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Time-Varying Formation Control For Second-Order Discrete-Time Multi-Agent Systems With Switching Topologies and Nonuniform Communication Delays

LYULONG HE[1,2], JIAQIANG ZHANG[2], YUEQI HOU[2], XIAOLONG LIANG[2], AND PENG BAI[1]

[1]Shaanxi Province Laboratory of Meta-synthesis for Electronic and Information Systems, Xi'an 710051, China

[2]National Key Laboratory of Air Traffic Collision Prevention, Air Force Engineering University, Xi'an 710051, China

Corresponding author: Jiaqiang Zhang (jiaqiang-z@163.com)

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ABSTRACT Time-varying formation control and velocity tracking problems for second-order discrete-time multi-agent systems with switching jointly-connected topologies and nonuniform communication delays are investigated. A local information-based distributed protocol is designed by utilizing the instantaneous states of the agent itself and the delayed states of its neighbors. Through model transformation and stability analysis, an explicit mathematical description of a feasible time-varying formation set is proposed. Necessary and sufficient conditions for the systems with switching topologies and nonuniform communication delays to achieve the feasible time-varying formation are obtained. The coupling constraints on the gain parameters and sampling period are proposed, so as to guide the design of parameters in the protocol. We include the effects of velocity tracking error damping gain in the protocol and derive milder conditions which allow for not only bounded nonuniform communication delays but also for dynamically switching directed graphs that are jointly connected. The numerical examples are further presented to illustrate the validity and effectiveness of the obtained results.

INDEX TERMS Multi-agent systems, consensus control, time-varying formation, switching directed topology, velocity tracking, communication delay.

I. INTRODUCTION

Multi-agent system is composed of many inexpensive simple individuals and can emerge much better performance and even new abilities through efficient coordination. Formation control, as one of the most fundamental distributed cooperative control problems for multi-agent systems, is a critical step of cooperation among agents [1], [2]. Therefore, cooperative formation control for multi-agent systems has become a research hotspot and accurate maintenance of geometric formation between agents has been studied extensively [3]–[5]. In general, the formation control problems for multi-agent systems are to design distributed coordination protocols for networks of agents such that they would approach and maintain some desired, possibly time-varying formation. The main challenge in cooperative formation control is that each agent has to use local information to achieve the desired formation, rather than rely on centralized coordination. It is more difficult when there are constraints such as time delay and switching topologies.

Recently, consensus control for multi-agent systems has attracted great attention from various domains and fruitful results have been achieved already [6]–[9]. Following the boom in the research of consensus control problems, consensus-based formation control approaches are developed, which provides the benefits of improving the flexibility, robustness and scalability of the formation. It has been proved in [10] that the classical leader-follower, behavior and virtual

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structure based approaches can be regarded as special cases of consensus-based ones, and the weaknesses of the previous approaches can be overcame to some extent.

Consensus based time-invariant formation control problems for first-, second- and high-order multi-agent systems have been studied extensively [11]–[16]. When the time-invariant formation is achieved, velocity components of all agents must be identical. This is not satisfactory because in many practical applications, such as target surveillance and formation reconfiguration, the desired formation requires the velocity components of agents to be different and time-varying. Besides, the formation of agents cannot remain stationary if certain tasks are to be executed, such as tracking a moving target with unmanned aerial vehicles (UAV) swarm. In these applications, not only the time-varying formation, but also reference velocity tracking are needed. In [17], a decentralized controller was proposed based on virtual leader structure to achieve predefined time-varying formation with fixed topology. In [18], a controller-observer approach for time-varying centroid tracking and formation control of multi-agent systems with switching topologies was proposed, provided that in each time instant the interaction graph was balanced and strongly connected (in the case of directed topology) or simply connected (in the case of undirected topology). Reference [19] dealt with the cooperative control problem for nonlinear multi-agent systems with undirected topology, whose objective was to stabilize a group of agents to time-varying formation and track a time-varying center. Time-varying formation tracking analysis and design problems for second-order multi-agent systems with switching interaction topologies were studied in [20], with the assumption that each of the possible interaction topologies had a spanning tree. The time-varying formation problems were studied in [21] and an adaptive approach was utilized to develop a fully distributed formation controller for general linear multi-agent systems with fixed topology. Reference [22] studied the time-varying formation tracking problems for multiple manipulator systems under fixed and switching directed graphs with a dynamic leader. Reference [23] considered the time-varying formation control problems for a class of networked systems with non-identical nonlinear dynamics and fixed topology. Time-varying formation analysis and design problems for general high-order swarm systems with communication constraints and fixed undirected topology were investigated in [24].

Considering the fact that time delay often exists in practical systems due to the limitation of bandwidth, abundant data transmission and asymmetry of communication links, formation control problems with time delay have been investigated in [25]–[27]. Reference [28] researched the distributed formation control problems for multi-agent systems with randomly switching undirected topologies and uniform time delay. Reference [29] considered the problems on formation tracking control of second-order multi-agent systems with communication delay and fixed topology. In [30], the containment control problems were considered for nonlinear multi-agent systems with fixed topology and uniform time-delay. Reference [31] investigated the leader-follower formation control problems for a group of networked robots that were subject to bounded time-varying communication delays and asynchronous clock.

It should be pointed out that in most recent literatures, the systems were described by continuous-time dynamics, such as the works mentioned above. However, in practical formation control applications via interaction networks, continuous states of agents (such as position and velocity) are always represented and updated by their sampled values at a certain interval, which results in discrete-time or sampled-data formulation. The conclusions obtained in continuous-time systems cannot be used to solve such problems directly. Thus, the formation control protocol design and analysis problems for multi-agent systems with discrete-time dynamics are necessary and of practical significance. In [32], the formation control problems without time delay were investigated for discrete-time multi-agent systems subject to unknown nonlinear dynamics by means of iterative learning approach. Reference [33] presented a distributed control law based on output regulation control framework to solve the formation control problems of first-order discrete-time nonlinear multi-agent systems without time delay. Reference [34] used a fault tolerant approach to control a group of wheeled mobile robots in a formation without time delay. In [35], the time-invariant formation control for high-order discrete-time multi-agent systems was achieved in the absence of time delay. The time-invariant formation control problems of second-order discrete-time systems with time delay and undirected topology were investigated in [36]. Reference [37] established the necessary and sufficient conditions for designing formation of discrete-time second-order multi-agent systems with only one sampling period delay and the desired formation cannot be time-varying. The asynchronous time-invariant formation control problems of second-order discrete-time multi-agent systems with time-varying delays were investigated in [38]. Time-varying formation control protocol design and analysis problems for second-order discrete-time multi-agent systems with directed interaction topology and communication delay were investigated in [39], but the topology was fixed and the communication delay was uniform.

Although some important results and approaches have been established in a few references, research on formation control for discrete-time systems is not as sufficient as for continuous systems, especially the time-varying formation control under conditions with time delay and switching topologies. This paper mainly focuses on the time-varying formation and velocity tracking control problems for second-order discrete-time multi-agent systems with nonuniform communication delays and switching jointly-connected topologies, which is meaningful yet still unresolved. The multi-agent systems in this paper are described by second-order discrete-time dynamics, where agents are governed by both position and velocity states. Many practical systems can be described by this model, such as UAV swarm and multi-robot system. It is more complicated and conforms to reality as the sampling period and gain parameters can be considered simultaneously. A distributed protocol is designed by utilizing the delayed states of neighbors and the instantaneous states of agent itself. In order to achieve reference velocity tracking, we include the effects of velocity tracking error damping gain in the protocol. Furthermore, both the desired formation and reference velocity can be time-varying and the switching interaction topologies are jointly-connected. Compared with the previous results, this paper aims to solve the following four problems: (i) How to design the control protocol to achieve desired time-varying formation and reference velocity tracking simultaneously; (ii) what are the conditions that guarantee the time-varying formation and velocity tracking can be achieved with switching jointly-connected topologies and nonuniform communication delays; (iii) how to design feasible desired time-varying formation in mission planning; (iv) how to design the parameters in the protocol to achieve the feasible time-varying formation and velocity tracking.

The remainder of this paper is organized as follows. In section II, some necessary preliminary results and lemmas are described together with problem description. Section III considers the time-varying formation control analysis and protocol design problems. Numerical examples are provided in section IV to illustrate the validity of the algorithm and section V summarizes this paper.

Notations: Throughout the paper, let $\mathbf{0}$ ($\mathbf{1}$) denotes the column vector of all 0 (1) with appropriate dimension. $\boldsymbol{I}_N$ denotes an identity matrix of dimension N.

II. PRELIMINARY AND PROBLEM DESCRIPTION

In this section, some necessary concepts and results on graph and matrix theory are introduced firstly, and then a detailed description of the problem is presented.

