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A Dynamic Traffic Assignment Method Based on Connected Transportation System

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ABSTRACT This paper proposed a dynamic traffic assignment method based on the connected transportation system to express the time-varied traffic flows caused by uncertain traffic demand and supply in the real traffic network accurately. In dynamic traffic assignment, because of the cumulative volumes of successive time intervals, a route-based traffic assignment algorithm is used for dynamic traffic assignment in this paper. Besides, a novel shortest path algorithm based actual and artificial link is proposed for the time-expanded network that consists of two kinds of arcs: the actual arc and the artificial one. Then, a simple example is used to illustrate the performance of the proposed method. The computational results show that the novel shortest path algorithm based actual and artificial link could be applied in the time-expanded networks with two types of arcs, and the route-based assignment method could achieve optimal balance solutions under different demand modes.

INDEX TERMS Connected transportation system, dynamic traffic assignment, route-based algorithm, shortest path algorithm based actual and artificial link, time-expanded network.

I. INTRODUCTION

The connected vehicle means the vehicle equipped with wireless communication technologies. Then, these vehicles can communicate with other vehicles or the road infrastructures during the trips. The connected transportation system contains the communication process between vehicles, the communication between vehicles and road infrastructures, and so on. Due to the in-trip communication, the travelers and traffic managers can collect the real-time information on the road. It will help them choose the optimal paths and manage the transportation system. Therefore, the connected transportation system is considered as the most promising technologies to reduce the delay and travel time. Based on the connected transportation system, the traffic evolution on recurrent factors in urban transportation systems is easily obtained, and the traffic assignments under periodic conditions are easily to implement. Urban road traffic network can be very complex and often faces traffic accidents and other emergencies, which can lead to road capacity degradation and the addition travel time of travelers [1]. During peak hours, the traffic demands increase first, and decrease later. The changing demands make that the equilibrium state of network could be broken. The new balance of traffic volume cannot be obtained instantaneously. Therefore, it is more complex to study the dynamic traffic assignment (DTA) considering the changing demand. However, based on connected transportation system, the information about the demand changing can be obtained by travelers and traffic managers immediately, they could change their paths of trips or assign the traffic flow in other routes. Therefore, studying the dynamic traffic assignment considering fluctuant demand make significant sense for connected transportation system to reduce the delay and travel time in paths.

Traffic assignment is a process of allocating the given origin-destination traffic demands onto the arcs in a road network. Two criteria: Wardrop's user-equilibrium (UE) and system optimum (SO), are usually accepted as the standard objective of traffic assignment model [2], [3]. The travel time on an arc is usually assumed to be a function of the traffic flow on the arc, not vice versa. The traffic assignment model, in which the origin-departure matrix (OD

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matrix) remains constant during a period, is called the static model.

However, the condition of urban transportation system is uncertain [4]–[8]. In fact, traffic demand is time varied and not fixed within a period, and road traffic capacity can be affected by traffic accidents, road maintenance, natural disasters and so on. Both uncertain traffic demand and supply can cause congestion [9]. Therefore, the static model could not describe the time-varying traffic flows in a real traffic network exactly. So, the DTA is proposed to describe the time-varying network.

The literatures on DTA could be classified into two groups: analytical approaches and simulation-based methods. In general, there are three major broad categories of analytical approaches: mathematical programme [11]–[16], optimal control theory [17]–[23], and variational inequality [24]–[27]. With the development of computer simulation technology, many papers have established simulation-based DTA models [28]–[30]. Generally, the stimulation-based models have advantages in solving the large scaled real-world DTA problems, in which the operational strategies come from individual decisions or personal choices. More comprehensive analysis and introduction about the history, the latest trend and main progresses about DTA could be found in literatures [28], [31].

Many researchers adopted the Frank–Wolfe (FW) algorithm [32] in mathematical programme or variational inequality method for DTA, such as Janson [14], Ran and Boyce [25], Jayakrishnan *et al.* [16], and Chen and Hsueh [26]. However, the FW algorithm is a kind of static traffic assignment method. Even though it can be applied in DTA, the convergence rate of FW algorithm is too slow to fit the assignment requirement of connected transportation system. Some scholars proposed several approaches to alternate the FW algorithm, the most widely used two approaches are column generation method (Dafermos and Sparrow [33]; Leventhal *et al.* [34]; Schittenhelm [35]; Han [36]) and simplicial decomposition method (Larsson and Patriksson [37]; Lee [38]). These methods use several extreme points to obtain the faster rate of convergence than FW algorithm. What is more, the column generation method approaches the equilibrium based on the traffic flows on the route, and it is different from the link-based methods, such as FW algorithm and simplicial decomposition method. The column generation method is also called route-based traffic assignment approach, because the solution is found by the route flow columns generated over iterations in this algorithm. The route-based assignment approach is more plausible than link-based approach [36]. Considering the characteristics of dynamic traffic assignment in connected transportation system, the route-based traffic assignment is studied in this paper. The reason why link-based traffic assignment method cannot be applied in this paper will be explained in detail in section 4. In recent years, Yao et al. first proposed a novel traffic assignment based on space-time reliability [39]. To our best known, Yao et al. firstly proposed an iterative assignment model considering the operation permit game between operators [40]. Han proposed a route-based solution algorithm to solve the DTA problem, and in this paper the authors pointed out that the route-based solution algorithm could attain high quality user equilibrium assignments under time varying conditions [36].

The contribution of this paper can be summarized as follows: Based on the characteristics of connected transportation system, we consider the dynamic traffic assignment, when road capacity degradation because of emergencies. Compared to most of the existing researches about DTA, this paper tends to establish a time-expanded network to describe the consideration of time dimension in the problem. It should be noticed that there are two kinds of arcs (the actual arcs and the artificial arcs) in a time expanded network, to describe the urban traffic network. The time expanded network is simplified by some strategies to decrease the scale of network. The conventional shortest path algorithm such as Dijkstra algorithm cannot find the shortest path in the network with heterogeneous arcs. Considering the real-time requirement of assignment in connected transportation system, it's almost impossible to have an actual application of the enumerated method. Thus, a novel shortest path algorithm based actual and artificial link is proposed to be applied on the time-expanded network with the actual arcs and the artificial arcs. The proposed algorithm can find the shortest path in the network with both actual and artificial link. Then, we used a route-based traffic assignment algorithm to solve the DTA problem.

The rest of this paper is organized as follows. In section II, the formulation of the dynamic assignment problem is presented and the definition of dynamic user equilibrium is described. Section III proposes the time-expanded network and a novel shortest path algorithm based actual and artificial link. An empirical study is conducted to validate the proposed algorithm in section IV. Finally, some conclusions of the paper are drawn in section V.

II. DYNAMIC TRAFFIC ASSIGNMENT

A. BASIC ASSUMPTIONS AND NOTATIONS

In this paper, we consider dynamic traffic assignment based on connected transportation system, and focus on the problem of traffic assignment with the fluctuant demand during the peak hours. Based on connected transportation system, the vehicles can communicate with each other. We assume that travelers can receive the information about the other vehicles and surrounding traffic condition. When the road capacity changes due to predictable traffic events, travelers can obtain the traffic changing information in advance. When the road capacity changes due to emergencies, travelers can obtain the traffic changing information immediately. Therefore, travelers can modify their path on time to approach the shortest travel time.

In proposed dynamic traffic assignment based on connected transportation system, N is a set of nodes and A is a set of arcs (with $a \in A$), which represents the demand nodes and roads, respectively. Let O and D represent the origin node set and destination node set, respectively. R_{od} is a set of all the feasible paths between o and d (with $o \in O, d \in D$). k donates the kth path between o and d, $K \in R_{od}$. t represents the time intervals with $t \in (1, 2, \cdots T)$, where the notation T represents the planning time horizon.

