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Adaptive Weapon-to-Target Assignment Model Based on the Real-Time Prediction of Hit Probability

JAEYEON JANG, HYUI GEON YOON, JEONG CHEOL KIM, AND CHANG OUK KIM
Department of Industrial Engineering, Yonsei University, Seoul 03722, South Korea

Corresponding author: Chang Ouk Kim (kimco@yonsei.ac.kr)

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ABSTRACT Weapon-to-target assignment (WTA), which minimizes the damage to our forces by launching interceptor missiles (weapons) against ballistic missiles of the enemy (targets), is a critical decision-making problem of ballistic missile defense missions. The primary objective is to launch an interceptor with a high hit probability for each target. The existing research on WTA assumes that the hit probability is known before an engagement regardless of whether the probability varies during the engagement. However, the hit probability in actual engagement situations is a time-dependent variable that changes in accordance with the flight states of the target and interceptor that are unknown in advance. Therefore, a rolling horizon-based decision approach is necessary. In this research, we propose an adaptive WTA (AWTA) model that makes WTA decisions at each radar scanning time based on the hit probability predicted using radar information about the engagement situation–for each target, an interceptor with a hit probability higher than a threshold is launched, thereby maximizing the total hit result. A machine learning model is suggested to learn the probabilistic relationship between the flight states and hit results, and this model is embedded in the solution procedure of the AWTA model. The performance of the AWTA model is evaluated via a simulation-based experiment, and the results confirm that the proposed AWTA model is appropriate for real-time engagement situations.

INDEX TERMS Weapon-to-target assignment, adaptive model, hit probability prediction, machine learning, simulation.

I. INTRODUCTION

A ballistic missile (BM) is an aerial threat with a warhead that is launched by its own propellant and has a high destructive power, thereby possibly causing large damage to our forces. BM (target) defense missions are carried out in the sequence of radar detection, target recognition, weapon-to-target assignment (WTA), engagement, and result verification. This sequence is repeated because enemy BMs attack at different locations at different times; in the sequence, the most important decision-making task is WTA, which assigns interceptors (weapons) that are appropriate for targets [1].

WTA has been proven to be an NP-complete problem [2]. Academically, WTA is classified into static WTA (SWTA) and dynamic WTA (DWTA) [3], [4]. SWTA assumes a single-stage engagement in which all available interceptors are launched simultaneously [3]. In contrast, DWTA is defined as launching interceptors in multiple stages [4], and this definition enables decision making about the launching time points of interceptors.

Research on the SWTA problem initiated by Manne [5], Ash [6], and Day [7] and the results of related studies have been published [3], [5]–[17]. Manne proposed a WTA model that can minimize the survival of targets, while Ash and Day proposed models that can minimize the total threat of targets. Matlin [8] reviewed various WTA models and proposed a linear WTA model that minimizes the total threat of targets. Soland [9] and Hosein and Athans [3] proposed an asset-based model using an objective function that maximizes the survival value of defense assets, and Ahuja *et al.* [10] defined a network flow-based WTA model and solved it by using the branch-and-bound algorithm. Karasakal [11]

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applied the WTA problem to the air defense of warships and proposed a target-based model using an objective function that maximizes the hit probability. Moreover, to efficiently solve large-scale problems, a genetic algorithm (GA) [12], tabu search (TS) [13], particle swarm optimization [14], ant colony optimization [1], Lagrange relaxation [15], [16], and hybrid algorithm [17] have been applied. However, these studies have limitations in application to actual BM defense missions due to the assumptions of simultaneous launching of all interceptors and using a fixed hit probability. In particular, the hit probability in actual engagement situations is a time-dependent variable that changes in accordance with the flight states of the target and interceptor.

DWTA has received attention from the academic field to resolve the limitations of SWTA [4], [18]–[24]. Hosein and Athans [4] defined DWTA as a multistage, asset-based model but did not consider the flight state and time of the missile. Khosla [18] considered the flight time of the missile and proposed a target-based model that maximizes the weighted sum of the total value of the hit targets and the total fitness of interceptors assigned to the targets. Karasakal [19] applied the concept of a time window and proposed an air defense model of a warship that can perform continuous missions as the time window progresses. Jinjun *et al.* [20] proposed a model to minimize the threats of surviving targets after an engagement is finished, and Naeem and Masood [21] proposed a model that regards the targets and interceptors as men and women and pairs them using the stable marriage algorithm with the consideration of preference scores. Due to the reality of engagement, these models have greatly increased constraints and computational complexity that are higher than those of SWTA models, and thus metaheuristic solution methods have been proposed [22]–[24]. DWTA is more effective than SWTA in BM defense missions. However, DWTA models still have limitations in actual missions because the hit probability has a random value within predefined upper and lower limits regardless of the missile flight state, and it is assumed that the number and launching time points of targets can be known in advance.

WTA must consider a variety of factors, including the type and number of targets, launching locations and times, interceptor performance, and time-varying hit probability. These factors mean that adaptive modeling that fits a dynamic engagement situation is required. The goal of this study is to develop an adaptive WTA (AWTA) model. The proposed AWTA model assumes that the number of targets and launching times are not known in advance and that a BM flies in a parabolic trajectory with varying velocity [25]. The BM defense procedure of the AWTA model is as follows. First, the flight state information about the locations and velocities of targets and interceptors is acquired via a radar at each scanning moment. An interceptor is assigned to a target at a moment (stage) when the hit probability is high. After the engagement, the hit result is verified, and if the target survives, an additional interceptor can be assigned at a subsequent moment considering the engagement constraints of the AWTA model.