A. GRAPH AND MATRIX THEORY

Consider a multi-agent system with N nodes labelled 1 through N, denote $\mathcal{I}_N = \{1, 2, \ldots, N\}$. The interaction topology among the agents can be modelled as a weighted directed graph $G = (W, Q, \boldsymbol{A})$, with $W = \{\varpi_1, \ldots, \varpi_N\}$, $Q \subseteq \{(\varpi_i, \varpi_j) : \varpi_i, \varpi_j \in W\}$ and $\boldsymbol{A} = [a_{ij}]_{N\times N}$ being the vertices set, edges set and weighted adjacency matrix, respectively. If agent ϖ_j can send its information to ϖ_i, then there exists a directed edge $(\varpi_j, \varpi_i) \in Q$ from vertex ϖ_j to ϖ_i, and ϖ_j is called a neighbour of ϖ_i. $N_i = \{\varpi_j | \varpi_j \in W : (\varpi_j, \varpi_i) \in Q\}$ represents the neighbours set of ϖ_i. The adjacency matrix $\boldsymbol{A}$ satisfies $a_{ij} > 0$ if and only if $\varpi_j \in N_i$, otherwise $a_{ij} = 0$. If $a_{ii} > 0$, we say that agent ϖ_i has self-loop. Laplacian matrix $\boldsymbol{L} = [l_{ij}]_{N\times N}$ plays an important role in description of the interaction relationship of agents in a graph and it is defined as $\boldsymbol{L} = \boldsymbol{\Delta} - \boldsymbol{A}$, where $\boldsymbol{\Delta} = [\Delta_{ij}]_{N\times N}$ is a diagonal matrix with $\Delta_{ii} = \sum_{k=1}^{N} a_{ik}$. An useful property of $\boldsymbol{L}$ is that all of its row sums are zero and $\mathbf{1}_N$ is an eigenvector of $\boldsymbol{L}$ associated with the zero eigenvalue, that is $\boldsymbol{L}\mathbf{1}_N = \mathbf{0}$. The directed graph of $\boldsymbol{A}$, denoted as $G(\boldsymbol{A})$, is a digraph with N vertices and satisfies there is an edge in $G(\boldsymbol{A})$ from vertex ϖ_j to ϖ_i if and only if $a_{ij} \neq 0$. A directed path is a sequence of ordered edges $(\varpi_{j_1}, \varpi_{j_2})$, $(\varpi_{j_2}, \varpi_{j_3})$, ..., $(\varpi_{j_{m-1}}, \varpi_{j_m})$ with $\varpi_{j_1}, \ldots, \varpi_{j_m} \in W$. A directed graph G is said to have a spanning tree if there is at least one vertex having directed paths to all the other vertices. A directed graph is said to be strongly connected if there is a directed path from every node to every other node. For a collection of graphs, its union is defined as a new graph whose nodes and edges are the union of nodes and edges of all the graphs in the collection.

Note that the interaction topology among agents in this paper can be switching. Let finite set S denotes all the possible graphs with an index set $\mathcal{I}_G \subset \mathbb{N}$, where $\mathbb{N}$ represents the set of natural numbers. Let $\sigma(t) : [0, +\infty) \to \mathcal{I}_G$ be a switching signal whose value is the index of the topology at t, $G(t)$, $\boldsymbol{A}(t)$ and $\boldsymbol{L}(t)$ stand for the corresponding graph, adjacency matrix and Laplacian matrix, respectively.

Given a matrix $\boldsymbol{M} \in R^{m\times m}$, $\boldsymbol{M}$ is nonnegative (positive) if all its elements are nonnegative (positive), denoted as $\boldsymbol{M} \geq 0$ ($\boldsymbol{M} > 0$). If a nonnegative matrix $\boldsymbol{M}$ satisfies $\boldsymbol{M}\mathbf{1}_N = \mathbf{1}_N$, then it is a stochastic matrix. A stochastic matrix is said to be indecomposable and aperiodic (SIA) if $\lim_{k\to\infty} \boldsymbol{M}^k = \mathbf{1}\boldsymbol{c}^T$, where $\boldsymbol{c} \in R^m$ is constant column vector. Let $\prod_{i=1}^{k} \boldsymbol{A}_i = \boldsymbol{A}_k\boldsymbol{A}_{k-1}\ldots\boldsymbol{A}_1$ denote the left product of matrices with compatible dimensions.

Lemma 1 [40]: Let $\boldsymbol{M}$ be a stochastic matrix. If $G(\boldsymbol{M})$ has a spanning tree with the property that the root vertex has self-loop, then $\boldsymbol{M}$ is SIA.

B. PROBLEM FORMULATION

Consider a multi-agent system with N nodes, the purpose of this paper is to design a protocol to steer the agents to form predefined time-varying formation and track desired velocity with switching directed topologies and nonuniform communication delays. In the discrete-time case, using the forward difference approximation as that employed in [41], the dynamics of each agent is described by

$$\begin{cases} \boldsymbol{x}_i(t+\delta) = \boldsymbol{x}_i(t) + \delta\boldsymbol{v}_i(t) \\ \boldsymbol{v}_i(t+\delta) = \boldsymbol{v}_i(t) + \delta\boldsymbol{u}_i(t), \end{cases} \tag{1}$$

where $\boldsymbol{x}_i(t) \in R^n$, $\boldsymbol{v}_i(t) \in R^n$ and $\boldsymbol{u}_i(t) \in R^n$ are the position, velocity and control input vectors of agent ϖ_i, $i = 1, \ldots, N$. $\delta > 0$ is analogous to the sampling period, in the following we will refer to it as such. The update time instants $t \geq 0$ will be the form $t = t_0 + q\delta$, $t_0 \geq 0$ is the initial moment, $q = 1, 2, \ldots$. $n \geq 1$ is the dimension of the states. In the following, for the sake of convenience in description, let n =1 if not otherwise specified. However, all the conclusions hereafter can be extended to higher dimensional cases directly by using Kronecker product.

The time-varying formation to be achieved is described by a set of bounded functions $\boldsymbol{h}_x(t) = [\boldsymbol{h}_{1x}(t), \ldots, \boldsymbol{h}_{Nx}^T(t)]^T$, $\boldsymbol{h}_v(t) = [\boldsymbol{h}_{1v}(t), \ldots, \boldsymbol{h}_{Nv}^T(t)]^T$ and $\boldsymbol{h}_a(t) = [\boldsymbol{h}_{1a}(t), \ldots, \boldsymbol{h}_{Na}^T(t)]^T$. $\boldsymbol{h}_x(t)$, $\boldsymbol{h}_v(t)$ and $\boldsymbol{h}_a(t)$ are so called formation offset functions with $\boldsymbol{h}_{ix}(t)$, $\boldsymbol{h}_{iv}(t)$ and $\boldsymbol{h}_{ia}(t)$ being the components of position, velocity and acceleration for agent ϖ_i, respectively. The formation cannot remain stationary if certain tasks are to be executed, such as tracking moving military targets with UAV swarm. The introduction of translational velocity (possibly time-varying) rectifies this and enables the proposed method to adapt to more practical applications. This can be accomplished by letting $\boldsymbol{h}_{iv}(t) = \tilde{\boldsymbol{h}}_{iv}(t) + \boldsymbol{v}_d(t)$, where $\tilde{\boldsymbol{h}}_{iv}(t)$ is the velocity component of the desired formation shape and $\boldsymbol{v}_d(t)$ is the desired translational velocity to be tracked. Define $\boldsymbol{h}_i(t) = [\boldsymbol{h}_{ix}(t), \boldsymbol{h}_{iv}(t)]^T$, $\boldsymbol{\xi}_i(t) = [\boldsymbol{x}_i(t), \boldsymbol{v}_i(t)]^T$, $\boldsymbol{h}(t) = [\boldsymbol{h}_1^T(t), \ldots, \boldsymbol{h}_N^T(t)]^T$ and $\boldsymbol{\xi}(t) = [\boldsymbol{\xi}_1^T(t), \ldots, \boldsymbol{\xi}_N^T(t)]^T$.

Definition 1: Multi-agent system (1) is said to achieve consensus if and only if for any given bounded initial states and $i \in \mathcal{I}_N$, there exists a vector-valued function $\bar{\boldsymbol{\xi}}(t)$ such that

$$\lim_{t\to\infty} \left(\boldsymbol{\xi}_i(t) - \bar{\boldsymbol{\xi}}(t)\right) = \boldsymbol{0}, \tag{2}$$

where $\bar{\boldsymbol{\xi}}(t) = [\bar{\boldsymbol{x}}(t), \bar{\boldsymbol{v}}(t)]^T$ is called the consensus states function. Obviously, (2) is equivalent to $\lim_{t\to\infty} \left(\boldsymbol{\xi}_i(t) - \boldsymbol{\xi}_j(t)\right) = \boldsymbol{0}$ for any $i, j \in \mathcal{I}_N$, $i \neq j$.

Definition 2: Multi-agent system (1) is said to achieve the time-varying formation specified by $\boldsymbol{h}(t)$ if and only if for any given bounded initial states and $i \in \mathcal{I}_N$, there exists a vector-valued function $\boldsymbol{r}(t)= [\boldsymbol{r}_x(t), \boldsymbol{r}_v(t)]^T$ such that

$$\begin{cases} \lim\limits_{t\to\infty} (\boldsymbol{x}_i(t) - \boldsymbol{h}_{ix}(t) - \boldsymbol{r}_x(t)) = \boldsymbol{0} \\ \lim\limits_{t\to\infty} (\boldsymbol{v}_i(t) - \boldsymbol{h}_{iv}(t) - \boldsymbol{r}_v(t)) = \boldsymbol{0}, \end{cases} \tag{3}$$

where $\boldsymbol{r}(t)$ is the formation reference point.

On this foundation, the formation is said to achieve velocity tracking if $\lim\limits_{t\to\infty} \boldsymbol{r}_v(t) = \boldsymbol{0}$, that is $\lim\limits_{t\to\infty} (\boldsymbol{v}_i(t) - \boldsymbol{h}_i(t)) = \boldsymbol{0}$ for any $i \in \mathcal{I}_N$.

Remark 1: Although the agents will eventually achieve the desired time-varying formation shape relative to $\boldsymbol{r}(t)$, the reference point is unknown to all agents, which is different from traditional centralized formation control method. Furthermore, it can be easily verified that (3) is equivalent to $\lim\limits_{t\to\infty} \left\{\left(\boldsymbol{\xi}_i(t) - \boldsymbol{h}_i(t)\right) - \left(\boldsymbol{\xi}_j(t) - \boldsymbol{h}_j(t)\right)\right\} = \boldsymbol{0}$ for any i, $j \in \mathcal{I}_N$, $i \neq j$.

Remark 2: Note that $\boldsymbol{h}_i(t)$ is the desired time-varying formation rather than the reference trajectory for each agent to follow, that is $\boldsymbol{h}_i(t)$ depicts the relative offset vector of $\boldsymbol{\xi}_i(t)$. In the case $\boldsymbol{h}_{ix}(t) = \boldsymbol{h}_{iv}(t) = \boldsymbol{0}$, Definition 2 becomes a consensus problem for discrete-time multi-agent systems.

