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Modeling the Enveloping Macroscopic Fundamental Diagram Based on the Traffic Assignment With Deterministic User Equilibrium

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ABSTRACT This paper aims to propose a new analytical method of deriving the enveloping macroscopic fundamental diagram (MFD) in the light of the well-defined sufficient condition and the route choice criterion in a general network. The enveloping MFD is defined to describe the boundary of all scatter points relating the flow rate (vehicles per unit time) to the vehicle accumulation (vehicles) in a road network. The theoretical framework consists of two parts. The first part is a static congested traffic assignment model which is based on the congested link performance function and the deterministic user equilibrium principle under the entirely congested condition. The second part is the new analytical method which is proposed based on the static uncongested and congested traffic assignments with the deterministic user equilibrium, in which the well-defined sufficient condition is satisfied by employing the static traffic assignment models, and the route choice criterion is fulfilled by the deterministic user equilibrium condition (Wardrop's principle). The main findings of this paper are summarized as follows: 1) the enveloping MFD delimits a region where all scatter points are located; 2) the existence and reproducibility of the enveloping MFD in a general network are verified through numerical examples; 3) the proposed method is suggested to be applied to transportation planning for describing and evaluating the macro-characteristics and the performance of road networks and evaluating whether the OD pattern is congruous with the network topology.

INDEX TERMS Transportation, mobility, macroscopic fundamental diagram (MFD), route choice, congested traffic assignment.

I. INTRODUCTION

In order to alleviate mass traffic congestions especially in the urban transportation networks, the theories about the macro-characteristics have played a significant role through traffic planning and the demand management in the road networks [1]. The traditional traffic forecasting models are complicated and fragile because numerous input variables are involved when dealing with congested conditions [2]–[4]. As a result, the methods for alleviating urban traffic congestion based directly on traffic data are getting increasing attention [5], [6]. A new concept of Macroscopic Fundamental Diagram (MFD) constructed from observed data is proposed to describe the macro-characteristics of a road network, in order to evaluate the network performance and traffic congestion. The MFD has attracted much attention in the research field of the macro-characteristics in road networks.

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A. THEORETICAL BACKGROUND OF MFD

The MFD is a type of diagram that describes the macroscopic and aggregated relation of traffic variables in the entire network, including average network flow (or equivalence variables: travel production, weighted network flow, outputs, outflow, trip completion rate, etc.), average network density (or equivalence variables: weighted network flow, network occupancy, vehicle accumulation, etc.) and speed.

The relative terms are explained as follows:

Vehicle accumulation (unit: vehicles): the number of vehicles on the network, which equals to the product of average density and network length, using the term "accumulation" for short.

Travel production (unit: vehicle-km traveled per unit time): the product of average flow rate and network length.

Trip completion rate (or output, outflow, unit: vehicles per unit time): the rate at which vehicles leave the network.

The well-defined MFD: The MFD with the low-scatter.

The high-scattering MFD: The MFD with high scattering phenomenon or uncertainty, i.e. contrary to the well-defined MFD.

The sufficient condition for well-defined MFDs: The network is in the state that all lanes or all links of the network are in a single regime (either uncongested or congested regime).

The development progress of published work about MFDs is shown in Figure 1. Daganzo [7] proposed an MFD model relating the travel production (or the trip completion rate) to the total vehicle accumulation in a road network, in which three different regimes are presented: undersaturated, saturated, and oversaturated state (see Figure 2 [7]), where the travel production increases, stabilizes and decreases correspondingly as the total accumulation grows.

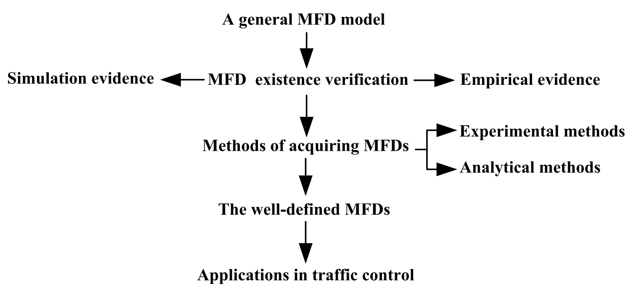

FIGURE 1. The development progress of published work about MFDs.

The MFD can become an effective theory and tool for the control strategies if it can be reproduced within the time-of-day and across days. For example, if aggregate accumulations in the road network can be maintained at the saturated state by controlling the inputs flow to the system, the traffic congestion can be relieved effectively and city mobility, in consequence, can be improved significantly.

The existence of MFD becomes the key first-step research, i.e., whether does the MFD exist in a real urban-scale network or not? Then simulation evidence and experimental findings based on detector data are used to prove the existence of urban-scale MFDs [8], [9]. Meanwhile, the application in traffic control is presented to improve the accessibility of network and relieve congestion by perimeter control strategies,

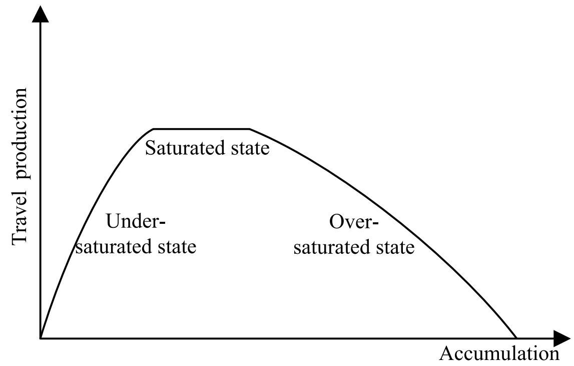

FIGURE 2. The MFD model relating travel production and accumulation.

i.e. through controlling the inputs flow to maintain the aggregate vehicle accumulations at the saturated state in a road network [8]–[10].

The research by Geroliminis and Daganzo [8] supports the existence mentioned above with micro-simulation, in which it shows that the MFD indeed exists independently of the demand for one-dimensional arterials and urban network in the downtown area of San Francisco with the CORSIM simulation model. Furthermore, Geroliminis and Daganzo [9] demonstrated that a well-defined MFD with low scatter actually exists and is examined in a realistic urban road network with a field experiment in the downtown of Yokohama (Japan). Afterwards, Daganzo and Geroliminis [11] presented an analytical method based on the moving observers and variational theory to obtain the MFD from the network infrastructure and control parameters, which is only applied to the networks with high regularity in topology and signalized settings in an arterial road.

However, some researches show there is much high scatter or uncertainty in MFDs (see Figure 3 [12]), in which many MFDs obtained from the field experiment or simulation data are rarely well-defined with low scatter.

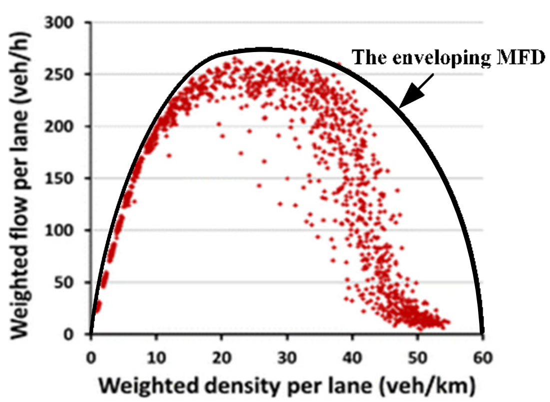

FIGURE 3. The high-scattering MFD and the enveloping MFD.

The existence condition of well-defined MFDs with low scatter becomes the key second step in the research, i.e., what is the sufficient condition for well-defined MFDs to exist in a real urban-scale network? Four conditions with regularity are conjectured to ensure a well-defined MFD with low scatter. Particularly, three of the four conditions should create a near-equilibrium state as in [13] with similar average speeds on all links.

B. LITERATURE REVIEW OF MFDS

After these pioneering studies, many further studies in theory and application about MFDs are conducted on the existence [14]–[16], the shape [17]–[19], the well-defined sufficient conditions [15], [20], [21], the method of acquiring MFDs [22]–[24], and the application method [12], [25]–[30].

As for MFD shapes, many studies are focused on the phenomenon of scatter or uncertainty. Heterogeneity in congestion distribution can affect the shape or even the existence of MFD [14], [17]. Furthermore, based on empirical and simulated data, Knoop *et al.* [19], Mazloumian *et al.* [20], Geroliminis and Sun [21] found that the spatial variability of the vehicle density in road networks is identified as one of the key factors to affect the scatter and the shape of MFDs. Moreover, they revealed that the MFDs between flow and density can be well-defined when link density variance is constant. They also demonstrate that the average network flow with a lower link density variance is consistently higher than that with higher link density variance for the same network density.

Gayah and Daganzo [31] summarized that the scattering phenomenon is inherent and cannot be eliminated, because naturally underlying instabilities in urban traffic networks can cause vehicles to tend towards heterogeneous spatial distributions, which results in scattered and unpredictable MFDs, especially when traffic congestion exists in the network. Cassidy *et al.* [15] suggested that reproducible MFDs with low scatter can be reasonably well-defined for freeway networks with the only precondition that data come from periods characterized by a single regime in all lanes or on all links of the network. In other words, it is implied that the well-defined MFDs only exist for a freeway network, when all links are either uncongested or congested in the network. Hence, the well-defined sufficient condition in this paper is defined as a situation that all lanes or all links of the network are in a single regime (either uncongested or congested regime) in the network.

Simultaneously, the analytical method of acquiring MFD is also a hot issue. In this research field, Daganzo and Geroliminis [11] proposed the fundamental analytical method based on variational algorithms, but the analytical method is suitable for the network composed of homogeneous links and regular signal settings. Geroliminis and Boyacı [22] further extended the analytical method to a more general case and investigated how variations in the signal settings and the link lengths affect the MFD. In order to consider the effect of route choice on the shape of network MFD, a more advanced analytical method for irregular topologies is proposed by Leclercq and Geroliminis [24], in which different route choice rules (such as Wardrop's principles, Logit model, and system optimum condition) are combined with the advanced analytical method to estimate MFDs in a simple parallel network. It is observed that the MFDs of the whole network for deterministic user equilibrium are identical under both the static conditions (inflow is equal to outflow for each route) and the dynamic conditions (network system is governed by the dynamic equations).