$v_a(t)$ is the traffic volume on arc a at time t (with $a \in A$, $t \in (1, 2, \cdots T)$)

$\tau_a(t)$ is the travel time on arc a at time t (with $a \in A$, $t \in (1, 2, \cdots T)$)

$g_a(t)$ is the preserved volume on arc a at time t (with $a \in A, t \in (1, 2, \cdots T)$)

$g_a^-(t)$ is the volume that entered arc a before time t and would enter next arc at time t (with $a \in A$, $t \in (1, 2, \cdots T)$)

$q_{od}(t)$ is the trip rate between origin o and destination d at time t (with $t \in (1, 2, \cdots T), o \in O, d \in D$)

$f_{od}^k(t)$ is the traffic volume on path k connecting O-D pair o-d at the time t (with $t \in (1, 2, \cdots T), o \in O, d \in D, K \in R_{od}$)

$\delta_{od,a}^k(t)$ is a binary variable which equals 1 if arc a belongs to path k between o and d at time t and 0 otherwise (with $t \in (1, 2, \cdots T)$, $o \in O$, $d \in D, K \in R_{od}$)

$\zeta_{od}^k(t)$ is the travel time of path k between o and d at time t (with $a \in A, t \in (1, 2, \cdots T), o \in O, d \in D, K \in R_{od}$). $\zeta_{od}^{k^*}(t) = \min\limits_{k \in R_{od}} \left(\zeta_{od}^k(t)\right)$ donates the minimal travel time among all feasible routes between o and d at time t (with $a \in A, t \in (1, 2, \cdots T), o \in O, d \in D, K \in R_{od}$)

B. THE ACTUAL TRAVEL TIME

The actual travel time ($\tau_a(t)$) represents the travel time of vehicles on arc a at time interval t. It should be noticed that in the model of DTA, the actual travel time of arc is unknown until vehicles traverse the arc, even in the connected transportation system, because the condition of the traffic flow on the arc is time dependent. Thus, it could be concluded that $\tau_a(t)$ could not be precisely gained at time t. However, based on connected transportation system, the actual perceptive travel time ($\hat{\tau}_a(t)$) can be obtained by the real time traffic conditions. To avoid ambiguity, in this paper both of the actual travel time ($\tau_a(t)$) and the actual perceptive travel time ($\hat{\tau}_a(t)$) are represented by $\tau_a(t)$ [25].

In this paper, the actual travel time of path k which is successively comprised of arc $a_1, a_2, \ldots, a_n$ between the origin o and destination d could be calculated by the following equation:

$$\zeta_{od}^k(t) = \tau_{a_1}(t) + \tau_{a_2}\left(t + \lceil \tau_{a_1}(t) \rceil\right) + \ldots + \tau_{a_n}\left(t + \lceil \tau_{a_1}(t) \rceil + \ldots + \lceil \tau_{a_{n-1}}(t) \rceil\right) \quad (1)$$

The sign "$\lceil \bullet \rceil$" is used to round up to an integer here.

C. DYNAMIC NETWORK LOADING

In DTA, dynamic network loading is a complex and significant step, which is used to compute flow distribution on a network. The dynamic loading at time interval t can be divided into two parts, the ingoing traffic volume and preserved volume. The ingoing volume should be the sum of all paths that pass through the arc in the same time interval, which can be formulated as $\sum\limits_{k \in R_{od}, od \in G} f_{od}^k(t) \times \delta_{od,a}^k(t)$. The preserved volume refers to the vehicles entered the arc before time interval t, and has not left the arc until this time interval.

For instance, if the actual travel time of arc a ($\tau_a(t1)$) at time $t-1$ is less than or equal to a single time increment, the traffic volume that is loaded on arc a at time $t-1$ will not remain on the arc at time interval t; The ingoing traffic volume of arc a at time $t-1$ will not leave arc a until time $t1 + \lceil \tau_a(t-1) \rceil$ at which the cumulative volume will be $(v_a(t1 + \lceil \tau_a(t-1) \rceil) - g_a^-(t1 + \lceil \tau_a(t-1) \rceil))$. Therefore, the amount of preserved volume could be measured as follows:

$$g_a(t - 1 + \lceil \tau_a(t-1) \rceil) = \begin{cases} 0 & \tau_a(t1) \leq 1 \\ v_a(t1 + \lceil \tau_a(t-1) \rceil) - \\ g_a^-(t1 + \lceil \tau_a(t-1) \rceil) & otherwise \end{cases} \quad (2)$$

The traffic volume on arc a at time interval t can be formulated as:

$$v_a(t) = \sum_{k \in R_{od}, od \in G} f_{od}^k(t) \times \delta_{od,a}^k(t) + g_a(t) \quad (3)$$

D. DYNAMIC USER EQUILIBRIUM

In this paper, the dynamic user equilibrium (DUE) assignment is an extension of the user equilibrium assignment (for further details, see Ran and Boyce [25]; Chen [41]; Kuwahara and Akamatsu [13]; Akamatsu [42]; Akamatsu and Kuwahara [43]). Under the user equilibrium condition, for any OD pairs at time t, if the volume of a path is not zero, the actual travel time of the path must equal to the minimal actual travel time of the paths connecting the OD pair; otherwise, the actual travel time of the path is greater than the minimal. We can write the above statement in a mathematical form according to the Wardrop's principle:

$$\begin{cases} \zeta_{od}^k(t) = \zeta_{od}^{k^*}(t) & if\ f_{od}^k(t) > 0 \\ \zeta_{od}^k(t) \geq \zeta_{od}^{k^*}(t) & otherwise \end{cases} \quad (4)$$

The gap function [44], [45] is used as the objective function of due assignment according to Han [36] , then dynamic user equilibrium can be formulated as follows:

$$\min \sum_t \sum_{k \in R_{od}, od \in G} \left(\zeta_{od}^k(t) - \zeta_{od}^{k^*}(t)\right) \times f_{od}^k(t) \quad (5)$$

$$\sum_{k \in R_{od}} f_{od}^k(t) = q_{od}(t) \quad t = 1, 2, \ldots, T \quad (6)$$

Because $\zeta_{od}^{k}(t) > \zeta_{od}^{k*}(t)$, thus the objective function value is greater than zero. If the gap is zero, it indicates a perfect dynamic user equilibrium flow pattern. It is very important to note that the dynamic traffic assignment model is a discrete assignment model. Formulation (6) is the flow conservation constraint.

III. SHORTEST PATH METHOD BASED ACTUAL-ARTIFICIAL LINKS

For the traffic assignment algorithm, identifying the shortest path for each OD pair is a critical step. In this section, we consider a time discrete network to fit the dynamic traffic assignment based on connected transportation system. The discrete network contains two kinds of arcs, the actual and artificial links, which will be described particularly in section 3.1. However, most of the traditional shortest path algorithms such as Dijkstra could not be used in the network consisting of several types of arcs. Besides, the enumeration method requires lots of computing time, and the shortest path algorithm applied in connected transportation system should satisfy the real-time requirement. Therefore, it is almost impossible to use an enumeration method to find feasible paths in proposed dynamic assignment algorithm based on connected transportation system. An effective shortest path algorithm, which could be applied into the network with heterogeneous arcs, is designed in this section.

A. *TIME-EXPANDED EXPANDED NETWORK WITH ACTUAL AND ARTIFICIAL ARCS*

By replicating all nodes in an actual traffic network for each time interval, the time-space network can be expanded to T sub-networks. The sub-networks could be linked by introducing artificial arcs with some features (e.g., zero travel time and unlimited capacity) in corresponding nodes. It should be notice that the size and the scale of the urban network would greatly expand after time-space expansion. In fact, each node at time interval t is only connected with corresponding node at a certain following time interval, which determined by the actual travel time $\tau_a(t)$. For example, assuming that it is $t = 1$ at present, certain traffic volume $u_o(1)$ departs from the corresponding node O and will arrive at node B after two time intervals. Then, the node B at time interval $t = 1$ should be only connected with node B at time $t = 3$. Therefore, it is unnecessary to establish a complete time-expanded network. This example is further illustrated with Fig.1. Instead of connecting with all the other layers ($(t \in (1, 2, \cdots T))$), node M in Fig. 1 is only linked with itself at the third time layer ($t = 3$) because only this artificial arc has physical meaning according to the above example. Thus, in order to downsize the scale of the time-expanded network, the actual travel time of all the useful artificial arcs are assumed to be zero, while those of the useless artificial arcs are assumed to be infinity. Based on the simplified time-expanded network the efficiency of searching shortest paths could be improved compared with the normal complete time-expanded network.

