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Optimal Multi-Objective Burn-In Policy Based on Time-Transformed Wiener Degradation Process

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ABSTRACT Burn-in is an effective and widely used means to improve product reliability by eliminating weak units before they are distributed in the market. Traditional burn-in that distinguishes weak units by failure during testing is inefficient and incompetent for degradation-failed products in which weak units degrade faster than normal individuals. Hence, the manufacturers have to turn to the degradation-based method. The mean lifetime to failure (MTTF) of a burnt-in population is diminished because of this type of burn-in increases the degradation level of all tested units. Ignoring the impact of burn-in leads to inferior test decisions. This study develops a multi-objective burn-in method that can simultaneously minimize the burn-in cost and maximize the burnt-in population's MTTF. We employ the time-transformed Wiener process with random effects to model the nonlinear degradation path of products and develop a burn-in scheme with two decision variables, namely, test duration and screening cutoff level. Cost expression and lifetime-based optimal objective are analytically developed. The optimal test policy is determined using the multi-objective evolutionary algorithm based on decomposition. A simulation study is conducted to demonstrate the usage and effectiveness of the multi-objective burn-in method.

INDEX TERMS Degradation, Wiener process, burn-in, expectation-maximization algorithm, multi-objective optimization.

I. INTRODUCTION

With increasing market competition, reliability has been regarded as one of the important quality indicators for modern products and exerts great effects on product price and manufacturer reputation. Therefore, manufacturers have great impulsion to design and produce products with high reliability to maintain market competitiveness. However, a small portion of poorly manufactured products, which are collectively called a weak subpopulation, inevitably exists due to the inherent variability in raw materials and accuracy fluctuation in the production process. These weak individuals are expected to fail earlier than normal individuals [1]. Although the proportion of the weak subpopulation can be reduced by improving the manufacturing technology and management level, these methods would entail high costs and long cycles. In comparison, burn-in has been proven to be one effective and low-cost means to identify weak units and improve products' onsite reliability from the customer's perspective [2], [3].

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Burn-in test can be classified into two types according to the failure mechanism. Traditional burn-in, which is based on catastrophic failure, is conducted by subjecting all units to normal or accelerated working load for a suitable duration. Then, the failed subjects are screened to prevent these weak subjects from being shipped to customers. Detailed discussions on the optimization of this type of burn-in policy can be found in Sheu and Chien [4], Cha and Finkelstein [5], Cha [2], and Ye *et al.* [6]. In reality, however, there exists a broad category of products or components whose reliability is directly related to the degradation of some quality characteristics [7]. Take the high-power laser diode (HPLD) as a example, which is widely used in laser processing industries. HPLD's main quality characteristic, the output optimal power, will constantly declines over time. If its output optimal power become less than the pre-specified level, the processing quality will no longer be able to meet the initial requirement, then it is regarded to have failed and should be replaced. With the aid of modern measurement techniques and Internet of Things, degradation data have become increasingly accessible at a relative low cost, and there will be a growing number of products whose failure can be defined by their degradation level [8]. For these degradation-failure products, although weak units have higher degradation rates than normal ones, they can still survive for a relatively long duration in traditional burn-in testing. Therefore, manufactures have to resort to the degradation measurement, that is, degradation-based burn-in, to perform screening.

The Wiener process and its extensions have been widely adopted to model degradation characteristics [9]–[13], and also successfully employed to construct degradation-based burn-in models. For example, Tseng and Peng [14] constructed a burn-in model using the integrated Wiener process and discussed burn-in scheme optimization. Ye *et al.* [15] studied the burn-in planning of products with two competing risks. Ye *et al.* [16] applied the Wiener process to model the measured degradation and discussed the optimal burn-in plan by jointly considering the burn-in test cost and maintenance expense. In the work of Peng [17], a degradation model with that considers random effects and measurement errors was proposed and the burn-in test is extended into a classification problem with several subpopulations. Zhai *et al.* [18] recently studied the optimization of degradation-based burn-in plan with considering the measurement errors. Moreover, gamma process has also been used in degradation-based burn-in optimization [1].

In many studies, test duration was considered as a main determinant of the burn-in scheme [1], [14], [18]. Long test duration increases the test cost and improves the burn-in effect and vice versa. However, on the other aspect, burn-in test will also increase the degradation level, thereby decreasing the remaining useful lifetime in field use. Its impact on the mean lifetime to failure (MTTF) of the burnt-in population should thus be taken into account. In addition, the previous studies usually adopted the single-objective optimization method from cost perspective, in which the test effect is measured by the misclassification probability [1], [14], [17], [18]. And the test expense and the misclassification probability are combined into a composite cost function. Nevertheless, as argued by Wang and Pham [19], specifying the weights of each sub-objective in the final composite objective remains challenging for decision makers. In fact, burn-in optimization essentially consists of two individual objectives, namely, minimizing the cost and maximizing the effect. The effect of burn-in test lie in the reliability improvement of the tested population, which include such aspects as increased MTTF, improvement of consistency and quality, enhancement of goodwill, etc. MTTF and quality are both two frequently used measures of reliability, and they are very useful in their respective application scenarios. When the warranty policy has been formulated before burn-in test, quality would be a feasible measure of the burn-in effect. However, when there is not yet an existing warranty or the manufacture tries to formulate a more optimal warranty policy than before, MTTF would play an important role in the decision-making process of the warranty period. In this study, we employ the Wiener process as a basic to develop the degradation model of the testing population, which consists of two sub-groups with different degradation rates. Then we develop a multi-objective optimization model for burn-in. We chose the per-unit cost increment and the burnt-in population's MTTF as the two objectives, and derive expressions for them. Additionally, a feasible multi-objective optimization method has also been studied.

The rest of the paper is organized as follows. In Section 2, we employ the Wiener process with random effects to model the performance degradation path of a type of product whose reliability is directly determined by the degradation level. Then, we derive the formulation of the product's lifetime distribution. Moreover, the burn-in test scheme is discussed, and parameter estimation using the expectation-maximization (EM) algorithm is presented. In Section 3, we briefly state the optimization problem from the two aspects of decision variables and objectives and derive the formulation of the burn-in cost and MTTF of the burnt-in population. The multi-objective optimization of the burn-in test scheme is established through an evolutionary algorithm. Section 4 provides a simulated example to demonstrate the availability and benefits of the proposed models. Section 5 presents the conclusion of the study and possible future work directions for this topic.

II. DEGRADATION-BASED BURN-IN MODEL

A. DEGRADATION PROCESS MODELLING

The Wiener process and its extensions have been widely used to describe performance degradation in reliability engineering practice and survival analyses. In this study, we adopt the non-homogeneous Wiener process to model degradation data, as it has been proven to be sufficient for describing the quality characteristics of many products with linear or nonlinear degradation paths. Let $\{Y(z), z > 0\}$ denotes the observed degradation values of a product's performance at time z. We assume that

$$Y(z) = \alpha\Lambda(z) + \sigma B(\Lambda(z)), \tag{1}$$

where α is the drift rate parameter, σ is a diffusion parameter, and $B(\cdot)$ is the standard Brownian motion which is used to represent the temporal variability of the degradation process. $\Lambda(z)$ is a monotonically increasing function with respect to time z and is specified according to the physical progress of degradation, such as fatigue growth, corrosion, and oxidation. This function is commonly referred to as a transformed time scale function [9], [11], that determines the path pattern of the degradation process. For example, the power low form of $\Lambda(z)$ is often used to describe the degradation of HPLD, and the linear form is widely adopted in many studies as well. Degradation process $Y(z)$ has independent increments $\Delta Y(z) = Y(z+\Delta z) - Y(z)$, and $\Delta Y(z)$ follows a normal distribution as follows:

$$\Delta Y(z) \backsim \mathcal{N}\left(\alpha \Delta\Lambda(z), \sigma^2 \Delta\Lambda(z)\right),$$

where $\Delta\Lambda(z) = \Lambda(z+\Delta z) - \Lambda(z)$.