Due to the limitation of bandwidth, abundant data transmission and congestion of network links, communication delay often exists in practical communication networks. Assume that the communication delay only exists in the actually transmitted information and every agent can use its own instantaneous states information (Fig. 1(a)). The packet loss is also ignored for convenience in description and analysis. In discrete-time systems, agents will only update their controller at certain time instants, hence the received data cannot be used immediately. A data buffer is introduced to temporarily store the latest received data packets from neighbouring agents and these data will be used at the next controller update time instant. The communication delay from agent ϖ_j to ϖ_i is denoted by τ_{ij} and it is reasonable to set the value of $\tau_{ij} = \tau\delta$ in discrete-time systems, $\tau \in \{0, 1, \ldots, \hbar\}$, where $\hbar$ is the smallest integer greater than or equal to $\tau_{\max}/\delta$, $\tau_{\max}$ is the upper bound of communication delay. It should be noted that for discrete-time systems, there exists at least one sampling period delay when the states are exchanged among agents, as shown in Fig. 1(b).

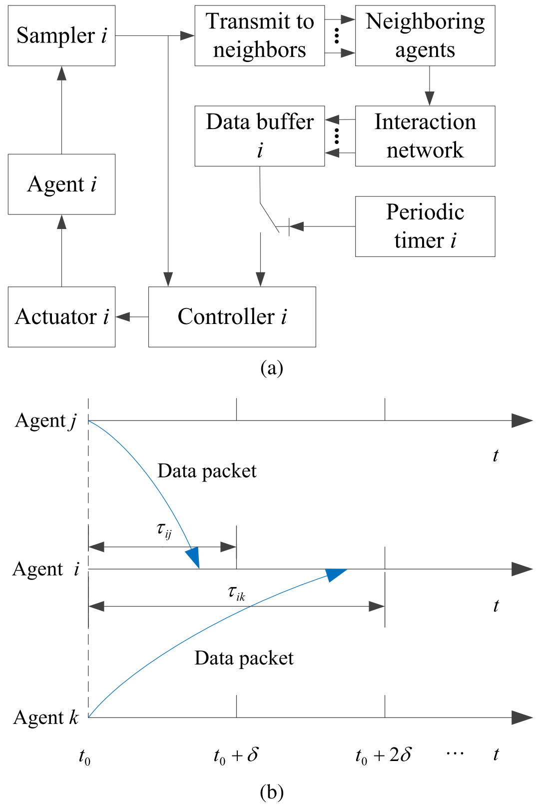

FIGURE 1. A distributed communication delay processing scheme and communication delay in discrete-time systems. (a) A distributed scheme to deal with the communication delay. (b) Communication delay in discrete-time systems.

To solve the time-varying formation control and velocity tracking problems with switching topologies and nonuniform communication delays, the control protocol using the instantaneous states information of agent itself and the delayed states information of its neighbours is designed as follows,

$$\begin{aligned} \boldsymbol{u}_i(t) = &-p_0 \left(\boldsymbol{v}_i(t) - \boldsymbol{h}_{iv}(t)\right) \\ &+ \boldsymbol{K}\sum_{j=1}^{N} a_{ij}(t)\left\{\left(\boldsymbol{\xi}_j(t-\tau_{ij}) - \boldsymbol{h}_j(t-\tau_{ij})\right)\right. \\ &\left. - \left(\boldsymbol{\xi}_i(t) - \boldsymbol{h}_i(t)\right)\right\} + \boldsymbol{h}_{ia}(t), \end{aligned} \tag{4}$$

where $\boldsymbol{K} = [p_1, p_2]$, $p_0 > 0$ is velocity tracking error damping gain, $p_1, p_2 > 0$ are formation control related gain parameters.

Remark 3: Equation (4) solves the problem (i) raised in the Introduction and the control protocol to achieve desired time-varying formation and reference velocity tracking is given. The design of parameters in the protocol will be further discussed in the next section.

III. TIME-VARYING FORMATION ANALYSIS AND CONTROL PROTOCOL DESIGN

In this section, we will address the time-varying formation control and velocity tracking problems for second-order discrete-time multi-agent systems with switching topologies and nonuniform communication delays.

Denote $\boldsymbol{E} = \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix}, \boldsymbol{F}_0 = \begin{bmatrix} 0 & 0 \\ 0 & -\delta p_0 \end{bmatrix}, \boldsymbol{F}_1 = \begin{bmatrix} \boldsymbol{0}^T \\ \delta \boldsymbol{K} \end{bmatrix}$. Under protocol (4), the closed-loop dynamics of multi-agent system (1) can be written in a compact form as

$$\begin{aligned} \boldsymbol{\xi}(t+\delta) &= (I_N \otimes \boldsymbol{E})\,\boldsymbol{\xi}(t) + (\boldsymbol{I}_N \otimes \boldsymbol{F}_0)\,(\boldsymbol{\xi}(t) - \boldsymbol{h}(t)) \\ &\quad - (\boldsymbol{L}_D(t) \otimes \boldsymbol{F}_1)\,(\boldsymbol{\xi}(t) - \boldsymbol{h}(t)) \\ &\quad + (\boldsymbol{A}_0(t) \otimes \boldsymbol{F}_1)\,(\boldsymbol{\xi}(t) - \boldsymbol{h}(t)) + \dots \\ &\quad + \left(\boldsymbol{A}_{\hbar}(t) \otimes \boldsymbol{F}_1\right)(\boldsymbol{\xi}(t-\tau_{\max}) - \boldsymbol{h}(t-\tau_{\max})) \\ &\quad + \left(\boldsymbol{I}_N \otimes [0, \delta]^T\right)\boldsymbol{h}_a(t), \end{aligned} \tag{5}$$

where $\boldsymbol{L}_D(t)$ is a diagonal matrix consists of the diagonal elements of matrix $\boldsymbol{L}(t)$, $\boldsymbol{A}_\tau(t) = [a_{\tau,ij}(t)]_{N\times N}$, element $a_{\tau,ij}(t) = a_{ij}(t)$ if $\tau_{ij} = \tau\delta$, otherwise $a_{\tau,ij}(t) = 0$, $\tau = 0, \dots, \hbar$. It is clear that $\boldsymbol{L}(t) = \boldsymbol{L}_D(t) - \sum_{\tau=0}^{\hbar} \boldsymbol{A}_\tau(t)$.

Let $\boldsymbol{\psi}_{ix}(t) = \boldsymbol{x}_i(t) - \boldsymbol{h}_{ix}(t)$, $\boldsymbol{\psi}_{iv}(t) = \boldsymbol{v}_i(t) - \boldsymbol{h}_{iv}(t)$, $\boldsymbol{\psi}_i(t) = [\boldsymbol{\psi}_{ix}(t), \boldsymbol{\psi}_{iv}(t)]^T$, $\boldsymbol{\psi}(t) = [\boldsymbol{\psi}_1^T(t), \dots, \boldsymbol{\psi}_N^T(t)]^T$, then one has

$$\boldsymbol{\psi}_i(t) = \boldsymbol{\xi}_i(t) - \boldsymbol{h}_i(t), \tag{6}$$

that is

$$\boldsymbol{\psi}(t) = \boldsymbol{\xi}(t) - \boldsymbol{h}(t). \tag{7}$$

Then it follows that

$$\boldsymbol{\xi}(t) = \boldsymbol{\psi}(t) + \boldsymbol{h}(t). \tag{8}$$

Substitute (8)into (5), it follows that

$$\begin{aligned} \boldsymbol{\psi}(t+\delta) &= (I_N \otimes \boldsymbol{E})\,\boldsymbol{\psi}(t) + (\boldsymbol{I}_N \otimes \boldsymbol{F}_0)\,\boldsymbol{\psi}(t) \\ &\quad - (\boldsymbol{L}_D(t) \otimes \boldsymbol{F}_1)\,\psi(t) + (\boldsymbol{A}_0(t) \otimes \boldsymbol{F}_1)\,\psi(t) \\ &\quad + \dots + \left(\boldsymbol{A}_{\hbar}(t) \otimes \boldsymbol{F}_1\right)(\boldsymbol{\psi}(t-\tau_{\max}) \\ &\quad - \boldsymbol{h}(t+\delta) + (\boldsymbol{I}_N \otimes \boldsymbol{E})\,\boldsymbol{h}(t) + \left(\boldsymbol{I}_N \otimes [0, \delta]^T\right)\boldsymbol{h}_a(t). \end{aligned} \tag{9}$$

Theorem 1: System (1) achieves the desired time-varying formation if and only if system (9) achieves consensus as $t \to \infty$, that is $\lim_{t\to\infty} \left(\boldsymbol{\psi}_i(t) - \boldsymbol{\psi}_j(t)\right) = \boldsymbol{0}$ for any $i, j \in \mathcal{I}_N$, $i \neq j$.

Proof: (Sufficiency) If system (9) achieves consensus, by substituting (6) into $\lim_{t\to\infty} \left(\boldsymbol{\psi}_i(t) - \boldsymbol{\psi}_j(t)\right) = \boldsymbol{0}$, it can be obtained that $\lim_{t\to\infty} \{(\boldsymbol{\xi}_i(t) - \boldsymbol{h}_i(t)) - (\boldsymbol{\xi}_j(t) - \boldsymbol{h}_j(t))\} = \boldsymbol{0}$ for any $i, j \in \mathcal{I}_N$, $i \neq j$. Then, based on Definition 2, one can find that the time-varying formation is achieved.