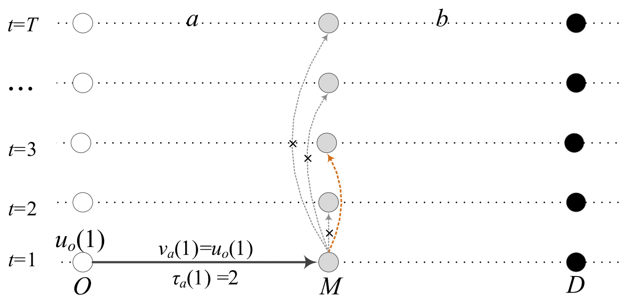


FIGURE 1. **Simplified time-expanded network.**

In DTA, there is traffic volume propagating and dispersing over time in the time-expanded network. Assuming that the traffic volumes arriving at node O at time $t = 1$ and time $t = 2$ is $u_o(1)$ and $u_o(2)$, respectively. The actual travel time of arc a at different time layers are $\tau_a(1) = 2$ and $\tau_a(2) = 1$; The actual travel time of arc b from M to D is $\tau_b(3) = 2$. Based on the data, the two trips could be described as Fig. 2.

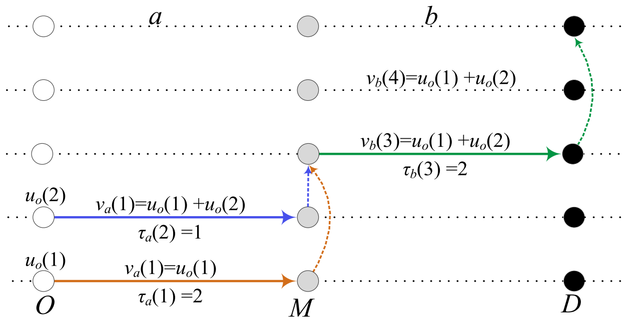


FIGURE 2. **Two feasible trips on time-expanded network.**

In the Fig. 2, it should be noticed that the volume $u_o(1)$ that enters arc a at the first time interval would spend 2 time intervals in traversing arc a, as a result, $u_o(1)$ should be accumulated to the volume on arc a at the second time interval. Similarly, volume $u_o(1)+u_o(2)$ that enters arc b at the third time intervals should be accumulated to the volume on arc b at the fourth time interval.

An example consists of 4 nodes and 5 arcs shown in Fig. 3. It could be used to explain the reasons why standard Dijkstra is not acceptable to the network with heterogeneous arcs. In the time-expanded network, because of the consideration of time, two actual arcs could not connect directly. For example, one car enters arc $A^{(1)} \rightarrow B^{(1)}$ at the first time interval, and it cannot enter arc $B^{(1)} \rightarrow D^{(1)}$ immediately after traversing arc $A^{(1)} \rightarrow B^{(1)}$. It should be firstly moved to node B at the second time interval via artificial arc $B^{(1)} \rightarrow B^{(2)}$, and then enters arc $B^{(2)} \rightarrow D^{(2)}$. Besides, traffic volume could not pass through two artificial arcs continuously because we assume that vehicles will not keep waiting at a node for more than one time-interval.

Assume the origin is node A and the destination is node D. There are two feasible paths: path $A^{(1)}B^{(1)}$-$B^{(1)}B^{(2)}$-$B^{(2)}C^{(2)}$and path $A^{(1)}C^{(1)}$-$C^{(1)}C^{(2)}$ from origin node A to

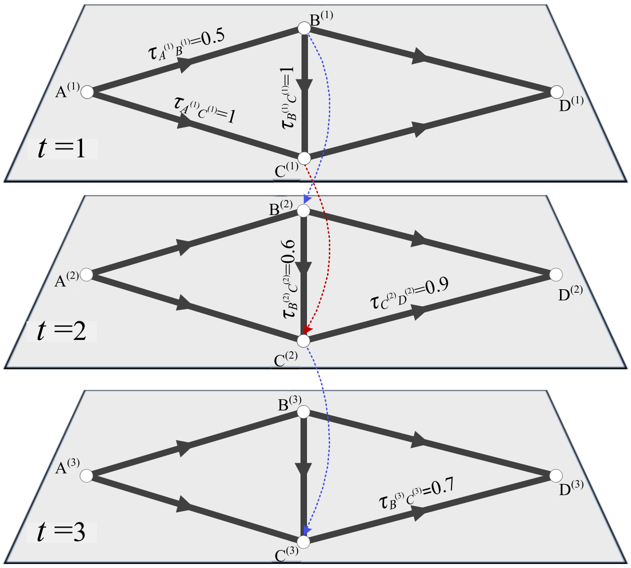


FIGURE 3. An example of standard Dijkstra for time-expanded network with actual and artificial arcs.

destination node C. Assume that the travel time of artificial arcs is zero, and then the travel time of these two paths are 2 and 1, respectively. The temporary label of $C^{(2)}$ will be updated to 1 by node $C^{(1)}$. Then, the permanent label of $C^{(2)}$ will also be recorded and its predecessor node is node $C^{(1)}$. Moreover, the travel time of $C^{(1)}$ - $C^{(3)}$ is infinite, since two artificial arcs cannot be continuously traversed. As node $C^{(2)}$ has be labeled by node $C^{(1)}$, path $A^{(1)}B^{(1)}$-$B^{(1)}B^{(2)}$-$B^{(2)}C^{(2)}$-$C^{(2)}C^{(3)}$ -$C^{(3)}D^{(3)}$ will not be considered. This example testify that the standard shortest path algorithm could not be applied into the time-expanded network with heterogeneous arcs.

B. THE SHORTEST PATH ALGORITHM BASED ACTUAL AND ARTIFICIAL LINK

An improved Dijkstra algorithm for network with actual and artificial link is proposed in this section. We combine the actual label in traditional Dijkstra algorithm with the artificial label. The actual label of a node, say node i, should be updated if node i and its predecessor arc should be an actual arc, while the artificial label of node i should be updated if node i and its predecessor are connected by an artificial arc. Both the two types of labels indicate the distance from the root node to node i along the (current) shortest path. In fact, each node is accessible to both the actual and artificial arcs and can be attached with the actual and the artificial label for no more than once, respectively.

When a node is labeled by an actual arc, its actual label would be determined and after that it could not be relabeled by other actual arcs. Similarly, if a node is labeled through an artificial arc, its artificial label would be decided and it could not be relabeled by any other artificial arcs. If a node has already been labeled by both the artificial and the actual arc, the node could be labeled by a permanent label. The shortest path consists of the nodes labeled by the permanent labels. For instance, having been visited by the traffic volume from $C^{(1)}C^{(2)}$, the node $C^{(2)}$ would be labeled by the artificial label, but the node could still be entered through actual arcs. In this way, the shortest path of the time-expanded network could be gained finally.

In the improved Dijkstra algorithm for network with actual and artificial link, the first and the last arc of the searched shortest path are required to be actual arcs, and each node would be alternately labeled by the actual label and the artificial label. This process continues until the shortest path is finally found. The process of the improved Dijkstra algorithm is described in details as follows:

Assume that $G = (V, E)$ is a time-expanded network diagram with weight, V represents all the nodes on the network, E presents the arc set including the artificial and actual arcs. According to the standard Dijkstra algorithm, V (the set of nodes) is divided into two groups. The first group, which is defined as S, is a set of nodes that belong to the shortest path. It should be noticed that every node in S has already been labeled by the artificial label and the actual label, in other words the nodes in S have gained a permanent label. The nodes without the permanent label are assigned to the other group T. If all the nodes are allocated to the set S, then the algorithm would stop. The detail of proposed improved Dijkstra algorithm is shown in Algorithm 1.

Algorithm 1 The Shortest Path Algorithm Based Actual and Artificial Link

```
//Initialization
$t_0 = 0, t_i = \infty \ (i = 1, 2, \ldots, n), cur\_node = 0,$
$S \leftarrow 0, T \leftarrow i \ (i = 1, 2, \ldots, n)$
While $T \neq \emptyset$ do
  For each node $i$ in $(i = 1, 2, \ldots, n)$
  $tra\_time = cal\_TT\ (i)$ //Calculate travel time
        from current to conjunctive node
  If $tra\_time < t_i$ then
   $t_i = tra\_time,$
End if
End for
 Find a node $i$ with the smallest travel time in $T$
 //Label the node
 $act_{cur\_node} = 1$ //If the last arc arced is an actual
        arc then label the actual label.
 $art_{cur\_node} = 1$ //If the last arc arced is an artificial
        arc then label the artificial label.
 $cur\_node = i$
 If $act_{cur\_node} == 1$ and $art_{cur\_node} == 1$ then
 $S \leftarrow S/\{cur\_node\}, T \leftarrow T \cup \{cur\_node\}$
 End if
 End while
```

In Algorithm 1, the $t_i\ (i = 0, 1, 2, \ldots, n)$ represents the travel time from origin node 0 to node i, $cal\_TT\ (i)$ denotes the function of calculating travel time from current node to next conjunctive node, act_{cur_node} and art_{cur_node} are the actual and artificial label of current node, respectively.