C. RESEARCH MOTIVATION

In the existing studies and results mentioned above, most studies are carried out mainly based on the simulation or field experiments, in which these indexes of the travel production and the network accumulation in MFDs can be easily measured by statistics. However, it is very likely to result in the scatter phenomenon or uncertainty for MFDs because a well-defined sufficient condition is rigid and hard to be satisfied in a realistic road network, which is not well applied to describe and evaluate the macro-characteristics and the performance of road networks. Therefore, it is significant to research the boundary of these scatter points (see Figure 3) and to acquire outer enveloping curves of MFDs, which can be defined as the enveloping MFD, for avoiding the inconvenience of application because of the high scattering phenomenon.

The analytical method is a promising approach to deriving the enveloping MFD. Although the published analytical methods have been presented to acquire the well-defined MFD, they are only applied to an arterial road instead of a general and complex road network, which could influence the application in realistic.

Simultaneously, it has been verified that the MFDs of the whole network for deterministic user equilibrium under static and dynamic conditions are identical [20]. Because dynamic traffic assignment models have a much higher computational complexity than static counterparts, a static traffic assignment model with use ease and mathematical rigor is a promising approach to be applied to the research of the enveloping MFD, especially when considering the route choice behavior (e.g. the deterministic user equilibrium principle).

Considering the analysis mentioned above, in this paper a new analytical method based on static traffic assignment models is provided to acquire the enveloping MFD (the outer boundary curves relating the OD flow rate to the vehicle accumulation) in a general network considering the route choice rule, in which the deterministic user equilibrium principle, the network topology, and link property data are needed.

First of all, the enveloping MFD must be a well-defined MFD, and the well-defined sufficient condition in Cassidy *et al.* [15] should be satisfied, i.e., all network links are required in a single regime (either uncongested or congested regimes). Though the sufficient condition is proposed based on freeway networks, the main aim of this paper is to estimate the enveloping MFD for a general network without considering the signalized intersection or the interaction of traffic flow through the merge or diverge nodes. Therefore, the sufficient condition on the freeway networks can also be used in this paper context.

Then, the traffic assignment under two single regimes should be respectively introduced to obtain the complete enveloping MFDs. The uncongested regime of enveloping MFDs can be obtained by using the conventional unconstrained models (i.e., Beckmann's model [39]). As for the congested regime of enveloping MFDs, the static congested traffic assignment model is introduced in a general network with all links congested, in which the congested link performance function is established to depict the relationship between travel time and flow rate according to the fact that both speed and flow rate decrease as the density increases under the congested regime (i.e., the regime that link density exceeds the critical density). Next, the congested user equilibrium principle is analyzed, and a congested traffic assignment model satisfying the congested user equilibrium principle is introduced.

Furthermore, by changing the quantity of and the distribution pattern of OD flow rate continually, we can obtain the equilibrium relationship of the OD flow rate and the accumulation based on both the static uncongested and the static congested traffic assignments without considering any time dynamics, and then the complete enveloping MFDs are obtained. It should be emphasized that the result of the static traffic assignment model here can be regarded as the magnitude of the steady flow rate, and its unit is ''vehicles per unit time''.

Finally, the existence and reproducibility of the enveloping MFD are verified in numerical examples, and the potential applications in transportation planning are discussed after considering the characteristics of the enveloping MFDs.

D. THE MAIN CONTRIBUTIONS

Different from the existing researches, the proposed analytical method in this paper is based on the static traffic assignment model. And we use the resulting enveloping MFDs to describe and evaluate the macro-characteristics and the performance of road networks, which can be applied to the field of transportation planning. In order to unify the variables between the MFD and the static traffic assignment, the relationship between the OD flow rate and the vehicle accumulation is employed to describe MFDs, because the OD flow rate and the vehicle accumulation are more common and convenient to calculate than the network average flow and the average network density in static traffic assignment models.

The main contributions of this paper include:

- A new concept of the enveloping MFD is defined to describe the boundary of all scattering points relating the OD flow rate to vehicle accumulation.
- A congested traffic assignment model is introduced to derive the congested branch of the enveloping MFD based on the congested link performance function and the congested deterministic user equilibrium principle.
- A new analytical method for acquiring the enveloping MFD considering the well-defined sufficient condition and route choice in a general network is proposed, which relates the MFD and traffic equilibrium.
- The new analytical method can verify the existence of the enveloping MFD in a general network from an analytical perspective, and it is suggested to be applied to transportation planning for describing and evaluating the macro-characteristics and the performance of road networks evaluating whether the OD pattern is congruous with the network topology.

The rest of this paper is organized as follows. In Section II, the notion of static traffic assignment is introduced, and a simple example is added to help to grasp the notions of the enveloping MFD. In Section III, the static congested traffic assignment theory is introduced. Then the methodology of the new analytical method is constructed based on static congested and uncongested traffic assignment models to acquire the enveloping MFDs, and the properties of the new analytical method are analyzed. Section IV verifies and discusses the existence and reproducibility of enveloping MFDs under various OD proportion patterns with the help of numerical examples, and a simple application example based on the enveloping MFD in transportation planning is illustrated. Section V concludes the paper.

II. NOTIONS AND A SIMPLE EXAMPLE

A. NOTIONS OF STATIC TRAFFIC ASSIGNMENT MODELS

The static traffic assignment (STA) is the mechanism which changes OD flow rates into link flow rates and obtains the travel time of links when the OD flow rate is time-stable with balanced inflows and outflows, including the static uncongested traffic assignment and the static congested traffic assignment.

The static uncongested traffic assignment (SUTA) is the conventional static traffic assignment with the increasing link performance function, which is only used for the case that the OD flow rate is time-stable and the density on all used links is less than the critical density.

The static congested traffic assignment (SCTA) is the static traffic assignment with the congested link performance function, which is only used for the case that the OD flow rate is time-stable and the density on all used links exceeds the critical density, and it is introduced in Section III in detail.

Link performance function is a function that describes the relationship between the travel time and the flow rate on a link, i.e., travel time function.

The deterministic user equilibrium (DUE) principle assumes that every user distinctly knows the state of the road network and tries to minimize his/her own travel time from origin to destination, and at equilibrium state the travel time on all used routes between one OD pair is equal and is shorter than that on unused routes.

The uncongested condition means that all used links are uncongested links.

The congested condition means that all used links are congested links.

Trip generation rate: the rate at which vehicles arrive in the network.

OD flow rate: the flow rates of trips from the origin node to the destination node (vehicles per unit time), which can describe the trip generation rate or the trip completion rate between OD pairs (the trip generation rate equals to the trip completion rate here).

B. A SIMPLE EXAMPLE

Consider a simple road network which includes only one OD pair connected by only one homogeneous link with the exit restriction in downstream (see Figure 4). The link with exit restriction in front of destination is common in realistic networks, such as near-saturated on-street and off-street parking spots in the urban network, or a toll station in the freeway network. A problem can be proposed, that is, what is the enveloping MFD relating the flow rate to the accumulation of this simple network?

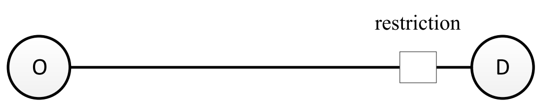

FIGURE 4. Traffic network.

According to [32]–[34], there are two traffic regimes along a link, i.e. uncongested regime and congested regime. Three fundamental traffic variables of flow rate, speed, and density are introduced to depict the traffic state of the network. The relation curves of the three variables are shown in Figure 5 (a) (b) and (c). The relevant curve of travel time versus flow derived by the flow rate-speed curve is shown in Figure 5 (d). In Figure 5, k_j means the maximum density, k_c means the critical density, v_f means the speed of free-flow, v_c means the critical speed and t_0 means the travel time of free-flow corresponding to the v_f, and C_a means the capacity of the link.

Under the steady state with homogeneous density distribution, the three fundamental variables can be respectively defined as homogeneous-flow rate, homogeneous-speed, and homogeneous-density. Thus, the curves in Figure 5 can respectively serve as the relation of the homogeneous-density versus the homogeneous-flow rate, the homogeneous-speed versus the homogeneous-flow rate, the homogeneous-speed versus the homogeneous-density, and the homogeneous-time versus the homogeneous-flow rate, which is also called the Fundamental Diagram (FD) of the link.

Figure 5 also reveals that one homogeneous-flow rate respectively corresponds to two homogeneous-densities, homogeneous-speeds, and homogeneous-times except for the capacity value. Taking homogeneous-density versus homogeneous-flow rate curve as an example, one homogeneous-flow rate corresponds to a smaller-density and a larger-density. If there exists a heterogeneous density distribution (defined as a heterogeneous-density) along the link that the average flow rate equals the

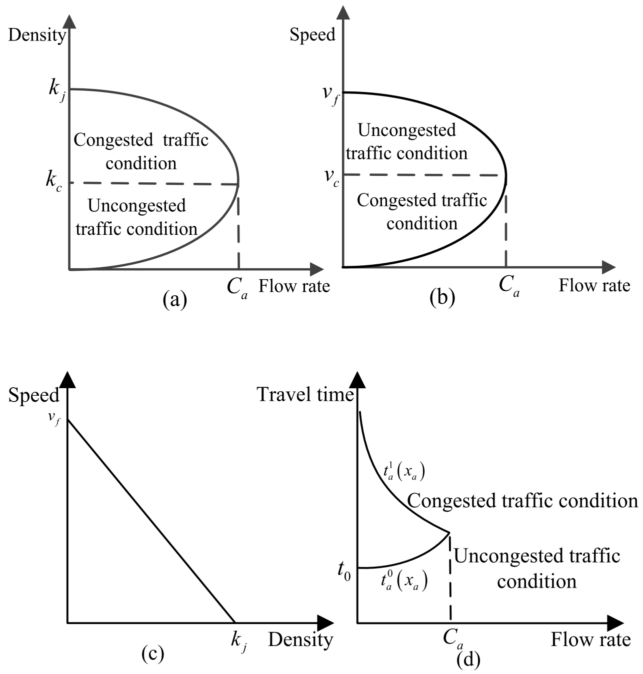

FIGURE 5. The relation curves between traffic variables: (a) Density-flow rate curve; (b) Speed-flow rate curve; (c) Speed-density curve; (d) Travel time-flow rate curve.

homogeneous-flow rate, the heterogeneous-density is larger than the smaller-density and smaller than the larger-density, which can serve as a scatter point in the region enclosed by the curve of homogeneous-density versus homogeneous-flow rate and the density-coordinate axis. Hence, the curve of homogeneous-density versus homogeneous-flow rate can serve as an enveloping FD, which encloses a region with the density-coordinate axis. And all heterogeneous points (i.e. scatter points) are located in the region. Similarly, the curves of homogeneous-speed versus homogeneous-flow rate, homogeneous-speed versus homogeneous-density and homogeneous-time versus homogeneous-flow rate can also be regarded as enveloping FDs.