(Necessity) If the time-varying formation is achieved by system (1), that is $\lim_{t\to\infty} \{(\boldsymbol{\xi}_i(t) - \boldsymbol{h}_i(t)) - (\boldsymbol{\xi}_j(t) - \boldsymbol{h}_j(t))\} = \boldsymbol{0}$. Taking account of (6), it follows that $\lim_{t\to\infty} (\boldsymbol{\psi}_i(t) - \boldsymbol{\psi}_j(t)) = \boldsymbol{0}$ for any $i, j \in \mathcal{I}_N$, $i \neq j$, that is the consensus of (9) is achieved. Thus the conclusion is obtained. $\square$

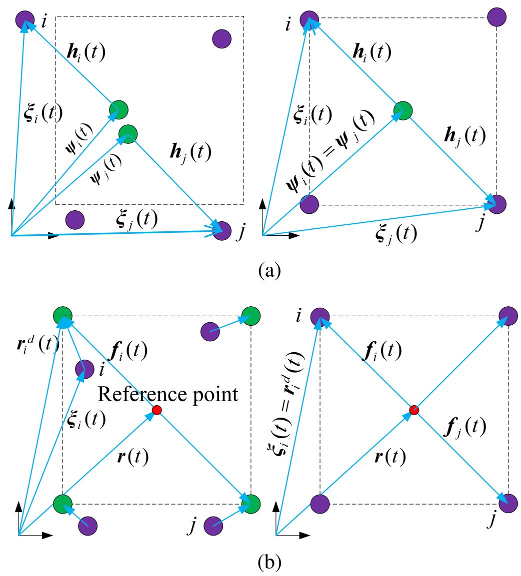

FIGURE 2. Illustration of the consensus-based and centralized formation control schemes. (a) The consensus-based formation control scheme. (b) Centralized formation control scheme.

Remark 4: Theorem 1 solves the problem (ii) raised in the Introduction and the time-varying formation control problem is transformed into a consensus problem. Fig.2(a) further illustrates the architecture of consensus-based formation control scheme. Under this scheme, each agent executes a simple motion and maintains desired offset relative to its respective reference point $\boldsymbol{\psi}_i(t) = \boldsymbol{\xi}_i(t) - \boldsymbol{h}_i(t)$. The condition for determining the multi-agent system has approached the desired formation is $\lim_{t\to\infty} \left(\boldsymbol{\psi}_i(t) - \boldsymbol{\psi}_j(t)\right) = \boldsymbol{0}$, that is the reference point of each agent has achieved consensus. From a geometric point of view, the formation is achieved if and only if the corresponding reference points of all agents coincide with each other. From Fig. 2(a), one can also find that the consensus-based formation control scheme depends only on the states of agent itself and the measurement of its neighbours' states. Fig.2(b) shows the architecture of traditional centralized formation control scheme, where $\boldsymbol{r}(t)$ and $\boldsymbol{\xi}_i(t)$ are formation reference point and the actual states of agent i, respectively. $\boldsymbol{f}_i(t)$ depicts the desired offset of agent i relative to $\boldsymbol{r}(t)$, then the desired states $\boldsymbol{r}_i^d(t)$ of agent i can be calculate by $\boldsymbol{r}_i^d(t) = \boldsymbol{r}(t) + \boldsymbol{f}_i(t)$. If every agent approaches its desired states accurately, that is $\boldsymbol{\xi}_i(t) = \boldsymbol{r}_i^d(t)$ for all $i \in \mathcal{I}_N$, then the desired formation is achieved accurately. This scheme relies on the assumption that each agent has the information of $\boldsymbol{r}(t)$, which is rather restrictive and may be unrealistic for many applications due to the limitations of communication bandwidth and range.

Because of the existing of communication delay and formation offset vector related items, it is difficult to analyse the stability of system (9) directly. Therefore, we first introduce the following proposition to simplify the problem.

Proposition 1: The predefined time-varying formation satisfies the following formation feasibility conditions for all $i \in \mathcal{I}_N$,

$$\begin{cases} \boldsymbol{h}_{ix}(t+\delta) = \boldsymbol{h}_{ix}(t) + \delta \boldsymbol{h}_{iv}(t) \\ \boldsymbol{h}_{iv}(t+\delta) = \boldsymbol{h}_{iv}(t) + \delta \boldsymbol{h}_{ia}(t) \end{cases}. \tag{10}$$

Remark 5: Proposition 1 gives answer to the problem (iii) raised in the Introduction. It proposes an explicit mathematical description of feasible time-varying formation set, which is not only important for deriving the conclusions of this paper, but also gives explicit criterions to design feasible formation in mission planning. Proposition 1 indicates that the desired formation has the same dynamics characteristic as system (1). The feasibility conditions are reasonable and intuitive, as it is obvious that a group of agents cannot accomplish all the formation due to their dynamics limitations. The formation that can be accomplished must meet some constraints such that the components of position, velocity and acceleration are compatible without any conflict or sharp change. Let $\delta \to 0$, (10) is equivalents to

$$\begin{cases} \dot{\boldsymbol{h}}_{ix}(t) = \boldsymbol{h}_{iv}(t) \\ \dot{\boldsymbol{h}}_{iv}(t) = \boldsymbol{h}_{ia}(t) \end{cases}, \tag{11}$$

which means that the formation offset functions are coordinated and second-order differentiable. In fact, (11) is similar to the formation feasibility constraints that given in [20] for continuous-time systems. On the other hand, (10) has the same form as system (1) and can be obtained by the forward difference approximation through (11).

Under the constraints of Proposition 1, the following theorem holds directly.

Theorem 2: Under Proposition 1, system (9) achieves consensus if and only if the consensus of system (12) is achieved as $t \to \infty$.

$$\begin{aligned} \boldsymbol{\psi}(t+\delta) &= (I_N \otimes \boldsymbol{E})\, \boldsymbol{\psi}(t) + (\boldsymbol{I}_N \otimes \boldsymbol{F}_0)\, \boldsymbol{\psi}(t) \\ &\quad - (\boldsymbol{L}_D(t) \otimes \boldsymbol{F}_1)\, \psi(t) + (\boldsymbol{A}_0(t) \otimes \boldsymbol{F}_1)\, \psi(t) \\ &\quad + \ldots + \left(\boldsymbol{A}_\hbar(t) \otimes \boldsymbol{F}_1\right)(\boldsymbol{\psi}(t - \tau_{\max}). \end{aligned} \tag{12}$$

This transformation simplifies the mathematical analysis by eliminating the formation offset vector related items with Proposition 1. However, due to the existence of nonuniform communication delays, it is still difficult to analyse the stability of system (12). In this kind of situation, we continue to perform a model transformation. It is easy to verify that (12) can be decomposed into (13) for any $i \in \mathcal{I}_N$,

$$\begin{aligned} \boldsymbol{\psi}_{ix}(t+\delta) &= \boldsymbol{\psi}_{ix}(t) + \delta \boldsymbol{\psi}_{iv}(t), \\ \boldsymbol{\psi}_{iv}(t+\delta) &= \boldsymbol{\psi}_{iv}(t) - p_0 \delta \boldsymbol{\psi}_{iv}(t) \\ &\quad + \delta \sum_{j \in N_i} a_{ij}(t) \left\{ p_1 \left(\boldsymbol{\psi}_{jx}(t-\tau_{ij}) - \boldsymbol{\psi}_{ix}(t) \right) \right. \\ &\quad \left. + p_2 \left(\boldsymbol{\psi}_{jv}(t-\tau_{ij}) - \boldsymbol{\psi}_{iv}(t) \right) \right\}. \end{aligned} \tag{13}$$

Let $\bar{\boldsymbol{\psi}}_{iv}(t) = \boldsymbol{\psi}_{ix}(t) + \gamma \boldsymbol{\psi}_{iv}(t)$, where $\gamma = p_2 / p_1$. Then one can obtain that

$$\begin{aligned} \boldsymbol{\psi}_{ix}(t+\delta) &= \boldsymbol{\psi}_{ix}(t) + \delta \frac{\left(\bar{\boldsymbol{\psi}}_{iv}(t) - \boldsymbol{\psi}_{ix}(t) \right)}{\gamma}, \\ \boldsymbol{\psi}_{iv}(t+\delta) &= \delta \frac{\bar{\boldsymbol{\psi}}_{iv}(t) - \bar{\boldsymbol{\psi}}_{ix}(t)}{\gamma} \\ &\quad + \bar{\boldsymbol{\psi}}_{iv}(t) - p_0 \delta \left(\bar{\boldsymbol{\psi}}_{iv}(t) - \boldsymbol{\psi}_{ix}(t) \right) \\ &\quad + p_2 \delta \sum_{j \in N_i} a_{ij}(t) \left(\bar{\boldsymbol{\psi}}_{jv}(t-\tau_{ij}) - \bar{\boldsymbol{\psi}}_{iv}(t) \right). \end{aligned} \tag{14}$$

Denote $\boldsymbol{\eta}_i(t) = [\boldsymbol{\psi}_{ix}(t), \bar{\boldsymbol{\psi}}_{iv}(t)]^T$, $\boldsymbol{\eta}(t) = [\boldsymbol{\eta}_1^T(t), \ldots, \boldsymbol{\eta}_N^T(t)]^T$, $\boldsymbol{\Lambda} = \begin{bmatrix} 1 - \delta/\gamma & \delta/\gamma \\ p_0\delta - \delta/\gamma & 1 + \delta/\gamma - p_0\delta \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & p_2\delta \end{bmatrix}$. Then (12) is turned into an equivalent system as follows

$$\begin{aligned} \boldsymbol{\eta}(t+\delta) &= \boldsymbol{\Gamma}(t)\boldsymbol{\eta}(t) + (\boldsymbol{A}_0(t) \otimes \boldsymbol{B})\, \boldsymbol{\eta}(t) \\ &\quad + \ldots + \left(\boldsymbol{A}_\hbar(t) \otimes \boldsymbol{B} \right) \boldsymbol{\eta}(t - \tau_{\max}), \end{aligned} \tag{15}$$

where $\boldsymbol{\Gamma}(t) = \boldsymbol{I}_N \otimes \boldsymbol{\Lambda} - \boldsymbol{L}_D(t) \otimes \boldsymbol{B}$.