When the actual label (artificial label) of current node is labeled, $act_{cur\_node}(art_{cur\_node})$ equals to 1, otherwise 0.

IV. SOLUTION ALGORITHM

A. THE CHOICE OF TRAFFIC ASSIGNMENT ALGORITHM

Route-based and link-based traffic assignment algorithms are two of the most widely used traffic assignment algorithms based on iterative procedures. In most previous studies, link-based traffic assignment algorithms are usually used. Among the link-based algorithms the F-W algorithm is the most frequently used and famous one. Since in the F-W algorithm only the traffic volumes on arcs are required to store, we could only gain the volume on each arc at equilibrium, and know neither the volumes' whence nor their whither. Thus, based on the F-W algorithm it's hard to measure the accumulated traffic flow [36].

However, in the model of DTA, due to the consideration of time dimension, the shortest path gained at a period based on static algorithm could be unfeasible for next period of time. For example, in Fig. 4, a simple network consisted of 3 nodes and 3 arcs is used to illustrate the limitations of the F-W algorithm in solving DTA problem.

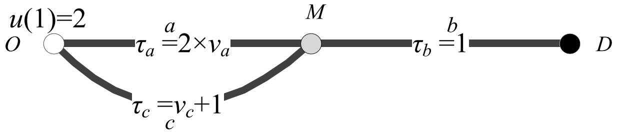


FIGURE 4. **A simple actual network.**

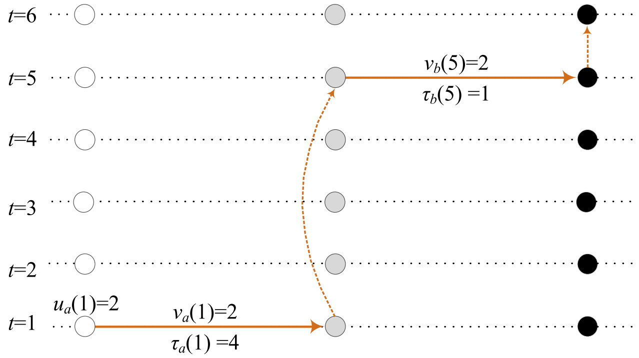


FIGURE 5. **The shortest path at the first time interval.**

Assume that 2 traffic volumes enter node o at time interval 1 and the travel time of each arc is shown in Fig. 5. Before time interval 1, the travel time of arc a is 0 and that of arc c is 1. Based on static traffic assignment algorithm, it is obvious that there must be a certain amount of volume that would travel by path a-b. According to the travel time of the 2 traffic volumes on arc a, the shortest path consists of two arcs, arc a at the first time interval $a(1)$ and arc b at the fifth time interval $b(5)$ at the first computation (Fig. 5). Then, a coefficient (0-1) is used to continually adjust the traffic volumes on each arc of the time-expanded network. Based on the link-based F-W algorithm, all the shortest paths gained during the process of searching are included in the equilibrium solutions. As a result, the traffic volume on $b(5)$ would not be reduced to 0. In fact, however, the equilibrium solution of the time-expanded network should be as what is shown in Fig.6, in which the traffic volume of$b(5)$ is 0. As mentioned before, in the F-W algorithm, the whence of traffic volume could not be known. Therefore, in a DTA model based on F-W algorithm, if a path is proved to be unfeasible, it is almost impossible to identify which part of the volume on an arc is from the unfeasible path, thus unlikely to transfer the volume to the feasible paths. Relying on the information of traffic volume gained by conventional traffic assignment algorithm the network could not approach equilibrium.

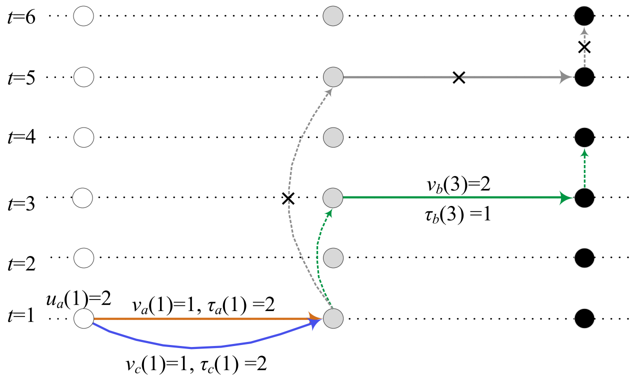


FIGURE 6. **The equilibrium solution of the network.**

In addition, in a time-varying network, part of traffic volume could not traverse an arc in a single time interval, and such volume is called the preserved volume which would be accumulated on the same arc at following time layers. Based on the F-W algorithm, the preserved volume of an arc would be directly added to the new volume of the arc, but in this way the preserved volume is calculated repeatedly and flow conservation constraints would be broken.

B. ROUTE-BASED TRAFFIC ASSIGNMENT APPROACH ON TIME-EXPANDED NETWORK

In link-based traffic assignment approach, the traffic volume of an arc is the linear combination of the auxiliary flow and the current flow. Because of the dynamic network loading, the updated traffic volume of the arc could not correctly present the flow propagation in DTA. A route-based solution algorithm, has been proposed in recent years to represent the flow propagation. Relying on the route-based traffic assignment approach, the repeated objective function evaluation could be avoided, and as a result the algorithm is particularly suitable for DTA, whose objective function need more computation time compared to the static one [36].

In this paper a route-based traffic assignment algorithm is used for DTA. There are two main benefits of using route-based algorithm. Firstly, in the process of searching for the shortest path, if a path is found to be unfeasible, the travel time of the path could be set to infinity and then the path with infinite travel time would be obsolete in the further searching process. Secondly, relying on the route-based algorithm, it's easy to bring out the whence and whither of the volume and always yields arc flow pattern that maintains correct flow propagation. The proposal route-based dynamic traffic assignment algorithm can be described in detail in Algorithm 2.

Algorithm 2 The Route-Based Dynamic Traffic Assignment Algorithm

```
1:      //Initialization
2:      $t = 0, n = 0, ROD = \emptyset$
3:      While n<Max iteration do
4:         For time interval t in T
5:            Update the time-space network topology based on $\tau_a(t)$ at time interval $t$
6:            //Traffic Assignment
7:            $UOD = \{1, 2, 3, \ldots, i, \ldots\}, AOD = \emptyset$
8:            For OD pair $k$ in Unassigned OD pair
9:               Apply Algorithm 1 in k, and find the shortest path $R_k^*$
10:              If$R_k^* \notin ROD$ then
11:              $ROD = ROD \cup \{R_k^*\}$
12:              $f_k^*(t) = 0$
13:              End if
14:              Find the path with longest travel time$f_k^-(t)$ from ROD
15:           $f_k^*(t) = f_k^*(t) + \lambda \times f_k^-(t), f_k^-(t) = (1-\lambda) \times f_k^-(t)$
16:              $UOD = UOD \backslash \{k\}, AOD = AOD \cup \{k\}$
17:           End for
18:        End for
19:        $x_a = \sum_i f_i * \delta_i^a, \tau_a = BPR(x_a)$
20:        Modify paths in $ROD$
21:        Adjust $ROD$
22:        $v_a^L(t+\Delta) = \begin{cases} v_a^L(t+\Delta) + v_a(t) & if\ \lceil \tau_a(t) \rceil > \Delta \\ v_a^L(t+\Delta) & otherwise \end{cases}$
23:  End while
```

In Algorithm 2, n is the iteration step of dynamic traffic assignment algorithm. ROD is the solution set of shortest path. UOD and AOD are the set of unassigned OD pair and the set of assigned OD pair, respectively. $f_k^*(t)$ is the traffic flow on shortest path R_k^* at time interval t, and $f_k^-(t)$ is the traffic flow on the path between OD pair k with the longest travel time at time interval t. In step 15, the variable λ $(\lambda \in (0, 1))$ in formulations $f_k^*(t) = f_k^*(t) + \lambda \times f_k^-(t)$ and $f_k^-(t) = (1-\lambda) \times f_k^-(t)$ should make $\zeta_k^*(t) = \zeta_k^-(t)$, and $\zeta(t)$ is the travel time of path. In the formulation of step 19, x_a is the traffic volume on arc a, δ_i^a is binary variable which equals 1 if arc a is in path i, 0 otherwise.