Due to the variability of raw materials quantity and manufacturing accuracy, it is common to see that the degradation rates of individuals in a population may show notable unit-to-unit variability [20]. This type of characteristic in the population's degradation pattern can be solved by introducing the random effect parameter into the Wiener process [11]. In this study, we assume that $\alpha \backsim \mathcal{N}(\mu, \gamma^2)$ to capture the heterogeneity across individuals in the same population, which means that random effect parameter α is fixed for each unit, but unknown, and follows a normal distribution. This type of normality assumption is widely adopted by many studies [20]–[22]. Conditional on the drift rate α, degradation increment $\Delta Y(z)$ follows the normal distribution $\mathcal{N}\left(\alpha\Delta\Lambda(z), \sigma^2\Delta\Lambda(z)\right)$. To obtain the unconditional distribution of $\Delta Y(z)$, we first derive the moment generating function(MGF) of $\Delta Y(z)$, which is given by

$$M_{\Delta Y(z)|\alpha}(s) = \exp\left(\alpha\Delta\Lambda(z)s - \frac{1}{2}\sigma^2 s^2 \Delta\Lambda(z)\right). \quad (2)$$

Integrating α out of (2), we can obtain the unconditional MGF of $\Delta Y(z)$ as

$$\begin{aligned} M_{\Delta Y(z)}(s) &= E_\alpha(M_{\Delta Y(z)|\alpha}(s)) \\ &= \exp\left(-\frac{1}{2}\sigma^2 s^2 \Delta\Lambda(z)\right)\int_{-\infty}^{+\infty} \exp(\alpha\Delta\Lambda(z)) f(\alpha)\, d\alpha \\ &= \exp\left(\mu\Delta\Lambda(z)s - \frac{1}{2}s^2\left(\sigma^2\Delta\Lambda(z) + \gamma^2\Delta\Lambda^2(z)\right)\right), \end{aligned} \quad (3)$$

where $f(\alpha)$ is the probability distribution function (PDF) of random variable α. Therefore, from the MGF in (3), it is easily known that the unconditional distribution of $\Delta Y(z)$ is also normally distributed, and the distribution of $\Delta Y(z)$ is given by

$$\Delta Y(z) \backsim \mathcal{N}\left(\mu\Delta\Lambda(z), \sigma^2\Delta\Lambda(z) + \gamma^2\Delta\Lambda^2(z)\right). \quad (4)$$

It is noticed that the introduction of random effect increases the variance, and the degradation model used by Ye *et al.* [16] can be treated as a special case of the proposed model when α is fixed.

B. LIFETIME DISTRIBUTION

When a product's degradation level reaches the per-defined threshold, which is denoted by $\mathrm{Y_{th}}$, the product is considered to have failed [23]. Take HPLD as an example, when the laser's accumulated degradation level of output optical power exceeds 30% of the initial value, it is regarded as not qualified for field use because its performance can no longer meet the original design requirement [24]. This situation also exists in the pantograph of high-speed rail. When the thickness of the pantograph pan is less than the required minimal value, it must be treated as having failed and replaced to prevent catastrophic failure. Therefore, we can derive the product lifetime distribution from above mentioned degradation model with the introduction of failure threshold $\mathrm{Y_{th}}$, which is usually given by industrial standards. The product lifetime is merely the first passage time (FPT) of the degradation process [25]. It has been proved that the FPT of the standard Wiener process follows the famous IG distribution [22].

As proposed by Wang in [26], with a time transformation function, the non-homogeneous Wiener process in (1) can be treated as a time-transformed Wiener process. Assuming that $t = \Lambda(z)$, where z is the actual operating calendar time, while t denotes a new chosen time scale for making the derivation process tractable, then it is readily obtained that $Y(z) = L(t) = \alpha t + \sigma B(t)$, where $L(t)$ is a standard Wiener process with drift and diffusion parameters α and σ respectively. Failure occurs when the accumulated amount of degradation reach the preset critical value $\mathrm{Y_{th}}$. Let $Z = \inf\{z\colon Y(z) \geq \mathrm{Y_{th}}\}$ and $T = \inf\{t\colon L(t) \geq \mathrm{Y_{th}}\}$ denote the FPTs of $Y(z)$ and $L(t)$ respectively. Owing to the monotonically increasing characteristic of $\Lambda(\cdot)$, there is a one-to-one relationship between each realization of Z and T, as $Z = \Lambda^{-1}(T)$ [11]. Thus the PDF of Z is $f_T(\Lambda(z))\,\Lambda'(z)$ for $z \geq 0$, where $f_T(\cdot)$ is the PDF of T. Particularly, the linear degradation model in [18] and [16] can be treated as a special case of this model. In this study, we will discuss the lifetime distribution and burn-in plan under the new t time scale to make the derivation process tractable.

Conditional on α, it is readily obtained that T follows the inverse Gaussian (IG) distribution $\mathcal{IG}\left(\mathrm{Y_{th}}/\alpha,\ \mathrm{Y_{th}}^2/\sigma^2\right)$ with mean $\mathrm{Y_{th}}/\alpha$ and variance $\mathrm{Y_{th}}\cdot\sigma^2/\alpha^3$. The conditional PDF and cumulative distribution function (CDF) are defined as

$$f_{T|\alpha}(t \mid \alpha) = \frac{\mathrm{Y_{th}}}{\sqrt{2\pi\sigma^2 t^3}}\exp\left(-\frac{(\alpha t - \mathrm{Y_{th}})^2}{2\sigma^2 t}\right),$$

and

$$F_{T|\alpha}(t \mid \alpha) = \Phi\left(\frac{\alpha t - \mathrm{Y_{th}}}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\mathrm{Y_{th}}\alpha}{\sigma^2}\right)\cdot\Phi\left(-\frac{\alpha t + \mathrm{Y_{th}}}{\sigma\sqrt{t}}\right),$$

respectively, where $\Phi(\cdot)$ is the CDF of the standard normal distribution. Given the normal prior distribution of α, we can further obtain the following marginal PDF and CDF of T by integrating α out of the joint distribution of α and T as

$$f_T(t) = \int_\Omega f_{T|\alpha}(t \mid \alpha) f(\alpha)\,\mathrm{d}\alpha = E_\alpha\left[f_{T|\alpha}(t \mid \alpha)\right],$$

and

$$F_T(t) = \int_\Omega F_{T|\alpha}(t \mid \alpha) f(\alpha)\,\mathrm{d}\alpha = E_\alpha\left[F_{T|\alpha}(t \mid \alpha)\right],$$

where Ω and $f(\alpha)$ represent the support set and PDF of α, respectively. As given in [27], when $\alpha \backsim \mathcal{N}\left(\mu, \gamma^2\right)$ and

$A, B, C \in \mathbb{R}$, then the following two results hold.