By holding the position-related variable $\boldsymbol{\psi}_{ix}(t)$ and velocity-related variable $\boldsymbol{\psi}_{iv}(t)$ together as a new variable $\bar{\boldsymbol{\psi}}_{iv}(t)$, the transformed input term $p_2\delta \sum_{j \in N_i} a_{ij}(t)(\bar{\boldsymbol{\psi}}_{jv}(t-\tau_{ij}) - \bar{\boldsymbol{\psi}}_{iv}(t))$ only contains the variables $\bar{\boldsymbol{\psi}}_{iv}(t)$, $i \in \mathcal{I}_N$. By defining a new states variable $\boldsymbol{\varsigma}(t) = [\boldsymbol{\eta}^T(t), \boldsymbol{\eta}^T(t-1), \ldots, \boldsymbol{\eta}^T(t-\tau_{\max})]^T$, system (15) can be equivalently represented by the following augmented system

$$\boldsymbol{\varsigma}(t+\delta) = \boldsymbol{\Xi}(t)\boldsymbol{\varsigma}(t), \tag{16}$$

where $\boldsymbol{\Xi}_{\sigma(t)}$ is given as (17), as shown at the bottom of this page.

It has been known that $\boldsymbol{L}(t) = \boldsymbol{L}_D(t) - \sum_{\tau=0}^{\hbar} \boldsymbol{A}_\tau(t)$, and $\boldsymbol{L}(t)\mathbf{1} = \mathbf{0}$, then it can be verified that $\boldsymbol{\Xi}(t)\mathbf{1} = \mathbf{1}$ by

$$\boldsymbol{\Xi}(t) = \begin{bmatrix} \boldsymbol{\Gamma}(t) + \boldsymbol{A}_0(t) \otimes \boldsymbol{B} & \boldsymbol{A}_1(t) \otimes \boldsymbol{B} & \ldots & \boldsymbol{A}_{\hbar-1}(t) \otimes \boldsymbol{B} & \boldsymbol{A}_\hbar(t) \otimes \boldsymbol{B} \\ \boldsymbol{I} & \mathbf{0} & \ldots & \ldots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \mathbf{0} & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \boldsymbol{I} & \mathbf{0} \end{bmatrix} \tag{17}$$

substitute $\boldsymbol{\Lambda}$ and $\boldsymbol{B}$ into (17). In order to introduce the main result of this subsection, we need the following proposition.

Proposition 2: The gain parameters p_0, p_1, p_2 and sampling period δ satisfy the following constraints

$$\begin{cases} 1-\delta\gamma\gamma > 0 \\ 1+\delta\gamma\gamma - p_0\delta > p_2\delta d_{\max} \\ p_0 - 1\gamma\gamma > 0, \end{cases} \tag{18}$$

where $d_{\max}$ denotes the largest diagonal element of all possible $\boldsymbol{L}(t)$.

Remark 6: Proposition 2 solves the problem (iv) raised in the Introduction. Coupling constraints on the gain parameters and interaction period are proposed, so as to guide the design of parameters in the protocol.

Lemma 2: If all the possible interaction topologies have spanning trees and the gain parameters p_0, p_1, p_2 and sampling period δ satisfy Proposition 2, then $G(\boldsymbol{\Xi}(t))$ also has a spanning tree. Furthermore, $\boldsymbol{\Xi}(t)$ is SIA.

Proof: It is obvious that all elements of $\boldsymbol{\Xi}(t)$ are non-negative under Proposition 2. Also note that $\boldsymbol{\Xi}(t)\mathbf{1} = \mathbf{1}$, then $\boldsymbol{\Xi}(t)$ is a stochastic matrix. Now, we are going to prove the graph associate with $\boldsymbol{\Xi}(t)$ has a spanning tree. Let $\boldsymbol{\Theta}(t) = \boldsymbol{\Gamma}(t) + \sum_{\tau=0}^{\hbar} \boldsymbol{A}_\tau(t) \otimes \boldsymbol{B}$. Denote the vertices in $G(\boldsymbol{\Theta}(t))$ as $\upsilon_1, \ldots, \upsilon_{2N}$ and the vertices in $G(\boldsymbol{\Xi}(t))$ as $\omega_1, \ldots, \omega_{2N(\hbar+1)}$. Since $\boldsymbol{\Gamma}(t) \geq 0$ and based on the assumption that all the possible interaction topologies have spanning trees (that is $G(\sum_{\tau=0}^{\hbar} \boldsymbol{A}^\tau(t))$ has spanning trees) and $\boldsymbol{B}_{22} > 0$, it can be obtained that the there exists a spanning tree among vertices $\upsilon_2, \upsilon_4 \ldots, \upsilon_{2(N-1)}, \upsilon_{2N}$. Further consider the form of $\boldsymbol{\Gamma}(t)$ and Proposition 2, it can be derived that vertices υ_{2i} and υ_{2i-1} are strongly connected since $\boldsymbol{\Gamma}_{(2i-1),2i}(t) > 0$ and $\boldsymbol{\Gamma}_{2i,(2i-1)}(t) > 0$, thus the graph $G(\boldsymbol{\Theta}(t))$ has a panning tree.

If their exists a directed edge (υ_i, υ_j) in $G(\boldsymbol{\Theta}(t))$, then there exists an edge $(\omega_{i+2Nl_{ij}}, \omega_j)$ in $G(\boldsymbol{\Xi}(t))$, where $l_{ij} = \tau_{ij}/\delta$ denotes the number of periods of communication delay from υ_i to υ_j. Recall the fact that $G(\boldsymbol{\Theta}(t))$ has a spanning tree and denote the root vertex as υ_i, then there must exist directed paths $(\upsilon_i, \upsilon_{j_1}), (\upsilon_{j_1}, \upsilon_{j_2}), \ldots, (\upsilon_{j_m}, \upsilon_j)$ for any $j \in \{1, \ldots, 2N\}, j \neq i$, thus there must exist directed edges $(\omega_{i+2Nl_{ij_1}}, \omega_{j_1}), (\omega_{j_1+2Nl_{ij_2}}, \omega_{j_2}), \ldots, (\omega_{j_m+2Nl_{j_mj}}, \omega_j)$. It is not difficult to see that there exist directed paths $(\omega_i, \omega_{i+2N}), (\omega_{i+2N}, \omega_{i+4N}), \ldots, (\omega_{i+2N(\hbar-1)}, \omega_{i+2N\hbar})$ for any $i \in \{1, .., 2N\}$. Therefore, $G(\boldsymbol{\Xi}(t))$ has a spanning tree rooted at ω_i. Furthermore, for any $i \in \{1, .., 2N\}$, it can be obtained that the diagonal element $\boldsymbol{\Xi}_{ii}(t) > 0$, that is the root vertex ω_i has self-loop. Then based on Lemma 1, $\boldsymbol{\Xi}(t)$ is SIA. □

Lemma 3: Let $G(t) \in S$ be the interaction graph at time t. If the union of the graphs $G(t_1), \ldots, G(t_2)$ for finite discrete time instants $t_1, \ldots, t_2, (t_2 > t_1)$ has a spanning tree and parameters p_0, p_1, p_2, δ satisfy Proposition 2, then $\prod_{t=t_1}^{t_2} \boldsymbol{\Xi}(t)$ is SIA.

Proof: Let $\boldsymbol{\Theta}(t) = \boldsymbol{\Gamma}(t) + \sum_{\tau=0}^{\hbar} \boldsymbol{A}_\tau(t) \otimes \boldsymbol{B}$. We are going to prove the graph $G(\sum_{t=t_1}^{t_2} \boldsymbol{\Theta}(t))$ has a spanning tree. Denote the vertices of $G(\sum_{t=t_1}^{t_2} \boldsymbol{\Theta}(t))$ as $\upsilon_1, \ldots, \upsilon_{2N}$. Since $\boldsymbol{\Gamma}(t) \geq 0$, based on the assumption that the union of graphs $G(t_1), \ldots, G(t_2)$ has a spanning tree (that is $G(\sum_{t=t_1}^{t_2} \sum_{\tau=0}^{\hbar} \boldsymbol{A}_\tau(t))$ has a spanning tree) and $\boldsymbol{B}_{22} > 0$, it can be obtained that the there exists a spanning tree among vertices $\upsilon_2, \upsilon_4, \ldots, \upsilon_{2(N-1)}, \upsilon_{2N}$. Further consider the form of $\boldsymbol{\Gamma}(t)$ and Proposition 2, it can be derived that $\boldsymbol{\Gamma}_{(2i-1),2i}(t) > 0$ and $\boldsymbol{\Gamma}_{2i,(2i-1)}(t) > 0$, that is the $2i^{th}$ and $(2i-1)^{th}$ vertices of graph $G(\sum_{t=t_1}^{t_2} \boldsymbol{\Theta}(t))$ are strongly connected, thus the graph $G(\sum_{t=t_1}^{t_2} \boldsymbol{\Theta}(t))$ has a panning tree. Furthermore, if parameters p_0, p_1, p_2, δ satisfy Proposition 2, there must exist positive constant μ such that $\sum_{t=t_1}^{t_2} \left(\boldsymbol{\Gamma}(t) + \boldsymbol{A}^0(t) \otimes \boldsymbol{B}\right) \geq \boldsymbol{\Gamma}(t) \geq \mu \boldsymbol{I}$. Then based on lemma 7 in [40], this lemma can be proved. □

Theorem 3: Under the Proposition 2, if all the possible interaction topologies $G(t) \in S$ have spanning trees, then system (1) achieves time-varying formation and velocity tracking as $t \to \infty$ with protocol (4).