The step 20 modifies the paths in ROD. The hypothesis of applying Dijkstra algorithm based actual and artificial link is that the traffic flow of a path is assumed to be independent of that of any other path. There may be some arcs on which traffic flows of several paths are accumulated. In that case, paths previously found to be shortest will become unfeasible. That can be specifically illustrated in Fig. 7. Path a(1) b(2) is initially found as the shortest one. However, arc a(1) is incorporated in more and more paths with the searching process conducted. The accumulation of traffic flows on several paths lead to an increment of travel time on arc a(1) to 3, so Path a(1) b(2) is no longer feasible. Therefore, it is necessary to adjust the path as a(1) b(2) according to the updated arc travel time. After the modification, arc travel time and path travel time should be updated.

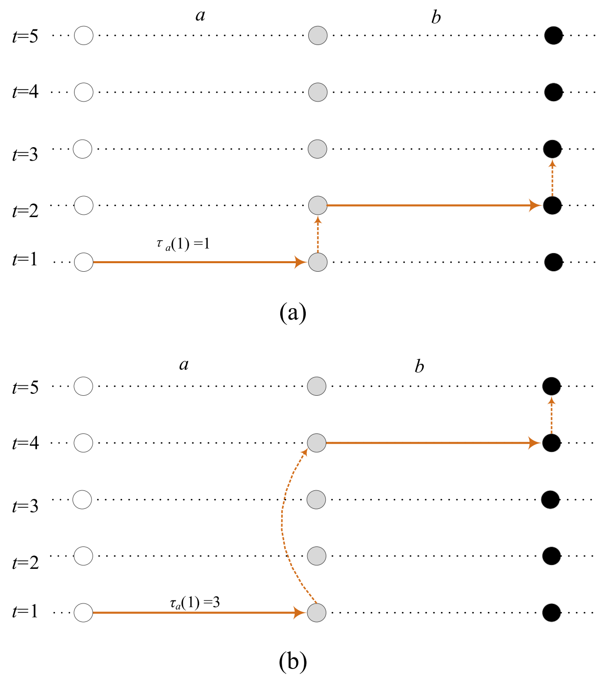


FIGURE 7. **The process of path modification. (a) The initial shortest path. (b) The adjusted shortest path.**

After path modification, the identical paths in *ROD* should be adjusted in step 21, so repetitive solutions should be removed from Rod. In addition, paths with no flows thereon should be removed as well. In step 22, the accumulation travel volumes on arcs are updated, if the travel time on arcs are more than one time-interval.

V. CASE STUDY

A substantial network [20], which consists of 5 nodes and 8 arcs, is depicted in Fig. 8. Detailed information of the network is shown in Table 1. In this paper, the travel time of arc is calculated based on BPR function (with $\alpha = 0.15$, $\beta = 4$). In the case, the time interval is set as 1 min, and in the model if changing the predetermined time interval into other values such as 0.5 min, 2 min or other larger value, would not make the algorithm unworkable. For ease of process description and result presentation, this paper chooses a relatively short duration, i.e. 20 min, as the test time period.

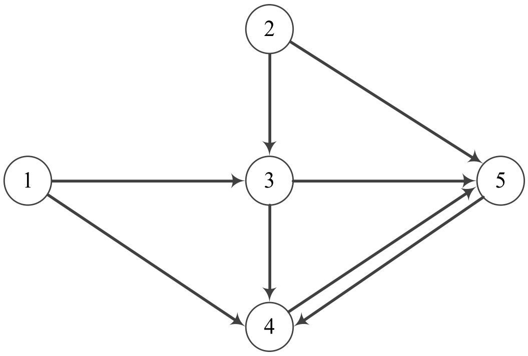


FIGURE 8. A substantial network.

TABLE 1. The information about the substantial network.

Origin node	Destination node	Free-flow travel time (min)	Capacity (vel/min)
1	3	1	40
1	4	1.25	30
2	3	1.1	40
3	4	1.1	60
2	5	1.15	30
3	5	1.2	60
4	5	0.5	25
5	4	0.5	40

A. VALIDATION OF THE METHOD

In this section, we adopt the 4 origin–destination pairs (1 → 4, 1 → 5, 2 → 4, 2 → 5) [20], [36] to test the method. In order to simplify calculation, the traffic demand is set to 30 vel/min and the number of time interval (time layer) is set to 3. The result is shown in Table 2 and the travel time of each arc at each time interval are presented in Fig. 9.

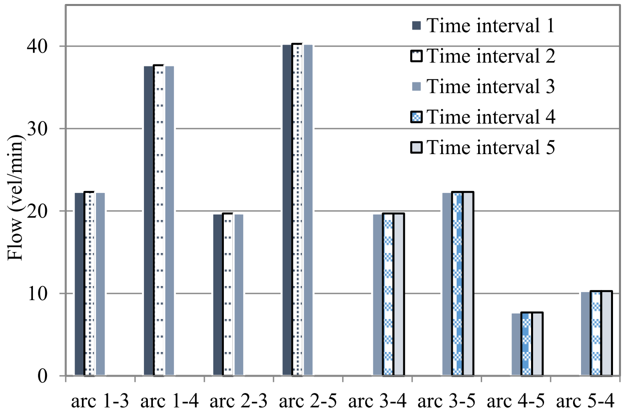


FIGURE 9. The travel time of each arc at each time interval.

TABLE 2. The result of traffic assignment.

Time interval	O	D	Flow (vel/min)	Travel time (min)	Path
1	1	4	30	1.72	1 4
1	1	5	7.69	2.22	1 4 5
1	1	5	22.31	2.22	1 3 5
1	2	4	7.7	2.22	2 5 4
1	2	4	22.3	2.22	2 3 4
1	2	5	30	1.72	2 5
2	1	4	30	1.72	1 4
2	1	5	7.69	2.22	1 4 5
2	1	5	22.31	2.22	1 3 5
2	2	4	7.7	2.22	2 5 4
2	2	4	22.3	2.22	2 3 4
2	2	5	30	1.72	2 5
3	1	4	30	1.72	1 4
3	1	5	7.69	2.22	1 4 5
3	1	5	22.31	2.22	1 3 5
3	2	4	7.7	2.22	2 5 4
3	2	4	22.3	2.22	2 3 4
3	2	5	30	1.72	2 5

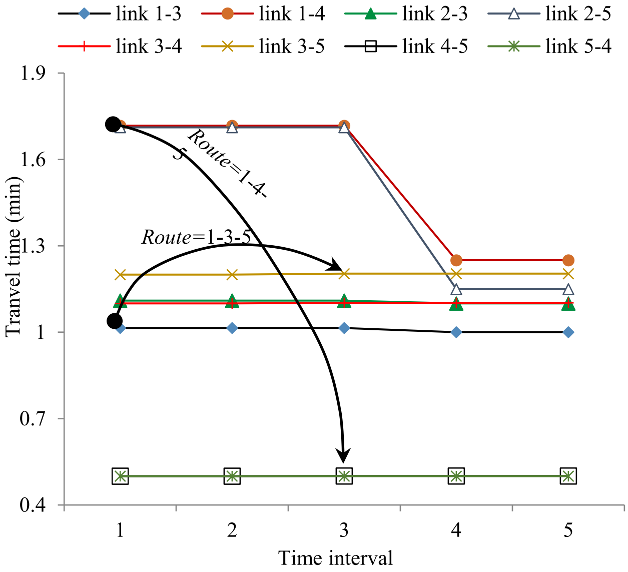


FIGURE 10. The travel time of each link at each time interval.

