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Self-Triggered Model Predictive Control for Linear Systems With Switched Cost Functions

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ABSTRACT This paper investigates the self-triggered model predictive control for networked linear systems. The self-triggered mechanism is designed based on switched cost functions, which can be used to enhance systems performance and are more appropriate for complex industrial requirements in contrast with a single cost function approach. The model predictive controller is designed by solving an optimization problem and the self-triggered condition is designed. With the proposed self-triggered mechanism, much network computation and communication burden are reduced and Zeno behavior can be excluded. Moreover, asymptotic stability of the networked systems with model predictive control strategy is shown via average dwell-time technique. Finally, the numerical simulation example is given to illustrate the effectiveness of the proposed methods.

INDEX TERMS Model predictive control, networked system, self-triggered, switched cost functions.

I. INTRODUCTION

In recent years, event-triggered control of networked control systems has attracted considerable attention due to its extensive applications [1]–[4]. In contrast with time-triggered control, there exist some advantages on event-triggered control in networked control systems, such as reduction of excessive-usage of communication resources and energy saving of sensors as well as controllers [5]. In particular, energy saving is important when wireless networks are taken into consideration [6], [7]. Event-triggered output tracking control is used in wireless networked control systems with communication delays and data dropouts to reduce energy consumption [8]. There are two main approaches on event-triggered control, i.e., event-based control and self-triggered control. A main characteristic of event-based control is that when to transmit information is determined by predefined event-triggering conditions. Specifically for networked control systems, states are measured at each sampling instant with event-based control approach. For self-triggered control, inputs are pre-determined based on the prediction of networked control systems. Though both data transmission and energy consumption are reduced by event-triggered control, system performance decreases in a certain degree for limited data. To address this issue, the event-triggered model predictive control (MPC) has been proposed in [9], [10]. Moreover, in [11], an event-triggered MPC is proposed for a continuous-time nonlinear systems subject to bounded disturbances. Though event-triggered MPC can improve system performance, there still exists room for enhancing system performance in networked linear continuous-time systems.

In practice, in order to improve overall system performance, it is desirable that multiple performance criteria are considered [12]. Usually, both response speed and energy consumption are taken into consideration by using related cost functions [13]. When disturbances or faults occur, quick react to the disturbances or faults is important for systems [14]. Therefore, corresponding weights of states and inputs are necessary in the related cost functions. To deal with different performances, switching controllers that corresponds to the different criteria were applied according to given switching rules [15]. In [16], a switched MPC was studied for a class of discrete-time switched linear systems with mode-dependent dwell time. A stage dwell-time was proposed to guarantee the persistent feasibility of MPC. In [17], a MPC algorithm under average dwell-time

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switching signals was proposed to guarantee recursive feasibility and asymptotic stability of a closed-loop switched system. In addition, switching functions has also been studied to enhance system performances. A time-dependent switching signal occurred in switching cost functions is designed for MPC of a nonlinear system [18], therefore, performance of the nonlinear system is improved by considering two performance criteria.

Motivated by the above discussion, a self-triggered MPC strategy based on switched cost functions is proposed for networked linear systems in this paper. For the networked linear systems, there exist switching cost functions and a continuous input trajectory that is obtained by solving an optimization problem corresponding to one of the cost functions. Then, the input trajectory is sampled into several discrete inputs by the designed self-triggered algorithm.

The main contributions of this paper are summarized as follows.

i A self-triggered MPC is proposed for networked linear systems.
ii The proposed self-triggered MPC is designed based on switched cost functions, which can adapt to changing environment parameters of the networked linear systems, and the switching signal satisfies average dwell-time.
iii With the proposed self-triggered MPC scheme, both recursive feasibility and asymptotic stability can be guaranteed while saving network resources.

The rest of this paper is organized as follows. In Section II, the problem statement and some preliminaries are given. In Section III, the main results including a self-triggered MPC scheme, a complete self-triggered switched MPC algorithm and a theorem to illustrate the asymptotic stability of the networked linear continuous-time systems and recursive feasibility of the proposed MPC optimization problem are presented. A numerical example is presented to illustrate the validity of the proposed self-triggered switched MPC algorithm in Section IV. Section V concludes this paper.

Notations: In the sequel, if not explicitly stated, matrices are assumed to have compatible dimensions. R is the set of real numbers and R_0^+ is the set of the nonnegative reals, respectively. For any $a \in R$, $|a|$ denotes the absolute value of a. R^n denotes the n-dimensional Euclidean space. Z denotes the set of positive integer. For any vector $x \in R^n$, let $\|x\|$ the Euclidean norm of vector or the induced two-norm of the matrix. For any matrix A, A^T denotes the transpose of matrix A, A^{-1} denotes the inverse of matrix A. I is the identity matrix of appropriate dimension. $\|x\|_P^2$ denotes $x^T Px$. For a function $\alpha(\cdot) : R_0^+ \rightarrow R_0^+$ denotes that function $\alpha(\cdot)$ is a $\mathcal{K}$ function. Moreover, function $\alpha(\cdot)$ is called a $\mathcal{K}_\infty$ function when function $\alpha(\cdot)$ is a $\mathcal{K}$ function with characters that it satisfies $\alpha(0) = 0$, continuous strictly increasing and $\alpha(s) \rightarrow \infty$ as $s \rightarrow \infty$.

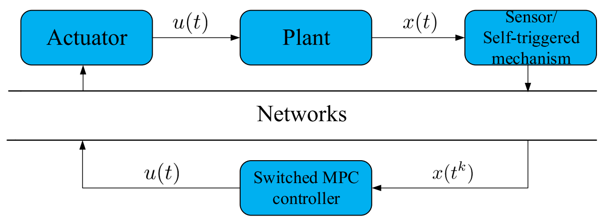


FIGURE 1. Structure of the networked system.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this paper, we consider self-triggered model predictive control for networked linear systems, and the control structure is shown in Fig.1.

A. PLANT MODEL

Consider linear time-invariant systems described as

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in R^n$ is the state and $u(t) \in R^m$ is the input. The matrices A, B have appropriate dimensions.