The core of the defense procedure is prediction of the hit probability at each time point to maximize the number of hit targets. In this study, we develop a hit probability predictor by using a machine learning model. In the experiment, three models (logistic regression (LR) [26], multilayer perceptron (MLP) [27], and stacked denoising autoencoder (SdA) [28]) are tested for this predictor. The predictor learns the probabilistic relationship between the flight states and hit result and is embedded in the defense procedure of the AWTA model. For every pair of targets and interceptors in an engagement situation, the predictor outputs a hit probability based on the radar information.

AWTA and DWTA are similar in that interceptors are assigned across multiple stages. However, DWTA determines the assignments of interceptors at all stages in the beginning stage of an engagement. AWTA can be regarded as similar to repetitive applications of SWTA because AWTA makes a decision at each radar scanning time considering the dynamic engagement situation (e.g., detection of new BMs, change in hit probability). However, SWTA is a single-stage problem in which all interceptors are assigned to targets at once. In contrast, the proposed AWTA model does not assign interceptors until the predicted hit probability reaches a threshold at each stage (radar scanning time). This assignment process is a fundamental difference between the two approaches.

The remainder of this paper is organized as follows. Section 2 presents the formal AWTA model and solution procedure. In addition, this section explains the machine learning-based scheme for hit probability prediction. Section 3 introduces the simulation-based experiments and presents the performance evaluation results of the hit probability predictor and AWTA model. Finally, Section 4 concludes this study.

II. PROPOSED MODEL

A. ADAPTIVE WTA MODEL

The proposed AWTA model is based on the following four assumptions.

- The number of targets and launching times are not known in advance, and information about targets is acquired through radar detection at every scanning time.
- A target flies in accordance with a parabolic trajectory, and the hit probability varies in accordance with the location and velocity of the target.
- Our forces have enough interceptors to defend against targets in flight.
- The interceptor launcher is limited to launching only one interceptor at a time.

The nomenclature used in this study is summarized in Table 1.

The hit probability has a time-varying nature and should be estimated based on information of the missile state collected

TABLE 1. Nomenclature.

t	Engagement time, $t = \{1, 2, \ldots, T\}$.
W	Set of interceptor launchers.
B_t	Set of surviving targets at time t.
CT_t	Set of candidate targets for which interceptors can be assigned at time t.
CI_t	Set of interceptor launchers that satisfy a launch condition at time t.
z_{ijt}	Assignment of an interceptor from launcher i to target j at time t.
p_{ijt}	Predicted hit probability from launcher i to target j at time t.
S_{jt}	Set of interceptors (launchers, launching times) chasing target j at time t.
M	Maximum number of interceptors that can be assigned to one target.
P_{min}	Minimum hit probability for launching an interceptor.
P_ε	Hit probability for the assignment of an additional interceptor.

at each cycle of radar scanning. Thus, the proposed AWTA model formulates a WTA problem at each engagement time t (t $= 1, \ldots, T$) corresponding to the radar scanning time. The objective function at time t is

$$\max \sum_{i \in W} \sum_{j \in B_t} p_{ijt} z_{ijt}. \tag{1}$$

Equation (1) determines the assignment (x_{ijt}) of an interceptor in launcher i for a target j that is alive at time t (j $\in B_t$) such that the total predicted hit result is maximized. The decision variable z_{ijt} is limited to one (assignment success) or zero (assignment failure).

The constraints at each time t are

$$\left(1 - p_{ijt}\right) z_{ijt} \leq 1 - P_{min} \forall i, j \tag{2}$$

$$Z_{ijt}\left(p_{ijt} - p_{ij(t+1)}\right) \geq 0 \forall i, j \tag{3}$$

$$P_\varepsilon z_{ijt} \leq 1 - \prod_{(i,t) \in S_{jt}} \left(1 - p_{ijt}\right) \forall i, j \tag{4}$$

$$\left|S_{jt}\right| + \sum_i z_{ijt} \leq M \forall j \tag{5}$$

$$\sum_i z_{ijt} \leq 1 \forall j \tag{6}$$

$$\sum_j z_{ijt} \leq 1 \forall i \tag{7}$$

$$Z_{ijt} \in \{0, 1\} \forall, i, j. \tag{8}$$

Equation (2) is the minimum launching condition and allows an interceptor to be launched only if the minimum probability (P_{min}) defined by the user is met. Equation (3) compares the predicted hit probability between the current and the next times and allows an interceptor to be launched after the predicted hit probability reaches the maximum. Equation (4) allows the assignment of an additional interceptor only for a target whose predicted hit probability did not reach P_ε. This condition reflects the opinion of military experts that additional interceptors can be assigned to hit a target in actual situations. Equation (5) limits the number of interceptors that can be assigned to one target. Equation (6) limits the assignment of only one launcher for one target at time t, and (7) limits the launching of only one interceptor from one launcher at time t. Equation (8) restricts the range of the decision variables.

B. DEFENSE PROCEDURE

The WTA problem defined above is solved through the following sequential procedure.

Step 1) Acquire radar information about the state (location and velocity) of every target and interceptor chasing that target (if any) at time t.

Step 2) Find CT_t, the set of targets in B_t for which an (additional) interceptor can be assigned (i.e., the target is chased by fewer than M interceptors). If $\mathrm{CT_t} \neq \emptyset$, go to the next step; otherwise, stop the procedure.

Step 3) For each target j in CT_t, predict the hit probabilities p_{ijt} and $p_{ij(t+1)}$ of each interceptor launcher i at the current time t and the next time $t+1$, respectively. The next state of the target at time $t+1$ can be estimated based on the current state of the target using the velocity equation in [25]; thus, we can predict the hit probability $p_{ij(t+1)}$.