According to the results illustrated in Fig. 9, arc 1 → 3, 1 → 4 *and*2 → 5 are always without traffic volume except the first third time intervals. Because the three arcs are unidirectional, and during the only three considered time intervals traffic volumes can enter and pass through the arcs. As a result, when it comes to the fourth and other following time intervals, no volume would still be preserved on the 3 arcs. As shown in Fig. 10, because the volumes have already gone, the travel time of arc 1 → 4 and arc 2 → 5 drop to

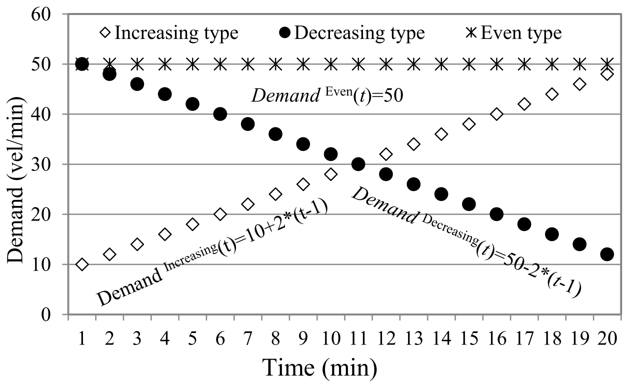


FIGURE 11. **Three demand type.**

free-flow travel time rapidly at time interval 4. While, there is no volume on arc 3 → 4, arc 3 → 5, arc 4 → 5 and arc 5 → 4 during the first two-time interval. This is because the volume set out from node 1 would need more than a single time interval to reach node 2, 4 and 5.

For example, as to route 1-3-5 and route 1-4-5 at time interval 1, the travel time of arc 1 → 3 and arc 1 → 4 is larger than 1 min, so when the vehicles enter arc 3 → 5 and 4 → 5, it has already come to the third time interval. The example could clearly prove that the method of this paper could correctly depict the time-varying travel time in the model of DTA.

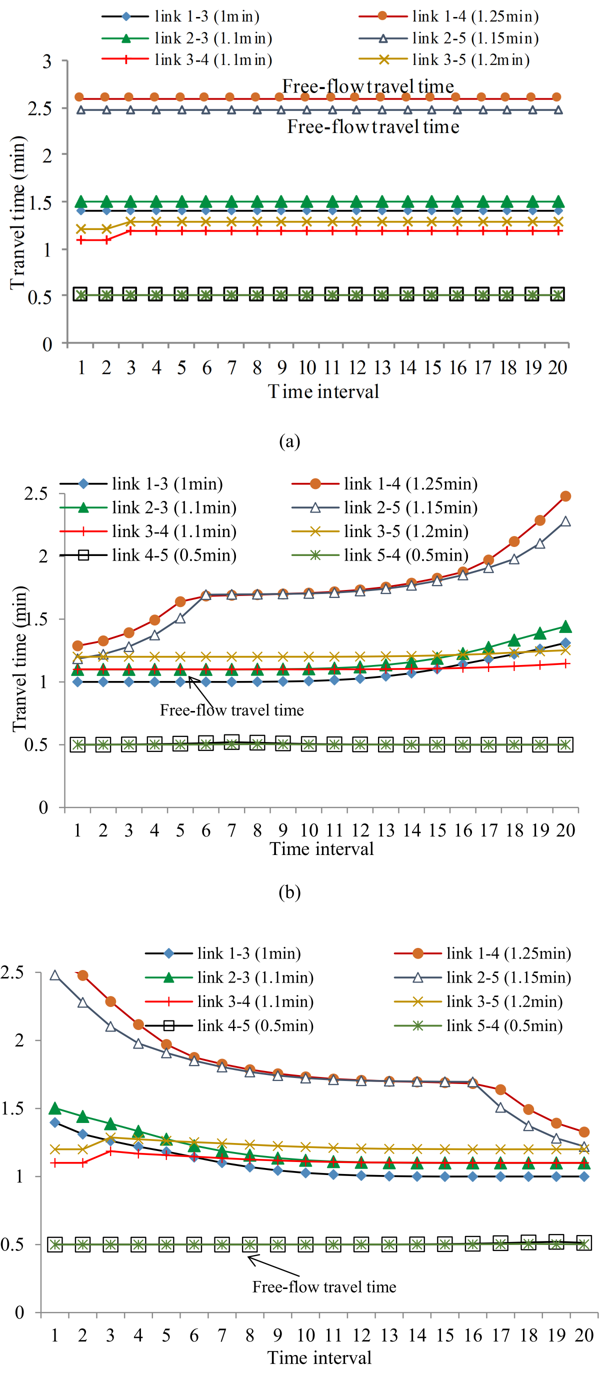
B. RESULTS UNDER DIFFERENT DEMAND MODES

In connected transportation system, the traffic demands always increase firstly and decrease latterly, especially during the peak hours. Therefore, in addition to the even traffic demand, the model is tested with two variable demand modes, including an increasing demand and diminishing demand. The three demand modes are depicted in Fig. 11.

The results of the three models are shown in Fig. 12(a), Fig. 12(b), and Fig. 12(c) respectively. The iterations of the three models are 17, 16 and 15 times. Although the free-flow travel time of arc 4 → 5 and 5 → 4 is short, the two arcs seem to keep empty till the end. This is because the upstream arcs i.e., arc 1 → 4 and 2 → 5 would be severely congested if the traffic volumes on them reach a certain value, so to avoid congestion no volume would choose the routes, where the arc 4 → 5 and 5 → 4 are located.

In Fig. 12(b), it is obvious that under the mode of diminishing demand before the 12th time interval there is a certain amount of traffic volume on arc 4 → 5 and 5 → 4. However, from the 13th time interval, as traffic volumes increases, the choosing probabilities of the two paths decrease to zero quickly. Similarly, in Fig. 12(c), the two arcs are not chosen till the 13th time interval, when the traffic volume is at a low level.

Besides, it could also be found in Fig. 12(a) and (c), at time interval 3 the curves of the travel time of arc 3 → 4 and 3 → 5 fluctuate obviously. Because, under the two modes, the initial demand is relatively large, as a result, the travel time of the two arcs would be more than one interval. In other words, the volume would spend two intervals getting access to arc 3 → 4 and 3 → 5. While as shown in Fig. 12(a), under the mode of increasing demand, the curve is flat because the initial demand is small. As to arc 1 → 4 and 2 → 5,



(a)

(b)

(c)

FIGURE 12. **The travel time of each link under the three demand type. (a) The travel time under the even type. (b) The travel time under the increasing type. (c) The travel time under the decreasing type.**

TABLE 3. Specific traffic assignment results under mixed demand mode on the standard network.