B. OPTIMIZATION PROBLEM

Denote the predicted state and control input as $\hat{x}(s; t^k)$ and $\hat{u}(s; t^k)$, $s \in [t^k, t^k + T]$, respectively, where T denotes the prediction horizon. The cost function at time t^k is defined as

$$J(x(t^k)) = \int_{t^k}^{t^k+T} (\|\hat{x}(s; t^k)\|_Q^2 + \|\hat{u}(s; t^k)\|_R^2) ds + \|\hat{x}(t^k + T)\|_P^2 \tag{2}$$

where $Q > 0$, $R > 0$ and $P > 0$. The sequence $\{t^k\}$, $k \in N$ are denoted the time instants when the optimization problem is solved. For simplicity, define $H(\hat{x}, \hat{u}) = \|\hat{x}(s; t^k)\|_Q^2 + \|\hat{u}(s; t^k)\|_R^2$, $F(\hat{x}) = \|\hat{x}(t^k + T; t^k)\|_P^2$. According to the cost function (2), the optimal control input can be computed by solving the following optimization problem.

Problem OP: At time instant t^k, calculate the optimal control $\hat{u}^*(s; t^k)$, $s \in [t^k, t^k + T]$ via cost function (2) based on $x(t^k)$

$$\hat{u}^*(s; t^k) = \arg\min J(x(t^k)) \tag{3}$$

$$\begin{aligned} \text{subject to: } & \dot{\hat{x}}(s; t^k) = A\hat{x}(s; t^k) + B\hat{u}(s; t^k), \\ & \qquad \forall s \in [t_k, t_k + T] \\ & \hat{u}(s; t^k) \in \mathcal{U}, \quad \forall s \in [t_k, t_k + T] \\ & \hat{x}(s; t^k) \in \mathcal{X}, \quad \forall s \in [t_k, t_k + T] \\ & \hat{x}(t^k + T) \in \mathcal{X}^f. \end{aligned} \tag{4}$$

where $\mathcal{X} \in R^n$ denotes the state constraint set. The input constraint set $\mathcal{U}$ is given as

$$\mathcal{U} = \{u(s) \in R^m : \|u(s)\| \leq u_{max}, \|\dot{u}(s)\| \leq K_u\} \tag{5}$$

where u_{max} and K_u are predefined constant parameters. The terminal constraint set $\mathcal{X}^f \in \mathcal{X}$ is a compact set with the origin in its interior. The following assumptions are needed in this paper.

Assumption 1: $H(\hat{x}(\cdot), \hat{u}(\cdot))$ and $F(\hat{x}(\cdot))$ are Lipschitz continuous with Lipschitz constants $L_H > 0$ and $L_F > 0$. Moreover, there exists $\mathcal{K}_\infty$ function β satisfying $H(\hat{x}(\cdot), \hat{u}(\cdot)) \geq \beta(\|x\|)$.

Assumption 2: There exists an auxiliary local controller $u = \kappa^{loc}(x) \in \mathcal{U}$ such that

$$\frac{\partial F}{\partial x}(Ax + B\kappa^{loc}(x)) \leq -H(x, \kappa^{loc}(x))$$

holds for all $x \in \mathcal{X}^f$.

C. MPC CONTROLLER

In this paper, the controller is designed as

$$u(s) = \begin{cases} \hat{u}^*(s), & for \ \ t^k \leq s < t^k + T \\ \kappa^{loc}(x(s)), & for \ \ s \geq t^k + T \end{cases}$$

where the controller $u(s) = \kappa^{loc}(x(s))$ satisfies the following constraint condition

$$K_u \geq \max_{x \in \mathcal{X}^f} \{\| \frac{\partial \kappa^{loc}(x)}{\partial x} \cdot (Ax + B\kappa^{loc}(x))\|\}$$

where K_u is computed off-line.

D. SELF-TRIGGERED CONDITION AND SAMPLING INSTANTS CHOOSING

Suppose that at time $t^k + \delta$, $\delta \in (0, T]$, the optimal state and practical state are denoted $\hat{x}^*(t^k + \delta)$ and $x(t^k + \delta)$, respectively. A feasible control input $\tilde{u}(s; t^k + \delta)$, $s \in [t^k + \delta, t^k + \delta + T]$ is applied

$$\tilde{u}(s; t^k + \delta) = \begin{cases} \hat{u}^*(s; t^k), & s \in [t^k + \delta, t^k + T] \\ \tilde{u}(s; t^k + T), & s \in (t^k + T, t^k + \delta + T]. \end{cases}$$

If $x(t^k + \delta) = \hat{x}^*(t^k + \delta)$, derived from [19], then with the feasible control input $\tilde{u}(t^k + \delta)$ the corresponding cost function at $t^k + \delta$ has the following form

$$\begin{aligned} &\tilde{J}(\hat{x}^*(t^k + \delta)) \\ &= \int_{t^k+\delta}^{t^k+\delta+T} (\|\tilde{x}(s; t^k + \delta)\|_Q^2 + \|\tilde{u}(s; t^k + \delta)\|_R^2)ds \\ &\quad + \|\tilde{x}(t^k + \delta + T; t^k + \delta)\|_P^2 \\ &\leq J^*(x(t^k)) - \int_{t^k}^{t^k+\delta} (\|\hat{x}^*\|_Q^2 + \|\hat{u}^*(s; t^k)\|_R^2)ds. \end{aligned}$$

Note that $\tilde{J}(\hat{x}^*(t^k + \delta)) \geq J^*(\hat{x}^*(t^k + \delta))$, where $J^*(\hat{x}^*(t^k + \delta))$ is the optimal cost function at $t^k + \delta$, and the following condition holds.

$$J^*(x(t^k+\delta))-J^*(x(t^k)) \leq J^*(x(t^k+\delta))-J^*(\hat{x}^*(t^k+\delta))-\Gamma_1 \tag{6}$$

where $J^*(x(t^k + \delta))$ is the optimal cost function if the current state is $x(t^k + \delta)$, $\Gamma_1 = \int_{t^k}^{t^k+\delta} (\|\hat{x}^*(s; t^k)\|_Q^2 + \|\hat{u}^*(s; t^k)\|_R^2)ds$. Then, if

$$J^*(x(t^k + \delta)) - J^*(\hat{x}^*(t^k + \delta)) - \Gamma_1 < 0 \tag{7}$$

holds, we have $J^*(x(t^k+\delta))-J^*(x(t^k)) < 0$, and the value δ^* which violates (7) determines the next triggering time instant $t^{k+1} = t^k + \delta^*$.