Step 4) Find CI_t, the set of interceptor launchers that satisfy $p_{ijt} \geq P_{min}$ and $p_{ijt} - p_{ij(t+1)} \geq 0$. If $\mathrm{CI_t} = \emptyset$, stop the procedure. Otherwise, for each interceptor launcher i in CI_t, assign the target j that has the maximum p_{ijt} among the targets in CT_t. Launch an interceptor from the assigned launcher to the target.

The above procedure is carried out repeatedly at each time point until the end point T is reached.

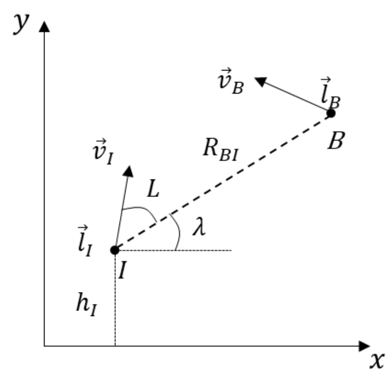


FIGURE 1. Features extracted from the states of the target and interceptor.

C. HIT PROBABILITY PREDICTION

1) PROPOSED FEATURES

The information that can be obtained from radar in a three-dimensional Cartesian coordinate space is the absolute locations $\vec{l}_B$ and $\vec{l}_I$ and absolute velocities $\vec{v}_B$ and $\vec{v}_I$ of target B and interceptor I, respectively. Using the state information, we extract six features that are useful for calculating the hit probability. As shown in Fig. 1, the first feature is the line-of-sight (LOS) distance, R_{BI}, between the target and

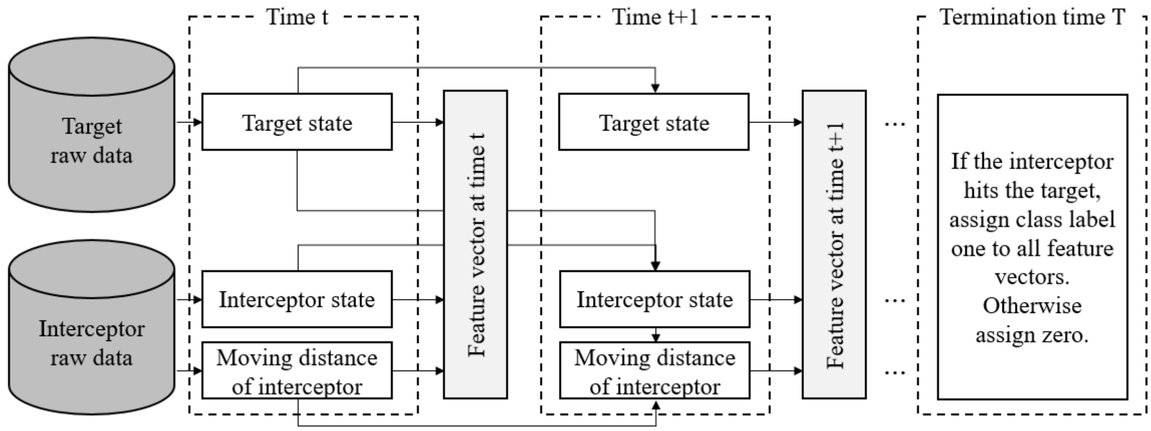


FIGURE 2. Outline of training data generation.

interceptor ($R_{BI} = \left\|\vec{l}_B - \vec{l}_I\right\|$). The LOS refers to the straight line connecting target B with interceptor I, as shown in Fig. 1. The shorter the LOS distance is, the higher the probability of hitting the target with the interceptor. The second feature is the relative velocity, $\vec{v}_{BI}$, between the target and interceptor ($\vec{v}_{BI} = \vec{v}_B - \vec{v}_I$). If the direction of the relative velocity is such that the distance between the target and interceptor becomes shorter, the hit probability increases.

We also extract features related to the hit angle because the angle at the point where the interceptor hits the target can influence the hit probability. Let the angle between the LOS and $\vec{v}_I$ be L and the angle between the LOS and the horizontal plane be λ. Then, L and λ are the third and fourth features, respectively. These four features represent the relative state information of the target and interceptor – the features do not directly use absolute locations and velocities of the two missiles to ensure that the machine learning model learns the hit probability independently of absolute geometry factors. Due to this property, the trained model and these features exhibit high performance for scenarios not used in the model training, and this result also enhances the generalized power of hit prediction. Among the extracted features, the first, second, and fourth features are also used for guided navigation used by the interceptor to chase the target [29]. The fifth feature is the moving distance of the interceptor, d_I. The interceptor cannot chase the target indefinitely because it has limited fuel. If the moving distance of the interceptor exceeds a threshold, the hit probability decreases rapidly. The sixth feature is the flight altitude of the interceptor, h_I. The minimum and maximum altitudes at which the target can be hit are determined by the interceptor type. The last two features are not related to the target. These features reflect the performance of the interceptor.

2) GENERATION OF TRAINING DATA

Consider an engagement scenario in which there is one target and one interceptor. The radar periodically collects the target information. Fig. 2 depicts the training data generation procedure. If the target state is $(\vec{l}_B^t, \vec{v}_B^t)$ and the interceptor state is $(\vec{l}_I^t, \vec{v}_I^t)$ at time t, then feature vector $\vec{x}_t$, consisting of the six proposed features, is extracted from the raw data repositories. The interceptor chases the target using a true proportional navigation method and thus can calculate its own state $(\vec{l}_I^{t+1}, \vec{v}_I^{t+1})$ at $t+1$ based on its own state $(\vec{l}_I^t, \vec{v}_I^t)$. The moving distance of the interceptor at $t+1$ is obtained by $d_I^{t+1} = d_I^t + \left\|\vec{l}_I^t - \vec{l}_I^{t+1}\right\|$. The radar then detects a new target state $(\vec{l}_B^{t+1}, \vec{v}_B^{t+1})$ at $t+1$. Thus, the target state, interceptor state, and moving distance of the interceptor at time $t+1$ are obtained, and the new feature vector $\vec{x}_{t+1}$ is calculated by using this information. If the interceptor repeats this process until the termination time T, the series of feature vectors for the engagement scenario is recorded. In this study, the termination time indicates when 1) the target reaches a defense asset of our forces, 2) the fuel of the interceptor is exhausted, or 3) the interceptor hits the target.