Time interval	O	D	Flow	Travel time (min)	Path	Time interval	O	D	Flow	Travel time (min)	Path	Time interval	O	D	Flow	Travel time (min)	Path
1	1	4	10.00	1.29	1-4	20	2	5	48.00	2.28	2-5	41	1	4	1.10	2.57	1-3-4
1	1	5	10.00	1.79	1-4-5	21	1	4	48.99	2.58	1-4	41	1	5	50.00	2.69	1-3-5
1	2	4	10.00	1.68	2-5-4	21	1	4	1.01	2.58	1-3-4	41	2	4	48.00	2.62	2-3-4
1	2	5	10.00	1.18	2-5	21	1	5	50.00	2.68	1-3-5	41	2	5	48.00	2.28	2-5
2	1	4	12.00	1.33	1-4	21	2	4	50.00	2.69	2-3-4	42	1	4	47.92	2.47	1-4
2	1	5	12.00	1.83	1-4-5	21	2	5	50.00	2.48	2-5	42	1	4	0.08	2.47	1-3-4
2	2	4	12.00	1.72	2-5-4	22	1	4	48.99	2.58	1-4	42	1	5	48.00	2.59	1-3-5
2	2	5	12.00	1.22	2-5	22	1	4	1.01	2.58	1-3-4	42	2	4	46.00	2.55	2-3-4
3	1	4	14.00	1.39	1-4	22	1	5	50.00	2.68	1-3-5	42	2	5	46.00	2.10	2-5
3	1	5	14.00	1.90	1-4-5	22	2	4	50.00	2.69	2-3-4	43	1	4	46.00	2.29	1-4
3	2	4	14.00	1.78	2-5-4	22	2	5	50.00	2.48	2-5	43	1	5	46.00	2.52	1-3-5
3	2	5	14.00	1.28	2-5	23	1	4	48.99	2.58	1-4	43	2	4	0.41	2.48	2-5-4
4	1	4	16.00	1.49	1-4	23	1	4	1.01	2.58	1-3-4	43	2	4	43.59	2.48	2-3-4
4	1	5	16.00	2.01	1-4-5	23	1	5	50.00	2.68	1-3-5	43	2	5	44.00	1.98	2-5
4	2	4	16.00	1.88	2-5-4	23	2	4	50.00	2.69	2-3-4	44	1	4	44.00	2.12	1-4
4	2	5	16.00	1.37	2-5	23	2	5	50.00	2.48	2-5	44	1	5	44.00	2.47	1-3-5
5	1	4	18.00	1.64	1-4	24	1	4	48.99	2.58	1-4	44	2	4	40.55	2.41	2-3-4
5	1	5	18.00	2.16	1-4-5	24	1	4	1.01	2.58	1-3-4	44	2	4	1.45	2.41	2-5-4
5	2	4	18.00	2.01	2-5-4	24	1	5	50.00	2.68	1-3-5	44	2	5	42.00	1.91	2-5
5	2	5	18.00	1.51	2-5	24	2	4	50.00	2.69	2-3-4	45	1	4	42.00	1.97	1-4
6	1	4	20.00	1.68	1-4	24	2	5	50.00	2.48	2-5	45	1	5	42.00	2.43	1-3-5
6	1	5	17.00	2.20	1-4-5	25	1	4	48.99	2.58	1-4	45	2	4	37.40	2.35	2-3-4
6	1	5	3.00	2.20	1-3-5	25	1	4	1.01	2.58	1-3-4	45	2	4	2.60	2.35	2-5-4
6	2	4	20.00	2.20	2-5-4	25	1	5	50.00	2.68	1-3-5	45	2	5	40.00	1.85	2-5
6	2	5	20.00	1.70	2-5	25	2	4	50.00	2.69	2-3-4	46	1	4	40.00	1.88	1-4
7	1	4	22.00	1.69	1-4	25	2	5	50.00	2.48	2-5	46	1	5	39.45	2.38	1-3-5
7	1	5	15.13	2.20	1-4-5	26	1	4	48.99	2.58	1-4	46	1	5	0.55	2.38	1-4-5
7	1	5	6.87	2.20	1-3-5	26	1	4	1.01	2.58	1-3-4	46	2	4	3.87	2.30	2-5-4
7	2	4	18.03	2.20	2-5-4	26	1	5	50.00	2.68	1-3-5	46	2	4	34.13	2.30	2-3-4
7	2	4	3.97	2.20	2-3-4	26	2	4	50.00	2.69	2-3-4	46	2	5	38.00	1.80	2-5
7	2	5	22.00	1.70	2-5	26	2	5	50.00	2.48	2-5	47	1	4	38.00	1.83	1-4
8	1	4	24.00	1.70	1-4	27	1	4	48.99	2.58	1-4	47	1	5	36.29	2.33	1-3-5
8	1	5	13.24	2.20	1-4-5	27	1	4	1.01	2.58	1-3-4	47	1	5	1.71	2.33	1-4-5
8	1	5	10.76	2.20	1-3-5	27	1	5	50.00	2.68	1-3-5	47	2	4	30.72	2.27	2-3-4
8	2	4	16.06	2.20	2-5-4	27	2	4	50.00	2.69	2-3-4	47	2	4	5.28	2.27	2-5-4
8	2	4	7.94	2.20	2-3-4	27	2	5	50.00	2.48	2-5	47	2	5	36.00	1.77	2-5
8	2	5	24.00	1.70	2-5	28	1	4	48.99	2.58	1-4	48	1	4	36.00	1.79	1-4
9	1	4	26.00	1.70	1-4	28	1	4	1.01	2.58	1-3-4	48	1	5	32.99	2.29	1-3-5
9	1	5	11.34	2.20	1-4-5	28	1	5	50.00	2.68	1-3-5	48	1	5	3.01	2.29	1-4-5
9	1	5	14.66	2.20	1-3-5	28	2	4	50.00	2.69	2-3-4	48	2	4	6.83	2.24	2-5-4
9	2	4	14.10	2.20	2-5-4	28	2	5	50.00	2.48	2-5	48	2	4	27.17	2.24	2-3-4
9	2	4	11.90	2.20	2-3-4	29	1	4	48.99	2.58	1-4	48	2	5	34.00	1.74	2-5
9	2	5	26.00	1.70	2-5	29	1	4	1.01	2.58	1-3-4	49	1	4	34.00	1.76	1-4
10	1	4	28.00	1.71	1-4	29	1	5	50.00	2.68	1-3-5	49	1	5	29.56	2.26	1-3-5
10	1	5	9.48	2.21	1-4-5	29	2	4	50.00	2.69	2-3-4	49	1	5	4.44	2.26	1-4-5

TABLE 3. ***(Continued.)*** **Specific traffic assignment results under mixed demand mode on the standard network.**