Some phenomena can be intuitively observed in this simple example. A link has two types of performance function (i.e., travel time function here): the uncongested and congested link performance functions. The former is a monotonically increasing function of flow rate, while the latter is a monotonically decreasing function of flow rate. The flow rate decreases with the density rising in the congested regime because users decrease speed more rapidly with density rising for driving safely.

From the view of a pair OD, the FD of the link can be regarded as the enveloping MFD for the OD pair. The enveloping MFD for one OD pair can be defined as the frontier curves about the flow rate to the accumulation for one OD pair, in which the accumulation of one OD pair is the vehicle numbers from the specific origin to the specific destination. Furthermore, from the view of the whole network, the FD of the link can also be regarded as the enveloping MFD for the whole network.

However, a general network is comprised of multiple links, routes, and pairs of OD, and how can the enveloping MFD for one OD pair and the whole network be derived in such a complex network?

In order to solve this problem, the STA program is adopted to compute the homogeneous-flow rate and the homogeneous-travel time on each link to obtain the link accumulation and the network accumulation. Considering that the SUTA is not suitable for the congested traffic network whose links are all congested, the SCTA is employed. The SCTA is a new concept and the key technology to derive the congested branch of the enveloping MFD. The properties of the SCTA are particularly analyzed in Section III, including the congested DUE principle and the mathematical model.

III. METHODOLOGY

A. THE STATIC CONGESTED TRAFFIC ASSIGNMENT THEORY

In this section, the congested link performance function, the congested DUE principle and the SCTA model with the DUE principle are proposed.

The idealized assumptions in this paper are concluded as follows.

Assumption 1: The density and speed are uniform on a link. An increase (or decrease) in inflow is instantaneously transformed as an equivalent increase (or decrease) in density on a link. It implies that inflow equals outflow on the link and that the traffic wave moves with the infinite speed at a steady state.

Assumption 2: The outflow of a link is not constant and does not stabilize at the bottleneck capacity on a uniform link with a bottleneck located at the end when demand is oversaturated, and the magnitude of link flow rate depends on the homogeneous density (or average density). The following subsection has explained this assumption in detail.

The two idealized assumptions are suitable in the static traffic assignment models and are convenient to research the enveloping MFD.

1) THE LINK PERFORMANCE FUNCTION

Since there is uncongested or congested traffic regime along a link, the uncongested and congested link performance function need to be constructed respectively in order to describe the different traffic regime and the complete FD. Under the uncongested condition, the speed decreases and the travel time increases as the flow rate increases on the link because of the interaction with other cars moving along the link [35]. It can be described by the uncongested link performance function. However, under the congested condition, as the density increases, the travel time increases, while the flow rate decreases. Hence, the travel time manifests as monotonically decreasing with the increasing flow rate, which cannot be depicted by the uncongested link performance function.

Compared with the uncongested link performance function, there are three special characteristics in the congested one.

(1) Congested link performance function is monotonically decreasing with the increasing flow rate (see Figure 5 (d)).

Some existing works [36] and [37] consider that the travel time of a link vertically increases because of queuing when the demand flow exceeds its capacity based on the queue theory, and the flow rate stabilizes at the bottleneck capacity consistently in a stationary state. However, in this paper, the assumption is that the flow rate is not a constant stabilizing at the link or the bottleneck capacity, but decreases as the density increases.

Density remains uniform along the roadway (i.e. queuing vehicles are regarded as the uniform flows along the roadway on average). The flow rate and travel time only depend on the density; hence, the travel time is manifested as the function of the flow rate. The assumption is similar to the instantaneous propagation model [38], in which shock waves move with the infinite speed. This idealized assumption also implies that the flows between upstream and downstream links do not interact with each other through the node. It is a reasonable assumption in the static model for researching the steady state of traffic and the enveloping MFD.

(2) The link flow rate can exceed its capacity a bit.

Theoretically, the flow rate on a link cannot exceed its capacity. But in practice of STA, it is convenient to simplify calculation when the flow rate on the link can exceed its capacity slightly.

(3) The flow rate of the intersection point between the uncongested and congested link performance function curves is the capacity of the link (see Figure 5 (d)).

According to all analyses above, construct the congested link performance function $t_a^1(x_a)$:

$$t_a^1(x_a) = t_a^0\left[\gamma c_a/x_a - \left(1 + \alpha (x_a/c_a)^\beta\right)\right] \tag{1}$$

In Eq.(1), t_a^0 is the travel time of free flow on link a; α, β, and γ are model parameters which satisfy $\alpha > 0$, $\beta > 0$, $\gamma > 0$ and can be obtained by fitting the data from the realistic network; x_a ($x_a > 0$) is the flow rate on congested link a, c_a is the capacity of link a.

The uncongested link performance function curve $t_a^0(x_a)$ and the congested link performance function curve $t_a^1(x_a)$ are shown in Figure 5 (d).

2) THE CONGESTED DUE PRINCIPLE

In both the SUTA and the SCTA with DUE, they assume that every user distinctly knows the state of the road network and tries to minimize his/her own travel time from origin to destination. Unlike economic equilibrium between supply and demand, the equilibrium discussed here is only between the travel times of all road users. Compared with the SUTA, the SCTA has different meanings. The traffic flow rates assigned on a road network under uncongested and congested conditions are respectively defined as traffic demand flow rate and traffic passing flow rate. The passing flow rate is defined to describe the actual flow rate of in congested links when the demand is larger than link capacity.

The SCTA is the mechanism that an oversaturated demand distribution between OD pairs is put on the road network and the link flow rate on every used link is solved under the condition of a high-density, low-speed and low-flow-rate network (the total demand flows do not equal the actual inflows or outflows of the whole network and the redundant demand flows queue at the origin but in the network). The formulations describe the process that the traffic passing flow rate decreases with the number of vehicles increasing because the speed decreases more rapidly at the higher density for safe driving, in which a decreasing link performance function is adopted. The results of the SCTA mean that the traffic passing flow rate on links in the oversaturated demand network, in which all used routes are in a congested state.

In the SCTA with the DUE principle, all used routes for each OD pair are congested to different extents. Assume that rigid oversaturated demands want to pass through the network and then a congested condition is generated, which is independent of the travel cost/supply-demand curve. Under the congested condition, the behavior of users choosing the shortest routes causes the travel time increasing and the passing flow rate decreasing on these shortest routes because of the increasing density. More users will drive on the shortest routes, which causes the density increasing but speed and flow rate decreasing on the shortest routes. Fewer users choose the other longer routes, which causes in the congestion of these routes relieving relatively, the density decreasing, the speed and the flow rate increasing. Consequently, since the users always choose the shortest routes, the travel time on all routes between one OD pair will be equal. Therefore, the congested DUE principle is that all used routes are congested and the travel time on all used routes connecting the same OD pair will be equal at congested equilibrium state if every user distinctly knows the state of the road network and tries to minimize his/her own travel time from origin to destination.

In order to describe the DUE state of the SCTA, consider the two-route (two-link) congested road network shown in Figure 6. This network represents one OD pair connected by route 1 and route 2. Let t_1 and t_2 represent the travel time on route 1 and route 2 under the congested condition respectively, and let x_1 and x_2 represent the passing flow rate on two routes respectively. Assume two routes are congested with fixed total passing flow rates Q (the demand information is unknown), where $Q = x_1 + x_2$.

As shown in Figure 7, the horizontal axis represents the total flow rates which are fixed to Q, and the left and the right vertical axes respectively represent t_1 and t_2. Note that the flow rates of two routes can increase or decrease because of the route choice of users, but their sum must be equal to Q. State A is the intersection point of two route performance curves and means the network is at a congested equilibrium state, i.e. the travel times of two routes are equal. State B and C are the disequilibrium states, i.e. the travel times of two routes are unequal.

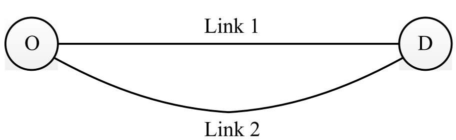

FIGURE 6. A road network with one OD pair and two routes.

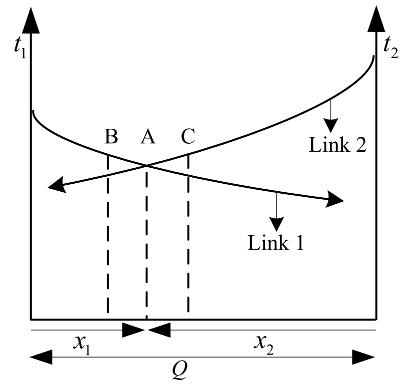

FIGURE 7. The DUE state of the SCTA.

