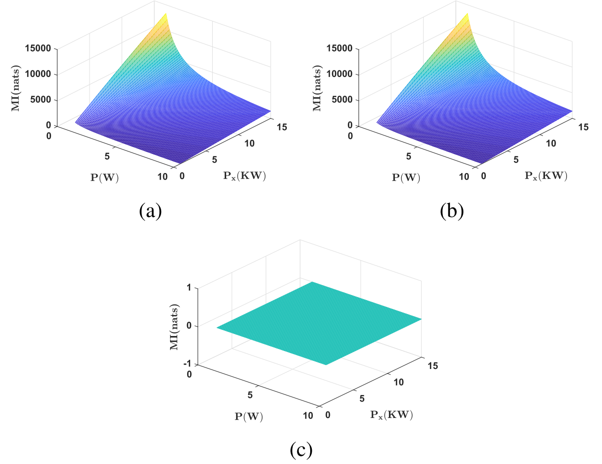

FIGURE 7. MI and their difference with respect to different P_x and P in situation 2. (a) I_{maxmin}. (b) I_{minmax}. (c) $I_{minmax} - I_{maxmin}$.

VI. CONCLUSION

In this paper, the strategies design problem for the monostatic radar and the jammer is investigated by modelling the interaction between them using game theory. According to whether the players are symmetric, two kinds of games including Stackelberg game and egalitarian game are considered. The MI between the received signal and the target impulse response is used as the utility function. Based on the MI utility function, the radar waveform and the jammer PSD are chosen as the strategies to be designed.

In the Stackelberg game, the radar and the jammer are asymmetric and the SE strategies are derived analytically. When the radar is the leader, it is concluded that their SE strategies both prefer to allocate more energy in the frequencies with larger target ESV. When the jammer is the leader, their SE strategies are both water-filling strategies.

In the egalitarian game, it is proven that if the target ESV is constant in the given bandwidth, then the NE exists. The NE strategies of the radar and the jammer are both distributing the energy or the power evenly. If the target ESV is not constant, the NE does not exist. However, from the perspective of game theory, we point out that the SE strategies, which provide the gain-floor for the radar and the loss-ceiling for the jammer, are still meaningful.

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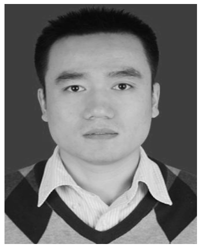

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