Each data element for the machine learning training model consists of input attributes and a response. In the proposed method, the feature vector $\vec{x}_t$ at time t corresponds to the attributes, and the engagement result at termination time T corresponds to the response y_t. In this study, y_t is set as follows:

$$y_t = \begin{cases} 1, & \textit{if interceptor hits the target} \\ 0, & \textit{otherwise} \end{cases} \qquad \forall t = 1, 2, \cdots, T. \tag{9}$$

The proposed method assigns the value one to y_t for every $\vec{x}_t$ $(t = 1, \ldots, T)$ if the interceptor hits the target at termination time T; otherwise, it assigns zero to y_t for every $\vec{x}_t$ $(t = 1, \ldots, T)$. The reason for this class label assignment is as follows. If the interceptor hits the target, we assume that every state of the interceptor contributed equally to the target elimination and give a reward ($y_t = 1$). If the interceptor fails to hit the target, every state of the interceptor is assumed to be responsible for the failure, so a penalty ($y_t = 0$) is given to every state. We do not consider a negative penalty because the hit probability model should output a value in [0, 1]. This class labeling mechanism enables the model to learn the individual state pairs of the target and interceptor, which are represented as feature vectors that have a high (or low) hit probability. In addition, a single simulation does not correspond to a data element but generates a dataset of size T, $\{(\vec{x}_t, y_t)\,|t = 1, \ldots, T\}$. Therefore, a large amount of training data can be obtained for the model if various engagement scenarios are simulated; this effect increases the generalized prediction power of engagement results for new scenarios.

3) MACHINE LEARNING MODELS

From the viewpoint of machine learning, if a model has a large bias (i.e., low model flexibility), it is likely that the complex nonlinear relationship between attributes and a response cannot be found. On the other hand, if a model has a large variance (i.e., high model flexibility), it is likely that the model is over-fitted to the training data and that the response of the test data cannot be predicted properly. Because the complexity of the relationship between attributes and responses is not known in advance, it is not easy to find a good model that has minimal bias and variance. In this study, LR [26], MLP [27], and SdA [28] are considered in the hit probability model. LR is known to have large bias and small variance; SdA has small bias and large variance; and MLP has bias and variance that are somewhere between those of LR and SdA. The best model can be found by comparing their performance.

LR defines the hit probability $P(\vec{x}_t)$ as the likelihood that the interceptor hits the target, as shown in (10) in which the regression coefficients α and $\vec{\beta}$ are determined so that the difference between the actual response y_t of $\vec{x}_t$ and the prediction probability $P(\vec{x}_t)$ are minimized [26].

$$P(\vec{x}_t) = \frac{e^{\alpha+\vec{\beta}\cdot\vec{x}_t}}{1+e^{\alpha+\vec{\beta}\cdot\vec{x}_t}}. \tag{10}$$

If LR is used for the classification task that predicts the success or failure of hitting the target in an engagement, then LR predicts $\hat{y}_t = 1$ (success) if $P(\vec{x}_t) \geq 0.5$ and $\hat{y}_t = 0$ (failure) if $P(\vec{x}_t) < 0.5$. LR is an appropriate hit probability model if the relationship between the feature vector $\vec{x}_t$ and the response y_t is not complex.

As shown in Fig. 3, MLP for the hit probability model is a neural network consisting of an input layer, a hidden layer, and an output layer [27]. The input layer has as many nodes as features to accept the feature vector $\vec{x}_t$, while the output layer has only one node for calculating the hit probability. The number of nodes in the hidden layer is set equal to the number of features throughout a preliminary experiment. The nodes of adjacent layers are fully connected, and each connection is weighted to learn using the training data. MLP uses an activator (also termed an activation function) for hidden nodes and an output node to express the nonlinear relationship between the attributes and response. In this study, a rectified linear unit (ReLU) [30] and a sigmoid function [31] are used

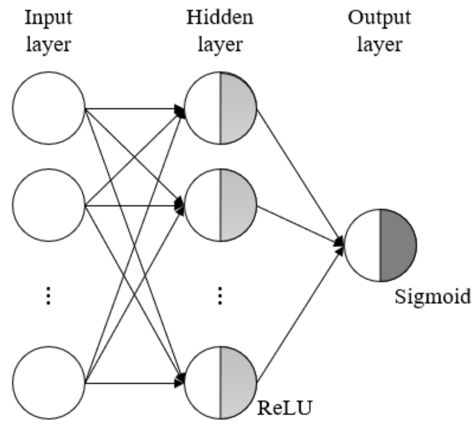


FIGURE 3. Network structure of multilayer perceptron.

as the activators of the hidden nodes and output nodes of MLP, respectively. To minimize the difference between the actual response, y_t, and the prediction probability, $P(\vec{x}_t)$, MLP adjusts the weights while propagating the squared error $\{y_t - P(\vec{x}_t)\}^2$ of the output layer to previous layers using a backpropagation algorithm.