The process of reaching and stabilizing at the equilibrium state under the congested condition is described as follows. Consider the case that the traffic state slightly deviates from the equilibrium state A in Figure 7, i.e. state B or state C. Taking the process of state B transforming into state A as an example, at state B, the travel time t_2 of route 2 is less than that of route 1, t_1, which will cause more vehicles choosing route 2 and hence aggravate the congestion of route 2, leading to higher density, fewer flow rates and longer travel time on route 2. Conversely, the density and congestion of route 1 will decrease and relieve because fewer users choose route 1 so that the speed and the passing flow rate increase, while the travel time decreases on route 1. Hence, the difference in the travel time on two routes draws closer and closer until becomes zero and the network reaches the equilibrium state A finally. As described above, the network gradually transforms from state B into state A. Similarly, state C will also change into state A eventually. Consequently, travel time on two routes will be equal and the network will be equilibrium. Here this phenomenon is called the congested DUE.

3) THE SUTA AND SCTA MODELS WITH DUE

Let $G = (N, A)$ be a road network defined by a set N of nodes and a set A of directed links. Each link $a \in A$ has a flow-rate-dependent uncongested link performance function $t_a^0\left(x_a^0\right)$ and a flow-rate-dependent congested link performance function $t_a^1\left(x_a^1\right)$. x_a^0 is the demand flow rate on the link a under the uncongested condition. x_a^1 is the passing flow rate on the link a under the congested condition, $x_a^1 \geq \varepsilon$ (i.e. $x_a^1 > 0$), where ε is a small positive number because the flow rate on congested links must be larger than 0. f_k^{rs} is the demand flow rate on route $k \in R_{rs}$ under the uncongested condition. g_l^{rs} is the passing flow rate on route $l \in R_{rs}$ under the congested condition. R_{rs} is the set of routes from origin r to destination s in the network. If link a is on route k between r and s, $\sigma_{a,k}^{rs} = 1$; else $\sigma_{a,k}^{rs} = 0$. If link a is on route l between r and s, $\delta_{a,l}^{rs} = 1$, else $\delta_{a,l}^{rs} = 0$.

The SUTA model is shown as follows [39], which will be used to obtain the uncongested branch of the enveloping MFD in the next section.

$$Minimize: \ Z(X^0) = \sum_a \int_0^{x_a^0} t_a^0(w)dw \tag{2}$$

$$\text{Subject to} \ \sum_k f_k^{rs} = q_{rs} \tag{3}$$

$$x_a^0 = \sum_r \sum_s \sum_k f_k^{rs} \sigma_{a,k}^{rs} \tag{4}$$

$$f_k^{rs} \geq 0 \tag{5}$$

The SCTA model with DUE is to find the link passing flow rate x_a ($a \in A$), which meets the DUE principle under the congested condition [40]. The link passing flow rate pattern can be obtained by solving the following mathematical program, and the SCTA model will be used to obtain the congested branch of the enveloping MFD in the next section.

$$Maximize: \ Z(X^1) = \sum_a \int_\varepsilon^{x_a^1} t_a^1(w)dw \tag{6}$$

$$\text{Subject to} \ \sum_l g_l^{rs} = q_{rs} \tag{7}$$

$$x_a^1 = \sum_r \sum_s \sum_l g_l^{rs} \delta_{a,l}^{rs} \tag{8}$$

$$g_l^{rs} > 0 \tag{9}$$

In Eq.(6), a small number ε is adopted, which can effectively ensure the practicality in engineering applications, because the link travel time is infinite when $x_a^1 = 0$ in the congested link performance function (see Eq.1).

Proposition: The SCTA model satisfies the congested DUE principle.

Proof: See the equivalency conditions of the model in **Appendix A**.

The comparisons between the SUTA and the SCTA models are summarized in **Appendix B**.

B. FORMULATIONS

In the new analytical method, the flow rate (vehicles per unit time) and the network accumulation (vehicles) are derived from the STA models. The route choice rule is embodied in the DUE principle.

In the STA with DUE, both the equilibrium link flow rate and the equilibrium travel time between OD can be calculated. Therefore, the relationship of the equilibrium travel time and distributed flow rate between OD can be established. The relationship curve of the equilibrium travel time u_{rs} and distributed flow rate q_{rs} under the uncongested condition is defined as the uncongested time-flow rate curve, which can be described by an increasing function $u_{rs} = T_{rs}^0(q_{rs})$ when flow rates of other OD pairs are invariable. Similarly, the congested time-flow rate curve is defined to indicate the relationship of the equilibrium travel time v_{rs} and distributed flow rate q_{rs} under the congested condition, which can be described by a decreasing function $v_{rs} = T_{rs}^1(q_{rs})$ when flow rates of other OD pairs are invariable.

Consequently, it can be generally received that, against a distributed flow rate q_{rs}, two different equilibrium travel time u_{rs} and v_{rs} can be obtained by the SUTA and the SCTA models respectively. Accordingly, there must be an intersection point at $q_{rs} = q_{rs}^c$ between the uncongested and congested time-flow rate curves making $v_{rs}^c = u_{rs}^c$ (see Figure 8). The distributed flow rate q_{rs}^c with $v_{rs}^c = u_{rs}^c$ is a critical point, by which the uncongested and congested condition can be distinguished.

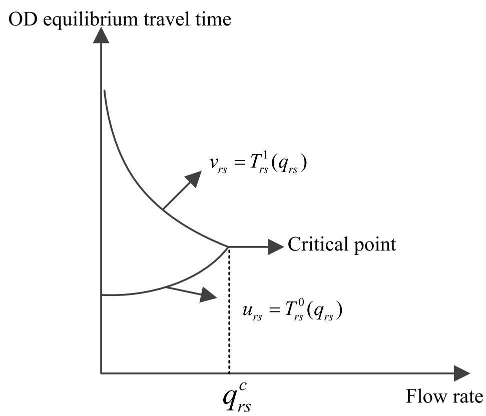

FIGURE 8. The intersection point between the uncongested and congested time-flow rate curves of one OD pair.

In the uncongested and congested time-flow rate curves, the parts of $q_{rs} \leq q_{rs}^c$ are adopted to compute the enveloping MFD for each OD pair. Because in the parts of $q_{rs} > q_{rs}^c$, the uncongested equilibrium travel time of an OD pair is larger than the congested one, which is contradicted with the fact (see Figure 8).

Based on the uncongested and the congested time-flow rate curves, the enveloping MFD relating the flow rate and the accumulation between one OD pair can be obtained. Based on the travel time and the flow rate on all links, the enveloping MFD relating the total OD flow rate and the total accumulation in the whole network can be obtained. Since the flow rate along the link is regarded as homogeneous-flow rate, the accumulation on the link and between OD pairs can be respectively computed by Eq.(10) and Eq.(11).[1]

$$N_a^0 = x_a^0 \cdot t_a^0(x_a^0), \quad N_a^1 = x_a^1 \cdot t_a^1(x_a^1), \ q_{rs} \leq q_{rs}^c \tag{10}$$

$$N_{rs}^0 = q_{rs} \cdot T_{rs}^0(q_{rs}), \quad N_{rs}^1 = q_{rs} \cdot T_{rs}^1(q_{rs}), \ q_{rs} \leq q_{rs}^c \tag{11}$$

[1] $N_a^0 = k_a \cdot L_a = \frac{x_a^0}{v_a} \cdot L_a = x_a^0 \cdot \frac{L_a}{v_a} = x_a^0 \cdot t_a^0(x_a^0)$, where v_a represents the speed of vehicles along link a, L_a represents the length of link a, and k_a represents the vehicle density of link a, similarly, $N_a^1 = x_a^1 \cdot t_a^1\left(x_a^1\right)$.

where N_a^0 and N_a^1 are the accumulation on uncongested and congested link a respectively, N_{rs}^0 and N_{rs}^1 are the accumulation from r to s under the uncongested and the congested condition respectively.

The total flow rate Q is the sum of all OD distributed flow rate q_{rs}. The total accumulation in the network is the sum of accumulation on all links.

$$Q = \sum_{rs} q_{rs}, \quad N = \sum_{a} N_a \tag{12}$$

The process of obtaining the enveloping MFD is summarized as follows:

Step 1: Set an appropriate initial value of Q and a given distribution proportion of the total flow rate p_{rs}.

$$q_{rs} = p_{rs} \cdot Q, \quad \sum_{rs} p_{rs} = 1 \tag{13}$$

where p_{rs} is the distribution proportion between origin node r and destination node s.

Step 2: Acquire the uncongested time-flow rate curves by the SUTA model with the Q increasing gradually.

The SUTA model (2)-(5), as well as the Frank-Wolfe algorithm [35], [41], is adopted to calculate the uncongested link flow rate and travel time; then, the uncongested time-flow rate curves can be acquired.

Step 3: Acquire the congested time-flow rate curves by the SCTA model with the Q increasing gradually.

The SCTA model (6)-(9), as well as the Frank-Wolfe algorithm [35], [41], is adopted to calculate the congested link flow rate and the travel time. Then the congested time-flow rate curves can be acquired. Note that only the routes used in Step 2 are adopted as the effective routes in SCTA model because the routes being assigned flow rate in congested condition must be used routes in SUTA model.

Step 4: Derive the enveloping MFD for OD pairs based on the uncongested and congested travel time-flow rate curves between each OD pair.

Find the intersection point of the uncongested and congested travel time-flow curves, and adopt the part of the distributed flow rate which is less than the critical-point flow rate, and then compute the accumulation by Eqs.(10) and (11). The enveloping MFDs of OD pairs are obtained based on the relationship between the distributed flow rate and the accumulation for each OD pair.

Step 5: Derive the enveloping MFD for the whole network based on the equilibrium link flow rate and link travel time.

The accumulation of the whole network under the uncongested and congested condition is computed by Eq. (12). The total OD flow rate Q and the total accumulation N in the network are adopted to derive the enveloping MFD of the whole network.

Like the uncongested and congested time-flow rate curves of the OD pair, the uncongested and congested flow rate-accumulation curves of the whole network can also be obtained. There is an intersection point at $Q = Q^c$ between the uncongested and the congested flow rate-accumulation curves, which is also defined as a critical point (see Figure 9).