$$E_\alpha\left[\exp\left(-\frac{(A-B\alpha)^2}{2C}\right)\right] = \sqrt{\frac{C}{B^2\gamma^2+C}}\cdot\exp\left(-\frac{(A-B\mu)^2}{2\left(B^2\gamma^2+C\right)}\right),$$
$$E_\alpha\left[\exp(A\alpha)\cdot\Phi(B+C\alpha)\right] = \exp\left(A\mu+\frac{1}{2}A^2\gamma^2\right)\cdot\Phi\left(\frac{B+C\mu+AC\gamma^2}{\sqrt{1+C^2\gamma^2}}\right),$$

where $E_\alpha[\cdot]$ is the expectation operator with respect to α. Then the PDF and CDF of T can be obtained as

$$f_T(t) = \frac{\Upsilon_{\text{th}}}{\sqrt{2\pi\left(\gamma^2t^4+\sigma^2t^3\right)}}\cdot\exp\left(-\frac{(\Upsilon_{\text{th}}-\mu t)^2}{2\left(\gamma^2t^2+\sigma^2t\right)}\right), \tag{5}$$

and

$$F_T(t) = \Phi\left(\frac{\mu t-\Upsilon_{\text{th}}}{\sqrt{\sigma^2t+\gamma^2t^2}}\right)+\exp\left(\frac{2\mu\Upsilon_{\text{th}}\sigma^2+2{\Upsilon_{\text{th}}}^2\gamma^2}{\sigma^4}\right)\cdot\Phi\left(-\frac{2tD\gamma^2+\sigma^2(\Upsilon_{\text{th}}+\mu t)}{\sigma^2\sqrt{\sigma^2t+\gamma^2t^2}}\right). \tag{6}$$

C. BURN-IN TEST MODEL BASED ON DEGRADATION

We consider a type of product whose degradation level $\{Y(z); z \geq 0\}$ follows the time-transformed Wiener process described in (1). After the manufacturing process of a product, the performance of a small portion called the inferior group deteriorates faster than that of normal ones. This inferior group always exists due to material defects or deviation in the manufacturing process. Hence, the product population is commonly assumed to consist of two groups with different degradation rates, i.e., normal and inferior subpopulations, as indicated in [14], [16], [18], [28]. Before the product population leaves the manufacture, burn-in test can be carried out to identify the inferior units from the main population, and prevent the inferior units from being delivered to customers. In this paper, normal and inferior group are assumed to share the same degradation model aside from the drift parameter that determines the degradation rate. Specifically, the time transformation function $\Lambda(\cdot)$ and variance coefficient σ are identical for these two subpopulations. We assume that the drift parameter of the normal group is α_1, which follows $\mathcal{N}(\mu_1, \gamma^2)$, and the drift parameter of the inferior group is α_2 which follows normal distribution $\mathcal{N}(\mu_2, \gamma^2)$, where $\mu_2 > \mu_1 > 0$. Before the burn-in test, the proportions of the normal and inferior groups are assumed to be fixed and known, as p_1 and p_2, respectively, where $p_1 + p_2 = 1$. This pair of value (p_1, p_2) is determined by the manufacturing technology level. For example, manufacturers with high manufacturing accuracy or raw material quality are likely to obtain a higher p_1 compared with their competitors.

We here construct a burn-in test model under the t time scale. For the degradation process $\{L(t); t \geq 0\}$, we suppose that $L(0) = 0$ without loss of generality. Hence, after the burn-in test with duration b under t time scale, the performance degradation increment is $L(b)$ which follows a Gaussian mixture model (GMM) and is given by

$$L(b) \backsim p_1\mathcal{N}\left(\mu_1 b, \sigma^2 b+\gamma^2b^2\right)+p_2\mathcal{N}\left(\mu_2 b, \sigma^2 b+\gamma^2b^2\right).$$

Let η denotes the cutoff point of degradation level to identify weak items from the main population. After the test, the probability of a tested unit being regarded as an acceptable one is

$$\Pr(L(b) < \eta) = p_1\Phi_1(b,\ \eta) + p_2\Phi_2(\text{b},\ \eta),$$

where $\Phi_1(b,\ \eta) = \Phi\left(\frac{\eta-\mu_1 b}{\sqrt{\sigma^2 b+\gamma^2 b^2}}\right)$, and $\Phi_2(b,\ \eta) = \Phi\left(\frac{\eta-\mu_2 b}{\sqrt{\sigma^2 b+\gamma^2 b^2}}\right)$. In the accepted population after burn-in, the proportions of normal and inferior units, denoted by $p_{b,1}$ and $p_{b,2}$, respectively, can be calculated by the condition probability method as

$$p_{b,1} = \Pr(\text{normal unit} \mid \text{accepted unit}) = \frac{p_1\Phi_1(b,\ \eta)}{\Pr(L(b) < \eta)}, \tag{7}$$
$$p_{b,2} = \Pr(\text{weak unit} \mid \text{accepted unit}) = \frac{p_2\Phi_2(b,\ \eta)}{\Pr(L(b) < \eta)}. \tag{8}$$

In general $p_1 > p_2$, thus there exist $p_{b,1} > p_1$ and $p_{b,2} < p_2$. This condition means that the normal unit proportion increases from original p_1 to $p_{b,1}$ through the burn-in test, and the inferior units proportion decreases from p_2 to $p_{b,2}$. As proposed by Ye [16], an interesting index called screening strength exists, and it is defined as the proportion of identified weak units through burn-in. Obviously, the screening strength is determined by cutoff η and test duration b and it can be calculated as

$$SS(\eta,\ b) = \frac{p_2 - p_{b,2}}{p_2}.$$

Another index of interest is the ratio of mis-rejection, which is defined as the normal unit proportion in the rejected subjects and given by

$$MS(b,\ \eta) = \Pr(\text{normal unit} \mid \text{rejected unit}) = \frac{p_1(1-\Phi_1(b,\ \eta))}{\Pr(L(b) > \eta)}.$$

The manufacturing cost of normal units that have been mis-rejected makes up the main component of the burn-in cost.

D. PARAMETER ESTIMATION

In actual application, model parameters should be estimated before burn-in. Two procedures are used to obtain the maximum likelihood estimator (MLE) of the parameters. First, the parametric form of $\Lambda(\cdot)$ should be determined based on degradation data or prior knowledge. If the information about the degradation pattern and physics of this type of products is sufficient, the parametric form of $\Lambda(\cdot)$ can be constructed by prior knowledge. As argued by Kawakubo *et al.* [29], analysis of the wear physics of the magnetic head in hard disk drivers reveals that its degradation level follows a power-law function. Meanwhile, if prior information is lacking, we can specify the function form for $\Lambda(\cdot)$ based on degradation data. The previous discussion indicates that the mean path of $Y(z)$ is $\mu\Lambda(z)$. Therefore, we can obtain the estimated mean path by averaging the degradation observations of all testing items and then employ a specific parametric form for $\Lambda(z)$ to fit the mean path. In addition, as proposed in [10], the semi-parametric inference method for Wiener process can also be used to determine the $\Lambda(\cdot)$ function type.