SdA is a neural network with multiple hidden layers that can identify a more complex relationship between attributes and responses than MLP [28]. The SdA model used in this study has four hidden layers. Each hidden layer has the same number of nodes as the number of input features. Each hidden node uses a ReLU as its activator. The output layer has one node, and this node uses a sigmoid function as an activator. Unlike MLP, weight training in SdA consists of pretraining and fine tuning steps. This training method can solve the vanishing gradient problem of the backpropagation algorithm in which the weights of the hidden layers close to the input layer are not trained well if the number of hidden layers becomes large. The pretraining step serves the role of training the initial weights. In this step, the output layer is temporarily placed next to the first hidden layer. Then, the weight of the first hidden layer is trained so that the input feature values are reconstructed in the output layer. Next, the temporary output layer is placed next to the second hidden layer. Then, the weight of the second hidden layer is trained so that the first hidden layer values are positioned in the output layer. Because there are four hidden layers, the weights of all layers are determined if this process is repeated four times. In the fine-tuning step, the weights are trained for each hidden layer. In the pretraining step, the weights are set as the initial weights of the total network. Then, the weights of the entire network are trained again by using the backpropagation algorithm.

III. COMPUTATIONAL RESULTS AND ANALYSIS

A. SIMULATOR

As the flight state information of a target and interceptor in an engagement situation is a military secret, related data were generated using a simulator in existing studies [32]–[34]. The simulator developed in this study consists of a target

TABLE 2. Performance evaluated at every radar scanning time.

	Model	Accuracy (train)	Accuracy (test)	TPR	TNR
Raw features	Logistic regression (LR)	0.930	0.928	0.927	0.929
	Multilayer perceptron (MLP)	0.854	0.870	0.761	0.923
	Stacked denoising autoencoder (SdA)	0.863	0.898	0.770	0.973
Proposed features	Logistic regression (LR)	0.956	0.967	0.969	0.934
	Multilayer perceptron (MLP)	0.786	0.763	1	0.746
	Stacked denoising autoencoder (SdA)	0.904	0.909	0.904	0.911
Whole features	Logistic regression (LR)	**0.979**	**0.973**	**0.984**	**0.953**
	Multilayer perceptron (MLP)	0.774	0.826	0.635	1
	Stacked denoising autoencoder (SdA)	0.891	0.895	0.819	0.928

simulation part and an interceptor simulation part. The target simulation part considered a SCUD-B as the BM. When the user inputs the launching point and targeting point of a BM, the simulation part generates three-dimensional location and velocity data during the flight of the BM [25]. The interceptor simulator chases a target based on proportional navigation guidance, which determines the location and velocity of the chasing interceptor at each time based on the location and velocity information of the target [29]. The maximum speed and acceleration of the interceptor were assumed to be 1.5 km/s and 20 gravity, respectively. The simulation was run on a desktop computer with an Intel Core i5 3.5 GHz processor and 16.0 GB RAM. Finally, the radar scanning cycle was set to one second.

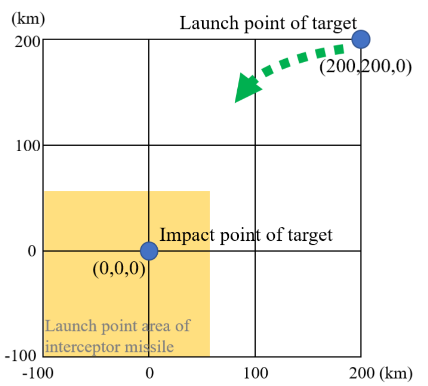


FIGURE 4. Engagement scenarios in the hit probability simulation.

B. HIT PROBABILITY PREDICTION EXPERIMENT

1) ENGAGEMENT SCENARIOS

Fig. 4 shows the engagement scenarios designed with a military expert's assistance. The target was launched at a fixed location (200, 200, 0) on the coordinate system and aimed at the defense asset of our forces at location (0, 0, 0) in every scenario. The distance between the target's launch point and impact point was approximately 280 km. The second-order polynomial relationship between the altitude of the target and the movement distance was determined.

The launch point of the interceptor was varied by changing the x and y coordinates in 10 km units from -100 km to 50 km. The launch time was changed in 10 second units from 100 seconds to 200 seconds after the target's launch. These experimental conditions generated a total of 2,816 ($16 \times 16 \times 11$) different engagement scenarios, some of which included situations where the interceptor failed to hit the target. Each simulation run corresponded to one engagement scenario, and approximately 1,000 feature vectors and hit results were generated. 70% of the data obtained from the 2,816 simulation runs were used to train the three machine learning models (training), and the remaining 30% were used to optimize the parameters of the model (i.e., normalization of the parameter for LR, number of hidden nodes, and the iteration number of the backpropagation algorithm for the two neural networks) (validation). In addition, 100 new engagement scenarios were created to test the performance of the trained models. In each test scenario, the interceptor's launch point was randomly determined within the launch point area in Fig. 4, and the launch time was randomly determined between 100 seconds and 200 seconds.

2) RESULT AND ANALYSIS

The three machine learning models (LR. MLP, and SdA) predict the hit result (success or failure) by rounding up the hit probability calculated at each radar observation time. As explained in Section II, we assume that the prediction is correct if the prediction result at any state of the interceptor is identical to the actual hit result at the termination time. The reason for this assumption is as follows. Consider a situation where an interceptor is chasing a target. A new interceptor is also available for the target. In this situation, launching the new interceptor at a time when the one in flight has a low chance to hit the target is a critical decision. To enable this decision, the proposed method calculates the hit probability while the interceptor is chasing the target as well as at the launch time of the interceptor.

Based on these criteria, the experiment measured the accuracy in terms of the true positive rate (TPR) and true negative rate (TNR) for all test scenarios. The experimental results are summarized in Table 2. The accuracy is the ratio of the number of correct prediction results (success or failure) to the total number of hit results. With respect to accuracy and compared to the trained MLP and SdA models, the trained LR model showed excellent overall performance. From the results, an LR model with a large bias and small variance is an appropriate model for hit probability estimation. This result implies that the probabilistic relationship between the attributes (feature vector) and the response (success or failure) is not so complex. The accuracy of LR for the test data exceeded 0.9 for the three feature selection methods. However, the accuracy of MLP was sensitive to the input data –MLP showed a high accuracy for the test data when the raw features were used as inputs (0.870), but its accuracy decreased when using the proposed features (0.763) or the whole features (0.826). SdA achieved stable accuracy for the feature selection methods (0.898, 0.909, and 0.895) compared with that of MLP, but SdA performance was inferior to that of LR. Based on these observations, the following experiments utilized the LR model.