Proof: By Lemma 2 and the definition of SIA matrix, it follows that

$$\prod_{t=0}^{+\infty} \boldsymbol{\Xi}(t) = \mathbf{1}\boldsymbol{c}^T, \tag{19}$$

where $\boldsymbol{c} \in R^{2N(\hbar+1)}$ is a constant vector. On the basis of (16), it can be obtained that

$$\lim_{t\to\infty} \boldsymbol{\varsigma}(t+\delta) = \prod_{t=0}^{+\infty} \boldsymbol{\Xi}(t)\boldsymbol{\varsigma}(0) = \mathbf{1}\boldsymbol{c}^T \boldsymbol{\varsigma}(0), \tag{20}$$

that is

$$\lim_{t\to\infty} \boldsymbol{\psi}_{ix}(t) = \lim_{t\to\infty} \bar{\boldsymbol{\psi}}_{iv}(t) = \boldsymbol{c}^T \boldsymbol{\varsigma}(0), (i \in \mathcal{I}_N). \tag{21}$$

Further considering the fact $\bar{\boldsymbol{\psi}}_{iv}(t) = \boldsymbol{\psi}_{ix}(t) + \gamma \boldsymbol{\psi}_{iv}(t)$, it can be obtained that $\lim_{t\to\infty} \boldsymbol{\psi}_{iv}(t) = \lim_{t\to\infty} \boldsymbol{\psi}_{jv}(t) = \mathbf{0}$ for all $i, j \in \mathcal{I}_N$. That is the consensus of system (12) is achieved and thus the time-varying formation is accomplished based on Theorem 1 and Theorem 2. When the consensus of system (12) is achieved, it follows that $\lim_{t\to\infty} \boldsymbol{\psi}_{iv}(t) = \lim_{t\to\infty} (\boldsymbol{v}_i(t) - \boldsymbol{h}_{iv}(t)) = \mathbf{0}$ $(i \in \mathcal{I}_N)$, that is the velocity tracking is also achieved. □

Theorem 4: Under the Proposition 2, if there exists an infinite sequence of bounded time interval $t_0, t_1, \ldots, t_k, \ldots$ with $0 < t_{k+1} - t_k \leq \bar{T}$, $k \in \mathbb{N}$, such that the union of graphs $G(t_k), \ldots, G(t_{k+1} - 1)$ has a spanning tree for any $k \in \mathbb{N}$, then system (1) achieves time-varying formation and velocity tracking as $t \to \infty$ with protocol (4).

Proof: For any $t > 0$, denote t_k the largest integer satisfies $t_k \leq t$. Then it follows that

$$\boldsymbol{\varsigma}(t+\delta) = \boldsymbol{\Xi}(t)\boldsymbol{\Xi}(t-1\ldots)\boldsymbol{\Xi}(t_k) \prod_{m=0}^{k-1} \boldsymbol{\Xi}(m)\boldsymbol{\varsigma}(0), \tag{22}$$

where $\boldsymbol{\Xi}(m) = \prod_{t=t_m}^{t_{m+1}-1} \boldsymbol{\Xi}(t)$. Since $0 < t_{m+1} - t_m \leq \bar{T}$ and the union of graphs $G(t_m), \ldots, G(t_{m+1} - 1)$ has a spanning tree, $\boldsymbol{\Xi}(m)$ is SIA based on Lemma 3, that is $\prod_{k=0}^{+\infty} \boldsymbol{\Xi}(m) = \mathbf{1}\boldsymbol{c}^T$. Note that $\boldsymbol{\Xi}(t)\mathbf{1} = \mathbf{1}$, then $\lim_{t\to\infty} \boldsymbol{\varsigma}(t+\delta) = \mathbf{1}\boldsymbol{c}^T$, thus the conclusion can be obtained similar to the proof of Theorem 3. □

Remark 7: The above theorems give us sufficient conditions for achieving formation and velocity tracking of system (1). Theorem 3 shows that system (1) can achieve time-varying formation and velocity tracking under the condition that all the possible topologies have spanning trees. Theorem 4 indicates that even though the interaction topology between agents is dynamically changing and the corresponding directed graphs may not have spanning trees all the time, the desired formation and velocity tracking can still be achieved if the union graph has a spanning tree.

IV. SIMULATION STUDY

In this section, two numerical simulations, namely Example 1 and Example 2, are presented to illustrate the effectiveness and validity of Theorem 3 and Theorem 4, respectively. In both of the examples, the multi-agent system consists of eight agents and the dynamics of each agent is described by system (1). The collections of all the possible topologies in Example 1 and Example 2 are respectively denoted as S_1 and S_2, and each contains four graphs, as shown in Fig. 3 and Fig. 4. It can be seen that all the graphs in S_1 have spanning trees. Although each individual graph in S_2 is disconnected, the union of the four graphs has a spanning tree. Fig. 5 shows the time-dependent switching topologies index $\sigma(t)$ and it changes randomly every 5 seconds. These eight agents are supposed to perform a combination of circular formation and translational motion, as shown in Fig. 6. On the basis of Proposition 1, the desired time-varying formation for agent i is described as follows

$$\boldsymbol{h}_{ix}(0) = \left[10\cos(\frac{2\pi(i-1)}{8}), 10\sin(\frac{2\pi(i-1)}{8})\right]^T,$$
$$\boldsymbol{h}_{iv}(t) = \left[-\frac{\pi}{3}\sin(\varphi_i), \frac{\pi}{3}\cos(\varphi_i)\right]^T + \boldsymbol{v}_d(t),$$

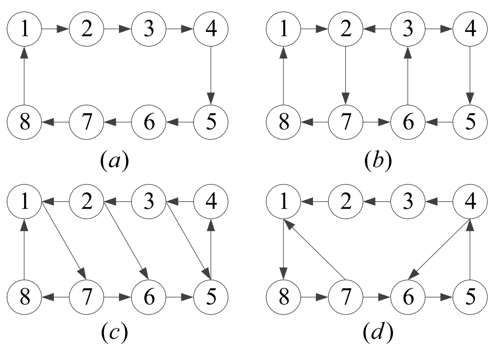

FIGURE 3. Interaction topologies in S_1.

where $\varphi_i = \frac{2\pi t}{60} + \frac{2\pi(i-1)}{8}$, $\boldsymbol{v}_d = \left[1, \frac{\pi}{3}\cos\left(\frac{2\pi t}{120}\right)\right]^T$, thus $\boldsymbol{h}_{ix}(t)$ and $\boldsymbol{h}_{ia}(t)$ for all $t > 0$ and $i \in \{1, \ldots, 8\}$ can be calculated by simple iteration. If the time-varying formation specified by $\boldsymbol{h}_i(t)$ is achieved, the eight agents will locate

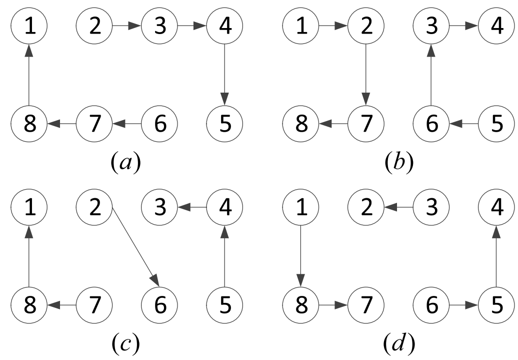

FIGURE 4. Interaction topologies in S_2.

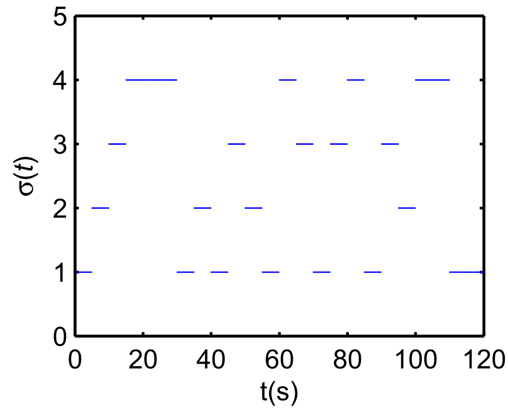

FIGURE 5. Time-dependent switching topologies index.

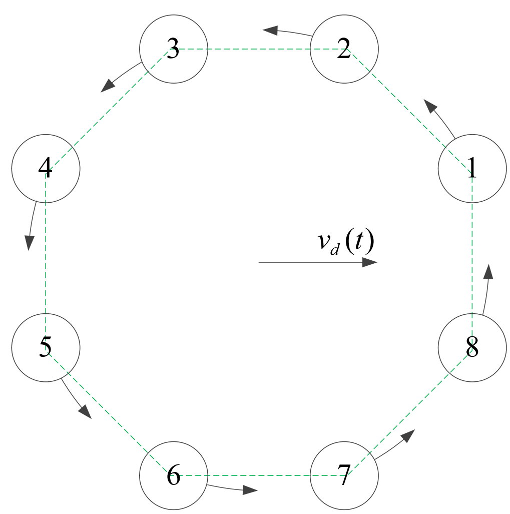

FIGURE 6. Desired formation.

on the vertices of an octagon respectively and keep rotation with a period of 60s. At the same time, the formation will also perform a translational motion as a whole with desired time-varying velocity $\boldsymbol{v}_d(t)$. For convenience in description, the initial positions and velocities of all the agents are set as

$$\boldsymbol{\xi}_i(t) = \left[12\cos(\varphi_i), 12\sin(\varphi_i), -\frac{2\pi}{5}\sin(\varphi_i), \frac{2\pi}{5}\cos(\varphi_i)\right]^T,$$

where $t \in [t_0 - \hbar\delta, t_0]$.