Remark 1: In order to avoid the occurrence of the Zeno phenomenon, the value of δ cannot be taken as 0. And, in order to ensure that the system can be triggered, we artificially design an upper limit T of the triggered interval. If there is no triggered between the current time instant t^k and the triggered upper time instant $t^k + T$, the system forces the trigger at the triggered upper time $t^k + T$.

E. SWITCHED MODEL PREDICTIVE CONTROL STRATEGY

In this subsection, a switched MPC strategy is proposed to enhance performance of responding to more complex environmental requirements. Suppose that, the networked linear system has h cost functions and only one cost function is active in a period of time. Let $\mathcal{V} \triangleq \{1, 2, \ldots, h\}$, $\nu \in \mathcal{V}$. Then the cost function (2) can be rewritten as

$$J_\nu(\hat{x}(t_k)) = \int_{t_k}^{t_k+T} H_\nu(\hat{x}(s; t_k), \hat{u}(s; t_k))ds + F_\nu(\hat{x}(t_k + T)). \tag{8}$$

Note that switches occur at some instants t_k when the controller receives the state $x(t_k)$. Take $\sigma(t)$ be an associated switched signal, where $\sigma(t) : [0, \infty) \to \nu$ is a right continuous function. Moreover, $\sigma(t_k)$ indicates that a cost function is active at instant t_k.

According to [20], there exists an average dwell-time τ_a for the switched signal $\sigma(t)$ if there exist $N_0 > 0$ and $\tau_a > 0$ such that

$$N_\sigma(T, t) \leq N_0 + \frac{T - t}{\tau_a}, \quad \forall\, 0 \leq t \leq T$$

where $N_\sigma(T, t)$ represents the number of switches during the time interval $(t, T]$ [22], and, it is possible to let some fast switches happen, which is characterized by the constant N_0.

Let $\tau_1, \tau_2, \ldots, \tau_{N_{\sigma(t,0)}}$ be the switched time instants in the time interval $(t, T]$. The active index of cost function (8) in the interval $[\tau_i, \tau_{i+1})$ is denoted as ν_i. In this paper, switched time instants τ_i is the design parameter which coincide with some instants t_k, i.e., $\tau_i = t_k$ for all $i \in \{0, 1, \ldots\}$ and some $k \in \{0, 1, \ldots\}$.

The control object is to asymptotically stabilize the system (1) by designing a self-triggered model predictive control strategy based on switched cost function, while minimizing the switched cost function.

III. MAIN RESULTS

In this section, self-triggered MPC scheme is firstly proposed, and an algorithm is given for implementing the proposed self-triggered MPC scheme. Then, we show that both the recursive feasibility of the optimization problem and asymptotic stability of the networked linear system (1) can be guaranteed with the proposed MPC scheme. Before presenting the main results, we assume that the following assumptions hold.

Assumption 3 [18]: For all $\nu \in \mathcal{V}$, there exist functions $W_{\nu,1}$ and $W_{\nu,2}$ with a mapping rule $\mathcal{X}^T \rightarrow \mathbf{R}_0^+$, where $\mathcal{X}^T \in \mathcal{X}$ is a set depending on the prediction horizon T. Furthermore, there exist constants $\beta_{\nu,1}, \beta_{\nu,2}, \upsilon_{\nu,1}, \upsilon_{\nu,2} > 0$ such that: (i) One has that $H_\nu(x(\cdot), u(\cdot)) \geq \upsilon_{\nu,1} W_{\nu,1}(x(\cdot))$ and $H_\nu(x(\cdot), u(\cdot)) \geq \upsilon_{\nu,2} W_{\nu,2}(x(\cdot))$ hold for all $x(\cdot) \in \mathcal{X}^T$ and $u(\cdot) \in \mathcal{U}$. (ii) For a feasible input $\hat{u}(s)$ with corresponding state trajectory $\hat{x}(t_k) = x(t_k)$ such that $H_\nu(\hat{x}(s), \hat{u}(s)) \leq \beta_{\nu,1} W_{\nu,1}(x(t_k))$ for all $t_k \leq s \leq t_k + T$ and $F_\nu(\hat{x}(t_k + T)) \leq \beta_{\nu,2} W_{\nu,2}(x(t_k))$.

Assumption 4 The initial state belongs to the feasible set, i.e. $x(t_0) \in \emptyset$.

Remark 2 Assumption 4 is standard. It is pointed out that feasible set is a set in which the optimization problem (3) has feasible solutions at initial time. If there is at least one initial state such that the optimization problem is solvable, then there must be a feasible set.

Remark 3 In this paper, self-triggered mechanism and switched model predictive control are designed based on switched cost functions, in particular, the optimal control trajectory can be computed by solving an optimization problem with respect to a specific cost function, then, the obtained control input is sent to the actuator via network.

The next theorem presents how to construct the self-triggered MPC scheme.

Theorem 1 Suppose that Assumptions 1-2 hold and the optimization problem (3) has a solution at time t^k. If there exist $\varrho \in (0, 1)$ and $\delta \in (0, T]$ such that

$$L_J \bar{e}(\delta) < \varrho \Gamma_1. \tag{9}$$

Then, the next triggering time t^{k+1} is determined by

$$t^{k+1} = t^k + \min\{\inf_{\delta^* > \delta}\{L_J \bar{e}(\delta^*) - \varrho\Gamma_1 = 0\}, T\} \tag{10}$$

where $L_J = TL_H$, $\bar{e}(\delta) = \frac{K_u}{2}\|B\|\delta^2 e^{\|A\|\delta}$. At t^{k+1}, the optimization problem (3) is solved, and $\hat{u}^*(t^{k+1})$ is applied to the system.