In general, the performance of the machine learning models depended on the quality of the input data. This experiment compared three input feature selection methods. The first method (raw features in Table 2) used the absolute locations and absolute velocities ($\vec{l}_B$, $\vec{v}_B$, $\vec{l}_I$, $\vec{v}_I$) of the target and interceptor detected by the radar as predictors. The second method (proposed features in Table 2) used six features (R_{BI}, $\vec{v}_{BI}$, L, λ, d_I, h_I) as predictors. The last method (whole features in Table 2) used all features (both the first and second methods). From the viewpoint of the three performance measures, the use of whole features as predictors for LR performed better than the use of raw features or proposed features.

A statistical hypothesis test was conducted to discover the input features that were significant for the response when using whole features. The test results indicated that the p-values of six features were smaller than 0.0001, implying that these features contributed to the prediction of the hit result. Among the raw features, l_{B_z} in $\vec{l}_B = (l_{B_x}, l_{B_y}, l_{B_z})$ and l_{I_x} and l_{I_y} in $\vec{l}_I = (l_{I_x}, l_{I_y}, l_{I_z})$ were verified to contribute significantly to the prediction. Their regression coefficients were 0.323, −0.310, and −0.311, respectively. Among the proposed features, the statistical test selected R_{BI}, L, λ, and d_I as significant features, and their regression coefficients were −0.752, 1.248, 18.405, and −0.398, respectively. The contribution of the proposed features to hit prediction was greater because there were more important variables in the proposed features than in the raw features, and the proposed features had greater absolute regression coefficient values than the selected raw features.

The angle λ between the horizontal plane and the LOS direction played the largest role in the hit probability calculation and had the largest absolute regression coefficient. The direction of the BM becomes almost perpendicular to the horizontal plane as the BM approaches the defense asset. The closer the missile is to the defense asset, the higher the hit probability. This result implies that the hit probability increases as λ approaches 90 degrees. For this reason, λ was selected as the most significant feature. The second important feature was L, which is the angle between the LOS and $\vec{v}_I$. This angle should be large enough for the interceptor to chase the target. The features R_{BI} and d_I are the LOS distance and moving distance of the interceptor, respectively. As the values of R_{BI} and d_I decrease, the hit probability becomes larger. Therefore, the regression coefficients of R_{BI} and d_I were negative. Among the proposed features, R_{BI} and λ were also used in the guided navigation of the interceptor. These results imply that the LR model was well-trained and that the significant features were selected appropriately.

When the whole set of features was input into the LR model, the TPR was 0.984. The TPR is the ratio of scenarios that correctly predicted hitting the target using the hit probability to the scenarios where the interceptor successfully hit the target. A high TPR implies that the chance of hitting the BM is very high if the interceptor missile is launched at a time when the hit probability is higher than 0.5. Thus, the hit probability can be used as a good target selection criterion in an engagement situation in which a limited number of interceptors exist for many targets. Furthermore, the TNR of LR was 0.953. The TNR is the ratio of scenarios that correctly predicted the failure to hit the target to the scenarios where the interceptor failed to hit the target. A high TNR has an advantage in the following situation. The hit probability was high at the time of launching an interceptor, but it dropped below 0.5 after the launch due to irregular state changes for the two missiles. In this case, launching an additional interceptor might be considered. The TNR of the hit probability model should be high (i.e., the confidence of the model about a failure to hit the target is high) to minimize the waste of an additional interceptor.

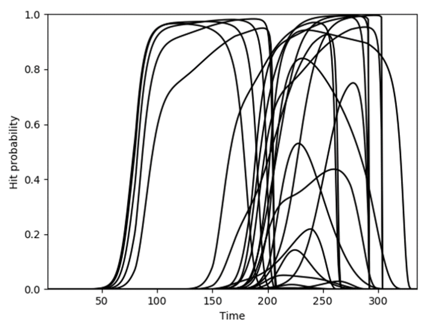


FIGURE 5. Hit probability over time.

Fig. 5 plots the curves of the hit probability of targets chased by interceptors launched at various locations. Each curve shows the time-varying probability from when an interceptor was fired to when it reached an impact position. Although the probability curves are different because of the various launching positions of the interceptors, the overall shapes are similar – the hit probability increases when the

TABLE 3. Performance measures.

Target assignment rate	$\frac{\text{Number of assigned targets}}{\text{Total number of targets}}$
Hit rate of assigned targets	$\frac{\text{Number of hit targets}}{\text{Number of assigned targets}}$
Number of interceptors per target	Average number of interceptors assigned to one target
Interceptor hit success rate	$\frac{\text{Number of interceptors that successfully hit the target}}{\text{Number of interceptors assigned to target}}$
Interceptor hit failure rate	$\frac{\text{Number of interceptors that failed to hit the target}}{\text{Number of interceptors assigned to target}}$
Interceptor mission incompletion rate	$\frac{\text{Number of additionally assigned interceptors lost during engagement}}{\text{Number of additionally assigned interceptors}}$

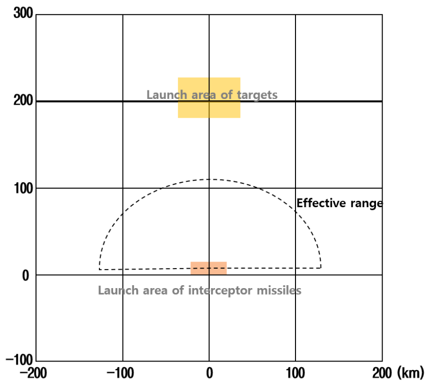


FIGURE 6. Engagement scenarios in the AWTA simulation.

target direction is upward and decreases when the target direction is downward.