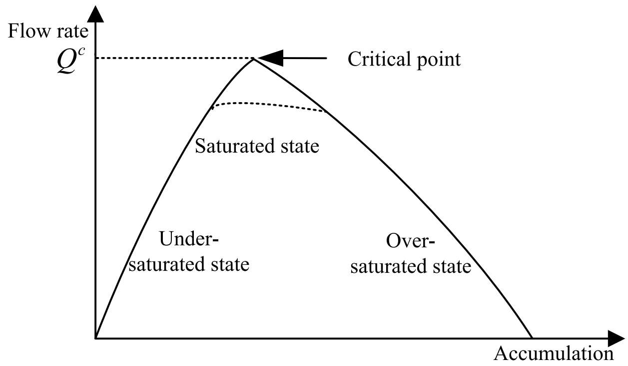

FIGURE 9. The enveloping MFD of the whole network with three states and the critical point.

The parts of $Q \leq Q^c$ in the uncongested and the congested flow rate-accumulation curves are adopted to derive the enveloping MFD for the whole network.

In the travel time-flow rate and flow rate-accumulation curves, the points whose flow rate are larger than the critical points flow rate q_{rs}^c and Q^c are both defined as the unqualified points, which stand for the impractical points, because the uncongested equilibrium travel time cannot exceed the congested travel time between the same OD pair in reality (or the accumulation under the uncongested condition cannot exceed that under the congested condition in reality). In some cases, the unqualified points for the enveloping MFD of OD pairs are allowed to exist for obtaining the complete enveloping MFD of the whole network.

C. PROPERTIES

The new analytical method has considered the well-defined sufficient condition and route choice in a general network. The well-defined sufficient condition is satisfied by the SUTA and the SCTA models. The route choice rule is embodied by the DUE principle. Although the link capacity constraints are not explicitly taken into account in the analytical method, the link flow rate can be implicitly restrained by the precondition that the uncongested equilibrium travel time cannot exceed the congested one between each OD pair. When the enveloping MFDs of the OD pair or the whole network are obtained, the equilibrium flow rate on some links may slightly exceed the capacity of these links around the critical point. This may be a weakness of the analytical method. Nevertheless, the overestimated results can be a rational upper limit and a valuable reference at saturated state. Hence the curves obtained by the analytical method can be regarded as the enveloping curve of the accurate MFD. What's more, the main aims of the new analytical method are to research the enveloping MFD and verify the existence of the enveloping MFDs in a general network via the analytical method based on traditional traffic assignment methods. The weakness is acceptable.

In fact, there are generally three types of traffic states about MFD: undersaturated, saturated and oversaturated

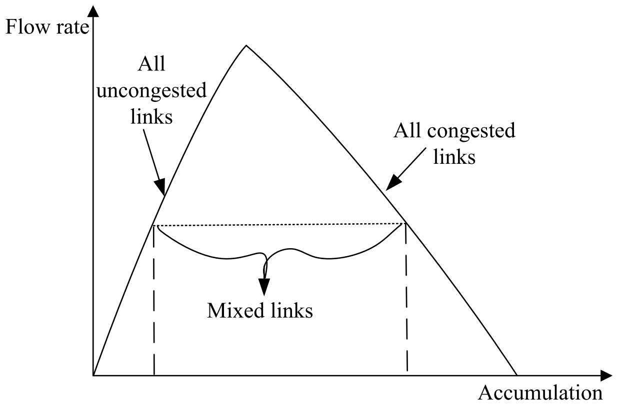

FIGURE 10. Comparison between the single and mixed state of the links.

state. When the road network is at undersaturated state with all links uncongested, the undersaturated part of the enveloping MFD can be obtained by the SUTA model. When the road network is at oversaturated state with all used links congested, the oversaturated part of the enveloping MFD can be obtained by the SCTA model. However, when the road network is at saturated state (including both uncongested links and congested links), the saturated part of the enveloping MFD is difficult to be obtained only by the STA model. Therefore, a compromise method is employed. The parts around the critical point in the uncongested and the congested flow rate-accumulation curves are used to replace the saturated part (see Figure 9, the dot lines represent saturated state).

Next, we will analyze the reasons why the enveloping MFD of the whole network can enclose all scatter points.

(1) The link flow rate and travel time obtained by the analytical method are homogeneous-flow rate and homogeneous-time. According to the statement of Section II, all heterogeneous points are located in the region enclosed by the enveloping MFD and the accumulation-coordinate axis.

(2) If the signalized intersections or the interaction of traffic flow through the node are considered in the networks, the mean speed of vehicles and the flow rate decrease at the same accumulation. Therefore, this can also create scatter points beneath the enveloping MFD.

(3) The compromise way of the analytical method overestimates the flow rate of the saturated state. Hence the saturated part of the actual MFD will be beneath the enveloping MFD (see Figure 9).

(4) If the road network is under any mixed state where both uncongested and congested links exist simultaneously, the accumulation is larger than the single state with all the uncongested links and is smaller than that with all the congested links under the same flow rate. This can create scatter points beneath the enveloping MFD. Geroliminis and Daganzo [8] draw a similar conclusion. (See Figure 10, the solid lines represent the single state and the dotted line represents the mixed state.)

To sum up, all scatter points are located in the region enclosed by the enveloping MFD and the accumulation-coordinate axis. Therefore, the enveloping MFD of the whole network obtained by this analytical method can be regarded as the outer boundary of all scatter points. It should be noted that the enveloping MFD of one OD pair can also be the outer boundary curves about the flow rate to the accumulation for this OD pair when the OD distribution pattern is fixed.

One merit of the analytical method is that not only the enveloping MFD of the whole network can be obtained, but the enveloping MFD of each OD pair can also be acquired. The concept "the enveloping MFD of each OD pair" is introduced to investigate that the influence of various OD distribution proportion patterns on the enveloping MFD of the whole network and verify whether it exists the best OD distribution pattern making the sum of flow rates of all OD pairs maximum (i.e., the OD distribution pattern under which the top of the enveloping MFD can be reached in reality, see Section IV). Furthermore, based on the enveloping MFD of the whole network, the macro-characteristics and the performance in a road network can be analyzed and evaluated from the view of the network level, which can be potentially applied to help to evaluate the schemes of network design (see Section IV).

In addition, the proposed method is based on the static traffic assignment models and the Frank-Wolfe algorithm, which have been widely applied to urban transportation planning. The discussions of CPU times and computational effort of the Frank-Wolfe algorithm are examined by many existing researches [42]–[45]. Therefore, it ensures that this method can be efficiently applied to urban-scale transportation networks.

Considering that the resulting enveloping MFD is the upper boundary curve, its current accuracy may limit the practical application of this method to traffic demand management and perimeter flow control. Therefore, we apply the enveloping MFDs to macroscopic transportation planning. There are two potential applications based on the method. The first is to help to evaluate the schemes of land-use development, and the other is to help to evaluate schemes of network design.

In future research, the advanced quasi-dynamic traffic assignment model considering the signalized intersection, link capacity constraints and the explicit queue will be introduced to improve the accuracy of this analytical method and obtain more accurate and practical MFDs instead of the enveloping MFDs in this paper. After improving its accuracy, we will attempt to apply it to traffic demand management and perimeter flow control.

D. COMPARISON WITH PUBLISHED ANALYTICAL METHODS

In order to further clarify the characteristics of the analytical method, we make comparisons with the analytical methods in Daganzo and Geroliminis (2008) [11] and Leclercq and Geroliminis (2013) [24].

Both the analytical methods in Daganzo and Geroliminis (2008) and Leclercq and Geroliminis (2013) are based on the variational theory (VT). The key of the VT is to express the kinematic wave (KW) model by a least-cost path problem and describe the homogeneous street by a relative cost function (CF) from the moving observers' frame of reference. The CF is assumed as a linear function, and the linear CF corresponds to the triangular FD. Therefore, traffic dynamics are defined by the KW model with a triangular FD. Then the MFD for an urban street composing of successive homogeneous links and the same signal setting is modeled to describe the relation between the density and the flow of the street based on the triangular FD. However, the analytical method in Daganzo and Geroliminis (2008) is only applied to the urban arterial with the regular topology and the same signal setting. Leclercq and Geroliminis (2013) improved this analytical method, and the regularity condition and the same signal settings are not needed. Thus, this improved method can be applied to more realistic and heterogeneous topologies. Nevertheless, the two methods based on the VT are only applied to a single urban arterial or a simple parallel network composing of successive links and signal settings without considering the turning movement. If applied to a more general and complex realistic network, the two methods are inefficient because a larger number of alternative paths have to be evaluated to identify the equilibrium condition [24]. Moreover, the challenging problems are how to obtain the network MFD from route MFDs and to relate correctly route and network MFDs based on the two analytical methods. These shortcomings lead to limited applications in more complex networks.

Thus, for the sake of applying to more complex realistic networks, the proposed method in this paper is based on the static traffic assignment considering the deterministic user equilibrium. The main comparisons with the analytical methods in Daganzo and Geroliminis (2008) and Leclercq and Geroliminis (2013) are summarized in Table 1.

TABLE 1. The comparisons between the previously published methods and the proposed method in this paper.

Items	The method in Daganzo and Geroliminis (2008)	The method in Leclercq and Geroliminis (2013)	Our method
Scope	The microcosmic network (an urban arterial) including the regular topologies and signal settings.	The microcosmic network (an urban arterial) including the irregular topologies and signal settings.	The relatively macroscopic network without considering the signalized intersection.
Key technology	The variational theory based on the kinematic wave model and a triangular FD.	The variational theory based on the kinematic wave model and a triangular FD.	The static traffic assignment models and the relationship between flow rate and travel time based on the concave FD.
Results	Effective MFDs for an urban arterial composing of homogeneous links and regular signal settings.	Effective MFDs for an urban arterial composing of successive links and irregular signal settings.	The enveloping MFD without the accurate saturated branch of MFDs for a general and complex network.
Advantages	More realistic and accurate MFDs for an urban arterial.	More realistic and accurate MFDs for an urban arterial.	The enveloping MFDs for a general network considering the route choice rule; Easy to use; Applicable to general networks.
Disadvantages	Not applicable to a general network or a complex network; No consideration for route choice.	Not applicable to a general network or a complex network.	Relatively low accuracy and no consideration for signalized intersections.