Remark 4 Although the form of the self-triggered condition is complex, based on the case of the specific system, T, L_H and K_u in (10) are calculated offline in advance. Therefore, the calculation of the self-triggered mechanism is not large, and the self-triggered mechanism can be applied to the real system.

Remark 5 Compared with the event triggered mechanism [23], [24], the outstanding advantage of the self-triggered mechanism is that the self-triggered mechanism can calculate the next triggered instant in advance, therefore, during the time from the current time instant to the next triggered time instant, the self-triggered mechanism does not calculate. Therefore, compared with the event triggered mechanism, the computing resources of the computer are greatly saved.

To prove the theorem, the following lemma is needed

Lemma 1 Under Assumption 1, the cost function (2) is Lipschiz continuous with Lipschitz constant L_J. Moreover, the following inequality holds.

$$J^*(x(t^k + \delta)) - J^*(\hat{x}^*(t^k + \delta)) \leq L_J \bar{e}(\delta) \tag{11}$$

where L_J and $\bar{e}(\delta)$ are defined in Theorem 1.

Proof of Lemma 1: Without loss of generality, the time instant t^k can be viewed as the initial time instant, e.g. $t^k = 0$. Consider the cost functions $J^*(x^1(0))$ and $J^*(x^2(0))$, where $x^1(0)$ and $x^2(0)$ are different initial states. Suppose that $x^{1*}(s)$, $x^{2*}(s)$, $u^{1*}(s)$, $u^{2*}(s)$, $s \in (0, T]$ are corresponding optimal state trajectories and control inputs, respectively. With the initial state $x^1(0)$, the optimal control $u^{1*}(s)$ is replaced by a feasible control input $\tilde{u}^1(s) = u^{2*}(s)$, then we have

$$\tilde{x}^1(s) = x^1(0) + \int_0^s [Ax(\tau) + Bu^{2*}(\tau)]d\tau$$
$$x^{2*}(s) = x^2(0) + \int_0^s [Ax(\tau) + Bu^{2*}(\tau)]d\tau.$$

Substituting the initial state $x^1(0)$ and $x^2(0)$ into the cost function, we have

$$\begin{aligned} &J^*(x^1(0)) - J^*(x^2(0)) \\ &\quad\leq \tilde{J}(x^1(0)) - J^*(x^2(0)) \\ &\quad\leq \int_0^T L_H\|\tilde{x}^1(s) - x^{2*}(s)\|ds + L_F\|\tilde{x}^1(T) - x^{2*}(T)\| \\ &\quad\leq (\int_0^T L_H ds)\|x^1(0) - x^2(0)\| \\ &\quad= L_J\|x^1(0) - x^2(0)\|. \end{aligned} \tag{12}$$

At time t^k, suppose that the state is $x(t^k)$. The practical control input $\hat{u}^*(t^k)$ and optimal control input $\hat{u}^*(s; t^k)$ are applied to the system, respectively, then the practical and optimal states at $t^k + \delta$ are obtained as follows

$$x(t^k + \delta) = x(t^k) + \int_{t^k}^{t^k+\delta} [Ax(s) + B\hat{u}^*(t^k))]ds$$
$$\hat{x}^*(t^k + \delta) = x(t^k) + \int_{t^k}^{t^k+\delta} [A\hat{x}^*(s) + B\hat{u}^*(s; t^k)]ds.$$

By means of triangle inequality and consider the control constraint (5), we obtain

$$\begin{aligned} &\|x(t^k + \delta) - \hat{x}^*(t^k + \delta)\| \\ &\quad= \|\int_{t^k}^{t^k+\delta} [A(x(s) - \hat{x}^*(s)) + B(\hat{u}^*(t^k) - \hat{u}^*(s; t^k))]ds\| \\ &\quad\leq \|A\|\int_{t^k}^{t^k+\delta} \|x(s) - \hat{x}^*(s)\|ds + \frac{\kappa}{2}\|B\|\delta^2. \end{aligned} \tag{13}$$

Applying the Gronwall-Bellman inequality to (13), we obtain

$$\|x(t^k + \delta) - \hat{x}^*(t^k + \delta)\| \leq \frac{K_u}{2}\|B\|\delta^2 e^{\|A\|\delta} = \bar{e}(\delta). \tag{14}$$

Let $x^1(0) = x(t^k + \delta)$ and $x^2(0) = \hat{x}^*(t^k + \delta)$. According to (12) and (14), we obtain $J^*(x(t^k + \delta)) - J^*(\hat{x}(t^k + \delta)) \leq L_J \bar{e}(\delta)$. The proof is complete.

Base on lemma 1,the proof of Theorem 1 is presented as follows.

Proof of Theorem 1: The critical value δ^* can be determined by (10), where (9) is always satisfied for $\delta < \delta^*$. Let the current triggered time be t^k, then the next triggering time is determined as $t^{k+1} = t^k + \delta^*$. According to Lemma 1, one has $J^*(x(t^{k+1})) - J^*(\hat{x}^*(t^{k+1})) \leq L_J \bar{e}(\delta^*)$. Combining with (9), $J^*(x(t^{k+1})) - J^*(\hat{x}^*(t^{k+1})) \leq \varepsilon\Gamma_1$ holds. Substituting it into (6) and with $\varepsilon \in (0, 1)$, $J^*(x(t^{k+1})) - J^*(\hat{x}^*(t^{k+1})) \leq 0$ can be obtained, which indicates that the Lyapunov-like function decreases at triggered time instants all the time. This completes the proof.

In the following, an algorithm is presented to implement the proposed self-triggered switched MPC scheme.

Algorithm 1 Self-Triggered Switched MPC

Step 1 Set $i = k = 0$ and $\tau_0 = t_0 = 0$.
Step 2 Measure the state $x(t_k)$ at time instant t_k.
Step 3 Choose a proper cost function to be optimized. If the cost function chosen at instant t_k is different to the one used at instant t_{k-1}, set $i = i + 1$ and $\tau_i = t_k$.
Step 4 Solve the optimization problem (8) with ν. Then calculate the next triggering time t^{k+1} according to Theorem 1.
Step 5 Send the first part of the computed optimal control input $u(s) = \hat{u}^*(s; t^k)$, $t^k \leq s \leq t^{k+1}$ to the actuator and apply them to the networked linear system (1) in the interval $[t_k, t_{k+1}]$ with $\tau_i \leq t_k < \tau_{i+1}$ in a sample-and-hold fashion.
Step 6 Return to **Step 2**.