C. AWTA EXPERIMENT

1) ENGAGEMENT SCENARIOS

Fig. 6 shows an engagement situation of multiple targets and multiple interceptors designed with the help of a military expert. In every engagement scenario, a BM is located at a random point in the area (x, y, z), where $x \in [-50, 50]$, $y\in [180, 230]$, and $z= 0$, and fired at a random time in [0, 120] (seconds) from the beginning of the engagement toward a random impact point in the area (x, y, z), where $x \in [-30, 30]$, $y\in [0, 30]$, and $z= 0$. Five interceptor launchers were deployed in the interval of 10 km to the left and right from location (0, 15, 0). In the figure, the effective range, which is approximately 100 km, means the area in which the interceptor can be hit. Ten different engagement scenarios were created for each number of targets (5, 10, 15, 20, 50, and 100 targets).

2) COMPARISON MODELS AND TEST MEASURES

In the experiment, the performance of the AWTA model was compared with that of the DWTA models using a TS [22] and GA [24]. However, unlike the AWTA model, which can be performed even if the number of targets, launching times of targets, and hit probability are unknown, existing DWTA models, including these two, require that information to perform WTA. Therefore, for every comparison case, a simulation of the targets without deploying interceptors was run, and then the outputs were fed into the two models. This process did not make sense as a defense procedure in an actual engagement, but it was necessary for performance comparison. The performance of the two comparison models was evaluated by applying the assignment result to the engagement scenario in which the same targets were applied but interceptors were launched this time.

The performance of each model was evaluated using the measures in Table 3. The parameter settings used in this experiment were as follows. For the AWTA model, $P_{min} = 0.5, M = 2$, and $P_{\varepsilon} = 0.9$. For the GA model, population size = 40, crossover size = mutation size = 10, selection size = 20, and an iteration count of 200 was set. For the TS model, an iteration count of 10 was set. The computation time was limited to 1,800 seconds (30 minutes). If the limited time was exceeded, the iteration was stopped, and the engagement simulation was conducted based on the assignment that was obtained at the end of the limited time.

3) RESULTS AND ANALYSIS

The performances of the AWTA, GA, and TS models are summarized in Table 4 for the target numbers of 5, 10, 15, and 20. The number in each cell indicates the average measurement of 10 different scenarios.

In terms of the target assignment rate, every model showed the same performance of 0.979 on average. In terms of the hit rate of assigned targets, the GA model showed the highest performance of 0.836. However, the performance of the AWTA model (0.799) was not much lower than that of the GA model and was better than that of the TS model (0.754). This result was very encouraging. Note that the WTA solutions of the GA model were obtained when future engagement scenarios were known in advance. In contrast, the AWTA model performed a series of WTAs with radar information that was obtained as the engagement scenarios progressed. From the results, it was verified that the AWTA model can be applicable to actual BM defense situations.

TABLE 4. Performance results for engagement scenarios.

	Model	5 targets	10 targets	15 targets	20 targets	Average
Target assignment rate	AWTA	0.920	0.970	0.993	0.975	**0.979**
	GA	0.920	0.970	0.993	0.975	**0.979**
	TS	0.920	0.970	0.993	0.975	**0.979**
Hit rate of assigned targets	AWTA	0.696	0.835	0.832	0.831	0.799
	GA	0.848	0.835	0.872	0.790	**0.836**
	TS	0.783	0.763	0.745	0.723	0.754
Number of interceptors per target	AWTA	1.109	1.113	1.074	1.082	**1.095**
	GA	1.739	1.649	1.591	1.605	1.646
	TS	1.761	1.732	1.436	1.174	1.526
Interceptor hit success rate	AWTA	0.627	0.750	0.775	0.768	**0.730**
	GA	0.488	0.506	0.549	0.492	0.509
	TS	0.444	0.440	0.519	0.616	0.505
Interceptor hit failure rate	AWTA	0.314	0.185	0.169	0.209	**0.219**
	GA	0.238	0.188	0.173	0.278	**0.219**
	TS	0.247	0.256	0.220	0.288	0.253
Interceptor mission incompletion rate	AWTA	0.059	0.065	0.056	0.024	**0.051**
	GA	0.275	0.306	0.278	0.230	0.272
	TS	0.309	0.304	0.262	0.096	0.243

With respect to the number of interceptors per target, the interceptor hit success and failure rates, and the interceptor mission incompletion rate, the AWTA model showed better performance than the comparison models. This performance gap was caused by the difference in problem-solving approaches between the AWTA model and the DWTA models. The AWTA model determined interceptor launching by carefully considering the hit probability – an interceptor (additionally) is launched when the probability becomes the highest. As a result, this model did not waste interceptors. In contrast, the comparison models allowed the assignment of multiple (M) interceptors to a target if the assignment improved the total hit probability – some interceptors could lose a target because previously launched interceptors might hit that target. The interceptor mission incompletion rate represents this case.

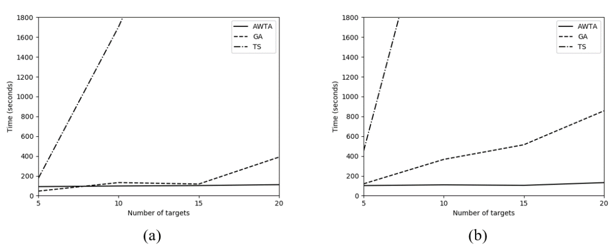


FIGURE 7. Computation time according to the number of targets: (a) average time and (b) maximum time.