IV. NUMERICAL EXAMPLES AND DISCUSSION

To begin with, we note that the definitions of the enveloping MFD in this paper are conceptually different from the published analytical methods. Moreover, the enveloping MFD is obtained under the steady state and the idealized condition without considering the signalized intersections (or the interaction of traffic flow through nodes). Therefore, it is meaningless to compare with published analytical methods through numerical examples. In numerical examples, the existence, potential applications and computational efficiency of the enveloping MFDs are investigated.

A. A SMALL TEST NETWORK

Consider a simple road network as shown in Figure 11. The test network is 4-nodes, 5-links network with 2 OD pairs (origin node 1, 2, destination node 3 and a merge node 4) and 4 distinct routes. The uncongested link performance function uses Eq.(14) and the congested link performance function uses Eq.(15). The values of parameters in Eq.(14) and Eq.(15) are assumed here to test the proposed method. Note that the parameters of uncongested and congested link performance functions can influence the shape of the enveloping MFD to a limited extent. In practice, the value of parameters can be accurately obtained by fitting the data from the realistic network. The given related data of links are shown in Table 2, where t_a^0 is the travel time of free flow on link a, and c_a is the capacity of link a. Different OD matrices (i.e., OD distribution patterns or OD proportion patterns) are loaded in

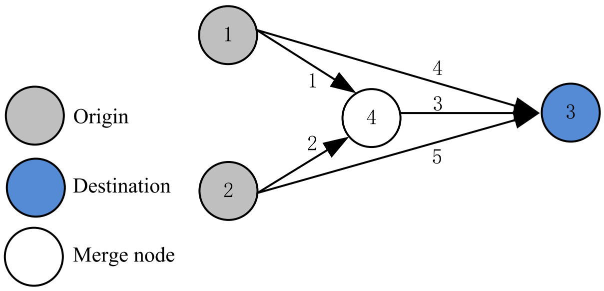

FIGURE 11. A small test network.

TABLE 2. Related data of road links.

Link number	t_a^0	c_a (vehicles/unit time)
1	6	1000
2	5	500
3	4	1200
4	10	500
5	9	500

the simple network to obtain the enveloping MFD for each OD pair and the whole network using the analytical method proposed in Section III.

Figure 12 (a), (b), (c) and (d) respectively show the crude enveloping MFDs of OD pairs and the whole network with some unqualified points under various OD proportion patterns $[q_{13} : q_{23}] = [1 : 1]$, $[q_{13} : q_{23}] = [1.5 : 1]$, $[q_{13} : q_{23}] = [2 : 1]$ and $[q_{13} : q_{23}] = [3 : 1]$. Figure 13 shows the comparison of the complete enveloping MFD for each OD pair under various OD distribution patterns. Figure 14 shows the comparison of the enveloping MFD for the whole network under various OD distribution patterns.

$$t_a^0(x_a) = t_a^0\left[1 + 0.5(x_a/c_a)^4\right] \tag{14}$$

$$t_a^1(x_a) = t_a^0[3c_a/x_a - \left(1 + 0.5(x_a/c_a)^4\right)] \tag{15}$$

Note that every point in the enveloping MFDs of Figure 12-14 stands for the steady state under different demand scenarios including variations in both the quantity and patterns of OD flow rates. Figure 12 presents that the enveloping MFDs for each OD pair and the whole network can be obtained when a network is in a single regime (either in the uncongested or in the congested regime).

According to Figure 12 (b), the OD proportion pattern with [1.5:1] is the best proportion in the four proportion matrices for a more complete enveloping MFD. That is to say, under the best proportion, when the enveloping MFDs for the whole network is obtained, no unqualified point exists in the enveloping MFDs for all OD pairs, and the maximum flow rate of the MFD for the whole network (i.e., the top of the enveloping MFD) may be reached in reality.

However, under the other proportion patterns, some unqualified points (i.e., the overlapping points in Figure 12 (a), (c) and (d)) for the enveloping MFD of OD pair are allowed to exist in order to obtain the complete enveloping MFD for the whole network. Therefore, the network under these OD distribution patterns cannot reach the maximum flow rate in fact. The unqualified points are introduced only to obtain the complete enveloping MFD for the whole network without gaps. These points are impractical or inexistent points in reality because their accumulation under the uncongested condition exceeds that under the congested condition. This is contradicted with the fact.

Taking Figure 12 (a) as an example, in order to obtain the complete enveloping MFD for the whole network, the unqualified points in the enveloping MFD of OD 2-3 are allowed to exist. If the OD flow rates are continuously increased to eliminate the gap in the enveloping MFD of OD 1-3, the MFD for the whole network will arise the unqualified points because the OD distribution proportion is fixed. Intuitively, the enveloping MFD for the OD pair possessing unqualified points means that its routes are in the oversaturated state. The enveloping MFD for the OD pair possessing gap means that its routes are in the unsaturated state. These phenomena imply that the OD pattern is not congruous with the network topology, which causes that some routes are overloaded while other routes are lower utilized (i.e. the unbalance of traffic demand and supply in space).

The similar phenomenon exists in Figure 12 (c) and (d). Only for the Figure 12 (b), the enveloping MFDs for OD 1-3, 2-3 and the whole network have no unqualified point or gap because this OD proportion pattern is most congruous with the network topology in the four patterns. Therefore, the concept ‘‘the enveloping MFD of each OD pair’’ can investigate that the influence of various OD proportion patterns on the whole network MFD and verify whether it exists the best OD proportion pattern making the sum of flow rate of all OD pairs maximum (i.e., the proportion pattern under which the top of the enveloping MFD can be reached in reality).

As we can see from the analysis above, these enveloping MFDs in Figure 12 can be beneficial to evaluate which pattern is better in given OD proportion patterns (i.e. which patter is more congruous with the network topology). Based on this information, the corresponding plans or adjustments for land-use development can be made so that the OD pattern is congruous with the existing network topology, which can be one of the potential applications of the enveloping MFDs. Specifically, this information may evaluate which scheme for land-use development is more congruous with the existing network topology, and then maximize the utilization of the existing road system.

Figure 13 shows the OD matrices have a significant influence on the shape of the enveloping MFDs for each OD pair, especially in the congested regime, but the triangular shape of enveloping MFD for each OD is similar. Figure 14 presents that the OD matrices affect the shape of the enveloping MFD for the whole network to a certain extent, which implies the enveloping MFD for the whole network can be reproducible with vastly different OD matrices to a certain extent.

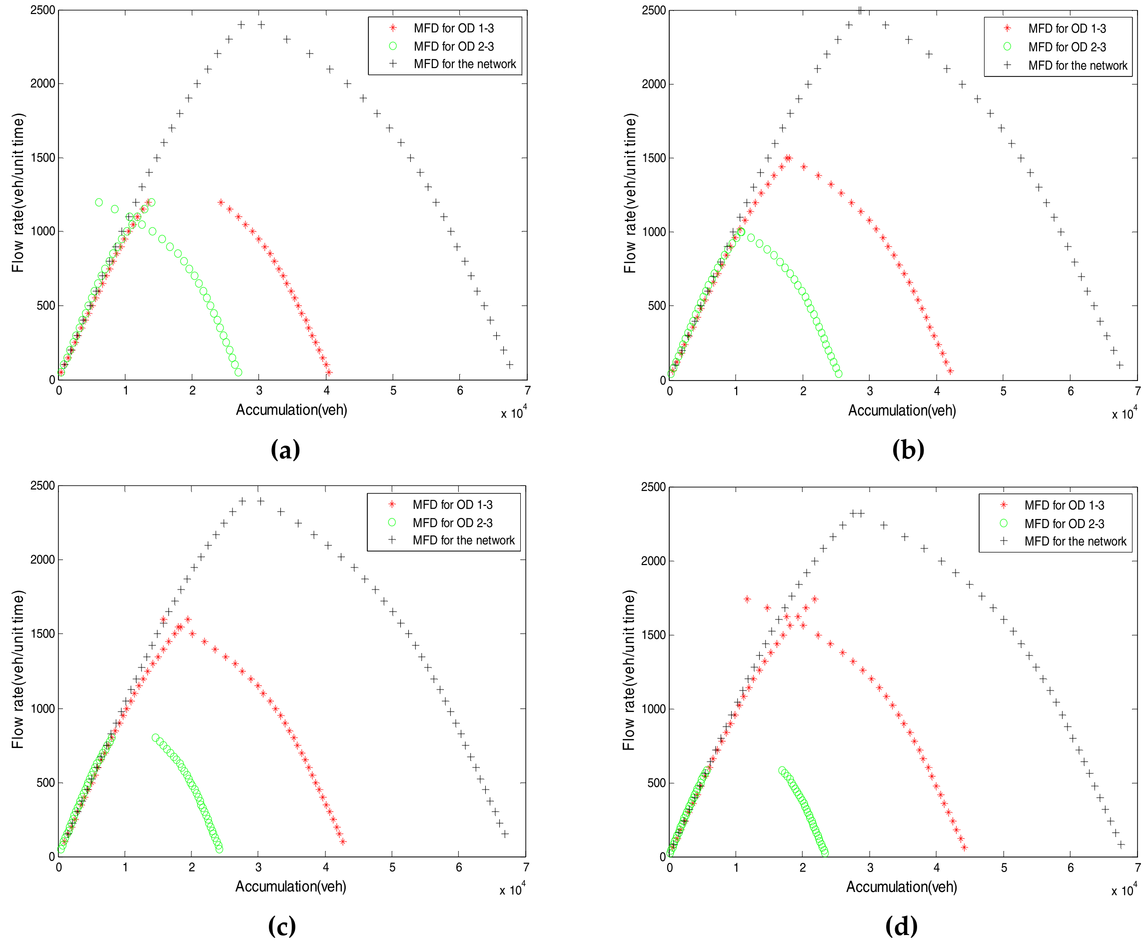

FIGURE 12. The enveloping MFDs with some unqualified points for different OD proportion patterns (a) $[q_{13} : q_{23}] = [1 : 1]$, (b) $[q_{13} : q_{23}] = [1.5 : 1]$, (c) $[q_{13} : q_{23}] = [2 : 1]$,(d) $[q_{13} : q_{23}] = [3 : 1]$.