Before presenting the asymptotic stability of the networked linear system (1) and recursive feasibility of the proposed MPC optimization problem, the following lemma is given.

Lemma 2 [18] For linear system (1) with multiple Lyapunov functions, the ratio of a new cost function to an old one at the switched times is bounded by μ, i.e.,

$$J_\nu(x(s)) \leq \mu J_\iota(x(s)), \quad \nu, \iota \in \mathcal{V}, \ \nu \neq \iota, \ \mu \geq 1.$$

Denote the optimal value of the cost function J_ν by $\mathbb{V}_\nu = J_\nu(x, \bar{u}_\nu^*)$, and the set for all the states (with feasible solutions of optimization problem (8) by $\mathcal{X}^T \subseteq \mathcal{X}$, where the subscript T indicates the dependence on the prediction horizon T. According to Lemma 2 and Assumption 3, the following theorem is given to prove both recursive feasibility of the optimization problem and asymptotic stability of the networked linear system (1).

Theorem 2 If there exists an initial feasible solution of optimization problem (8) for $x(0) \in \mathcal{X}^T$ with an local controller $u(s) = \kappa_\nu^{loc}(x(s))$ for all $\nu \in \mathcal{V}$, then the optimization problem (8) is feasible in Step 4 of Algorithm 1 for all $t \geq 0$. Moreover, there exist μ and λ_0 such that

$$\mathbb{V}_\nu(x(t_{k+1})) - \mathbb{V}_\nu(x(t_k)) \leq -\lambda_0 \int_{t_k}^{t_{k+1}} \mathbb{V}_\nu(x(s))ds \quad \forall\tau_i \leq t_k \leq t_{k+1} < \tau_{i+1}, \quad \forall i = 0, 1, \cdots \tag{15}$$

holds for all $x \in \mathcal{X}^T$. Moreover, the networked linear system (1) is asymptotically stable, if $\sigma(t)$ is a switched signal with average dwell-time satisfying

$$\tau_a > \frac{\ln \mu}{\lambda_0}. \tag{16}$$

Remark 6 Not that condition (15) only holds for ν whose associated cost functional J_ν is active during the time interval $[\tau_i, \tau_{i+1})$. As for all other $\nu \in \mathcal{V}$, $\mathbb{V}_\nu$ also can increase in this time interval.

Proof of Theorem 2: Note that $\hat{u}(s) = u_\nu^*(s; t_k)$ is a feasible solution during the time interval $[t_k, t_k + T]$ for $\nu \in \mathcal{V}$, where $u_\nu^*(s; t_k)$ represents an input computed at instant t_k. Then

$$\hat{u}(s) = \begin{cases} u_\nu^*(s; t_k) & s \in [t_{k+1}, t_k + T] \\ \kappa_\nu^{loc}(x(s; t_{k+1})) & s \in (t_k + T, t_{k+1} + T] \end{cases}$$

is a feasible solution in the time interval $[t_k, t_k + T]$ [19]. The recursive feasibility of the proposed MPC algorithm is established.

Let the optimal cost $J_\nu^*(x(\cdot), u^*(\cdot)) = J_\nu^*(x^*(\cdot))$ be denoted as $\mathbb{V}_\nu(x(\cdot)) = J_\nu^*(x^*(\cdot))$. If Assumption 3 holds, there exists

$$\begin{aligned}
&\mathbb{V}_\nu(x(t_k)) \\
&\quad = J_\nu^*(x(t_k), u^*(\cdot)) \\
&\quad \leq J_\nu(x(t_k), \hat{u}(\cdot)) \\
&\quad = \int_{t_k}^{t_k+T} H_\nu(\hat{x}(s), \hat{u}(s))ds + F_\nu(\hat{x}(t_k + T)) \\
&\quad \leq \beta_{\nu,1} T W_{\nu,1}(x(t_k)) + \beta_{\nu,2} W_{\nu,2}(x(t_k)) \\
&\quad \leq \left(\frac{\beta_{\nu,1} T}{\upsilon_{\nu,1}} + \frac{\beta_{\nu,2}}{\upsilon_{\nu,2}}\right) H_\nu(x(t_k), u(t_k)) \\
&\quad = \frac{1}{\lambda_{0,\nu}} H_\nu(x(t_k), u(t_k))
\end{aligned}$$

where

$$\frac{\beta_{\nu,1} T}{\upsilon_{\nu,1}} + \frac{\beta_{\nu,2}}{\upsilon_{\nu,2}} = \frac{1}{\lambda_{0,\nu}}.$$

Obviously, it follows that

$$H_\nu(x(t_k), u(t_k)) \geq \lambda_{0,\nu} \mathbb{V}_\nu(x(t_k)). \tag{17}$$

Combining (17) with the following inequality

$$J_\nu^*(x^*(t_{k+1})) - J_\nu^*(x^*(t_k)) < -\int_{t_k}^{t_{k+1}} H_\nu(x^*(s), u^*(s))ds \quad \forall\ \tau_i \leq t_k < t_{k+1} < \tau_{i+1}$$

one has that

$$J_\nu^*(x^*(t_{k+1})) - J_\nu^*(x^*(t_k)) \leq -\lambda_0 \int_{t_k}^{t_{k+1}} \mathbb{V}_\nu(x(s))ds$$

with

$$\lambda_0 = \min_\nu \lambda_{0,\nu} \tag{18}$$

for all $\nu \in \mathcal{V}$. That is,

$$\mathbb{V}_\nu(x(t_{k+1})) - \mathbb{V}_\nu(x(t_k)) \leq -\lambda_0 \int_{t_k}^{t_{k+1}} \mathbb{V}_\nu(x(s))ds.$$