The computation times required to obtain the assignment results are compared in Fig. 7. For each engagement scenario, the computation time of the AWTA model is the sum of the WTA calculation times at all points of time from the start to the end time points of the engagement. For example, if the total engagement time is 250 seconds, the model is performed at each time point, and 250 WTA calculations are carried out. The total of these 250 calculation times becomes the total computation time of the AWTA model for that scenario. The computation times of the GA and TS models correspond to the algorithm execution times at the beginning of the engagement scenario, not including the time required to acquire input data from the simulation.

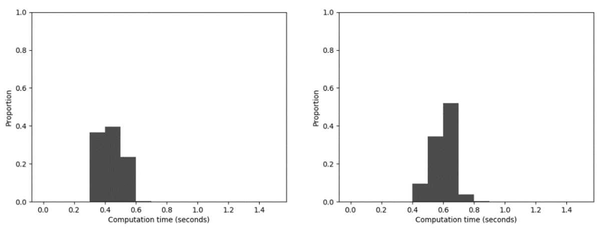
As the number of targets increased, the AWTA model showed better performance than the comparison models in terms of the computation time (average and maximum). In every model, the time increased almost linearly as the number of targets increased, but the amount of the increase for the AWTA model was the smallest. In the case of TS, when the number of targets exceeded 10, the model could not calculate the optimal assignment plan within the limited time (1,800 seconds). This result means that the DWTA models are hard to deploy in actual engagement situations due to the computation burden even if the models know the time-varying hit probability, number of targets, and launching times of targets.

TABLE 5. Performance results for large-scale engagement scenarios.

Performance measures	50 targets	100 Targets
Target assignment rate	0.968	0.973
Hit rate of assigned targets	0.816	0.852
Number of interceptors per target	1.095	1.095
Interceptor hit success rate	0.745	0.778
Interceptor hit failure rate	0.211	0.170
Interceptor mission incompletion rate	0.043	0.052
Maximum computation time (seconds)	165.90	212.63
Average computation time (seconds)	139.97	188.59

In real situations, many BMs are in flight to attack our forces during an engagement period. Thus, we conducted experiments for large-scale engagement problems (number of targets is 50 and 100). The results are shown in Table 5.

From the results in Tables 4 and 5, we observed that the performance of the AWTA model was stable even if the number of targets increased. In particular, when the number of targets was 5, the average computation time was 91.46 seconds (see Fig. 7), but when the number of targets increased to 100, the average computation time was 188.59 seconds. That is, although the number of targets increased by 20 times, the average computation time only doubled.



(a) (b)

FIGURE 8. **Distribution of computation time for the assignment at a time point: (a) 50 targets and (b) 100 targets.**

The AWTA model determines WTA at each radar scanning time. If the computation time for the assignment at a time point exceeds the radar detection cycle (one second), it is impossible to launch the interceptor at the current time. Therefore, the time required for the assignment is an important criterion for judging the possibility of using the AWTA model in actual situations. The distributions of the computation time are shown in Fig. 8. Even for large-scale problems, the assignment time was within one second. Consequently, the AWTA model is a promising tool for BM defense missions in large-scale, dynamic engagement situations.

IV. CONCLUSION

In WTA problems, it is unrealistic that the number of targets, launching times, and hit probability are known to obtain WTA solutions. In particular, the hit probability varies according to the flight states of the target and the interceptor chasing the target. To overcome such limitations, we developed an AWTA model. In this model, WTA is performed at each radar scanning moment based on the hit probability calculated using the radar information acquired at that moment. An interceptor whose hit probability exceeds a threshold and is higher than that of the other interceptors is assigned to each target. We created the hit probability predictor by training a machine learning model. The input features for the training model were developed to improve the accuracy of the hit probability. Among LR, MLP, and SdA machine learning models, the performance of LR was the highest. Simulation-based experiments with two comparison DWTA models employing TS and GA confirmed that the AWTA model had excellent performance and could be applied to actual BM defense missions for small- and large-scale WTA problems.

In this study, we showed the validity of the AWTA model in an engagement situation where only one type of BM (SCUD-B) is considered. Future studies should consider engagement situations of different types of BMs and interceptors that are more similar to reality. In addition, it is necessary to consider the target reassignment problem, which allows an interceptor in flight to chase a new target when the old target is lost, hit by another interceptor, or already reaches its impact point. This reassignment capability will decrease the mission incompletion rate of interceptors, enabling efficient utilization of resources. The AWTA model outfitted with the reassignment function will significantly contribute to future BM defense missions.

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JAEYEON JANG received the B.S. degree in industrial engineering from Yonsei University, South Korea, in 2015, where he is currently pursuing the Ph.D. degree in industrial engineering. His current research interests include machine learning for manufacturing and defense analysis.

HYUI GEON YOON received the B.S. degree in industrial engineering from Yonsei University, South Korea, in 2014, where he is currently pursuing the Ph.D. degree in industrial engineering. His current research interest includes data science for manufacturing industries.

JEONG CHEOL KIM received the B.S. degree in industrial engineering from the Republic of Korea Airforce academy, South Korea, in 2008, and the M.S. degree from Yonsei University, South Korea, in 2019. He is currently serving as an Officer of the Airforce. His current research interests include data science for defense analysis and intelligent decision-making.

CHANG OUK KIM received the Ph.D. degree from Purdue University, West Lafayette, IN, USA, in 1996, all in industrial engineering. He is currently a Professor with the Department of Industrial Engineering, Yonsei University, South Korea. He has published more than 100 papers in journals and conference proceedings. His current research interests include data science for manufacturing and defense analysis.