Let $t_0 = \hbar\delta, p_0 = 0.25, p_1 = 0.20, p_2 = 1.0$ and $\delta = 0.1$ s, after some calculation, we know that these parameters satisfy Proposition 2. Simulations were carried out with $\tau_{\max} = 0.5$ s and the nonuniform communication delays are set as

$$[\tau_{ij}] = 0.1 * \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 4 & 3 \\ 2 & 0 & 1 & 2 & 3 & 4 & 5 & 4 \\ 3 & 2 & 0 & 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 0 & 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 & 0 & 1 & 2 & 3 \\ 4 & 5 & 4 & 3 & 2 & 0 & 1 & 2 \\ 3 & 4 & 5 & 4 & 3 & 2 & 0 & 1 \\ 2 & 3 & 4 & 5 & 4 & 3 & 2 & 0 \end{bmatrix}.$$

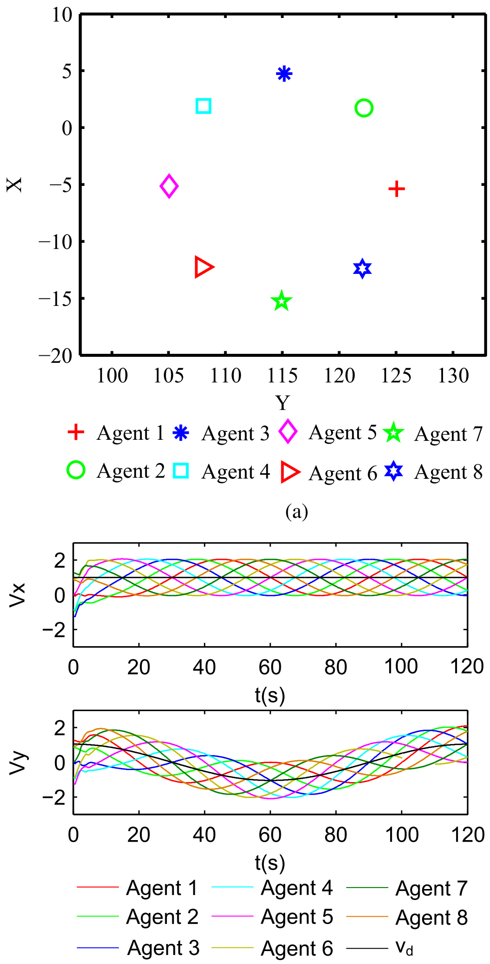

FIGURE 7. **Simulation results of Example 1. (a) Snapshot of the formation at $t = 120$ s. (b) Velocities of agents.**

The snapshots of the formation at $t = 120$ s and velocities of all the agents in Example 1 and Example 2 are shown in Fig. 7 and Fig. 8, respectively. Fig. 9 depicts the curves of the total formation error ($F\_error(t) = \sum_{i=2}^{8} \left((\boldsymbol{\psi}_i(t) - \boldsymbol{\psi}_1(t))^T(\boldsymbol{\psi}_i(t) - \boldsymbol{\psi}_1(t))\right)$) and velocity tracking error ($Vel\_error(t) = \sum_{i=1}^{8} \left(\boldsymbol{\psi}_{iv}^T(t)\boldsymbol{\psi}_{iv}(t)\right)$) in Example 1 and Example 2. From Figs. 7-9, one sees that both

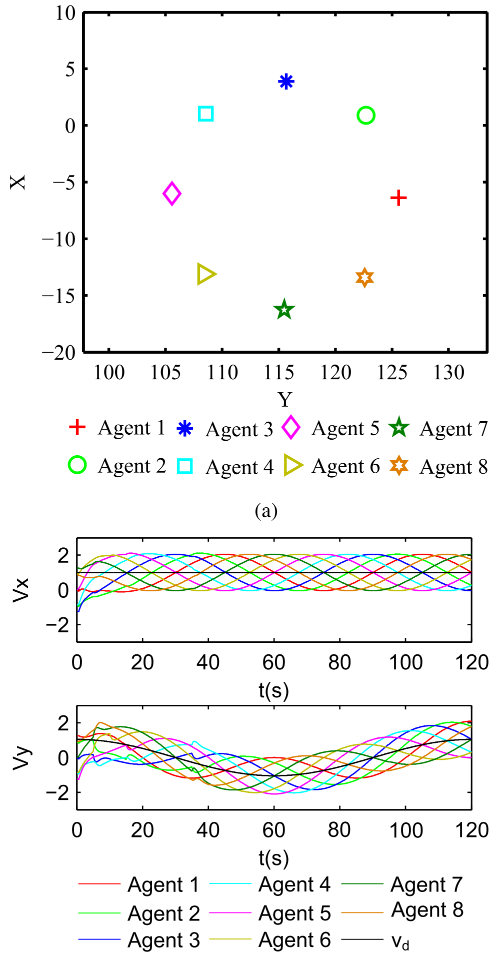

FIGURE 8. **Simulation results of Example 2. (a) Snapshot of the formation at $t = 120$ s. (b) Velocities of agents.**

the time-varying formation and velocity tracking are accomplished under the influence of nonuniform communication delays and switching directed topologies using the protocol proposed in the current paper. Through comparison and analysis of Fig. 7(b), Fig. 8(b) and Fig. 9, we can find that the formation is accomplished within about 20 s and 50 s, respectively. This indicates that although the connectivity of the interaction topologies may affect the convergence rate of the system, the multi-agent system can achieve the desired time-varying formation as long as the union of the interaction graphs has a spanning tree. Further analysis of Fig. 7(b), Fig. 8(b) and Fig. 9(b) shows that the whole formation can track desired time-varying translational velocity. It is also important to note that the translational velocity is that of the coincident reference points of agents, the absolute velocity of each agent is the combination of the velocity components of the desired formation and desired translational velocity, as shown in Fig. 7(b) and Fig. 8(b).

Compared with existing technology, we include the effects of velocity error damping gain in the protocol and derive milder conditions which allow for not only bounded

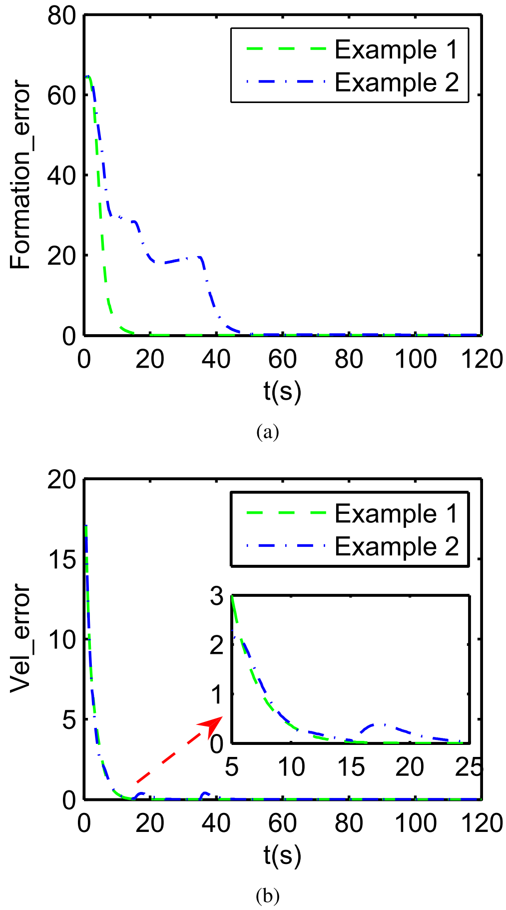

FIGURE 9. Curves of the total formation error and velocity tracking error. (a) Formation error. (b) Velocity tracking error.

nonuniform communication delays but also for dynamically switching directed graphs that may not have a spanning tree.

V. CONCLUSIONS

Time-varying formation control and velocity tracking problems for second-order discrete-time multi-agent systems with directed topology and communication delay are investigated, where the interaction topology can be switching and communication delays are nonuniform. A local information based distributed control protocol is proposed by utilizing the delayed states of neighbours and instantaneous states of agent itself. An explicit mathematical description of the feasible time-varying formation set is given. Constraints on the gain parameters and sampling period are proposed as guidance to the design of parameters in the protocol. Numerical simulations show that nonuniform communication delays can be safely tolerated, even though the interaction topologies between agents are dynamically switching over time and the corresponding directed graphs may not have spanning trees.

There are still a number of issues need to be further investigated and extensions to heterogeneous multi-agent systems with asynchronous updates are currently under investigation. Another thing needs to be discussed in the future is that other constraints such as measurement error, external disturbance and input saturation should be taken into account.

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REFERENCES

[1] R. R. Nair, L. Behera, V. Kumar, and M. Jamshidi, "Multisatellite formation control for remote sensing applications using artificial potential field and adaptive fuzzy sliding mode control," *IEEE Syst. J.*, vol. 9, no. 2, pp. 508–518, Jun. 2015.

[2] L. He, P. Bai, X. Liang, J. Zhang, and W. Wang, "Feedback formation control of UAV swarm with multiple implicit leaders," *Aerosp. Sci. Technol.*, vol. 72, pp. 327–334, Jan. 2018.

[3] K. K. Oh, M. C. Park, and H. S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, Mar. 2015.

[4] R. Tron, J. Thomas, G. Loianno, K. Daniilidis, and V. Kumar, "A distributed optimization framework for localization and formation control: Applications to vision-based measurements," *IEEE Control Syst.*, vol. 36, no. 4, pp. 22–44, Aug. 2016.

[5] J.-H. Wang, Y.-L. Xu, J. Zhang, and D.-D. Yang, "Time-varying formation for general linear multi-agent systems via distributed event-triggered control under switching topologies," *Chin. Phys. B*, vol. 27, no. 4, 2018, Art. no. 040504.

[6] L. Ding, Q.-L. Han, X. Ge, and X.-M. Zhang, "An overview of recent advances in event-triggered consensus of multiagent systems," *IEEE Trans. Cybern.*, vol. 48, no. 4, pp. 1110–1123, Apr. 2018.

[7] X. Ge, Q.-L. Han, D. Ding, X.-M. Zhang, and B. Ning, "A survey on recent advances in distributed sampled-data cooperative control of multi-agent systems," *Neurocomputing*, vol. 275, pp. 1684–1701, Jan. 2017.

[8] B. Wang, W. Chen, J. Wang, B. Zhang, Z. Zhang, and X. Qiu, "Accurate cooperative control for multiple leaders multiagent uncertain systems: A two-layer node-to-node communication framework," *IEEE Trans. Ind. Informat.*, vol. 14, no. 6, pp. 2395–2405, Jun. 2017.

[9] P. Lin and Y. Jia, "Consensus of second-order discrete-time multi-agent systems with nonuniform time-delays and dynamically changing topologies," *Automatica*, vol. 45, no. 9, pp. 2154–2158, 2009.

[10] W. Ren, "Consensus strategies for cooperative control of vehicle formations," *IET Control Theory Appl.*, vol. 1, no. 2, pp. 505–512, Mar. 2007.

[11] Y. Dong and X. Hu, "Distributed control of periodic formations for multiple under-actuated autonomous vehicles," *IET Control Theory Appl.*, vol. 11, no. 1, pp. 66–72, Jun. 2017.