By the comparison principle [21], it is obtained that

$$\mathbb{V}_\nu(x(\tau_{i+1})) \leq e^{-\lambda_0(\tau_{i+1}-\tau_i)} \mathbb{V}_\nu(x(\tau_i)) \tag{19}$$

with $\tau_i \leq t_k < t_{k+1} < \tau_{i+1}$. From Lemma 2 and (19), one has that

$$\begin{aligned}\mathbb{V}_{\sigma(\tau_{i+1})}&(x(\tau_{i+1}))\\ &\leq \mu\mathbb{V}_{\sigma(\tau_i)}(x(\tau_{i+1}))\\ &\leq \mu e^{-\lambda_0(\tau_{i+1}-\tau_i)}\mathbb{V}_{\sigma(\tau_i)}(x(\tau_i))\end{aligned} \tag{20}$$

for any two switched instants τ_i and τ_{i+1}. By iterating (20) from $i = 0$ to $i = N_\sigma(t, 0)$, it follows that

$$\begin{aligned}\mathbb{V}_{\sigma(t)}&(x(t))\\ &\leq \mu^{N_\sigma(t,0)}e^{-\lambda_0 t}\mathbb{V}_{\sigma(0)}(x(0))\\ &= e^{N_\sigma(t,0)\ln(\mu)-\lambda_0 t}\mathbb{V}_{\sigma(0)}(x(0))\\ &\leq \mu^{N_0}e^{(\ln(\mu)/\tau_a-\lambda_0)t}\mathbb{V}_{\sigma(0)}(x(0))\\ &\leq \mu^{N_0}e^{-\lambda t}\mathbb{V}_{\sigma(0)}(x(0))\end{aligned} \tag{21}$$

for $\tau_a > \ln(\mu)/\lambda_0$ and $\lambda \in (0, \lambda_0)$. Because $H_\nu(x(t_k), u(t_k))$ is continuous and positive definite by Assumption 1, there exists $H_\nu(x(t_k), u(t_k)) \geq \alpha_\nu(\|x(t_k)\|)$, where $\alpha_\nu(\cdot)$ is a $\mathcal{K}_\infty$ function for all $\nu \in \mathcal{V}, x \in \mathcal{X}^T$ and $u \in \mathcal{U}$. There exists a $\mathcal{K}_\infty$ function $\bar{\alpha}(\cdot)$ such that $\mathbb{V}_\nu(x(t_k)) \geq \bar{\alpha}(t_k)$ holds for all $\nu \in \mathcal{V}$ and $x \in \mathcal{X}^T$. Thus, (21) is rewritten as

$$\|x(t)\| \leq \bar{\alpha}^{-1}(\mu^{N_0}e^{-\lambda t}\mathbb{V}_{\sigma(0)}(x(0))).$$

Obviously, $\bar{\alpha}^{-1}(\cdot)$ is also a $\mathcal{K}_\infty$ function, which indicates that $x(t)$ asymptotically converges to the origin. This completes the proof.

Remark 7 In this paper, a Lyapunov-like function is applied. The Lyapunov-like function is a kind of energy function. Because the system is a linear system and there is a self-triggered upper limit T, although the Lyapunov-like function at the triggered interval may rise or fall, it cannot rise or fall to infinity, it will always have a certain limit. Through the proof of theorem 2, it can be seen that with the increasing of k, the Lyapunov-like function decreases strictly to zero. At the same time, the stability of the close-loop system can be guaranteed.

Remark 8 Based on the obtained results, since the proposed self-trigged MPC scheme is designed based on switched cost functions, which can not only save network resources, but also meet the requirements of more complex industrial systems compared with the MPC algorithm using single cost function. Therefore, the proposed self-triggered MPC scheme is more practical.

IV. NUMERICAL EXAMPLE

In this section, a simulation example of pneumatic artificial muscle platform is provided to show the effectiveness of the proposed method.

The pneumatic artificial muscle platform is considered as

$$\begin{cases}\dot{\theta}(t) = k_u\theta(t) + k_u L_0 u\\ \ddot{\theta}(t) = -\dfrac{4b_v}{mr^2}\dot{\theta}(t) + \dfrac{8h_0u_0(2\iota_1\psi_0 + \iota_2)L_0^{-1}}{m}\theta(t)\\ \qquad + \dfrac{8h_ok_u(\iota_1\psi_0^2 + \iota_2\psi_0 + \iota_3)}{mr}u(t)\end{cases}$$

where $\theta(t)$ is a tilt angle of the upper platform, $u(t)$ is a voltage controlling two pneumatic valves, b_v is a damping factor, r is a radius of the disc, m is weight of a load, h_0 is a coefficient to control a proportion of pressure and voltage, $\iota_1 - \iota_3$ and k_u are given parameters of the pneumatic artificial muscle platform, ψ_0 is an initial length of the pneumatic artificial muscle, and u_0 is an early warning voltage.

Define $x_1(t) = \theta(t)$, $x_2(t) = \dot{\theta}(t)$, therefore, we can get the state space equation as following

$$\begin{aligned}\dot{x}_1(t) &= k_u x_1(t) + k_u L_0 u(t)\\ \dot{x}_2(t) &= b_1 x_1(t) + b_2 x_2(t) + b_0 u(t)\end{aligned}$$

where $b_0 = \frac{8h_ok_u(\iota_1\psi_0^2+\iota_2\psi_0+\iota_3)}{mr}$, $b_1 = \frac{8h_0u_0(2\iota_1\psi_0+\iota_2)L_0^{-1}}{m}$ and $b_2 = -\frac{4b_v}{mr^2}$. Define

$$x(t) = \begin{bmatrix} x_1(t)\\ x_2(t)\end{bmatrix}, \quad A = \begin{bmatrix} k_u & 0\\ b_1 & b_2\end{bmatrix}, B = \begin{bmatrix} k_u L_0\\ b_0\end{bmatrix},$$

therefore, there exists

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{22}$$

with

$$A = \begin{bmatrix} 0.57 & 0\\ -14.3 & -0.4\end{bmatrix}, \quad B = \begin{bmatrix} 479.8\\ 11.5\end{bmatrix}$$

gotten by selecting appropriate parameters. The constraint sets of the linear system (22) are given as follows

$$\begin{aligned}u(t) &\in \mathcal{U} \triangleq \{u(t) : |u(t)| \leq 60\}\\ x(t) &\in \mathcal{X} \triangleq \{x(t) : |x_1(t)| \leq 10, \ |x_2(t)| \leq 10\}.\end{aligned}$$