The second procedure is statistical inference for model parameters Θ, that are $\mu_1, \mu_2, \gamma, \sigma, p$ and Λ which represents the parameters involved in the time transform function. Suppose that N units are subject to a degradation test. For simplicity, in this study we only discuss the case when all testing units are inspected with the same inspection times. For the case of different inspection times among various units,the inference method proposed in [26] can be employed to obtain parameter estimations. Assume that the i-th unit is inspected at time z_j with observation $Y_i(z_j), i = 1, \ldots, N, j = 1, \ldots, J$. Let $y_{ij} = y_i(z_j) - y_i(z_{j-1})$ be the degradation increments and $\lambda_j = \Lambda(z_j) - \Lambda(z_{j-1})$ be the time increment in z time scale. Then, y_{ij} is independent and follows mixed Gaussian distribution. The PDF of y_{ij} is

$$f(y_{ij} \mid \Theta) = p_1 f_1(y_{ij} \mid \Theta) + p_2 f_2(y_{ij} \mid \Theta),$$

where $f_1(\cdot)$ and $f_2(\cdot)$ denote the PDF of normal and weak subpopulations, respectively. Considering that the PDF of y_{ij} is in the form of a sum, the corresponding log-likelihood function, which is composed of this type of PDF, can not be directly maximized for MLE. Here, we will introduce a latent variable and employ the EM algorithm to find MLE iteratively. We define a random vector $h_i = (h_{i,1}, h_{i,2})$, where h_{i1} and h_{i2} are class indexes for the i-th unit, $h_{i,1}, h_{i,2} \in \{0, 1\}$ and $h_{i,1} + h_{i,2} = 1$. For example, $\Pr\{h_{i1} = 1, h_{i2} = 0\} = p_1$ indicates that the i-th unit is a normal item with probability p_1. With the help of latent variable h_i, the joint density function of complete data (y_{ij}, h_i) can be obtained as follows:

$$f(y_{ij}, h_i \mid \Theta) = \prod_{k=1}^{2}\left(p_k \prod_{j=1}^{J} f_k(y_{ij} \mid \Theta)\right)^{h_{ik}}.$$

Hence, we can form the log-likelihood function for Θ up to a constant as

$$\begin{aligned} L(\Theta \mid y_{ij}, h_i) &= \sum_{i=1}^{N}\sum_{k=1}^{2} h_{ik}\left(\ln p_k + \sum_{j=1}^{J}\ln f_k(y_{ij} \mid \Theta)\right) \\ &\propto \sum_{i=1}^{N}\sum_{k=1}^{2} h_{ik}\left(\ln p_k - \frac{1}{2}\sum_{j=1}^{J}\left(\ln\left(\sigma^2\lambda_j + \gamma^2\lambda_j^2\right)\right.\right. \\ &\quad \left.\left. + \frac{(y_{ij} - \mu_k\lambda_j)^2}{\sigma^2\lambda_j + \gamma^2\lambda_j^2}\right)\right) \end{aligned}$$

E-step According to the definition of latent variable h_i, we have

$$\begin{aligned} E\left(h_{ik} \mid y_{ij}, \hat{\Theta}^{(m)}\right) &= \Pr\left\{h_{ik} = 1 \mid y_{ij}, \hat{\Theta}^{(m)}\right\} \\ &= \frac{\hat{p_k}^{(m)} \sum_{j=1}^{J} f_k\left(y_{ij} \mid \hat{\Theta}^{(m)}\right)}{\sum_{k=1}^{2} \hat{p_k}^{(m)} \sum_{j=1}^{J} f_k\left(y_{ij} \mid \hat{\Theta}^{(m)}\right)}. \end{aligned} \tag{9}$$

Then, the Q-function can be obtained by taking the conditional expectation of log-likelihood function with respect to h_{ik} as follows:

$$\begin{aligned} Q(\Theta \mid \Theta^{(m)}) &= E\left(L(\Theta) \mid y_{ij}, \hat{\Theta}^{(m)}\right) \\ &= \sum_{i=1}^{N}\sum_{k=1}^{2} E(h_{ik}) \ln p_k - \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{2}\sum_{j=1}^{J} E(h_{ik}) \\ &\quad \cdot \left(\ln\left(\sigma^2\lambda_j + \gamma^2\lambda_j^2\right) + \frac{(y_{ij} - \mu_k\lambda_j)^2}{\sigma^2\lambda_j + \gamma^2\lambda_j^2}\right). \end{aligned} \tag{10}$$

M-step Through maximizing the Q-function (10) with respect to Θ, we can obtain the $\hat{\Theta}^{(m+1)}$ for the next iteration. As we can see, p_k in (10) is only involved in the first term on the left hand side, denoted by $Q_1(p_k \mid y_{ij}, \Theta^{(m)})$. Analytically maximizing this term yield

$$\hat{p}_k^{(m+1)} = \frac{\sum_{i=1}^{N} E(h_{ik})}{N}.$$

The other parameters are all involved in the second part of the Q-function. Hence we can obtain the remaining solutions by minimizing the second part, denoted by $Q_2(\mu_k, \sigma, \gamma, \Lambda \mid y_{ij}, \Theta^{(m)})$. All test units are assumed to share the same inspection times, which is easily to meet in the degradation test by manufacturers. Under this assumption, when $\Lambda(t)$ take a certain parametric form, such as linear, power law or exponential function, the parameter of $\Lambda(\cdot)$ can be easily and simultaneously obtained with the other ones. Here, we employ a multi-dimensional unconstrained optimization algorithm("fminsearch" function in MATLAB) to obtain $\hat{\mu}_k^{(m+1)}, \hat{\sigma}^{(m+1)}, \hat{\gamma}^{(m+1)}$ and $\hat{\Lambda}^{(m+1)}$. Specifically,

$$\begin{aligned} &\left(\hat{\mu}_k^{(m+1)}, \hat{\sigma}^{(m+1)}, \hat{\gamma}^{(m+1)}, \hat{\Lambda}^{(m+1)}\right) \\ &\qquad = \arg\max Q_2\left(\mu_k, \sigma, \gamma, \Lambda \mid y_{ij}, \Theta^{(m)}\right) \end{aligned}$$

Substituting these MLEs into (9), the updated value of $E(h_{ik})$ is acquired. Accordingly, the new Q-function is obtained. Therefore the EM algorithm can be performed by iteratively processing the E-step and M-step until all MLEs converge. Additionally, the bootstrap method can be used here to assess the uncertainty of these parameter estimators [26].

III. MULTI-OBJECTIVE OPTIMIZATION OF THE BURN-IN SCHEME

A. ASSUMPTIONS

To further specify the application scenario of the burn-in scheme optimization, the following explanations are established.

(1) Burn-in is an essential classification problem based on the degradation level of tested units. Burn-in duration b is a key decision variable. If the test duration is inadequate, degradation measurements of the weak units will be close to that of the normal ones. Hence, achieving a presentable classification result is difficult, especially when the products exhibit low degradation rates. By contrast, if the test duration is too long, the normal population will waste its lifetime in the burn-in test although many weak units can be successfully removed.

(2) After burn-in duration b, optimal cutoff value η is another key decision variable that determines the screening strength. Units whose degradation level exceeds cutoff level η will be regarded as inferior individuals and disposed without any reward. Therefore, a low η represents a strict standard for accepting a tested unit as a qualified item.

(3) The principal aim of burn-in is to improve field-use reliability. In this study, we choose the burnt-in population's MTTF, which is crucial to product price and brand value, to assess the burn-in effect. On the other side, burn-in cost, including the test expenditure itself and the manufacturing cost of those rejected units, is another important factor to be considered. Burn-in optimization is to balance the test effect with the incurred cost, and offers support means for decision-making.

B. BURN-IN COST FORMULATION

From the manufacturer's point of view, burn-in cost mainly consists of two parts. The first part, which is the direct test cost, is the burn-in test expense. It is calculated by $c_0 + c_t\Lambda^{-1}(b)$, where c_0 is the fixed burn-in cost for each unit, c_t represents the unit time burn-in cost for individuals on the average, and b is the burn-in duration under t time scale. In practice, c_0 and c_t are determined by the test method that is conducted according to corresponding industrial standards. For a given class of products, these two cost parameters are constant; thus the direct test cost is directly proportional to the test duration. The second part is called the indirect burn-in cost, which is defined as the per-unit cost increment incurred by the burn-in test. Assuming that the original per-unit manufacturing cost is c_m and increase to $c_m/(1-MS(b,\eta))$ after burn-in, hence the indirect cost for each unit is $c_m/(1-MS(b,\eta))-c_m$, and the per-unit burn-in cost can be calculated as follows:

$$C_b(b,\eta) = c_0 + c_t\Lambda^{-1}(b) + \frac{c_m \cdot MS(b,\eta)}{1-MS(b,\eta)}. \tag{11}$$

According to (11), the direct test cost increases as the test duration b increases. The indirect cost is jointly determined by test duration b and classification cut-off η. A small η results in the removal of many tested units, indicating a rigorous screening standard and high corresponding test cost of burnt-in units.