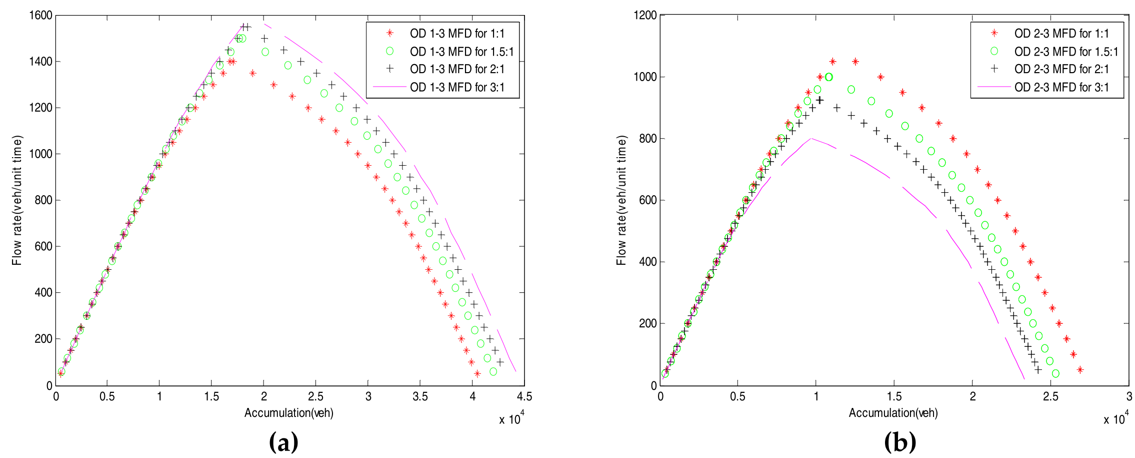

FIGURE 13. The comparison of the enveloping MFDs for each OD with different OD distribution patterns.

From Figure 13 and Figure 14, it can be found that though the enveloping MFDs for OD pairs are significantly influenced by the OD distribution patterns, the shape of the enveloping MFD for the whole network is insensitive to the OD patterns to a certain degree. It implies that the enveloping MFD for the whole network is dependent on the physical characteristics of the network, but not the OD patterns. Therefore, the enveloping MFD for the whole network can be applied to evaluate the macro-characteristics and the performance of the network. Next, a potential application about

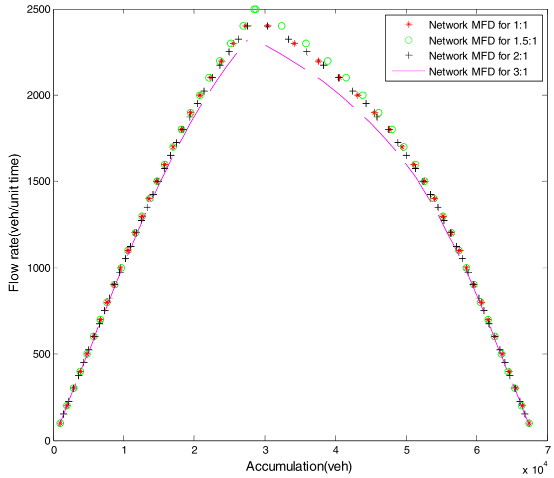

FIGURE 14. The comparison of enveloping MFDs for the whole network with different OD distribution patterns.

scheme evaluation of network design is illustrated by a simple example as follows.

Three schemes for the enhancement of link capacity of the original network in Figure 11 are given considering the limited investment. Plan A: the capacity of link 3 is increased by 200; Plan B: the capacity of link 4 is increased by 200; Plan C: the capacity of link 5 is increased by 200. The three plans can be evaluated by their corresponding enveloping MFD of modified networks. The results of comparison based on the OD proportion pattern with [1.5:1] are illustrated in Figure 15. The network performance at different demand levels can be observed by these enveloping MFDs. It can be obviously drawn that in the three plans, plan B can get the largest sum of OD flow rates and the flow rates of plan B are the largest one at the same accumulation. Therefore, plan B will make all vehicles at the network have better mobility and accessibility, especially in the congested regime. Similarly, other network design problem can also apply this method to evaluate alternative schemes.

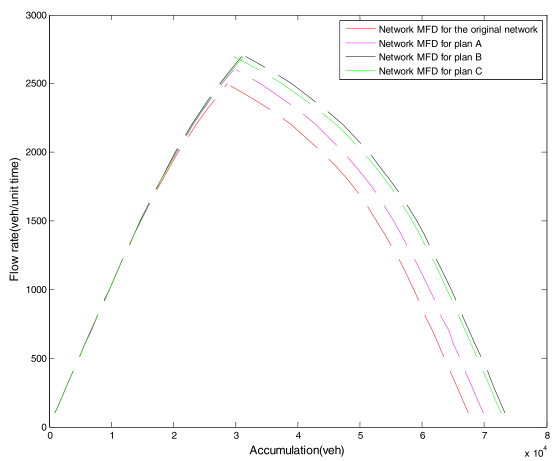

FIGURE 15. The comparison of enveloping MFDs for three plans of the capacity enhancement.

B. NGUYEN-DUPUIS NETWORK

To better test the effectiveness of the new analytical method and analyze the influence of OD matrices to the enveloping MFD, we apply the new analytical method to Nguyen-Dupuis network shown in Figure 16. Nguyen-Dupuis network [42] consists of 4 OD pairs (origin node 1, 4 and destination node 2, 3), 19 links, and 25 routes. The given link data are shown in Table 3. The uncongested link performance function uses Eq.(14) and the congested link performance function uses Eq.(15).

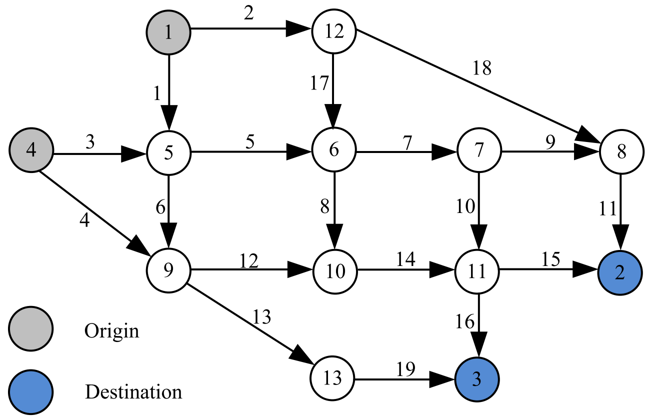

FIGURE 16. Nguyen-Dupuis network.

TABLE 3. Related data of links in Nguyen-Dupuis network.

Link number	t_a^0 (min)	c_a (vehicles/min)	Link number	t_a^0 (min)	c_a (vehicles/min)
1	7	75	11	12	75
2	9	75	12	10	135
3	9	75	13	9	50
4	15	50	14	3	100
5	6	85	15	9	75
6	9	100	16	7	75
7	4	75	17	5	40
8	13	50	18	14	75
9	8	50	19	11	50
10	11	55			

Various OD distribution patterns are loaded into Nguyen-Dupuis network, including $[q_{12}:q_{13}:q_{42}:q_{43}] = [1:1:1:1]$, $[q_{12}:q_{13}:q_{42}:q_{43}] = [2:2:1:1]$, $[q_{12}:q_{13}:q_{42}:q_{43}] = [3:3:2:2]$, $[q_{12}:q_{13}:q_{42}:q_{43}] = [1:1:2:2]$. With the total quantity of OD flow rates gradually increasing (from 40 vehicles/min to 280 vehicles/min), we obtain the enveloping MFDs of Nguyen-Dupuis network as shown in Figure 17.

Figure 17 further shows the different OD proportion matrices slightly modify the shape of enveloping MFDs for the whole network, which can effectively support the standpoint from Figure 14.

C. SIOUX-FALLS NETWORK

In order to evaluate the practicality of the proposed method, it is applied to a famous test network, Sioux–Falls network. Figure 18 shows the topology of Sioux–Falls network.

TABLE 4. Related data of links in sioux falls network.

Link number	t_a^0 (min)	c_a (vehicles/min)	Link number	t_a^0 (min)	c_a (vehicles/min)	Link number	t_a^0 (min)	c_a (vehicles/min)	Link number	t_a^0 (min)	c_a (vehicles/min)
1	6	75	21	10	50	41	5	75	61	4	75
2	4	125	22	5	50	42	4	100	62	6	50
3	6	75	23	5	50	43	6	125	63	5	50
4	5	100	24	10	100	44	5	50	64	6	100
5	4	75	25	3	75	45	3	75	65	2	75
6	4	125	26	3	75	46	3	75	66	3	75
7	4	75	27	5	50	47	5	50	67	3	75
8	4	75	28	6	75	48	4	50	68	5	75
9	2	100	29	4	50	49	2	50	69	2	75
10	6	75	30	8	125	50	3	75	70	4	50
11	2	100	31	6	50	51	8	50	71	4	50
12	4	100	32	5	125	52	2	100	72	4	100
13	5	50	33	6	75	53	2	75	73	2	50
14	5	100	34	4	75	54	2	75	74	4	50
15	4	75	35	4	50	55	3	50	75	3	125
16	2	100	36	6	125	56	4	75	76	2	100
17	3	75	37	3	55	57	3	75			
18	2	100	38	3	125	58	2	50			
19	2	75	39	4	125	59	4	50			
20	3	50	40	4	100	60	4	100			

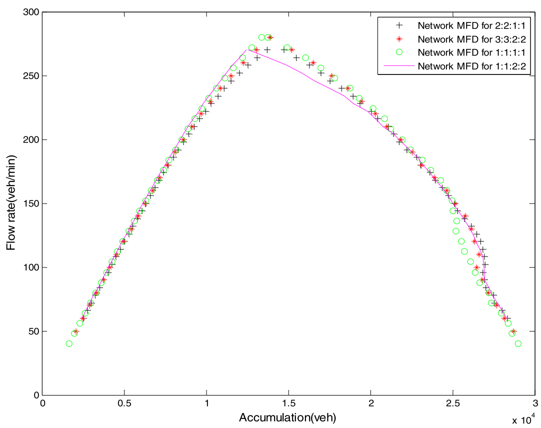

FIGURE 17. The comparison of enveloping MFDs for Nguyen-Dupuis network with different OD matrices.