10	1	5	18.52	2.21	1-3-5
10	2	4	12.16	2.20	2-5-4
10	2	4	15.84	2.20	2-3-4
10	2	5	28.00	1.70	2-5
11	1	4	30.00	1.72	1-4
11	1	5	7.69	2.22	1-4-5
11	1	5	22.31	2.22	1-3-5
11	2	4	10.29	2.21	2-5-4
11	2	4	19.71	2.21	2-3-4
11	2	5	30.00	1.71	2-5
12	1	4	32.00	1.73	1-4
12	1	5	6.00	2.23	1-4-5
12	1	5	26.00	2.23	1-3-5
12	2	4	8.51	2.22	2-5-4
12	2	4	23.49	2.22	2-3-4
12	2	5	32.00	1.72	2-5
13	1	4	34.00	1.76	1-4
13	1	5	4.44	2.26	1-4-5
13	1	5	29.56	2.26	1-3-5
13	2	4	6.83	2.24	2-5-4
13	2	4	27.17	2.24	2-3-4
13	2	5	34.00	1.74	2-5
14	1	4	36.00	1.79	1-4
14	1	5	3.01	2.29	1-4-5
14	1	5	32.99	2.29	1-3-5
14	2	4	5.28	2.27	2-5-4
14	2	4	30.72	2.27	2-3-4
14	2	5	36.00	1.77	2-5
15	1	4	38.00	1.83	1-4
15	1	5	36.29	2.33	1-3-5
15	1	5	1.71	2.33	1-4-5
15	2	4	3.87	2.30	2-5-4
15	2	4	34.13	2.30	2-3-4
15	2	5	38.00	1.80	2-5
16	1	4	40.00	1.88	1-4
16	1	5	39.45	2.38	1-3-5
16	1	5	0.55	2.38	1-4-5
16	2	4	2.60	2.35	2-5-4
16	2	4	37.40	2.35	2-3-4
16	2	5	40.00	1.85	2-5
17	1	4	42.00	1.97	1-4
17	1	5	42.00	2.43	1-3-5
17	2	4	40.55	2.41	2-3-4
17	2	4	1.45	2.41	2-5-4
17	2	5	42.00	1.91	2-5
18	1	4	44.00	2.12	1-4
18	1	5	44.00	2.47	1-3-5
18	2	4	43.59	2.48	2-3-4
29	2	5	50.00	2.48	2-5
30	1	4	48.99	2.58	1-4
30	1	4	1.01	2.58	1-3-4
30	1	5	50.00	2.68	1-3-5
30	2	4	50.00	2.69	2-3-4
30	2	5	50.00	2.48	2-5
31	1	4	48.99	2.58	1-4
31	1	4	1.01	2.58	1-3-4
31	1	5	50.00	2.68	1-3-5
31	2	4	50.00	2.69	2-3-4
31	2	5	50.00	2.48	2-5
32	1	4	48.99	2.58	1-4
32	1	4	1.01	2.58	1-3-4
32	1	5	50.00	2.68	1-3-5
32	2	4	50.00	2.69	2-3-4
32	2	5	50.00	2.48	2-5
33	1	4	48.99	2.58	1-4
33	1	4	1.01	2.58	1-3-4
33	1	5	50.00	2.68	1-3-5
33	2	4	50.00	2.69	2-3-4
33	2	5	50.00	2.48	2-5
34	1	4	48.99	2.58	1-4
34	1	4	1.01	2.58	1-3-4
34	1	5	50.00	2.68	1-3-5
34	2	4	50.00	2.69	2-3-4
34	2	5	50.00	2.48	2-5
35	1	4	48.99	2.58	1-4
35	1	4	1.01	2.58	1-3-4
35	1	5	50.00	2.68	1-3-5
35	2	4	50.00	2.69	2-3-4
35	2	5	50.00	2.48	2-5
36	1	4	48.99	2.58	1-4
36	1	4	1.01	2.58	1-3-4
36	1	5	50.00	2.68	1-3-5
36	2	4	50.00	2.69	2-3-4
36	2	5	50.00	2.48	2-5
37	1	4	48.99	2.58	1-4
37	1	4	1.01	2.58	1-3-4
37	1	5	50.00	2.68	1-3-5
37	2	4	50.00	2.69	2-3-4
37	2	5	50.00	2.48	2-5
38	1	4	48.99	2.58	1-4
38	1	4	1.01	2.58	1-3-4
38	1	5	50.00	2.68	1-3-5
38	2	4	50.00	2.69	2-3-4
38	2	5	50.00	2.48	2-5
39	1	4	48.99	2.58	1-4
39	1	4	1.01	2.58	1-3-4
49	2	4	8.51	2.22	2-5-4
49	2	4	23.49	2.22	2-3-4
49	2	5	32.00	1.72	2-5
50	1	4	32.00	1.73	1-4
50	1	5	26.00	2.23	1-3-5
50	1	5	6.00	2.23	1-4-5
50	2	4	10.29	2.21	2-5-4
50	2	4	19.71	2.21	2-3-4
50	2	5	30.00	1.71	2-5
51	1	4	30.00	1.72	1-4
51	1	5	22.31	2.22	1-3-5
51	1	5	7.69	2.22	1-4-5
51	2	4	12.16	2.20	2-5-4
51	2	4	15.84	2.20	2-3-4
51	2	5	28.00	1.70	2-5
52	1	4	28.00	1.71	1-4
52	1	5	9.48	2.21	1-4-5
52	1	5	18.52	2.21	1-3-5
52	2	4	14.10	2.20	2-5-4
52	2	4	11.90	2.20	2-3-4
52	2	5	26.00	1.70	2-5
53	1	4	26.00	1.70	1-4
53	1	5	11.34	2.20	1-4-5
53	1	5	14.66	2.20	1-3-5
53	2	4	16.06	2.20	2-5-4
53	2	4	7.94	2.20	2-3-4
53	2	5	24.00	1.70	2-5
54	1	4	24.00	1.70	1-4
54	1	5	13.24	2.20	1-4-5
54	1	5	10.76	2.20	1-3-5
54	2	4	18.03	2.20	2-5-4
54	2	4	3.97	2.20	2-3-4
54	2	5	22.00	1.70	2-5
55	1	4	22.00	1.69	1-4
55	1	5	15.13	2.20	1-4-5
55	1	5	6.87	2.20	1-3-5
55	2	4	20.00	2.20	2-5-4
55	2	5	20.00	1.70	2-5
56	1	4	20.00	1.68	1-4
56	1	5	17.00	2.20	1-4-5
56	1	5	3.00	2.20	1-3-5
56	2	4	18.00	2.01	2-5-4
56	2	5	18.00	1.51	2-5
57	1	4	18.00	1.64	1-4
57	1	5	18.00	2.16	1-4-5
57	2	4	16.00	1.88	2-5-4
57	2	5	16.00	1.37	2-5
58	1	4	16.00	1.49	1-4

TABLE 3. ***(Continued.)*** **Specific traffic assignment results under mixed demand mode on the standard network.**

18	2	4	0.41	2.48	2-5-4	39	1	5	50.00	2.68	1-3-5	58	1	5	16.00	2.01	1-4-5
18	2	5	44.00	1.98	2-5	39	2	4	50.00	2.69	2-3-4	58	2	4	14.00	1.78	2-5-4
19	1	4	46.00	2.29	1-4	39	2	5	50.00	2.48	2-5	58	2	5	14.00	1.28	2-5
19	1	5	46.00	2.52	1-3-5	40	1	4	48.99	2.58	1-4	59	1	4	14.00	1.39	1-4
19	2	4	46.00	2.55	2-3-4	40	1	4	1.01	2.58	1-3-4	59	1	5	14.00	1.90	1-4-5
19	2	5	46.00	2.10	2-5	40	1	5	50.00	2.68	1-3-5	59	2	4	12.00	1.72	2-5-4
20	1	4	48.00	2.48	1-4	40	2	4	50.00	2.69	2-3-4	59	2	5	12.00	1.22	2-5
20	1	4	0.00	2.48	1-3-4	40	2	5	50.00	2.48	2-5	60	1	4	12.00	1.33	1-4
20	1	5	48.00	2.58	1-3-5	41	1	4	48.90	2.57	1-4	60	1	5	12.00	1.83	1-4-5
20	2	4	48.00	2.61	2-3-4												

under the condition of even traffic demand, the travel times are fixed in a relatively high level from the first interval to the last. When it comes to the increasing mode, the travel times of the two arcs sustainably increase especially during the first six intervals. For the decreasing type, the travel time of the two arcs changes in an inverse direction from that of the increasing type.

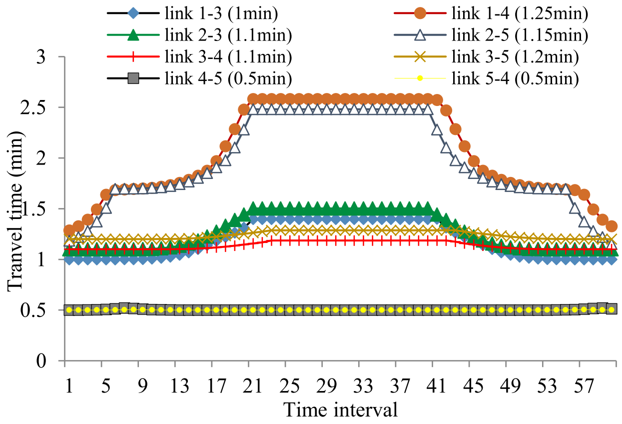


FIGURE 13. **The travel time under a mixed demand mode.**

On the basis of the comparison of the results of three demand modes, the method of this paper is tested with a more complex demand mode, in which the demand would increase in the way of the increasing demand mode at first, then would keep flat like the fixed demand mode, and gradually drop just as the diminishing demand mode finally. Besides, the testing time is prolonged into 60-time intervals at the same time. The result after 17 iterations is presented in Fig. 13. In general, the travel time of each arc would increase with the increasing traffic volume, and when a region has almost reached its full capacity, the travel time of the arcs on the region would increase obviously. Moreover, the iteration times are not remarkable increased with increasing amount of time interval. Therefore, the algorithm of this paper could be stable. More specific traffic assignment results under mixed demand mode are listed in Table 3 of Appendix.

VI. CONCLUSIONS

In this paper, a dynamic traffic assignment algorithm is designed for connected transportation system. The time-expanded network is used in the route-based traffic assignment algorithm to fit the spatial dimension and time dimension of the process of dynamic traffic assignment. In the time-expanded network, each node at a certain time layer could only connect with one other node at a future time layer, and in this respect a simplified time-expanded network is proposed to downsize the network scale. The simplified network could correctly represent flow propagation.

Besides, the time-expanded network is composed of two types of arcs, which are the actual arc and the artificial one. The conventional shortest path algorithm could not be applied to the type of time-expended network. A novel Dijkstra algorithm based actual and artificial link is proposed to search for the shortest path in the time-expanded network. The experimental results show that the algorithm function implemented well on the time-expanded network and could approach equilibrium under different demand modes. Comparing the iterations of the examples with different numbers of time intervals, it could be found that the iteration would not increase with increasing number of time intervals under the same demand mode.

Furthermore, in this paper our method is tested in the relatively small traffic networks. In future studies, the method would be applied to the actual traffic network to test its feasibility.

APPENDIX

See Table 3.

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