Note that $x_1(t)$ and $x_2(t)$ represent the first state and the second state of the linear system(22), respectively. Two cost functions are considered for the linear system (22) and the stage cost and terminal cost are chosen as

$$\begin{aligned}H_\nu(x(t), u(t)) &= x^T(t)Q_\nu x(t) + u^T(t)R_\nu u(t)\\ F_\nu(x(t), u(t)) &= x^T(t)P_\nu x(t)\end{aligned}$$

with $\nu \in \{1, 2\}$. The weighting matrices are chosen as

$$Q_1 = \begin{bmatrix} 1 & 0\\ 0 & 10\end{bmatrix}, \quad R_1 = 1$$

$$Q_2 = \begin{bmatrix} 1 & 0\\ 0 & 8\end{bmatrix}, \quad R_2 = 1.5.$$

The initial state of the linear system (22) is chosen as

$$x(0) = [3 \quad 3]^T.$$

The prediction horizon is $T = 8$. The local controller for the linear system (22) is designed as

$$u(t) = \kappa^{loc}(x(t)) = -2.0246x_1(t) + 19.2570x_2(t)$$

which is calculated as the solution of the linear quadratic regulator problem for the linear system (22) with weighting matrices Q_1 and R_1. The terminal region of the linear system (22) is obtained as

$$\mathcal{X}^f = \{x(t) : x^T(t)Px(t) \leq \alpha^f\}$$

with

$$P = \begin{bmatrix} 4.877 & 1.494 \\ 1.494 & 5.088 \end{bmatrix}$$

and $\alpha^f = 7.62$ is calculated according to Remark 2 in [18]. The parameter K_u is given as $K_u = 0.3$, and ε is chosen according to $\varrho = 0.8$. Besides, parameters $\mu = 10$ and $\tau_a = 25.1$ are obtained using the method in [18]. Then the switching signal $\sigma(t)$ is chosen such that the switching occurs in the two cost functions every 30 time units which is satisfied with the condition (16).

We first compute the value of λ_0 and design $v_1 = v_2 = 1$, $\beta_{1,1} = 4.16 \times 10^3$, $\beta_{2,1} = 1.04 \times 10^4$, and $\beta_{1,2} = \beta_{2,2} = 10.15$. Using Remark 9 in [18], we obtain that

$$\lambda_{0,1} = 0.1052$$
$$\lambda_{0,2} = 0.1013$$

thus, according to (18)

$$\lambda_0 = \min_{\nu \in \{1,2\}} \lambda_{0,\nu} = 0.1013.$$

Furthermore, according to [18], we obtain $\mu = 10$, which according to (16) finally leads to

$$\tau_a > \frac{\ln \mu}{\lambda_0} = 25.1. \tag{23}$$

The average run time of a self-triggered program in a simulation program is 0.020104 seconds, therefore, the self-triggered condition is applicable for real control systems. As shown in Fig. 2, we can see that about 50% bandwidth resources are saved, until the system (22) is stabilized. The state response of the linear system (22) is shown in Fig. 3.

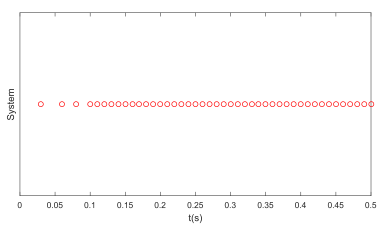


FIGURE 2. Triggering times of system with self-triggered MPC scheme.

Fig. 3 shows that the convergence rate of state is fast and the stability of the linear system (22) is achieved, although, there exist periodic changes of state which is induced by the switched cost function. The input of the linear system (22) is shown in Fig. 4.

From Fig. 3 and Fig. 4, it is obvious that the constrains on both state and input of the linear system (22) are satisfied. Due to the existence of switched cost functions in Algorithm 1, there are periodic fluctuations in Fig. 3 and Fig. 4.

For comparison, the cost values with different single cost functions and switched cost functions are shown in Fig.5.

In Fig. 5, the red dot line represents the cost of the linear system (22) by applying Algorithm 1, the blue solid line and the green dashed line represent costs of the linear

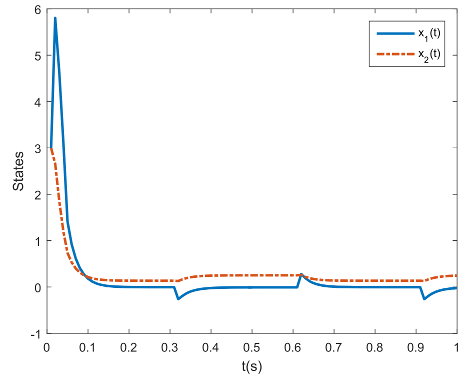


FIGURE 3. State response of the linear system.

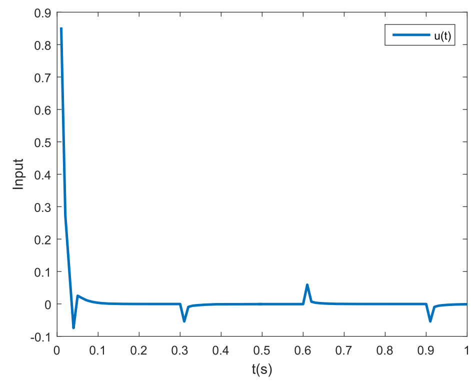


FIGURE 4. Input of the linear system.