Table 4 shows the link data of Sioux-Falls network. We assume there are 16 OD pairs and 4 OD distribution proportions in Sioux-Falls network, and Table 5 shows the information of OD pairs, and p_{rs} is distribution proportion of each OD pair. The uncongested link performance function uses Eq.(14) for testing and the congested link performance function uses Eq.(15) for testing. Moreover, it is should be noted that only the acyclic routes used in uncongested condition are effective routes in the congested condition and can be assigned flow rates in the congested condition, as described in Step 3 of the process of obtaining the enveloping MFD.

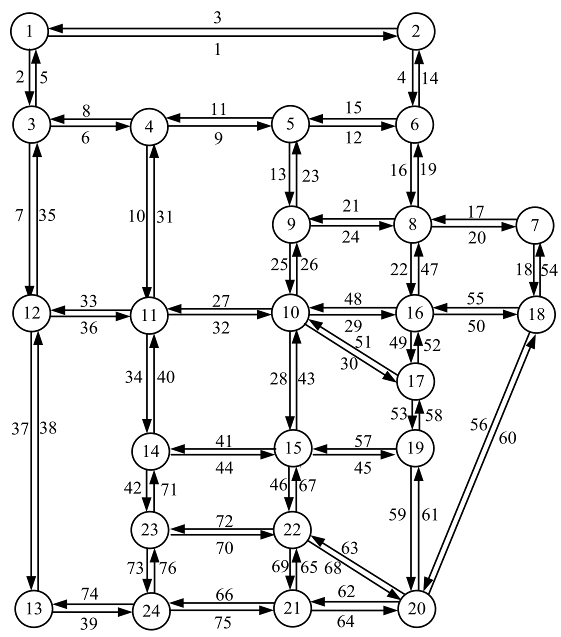

FIGURE 18. Sioux falls network.

The enveloping MFDs of Sioux Falls network under various OD distribution proportions are shown in Figure 19, which can also effectively support the standpoint from Figure 14. The method was run on a computer with Core i7@3.4Ghz running Windows 10 64-bit 8G RAM, and it takes 8.85 and 78.52 CPU seconds to reach the DUE satisfying the convergence criterions ($\sqrt{\sum_a \left(x_a^{n+1} - x_a^n\right)} \Big/ \sum_a x_a^n \le \bar{\varepsilon}$)

TABLE 5. **Related data of OD pairs in sioux falls network.**

OD pair number	OD distribution pattern A (p_{rs})	OD distribution pattern B (p_{rs})	OD distribution pattern C (p_{rs})	OD distribution pattern D (p_{rs})
1-18	0.0833	0.0750	0.0625	0.0500
7-3	0.0833	0.0750	0.0625	0.0500
20-12	0.0417	0.0500	0.0625	0.0750
12-21	0.0417	0.0500	0.0625	0.0750
12-2	0.0833	0.0750	0.0625	0.0500
7-20	0.0833	0.0750	0.0625	0.0500
1-10	0.0417	0.0500	0.0625	0.0750
21-7	0.0417	0.0500	0.0625	0.0750
13-18	0.0833	0.0750	0.0625	0.0500
13-4	0.0833	0.0750	0.0625	0.0500
2-12	0.0417	0.0500	0.0625	0.0750
14-16	0.0417	0.0500	0.0625	0.0750
11-21	0.0833	0.0750	0.0625	0.0500
10-20	0.0833	0.0750	0.0625	0.0500
24-10	0.0417	0.0500	0.0625	0.0750
19-9	0.0417	0.0500	0.0625	0.0750
$\sum_{rs} p_{rs}$	1	1	1	1

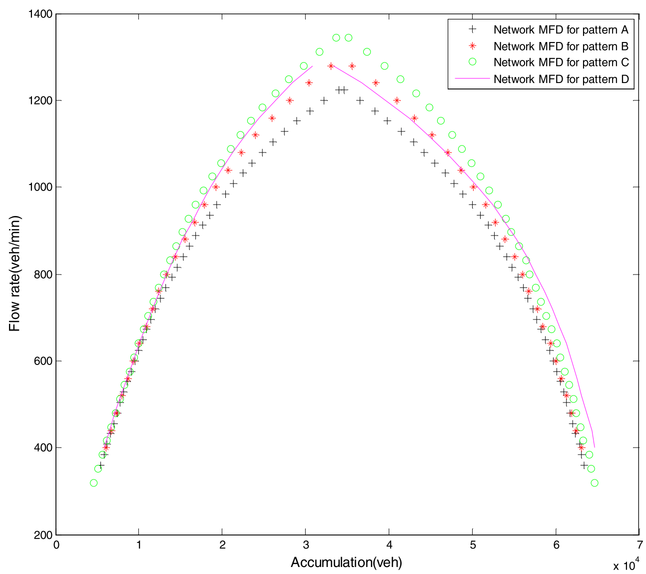

FIGURE 19. **The enveloping MFD of sioux falls network.**

of 10^{-4} and 10^{-5} respectively in the test network. When precision is better, the computational time is longer. It means that the proposed method can be used on the large-scale transportation network.

V. CONCLUSIONS

In this paper, a new analytical method to obtain the enveloping MFD is presented, which considers the well-defined sufficient condition and the route choice rule. The method can be used to verify the existence and reproducibility of enveloping MFDs in a general network and analyze and evaluate the macro-characteristics and the performance of the network. The well-defined sufficient condition is met by the STA models and the route choice is embodied by DUE principle. Considering that the SUTA model with the increasing link performance function cannot effectively depict the characteristics of congested traffic flows, the SCTA model based on the congested link performance function is introduced. Then, the congested DUE principle is analyzed, and the SCTA model is set up in detail. Finally, the new analytical method is proposed based on the SUTA and the SCTA models.

The example results reveal that the analytical method not only acquires the enveloping MFD but also verifies the existence and the reproducibility of enveloping MFDs for the whole network. It indicates that the enveloping MFD of the network can analyze and evaluate the macro-characteristics and the performance of the network. The proposed method can be potentially applied to macroscopic transportation planning, such as the evaluation of OD proportion patterns derived from land-use development and the evaluation of road network design. It is beneficial to maximize the utilization of the existing urban road system and avoid the misuse of land in transportation infrastructure.

In the enveloping MFD, the flow rate around the saturated state may be slightly larger than the actual one at the same accumulation. Hence, there are rooms for further improvement to this analytical method to gain more accuracy MFDs. Nevertheless, the obtained enveloping MFD will not deviate from the actual value too much and can be a valuable reference.

In the future, the analytical method should be further researched, to consider the saturated network including both the uncongested and congested links. Meanwhile, the

TABLE 6. The comparisons between the SUTA and the SCTA models.

Items	SUTA	SCTA
Traffic condition	All links are uncongested	All used links are congested
Flow rates on the link	Traffic demand flow rate	Traffic passing flow rate
OD matrix	Traffic demand distribution flow rates between OD pairs	Traffic passing distribution flow rates between OD pairs
Travel routes	Used routes and unused routes	Congested routes
Link performance function	A monotonically increasing function of flow rate	A monotonically decreasing function of flow rate
DUE principles	For each pair of OD, the travel time of all used routes is equal and is less than or equal that of any unused route.	For each pair of OD, all used routes are congested and the travel time of all used routes is the same.
The object function of the DUE model	$\min \sum_a \int_0^{x_a^0} t_a^0(w)dw$	$\max \sum_a \int_\varepsilon^{x_a^1} t_a^1(w)dw$
The link capacity	The flow rate of the intersection point between uncongested and congested link performance function curves is the capacity of a link.	
Route choice rule	Each user distinctly knows the state of the road network and tries to minimize his/her own travel time from the origin node to the destination node.	

signalized intersection and link capacity constraints will be considered in this proposed method to obtain accurate and practical MFDs rather than the enveloping MFD. In addition, the method of deriving the optimal OD proportion pattern, which is most congruous with the network topology, will be intensively investigated. Furthermore, empirical data will also be used to support this method. We believe that this analytical method is the first and an important step towards developing more advanced analytical methods for acquiring the accurate MFD in general networks based on the static assignment method.

APPENDIX A
EQUIVALENCY CONDITIONS

The Lagrangian of the equivalent maximization problem (6) with respect to the equality constraints (7) can be formulated as

$$L = Z(X^1) + \sum_{rs} v_{rs}\left(q_{rs} - \sum_l g_l^{rs}\right). \quad (A.1)$$

At the stationary point of the Lagrangian, the following conditions have to hold with respect to the route flow variables:

$$g_l^{rs}\frac{\partial L}{\partial g_l^{rs}} = 0 \quad \text{and} \quad \frac{\partial L}{\partial g_l^{rs}} \le 0\ \forall\, l \in R_{rs},\ r \in N,\ s \in N \quad (A.2)$$

Hence, the general first-order conditions for the maximization program are

$$\begin{cases} g_l^{rs}\left(c_l^{rs} - v_{rs}\right) = 0 \\ c_l^{rs} - v_{rs} \le 0, \end{cases} \quad \forall\, l \in R_{rs},\ r \in N,\ s \in N. \quad (A.3)$$

These conditions Eq.(A.3) hold for each used route between any OD pair in the network. Considering that all used routes must be congested routes and have positive flow rates, $g_l^{rs} > 0$ must hold for all congested routes in the network, so $c_l^{rs} = v_{rs}$ for all used routes. Thus, the Eq.(A.3) means that the travel time on all used routes connecting each OD pair will be the same at congested DUE state. Thus, the general first-order conditions satisfy the congested DUE principle, and the results of the SCTA model satisfy the congested DUE principle.

APPENDIX B

See Table 6.

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