C. MTTF OF BURNT-IN UNITS

The advantage of burn-in is its capability to pick out weak units and increase the burnt-in population's MTTF. In this section, we derive the field-use reliability from the customer's point of view. We let random variable u be the degradation level of an accepted unit right after the burn-in test with duration b and screening cutoff η. It can be verified that u follows the truncated normal distribution, and the PDF is given by

$$g_k(u) = \frac{1}{\sigma_b \cdot \Phi\left(\frac{\eta-\mu_k b}{\sigma_b}\right)} \cdot \phi\left(\frac{u-\mu_k b}{\sigma_b}\right), \tag{12}$$

where $\sigma_b = \sqrt{\sigma^2 b + \gamma^2 b^2}$ is the degradation variance of the units right after burn-in, and k is an index for normal ($k=1$) and weak ($k=2$) units. We discuss the lifetime under t time scale. Let T_b represents the remaining lifetime of the burnt-in population, and $T_{b,k}$ is the corresponding lifetime of the normal or weak subpopulation. Conditional on the initial degradation level u, the PDF and CDF of $T_{b,k}$, written as $f_{T_{b,k}}(t|u)$ and $F_{T_{b,k}}(t|u)$, can be obtained by replacing μ with μ_k and Υ_{th} with $\Upsilon_{\text{th}} - u$ in (5) and (6) respectively. Then the PDF of $T_{b,k}$ is given by

$$f_{T_{b,k}|u}(t|u) = \frac{\Upsilon_{\text{th}} - u}{\sqrt{2\pi\left(\gamma^2 t^4 + \sigma^2 t^3\right)}} \cdot \exp\left(-\frac{(\Upsilon_{\text{th}} - u - \mu_k t)^2}{2\left(\gamma^2 t^2 + \sigma^2 t\right)}\right).$$

By using the law of total probability, the marginal PDF of $T_{b,k}$ can be derived as

$$f_{T_{b,k}}(t) = E_u\left[f_{T_{b,k}}(t,\ u)\right] = \int_{-\infty}^{\eta} f_{T_{b,k}}(t|u) \cdot g_k(u)\,\mathrm{d}u. \tag{13}$$

The lifetime PDF of the burnt-in subjects then can be calculated by conditional on the class index of this subject, which is given by

$$f_{T_b}(t;\ b,\ \eta) = \sum_{k=1}^{2}\left(p_{b,k} \cdot f_{T_{b,k}}(t)\right). \tag{14}$$

The PDF of T_b then can be obtained by substituting (7), (8) and (13) into (14) as follows:

$$f_{T_b}(t;b,\eta) = \frac{1}{\sigma_b \Pr(L(b) < \eta)} \sum_{k=1}^{2} p_k \cdot \int_{-\infty}^{\eta} \frac{(\Upsilon_{\text{th}} - u)}{\sqrt{2\pi\left(\gamma^2 t^4 + \sigma^2 t^3\right)}} \cdot \exp\left(-\frac{(\Upsilon_{\text{th}} - u - \mu_k t)^2}{2\left(\gamma^2 t^2 + \sigma^2 t\right)}\right) \cdot \phi\left(\frac{u-\mu_k b}{\sigma_b}\right)\mathrm{d}u. \tag{15}$$

Let Z_b denotes the burnt-in population lifetime under the actual operating calendar time scale of z. As proposed by Wang in [11], considering the monotonically increasing characteristic of $\Lambda(\cdot)$, the function relationship between Z_b and T_b can be obtained as

$$Z_b = \Lambda^{-1}(b + T_b) - \Lambda^{-1}(b).$$

Eventually, the actual operating calendar time MTTF of burnt-in units can be calculated as

$$MTTF(b,\eta) = \int_0^{\infty}\left(\Lambda^{-1}(b+t) - \Lambda^{-1}(b)\right) f_{T_b}(t;\ b,\ \eta)\,\mathrm{d}t. \tag{16}$$

Equation (16) have no close-form expression. Here we use the numerical integration method to calculate the MTTF under a specific burn-in scheme (b, η). For each t with a fixed value, $f_{T_b}(t;\ b,\ \eta)$ in (15) can be effectively computed using some well-developed numerical meanings, such as the Riemann-Stieltjed method [30] and the integral method provided in [16]. It is not difficult to verify that, under an appropriate burn-in scheme, the MTTF of burnt-in units will gain a significant increment compared with the original population and the mean lifetime increment can be regarded as the profit of the burn-in test.

D. MULTI-OBJECTIVE OPTIMIZATION DESIGN

As discussed above, the burn-in scheme involves two decision variables: burn-in duration b and degradation cut-off η; and its scheme optimization consist of two individual objectives: minimizing per-unit the cost increment caused by burn-in and maximizing MTTF of the burnt-in population, which can be regarded as reliability improvement through burn-in application. These objectives are in conflict with each other. Hence, there does not exist a single feasible solution that can simultaneously make all objectives absolutely optimal. Optimization aims to seek out the trade-offs for balancing the cost increment and burn-in effect under some resource restrictions and provide decision makers with a non-inferior solution set, which is called the Pareto optimal solutions in the multi-objective optimization problem (MOP) [19]. Then manufactures can select a proper scheme from the non-inferior solution set according to their own situation, such as market positioning and competitiveness. The expected MOP problem can now be formulated as

$$
\begin{aligned}
&Min\ \ C_b(b, \eta) \text{ and } Max\ \ MTTF(b, \eta)\\
&\text{Subject to: } b \le \mathrm{T_{max}}\\
&\qquad\qquad C_b(b, \eta) \le \mathrm{C_{max}}\\
&\qquad\qquad MTTF(b, \eta) \ge \mathrm{MTTF_0}
\end{aligned}
$$

where $\mathrm{T_{max}}$ is the maximum test duration in t time scale, $\mathrm{C_{max}}$ is the affordable maximum cost for the burn-in test, and $\mathrm{MTTF_0}$ is the required minimum mean lifetime. The solutions out of these three constraint conditions are useless for the field application. The optimization algorithm is not an important topic in this study, and many existing methods can be used to perform optimization efficiently. Evolutionary algorithms are proved to be highly effective in multi-objective optimization [31], [32]. In this study, we will solve the burn-in scheme optimization problem using the multi-objective evolutionary algorithm based on decomposition (MOEA/D) which has been verified as an excellent representative of evolutionary algorithms and is famous for its low computational complexity and even distributed solutions [33]. The source codes and application note of the MOEA/D algorithm are available on the its homepage (https://dces.essex.ac.uk/staff/zhang/webofmoead.htm).

IV. ILLUSTRATIVE EXAMPLE

In this section, the MEMS device example provided by Peng et al. [34] is utilized for illustration. The MEMS device equipped with a micro-engine is a typical product that suffers from degradation-type failure. Similar to Ye et al. [16] and Zhai et al. [18], we assume that the MEMS device population consists of two subpopulations, namely, majority of normal units with the proportion $p_1 = 0.85$ and a small part of inferior units with the proportion $p_2 = 0.15$. We assume that the degradation path of the two subpopulations can be modeled by the time-transformed Wiener process with a distinct drift rate that is represented by the random effect parameter. We here set the two subpopulation's random effect parameters α_1 and α_2 in accordance with the normal distribution with mean $\mu_1 = 0.54 \times 10^{-4}$ and $\mu_2 = 1.28 \times 10^{-4}$ respectively, and the common variance coefficient $\gamma = 0.12 \times 10^{-4}$. The two subpopulations share an identical diffusion coefficient with $\sigma = 1.82 \times 10^{-4}$. We adopt the power law time transform function as $\Lambda(z) = z^q$ and let $q = 1.3$. The degradation failure threshold $\mathrm{Y_{th}}$ is 6.8. Fig. 1 shows the degradation path simulation result of 100 units.