[12] M. I. El-Hawwary, "Three-dimensional circular formations via set stabilization," *Automatica*, vol. 54, no. 2, pp. 374–381, Apr. 2015.

[13] Y. Kim and A. Mahmood, "Collision-free second-order vehicle formation control under time-varying network topology," *J. Franklin Inst.*, vol. 352, no. 10, pp. 4595–4609, Oct. 2015.

[14] M. Guo, M. M. Zavlanos, and D. V. Dimarogonas, "Controlling the relative agent motion in multi-agent formation stabilization," *IEEE Trans. Autom. Control*, vol. 59, no. 3, pp. 820–826, Mar. 2014.

[15] H. Du, S. Li, and X. Lin, "Finite-time formation control of multiagent systems via dynamic output feedback," *Int. J. Robust Nonlinear Control*, vol. 23, no. 14, pp. 1609–1628, Jun. 2012.

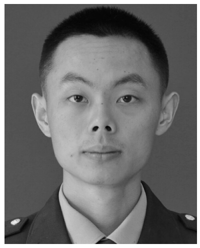
[16] Z. Li, W. Ren, X. Liu, and M. Fu, "Distributed containment control of multi-agent systems with general linear dynamics in the presence of multiple leaders," *Int. J. Robust Nonlinear Control*, vol. 23, no. 5, pp. 534–547, 2013.

[17] R. Rahimi, F. Abdollahi, and K. Naqshi, "Time-varying formation control of a collaborative heterogeneous multi agent system," *Robot. Auton. Syst.*, vol. 62, no. 12, pp. 1799–1805, Dec. 2014.

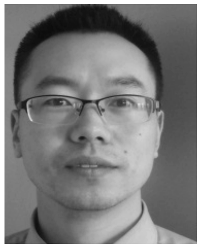
[18] G. Antonelli, F. Arrichiello, F. Caccavale, and A. Marino, "Decentralized time-varying formation control for multi-robot systems," *Int. J. Robot. Res.*, vol. 33, no. 7, pp. 1029–1043, 2014.

[19] L. Briñón-Arranz, A. Seuret, and C. Canudas-de-Wit, "Cooperative control design for time-varying formations of multi-agent systems," *IEEE Trans. Autom. Control*, vol. 59, no. 8, pp. 2283–2288, Aug. 2014.

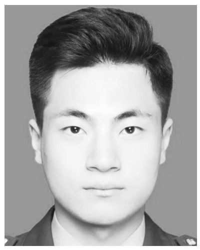
[20] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, "Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application to quadrotor formation flying," *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 5014–5024, Jun. 2017.
[21] Y. Zhao, Q. Duan, G. Wen, D. Zhang, and B. Wang, "Time-varying formation for general linear multiagent systems over directed topologies: A fully distributed adaptive technique," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published.
[22] M.-F. Ge, Z.-H. Guan, C. Yang, T. Li, and Y.-W. Wang, "Time-varying formation tracking of multiple manipulators via distributed finite-time control," *Neurocomputing*, vol. 202, pp. 20–26, Aug. 2016.
[23] L. He, X. Sun, and Y. Lin, "Distributed adaptive control for time-varying formation tracking of a class of networked nonlinear systems," *Int. J. Control*, vol. 90, no. 7, pp. 1319–1326, Jul. 2017.
[24] L. Wang, J. Xi, M. Yuan, and G. Liu, "Guaranteed-performance time-varying formation control for swarm systems subjected to communication constraints," *IEEE Access*, vol. 6, pp. 45384–45393, 2018.
[25] L. Han, X. W. Dong, Q. Li, and Z. Ren, "Formation tracking control for time-delayed multi-agent systems with second-order dynamics," *Chin. J. Aeronaut.*, vol. 30, no. 1, pp. 348–357, Feb. 2017.
[26] H. Xia, T.-Z. Huang, J.-L. Shao, and J.-Y. Yu, "Leader-following formation control for second-order multi-agent systems with time-varying delays," *Trans. Inst. Meas. Control*, vol. 36, no. 5, pp. 627–636, Jul. 2014.
[27] R. Cepeda-Gomez and N. Olgac, "Stability of formation control using a consensus protocol under directed communications with two time delays and delay scheduling," *Int. J. Syst. Sci.*, vol. 47, no. 2, pp. 433–449, Mar. 2016.
[28] D. Xue, J. Yao, J. Wang, Y. Guo, and X. Han, "Formation control of multi-agent systems with stochastic switching topology and time-varying communication delays," *IET Control Theory Appl.*, vol. 7, no. 13, pp. 1689–1698, Sep. 2013.
[29] T. Li, Z. Li, H. Zhang, and S. Fei, "Formation tracking control of second-order multi-agent systems with time-varying delay," *J. Dyn. Syst. Meas. Control*, vol. 140, no. 11, pp. 111015-1–111015-11, 2018.
[30] J. Hu, J. Yu, and J. Cao, "Distributed containment control for nonlinear multi-agent systems with time-delayed protocol," *Asian J. Control*, vol. 18, no. 2, pp. 747–756, Mar. 2016.
[31] L. Peng, F. Guan, L. Perneel, H. Fayyad-Kazan, and M. Timmerman, "Decentralized multi-robot formation control with communication delay and asynchronous clock," *J. Intell. Robot. Syst.*, vol. 89, nos. 3–4, pp. 465–484, Mar. 2018.
[32] Y. Liu and Y. Jia, "Formation control of discrete-time multi-agent systems by iterative learning approach," *Int. J. Control, Automat. Syst.*, vol. 10, no. 5, pp. 913–919, Oct. 2012.
[33] S. Li, J. Zhang, X. Li, F. Wang, X. Luo, and X. Guan, "Formation control of heterogeneous discrete-time nonlinear multi-agent systems with uncertainties," *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 4730–4740, Jun. 2017.
[34] M. E. Nakai, R. S. Inoue, M. H. Terra, and V. Grassi, Jr., "Robust discrete-time Markovian control for wheeled mobile robot formation: A fault tolerant approach," *J. Intell. Robot. Syst.*, vol. 91, no. 2, pp. 233–247, Aug. 2018.
[35] W. Zhang, J. Liu, and H. Wang, "Ultra-fast formation control of high-order discrete-time multi-agent systems based on multi-step predictive mechanism," *ISA Trans.*, vol. 58, pp. 165–172, Sep. 2015.
[36] J. Xu, G. Zhang, J. Zeng, and W. Tang, "Consensus based second order discrete-time multi-agent systems formation control with time-delays," in *Proc. IEEE Int. Conf. Inf. Automat.*, Aug. 2015, pp. 2626–2631.
[37] G. Xu, C. Huang, and G. Zhai, "A necessary and sufficient condition for designing formation of discrete-time multi-agent systems with delay," *Neurocomputing*, vol. 315, pp. 48–58, Nov. 2018.
[38] L. Shi, J. Shao, W. X. Zheng, and T.-Z. Huang, "Asynchronous containment control for discrete-time second-order multi-agent systems with time-varying delays," *J. Franklin Inst.*, vol. 354, no. 18, pp. 8552–8569, Dec. 2017.
[39] L. He, J. Zhang, Y. Hou, X. Liang, and P. Bai, "Time-varying formation control for second-order discrete-time multi-agent systems with directed topology and communication delay," *IEEE Access*, vol. 7, pp. 33517–33527, 2019.
[40] F. Xiao and L. Wang, "State consensus for multi-agent systems with switching topologies and time-varying delays," *Int. J. Control*, vol. 79, no. 10, pp. 1277–1284, 2006.
[41] J. Qin, H. Gao, and X. Z. Wei, "Consensus strategy for a class of multi-agents with discrete second-order dynamics," *Int. J. Robust Nonlinear Control*, vol. 22, no. 4, pp. 437–452, Mar. 2012.



LYULONG HE was born in 1990. He received the B.S. degree from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2013, and the M.S. degree from Air Force Engineering University, Xi'an, China, in 2015, where he is currently pursuing the Ph.D. degree. He is also an Assistant Engineer with the Shaanxi Province Laboratory of Meta-Synthesis for Electronic and Information Systems. His current research interest includes consensus and formation control of multi-agent systems with applications to aircraft swarm formation flying.



JIAQIANG ZHANG was born in 1984. He received the M.S. degree in weapon system and application engineering and the Ph.D. degree in armament science and technology from the School of Aeronautics and Astronautics Engineering, Air Force Engineering University, in 2009 and 2012, respectively, where he is currently a Lecturer with the School of Air Traffic Control and Navigation College. He has published over 20 journal papers and has finished over ten projects. His research interests include aviation cluster technology and airspace management intelligence.



YUEQI HOU was born in 1995. He received the B.S. degree with the School of Air Traffic Control and Navigation College, Air Force Engineering University, in 2017, where he is currently pursuing the M.S. degree in control science and engineering. His research interests are intelligent air combat of aircraft swarm and electronic warfare applications.

XIAOLONG LIANG was born in 1981. He received the master's degree in operational research and cybernetics, and the Ph.D. degree in armament science and technology from the Air Force Engineering University, where he is currently a Ph.D. Supervisor and a Professor with the School of Air Traffic Control and Navigation College. He has published over 50 journal papers and finished over 20 projects. He is a major of several national scientific research projects. His research interests are aircraft swarm technology, airspace management intelligence, and intelligent aviation systems.

PENG BAI was born in 1961. He received the bachelor's degree in radar engineering from the School of Aeronautics and Astronautics Engineering, Air Force Engineering University, in 1983, and the master's degree in information and communication engineering from the School of Northwestern Polytechnical University, in 1989. He is currently the Chief Scientist of the Shaanxi Province Laboratory of Meta-Synthesis for Electronic and Information System. He has published over 120 journal papers as the major author; among them, 30 articles were retrieved by SCI and 37 by EI. He is the Chief Expert and the main person responsible for a number of national key scientific research projects. His current research interests include advanced electronic science and technology in the future, and science and technology of network information system in the future. He received the National Science and Technology Progress Award several times. He was commended by the President of China.