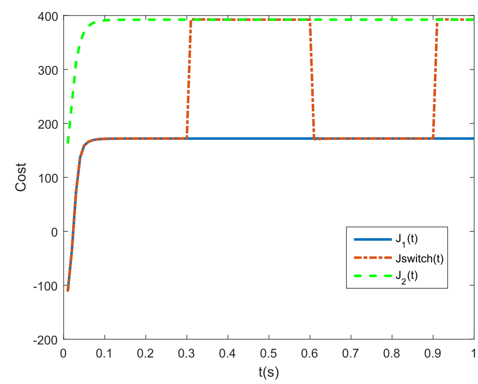


FIGURE 5. Cost.

system (22) with cost J_1 and cost J_2, respectively. From Fig. 5, we can see that the cost with Algorithm 1 (based on switching cost functions) is between the costs by using separate cost functions. However, because the switched cost functions are used in the proposed algorithm, it can achieve more complex specifications in practical industrial systems compared with the MPC scheme based on single cost function.

V. CONCLUSION

In this paper, a self-triggered MPC scheme based on switched cost functions has been proposed for networked linear systems. The optimization problem is solved when the self-triggered condition is satisfied. The proposed scheme reduces the computing burden and communication load. The proposed self-triggered MPC is designed based on switched cost functions, hence, more complex specifications can be met compared to strategies with single cost function. Moreover, by using the proposed algorithm, both recursive feasibility and asymptotically stability can be guaranteed. In the end, a numerical example is given to show the effectiveness of the proposed method.

REFERENCES

[1] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1680–1685, Sep. 2007.

[2] W. P. M. H. Heemels and M. C. F. Donkers, "Model-based periodic event-triggered control for linear systems," *Automatica*, vol. 49, no. 3, pp. 698–711, 2013.

[3] L. Dai, Y. Xia, Y. Gao, and M. Cannon, "Distributed stochastic MPC of linear systems with additive uncertainty and coupled probabilistic constraints," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3474–3481, Jul. 2017.

[4] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1291–1297, May 2012.

[5] X. Wang and M. D. Lemmon, "On event design in event-triggered feedback systems," *Automatica*, vol. 47, no. 10, pp. 2319–2322, Oct. 2011.

[6] Y. Sadi, S. C. Ergen, and P. Park, "Minimum energy data transmission for wireless networked control systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 2163–2175, Apr. 2014.

[7] Y. Sadi and S. C. Ergen, "Joint optimization of wireless network energy consumption and control system performance in wireless networked control systems," *IEEE Trans. Wireless Commun.*, vol. 16, no. 4, pp. 2235–2248, Apr. 2017.

[8] C. Peng, Y. Song, X. P. Xie, M. Zhao, and M.-R. Fei, "Event-triggered output tracking control for wireless networked control systems with communication delays and data dropouts," *IET Control Theory Appl.*, vol. 10, no. 17, pp. 2195–2203, Jul. 2016.

[9] E. Henrikssonl, D. Quevedo, H. Sandberg, and K. H. Johansson, "Self-triggered model predictive control for network scheduling and control," in *Proc. 8th Int. Symp. Adv. Control Chem. Processes*, Jul. 2012, pp. 432–438.

[10] D. Lehmann, E. Henriksson, and K. H. Johansson, "Event-triggered model predictive control of discrete-time linear systems subject to disturbances," in *Proc. Eur. Control Conf. (ECC)*, Jul. 2013, pp. 1156–1161.

[11] H. Li and Y. Shi, "Event-triggered robust model predictive control of continuous-time nonlinear systems," *Automatica*, vol. 50, no. 5, pp. 1507–1513, May 2014.

[12] A. Bemporad and D. Muñoz de la Peña, "Multiobjective model predictive control," *Automatica*, vol. 45, pp. 2823–2830, Dec. 2009.

[13] Y. Pang, K. Zhang, Y. Yuan, and K. Wang, "Distributed object detection with linear SVMs," *IEEE Trans. Cybern.*, vol. 44, no. 11, pp. 2122–2133, Nov. 2014.

[14] Y. Yuan, F. Sun, and H. Liu, "Resilient control of cyber-physical systems against intelligent attacker: A hierarchal Stackelberg game approach," *Int. J. Syst. Sci.*, vol. 47., no. 9, pp. 2067–2077, 2016.

[15] F. Khani and M. Haeri, "Smooth switching in a scheduled robust model predictive controller," *J. Process Control*, vol. 31, pp. 55–63, Jul. 2015.

[16] L. Zhang, S. Zhuang, and R. D. Braatz, "Switched model predictive control of switched linear systems: Feasibility, stability and robustness," *Automatica*, vol. 67, pp. 8–21, May 2016.

[17] M. A. Müller, P. Martius, and F. Allgöwer, "Model predictive control of switched nonlinear systems under average dwell-time," *J. Process Control*, vol. 22, no. 9, pp. 1702–1710, Oct. 2012.

[18] M. A. Müller and F. Allgöwer, "Improving performance in model predictive control: Switching cost functionals under average dwell-time," *Automatica*, vol. 48, no. 2, pp. 402–409, Feb. 2012.

[19] H. Chen and F. Allgöwer, "A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability," *Automatica*, vol. 34, no. 10, pp. 1205–1217, 1998.

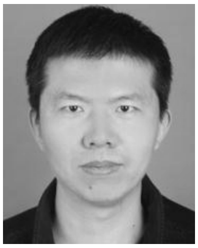
[20] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," in *Proc. 38th IEEE Conf. Decis. Control*, Dec. 1999, pp. 2655–2660.

[21] H. K. Khalil, *Nonlinear System*, Upper Saddle River, NJ, USA Prentice-Hall, 2002.

[22] J. P. Hespanha and A. S. Morse, "Stabilization of nonholonomic integrators via logic-based switching," *Automatica*, vol. 35, no. 3, pp. 385–393, Mar. 1999.

[23] Z. Wang, L. Shen, J. Xia, H. Shen, and J. Wang, "Finite-time non-fragile control for jumping stochastic systems subject to input constraints via an event-triggered mechanism," *J. Franklin Inst.*, vol. 355, no. 14, pp. 6371–6389, Sep. 2018.

[24] B. Xu, G. Wang, H. H-C. Iu, S. Yu, and F. Yuan, "A memristor—Meminductor-based chaotic system with abundant dynamical behaviors," *Nonlinear Dyn.*, vol. 96, no. 1, pp. 765–788, 2019. doi: 10.1007/s11071-019-04820-1.



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