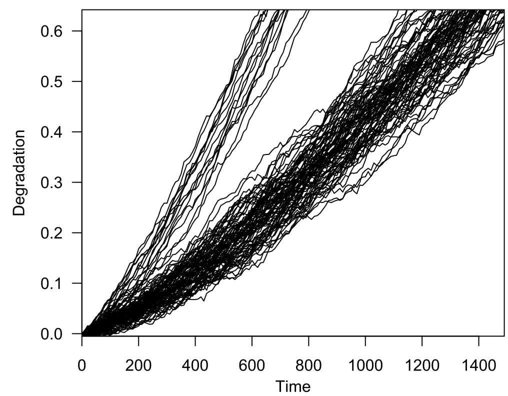

FIGURE 1. Simulated degradation paths.

It is obvious noticed that there are a small proportion degrading faster than the majority of the population, and the units in this population cannot meet the application requirement due to their short service life, which will compromise the manufacture's brand reputation. Therefore, burn-in test is essential for manufacturers to eliminate weak units and increase the field-use lifetime of their products from the customer's perspective. The following cost profiles are adopted to construct the burn-in cost formulation: $c_0 = 5\$$ /unit, $c_t = 0.3\$$ /unit time, and $c_m = 2000\$$ /unit.

A. PARAMETER ESTIMATION

To assess the performance of the proposed parameter estimation algorithm, a Monte Carlo simulation study is carried out. We use the identical values of model parameters as given in above section, and choose the number of sample units $N = 30,\ 60,$ and 90, respectively. The number of observation $J = 500$, the observation interval $\lambda_j = 2$. We generate

TABLE 1. Bias and RMSE of the MLE estimates based on 2000 simulation replications.

Parameters	N = 30		N = 60		N = 90	
	Bias	RMSE	Bias	RMSE	Bias	RMSE
$\mu_1(\times 10^{-5})$	0.574	1.203	0.294	0.642	0.122	0.475
$\mu_2(\times 10^{-5})$	1.342	2.602	0.418	1.262	0.281	0.558
$\gamma(\times 10^{-5})$	0.188	0.521	0.122	0.414	0.121	0.402
$\sigma(\times 10^{-5})$	1.470	2.250	0.852	1.564	0.588	0.554
q	0.253	0.752	0.085	0.127	0.078	0.112
p_1	0.028	0.105	0.011	0.038	0.010	0.034

the degradation paths and then perform the EM algorithm to estimate the parameters. Under each setting of sample size, the simulation is respectively repeated 2000 times, based on which the bias and the root mean square errors (RMSE) of the estimations are obtained. The results are presented in Table 1. It can be observed that the estimation accuracy is quite high, and the estimation biases and RMSEs decrease as the sample size increases. From the results of subsection IV-C, we further know that the biases of estimated parameters could meet the requirement of burn-in scheme optimization with a reasonable quantity of testing units.

B. OPTIMAL BURN-IN SCHEME

We employ the MOEA/D algorithm provided in [35] to obtain the optimal burn-in test scheme. The MOEA/D algorithm decomposes an MOP into a number of scalar single-objective optimization subproblems by using the Tchebycheff aggregation approach. These subproblems are simultaneously optimized based on the information of their neighborhood with a relatively low computational cost. In the implementation, the algorithm settings are as follows: population size $N_p = 100$, neighborhood size $T = 20$, the probability of selecting mating parents from the neighborhood is 0.9, the crossover rate is 1.0, and the mutation rate is 0.5. The initial populations are generated by performing uniform random sampling from the feasible search space, and the initial weight vectors $\lambda^1, \ldots, \lambda^{N_p}$ are generated by using the method provided in subsection IV-E of [35]. The simulated binary crossover method and polynomial mutation are used to produce the new generation of population. The achieved population size is set to 30, and the maximal generation number is 300. MOEA/D algorithm is able to search out the non-inferior solutions from a given solution space. In practice, the solution space should be specified by the decision makers based on the realistic restriction of decision variables, such as the physical meaning, the actual Implementation conditions. In this study, the required mean lifetime MTTF_0 is set to 7600 hours, and the affordable burn-in duration and test cost are 500 hours and 200$, respectively.

The obtained Pareto solutions that simultaneously minimize the burn-in cost and maximize the MTTF of the burnt-in population are indicated in Fig. 2. It is clear that the obtained solutions are evenly distributed, and these 30 solutions are called the Pareto frontier. For illustrative purposes, we evenly sample 10 solutions from the Pareto frontier to further demonstrate the usage of the optimization result. The selected solutions are denoted by a red star in Fig. 2 and presented in Table 2.

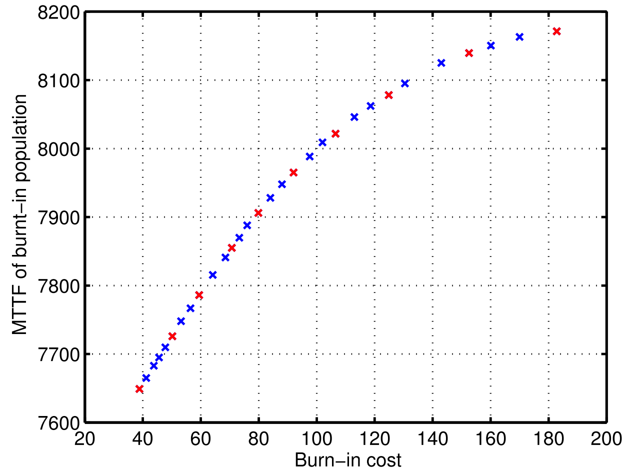

FIGURE 2. Pareto solutions.

TABLE 2. Selected Pareto solutions.

Solution number	Objectives		Decision variables		SS	MS
	Cost	MTTF	b^*	η^*		
1	193.6	8173.1	165.7	0.065	0.9727	0.3014
2	152.5	8139.6	164.2	0.069	0.9498	0.2026
3	124.8	8078.2	154.8	0.066	0.9399	0.2675
4	106.5	8021.7	148.8	0.062	0.8795	0.1742
5	92.1	7965.1	121.8	0.055	0.7535	0.1829
6	86.5	7905.2	118.7	0.061	0.6825	0.1651
7	70.7	7854.1	98.8	0.057	0.6277	0.1724
8	59.4	7786.5	95.2	0.058	0.5846	0.1248
9	50.2	7725.9	82.8	0.042	0.5471	0.1027
10	38.8	7649.1	70.9	0.041	0.4915	0.092

As shown in Fig. 2, this type of multi-objective optimization is only able to provide a decision set which is consist of Pareto solutions. The final decision will be made by the decision makers based on their own specific situation. Specific to the burn-in test, manufacturers could finally make the test scheme based on their products' circumstances, such as market positioning, cost, and their profit, etc. For example, in cases where manufacturers focus on product MTTF, selecting the first solution with 165.7 test duration and 0.065 classification cutoff is feasible. Accordingly, the burnt-in population MTTF will reaches to 8172.1 hours and the burn-in cost is 193.6$. By contrast, if the test cost is limited, selecting the 10th solution it is suggested to improve the field-use MTTF to 7649.1 hours with a relatively low burn-in cost of 38.8$.

In addition to MTTF, the proportion of weak individuals in the burnt-in population is another important factor to be considered. The SS of the first solution is as high as 97.27%, which means that the most weak units has been picked out.

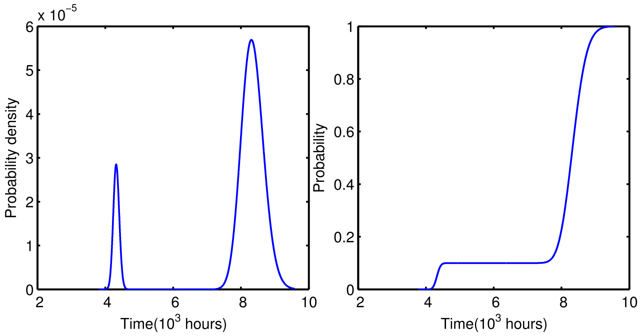

FIGURE 3. PDF and CDF of products without burn-in.

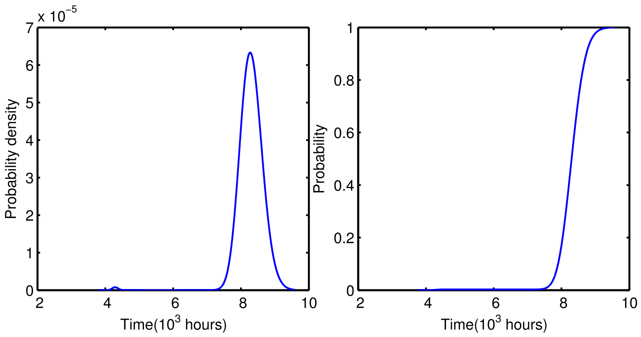

FIGURE 4. PDF and CDF of products with 1st burn-in policy.

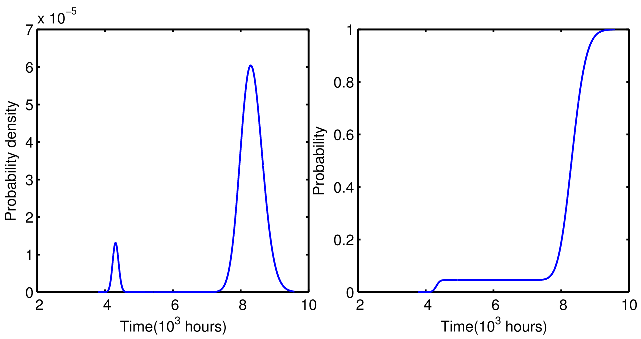

FIGURE 5. PDF and CDF of products with 10th burn-in policy.

With the 10th solution, the SS is 49.15%, indicating that only half of the weak units can be eliminated.

We further demonstrate the burn-in effect from the product lifetime distribution aspects. In accordance with the simulation assumption, we draw the lifetime PDF and CDF of the products before the burn-in test in Fig. 3. A relatively high proportion of weak units exists, and their MTTF is approximately 4000 hours. The lifetime distributions of the burnt-in population under the 1st and 10th solutions in Table 2 are presented in Figs. 4 and 5 respectively. It is readily seen that the 10th test policy with a cost of 38.8$ effectively decreased the weak unit proportion, and the 1st policy with a cost of 193.6$ is able to pick out almost all weak individuals. Additionally, manufacturers could significantly improve their products' consistency on field use lifetime with the 1st test policy, which is very useful in formulating warranty policies.

TABLE 3. Sensitivity of optimal policies considering the estimation biases for $\mu_{1,2}$, γ, and σ.

ϵ_1	ϵ_2	ϵ_3	1st solution			10th solution		
			Cost	MTTF	SS	Cost	MTTF	SS
+5%	+5%	+5%	181.5	7768.2	0.9788	36.5	7250.1	0.5542
+5%	0	0	180.8	7728.6	0.9782	36.9	7266.5	0.5601
+5%	-5%	-5%	179.7	7770.5	0.9800	36.8	7268.5	0.5554
0	+5%	0	190.5	8168.5	0.9739	38.2	7645.8	0.4988
0	0	-5%	189.2	8170.5	0.9730	38.4	7612.8	0.4968
0	-5%	+5%	188.7	8169.2	0.9723	38.1	7600.8	0.4972
-5%	+5%	-5%	205.5	8558.4	0.9695	43.2	8171.5	0.4602
-5%	0	+5%	208.7	8572.5	0.9689	42.8	8105.8	0.4650
-5%	-5%	0	206.4	8577.8	0.9697	42.5	8102.5	0.4638
0	0	0	193.6	8173.1	0.9727	38.8	7649.1	0.4915

C. SENSITIVITY ANALYSIS

The optimal policy is obtained depending on the true values of the model parameters. However, the parameter values would be subject to estimation bias especially when the degradation observation is inadequate in the pilot study. Hence this type of estimation errors must be considered, and sensitivity analysis should be carried out to observe the robustness of the optimal burn-in. The parameter vector Θ can be divided into two groups, the first group is model-related, μ_k, γ, and σ; the other parameters, p_1 and p_2, are determined by the manufacturing technology which is generally considered to be stable for some time. We here only conduct sensitivity analysis for the model parameters. Without loss of generality, we assume that ϵ_1, ϵ_2, and ϵ_3 denote the estimation bias for $\mu_{1,2}$, γ, and σ, respectively. Under the 1st and 10th policy, burn-in tests are re-conducted for various model parameter combinations of $(1+\epsilon_1)\mu_{1,2}$, $(1+\epsilon_2)\gamma$, and $(1+\epsilon_3)\sigma$. Table 3 presents the cost, MTTFs, and SS for each parameter combination. From these results, see the burn-in policy is quite robust for the estimation bias on γ and σ, and relatively sensitive to $\mu_{1,2}$ departure.

V. CONCLUSION

With the rapid development of measurement techniques and further understanding of failure mechanisms, a growing number of products will be subjected to burn-in tests via the degradation-based method, which can significantly improve test efficiency with a relatively low cost. In this type of degradation-based burn-in, the test duration is an important contributor to the burn-in effect. An extremely long test duration will considerably increase the degradation level of test units and hence shorten their remaining useful lifetime. Therefore, burn-in optimization should consider its negative effect on the remaining mean lifetime of the burnt-in population aside from the classification result. This work proposes a degradation-based burn-in model and constructs a test policy with two individual objectives. The first objective is the test cost, which includes the direct test expenditure and the cost increment caused by those rejected units. The other objective is the MTTF of the accepted population, which represents the burn-in profit. The optimal solution set of test duration and classification cutoff is obtained using the MOEA/D algorithm. The results of this study can provide manufacturers a decision set and help them design an appropriate burn-in test according to their own product strategy.

Future work could tackle several topics of interest. In the case where the degradation path is monotonic, the gamma or inverse Gaussian process is highly suitable for degradation modeling, and the corresponding burn-in policy optimization deserves further investigation. In practice, besides of faster degradation rate, the weak subpopulation would also suffer from larger degradation variance than normal ones. In such situation, the method of using different γ for two subpopulations would become a reasonable choice. The proposed burn-in optimization should be classified as an off-line method, in which the test duration is determined based on the burn-in model before testing. Online optimization, which utilizes field degradation data during a test to obtain the minimal duration, is another topic of interest. In this study, only the degradation value at the end time point are utilized to perform burn-in screening, this may result in the loss of substantial information on degradation process characteristics. By contrast, the burn-in method of using the measured degradation sequence may be a valuable direction for further research, which would further shorten the test duration and obtain more accurate burn-in result.

REFERENCES

[1] C.-C. Tsai, S.-T. Tseng, and N. Balakrishnan, "Optimal burn-in policy for highly reliable products using gamma degradation process," *IEEE Trans. Rel.*, vol. 60, no. 1, pp. 234–245, Mar. 2011.

[2] J. H. Cha, "A survey of burn-in and maintenance models for repairable systems," in *Replacement Models With Minimal Repair*. London, U.K.: Springer, 2011, pp. 179–203.

[3] J. H. Cha and G. Pulcini, "Optimal burn-in procedure for mixed populations based on the device degradation process history," *Eur. J. Oper. Res.*, vol. 251, no. 3, pp. 988–998, 2016.

[4] S.-H. Sheu and Y.-H. Chien, "Minimizing cost-functions related to both burn-in and field-operation under a generalized model," *IEEE Trans. Rel.*, vol. 53, no. 3, pp. 435–439, Sep. 2004.

[5] J. H. Cha and M. Finkelstein, "Stochastically ordered subpopulations and optimal burn-in procedure," *IEEE Trans. Rel.*, vol. 59, no. 4, pp. 635–643, Dec. 2010.

[6] Z. S. Ye, L. C. Tang, and M. Xie, "A burn-in scheme based on percentiles of the residual life," *J. Qual. Technol.*, vol. 43, no. 4, pp. 334–345, 2017.

[7] W. Peng, Y.-F. Li, J. Mi, L. Yu, and H.-Z. Huang, "Reliability of complex systems under dynamic conditions: A Bayesian multivariate degradation perspective," *Rel. Eng. Syst. Saf.*, vol. 153, pp. 75–87, Sep. 2016.

[8] W. Peng, Y.-F. Li, Y.-J. Yang, J. Mi, and H.-Z. Huang, "Bayesian degradation analysis with inverse Gaussian process models under time-varying degradation rates," *IEEE Trans. Rel.*, vol. 66, no. 1, pp. 84–96, Mar. 2017.

[9] G. A. Whitmore and F. Schenkelberg, "Modelling accelerated degradation data using Wiener diffusion with a time scale transformation," *Lifetime Data Anal.*, vol. 3, no. 1, pp. 27–45, 1997.

[10] X. Wang, "Semiparametric inference on a class of Wiener processes," *J. Time Ser. Anal.*, vol. 30, no. 2, pp. 179–207, 2009.

[11] X. Wang, "Wiener processes with random effects for degradation data," *J. Multivariate Anal.*, vol. 101, no. 2, pp. 340–351, 2010.

[12] Z. Pan, N. Balakrishnan, Q. Sun, and J. Zhou, "Bivariate degradation analysis of products based on Wiener processes and copulas," *J. Stat. Comput. Simul.*, vol. 83, no. 7, pp. 1316–1329, 2013.

[13] Z.-S. Ye, N. Chen, and Y. Shen, "A new class of Wiener process models for degradation analysis," *Rel. Eng. Syst. Saf.*, vol. 139, pp. 58–67, Jul. 2015.

[14] S.-T. Tseng and C.-Y. Peng, "Optimal burn-in policy by using an integrated Wiener process," *IIE Trans.*, vol. 36, no. 12, pp. 1161–1170, 2004.

[15] Z.-S. Ye, M. Xie, L.-C. Tang, and Y. Shen, "Degradation-based burn-in planning under competing risks," *Technometrics*, vol. 54, no. 2, pp. 159–168, 2012.

[16] Z.-S. Ye, Y. Shen, and M. Xie, "Degradation-based burn-in with preventive maintenance," *Eur. J. Oper. Res.*, vol. 221, no. 2, pp. 360–367, 2012.

[17] C.-Y. Peng, "Optimal classification policy and comparisons for highly reliable products," *Sankhya B*, vol. 77, no. 2, pp. 321–358, 2015.

[18] Q. Zhai, Z.-S. Ye, J. Yang, and Y. Zhao, "Measurement errors in degradation-based burn-in," *Rel. Eng. Syst. Saf.*, vol. 150, pp. 126–135, Jun. 2016.

[19] Y. Wang and H. Pham, "A multi-objective optimization of imperfect preventive maintenance policy for dependent competing risk systems with hidden failure," *IEEE Trans. Rel.*, vol. 60, no. 4, pp. 770–781, Dec. 2011.

[20] P. Chen and Z.-S. Ye, "Random effects models for aggregate lifetime data," *IEEE Trans. Rel.*, vol. 66, no. 1, pp. 76–83, Mar. 2017.

[21] Z.-S. Ye, L.-C. Tang, and M. Xie, "Bi-objective burn-in modeling and optimization," *Ann. Oper. Res.*, vol. 212, no. 1, pp. 201–214, 2013.

[22] P. Chen and Z.-S. Ye, "Estimation of field reliability based on aggregate lifetime data," *Technometrics*, vol. 59, no. 1, pp. 115–125, 2017.

[23] C.-Y. Peng, "Inverse Gaussian processes with random effects and explanatory variables for degradation data," *Technometrics*, vol. 57, no. 1, pp. 100–111, 2014.

[24] N. Trivellin, M. Meneghini, E. Zanoni, K. Orita, M. Yuri, T. Tanaka, D. Ueda, and G. Meneghesso, "A review on the reliability of GaN-based laser diodes," in *Proc. Rel. Phys. Symp.*, 2010, pp. 1–6.

[25] M. Abundo, "An inverse first-passage problem for one-dimensional diffusions with random starting point," *Statist. Probab. Lett.*, vol. 82, no. 1, pp. 7–14, 2012.

[26] X. Wang and D. Xu, "An inverse Gaussian process model for degradation data," *Technometrics*, vol. 52, no. 2, pp. 188–197, May 2010.

[27] X.-S. Si and D. Zhou, "A generalized result for degradation model-based reliability estimation," *IEEE Trans. Autom. Sci. Eng.*, vol. 11, no. 2, pp. 632–637, Apr. 2014.

[28] J. H. Cha and M. Finkelstein, "Burn-in and the performance quality measures in heterogeneous populations," *Eur. J. Oper. Res.*, vol. 210, no. 2, pp. 273–280, 2011.

[29] Y. Kawakubo, S. Miyazawa, K. Nagata, and S. Kobatake, "Wear life prediction of contact recording head," *IEEE Trans. Magn.*, vol. 39, no. 2, pp. 888–892, Mar. 2003.

[30] M. Xie, "On the solution of renewal-type integral equations," *Commun. Statist.-Simul. Comput.*, vol. 18, no. 1, pp. 281–293, 1989.

[31] Y. Lü and Y. Zhang, "Reliability modeling and maintenance policy optimization for deteriorating system under random shock," *J. Shanghai Jiaotong Univ., Sci.*, vol. 23, no. 6, pp. 791–797, 2018.

[32] A. Konak, D. W. Coit, and A. E. Smith, "Multi-objective optimization using genetic algorithms: A tutorial," *Rel. Eng. Syst. Saf.*, vol. 91, no. 9, pp. 992–1007, Sep. 2006.

[33] Q. Zhang, W. Liu, E. Tsang, and B. Virginas, "Expensive multiobjective optimization by MOEA/D with Gaussian process model," *IEEE Trans. Evol. Comput.*, vol. 14, no. 3, pp. 456–474, Jun. 2010.

[34] H. Peng, Q. Feng, and D. W. Coit, "Simultaneous quality and reliability optimization for microengines subject to degradation," *IEEE Trans. Rel.*, vol. 58, no. 1, pp. 98–105, Mar. 2009.

[35] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Trans. Evol. Comput.*, vol. 11, no. 6, pp. 712–731, Dec. 2007.

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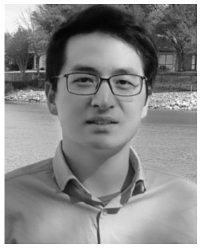

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